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PREFACE

This working paper is a draft of a chapter in a larger manuscript which is concerned with the time series variations in fertility in the United States since 1920. This chapter asks how economic models of fertility aid our understanding of our demographic history. Thus little attention is given here to the suitability of economic models for the explanation of cross-sectional fertility differentials.

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ECONOMIC THEORIES OF FERTILITY: WHAT DO THEY EXPLAIN?

by

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The economic theory of fertility, in its current state, is a product of two broad strands of influence. One such strand can be found in the works of Becker,[1] Mincer[2] and Willis.[3] The other strand winds its way through the works of Easterlin.[4] The Becker- and Easterlin-type approaches are fundamentally quite distinct, not in the least part because of the differences in what the authors try to accomplish. The main thrust of Becker's work (and that of his followers) is to show how economic models may be used to aid our understanding of fertility variations and differentials. The spirit of the Becker-type analysis, is the spirit of the economic theorist who is demonstrating the strength and breadth of the analytical framework by showing how it may be used to analyze a complex and unresolved problem. Easterlin came to the study of fertility with a quite different background, that of an economic historian. The problem for Easterlin was to understand fertility variations within a historical context. Thus, for Easterlin, the main problem was to provide a framework in which to comprehend the available information on the variations in fertility over time. The spirit of the Easterlin-type analysis is the spirit of the economic historian who finds himself compelled to modify economic models and concepts for the purpose of understanding some observed behavior. Of course, there is much in common between these two approaches.

One problem which is shared by both approaches to the economics of fertility is how to account for the inverse relationships between fertility and income which are so frequently observed.[5] If the relationships were always negative there would be some temptation to label children as "inferior goods", but in reality the relationships vary in sign.[6] Furthermore, treating children as if they were inferior goods does not explain observed behavior, but rather just gives it a name after the fact. So economists have shied away from assuming that children could be viewed as inferior goods and have regarded the negative associations between income and fertility as something which must be explained in other terms. Thus all the models of fertility discussed below have some mechanism which can transform a nominally positive association between income and fertility into a negative one.

The Economic Theory of Fertility Before Becker: Leibenstein

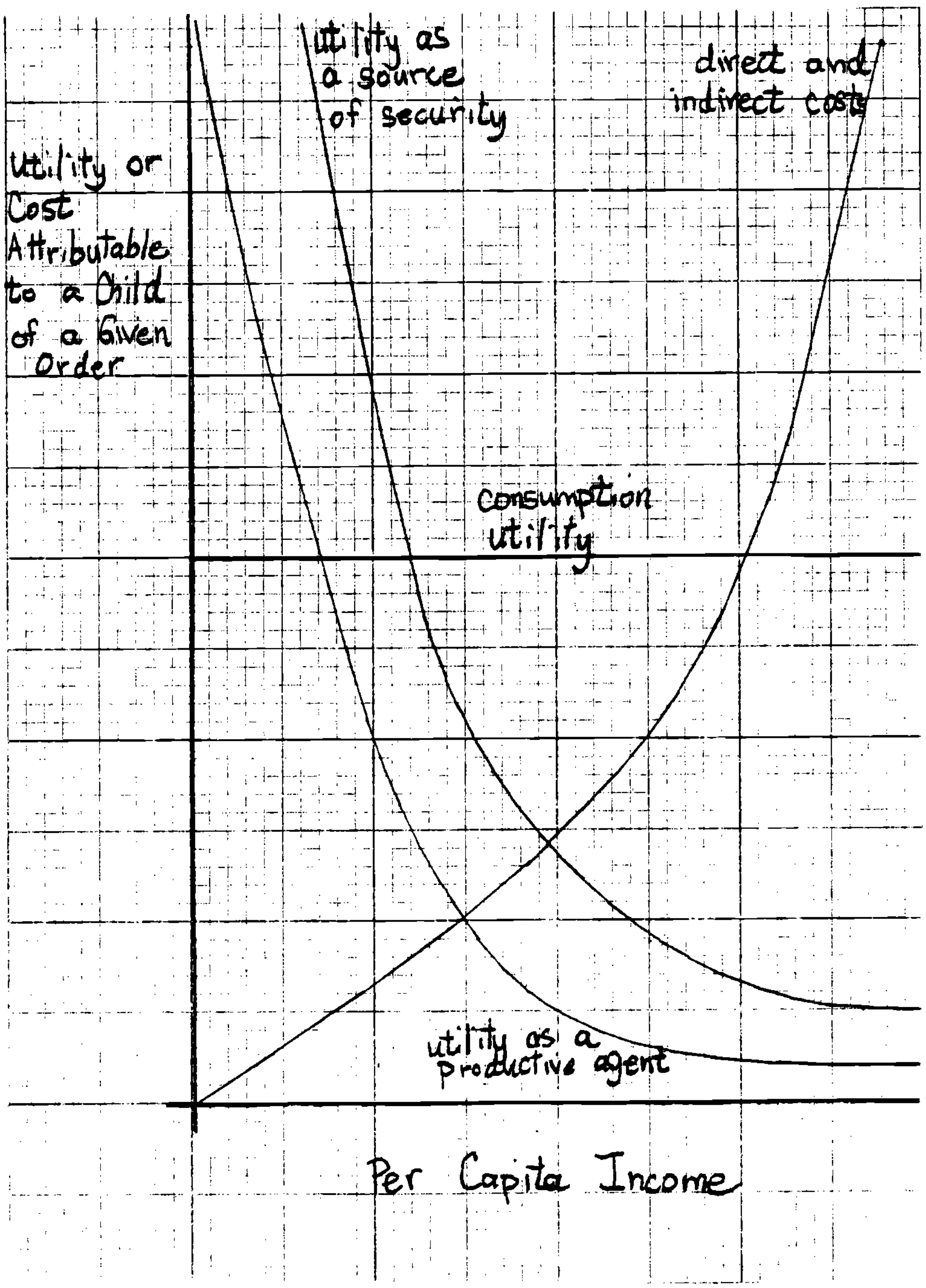
Becker is usually considered the father of the contemporary economic theory of fertility although his work was not the first to analyze the demand for children within the framework economic theory. His work was predated at least by the contributions of Leibenstein[7] and Okun.[8] What differentiated Becker's work from that of Leibenstein's and Okun's was Becker's use of the well known demand theory approach to the problem of fertility without the introduction of *ad hoc* or unfamiliar notions into the structure of model in order to explain the possibility of a negative relationship between icnome and fertility.

The essence of Leibenstein's contribution is presented in the following quotation.

> Our central notion is that people behave in the same way as they would if they applied rough calculations to the problem of determining the number of births they desire. And such calculations would depend on balancing the satisfactions or utilities to be derived from an additional birth as against the "cost," both monetary and psychological, of having an additional child. We distinguish among three types of utility to be derived from an additional birth and two types of cost. The types of utility are: (1) the utility to be derived from the child as a 'consumption good,' namely, as a source of personal pleasure to the parents; (2) the utility to be derived from the child as a productive agent, that is, at some point the child may be expected to enter the labor force and contribute to family income; and (3) the utility derived from the prospective child as a potential source of security, either in old age or otherwise.
> The costs of having an additional child can be divided into direct and indirect costs. By direct costs we refer to the conventional current expenses of maintaining the child, such as feeding and clothing him at conventional standards until the point is reached when the child is self-supporting. By indirect costs we refer to the opportunities foregone due to the existence of an additional child. These are represented by such lost opportunities as the inability of mothers to work if they must tend to children, lost earnings during the gestation period, or the lessened mobility of parents with large family responsibilities.[9]

Many of the ideas subsequently developed in the economic theory of fertility can be found in Leibenstein's work. Indeed, those paragraphs suggest at least one line of approach which has not yet been explored. Leibenstein's main interest in fertility in his 1957 volume, was why fertility declined as per capita income rose. He summarized his argument on this score in the following figure.

Figure 1


utility as
a source
of security
direct and
indirect costs
Utility or
Cost
Attributable
to a Child
of a Given
Order
consumption
utility
utility as a
productive agent
Per Capita Income

In the Leibenstein model fertility fell as per capita income rose for two main reasons. First, as per capita income increases there are associated changes in the structure of economic activities which tend to reduce the value of children to their parents. Leibenstein suggests that the main causes of this decline are the decreasing utility of children of a given order in providing old-age security for their parents and the decreasing potential of children contributing to family income through work activities. On the other side of the coin, Leibenstein views the costs of children of a given order as increasing as per capita income increases because "the style in which a child is maintained depends on the position and income of the parents."[10] Leibenstein also thought that the indirect costs of children of a given order would rise because he considered "opportunities for (parents) engaging in productive or in various time-consuming activities as likely to grow as income increases."[11]

Leibenstein's analysis is focused on explaining variations in fertility over time. It suggests that students of fertility pay attention to two broad sets of forces in determining fertility movements. The first is the set of structural transformations which accompany the rise in per capita income and which (according to Leibenstein) decrease the value of children to their parents. The second is a set of forces which increases the cost of children as income increases. Thus, the Leibenstein model differs from a simple model of constrained optimization in that per capita income affects fertility through a variety of mechanisms in addition to its effect on the budget constraint. Leibenstein solved the problem of explaining the negative secular relationship between income and fertility by positing that the utility function shifts with changes in per capita income and that the price of children varies with family income. In the Leibenstein model secular increases in income are associated with changes in tastes and changes in the relative price of children which dominate the pro-natal effect of income increases and cause the observed negative relationship. I think it is fair to say that the Leibenstein model represented, in somewhat more formal terms, the main lines explanation of the secular decline in fertility which were widely accepted at the time of his writing.

An Explanation of Fluctuations in American Fertility: The Work of Easterlin

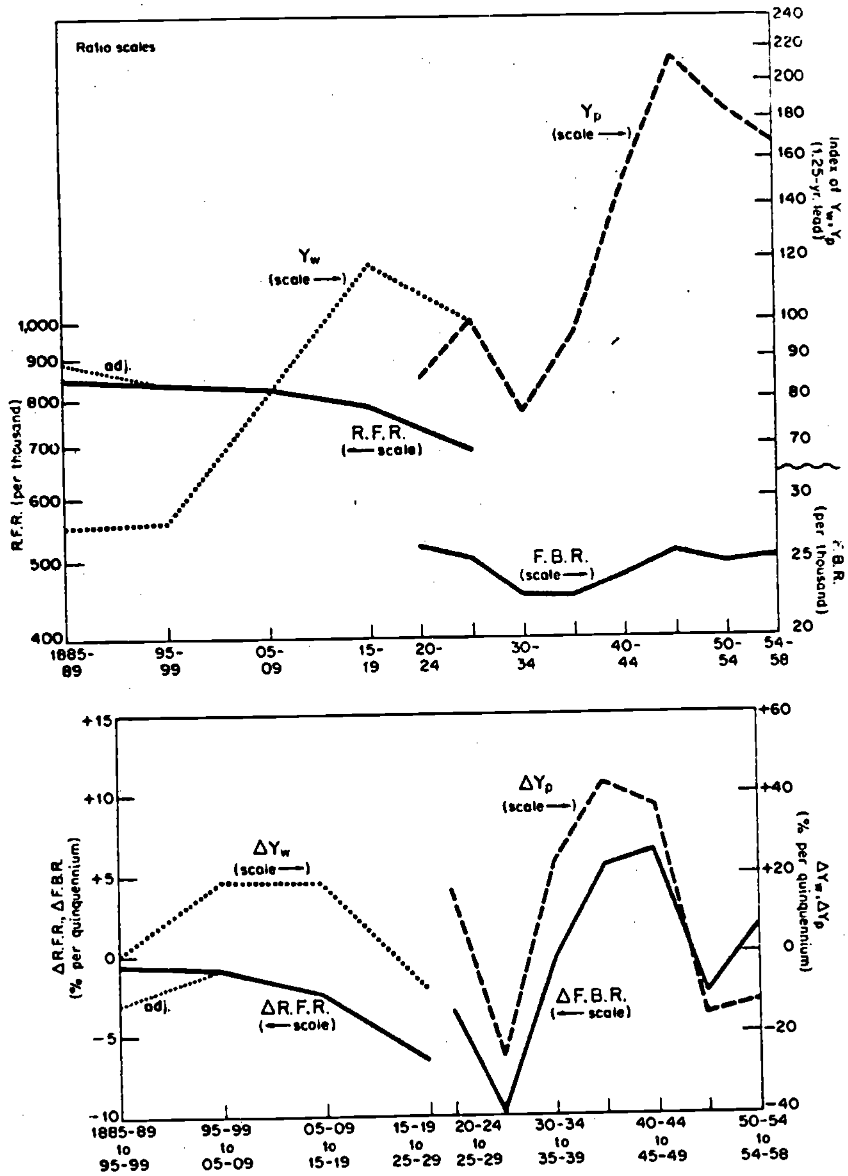
Like Leibenstein's ideas, the main focus of Easterlin's work has been on fertility variations over time. Easterlin has made a number of contributions to the economic analysis of fertility[12], but in this chapter we shall be eclectic and treat only those which are directly relevant to the present discussion. Whereas Leibenstein considered the problem of fertility variations in the context of developing countries, the works of Easterlin, which will be discussed here, deal with fertility fluctuations over time in the United States. In 1961, Easterlin proposed an explanation for the American baby boom[13] which suggested that the baby boom was a manifestation of the same sort of forces which, in an earlier era, had produced long swings in migration. In 1966, Easterlin suggested an integrated explanation of both the baby boom and the following fertility decline.[14]

In the earlier article, Easterlin focused on the time profile of white fertility which he disaggregated into the time paths of the fertility of foreign-born whites, rural native whites, and urban native white. He found through this process, that most of the temporal decline in fertility in white fertility in the United States was due to the declining fertility of foreign-born whites, the declining fertility of rural native whites, and the rural-urban shift. The fertility of urban native whites decreased only slightly from 1885-89 to 1925-29. After 1925-29, however, urban white fertility shows a marked alteration from its relative constancy of the previous 40 years. The great depression caused a substantial decrease in fertility and the postwar period, through to the middle '50s, saw an unprecendented fertility increase.

Easterlin suggests a separate explanation for the fertility patterns of each of the three groups. For our present purposes it is sufficient to examine his explanation of the course of urban native fertility. In order to explain these fertility changes Easterlin concentrates on the fertility of young people. Easterlin's explanation of changes in the fertility of urban native whites hinges on the interaction of two factors: changes in the aggregate unemployment rate and changes in the rate of growth of the total white male population 20-29. Both increases in the aggregate unemployment rate and the rate of growth of the total white male population are thought to be negatively associated with fertility changes. The changes in these two factors taken together broadly reflect changes in the economic well-being of young people, if we abstract from the secular upward trend in income. This explanation of fertility changes is supported by evidence shown in Figure 2. There is can be seen that changes in the unemployment rate and changes in the rate of growth of the total white male population were inversely

Figure 2

LEVEL AND RATE OF CHANGE OF RURAL WHITE FERTILITY RATIO (R.F.R.) AND REAL GROSS FARM INCOME PER ENGAGED (Y_w), 1885–1929; AND OF FARM BIRTH RATE (F.B.R.) AND REAL NET FARM INCOME PER HEAD (Y_p), 1920–58



Source: Easterlin (1968), p. 98.

associated before the Great Depression. This inverse relationship is caused by a negative association between the unemployment rate in the United States and international migration to the United States.[15] According to Easterlin, the rapid response of migration to changes in labor market conditions which prevented pronounced cyclical movements in the economic well-being of young adults was the reason for the relative constancy of urban native white fertility before the Depression of the 1930's. After the statutory restriction of international migration in the early 1920's the stage was set for the native white population to bear the full brunt of economic fluctuations. In particular, the baby boom period of the 'fifties was a period of relatively low unemployment rates and a relatively low growth rate of the white male population 20-29 because of the low fertility in the 1930's. The combination of these two circumstances was unusual and Easterlin suggested that it was this fortuitous combination which caused the baby boom.

In his later article on the baby boom, Easterlin added another element to his explanation of fertility changes. Easterlin argued that it was incorrect to create economic models of fertility based on the assumption that tastes remained fixed. He argued that young adults become acquainted with a certain level of consumption when they are teenagers in their parents' households and that this level of consumption affects their tastes and aspirations. When the young adults become married, so the argument goes, the tastes and aspirations formed in their adolescence remain with them. If their income is such that their aspirations are satisfied they will have higher fertility than if they are struggling to attain their desired level of consumption.[16] Thus, Easterlin suggests not only that we study changes in the economic position of young people in order to understand changes in fertility, but that we also study changes in their economic position relative to that of their parents.

Easterlin presented a table to support the intergenerational relative income hypothesis, which showed the relationship between median family incomes of families whose heads were 14-24 in a given year with those of families whose heads were 35-44 five years earlier. That table contained data for the years 1953-1962. Table 1 shows similar data for the years 1953-1972 and these data together with the age specific fertility rates of married women 15-19 and 20-24 lagged one year are plotted in Figure 3A.

The peak of the income ratio is in 1956 and the ratio declines rapidly to a local trough in 1963. The two years of marked increase which follow 1963 give way to a continued decline through 1972. Thus, at first glance, the income ratio series does not seem to do well in explaining the fertility of young women. While both the lagged age-specific marital fertility rates and the income ratios have local peaks in 1956, the income ratio declines through 1963, while the lagged age-specific martial fertility rate for women 15-19 is higher in 1960 than in 1957 and the rate for women 20-24 is higher in 1959 than in 1957. After 1963, the three series hardly seem related at all. In Figure 3-B, the intergenerational relative income ratio and the marriage rate for unmarried women 15-44 are plotted. It can be seen from that figure that the marriage rate and the income ratio are quite closely related. Indeed, apparently they are more closely related than the income ratio and age-specific fertility rates. However, the relationship between the income ratio and the marriage rate is not so close as to obviate the need for further discussion. This is not the place for a complete test of the intergenerational relative income hypothesis. Here we simply want to suggest the possibility that, to the extent that intergenerational relative income is an important determinant of fertility, its impact is chiefly through its influence on marriage rates and only secondarily through it influence on the completed fertility of married women.

It would be quite naive to believe that all fertility variations could be understood with reference to a single income ratio and this is not what Easterlin intended. For one thing, contraceptive technology was changing rapidly in the '60's and this could possibly account for some of the deviations between the income ratio series and the lagged age-specific marital fertility series in the middle of the decade. A balanced view of the matter would suggest that the intergenerational relative income effect may be quite important in understanding fertility changes, but that a definitive test of that hypothesis has not yet been made.

Relative income and age specific birth rates for married women 15-19 and 20-24.

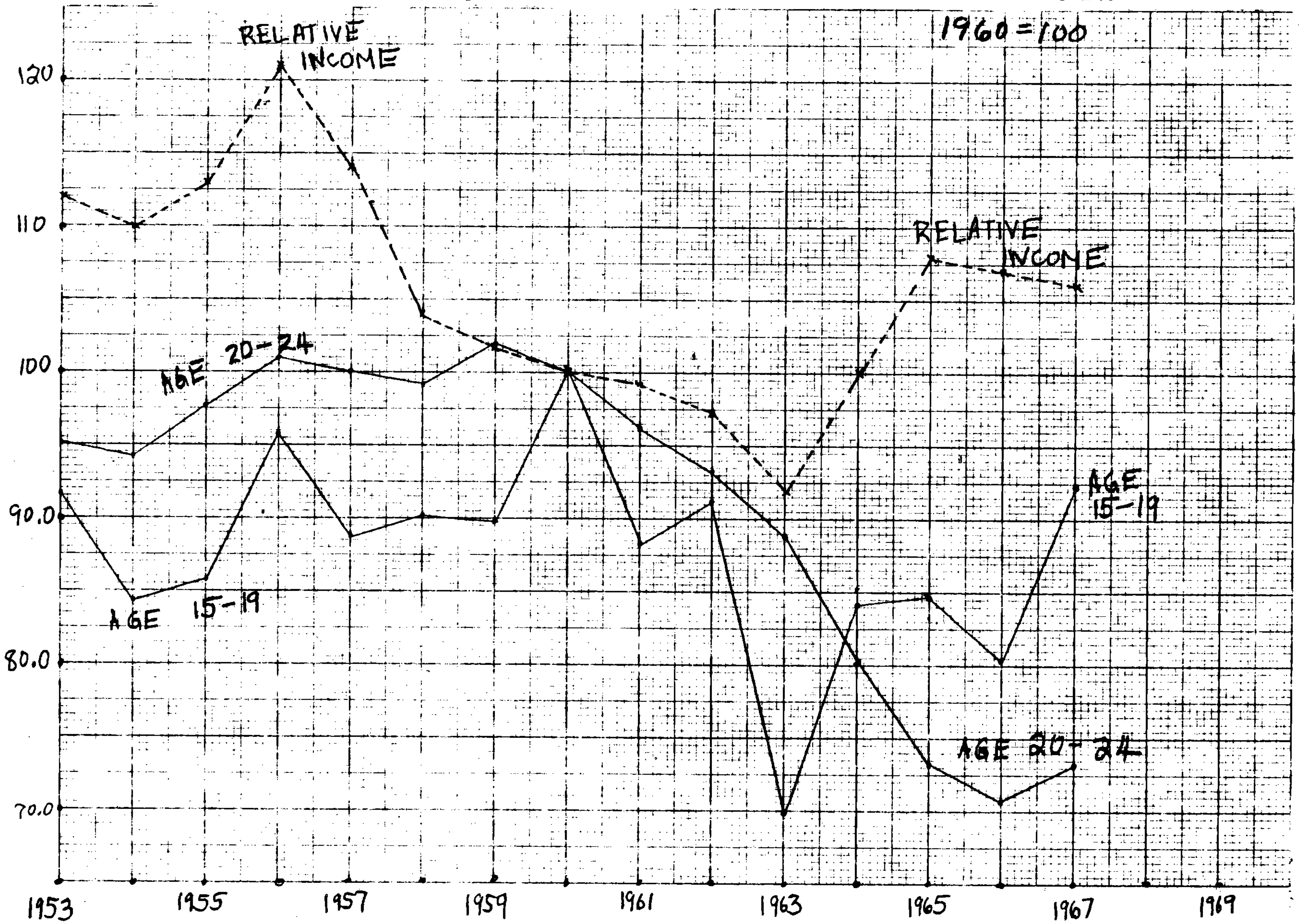


Figure 3-B

Relative income and the marriage rate for unmarried women 15-44.

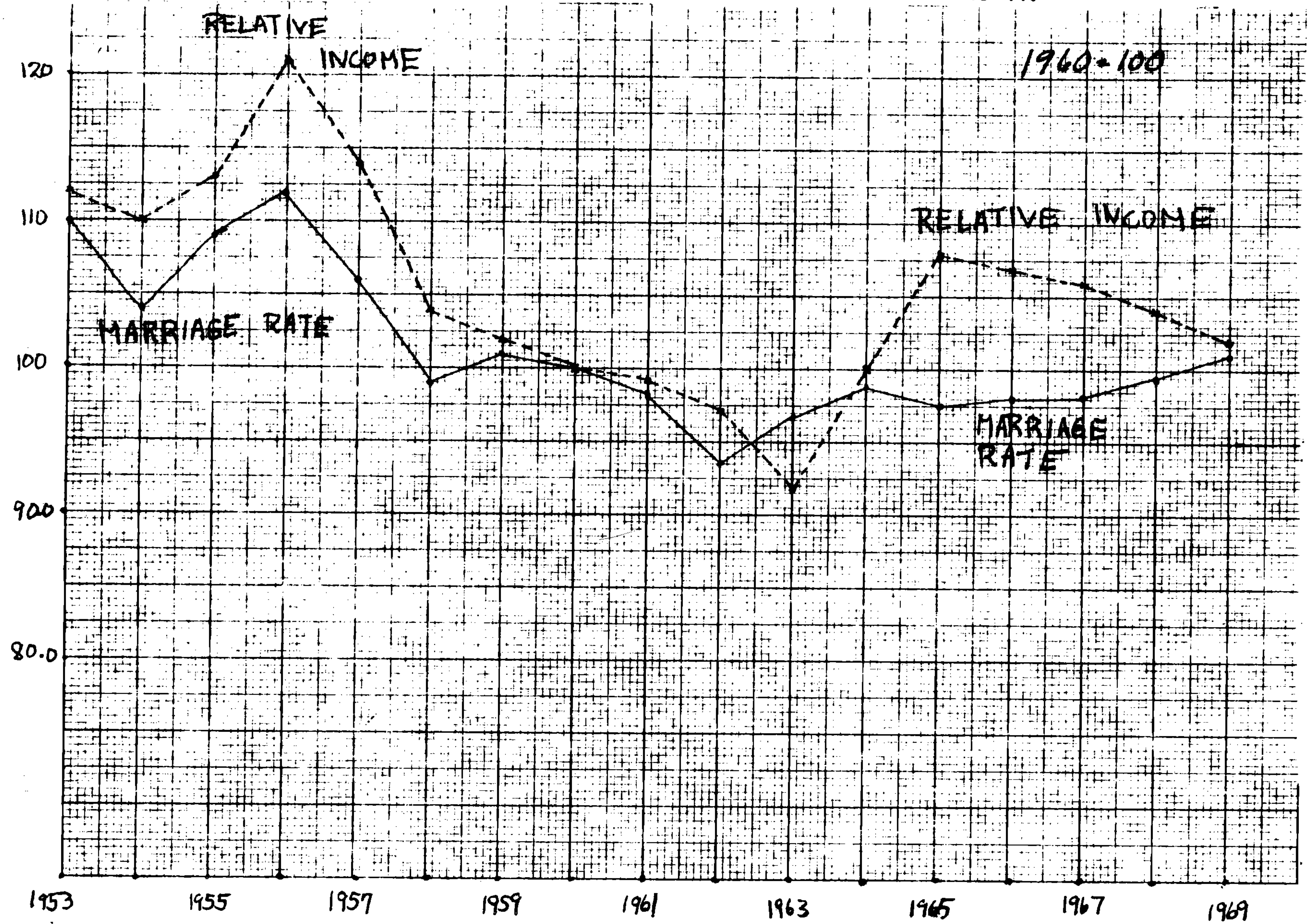


Table 2: Median Real Income of Families with Head Aged 14-24 Divided by Median Real Income of Families with Head Aged 35-44, Five Years Earlier; 1953-1972.
(income figures for families with head aged 35-44 are three year averages centered at indicated date)

Year	Median Income Ratio (in percent)
1953	82.2
1954	80.9
1955	83.2
1956	89.0
1957	84.1
1958	76.7
1959	75.0
1960	73.7
1961	73.2
1962	71.6
1963	67.7
1964	73.7
1965	79.7
1966	78.7
1967	78.0
1968	76.7
1969	75.3
1970	71.7
1971	64.6
1972	64.0

Source: 1947-1964: Trends in the Income of Families and Persons in the United States, by Mary F. Henson, U. S. Department of Commerce, Bureau of the Census, Technical Report 17, USGPO, Washington, D. C. 1967, Table 3.

1965-1972: Current Population Reports, P-60, various issues.

Easterlin, like Leibenstein, views tastes as changing in an antinatal direction as income increases. At first glance, this might seem to be a noneconomic explanation of fertility variation. However, the Easterlin-Fuchs intergenerational relative income hypothesis puts economics back into the picture by claiming that economics can aid in the understanding of intergenerational taste differences.

Becker's Economic Theory of Fertility: Leibenstein Formalized?

In 1960, Becker formally applied demand theory to task of understanding fertility.[17] The model suggested by Becker dealt with a single consumer and may be written as follows:

Model I: The Becker Model

$$\text{Maximize } U = f(n, e, s)$$

$$\text{subject to } I = nep_c + p_s s$$

where f is a utility function, n is the number of children, e is the average real expenditure per child,[18] p_c is a price index for the child expenditure bundle, s is an index of the quantity of everything else consumed by the houshold, p_s is a price index for s, and I is money income. Thus, in the Becker model the utility function is defined over the number of children, the average real expenditures per child, and the quantity of everything else. The distinction between p_c and e

is very important for Becker's argument. He notes that it is quite plausible that increases in income are associated with increases in the average real expenditures per child, but that this does not by itself indicate that the cost of children rises with income. Becker wrote:

> A change in the cost of children is a change in the cost of children of <u>given quality</u>, perhaps due to a change in the price of food or education. . . One would not say that the price of cars has risen over time merely because more people now buy Cadillacs and other expensive cars. A change in price has to be estimated from indexes of the price of a given quality. Secular changes in real income and other variables have induced a secular increase in expenditures on children, often interpreted as a rise in the cost of children. The cost of children may well have risen . . . but the increase in expenditure on children is no evidence of such rise since the quality of children has risen. Today children are better fed, housed, and clothed, and in increasing numbers are sent to nursery schools, camps, high schools, and colleges. For the same reason, the price of children to rich parents is the same as that to poor parents even though rich parents spend more on children. The rich simply choose higher quality children as well as higher qualities of other goods.[19]

Armed with the distinction between cost and expenditure, Becker attacked Leibenstein and others for assuming that the cost of children necessarily rose with income. He argued that it was preferable to make an analogy between children and consumer durables. In the case of consumer durables, people can not only choose the quantity of these items they wish to purchase, but also, to some extent, the amount of money they wish to spend on each unit. Becker claimed that in the world of consumer durables quality and quantity income elasticities tend to be positive with the former exceeding the latter and, arguing by analogy, he claimed that this would also be true for the demand for children, were there no contraceptive costs.

Becker's distinction between expenditure and cost was quite persuasive and for 13 years after Becker's initital article on fertility most economists working in the field were convinced that "the price of children to rich parents is the same as that to poor parents even though rich parents spend more on children." However, in 1973, Becker and Lewis[20] discovered that this statement was an <u>obiter dictum</u> and not a true implication of the Becker model. Indeed, income changes generally produced endogeneous relative price changes in that model whose effects had not been previously analyzed. In analyzing these induced relative price changes Becker and Lewis found that Becker had been wrong in his earlier article. They wrote:

> This price effect, however, does offer a correction to the argument advanced by Becker (1960), and followed by many others, that the price of children is the same for the rich as for the poor (aside from the cost-of-time argument), even though the rich choose more expensive children. The relevant price of children with respect to their number <u>is</u> higher for the rich precisely because they choose more expensive children. Similarly, the relevant price of cars, houses, or other goods is higher for the rich because they choose more expensive varieties.[21]

In some regards the Becker-Lewis article in 1973 brings us full circle to Leibenstein's 1957 arguments, albeit with a considerable increase in the level of analytic sophistication. Nonetheless, the initial Becker article still has some appeal and it is difficult to accept the proposition that the prices of goods are higher for the wealthy than for the poor just because the wealthy purchase higher quality goods. Are the price of automobiles, houses, children and other goods really higher for the rich than for the poor? In order to discuss this question let us consider two alternative models.

Model I: The Becker Model

Maximize $U = f(n,e,s)$

Subject to $I = p_c ne + p_s s$

where the symbols are defined as above on page 11.

Model II: A Model of Expenditures on Each Child

Maximize $U = g(e_1, e_2, \ldots, e_m, s)$

Subject to $I = p_c e_1 + p_c e_2 + \ldots + p_c e_m + p_s s$

where g is a utility function, e_j $(j=1, \ldots, m)$ is the real expenditure on the jth child, s is a quantity index of everything else consumed by the household, p_c is a price index for the child expenditure bundle[22], p_s is a price index for s, I is money income, and m is the biologically determined maximum number of children a couple can have. If the couple has n children, then $e_{n+1} = e_{n+2} = \ldots = e_m = 0$.

The main difference between the Becker Model and Model II is that in the second model the real expenditure of each child is treated as a separate argument in the utility function. In Becker's model these arguments are aggregated together into the number of children, and the <u>average</u> real expenditure per child. Thus, parents are assumed to be indifferent between the situation in which they have two children with a real expenditure of \$10,000 on the first and \$2 on the second and the situation in which the parents have two children with a real expenditure of \$5,001 on each of them. No such assumption is made in Model II. Neither of these models has any implications for the number of children or real expenditures on them as it is written. In order to derive some implications it is customary to assume that all the arguments in the utility functions are normal goods.[23] With these additional assumptions in mind let us consider the Becker model first.

It can be seen immediately that the Becker model is not the standard model of demand theory. There are several very important differences. First of all, the arguments of a utility function are generally quantities of goods and services. In the Becker model, n and s meet this criteria, but e does not because it represents an average of quantities, the average being taken over another argument in the utility function. This specification leads to the other important departure from the standard model; the budget constraint is not linear in the arguments of the utility function. The consequences of these differences can be seen most clearly when the first-order conditions for a utility maximum are written down. They are

$$(1) \qquad \frac{f_n}{p_c e} = \frac{f_e}{p_c n} = \frac{f_s}{p_s}$$

where $f_k = \frac{\partial f}{\partial k}$ for $k = n$, e, and s . The shadow price of n in this formulation is $p_c e$ and the shadow price of e is $p_c n$. Not only do the shadow prices of n and e change with income, the shadow prices are functions of the arguments of the utility function.

Now in what sense is it true that children are more expensive to rich people than to poor people in the context of the Becker model? To answer this question, let us suppose that, subject to the budget constraint

$$(2) \qquad I^{(1)} = p_c ne + p_s s ,$$

utility is maximized at the point $(n^{(1)},e^{(1)},s^{(1)})$. At this point let the shadow price of n be defined as $\Pi_n^{(1)} = p_c e^{(1)}$ and the shadow price of e be defined as $\Pi_e^{(1)} = p_c n^{(1)}$. Using these shadow prices, the budget constraint may be rewritten as follows:

$$(3) \quad R^{(1)} = \Pi_n^{(1)} n^{(1)} + \Pi_e^{(1)} e^{(1)} + p_s s^{(1)}$$

where $R^{(1)} = I^{(1)} + p_c e^{(1)} n^{(1)}$. As in the standard analysis, holding the shadow prices constant, equation 3 is linear in the arguments of the utility function. Now let income increase from $I^{(1)}$ to $I^{(2)}$. In this case there will exist an $R^{(2)} > R^{(1)}$ such that at the point $(n^{(2)},e^{(2)},s^{(2)})$ utility is maximized subject to the <u>two</u> constraints

$$(4) \quad R^{(2)} = \Pi_n^{(1)} n^{(2)} + \Pi_e^{(1)} e^{(2)} + p_s s^{(2)}$$

and $$(5) \quad I^{(2)} = p_c n^{(2)} e^{(2)} + p_s s^{(2)} \quad .$$

Since there are two constraints[24] passing through point $(n^{(2)},e^{(2)},s^{(2)})$ there are two sets of shadow prices. The shadow prices for n, e, and s along equation 4 are $\Pi_n^{(1)}$, $\Pi_e^{(1)}$, and p_s respectively. The shadow prices for n, e, and s along equation 5 are $\Pi_n^{(2)}(=p_c e^{(2)})$, $_e^{(2)}(=p_c n^{(2)})$, and p_s respectively. Before we compare shadow price ratios along the constraints, it is important to note that Becker and Lewis believe that the pure income elasticity of demand for e is greater than the pure income elasticity of demand for n. If this is the case, $\frac{e^{(2)}}{n^{(2)}} > \frac{e^{(1)}}{n^{(1)}}$. Now how do the relative shadow prices of n and e compare along the two constraints?

Clearly,

$$(6) \quad \frac{\Pi_n^{(1)}}{\Pi_e^{(1)}} = \frac{e^{(1)}}{n^{(1)}} < \frac{e^{(2)}}{n^{(2)}} = \frac{\Pi_n^{(2)}}{\Pi_e^{(2)}} .$$

Therefore, the shadow price of n relative to the shadow price of e at point $(n^{(2)}, e^{(2)}, s^{(2)})$ along equation 5 is greater than the shadow price of n relative to the shadow price of e at point $(n^{(1)}, e^{(1)}, s^{(1)})$ along equation 2. It is in this sense that Becker and Lewis mean that children are more expensive to rich people than to poor people. However, continuing the line of argument, there is another sense in which children are more expensive for the rich than for the poor. The shadow prices of both n and e relative to the price of s are higher at point $(n^{(2)}, e^{(2)}, s^{(2)})$ along equation 5 than at $(n^{(1)}, e^{(1)}, s^{(1)})$ along equation 2 even if the true income elasticity of demand for n is greater than the true income elasticity of demand for e.[25]

The complications caused by the induced relative price effects in the Becker model contrast markedly with the simplicity of the analysis of Model II, the model in which each child is treated separately. Model II is formally identical to the standard model of demand theory and therefore the first-order conditions for a maximum are:

$$(7) \quad \frac{g_{e_1}}{p_c} = \frac{g_{e_2}}{p_c} = \ldots = \frac{g_{e_n}}{p_c} = \frac{g_s}{p_s}$$

and $$(8) \quad \frac{g_{e_j}}{p_c} < \frac{g_{e_1}}{p_c} \qquad \text{for } j = n+1, \ldots, m,$$

where $g_{e_i} = \frac{\partial g}{\partial e_i}$ for $i = 1,\ldots,m$, and where $g_s = \frac{\partial g}{\partial s}$,

and where n is the number of children in the family. In Model II shadow prices are fixed parameters independent of income. Therefore, the shadow prices are the same to rich and poor alike. There is no sense in which the price of children or automobiles or houses, for that matter, varies with income in this model.

Thus, it is the standard demand model which yields the assertion that prices are invariant to income, whereas the original Becker model itself leads to the Leibensteinian position taken on this question in the Becker-Lewis 1973 article. Clearly, whether or not the price of children varies with income depends upon the model chosen to represent the decision-making process. The same is certainly true for houses, automobiles, and other goods.

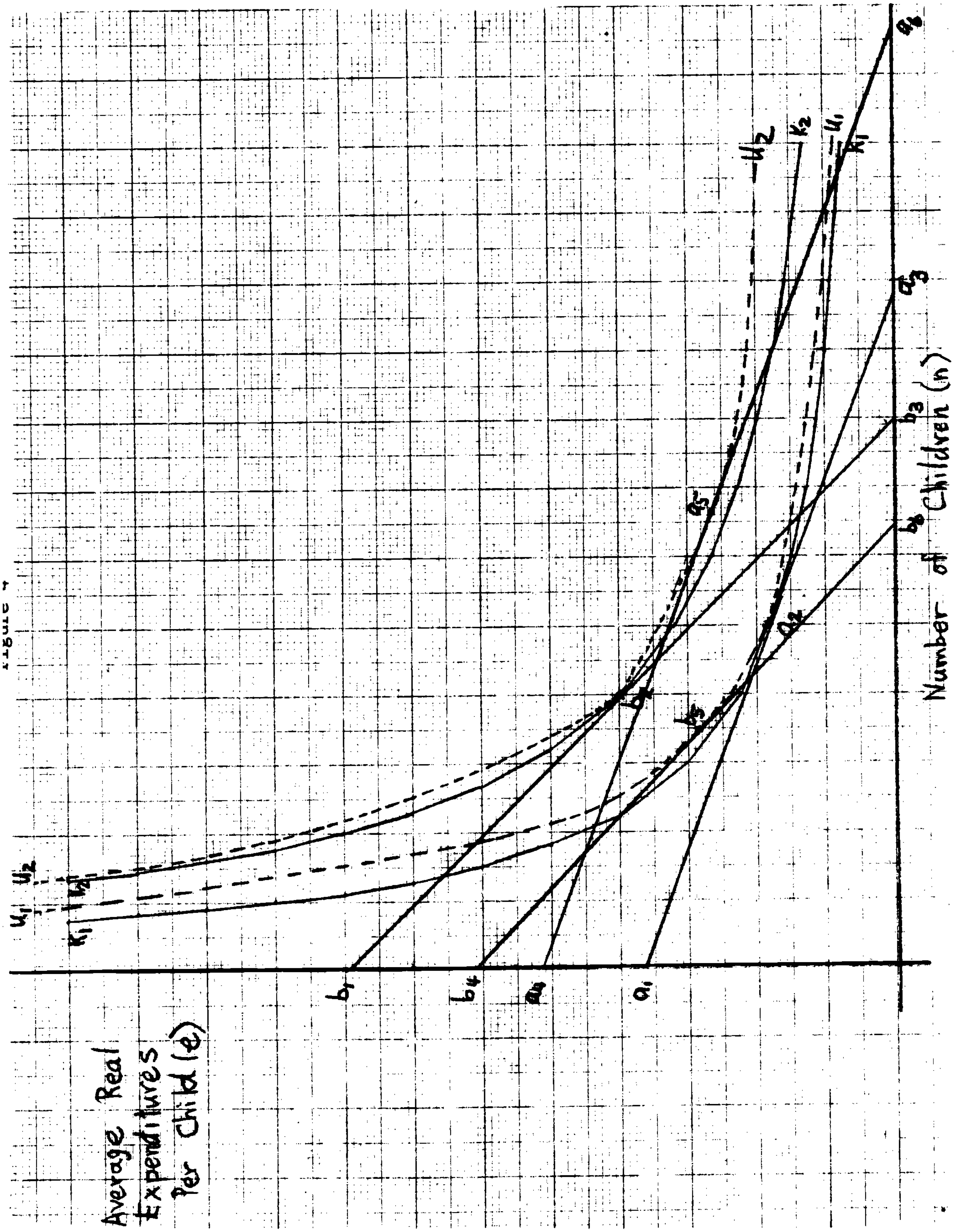
Now that there are two models proposed to explicate the same phenomena, the question naturally arises: which model is preferable?

For those who would resolve this question by an empirical test of the implications of the models, the absence of any testable implications from the Becker model poses something of a problem. The Becker model can accomodate virtually any fertility-income pattern, but it predicts no particular association. Becker and Lewis have shown that even though n and e are assumed to be normal goods, their observed elasticities with respect to income may be negative. This can be seen in Figure 4, where we have assumed that utility is initially maximized at the point $(n^{(1)},e^{(1)},s^{(1)})$ and given an increase in income at the point $(n^{(3)},e^{(3)},s^{(3)})$.

In that figure the constraint K_1K_1 and the indifference curve U_1U_1 are both drawn holding the quantity of s consumed at $s^{(1)}$. Similarly the constraint K_2K_2 and the indifference curve U_2U_2 are both drawn holding the quantity of s consumed at $s^{(3)}$.

Figure 4 illustrates the case where fertility decreases with income even though n is a normal good in the utility function. The initial point which maximizes utility is at a_2, the point where the budget constraint K_1K_1 and the indifference curve U_1U_1 are tangent. With the increase in income the constraint may shift upward to K_2K_2 which is tangent to U_2U_2 at point b_2. Clearly the increase in income in that representation causes fertility to decline. Nonetheless, n is not an inferior good. In order to show this, consider the straight line $a_1a_2a_3$ which is tangent to both U_1U_1 and K_1K_1 at a_2. Shifting this line upward without changing its slope until it becomes tangent to U_2U_2 we obtain the line $a_4a_5a_6$. The point of tangency between the line $a_4a_5a_6$ and the indifference curve is a_5. However, a_5 is above and to the right of a_2 indicating that holding relative prices constant an increase in income would be associated with both an increase in n and e. Essentially, the same argument can be made starting at any point on either indifference curve. For example, let us consider the straight line $b_1b_2b_3$ which is tangent to U_2U_2 and K_2K_2 at point b_2. A parallel downward shift in that line such that the resulting line is tangent to U_1U_1, yields the line $b_4b_5b_6$. Clearly, b_2 is above and to the right of b_5 again indicating that both n and e are normal goods.

Figure 4


Average Real Expenditures Per Child (e)
Number of Children (n)
u_1
u_2
k_1
k_2
a_1
a_2
a_3
a_4
a_5
a_6
b_1
b_3
b_4
b_5
b_6

In addition to having no implications for the relationship between income and fertility, the Becker model also has no implications for the effects of market price changes on fertility. Changing p_s has a pure cross-substitution effect[26] on n of unknown sign which is now compounded by an income effect of unknown sign. Decreasing p_c is equivalent to increasing income and increasing p_s, but since both of the latter two changes result in ambiguous changes in n, even decreasing p_c has no clear effect on fertility in the Becker model.[27] Therefore, since the Becker model has no implications for fertility it cannot easily be falsified by observed data. Rather than providing implications concerning fertility the Becker model provides a framework for the analysis of fertility into which practically all observed data can fit.

There is one particularly interesting subtelty of the Becker (1960) model that, as Becker and Lewis (1973) have shown, deserves attention. In the context of the Becker model, the ordering of the observed income elasticities of demand for n and e may be the reverse of the ordering of the true income elasticities of demand for n and e. Thus, we might observe that the income elasticity of demand for e is greater than the income elasticity of demand for n, but nonetheless it may be the case that the true income elasticity of demand for n exceeds the true income elasticity of demand for e. One matter which is clearly implied by the Becker model is that in dealing with that model we must be cautious about assuming that the signs or the ordering of true income elasticities is identical to the signs or the ordering of observed income elasticities.

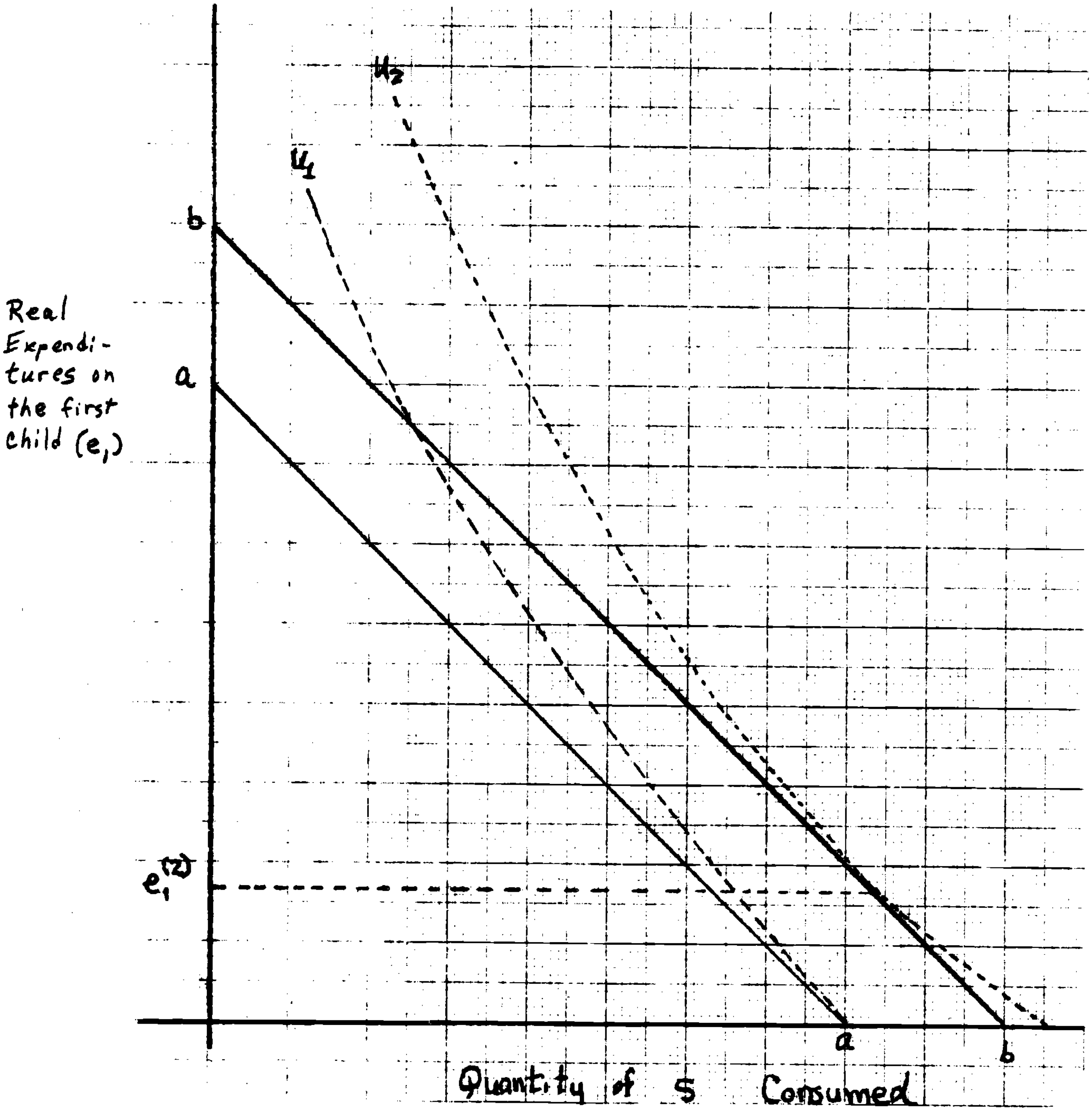
As a framework for the analysis, the Becker model tells its user that he cannot learn much about fertility from simply studying income and market prices. In order to understand even the direction in which fertility responds to an economic change, the Becker model requires quite a bit of information about the individual's tastes. Thus, the framework of analysis suggested by Becker and Lewis is one in which the differences in the structures of individuals' taste patterns are crucial to an understanding of fertility differentials.

In contrast to the Becker model, in Model II there are implications for fertility of changes in economic variables. The implications are

1. increasing (decreasing) income never causes fertility to fall (rise) and may cause it to rise (fall).
2. decreasing (increasing) p_c never causes fertility to fall (rise) and may cause it to rise (fall).
3. decreasing (increasing) p_s may cause fertility either to rise or fall.

The first implication may be easily demonstrated. Let us suppose at some initial income level $I^{(1)}$ utility is maximized with 3 children. In other words, $e_1^{(1)}$, $e_2^{(1)}$, and $e_3^{(1)}$ are positive and $e_4^{(1)},\ldots,e_m^{(1)}$ are zero. As income increases more money is spent on each of the three children so that at least three children must be desired after the income increase. However, the family may choose to have even more children after the income increase. This possibility is shown in Figure 5.

Figure 5



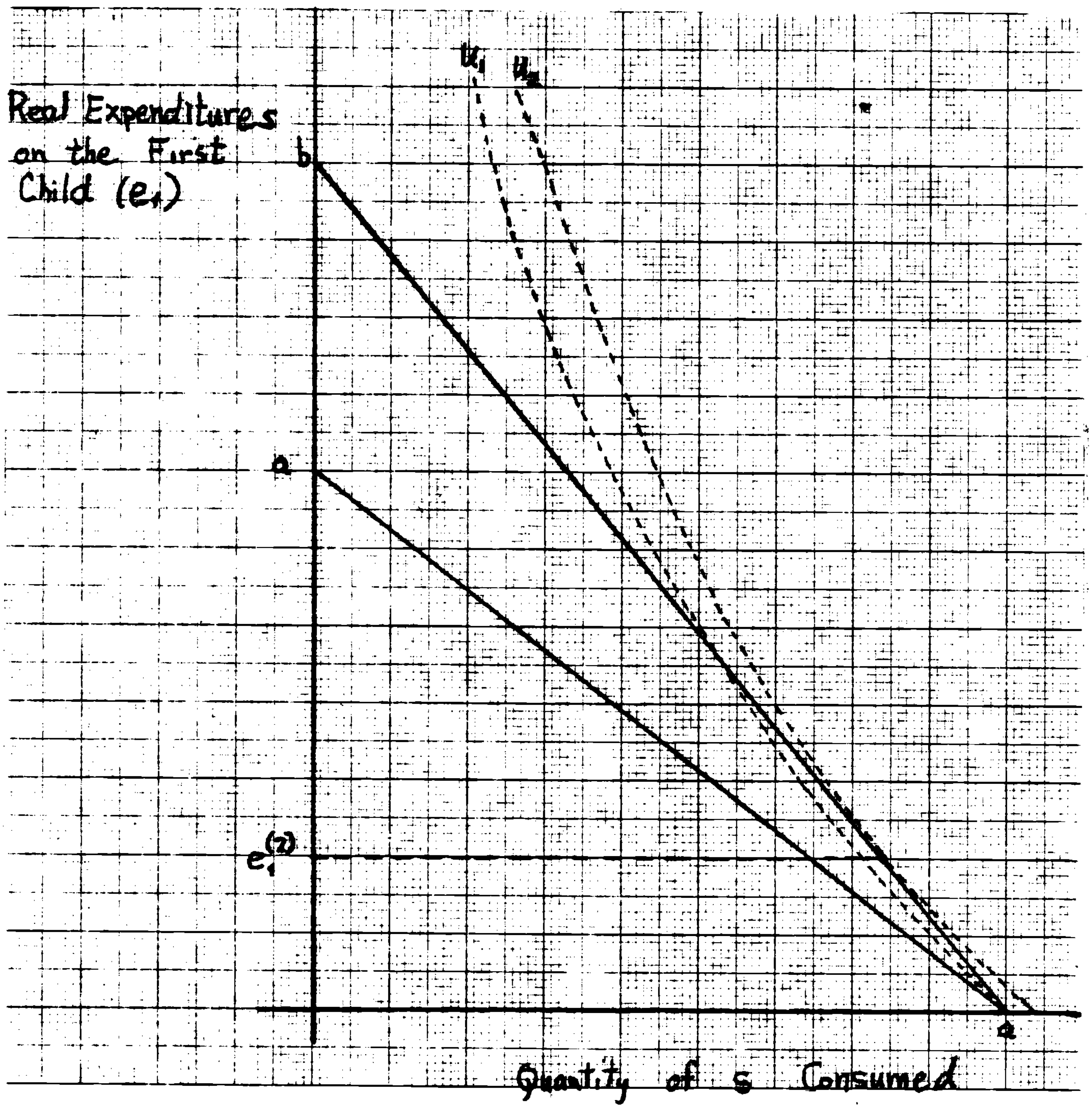
U_2
U_1
b
a
Real Expenditures on the first child (e_1)
$e_1^{(2)}$
a
b
Quantity of S Consumed

In Figure 5, at income level aa , the family chooses not to have any children, i.e., $e_1=0$. However, increasing income to bb induces the family to have their first child and e_1 becomes positive.

A similar diagram can be used to illustrate the second implication. When p_c decreases fertility cannot fall because more is spent on each existing child, but fertility may rise. This is shown in Fig. 6 again for the case where the family initially has no children. Given the constraint aa , e_1 is zero. When the constraint shifts to ba , because of the decrease in p_c , the family is induced to have their first child and e_1 becomes positive. A decrease in p_s is equivalent to an increase in income and an increase in p_c . The increase in income taken by itself could have a positive effect on fertility and the increase in p_c , taken by itself could have a negative effect on fertility and it is impossible to determine a priori which of the two effects would dominate. Therefore, the effect of a change in p_s on fertility is indeterminant.

Becker and Lewis proved, in the context of the Becker model, that although n , e , and s are assumed to be normal goods, n or e may decrease as income increases. In Model II, which treats each child separately, when the arguments of the utility function are all normal goods, n can never decrease as income increases. Perhaps

Figure 6

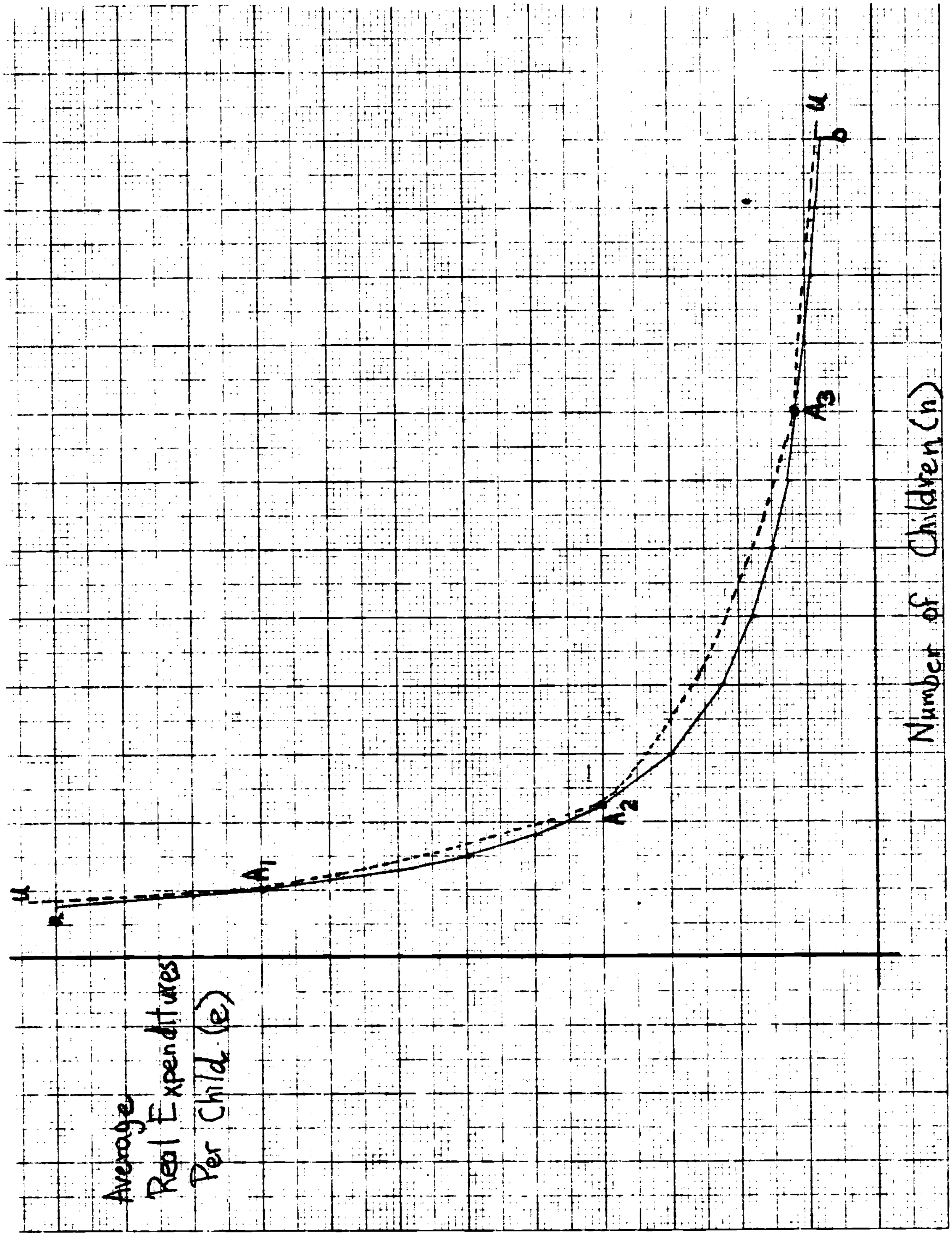


Real Expenditures on the First Child (e_1)
b
a
$e_1^{(2)}$
u_1
u_2
a
Quantity of s Consumed

this added ambiguity in the Becker model concerning the effect of observed income changes on fertility is a point in its favor because observed fertility variations have indeed been both positively and negatively associated with income. But some caution is suggested here. Any associations may be rationalized within a framework which does not restrict the direction or magnitude of association. This is one reason economists have chosen to assume children are not an inferior good and have reasoned, therefore, that children ought to be treated in our models as if they were consumer durables. If the Becker model of quantity-quality interaction is appropriate for both children and consumer durables, then why do we often observe negative income elasticities of "demand" for children, whereas in the case of consumer durables positive observed income elasticities are almost invariably observed. Within the context of the model the answer could only be that tastes for children are really different from tastes for durables. While this answer is better than no answer, there will be those who would argue that the same answer could have obtained without a model of quanity-quality interaction.

Before completing the comparison of the Becker model with the standard demand theory model of fertility, there are a few minor points to be mentioned. In the Becker model, given fixed money income and fixed prices, p_c and p_s, the values of n, e, and s which maximize the utiliity function need not be unique. This is shown in Figure 7. In that figure it is assumed that utility is mazimized when the quantity s_0 of s is consumed and that assumption underlies the drawing of

Figure 7


u
A1
A2
A3
b
u
Average
Real Expenditures
Per Child (e)
Number of Children (n)

both the constraint ab and the indifference curve UU . Clearly a multiplicity of solutions are possible in this case and, as it is drawn, utility is maximized at points A_1 , A_2 , and A_3 . Thus far we have ignored this problem and we shall assume it away in the remainder of the chapter.

Another minor point is that in the Becker model there is a quality dimension in only one of the two goods, children. Adding a quality dimension to the other good complicates the model considerably and weakens whatever hope there was of ever putting the Becker model to an empirical test.[28] Another aspect of the Becker model is that, although both n and e are assumed to be normal goods, even their product, total real expenditures on children $(c=n \cdot e)$, need not be normal with respect to observed income changes. It is possible, in the Becker model, for total real expenditures on children to decrease with an increase in income[29] while in Model II total real expenditures on children must increase when income increases.

In the end, the a priori choice between the Becker model and the alternative model boils down to the question of how relevant one believes the form of quality-quantity interaction (specified in the Becker model) to be in the case of fertility decisions. Model II, where relative prices do not change with income, is certainly easier to analyze and manipulate, but it treats each child as a separate entity just like each item in the household's consumption basket. Model II, which is, in fact, the standard demand theory model, thus fails to reflect certain sociological or psychological constraints upon the freedom of a family to select a "consumption mix."

Ironically in capturing the influence of such constraints upon fertility decisions, the Becker model comes closer to recognizing the range of considerations which underlay Leibenstein's treatment of fertility. One source of quantity-quality interaction, for example, may lie in parents' desire to make the same real expenditures on each child. In today's environment in the United States of small families in which the children are relatively closely spaced, it seems plausible to believe that real expenditures per child do not differ much across children in the same family. This would perhaps justify treating the average real expenditure per child as a separate argument of the utility function --

it would be an index of the family's (uniform) child-rearing style.

Taking a more historical view, however, there seems to be reasons to doubt that expenditures per child were equalized across children in the family. Inheritance practices, of course, varied across time and space, but it was not unusual for a farm to be divided unequally between a farmer's heirs.[30] In the United States earlier in this century it was not uncommon for some children of the family to be sent to work so that other children (brothers, usually) could be sent to school.[31] Therefore, it is not clear that observed regularities in behavior at one time are most usefully treated as sociological or psychological constraints operating on utility functions for all times. If there is a modern social convention which influences parents to spend equally on each child, it certainly owes much to the small size of the modern family. To accept the convention as operating at all times can add an element to circularity to any explanation of the decline in fertility over time. An appropriate historical model should be able to account for diversity in expenditures per child and the possible convergence of these expenditures over time. In this regard, Model II, being more general, must be judged to be preferable to the Becker model as a framework for analysis.

The Household Viewed as a Production Structure: The Willis Model of Fertility Decisions

The analysis of household fertility decisions recently advanced by Willis[32] represents the combination and development of two lines of approach, both suggested by Becker. The result is the most sophisticated and powerful model of fertility choice generally known to economists. Willis combined the Becker model of fertility discussed above with the concepts of household production and time allocation pioneered by Muth[33] and Becker.[34] The two threads, however, can exist separately or in combination with other strands of thought. Becker and Lewis[35] have shown that it is possible to analyze quantity-quality interactions in the Becker model without considering the structure of household production and below it will be shown that the structure of household production may be usefully placed in a model without the specific form of quantity-quality interaction envisioned in the Becker model.[36]

The structure of the Willis model is as follows:

Model III: The Willis Model

(9)	Maximize	$U = f(n,e,s)$	utility function
(10)	subject to	$s = G(t_s,x_s)$	household produc-
(11)		$c = H(t_c,x_c)$	tion functions
(12)		$e \equiv c/n$	definition of e
(13)		$T = t_c + t_s + t_\ell$	wife's time constraint
(14)		$V + w't_\ell = p_x(x_s + x_c)$	budget constraint

Assumptions: (i) $G(\ )$ and $H(\ )$ are homogeneous of degree one.

(ii) $H(\ )$ is more intensive in the wife's time than $G(\ )$.[37]

(iii) $n, e,$ and s are all normal goods.

where f( ) is a utility function; G( ) is a household production function whose output s is considered as a composite of all activities not associated with the production of child services, c; t_s is the wife's time spent in the production of s and x_s is the quantity of goods used in producing s ; H( ) is a household production function whose output is childservices, c ; t_c and x_c are the quantities of the wife's time and market goods respectively used in the production of childservices; e is the average quantity of childservices per child;[38] T is the total amount of time the wife has available during the period in question[39] and t_ℓ is the amount of time she spends in the labor market; V is total family money income during the period in question excluding the wife's earnings in the labor market; w' is the wife's wage rate if she participates in the labor market[40] and p_x is the market price of the pruchased good x .

A few words of comment are necessary here. Clearly, the Willis model is not a complete model of fertility in the sense that there are many plausible additions wich might still be introduced, even though as it stands it constitutes a significant elaboration of the original Becker model. For the purpose of creating models which have analytic implications, it is certainly important to abstract from all but the most important aspects of the problem, but this does not mean that the specification of the model is beyond question. Rather than probing the model for those assumptions which are crucial and for those which are not, let us proceed to a brief discussion of the model's properties. After the model has been put through its paces we shall return to the question of whether it would have performed any differently had its assumptions been altered.

At first glance the only portion of the Willis model which resembles the Becker model is the utility function. However, their similarities run considerably deeper. Since G and H are homogeneous of degree one, the derived demand functions for their inputs may be written as follows:[41]

(15) $$t_s = g_t(w,p_x)s$$

(16) $$x_s = g_x(w,p_x)s$$

(17) $$t_c = h_t(w \cdot p_x)c$$

(18) $$x_c = h_x(w,p_x)c$$

where among other restrictions on g_t, g_x, h_t, h_x they are all homogeneous of degree zero.[42] If the wife is in the labor force we know that:

(19) $$t_\ell = T - t_c - t_s \neq 0$$

and substituting this value of t_ℓ into the budget constraint yields

(20) $$V + w'T = w't_c + p_x x_c + w't_s + p_x x_s \ .$$

If the wife is not in the labor market multiplying equation (13) by her "shadow wage rate", w*, and adding the resulting equation to equation (14) yields

(21) $$V + w^*T = w^*t_c + p_x x_c + w^*t_s + p_x x_s \ .$$

Equation (20) is the constraint binding the household if the wife is in the labor force and equation (21) is the constraint if she is not.

Substituting equations (15) - (18) first into equation (20) and then into equation (21) we obtain

(22) $$V + w'T = s(w'g_t(w',p_x)+p_xg_x(w',p_x)) + c(w'h_t(w',p_x)+p_xh_x(w',p_x)) .$$

and

(23) $$V + w^*T = s(w^*g_t(w^*,p_x)+p_xg_x(w^*,p_x)) + c(w^*h_t(w^*,p_x)+p_xh_x(w^*,p_x)) .$$

If we write

(24) $$I = V + wT$$

(25) $$p_s = wg_t(w,p_x) + p_xg_x(w,p_x)$$

(26) $$p_c = wh_t(w,p_x) + p_xh_x(w,p_x)$$

then we obtain

(27) $$I = p_ss + p_cc$$

and making use of equation (12)

(28) $$I = p_ss + p_cne ,$$

which is identical in form to the budget constraint in the Becker model.[43] Thus both the Willis model and the Becker model may be written

(29) Maximize $U = f(n,e,s)$

(30) subject to $I = p_ss + p_cne$.

However, in the Becker model p_s , p_c and I are exogenous, whereas in the Willis model they are endogenous.[44]

There is somewhat less than meets the eye in this similarity between the Becker and Willis models because the budget constraint in the Willis model is really a combination of one constraint which is binding when the wife is in the labor market and one which is binding when she is not.[45] This situation is shown in Figure 8.

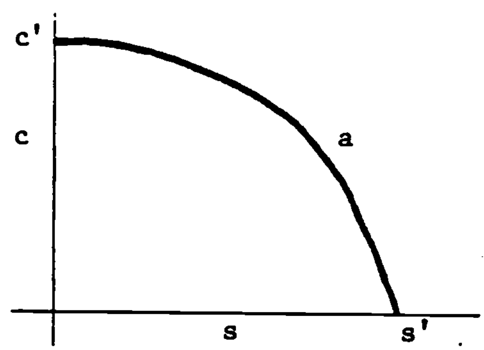


Figure 8

The curve c'as' is the production possibilities frontier associated with equations (10) (11) (13) and (14). The nonlinear portion c'a shows the situation in which the wife is not working in the labor market and the linear portion as' shows the situation in which she is working in the labor market. The cause of this rather unusual production possibilities curve may be stated simply. When the wife participates in the labor market her wage rate is fixed (by assumption) at w' regardless of how many hours she works and therefore p_s and p_c remain fixed. Moving upwards and to the left along the production possibilities frontier from s'a causes the wife's hours of work to decline steadily[46] until at point a the wife stops working in the labor market. Continuing along the curve from a to c', the wife's shadow wage rate rises because of the assumption that c production is more time intensive than s production.[47] The increase in the wife's shadow wage causes the price of c to rise relative to the price of s and this is reflected in the curvature of the constraint between a and c'.

The Willis model has four exogenous variables, V, w, T, and p_x. Unfortunately, without further assumptions the model has no implication for the direction of the change in fertility for any given change in one of the exogenous variables. Changes in V, w, T, and p_x produce income effects and alter p_c and p_s. Once the income and relative price effects are identified, the analysis of the model is identical to that of the Becker model. However, the Becker model as we have seen has no implications for fertility. In the Willis model a change in a single exogenous variable often produces both income and relative price effects and, in this case, the amalgam of changes, each of which may be positively or negatively related to fertility, still produces a fertility response whose direction is indeterminant. Nonetheless, the framework of the model is quite useful and Willis has been able to extract from it quite a plausible explanation of observed cross-sectional fertility differentials. This explanation will be discussed in connection with Model IV below.[48]

A Standard Demand Theory Version of the Willis Model

The ambiguity with regard to implications for fertility in the Willis Model has two roots: the specification of the utility function and the specification of the production structure. It is helpful in understanding the Willis model to separate the influences of these two segments of the structure and to consider a modified model with the production structure of the Willis model and the utility formulation of Model II above. This model may be written

Model IV: A Willis-type Model Which Treats Each Child Separately

(31) Maximize $U = f(c_1, c_2, \ldots, c_m, s)$

(32) subject to $s = G(t_s, x_s)$

(33) $c_1 = H(t_{c1}, x_{c1})$

(34) $c_2 = H(t_{c2}, x_{c2})$

$\vdots$

(35) $c_m = H(t_{cm}, x_{cm})$

(36) $$T = t_s + \sum_{i=1}^{m} t_{c_i} + t_\ell.$$

(37) $$V + wt_\ell = p_x(x_s + \sum_{i=1}^{m} x_{c_i})$$

Assumptions: (i) $G(\ )$ and $H(\ )$ are homogeneous of degree one;
(ii) $H(\ )$ is more time intensive than $G(\ )$;[49]
(iii) s and c_1 through c_m are normal goods;[50]

where c_i is the output of childservices from the i-th child, t_{c_i} is the wife's time spent on the i-th child, and x_{c_i} is the quantity of goods spent on the i-th child. Otherwise, all the variables are defined as above for the Willis model. As in the Willis model, it is assumed that G and H are homogeneous of degree one and that the production of childservices is more intensive in the wife's time than is the production of s. However, Model IV makes the additional assumption that the production functions for childservices are independent of parity. This is an assumption whose merits can scarcely go unquestioned.[51] Nonetheless, as a simplification which aids us in understanding the Willis model it is quite useful.

Model IV combines the endogeneous relative prices of the Willis model with the utility function of Model II. Once the income and relative price effects due to some change in the exogenous variables are determined, the analysis of Model IV becomes identical to the analysis of Model II. In Model IV, the constraint facing the household may be written

(38) $$V + wT = p_c c_1 + p_c c_2 + \ldots + p_c c_m + p_s s \ ,$$

(39) where $$p_c = wh_t(w,p_x) + p_x h_x(w,p_x)$$

(40) and $$p_s = wg_t(w,p_x) + p_x g_x(w,p_x) \ .$$

This constraint differs from that in Model II only in that the income concept is full income rather than money income and in that prices are endogenous. The distinction between whether the wife is in or out of the labor force remains important in Model IV. Let us make the following definition:

(41) $$c \equiv c_1 + c_2 + \ldots + c_m$$

or, in words, c is equal to total childservices. Now the constraint faced by the household may be simply written[52]

(42) $$V + wT = p_s s + p_c c \ .$$

The constraint is identical to the constraint in the Willis model and has nonlinear and linear segments as shown in Figure 8.

Because of the differences in the utility functions between Model III and Model IV the latter has some implications for fertility. A discussion of the implications of Model IV follows. If the wife is in the labor force both before and after an increase in husband's income, V, then this increase must cause fertility to increase or remain constant, but it can never cause it to decline. An analogous statement is also true for an increase in T .

If the wife is in the labor force both before and after an increase in either V or T , full income increases and relative prices remain fixed.[53] This is identical to the situation in Model II where income increased and it was shown below that in that situation fertility could not decrease and possibly could increase.[54] If the wife is out of the labor force both before and after an increase in V , then the sign of the change in fertility is indeterminant. Why are the implications for fertility of an increase in V different for women in and out of the labor market? They differ because if the wife is in the labor force an increase in V is essentially identical to an increase in I in Model II. However, if the wife is not in the labor force and increase in V has both an income and a relative price effect. This is due to a result known as Rybczynski's Theorem.[55] For our present purposes Rybczynski's Theorem states that if G( ) and

H( ) are both homogeneous of degree one, if H( ) is more time intensive than G( ) , if the wife is not in the labor market and if the family initially consumed at (s_0c_0) with the relative price ratio $\frac{p_s}{p_c} = P^*$, then

i) increasing V , the relative price ratio, $\frac{p_s}{p_c}$, will equal P^* at a point (s_1,c_1) such that $s_1 > s_0$ and $c_1 < c_0$;

ii) increasing T , the relative price $\frac{p_s}{p_c}$ will equal P^* at a point (s_2,c_2) such that $c_2 > c_0$ and $s_2 < s_0$. This may be seen graphically in Figures 9 and 10.[56] In Figure 9 the relative price ratio, p_s/p_c , is identical at points a and b . However, if the family initially consumed at point a it could not consume at point b after the increase in V without violating the assumption that c was a normal good. All points on the constraint which do not violate this normality assumption must be above and to the left of point b and have a relative price ratio greater than the relative price ratios at points a and b .[57] Therefore, if c is a normal good its relative price must increase as V increases. Thus, increasing V has an income effect which <u>cet</u>. <u>par</u>. would cause fertility to increase and a relative price effect which <u>cet</u>. <u>par</u>. would cause fertility to decline and the combination of these two effects has an indeterminant sign.

In the case of increasing T , both the income and the relative price effects work in the same direction, toward increasing fertility. In Figure 10, which shows the effect of increasing T , the relative price ratios $\frac{p_s}{p_c}$ are identical at points a and b . If the family

Figure 9

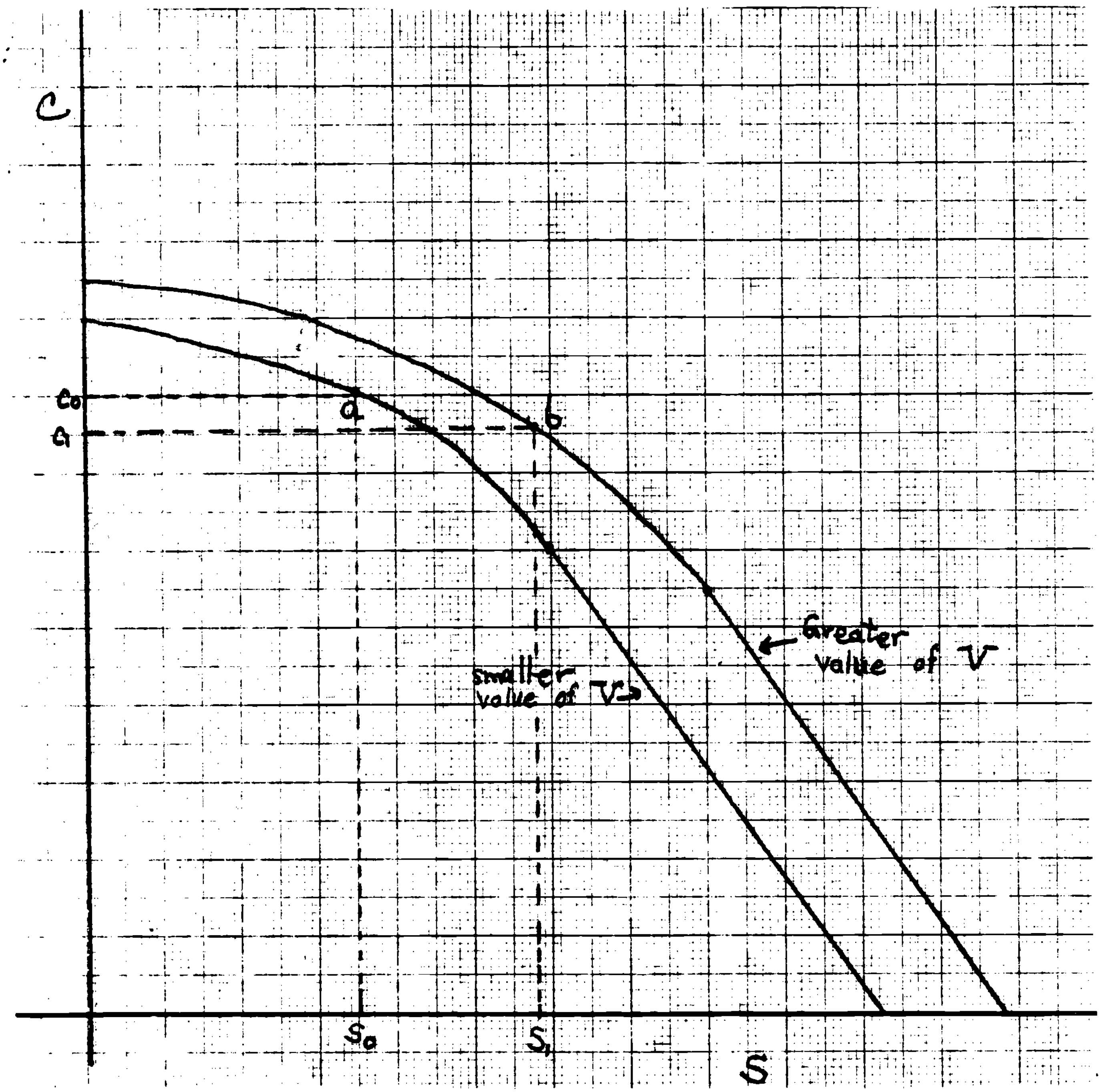
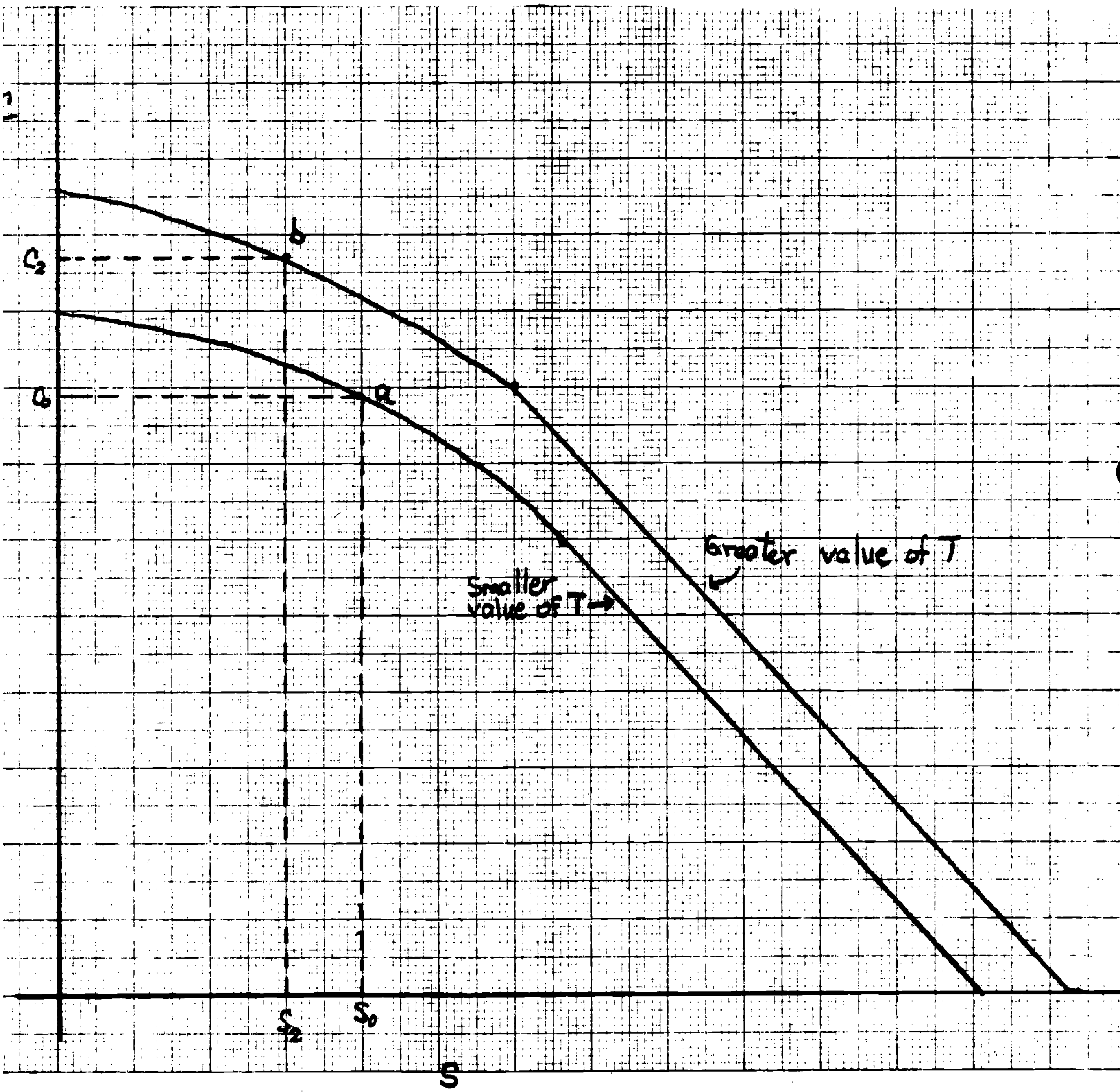


Figure 10



initially consumed at point a , it could not consume at point b after the increase in T without violating the assumption that s was a normal good. Therefore, all points consistent with the assumption that s is a normal good must lie below and to the right of point b along the constraint. At all such points, however, the relative price ratio $\frac{p_s}{p_c}$ must be lower than it was at point a .[58] The income effect and the relative price effect reinforce each other in this instance and therefore fertility cannot fall.

If the wife is out of the labor force before and after an increase in the market wage rate w' , her fertility is unaffected by that change. If the wife is in the labor market both before and after an increase in the market wage rate, the effect of this change upon fertility is indeterminant. The increase in the wife's market wage has an income effect which cet. par. would (by assumption) cause fertility to increase. However, the increase in w' , causes p_c to rise relative to $p_s t$,[59] and other things being equal, this, in turn, would cause a decline in fertility and the resultant of these two forces acting in opposite directions is of indeterminant sign.

From considerations similar to these Willis derives an empirically testable "mixture model" of fertility which he uses to explain nonlinearities in observed cross-sectional fertility patterns.[60] The fundamental assertions of the "mixture model" are, cet. par.;

i) the effect on fertility of increasing the husband's income is greater (i.e. more positive or less negative) when the wife's wage rate is higher than when it is lower;

ii) the effect on fertility of increasing the wife's wage rate is greater (i.e. more positive or less negative) when the husband's income is lower than when it is higher.

The assertions which comprise the "mixture model" are not implications of either Model III or Model IV, but rather are plausible interpretations of these models. Statement i) is derived from the following argument. Holding everything else constant increasing wives' wage rates tends to increase wives' labor force participation rates.[61] The effect of an increase in the husband's income has a positive effect on fertility if the wife is in the labor force and an ambiguous effect if she is not. It is plausible, although not certain, that the effect of an increase is V on fertility is smaller if the wife is not in the labor force than if she is. Indeed, it is quite plausible that the effect of increasing V is to increase the fertility when the wife is in the labor force and to decrease it when she is not. If it is true that the effect on fertility of increasing V is greater when the wife is in the labor market then when she is not, then the impact of an increase in V is greater the greater the wive's labor force participation rate.

Statement ii) is derived from the following argument. Holding everything else constant, increasing husband's incomes causes wives' labor force participation rate to decrease.[62] Increasing the wives' wage rates only affects the fertility of women in the labor market. If increasing wives' wage rates causes fertility to decline then as wives' labor force participation rates decrease, due to increases in V , it is likely the negative effect of increasing w would diminish. In the extreme case where wives' labor force participation rates were zero there would be no negative effect at all. Nonlinearities in fertility differentials of the sort described in statements i) and ii) have been found in a wide array of fertility data.[63]

The implications sketched above would be essentially identical to the considerations in the Willis model for determining fertility if c entered the utility function as a normal good and fertility was assumed to vary directly with c . However, c does not enter the utility function at all; in its place are the separate arguments n , and e . The added difficulty involved in such an approach, discussed above in terms of the Becker model, turns all the sign implications of the Model IV into ambiguous results and complicates the ambiguity of those situations in which the sign implications for fertility are already unclear. Thus entertaining the Becker form of the utility function as opposed to a utility structure which treats each child as a separate entity has a substantial cost within the context of the Willis model. One would exepct economists to accept it only if there was some substantial benefit from doing so. To date, the existence of such benefits remains to be established.

There are clearly a number of facets of the Willis model which can be elaborated and extended. However, sociologists frequently argue that there is one particular portion of the model which is not in need of elaboration or extension, but rather a candidate for removal: the utility function. The core of this objection may be separated into two parts. The first half of the objection is that fertility control is not perfect and parents often complete their childbearing years with a number of children different from their preferred number.[64] The second half of the objection is that people may not choose their preferred number of children in a manner which seems rational or consistent from the point of view of an economic decision-making model.[65] Acceptance of the latter conetntion, it should be noted, would undercut Leibenstein's approach no less than Becker's -- the presence of some significant degree of implicit sociology in both formulations notwithstanding.

The first objection can be eliminated within the framework of models of utility-maximizing behavior by assuming that families choose a contraceptive strategy rather than directly choosing a number of children.[66] The normative model then involves maximization of the family's exepcted utility over possible contraceptive strategies. This formulation may satisfy the first objection, but it certainly would not satisfy the second. In order to satisfy the second objection altogether another framework for analysis is needed.

Modelling Fertility Behavior Without the Utility Maximization Hypothesis

There is another framework in which models of household production may fruitfully be viewed and which satisfies the second objection. That framework includes a statistical representation of the behavior of a group of households. I have shown elsewhere[67] that it is possible to develop a demand theoretic structure for group behavior which did not presume that individual consuming units maximized a utility function or acted according to a consistent set of preferences. In order to accomplish this, assumptions about distributions of purchases and how they change when economic conditions change were substituted for assumptions about individuals' maximizing behavior.

Let us now consider another model of fertility which makes use of the household production framework; this time a statistical model of aggregate fertility behavior which does not assume anything about the utility functions of individual families. First however, some introductory comments are needed.

Standard demand theory envisions a single household making choices subject to a constraint. These models, then, have two main parts, the constraint and the representation of the individual's preferences. All the four models of fertility which have been discussed above are of this nature. The statistical model of fertility behavior which shall be considered below envisions a large number of households, all of whom are subject to the same constraint. Using Model IV as an example, in Figure 11 the standard demand analysis is concerned with how much c and s a single household will choose to consume when it is subject to the constraint $a_1a_2a_3$. If the household chooses a point on the arc a_1a_2 the wife will not participate in the labor market. If the household chooses a point on the line segment a_2a_3, then the wife will participate in the labor market. In contrast, the statistical model of fertility behavior considers a large number of households distributed along the constraint. In general, there will be some households in which the wife participates in the labor market and some in which she does not. Each household, though, consumes a particular combination of s and c. Let us suppose that there are m households whose consumption bundles are located at various points along the constraint. The quantities of s and c consumed may be ordered from the largest to the smallest so that

$$s_1 \geq s_2 \geq \cdots \geq s_m$$

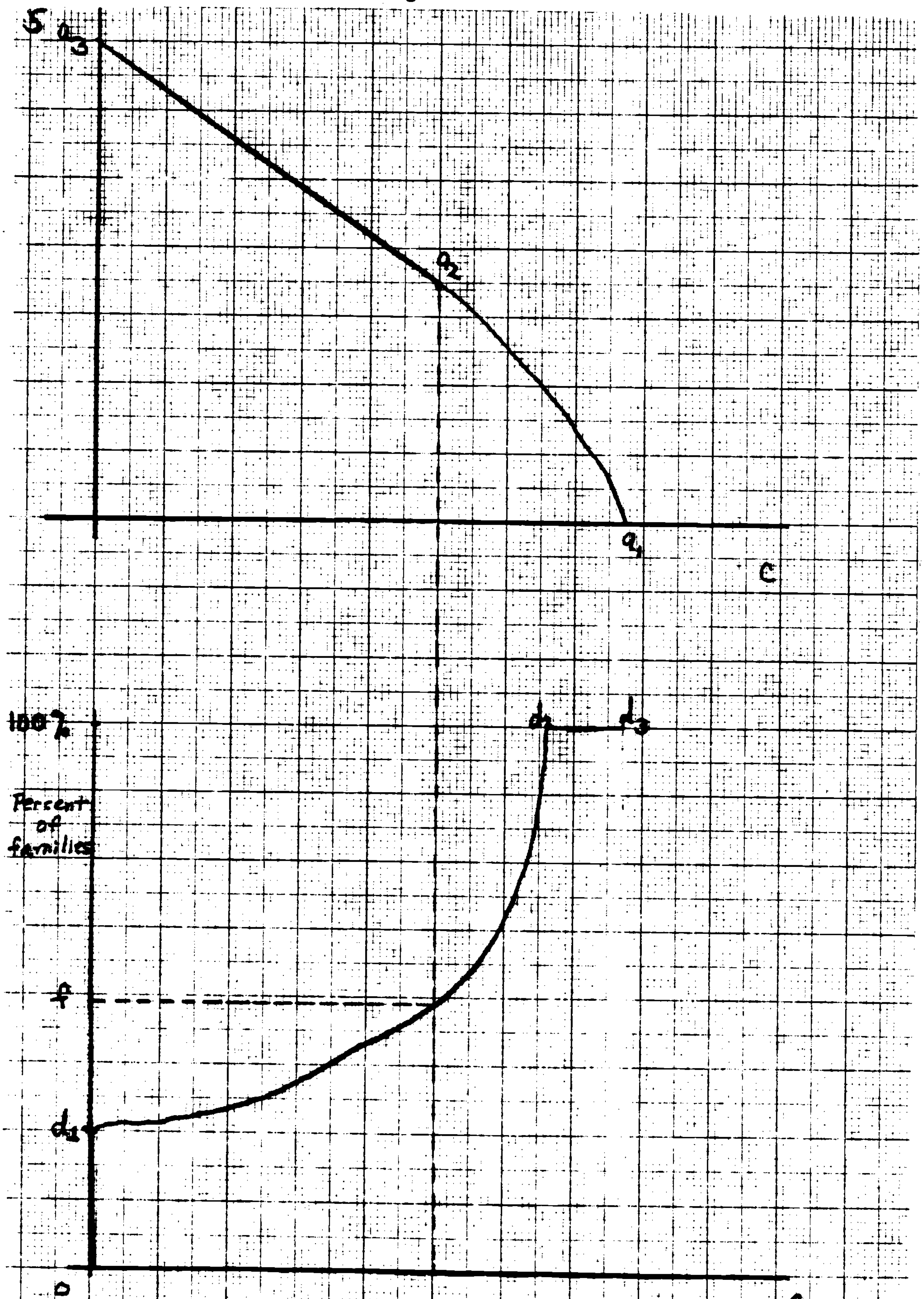
and

$$c_1 \geq c_2 \geq \cdots \geq c_m .$$

We denote the ordered set of the s_i by $\{s\}$ and the ordered set of the c_i by $\{c\}$ and call them the distribution of s and the distribution of c respectively.

The distributions of s and c may be represented graphically by their distribution (or cumulative density) functions. Figure 11 shows a constraint and its associated distribution of c.[68] In Figure 11, d_1 is the proportion of families, subject to the constraint $a_1a_2a_3$, which are childless. Generally, for any given quantity of childservices consumed, say c_2, the distribution function tells what percentage of the families subject to the constraint consume a quantity of childservices less than or equal to c_2. Thus in Figure 11 f percent of the households in question consume a quantity to c less than or equal to c_2. Since all women for whom $c < c_2$ are in the labor force, f is also the wives' labor force participation rate. In Figure 11, a graph of

Figure 11



the distribution of s was not included, but it is logically no less important than the distribution of c; it can be derived from the constraint and the distribution of c in this two-dimensional case.

The major difference between the standard demand theory model and the new demand theory model being considered here is that the standard demand theory model consists of a constraint and a representation of the tastes of a single household, while the new demand theory model consists of a constraint and representations of purchase distributions of a large group of households. Standard demand theory does not get very far on constraints and utility functions alone and it has become standard practice to assume that the arguments of the utility function are normal goods. Similarly the new demand theory does not get very far until an analogous normality assumption is made. Before such an assumption can be made, however, we must make the following definition.

<u>Definition of an increase in the opportunity to consume a household commodity</u>:

Let us consider a n-dimensional production possibilities curve, given the prices and income in situation $i(i=1 \text{ or } 2)$[69] which we write

$$Z_{jk}^{(i)} = L^{(i)} \left(Z_{1k}^{(i)}, Z_{2k}^{(i)}, \ldots , Z_{j-1,k}^{(i)} , Z_{j+1,k}^{(i)} , \ldots , Z_{nk}^{(i)}\right)$$

where $Z_{jk}^{(i)}$ is the maximum amount of household commodity j which can be produced by household k given the resources in situation i and given that it also produces the quantities of the other household commodities specified as arguments in the $L^{(i)}$ function. If $(Z_{1k}^{(i)}, Z_{2k}^{(i)}, \ldots,$

$(Z^{(i)}_{j-1,k})$ $(Z^{(i)}_{j+1,k})$ $\ldots,$ $Z^{(i)}_{nk})$ is infeasible[70] then $Z^{(i)}_{jk}$ is zero. If the production possibilities curves in situations 1 and 2 differ such that

$$\text{i)}\quad Z^{(2)}_{jk} \geq Z^{(1)}_{jk} \qquad (Z^{(2)}_{jk} \leq Z^{(1)}_{jk})$$

over all points $(Z^{(i)}_{1k}, Z^{(i)}_{2k}, \ldots, Z^{(i)}_{j-1,k}, Z^{(i)}_{j+1,k}, \ldots, Z^{(i)}_{nk})$ which are feasible either in situation 1 or in situation 2

and

ii) there exists some point $(Z^{(i)}_{1k}, Z^{(i)}_{2k}, \ldots, Z^{(i)}_{j-1}$ $Z^{(i)}_{j+1}$ $\ldots, Z^{(i)}_{nk})$ such that $Z^{(2)}_{jk} > Z^{(1)}_{jk}$, $(Z^{(2)}_{jk} < Z^{(1)}_{jk})$

then we say that the opportunity to consume household commodity j is greater (smaller) in situation 2 than in situation 1 .

<u>Normality Assumption</u>: If the opportunity to consume Z_j is greater (smaller) in situation 2 than in situation 1 then

$$Z^{(2)}_{jk} \geq Z^{(1)}_{jk} \quad (Z^{(2)}_{jk} \leq Z^{(1)}_{jk}) \qquad \text{for} \qquad k = 1, \ldots, m\,.$$ [71]

In essence, the normality assumption asks our forebearance in the assertion that whenever the constraint shifts upward and to the right, the distribution functions of the quantities purchased of each good also shift to the right.

Now we may state a household production model of fertility behavior which does not assume that each family maximizes a utility function.

Model V: <u>A Model of Fertility Without Utility Maximization</u>

(43) $$s = G(t_s, x_s)$$

(44) $$c = H(t_c, x_c, n)$$

(45) $$T = t_c + t_s + t_\ell$$

(46) $$V + wt_\ell = p_x(x_c + x_s) \ .$$

Assumptions: (i) G and H are homothetic production functions,

(ii) H is more intensive in the wives' time than G ,

(iii) there are m households consuming at various points along the households' production possibilities frontier,

(iv) s and c are normal goods in the sense of normality given above,

(v) wives' time and numbers of children are complementary in the production of childservices.

Model V differs from the original Willis model in a number of ways. The most obvious difference is that there is no utility function in Model V. It has been replaced by the assumption that there are m households consuming at various points on their production possibilities frontier. Another difference is that in the Willis model G( ) and H( ) were assumed to be homogeneous of degree one while in Model V G( ) and

H( ) are just assumed to be homothetic production functions, a considerable generalization of the initial Willis specification. The childservices production function here differs from that in the original Willis model in that here the childservices production function includes the number of children, whereas in the Willis model the quantity of childservices did not depend on the number of children. Indeed, in all four previous models, it was assumed that holding the number of children constant it was always possible to increase childservices by any mulitple so long as time and goods inputs into children were increased by that multiple. In these models it is never envisioned that spending ever more time and goods on a fixed number of children could ever lead to diminishing returns. In the specification used in Model V, if G( ) and H( ) were homoegeneous of degree one, a doubling, for example, of c could be obtained by doubling the time and goods spent on children <u>and</u> the number of children; doubling the time and goods spent on children without increasing the number <u>would not</u> double childservices, but rather increase it by a smaller multiple.

Putting the number of children into the childservices production function is not without some difficulty, however. If n is to be treated as an input, it must have a price either in terms of wives' time, market goods or both. The question then becomes, "how much does a live birth cost in terms of market goods and the wife's time?" In Model V, n appears in neither the wives' time constraint nor the budget constraint. In other words, it is assumed that the time and money costs of producing a live birth are zero. In reality of course, this is not the case, but I have excluded them from this particular model for ease of exposition. Further, I do not believe their inclusion would aid in the explanation of either time series trends or cross-sectional differentials in fertility. This is not a crucial simplification, however; the reader may easily produce the modifications needed to treat both a time and a money cost of producing a live birth. Since, in this model live births are costless, the production function H must be one such that the marginal product of children can become zero when n is within a plausible range.[72]

The derived demand functions drawn from equations (43) and (44) may be written

$$t_s = g_t(w,p_x)\,\Phi(s) \tag{47}$$

$$x_s = g_x(w,p_x)\,\Phi(s) \tag{48}$$

$$t_c = h_t(w,p_x)\,\Psi(c) \tag{49}$$

$$x_c = h_x(w,p_x)\,\Psi(c) \tag{50}$$

$$n = h_n(w,p_x)\,\Psi(c)\ . \tag{51}$$

where, among other restrictions, g_t, g_x, h_t, h_x, and h_n are homogeneous of degree zero and where $\Phi(s)$ and $\Psi(c)$ are strictly increasing functions of their arguments.[73]

The time and budget consgraints, equations (45) and (46) may be combined into a single equation as was done in equation (27) above. Thus we obtain:

$$V + wT = \Phi(s)\,(wg_t(w,p_x) + p_x g_x(w,p_x)) + \Psi(c)(wh_t(w,p_x) + p_x h_x(w,p_x))\ , \tag{52}$$

and

$$(53) \qquad V + wT = \Phi(s)\, p_s + \Psi(c)\, p_c$$

where p_s and p_c are defined as in equations (25) and (26) above. Clearly, if this constraint is interpreted as relating $\Phi(s)$ and $\Psi(c)$ rather than s and c, it is identical to the constraint relating s and c in the case of linear homogeneous production functions. Equation (53), which is graphically represented in Figure 8, has a linear segment for the situation in which women are in the labor force and a nonlinear segment covering the situation in which women are not in the labor force.

In Model V average fertility, in situation i[73], $\bar{n}^{(i)}$, may be written as follows

$$(54) \qquad \bar{n}^{(i)} = \frac{1}{m} \sum_{j=1}^{m} h_n \left(w_j^{(i)}, p_x\right) \psi\left(c_j^{(i)}\right)$$

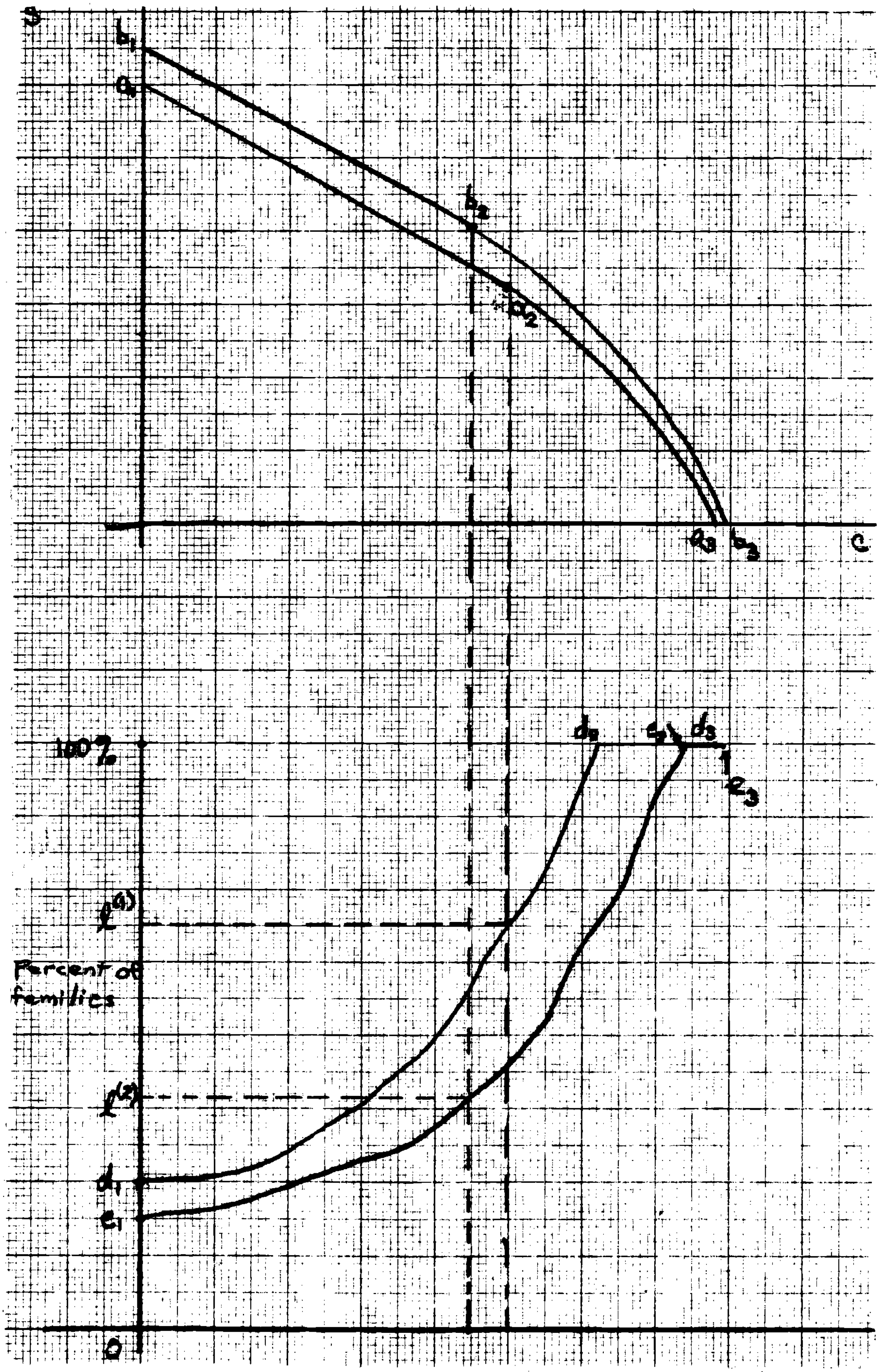
where $c_1^{(i)} \geq c_2^{(i)} \geq \ldots \geq c_m^{(i)}$

and where $w_j^{(i)}$ is the (shadow) wage rate of the wife in the jth family in situation i. The wage rate $w_j^{(i)}$ is the market wage rate if the wife is in the labor market; if she is not, it is her shadow wage rate.

Now let us consider the effect on fertility of an increase in V. This is shown graphically in Figure 12.

In Figure 12, the constraint $a_1a_2a_3$ is associated with the distribution function $d_1d_2d_3$ and the constraint $b_1b_2b_3$ is associated

Figure 12


S
b_1
a_1
b_2
a_2
a_3
b_3
C
100%
d_2
e_2
d_3
e_3
$\ell^{(4)}$
Percent of families
$\ell^{(2)}$
d_1
e_1
O

with the distribution function $e_1e_2e_3$. Rybczynski's Theorem[74] tells us that the point b_2 must always be above and to the left of a_2 . Since by the Normality Assumption, the distribution function $e_1e_2e_3$ can never lie to the left of the distribution function $d_1d_2d_3$, the wives' labor force participation rate after the increase in V , $\ell^{(2)}$ in Figure 12, must be below $\ell^{(1)}$, the wives' labor force participation rate before the increase in V .[75] In the case of m discrete households it is only possible to show that the labor force participation rate of wives' cannot increase when V increases.[76]

Let us suppose, then, that $q^{(1)}$ women are in the labor market before the increase in V and that $q^{(2)}$ women are in the labor market after the increase in V . Clearly $q^{(2)} \leq q^{(1)}$. Let us write

$$\bar{n}_1^{(1)} = \frac{1}{q^{(2)}} \sum_{j=m-q^{(2)}+1}^{m} h_n(w',p_x)\ \psi(c_j^{(1)}) \tag{55}$$

and

$$\bar{n}_1^{(2)} = \frac{1}{q^{(2)}} \sum_{j=m-q^{(2)}+1}^{m} h_n(w',p_x)\ \psi(c_j^{(2)}) \ . \tag{56}$$

where w' is the market wage rate.

The terms $\bar{n}_1^{(1)}$ and $\bar{n}_1^{(2)}$ are the average fertility analogs of the Willis model concepts of the fertility of a woman who was in the labor force both before and after an increase in V . In this case, however, it is not necessary to assume that we have observed the same women both before and after an increase in V . Even if the same women are observed the $q^{(2)}$ women considered in computing $\bar{n}_2^{(1)}$ need not be the same as those considered in computing $\bar{n}_1^{(2)}$.[77] It is important to note that although $\bar{n}_1^{(2)}$ is the average fertility of women in the labor force after an increase V , $\bar{n}_1^{(1)}$ is not the average fertility of women in the labor force before the increase. Rather $\bar{n}_1^{(1)}$ is the average fertility of the $q^{(2)}$ women in the labor market with the lowest values of c or what amounts to the same thing in this model, the highest numbers of hours worked in the labor force.[78]

Now it is easy to prove that $\bar{n}_1^{(2)} \geq \bar{n}_1^{(1)}$. The proof proceeds as follows

$$(57) \qquad \bar{n}_1^{(1)} = \frac{h_n(w',p_x)}{q^{(2)}} \sum_{j=m-q^{(2)}+1}^{m} \Psi(c_j^{(1)})$$

and

$$(58) \qquad \bar{n}_1^{(2)} = \frac{h_n(w',p_x)}{q^{(2)}} \sum_{j=m-q^{(2)}+1}^{m} \psi(c_j^{(2)}) \ .$$

Therefore to complete the proof it is only necessary to show that

$$(59) \qquad \sum_{j=m-q_i^{(2)}+1}^{m} \psi(c_j^{(2)}) \geq \sum_{j=m-q_i^{(2)}+1}^{m} \psi(c_j^{(1)}) \ .$$

However, statement (59) follows directly from

(i) $\Psi(\ )$ is a strictly increasing function of c

and

(ii) $c_j^{(2)} \geq c_j^{(1)}$ $j=1,\ldots,m$ (because of the Normality Assumptiton[79]).

Thus, we have demonstrated that the average fertility of comparable groups of women in the labor market increases as husbands' income increases. This result is clearly similar to the result in Model IV, for increasing V while the wife remains in the labor market.

In Model IV, it was shown that if the wife was not in the labor force both before and after an increase in V, the resulting direction of the fertility change could not be predicted. A similar statement concerning average fertility can be made for Model V. Let $r^{(1)}$ be the number of women not participating in the larbor market at the lower level of V and let $r^{(2)}$ be the number not participating at the higher level of V. Clearly, $r^{(2)} \geq r^{(1)}$. Now let

$$(60) \qquad \bar{n}_0^{(2)} = \frac{1}{r^{(1)}} \sum_{j=1}^{r^{(1)}} h_n(w_j^{(2)}, p_x) \quad \Psi(c_j^{(2)})$$

and

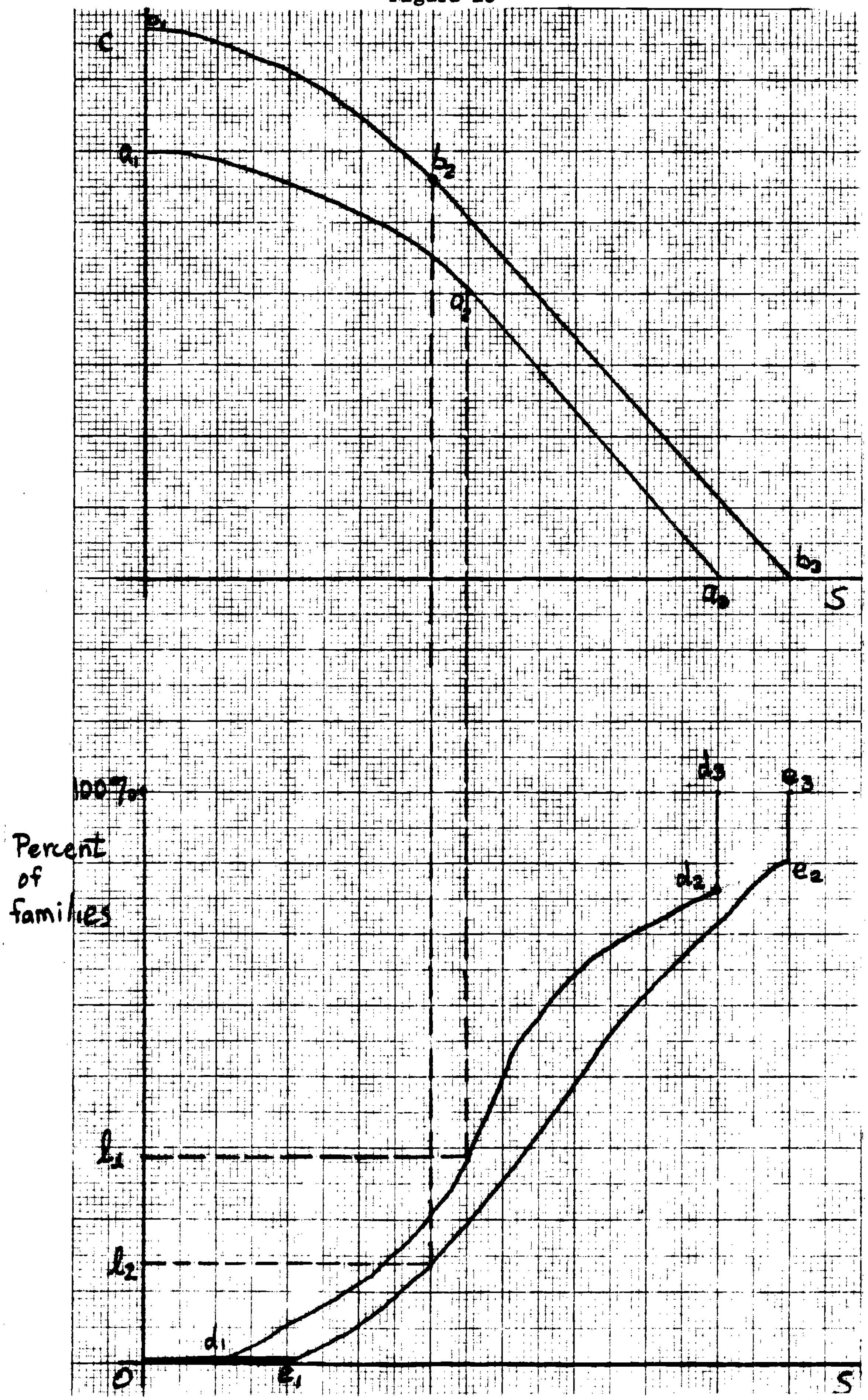
$$(61) \qquad \bar{n}_0^{(1)} = \frac{1}{r^{(1)}} \sum_{j=1}^{r^{(1)}} h_n\ (w_j^{(1)}, p_x)\ \Psi(c_j^{(1)})\ .$$

The average fertilities of comparable groups of women not in the labor force before and after an increase in V, $\bar{n}_0^{(1)}$ and $\bar{n}_0^{(2)}$, respectively, have no determinant relationship to one another. In order to see this clearly it is sufficient to note that

$$\left.\begin{array}{lll} \text{(i)} & \Psi(c_k^{(2)}) \geq \Psi(c_k^{(1)}) & \text{since } c_k^{(2)} \geq c_k^{(1)} \\ \text{(ii)} & h_n(w_k^{(2)}, p_x) \leq h_n(w_k^{(1)}, p_x) & \text{since } w_k^{(2)} > w_k^{(1)}\ . \end{array}\right. \quad k=1,\ldots,r_1$$

The analysis of Model V can continue in the same vein until it is shown that for every implication for fertility in Model IV is matched by an analogous result for average fertility in Model V. Let us consider next what happens when T is greater in situation 2 than in situation 1. This case is shown in Figure 13.

Figure 13


C
b_1
a_1
b_2
a_2
a_3
b_3
S
100%
Percent
of
families
d_3
e_3
d_2
e_2
l_1
l_2
d_1
e_1
O
S

In Figure 13 the axes in the upper panel are the reverse of what they were in Figure 12. The constraint $a_1a_2a_3$ is associated with the distribution function $d_1d_2d_3$ and the constraint $b_1b_2b_3$ is associated with the distribution function $e_1e_2e_3$.

By Rybczynski's Theorem b_2 must be above and to the left of a_2 . Since $e_1e_2e_3$ can never be to the left of $d_1d_2d_3$, the wives' labor force participation rate in situation 2, which $100 - \ell_2$ per cent, must be greater than $100 - \ell_1$ per cent, the wives' labor force participation rate in situation 1. In the case of a discrete number of families, if T is higher in situation 2 than in situation 1, then the wive's labor force participation rate must be at least as great in situation 2 as in situation 1.

Let $q^{(1)}$ be the number of women in the labor force in situation 1 and let $q^{(2)}$ be the number of women in the labor force in situation 2.[79] Clearly $q^{(2)} \geq q^{(1)}$.

Let us define

$$(62) \qquad \bar{n}_1^{(1)} = \frac{1}{q^{(1)}} \sum_{j=m-q^{(1)}+1}^{m} h_n(w', p_x)\ \psi\ (c_j^{(1)})$$

and

$$(63) \qquad \bar{n}_1^{(2)} = \frac{1}{q^{(1)}} \sum_{j=m-q^{(1)}+1}^{m} h_n(w', p_x)\ \psi\ (c_j^{(2)})$$

where w' is the wives' market wage rate.

The observation that $\bar{n}_1^{(2)} \geq \bar{n}_1^{(1)}$ follows directly from the Normality Assumption which tells us that $c_j^{(2)} \geq c_j^{(1)}$, $j=1$, ... , m , and from the fact $\psi(c_j)$ is a strictly increasing function of c_j. Thus the average fertility of comparably defined groups of women in the labor force never decreases as T increases.

Let $r^{(1)}$ and $r^{(2)}$ denote the numbers of women not in the labor force in situation 1 and 2 respectively. We know that $r^{(1)} \geq r^{(2)}$. Let us define

$$\bar{n}_0^{(1)} = \frac{1}{r^{(2)}} \sum_{j=1}^{r^{(2)}} h_n(w_j^{(1)}, p_x)\, \psi(c_j^{(1)}) \tag{64}$$

and

$$\bar{n}_0^{(2)} = \frac{1}{r^{(2)}} \sum_{j=1}^{r^{(2)}} h_n(w_j^{(2)}, p_x)\, \psi(c_j^{(2)}) \tag{65}$$

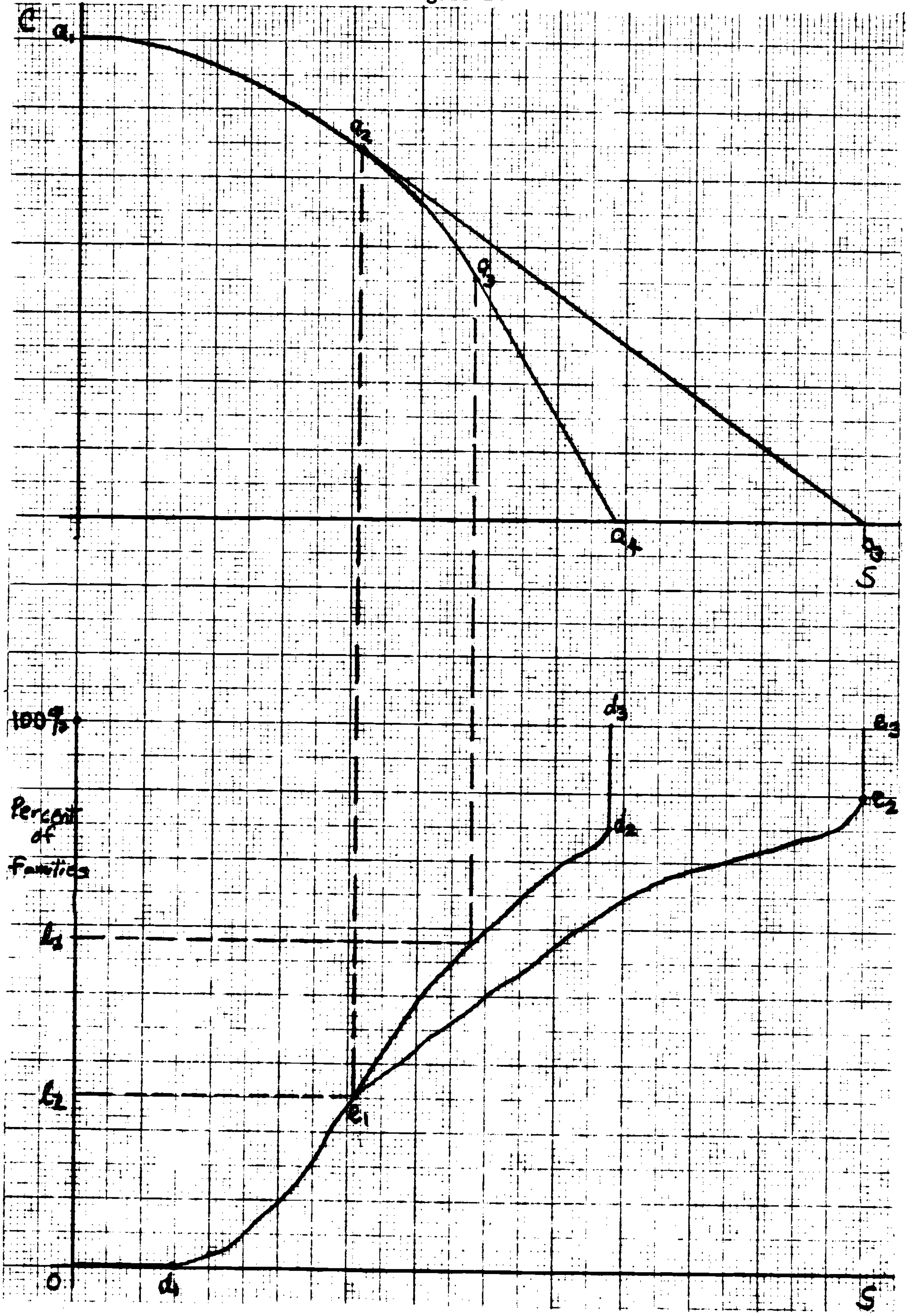
where $w_j^{(1)}$ is the shadow wage of the wife in family j in situation i. It may be simply shown that $\bar{n}_0^{(2)} \geq \bar{n}_0^{(1)}$. Let us consider the kth term in each of the summations. We know:[80]

$$\left.\begin{array}{lll} \text{(i)} & \Psi(c_k^{(2)}) \geq \psi(c_k^{(1)}) & \text{Since } c_k^{(2)} \geq c_k^{(1)} \\ \text{(ii)} & h_n(w_k^{(1)}, p_x) \geq h_n(w_k^{(1)}, p_x) & \text{Since } w_k^{(2)} < w_k^{(1)} \end{array}\right\} k=1, \ldots, r^{(2)}.$$

Therefore $\bar{n}_0^{(2)} \geq \bar{n}_0^{(1)}$. For comparably defined groups of women not in the labor force, if T is greater in situation 2 than in situation 1 then average fertility can be no lower in situation 2 than in situation 1.

The implications of Model V for the signs of changes in average fertility due to changes in the wives' market wage rate are again analogous to the implications of Model IV. Let the wives' market wage rate be greater in situation 2 than in situation 1. This case is shown in Figure 14.

Figure 14


C
a1
a2
a3
a4
a5
S
100%
Percent of Families
d3
e3
d2
e2
L1
L2
e1
0
d4
S

The constraint $a_1a_2a_3a_4$ is associated with the distribution function $d_1e_1d_2d_3$ and the constraint $a_1a_2b_3$ is associated with the distribution function $d_1e_1e_2e_3$. The distribution function beneath that portion of the constraint which does not shift cannot shift either.[81] Thus both situation 1 and 2 share the arc a_1a_2 on the constraint and the arc d_1e_1 on the distribution function. Since point a_2 must lie to the right of a_3 , with a higher market wage rate, the wive's labor force participation rate, $100 - \ell_2$ per cent must be greater than $100 - \ell_1$ per cent, the wive's labor force participation rate when their market wage rate is lower. In the case of a discrete number of families, increasing the wive's wage rate can never cause a decrease in their labor force participation rate.

Let us suppose that in situation 1, with the lower wage rate, $r^{(1)}$ women do not work and that in situation 2, $r^{(2)}$ women do not work. Clearly $r^{(1)} \geq r^{(2)}$. In this case it may easily be shown that

$$(66) \qquad \frac{1}{r^{(2)}} \sum_{j=1}^{r^{(2)}} h_n(w_j^{(2)}, p_x) \, \Psi (c_j^{(2)}) = \frac{1}{r^{(2)}} \sum_{j=1}^{r^{(1)}} h_n(w_j^{(1)}, p_x) \, \psi (c_j^{(1)})$$

Equation must hold because

$$\left.\begin{array}{ll} \text{(i)} & c_j^{(2)} = c_j^{(1)} \\ \text{(ii)} & w_j^{(2)} = w_j^{(1)} \end{array}\right. \quad j=1 , \ldots , r^{(2)}$$

In other words, equation (66) must be true because the first $r^{(2)}$ points on the constraints in both situations must be identical. We have just shown that the average fertilities of comparably defined groups of women who do not participate at either the higher or the lower wage rate remain identical regardless of the wage rate change.

The average fertility of comparably defined groups of women in the labor force may either be greater or smaller in situation 2 as compared to situation 1. Let $q^{(1)}$ be the number of women who participate in the labor market in situation 1 and $q^{(2)}$ be their number in situation 2. Clearly $q^{(2)} \geq q^{(1)}$. Let us define the following average fertilities:

(67) $$\bar{n}_1^{(1)} = \frac{1}{q^{(1)}} \sum_{j=m-q^{(1)}+1}^{m} h_n(w^{(1)}, p_x) \; \Psi(c_j^{(1)}$$

and

(68) $$\bar{n}_1^{(2)} = \frac{1}{q^{(1)}} \sum_{j=m-q^{(1)}+1}^{m} h_n(w^{(2)}, p_x) \quad \psi(c_j^{(2)})$$

where $w^{(i)}$ is the market wage rate in period i and where $w^{(2)} > w^{(1)}$. It is impossible to determine whether $\bar{n}_1^{(2)}$ is greater than, less than or equal to $\bar{n}_1^{(1)}$ because

(i) $(c_k^{(2)}) \geq \psi(c_k^{(1)})$ since $c_k^{(2)} \geq c_k^{(1)}$ for $k = m-q^{(1)}+1,\ldots,m$

and (ii) $h_n(w^{(2)}, p_x) \leq h_n(w^{(1)}, p_x)$ since $w^{(2)} > w^{(1)}$.

This completes the discussion of the implications of Model V .

We have just demonstrated that without the assumption of utility maximization it is possible to construct a household production model which comes to essentially the same conclusions as the original Willis model. The main differences between the Willis model and the statistical aggregate model presented here are:

1. The Willis model assumes that a single household maximizes its utility function whereas the present model deals with aggregates and substitutes postulates on group behavior for the postulates on individual behavior used in the Willis model.

2. The present model yields implications for changes in average behavior for discrete changes in the independent variables while the Willis model yields implications for a single family's behavior for infinitesimal changes in the independent variables.

3. More general homothetic production functions were used in the present model as opposed to linear homogeneous production functions in the Willis model.

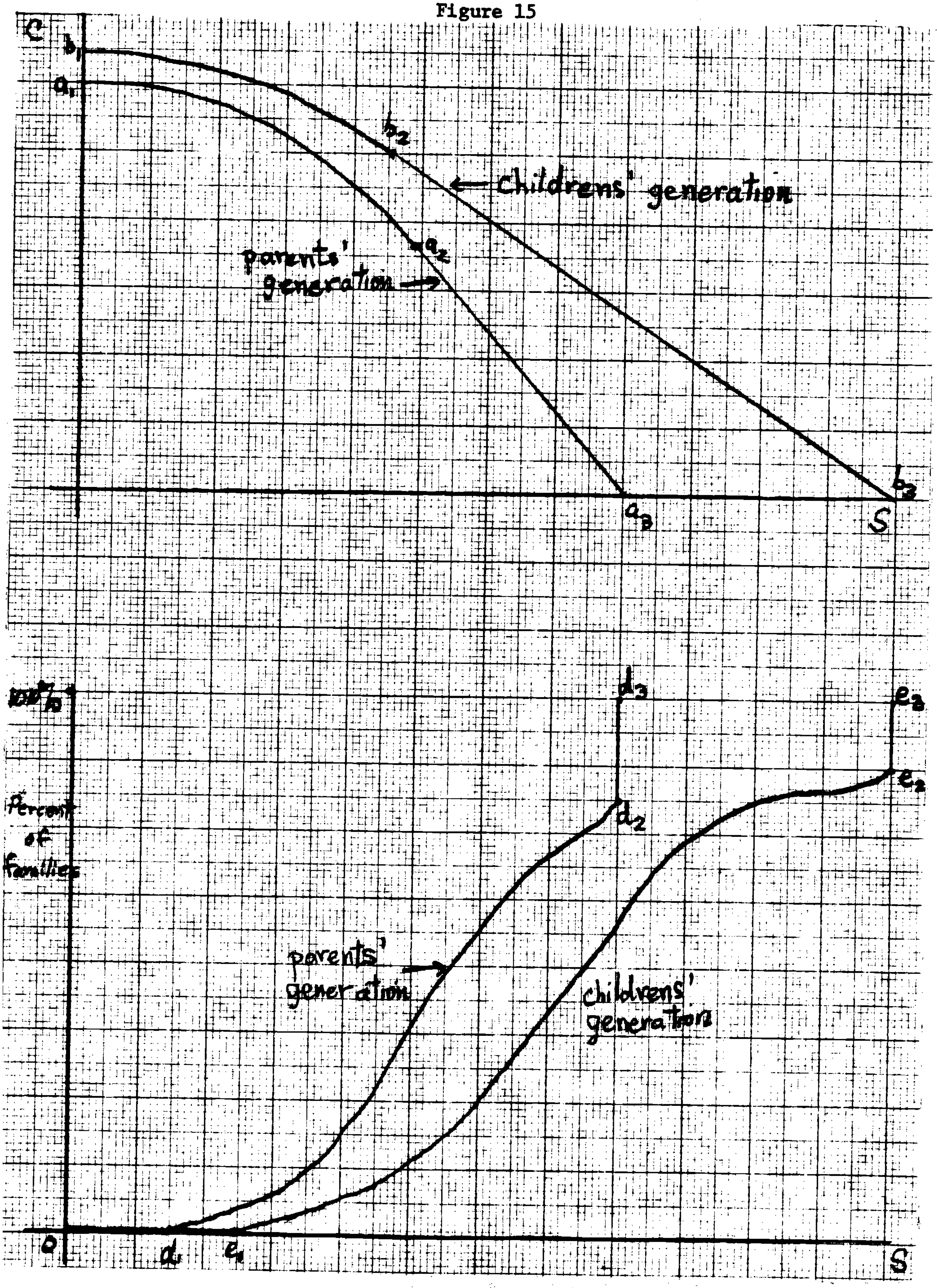
That with all these changes the implications of the Willis model remain essentially unchanged shows that the features of the production structure embedded in the original Willis model give rise to conclusions which are quite robust.

The Non-Utilitarian Model of Fertility: Some Future Developments

In addition to implications analogous to those of the Willis model, Model V potentially has implications which go quite beyond anything in the Willis model. Of all the models considered in this chapter only Model V could have implications for changes in the partiy distribution of fertility. The other models deal only with a single family so that their parity distributions of fertility are nothing but single points. In contrast, Model V deals with a large group of families; it envisions a situation in which, given the same values of the exogenous variables, some families will have many children, some will have a few, and some will have none at all. Since marked changes over time have been observed in parity distributions of fertility in the U.S., it would be interesting to discover if the implications of Model V for changes in the parity distribution accord with the observed changes. This is a new and complex topic whose full treatment would require a separate monograph. Rather than treating it here, the derivation of these implications shall be deferred until after the observed changes in parity distributions have been discussed in Chapters 3 and 4.

Another possible line of development of Model V is toward a reinterpretation of the Easterlin-Fuchs intergenerational relative income hypothesis. Since Model V allows for the consideration of discrete changes in exogenous variables, we may use it as an intergenerational model of fertility. Let us consider two situations, the first representing the lifetime economic conditions facing the parent generation and the second representing the lifetime economic conditions facing their children's generation. We assume that the husbands' real income and the wives' real wage rate are higher in the children's generation than in the parents;, but that the wives' time constraint is identical in both generations. The production possibilities frontiers for the parents' and the children's generations are shown below in Figure 15. As it stands, this intergenerational model of fertility has no implications for fertility either for women who are in the labor market or for those who are not.

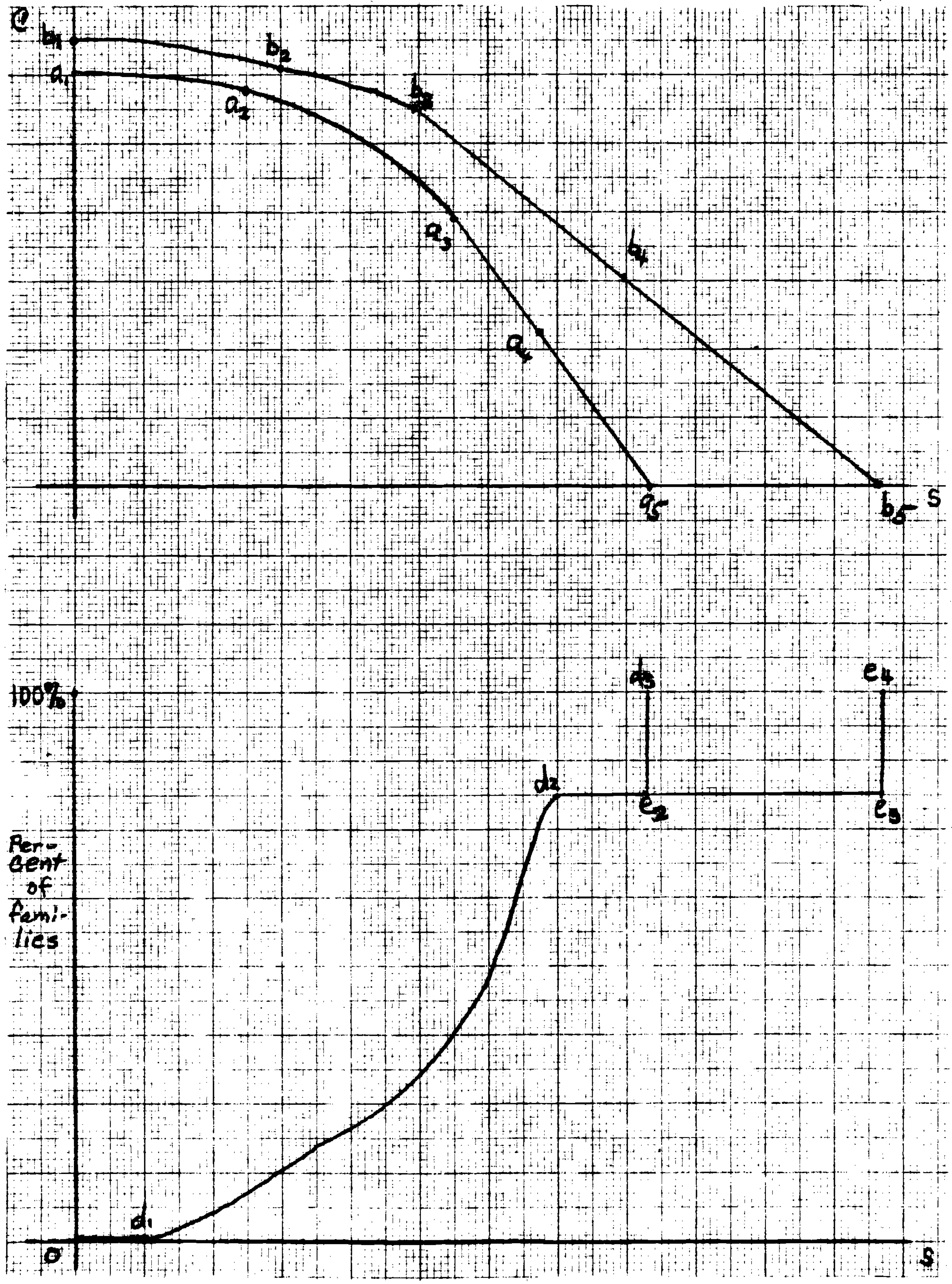
Figure 15


C
b_1
a_1
b_2
Childrens' generation
a_2
parents' generation
a_3
b_3
S
100%
d_3
e_3
Percent of families
d_2
e_2
parents' generation
childrens' generation
o
d_1
e_1
S

The reason for this lack of implications is not because of a change in tastes, for in Model V this would have to be manifested by a violation of the Normality Assumption stated above. Rather, with the Normality Assumption inviolate, the absence of implications arises because increases in husbands' income and the wives' wage rates generate conflicting income and substitution effects, both in the case where the wife is in the labor market and in the case where she is not.[82] Must the model be discarded as unilluminating, or is it possible to obtain from it some reasonably firm implications about fertility behavior? In order to obtain some implications, it is necessary to say something about the distributions of c and s and it is here that the intergenerational relative income hypothesis plays a role.

The Easterlin-Fuchs intergenerational relative income hypothesis suggests that young people's tastes and expectations regarding their material standards of living are formed when they are adolescents in their parents' households. For ease of exposition let us consider an extreme form of this hypothesis which states that the distribution of s in the children's generation depends the economic conditions they experienced as adolescents, but not on their own economic conditions.[83] If the economic conditions of the children's generation differ from what they expected on the basis of their adolescent experiences, the entire difference is absorbed by changes in their production of child-services. Clearly in this extreme form, the Easterlin-Fuchs hypothesis is analogous to Friedman's permanent income hypothesis,[84] with child-services playing the role of savings in the Friedman model.

Figure 16


C
b_1
b_2
a_1
a_2
b_3
a_3
b_4
a_4
a_5
b_5
S
100%
e_1
e_4
d_2
e_2
e_3
Per-cent of fami-lies
d_1
O
S

However, even given the extreme version of the Easterlin-Fuchs hypothesis Model V is short on implications for intergenerational fertility differentials. In order to see this let us consider the following conceptual experiment. Let us consider dividing a cohort of adults who have had the same adolescent background into two groups, one of which experiences lower economic growth and therefore faces the lifetime constraint a_1a_5 and the other experiences more rapid economic growth and therefore faces the lifetime constraint b_1b_5. This situation is shown in Figure 16. If the distribution of s is the same for both groups because their adolescent experiences were identical then the jth (kth) family (in the distribution of s) would consume at $a_2(a_4)$ given the lower rate of economic growth and at $b_2(b_4)$ given the higher rate of economic growth. Nonetheless, the model has no implications for fertility since without further restrictions it is impossible to tell whether fertility is higher or lower at $b_2(b_4)$ than at $a_2(a_4)$ since not only is (c) higher at $b_2(b_4)$ than at $a_2(a_4)$ but the wives' wage rate is also higher at $b_2(b_4)$ than at $a_2(a_4)$. The positive income effect and the negative relative price effect offset one another leaving no *a priori* sign for the change in fertility.

Thus even with the addition of the Easterlin-Fuchs hypothesis in its extreme form Model V is not sufficiently articulated to produce implications for intergenerational fertility changes. Clearly, additional specifications must be added to the model if it is to produce such implications. But which ones? Perhaps the best approach would be to consider known temporal and cross-sectional fertility variations and to try to find restrictions on the model which cause it to reproduce those patterns of variation. However, it was our original intent to discover what economic models of fertility told us about time series movements in fertility. Clearly, we have come full circle. In order to make our models better, we must have a better and more detailed knowledge of the temporal behavior of fertility.

FOOTNOTES

1. Becker's contributions to the economic theory of fertility include: (i) "An Economic Analysis of Fertility," in Demographic and Economic Change in Developed Countries, Universities-National Bureau Conference Series, 11, Princeton, N. J.: Princeton University Press, 1960; (ii) "A Theory of the Allocation of Time," Economic Journal, 75 (September 1965), pp. 493-517; and (iii) with Gregg Lewis, "On the Interaction Between the Quantity and Quality of Children," Journal of Policital Economy, Vol. 81, No. 2, Part II, March/April 1973, pp. S279-S288.

2. Mincer, Jacob, "Market Prices, Opportunity Costs, and Income Effects," in Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld, C. Christ et al, eds., Stanford, California: Stanford University Press, 1963.

3. Willis, Robert J., "A New Approach to the Economic Theory of Fertility Behavior," Journal of Political Economy, Vol. 81, No. 2, Part II, March/April 1973, pp. S41-S16.

4. Easterlin's contributions to an economic understanding of fertility variations include: (i) Population, Labor Force, and Long Swings in Economic Growth: The American Experience, General Series 86, New York: National Bureau of Economic Research, 1968; (ii) "Towards A Socio-Economic Theory of Fertility: A Survey of Recent Research on Economic Factors in American Fertility," in Fertilty and Family Planning: A World View, S. J. Behrman, Leslie Corsa, Jr., and Ronald Freeman, Ann Arbor: University of Michigan Press, 1969.

5. For example, see Grabill, Kiser and Whelpton (1958), pp. 274-277. and Kiser, Grabill and Campbell (1968), pp. 208-212.

6. There are numerous findings U-shaped fertility-income relationships. An interesting relatively early example can be found in Ruggles and Ruggles (1960). A more contemporary observation can be found in Bernhardt (1972).

7. Leibenstein (1952) and Leibenstein (1957).

8. Okun (1958).

9. Leibenstein (1957), p. 161.

10. Leibenstein (1957), p. 162.

11. Leibenstein (1957), p. 162.

12. Particularly Easterlin (1961), Easterlin (1966) and Easterlin (1968).

13. Easterlin (1961).

14. Easterlin (1966).

15. This inverse relationship has been explored in a number of studies. Particularly, Easterlin (1961) and Thomas (1972).

16. This hypothesis was first suggested in a remarkably prescient analysis by Fuchs (1956).

17. Becker (1960).

18. Becker calls average real expenditure per child "child quality". Generally, definitions are not subject to arguments pro and con but this particular definition, I believe, is particularly misleading. Among the general populace child quality is usually associated with health, beauty, talent, certain pleasing personality traits, and other such

18. (continued)

characteristics of children to which the contribution of real expenditures is often quite minimal. Moreover, it is not generally accepted that average real expenditure per unit is a particularly good measure of quality for durable goods in general. Suppose two people bought identical automobiles one of which worked beautifully and the other of which was a lemon. The person who purchased the lemon is likely to spend considerably more money on his auto than the person who bought the car which functioned perfectly. Yet, we are hardly likely to say that the person who bought the lemon had the higher quality automobile just because his real expenditure per unit was higher than that of the other person. Indeed, we would say just the reverse.

Let us consider another example of how Becker's definition of child quality could be misleading. Suppose someone had a professional photographer come to his home to take photographs of his children and his automobile. Most people would not think that either the quality of the car or the quality of the children would be enhanced thereby. Nonetheless, Becker's definition of child quality would lead us to say that the quality of the children and the quality of the automobile increased because their pictures were taken. In footnote 38, we argue that the confusion in terminology grows even worse in the context of models of household production.

Economists have no monopoly on the study of fertility and if they are to interact fruitfully with people in other disciplines it is incumbent upon them to keep <u>nonessential</u> semantic difficulties to a minimum. For this reason, I do not accept Becker's definition of child quality and until something better comes along whenever I wish to say "average real expenditure per child" I shall use the inelegant phrase "average real expenditure per child."

19. Becker (1960), p. 214.

20. Becker and Lewis (1973).

21. Becker and Lewis (1973), p. S281 fn. 1.

22. It is assumed in Model II that the price index for the child expenditure bundle is invariant with respect to birth order. This is not a crucial assumption of the model and its implications remain intact even if the price index varies with parity.

23. A normal good is defined as one for which the demand increases as income increases if all the relevant relative prices are held constant. There is one exception to this rule. If there was none of a good consumed before and after an income increase, it still may be considered a normal good.

24. The reason that the _two_ constraints both pass through the same point is that $R^{(2)}$ was _chosen_ such that the point of tangency of equation (4) with an indifference was a point on equation (5).

25. The shadow prices of n and e relative to the shadow price of s are $p_c e/p_s$ and $p_c n/p_s$ respectively. Since $e^{(2)} > e^{(1)}$ and $n^{(2)} > n^{(1)}$, both relative shadow prices must be higher at $(n^{(2)}, e^{(2)}, s^{(2)})$ along equation 5 than at $(n^{(1)}, e^{(1)}, s^{(1)})$ along equation 2.

26. By pure cross-substitution effect of a price change, we mean the

26. (continued)

effect on other goods of changing the price of one good while holding utility constant.

27. Another way to understand why even changes in p_c have an ambiguous effect on fertility in the Becker model is to notice that if the pure price elasticities with respect to simultaneous proportional changes in the shadow prices of n and e are not identical then the differences in the price elasticities cause induced relative shadow price changes between n and e which could offset the initial price change.

28. If both goods in the Becker model had a quality dimension then there would be four arguments in the utility function, the quantity and quality of each of the two goods. In addition, there would be three endogenous shadow price ratios as opposed to two in the simpler model. There would also be considerably greater difficulties because the ambiguity is compounded by the interaction of yet another variable about which little is known <u>a priori</u>.

29. This result can simply be obtained by adding the first two equations labeled (A13) in Becker-Lewis (1973), p. S286.

30. For example see Berkner and Mendels (1973).

31. The most common form of differentiation between siblings was on the basis of sex. Traditionally, only males were sent to college while their sisters either married or went out to work.

32. Willis (1973).

33. Muth (1966).

34. Becker (1965).

35. Becker and Lewis (1973).

36. Models IV and V below are such models. The discussion of Model IV begins on page 34.

37. Assumption ii) may be restated as follows: given any values of the wife's wage rate and the price of goods the ratio of wife's time to market goods used in the production of c is greater than the ratio of wife's time to goods used in the production of s.

38. Willis follows Becker and calls e child quality. However, this term is quite misleading in the context of a model of household production. Although it is convenient to speak of household production functions and a household production structure, the activities of the household which are represented in this fashion partake both of the character of production activities and consumption activities. Suppose parents take their children to a park on a Sunday for the purposes of admiring the scenery, drinking in some sunshine and playing with their children in the park environment. Some of the value of the parents' time spent at the park would be correctly allocated to the childservices production function, as would a portion of the cost of transporting the family to the park. Nonetheless, it is quite confusing to claim that child quality has increased just because the parents have played with their children. We do not claim that the quality of phonograph records increases as they are played more and more. We do not claim that the quality of an automobile increases as it is driven more miles. Why should we assume that all

38. (continued)

time and goods spent on children increase child quality? But then again perhaps children are not like other consumer durables. See fn. 18.

39. The model is meant to be a one-period lifetime model of fertility where T is the length of a couple's married life together.

40. To avoid confusion, the wife's market wage rate is denoted w' and the wife's shadow wage rate when she does not participate in the labor market is denoted w^*. In those contexts where either wage concept may be appropriate the symbol w is used.

The implicit assumption is that the wife is free to work any number of hours she chooses at the market wage rate w'. If the wife's wage rate rose as her lifetime hours of work rose, this would complicate the model, but would not change its thrust. See Willis (1973), p. S38.

41. The derived demand functions are simply functions which tell us the cost minimizing input mix for any quantity of output and any set of relative factor prices. In the case of production functions which are homogeneous of degree one, these derived demand functions must be multiplicative in output and a function of relative prices. If it were not mutiplicative then the ratio of factor inputs would depend on the scale of output and that cannot be the case for any homogeneous production function.

42. It must be true that g_t, g_x, h_t, and h_x are homogeneous of degree zero because the cost minimizing quantities of inputs demanded depends only on relative prices not the absolute level of prices. This is just one of the restrictions on the functions of the input prices.

42. (continued)

Another restriction, for example, would be that $g_t(w^{(1)},p_x) \leq g_t(w^{(2)},p_x)$ whenever $w^{(1)} > w^{(2)}$. Clearly there are more restrictions on these functions, but this need not concern us here.

43. See page 14.

44. In the Becker model I, p_c, and p_s are fixed exogenous variables. In the Willis model the exogenous variables are V, w', and p_x. The values of I, p_c and p_s in that model depend on the values of V, w, and p_x and in that sense are endogenous.

45. The constraint facing the household may be written as

$$(1) \quad V + w'T = s(w'g_t(w',p_x) + p_xg_x(w',p_x)) + c(w'h_t(w',p_x) + + p_xh_x(w',p_x)$$

if the wife is in the labor force. If the wife is not in the labor force the following two constraints must be satisfied simultaneously.

$$(2) \quad V = sp_xg_x(w^*,p_x) + cp_xh_x(w^*,p_x)$$

and

$$(3) \quad T = sg_t(w^*,p_x) + ch_t(w^*,p_x) \quad .$$

46. An expression for the wife's hours of labor market work may be derived from equations 13, 15, 17, and 22. First, solving for c in equation 22 we obtain

$$(1) \quad c = \frac{V+w'T-s(w'g_t(w',p_x)+p_xg_x(w',p_x))}{w'h_t(w',p_x)+p_xh_x(w',p_x)} \quad .$$

46. (continued)

Substituting the values of t_c and t_s from equations 15 and 17 into equation 13 and solving for t_ℓ we obtain

$$(2) \quad t_\ell = T - sg_t(w',p_x) - ch_t(w',p_x) \quad .$$

Substituting the value of c derived above in equation 1 into equation 2, we obtain the following expression for t_ℓ

$$(3) \quad t_\ell = \frac{1}{w'h_t(w',p_x)+p_xh_x(w',p_x)}[T(p_xh_x(w',p_x)) - V(h_t(w',p_x)) + + sp_x[g_x(w',p_x)h_t(w',p_x) - h_x(w',p_x)g_t(w',p_x)]] \quad .$$

Now, we can answer the question of whether t_ℓ increases or decreases with s. The wife's time in the labor force must increase as s production increases (holding w' constant) because the coefficient of s in equation 3 must be positive. The positivity of the coefficient of s would follow if

$$(4) \quad g_x(w',p_x)h_t(w',p_x) - h_x(w',p_x)g_t(w',p_x) > 0 \quad .$$

However, equation (4) must be true because of the time intensity assumption made on page 29. The time intensity assumption requires that

$$(5) \quad \frac{h_t(w',p_x)}{h_x(w',p_x)} > \frac{g_t(w',p_x)}{g_x(w',p_x)}$$

which is sufficient to prove (4).

46. (continued)

It is also useful to note at this point that the value of s at point a in figure 8, the point at which the wife stops (starts) working in the labor market, may be expressed

$$(6) \quad \bar{s} = \frac{Vh_t(w',p_x)-Tp_x h_x(w',p_x)}{p_x[g_x(w',p_x)h_t(w',p_x)-h_x(w',p_x)g_t(w',p_x)]} \quad .$$

47. See Samuelson (1949).

48. See pages 40-42.

49. See footnote 37.

50. See footnote 23.

51. It is not strictly necessary in Model IV to assume that the production functions for childservices are independent of parity. Hicks-nuetral technological differences between production functions for different parities can be assumed without altering the analysis. The assumption made in the text that production functions for childservices are independent of parity is made purely for expositional ease.

52. The flows of childservices from the m possible children may be aggregated into a single composite commodity c because since the shadow price of childservices does not vary across parities, the flows $c_1, \ldots , c_m$ meet the conditions for Hicks' composite commodity theorem. See Hicks (1962).

53. The relative price of s and c only depend on the ratio of the wife's wage to the price of goods. If the latter is constant so is the former.

54. See pages 22 and 23.

55. Rybczynski (1955).

56. It is important to note that Rybczynski's Theorem holds in particular for the points of transition between the linear and nonlinear portions of the constraint. An algebraic representation of s at the point of transition, which illustrates Rybczynski's Theorem, can be found in equation 6 in footnote 46.

57. This line of reasoning utilizes the fact that along the nonlinear portion of the constraint p_s/p_c falls as a increases. See Samuelson (1949).

58. See footnote 57.

59. Increasing w' relative to p_x must cause p_c to rise relative to p_s if c production is more time intensive than s production. See Samuelson (1949).

60. Willis (1973), pp. S41-S53.

61. This familiar result is proved in the context of Model V on page 61.

62. This familiar result is proved in the context of Model V on page 54.

63. For example Ruggles and Ruggles (1960), Table A2.

64. See Bumpass and Westoff (1970) for a brief discussion of the prevalence of "unwanted" children.

65. Blake (1968). See particularly pp. 15-17.

66. This has already been done in a study by Michael and Willis (1973).

67. Sanderson (1974).

68. The distribution of s is implied by the distribution of c and the constraint. In figure 11, d_1 is the proportion of families who have no children. Since as figure 11 is drawn the distribution

68. (continued)

of c reaches 100% at d_2, it indicates that no families consume more childservices than c^* nor less s than s^*.

69. By situation i, we simply mean some fixed configuration of income and prices. Situation 1 may differ from situation 2, for example, if the wives' wage rates differ.

70. A point $(Z_{1k}^{(i)}, Z_{2k}^{(i)}, \ldots, Z_{j-1,k}^{(i)}, Z_{j+1,k}^{(i)}, \ldots, Z_{nk}^{(i)})$ is infeasible in situation i if given the resources available in situation i, it is impossible to produce those quantities of the household commodities.

71. Since, in reality, we usually deal with a finite number of families, we have written the normality assumption in terms consistent with this observation. However, it usually is more aesthetically pleasing to draw the figures on the basis of a continuous distribution of families. To put the normality assumption in terms of continuous distribution functions, let us consider two distribution functions $F^{(1)}(\ )$ and $F^{(2)}(\ )$ defined over the closed interval $[o,a]$. The normality assumption may now be restated:

> if the opportunity to consume Z_j is greater (smaller) in situation 2 than in situation 1 then $F^{(2)}(Z_{jk}) \leq F^{(1)}(Z_{jk})$ $[(F^{(2)}(Z_{jk}) \geq F^{(1)}(Z_{jk})]$ for $o \leq Z_{jk} \leq a$. In addition, there exists some value of Z_j in the interval $[o,a]$, say Z_{jk}^* such that $F^{(2)}(Z_{jk}^*) < F^{(1)}(Z_{jk}^*)$ $(F^{(2)}(Z_{jk}^*) > F^{(1)}(Z_{jk}^*))$.

In this definition $F^{(i)}$ is the distribution function for situation i.

72. It is not beyond belief that, holding other inputs into childservices fixed, the number of children which parents have is such that the contribution of an additional child to the production of childservices would be negative.

73. In order to understand why $\Phi(\ )$ and $\Psi(\ )$ must be strictly increasing functions of their arguments, let us consider the implications of that not being so. Suppose $s_1 > s_2$ and $\Phi(s_1) = \Phi(s_2)$. That would imply that it is possible to produce more output with no increase in inputs which would be inconsistent with G being a production function.

74. See pages 37 and 38.

75. All the following figures are drawn on the assumptions that i) the wives' labor force participation rate lies between zero and unity, and ii) the distribution function never has a slope of zero except at 100%.

76. In the case of a finite number of discrete households it is possible that there exist sufficiently small changes in the exogenous variables such that the number of women in the labor force does not change.

77. Generally speaking, since the ordering of the families can change between situation 1 and situation 2, the same women need not be in the labor force in both periods. However, since people only have one lifetime, it is impossible to observe the same family before and after changes in lifetime exogenous variables.

78. This may be clearly seen from equation 3 in footnote 46.

79. When V increases the opportunity to consume c increases and therefore, by the Normality Assumption $c_j^{(2)} \geq c_j^{(1)}$ for $j = 1, \ldots, m$.

80. We must have $w_k^{(2)} < w_k^{(1)}$ since $\frac{p_s^{(2)}}{p_c^{(1)}} < \frac{p_c^{(1)}}{p_c^{(1)}}$ (see the discussion on pages 37-39) and since there is a one to one positive association between the wives' shadow wage and the relative price of c See. Samuelson (1949).

81. The distribution beneath the portion of the constraint which does not shift because if it did it would necessarily violate either the Normality Assumption as it applies to c or as it applies to s.

82. Below, the conflicting income and substitution effects which arise when the wife is out of the labor force have been discussed in some detail. Conflicting income and substitution effects now arise even when the wife is in the labor force because we have assumed that the children's generation has both a higher husband's income and a higher wives' wage rate than the parents' generation.

83. In essence this assumption states that the distribution of s in the childrens' generation is invariant to their own economic conditions.

84. Friedman (1957).

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