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ON THE DIFFERENCE BETWEEN TAX AND SPENDING POLICIES IN MODELS WITH FINITE HORIZONS

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ABSTRACT

This paper uses the Blanchard (1985) finite horizon model to study how taxes and government spending can be managed to stabilize aggregate demand.

It is shown that tax policy cannot stabilize demand in less time than it stabilizes the public debt, but that, if government spending is the instrument of policy, demand can be stabilized independently of the dynamics of the debt. These results imply that if the objective is to stabilize the debt while maintaining demand as close as possible to a pre-determined target path, and taxes are the instrument, taxes would have to be changed temporarily as much as feasible. On the other hand, if the instrument is government spending, it can be changed gradually to achieve the objectives.

The dynamic effects of taxes are a straightforward implication of the intertemporal budget constraint, when it is assumed that agents cannot be surprised by government policies. More traditional dynamics can be obtained if it is assumed that the government succeeds in announcing a policy and implementing a different one. If however the announcement is no credible, discretion is inferior to a predetermined tax rule.

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ON THE DIFFERENCE BETWEEN TAX AND SPENDING POLICIES IN MODELS WITH FINITE HORIZONS

William H. Branson and Giampaolo Galli

1. Introduction and Summary

It is well known that a sufficient condition for tax policy to affect consumption is that agents' horizon be shorter than that of government. In this case a decrease in taxes today boosts consumption of the present generation because some other generation will pay the higher taxes which will have to be raised in the future to finance the deficit. If agents' life is stochastic, consumption will be affected because there is some probability that those currently living will not be there to pay for future taxes.

The main implication of these propositions for macroeconomic modelling is that the time pattern of consumption depends on the time profile of taxes and of the public debt. This fact has of course been recognized at least since Modigliani and Brumberg laid down the life-cycle theory of household behaviour in 1954 and Ando and Modigliani (1963) estimated the resulting equation for consumption expenditure.

A difficulty that has always been recognized with the life cycle theory concerns aggregation; even with the most convenient assumptions concerning individual behaviour, exact aggregation over agents of different ages proves to be an impossible task. Blanchard (1985) solved the problem of aggregation by assuming that every period each agent faces the same constant probability of death. While this assumption has its limitations, Blanchard's finite horizon model provides an interesting framework to inquire with a new blend of rigor into issues which have long been central to macroeconomic theory and policy. In a recent paper Galli and Masera (1988) have used an open economy version of Blanchard's model to answer the following question. Suppose an exogenous shock (e.g. an increase in the world rate of interest) causes the public debt to start rising: does a combination of fiscal and monetary policy exist that stabilizes the public debt while keeping income along some predetermined path (so as to avoid both unemployment and inflation)?

The core of the problem is the following. Aggregate demand is a function of both the flows (of taxes and government spending) and the stock of the debt. When the latter starts to rise and the government reduces spending or increases taxes in order to meet the solvency requirement there are two conflicting forces on the level of activity originating from a rising stock and a declining flow. Galli and Masera show that a under rather general conditions it is possible to reduce <u>government spending</u> at a rate which just compensates the effects of a growing debt on the level activity. The ensuing policy prescription is that spending be reduced gradually.

The work of Galli and Masera focussed on government spending as the instrument; nothing is said about taxes, which are held constant in the analysis. It turns out that in this model the economics of tax policy is very different from that of public spending. The effects of these two policies differ for reasons which go well beyond those popularized by Haavelmo's (1945) analysis of the balanced budget multiplier.

The basic point that we make here is that there exists no tax policy that can decouple the growth of consumption from that of the debt. No matter how fast taxes are <u>increased</u>, consumption continues to rise as long as the stock of debt rises; in other words in this model, independently of the values of the parameters, stock effects always dominate.

In order to maintain income as close as possible to a predetermined path in the face of shocks, taxes must temporarily be changed as much as is feasible so as to quickly stabilize the debt. In formal terms, the solution to the problem of minimizing a loss function in the deviation of income from target is to achieve a discrete change in the stock of debt, which requires an infinite instantaneous flow rate of taxation. This policy allows the authorities to immediately attain the target. In economic terms this means changing the stock of the debt by the stroke of a pen. Even if one rules out extreme solutions, it still remains that shock treatment, rather than gradualism, is the prescription of this model when taxes, rather than spending, are the instrument of policy.

The following proposition further highlights the difference between tax and spending policies. When government spending is increased, there exists no sequence of present and future taxes of finite size that can avoid fluctuations of the level of activity. When spending is increased, the debt starts to rise: until it reaches a position of rest, consumption continues to rise. If taxes are raised as much as spending so as to keep the budget in balance, income rises as in Haavelmo's model. In order to avoid the increase in income, taxes must be increased by more: the debt then starts to fall and so does consumption until the system has reached a new steady state. In substance, if taxes are raised so as to keep the debt constant, income varies; if they are raised by the amount that is necessary to stabilize income, the debt starts to vary and so do consumption and income.

The sharp asymmetry between the dynamic effects of tax and spending policies in this model is, in our view, rather puzzling and worth some investigation in order to understand its economic significance and the extent to which it is specific to the chosen model.

In section 2 we set up the simplest version of the model which is necessary to obtain the results: the economy is closed, labour is the only factor of production and prices are fixed. In section 3 we derive the basic results. In section 4 we give an economic interpretation of the results. The basic suggestion is that these results are straightforward implications of the intertemporal budget constraint when agents know the policy rule and expectations about future non-interest income are consistent with the model. More traditional dynamics can be obtained in two ways. The first one is to introduce liquidity constraints: in this case the intertemporal budget is not the relevant constraint for the maximization problem. This approach has been extensively developed in the literature.

The second possibility is to assume that agents can be surprised by government policies. We develop this second approach and construct an example in which agents may have incorrect forecasts about taxes but are rational in the sense that they compute future income in a way which is consistent with the model subject to the expected policy. It is shown that, given a loss function in the deviation of income from target, a tax policy exists that hits the target in every period. Furthermore, under these assumptions, it is possible to derive an expression which closely resembles the consumption function estimated by Ando and Modigliani (1963).

In section 5 we revert to the hypothesis that agents know the future policy and characterize the dynamic behaviour of the economy under the two assumptions of precommittment and discretion. With precommittment, the loss function, while positive, attains a lower value than under discretion because the government decides its strategy using the full set of expectational constraints implied by the model.

The main point is that the assumption of time consistency acts as a constraint on the public authorities which, although not powerless as in a Ricardian world, still cannot achieve the full range of macroeconomic objectives which traditional analysis associates with tax policy.

This conclusion is much more general than the model which is used in this paper. It holds as well if investment and a foreign sector are introduced or if prices are assumed to be market clearing. In different or more complex models, the lack of controllability of the demand for consumption will be reflected on different variables or on the same variables in different ways.

2. The Theoretical Framework

We consider the simplest IS-LM economy in which labour is the only factor of production and the price of goods in terms of money is fixed and normalized to 1. The government budget constraint is explicitly considered and states that current expenditure (including interest) is financed through taxes, short term bonds or money. The only complication relative to the text-book model concerns the consumption function whose derivation follows Blanchard (1985).

Denote by $c(s,t)$, $y(s,t)$, $m(s,t)$, $w(s,t)$, $h(s,t)$ consumption, non-interest income, money balances, assets (money plus short term government bonds) and human wealth of an agent born at time s, as of time t. Let $r(t)$ and $\tau(t)$ be the interest rate and lump sum taxes at time t. p and θ denote the probability of death and the discount rate and are both constant. Under the assumption that the instantaneous utility is logarithmic, the agent maximizes

$$(1) \qquad E_t \int_t^{\infty} \left[\alpha \ln c\,(s,v) + (1-\alpha) \ln m\right] e^{\theta(t-v)} dv \;.$$

The individual has a contract with an insurance company according to which the company inherits the agent's wealth (including money) in exchange for the payment of a sum pw(s,t) to the agent while he is alive. The dynamic budget constraint of the individual is thus

$$(2) \quad \frac{dw(s,t)}{dt} = [r(t)+p]w(s,t)+y(s,t)-\tau(t)-c(s,t)-r(t)m(s,t) .$$

The term r(t)m(s,t) is subtracted from the RHS of (2) because only the bond component of w(s,t) yields interest. Subject to the appropriate transversality condition, the solution to this problem from the first-order conditions when the only uncertainty concerns the time of death can be written as

$$(3) \quad c(s,t) = \alpha(\theta+p)[w(s,t)+h(s,t)] , \text{ and}$$

$$(4) \quad m(s,t) = \frac{(1-\alpha)(\theta+p)}{r(t)} [w(s,t)+h(s,t)] ,$$

where h(s,t) is human wealth defined as

$$(5) \quad h(s,t) = \int_t^{\infty} [y(s,v)-\tau(v)]^{-\int_t^v [r(\mu)+p]d\mu} dv .$$

Aggregation over consumers can be done the hard way as in Blanchard (1985) or in the following way. If no agents died at time t (in which case the insurance industry would go brankrupt) the evolution of aggregate wealth would be given by (2) with aggregate variables substituting for individual ones. However every period p individuals die; since the probability of death is independent of age, p will also be the fraction of aggregate wealth that is tranferred to the insurance industry. Hence in aggregation the term pW(t) (upper case letters denoting aggregates over consumers) must be subtracted from the budget constraint, which becomes:

$$(6) \quad \overset{\circ}{W}(t) = r(t)W(t)+Y(t)-\tilde{T}(t)-C(t)-r(t)M(t) ,$$

where $\tilde{T}(t)$ is total tax payments.

Similar reasoning can be applied to human wealth. In this case, however, the fraction of wealth that vanishes because p individuals die every period is immediately replaced by that of p individuals who are born. The replacement is one to one because, by assumption, income and taxes are evenly distributed among consumers of different age. The evolution of aggregate human wealth is hence

$$(7) \quad \overset{\circ}{H}(t) = [r(t)+p]H(t)-[Y(t)-\tilde{T}(t)] .$$

Aggregate money demand and consumption will be given by

$$(8) \quad C(t) = \alpha(\theta+p)\ A(t) \text{ , and}$$

$$(9) \quad M(t) = \frac{(1-\alpha)(\theta+p)}{r(t)} A(t) .$$

where $A(t)$ is total wealth defined as

$$(10) \quad A(t) = H(t) + A(t) .$$

The model is closed by the goods market equilibrium condition

$$(11) \quad Y = C + G ,$$

where G is government consumption. We assume that the central bank intervenes in the bond market so as keep the interest rate constant ($r(t) = r$ for all t); this assumption allows us to concentrate on movements of the IS schedule of the system neglecting the feedbacks from the LM. We finally write

(12) $\tilde{T}(t) = T(t) + \gamma W(t)$.

Eq. (12) is a convenient way to parameterize a policy reaction function for lump sum taxes. In the preliminary analysis of this paragraph we set T(t) constant for all t: in this case eq. (12) states that taxes are increased as the level of the public debt, W(t), rises. In the following sections we will need to consider more general paths for T(t).

To characterize the dynamic behaviour of the system, we sum (6) and (7) using (8), (9) and (10): this yields

(13) $\overset{\circ}{A} = (r-\theta)A-pW$

Using (8), (9), (11) and (12), eq. (6) becomes

(14) $\overset{\circ}{W} = -(\gamma-r)W+G-T-(1-\alpha)(\theta+p)A$.

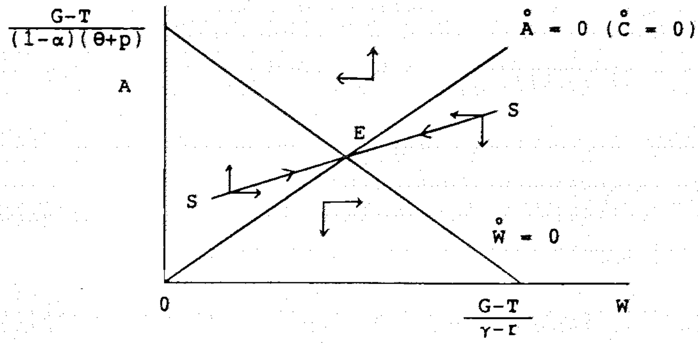
On the system (13) and (14), we impose

(15) $r - \theta > 0$, and

(16) $\gamma - r > 0$.

Later in this section we will discuss the reasons why these restrictions are imposed. For the moment, we take them as given and describe the phase diagram of the system (Fig. 1)

Fig. 1



The $\mathring{A} = 0$ line is the locus along which total wealth (A) is constant; since from equation (8) consumption is proportional to A, along this schedule consumption is constant as well. The equation for this schedule is obtained by setting $\mathring{A}$ equal to zero in eq. (13) and solving for A:

$$(17) \quad A = \frac{p}{r-\theta} W \, .$$

Eq. (17) has intercept at zero and a positive slope (since $r-\theta>0$). The partial derivative of $\mathring{A}$ with respect to A is positive: therefore the direction of motion around the $\mathring{A} = 0$ schedule is unstable. This is represented in Figure 1 by vertical arrows pointing away from the $\mathring{A} = 0$ line.

The downward sloping line labelled $\mathring{W} = 0$ represents the combinations of A and W which maintain the debt constant. From eq. (14), an increase in the debt reduces $\mathring{W}$ because of the assumption that taxes are increased by a factor γ which is greater than the rate of interest. Also an increase in A reduces $\mathring{W}$. From eq. (9) the term $(1-\alpha)(\theta+p)A$ on the RHS of (14) is equal to rM: an increase in A shifts the financing of the deficit from bonds to money, thus allowing the government to save on interest payments. Setting $\dot{W} = 0$ in equation (14) yields the expression for the constant debt line:

$$(18) \quad A = -\frac{1}{(1-\alpha)(\theta+p)}\left[(\gamma-r)\,W + G - T\right] .$$

Around the $\dot{W} = 0$ line, the direction of motion is stable: horizontal arrows point towards the line.

In conclusion, the dynamics of the system is saddle point stable: the saddle path is line SS in the figure.

We now explain why we impose restriction (15) and (16). Restriction (15) ensures that we are considering cases in which the public debt is positive in the steady state. This is seen from eq. (17): if consumption and therefore A are positive, the debt is positive only if $r-\theta>0$. We could of course consider the case in which the government is a net creditor in steady state, but this is a less interesting case and will be neglected.

Restriction (16) is sufficient, but by no means necessary, to ensure that the system be a saddle point stable. Solving for W from (17) and (18) yields

$$(19) \quad W = -\frac{r-\theta}{\Delta}(G-T) ,$$

where Δ is the determinant of the system:

$$(20) \quad \Delta = -(\gamma-r)(r-\theta) - p(1-\alpha)p(\theta+p) .$$

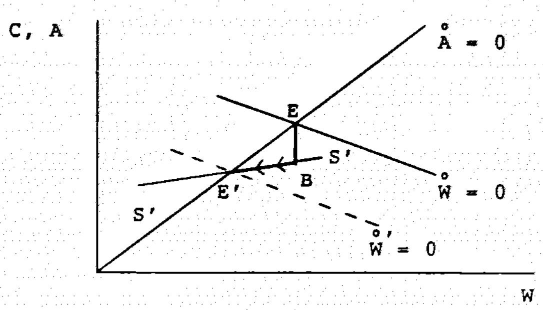
Saddle point stability requires that the determinant be negative. Hence for W to be positive, G-T must also be positive. When $\gamma=0$, G-T is the primary deficit; if it is positive the debt may be stable only if in steady state the government <u>receives</u> net interest from the private sector, i.e. money is larger than wealth (α small in (20)) which implies that bonds are negative. If γ is positive the system may both be stable and have positive bonds in steady state (for which a necessary and sufficient condition is $\gamma > -\frac{\Delta}{r-\theta}$) ; if it is greater than r, Δ is negative and the system is saddle point stable regardless of the value of α (including the case when $\alpha=1$ and our unbacked currency has no value, as in Sargent, 1987, chapt. 4.1).

3. The Basic Results

The system defined by eqs. (13) and (14) is invariant with respect to equal changes in G and T. However when G and T are increased by equal amounts income rises from eq. (11), the goods market equlibrium condition. The balanced budget multiplier is 1 as in the simple textbook model (on this point see also Rankin (1987) and Frenkel and Razin (1987)).

If T rises more than G, the $\mathring{W} = 0$ schedule shifts down as in Figure 2.

Fig. 2



Since expected net non-interest disposable income falls, the human wealth component of A immediately falls from E to B. Over time the public debt shrinks and so do total wealth and consumption.

If at point B income is unchanged relative to point E (because taxes have been raised in such a way as to cause a fall in consumption which equals the increase in G), it will necessarily be falling further from point B to the new steady state E'.

This example shows that a once and for all change in T does not stabilize income continuously when G is changed. This point can be made more general in two ways. First a single change in T does not stabilize income, in the face of any shock (interest rate, α, p, θ). This can easily be checked graphically. The more interesting point is there exists no pattern of T(t) which can stabilize income continuously; furthermore the time it takes to stabilize income is no smaller than the time it takes to stabilize the public debt. The reverse is not true; the debt may be fixed while consumption and income change.

These points are easily proved. From eq. (13) we know that $\dot{A} = 0$ implies that the level of the debt be constant. Recalling that consumption depends only on A, this means that consumption will always vary unless the debt is constant. Since A is the sum of human wealth plus the debt, another way of stating this point is to say that under no circumstance can the variation of human wealth be equal in size and opposite in sign to that of the debt. To illustrate this point, suppose for a moment that there is no money ($\alpha = 1$): in this case the dynamics of the debt (eq. 13) is independent of A. It may then be thought that it should be possible to have separate control over the dynamics of W and H. <u>Current</u> taxes can be used to control the dynamics of the debt, which is completely independent of future taxes. Given present taxes, it may seem that there should exist a <u>future</u> path of taxes that causes current human wealth to vary, at least for some time, in such a way as to keep the sum of human wealth and the debt constant. Eq. (13) tells us that this is not possible (regardless of whether α is equal or greater than zero).

This point is much more general than the model used in this paper: eq. (13) is derived exclusively from aggregation of the first-order conditions of individual consumers. It holds as well in models with flexible prices and market clearing, or in models in which there is a foreign sector or labour is not the only factor of production. The only thing that would change is the definition of W, which, in more complex models, could no longer be indentified with the public debt.

In this model, an immediate implication is that if G changes, consumption will start to vary over time, unless T is changed by the same amount (so as to keep $\mathring{W} = 0$); if this is done however income increases.

Concerning the sign of the change in consumption, it can be shown that if r, p and θ are constant any fiscal policy which causes the debt not to fall at any time (rise) and to increase (decrease) at least <u>sometimes</u>, will cause consumption to <u>continuously</u> increase (decrease) at all times. This is the following

<u>proposition</u>: let $w(t_0,v)$ be the level of the debt expected for time v, as of time t_0 and $\Delta(v, v+\delta)$ be the change in the debt expected to occur between v and $v+\delta$, i.e.

$$\Delta(v,v+\delta) \equiv W(t_0,v+\delta) - W(t_0,v) .$$

If

$$\Delta(v,v+\delta) \geq 0 \text{ for all } \delta>0 \text{ and } v\geq t_0$$

and $\Delta(v,v+\delta) > 0$ for some $\delta>0$ and $v\geq t_0$,

then

$$A(t+\delta) - A(t) > 0 \text{ for all } t_0 < t < \tilde{v}+\tilde{\delta} \text{ and } \delta > 0 ,$$

where $\tilde{v}+\tilde{\delta}$ is the last period in which wealth rises.

<u>Proof</u>. Integrating (13) forward yields

$$A(t) = \int_t^\infty e^{(r-\theta)(t-v)} pW(t_0,v)dv = \int_0^\infty e^{-(r-\theta)v} pW(t_0,t+v)dv \ .$$

Similarly

$$A(t+\delta) = \int_0^\infty e^{-(r-\theta)v} pW(t_0,t+\delta+v)dv \ ,$$

which can be written as

$$(21) \quad A(t+\delta) = \int_0^\infty e^{-(r-\theta)v} p[W(t_0,t+v) + \Delta(t+v,t+v+\delta)]dv$$

$$= A(t) + \int_0^\infty e^{-(r-\theta)v} p\Delta(t+v,t+v+\delta)dv \ .$$

Since $\Delta(.,.)$ is never negative $A(t+\delta) \geq A(t)$. If t is smaller than the last time in which $\Delta(.,.)$ is positive $A(t+\delta) > A(t)$.

4. Interpretation

The results of the previous section are somewhat puzzling for two reasons.

First, they are at variance with what one can obtain from a traditional consumption function; this is often written in a form like

$$(22) \quad C = c[Y-\tilde{T},W] \ .$$

In (22) there always exist a level and rate of change of $\tilde{T}$ that holds C constant at any desired level when $\overset{\circ}{W} \neq 0$.

Second, from a mathematical point of view, there are as many independent instruments (one tax for each period) as there are targets (income in each period). The mathematical puzzle is easily resolved noting that the target can be attained continuously moving the instruments by an infinite amount: if wealth taxes can be levied (which is the same as setting the flow of T equals to infinity), then wealth can instantly be brought to the desired steady state. A surprise wealth tax (or subsidy) solves the problem of the debt and that of the stabilization of income at the same time.

From the economic point of view, the lack of controllability of the system derives critically from the assumption that agents know future policies.

If we relax this assumption, agents will maximize (1) subject to (2) and aggregate consumption and wealth will still be given by (8) and (14). Human wealth will not however accumulate as in eq. (7). The discounted value of expected net income will have to be written as

$$(23)\quad H(t) = \int_t^{\infty} [Y(t,v) - T(t,v)]e^{(r+p)(t-v)}dv \ ,$$

where $Y(\cdot)$ and $T(\cdot)$ now depend on t, the time when the expectation is taken. Differentiating (23) with respect to t and assuming that the present is known (so that $Y(t,t) = Y(t)$ etc.), yields

$$(24)\quad \overset{\circ}{H}(t) = - [Y(t) - T(t)] + (r+p)\ H(t) + \Psi(t)\ , \text{ where}$$

$$(25)\quad \Psi(t) = \int_t^{\infty} \frac{d}{dt}\ [Y(t,v) - T(t,v)]\ e^{(r+p)(t-v)}dv\ .$$

$\Psi(t)$ represents the revision of expectations as new information comes in. Proceeding as before we can add (6) and (24), using (8) and (9); this yields

$$(26) \quad \mathring{A} = (r-\theta)\,A - pW + \Psi(t) \;.$$

The presence of the surprise term $\Psi(t)$ in (26) suggests that it should now be possible to control the level and the dynamics of A independently of W. We pursue this idea and assume that agents have rational expectations in the sense that they compute future income in a way that is consistent with the model; they may however be wrong in their forecasts of future taxes. Under these assumptions, we can derive a consumption function which is very similar to (22).

The solution is obtained as follows. We first compute the value of A(t) which is consistent with the model, given W(t) and agents expectations about future taxes. In the Appendix, we show that this is given by

$$(27) \quad A(t) = \frac{1}{v_1}\left[W(t)+f(t)\right] \text{, and}$$

$$(28) \quad f(t) = \int_t^{\infty} e^{-\lambda_2(v-t)}[G-T(t,v)]dv \;,$$

where $T(t,v)$ is the expectation of T at time v, as of time t. λ_2 is the unstable root of the system (eqs. (13) and (14)). v_1 is the first element of the right eigenvector associated with the stable root (λ_1)

$$(29) \quad v_1 = \frac{r-\theta-\lambda_1}{p} \;.$$

Since $r-\theta>0$ and $\lambda_1<0$, v_1 is positive.

Equations (27) and (28) determine consumption at each point in time. (13) determines the evolution of the debt.

Next we formulate a rule according to which agents revise their expectations. For instance, suppose that agents believe that T will gradually evolve from its current value to a fixed value $\bar{T}$:

$$(30)\quad \frac{d}{dv} T(t,v) = -\lambda[T(t)-\bar{T}] \;;\quad \lambda > 0 .$$

Since total taxes $\tilde{T}$ are given by T plus the term γW, eq. (30) implies that agents believe that taxes tend toward a value which is given by a constant $\bar{T}$ plus an increasing function of the level of the debt. Integrating (30), substituting the result in (28) and integrating again yields

$$(31)\quad f(t) = \frac{G}{\lambda_2} - \frac{T(t)}{\lambda_2+\lambda} - \frac{\bar{T}\lambda}{\lambda_2(\lambda_2+\lambda)} .$$

If $\lambda = 0$, agents simply consider current T as permanent.

Substituting (31) and (27) into the consumption function yields

$$(32)\quad C(t) = \pi_0 \left[W(t)+\frac{G}{\lambda_2} - \frac{T(t)}{\lambda_2+\lambda} - \frac{\bar{T}\lambda}{\lambda_2(\lambda_2+\lambda)}\right] \text{, where}$$

$$\pi_0 = \frac{\alpha}{1-\alpha}(r-\gamma-\lambda_1) = \frac{\alpha}{1-\alpha}\,\frac{p(1-\alpha)(\theta+p)}{r-\theta-\lambda_1} = \frac{\alpha(\theta+p)}{v_1} > 0 .$$

Substituting (11) and (12) in (32) yields

$$(33)\quad C = \pi_1 W + \pi_2 (Y-\tilde{T}) + \pi_3 (\tilde{T}-\bar{T}) \text{, with}$$

$$\pi_1 = \frac{\lambda_2 \pi_0}{\lambda_2+\pi_0} > 0 \qquad \pi_2 = \frac{\pi_0}{\lambda_2+\pi_0} > 0 \qquad \pi_3 = \frac{\pi_0 \lambda}{(\lambda_2+\pi_0)(\lambda_2+\lambda)} > 0 .$$

With constant parameters and $\lambda = 0$, (33) is a particular functional form of (22) and closely resembles the consumption function estimated in Ando and Modigliani (1963).

The main point is that in (33), as well as in (22), wealth and taxes appear separately so that the system can be controlled continuously through taxes at any desired level of consumption and therefore of income.

From (32) and the goods market condition (11), one can solve for T(t) as a function of the desired level of income (Y^*). Substituting then into the debt accumulation equation (14), after A has been expressed as a function of Y^* and G, yields the following stable differential equation, which completely characterizes the dynamics of the system under discretionary policies.

$$\overset{\circ}{W} = -\pi_4 W - (G-\bar{T})\frac{\lambda}{\lambda_2} + \left(Y^*-G\right)\frac{\pi_4+\lambda_1}{\alpha\pi_0},$$

where $\pi_4 = \gamma + \lambda_2 + \lambda - r > 0$.

The implication is that a loss function of the form

$$(34) \quad L(t) = \int_t^\infty [Y(t)-Y^*]^2 e^{\xi(t-v)} dv$$

can be set equal to its minimum value (zero), through appropriate tax policies, for any Y^* and any shock that hits the system. As in Kydland and Prescott (1977) and Calvo (1978) the optimal plan is time inconsistent.

5. The Suboptimality of Discretion

We now revert to the assumption that agents have correct forecasts of future taxes and show that the path for taxes which has been derived in section 4 is suboptimal.

Consider the following expression for T(t):

(35) $T(t) = \beta_0 + \beta_1 W(t)$.

Substituting (35) into (32), using (11), we can solve for the values β_0 and β_1, as a function of target income (Y^*), which result under the assumption that the government reoptimizes every period.

(36) $\beta_0 = G + [G-Y^*] \frac{v_1\lambda_2}{\alpha(\theta+p)}$.

(37) $\beta_1 = \lambda_2$.

We now show that setting $\beta_1 = \lambda_2$ is not optimal in the sense that it does not minimize the loss function (eq. 34).

Suppose that the government commits itself to a tax rule of the form of (35). Using (27) the equation for the saddle path can then be written as

(38) $A(t) = v_0 + v_1^{-1}W(t)$,

where v_0 is a constant. We can hence write

(39) $A(t) - A^* = e^{\lambda_1(t-t_0)} v_1^{-1} \left[W(t_0)-W^*\right]$,

where A^* and W^* are the steady state levels of A and W when $Y = Y^*$. Given the relation between λ_1 (the speed of adjustment of the system) and v_1 (eq. 29), for any t we have that $A(t) - A^*$ (hence $Y(t) - Y^*$) is smaller the larger λ_1 in absolute value. In turn the relation between λ_1 and β_1 is given by

(40) $\lambda_1 = \frac{1}{2} \left\{(r-\gamma-\beta_1) + (r-\theta) - \sqrt{(\theta-\gamma-\beta_1)^2 + 4p(1-\alpha)(\theta+p)}\right\}$.

λ_1 is a monotonically decreasing function of β_1; moreover it is unbounded, i.e.

$$(41) \quad \lim_{\beta_1 \to \infty} \lambda_1 = - \infty .$$

The limiting case in which β_1 is set equal to infinity corresponds to a wealth tax; in this case the system jumps immediately to the steady state. Wealth is no longer a predetermined variable because the flow of taxes is unbounded.

In conclusion the formal solution to the problem of minimizing the loss function is to set β_1 equal to infinity. If this is not feasible it still remains true that if the authorities commit themselves to a rule like (35) with β_1 larger than λ_2 they can attain a lower value of the loss function than under discretion.

Appendix

To derive eqs. (27) and (28) of section 4 we first compute the standard rational expectation solution of the model and show how it should be modified to allow for suprises.

Consider the following differential system

(1) $Dx(t) = Bx(t) + C\,z(t)$.

$x(t)$ is a 2x1 vector of state variables, $z(t)$ an mx1 vector of forcing or exogenous variables. B and C are constant matrixes and D is the linear differential operator. For the model of this paper the elements of $x(t)$ are $W(t)$ and $A(t)$, $z(t)$ is the scalar $G-T(t)$ and C = transpose [1,0].

Since the model has two distinct roots, B can be diagonalized by a similarity transformation

(2) $\Lambda = V^{-1}\,BV$.

V is a 2x2 matrix of eigenvectors of B and Λ is a diagonal matrix whose diagonal elements are the eigenvalues ($\lambda_1 < 0$ and $\lambda_2 > 0$).

Let

(3) $p = V^{-1}\,x$.

Then (1) can be written as

(4) $Dp(t) = \Lambda p(t) + V^{-1}\,C\,z(t)$,

or

(5a) $Dp_1(t) = \lambda_1 p_1(t) + u_1\,C\,z(t)$, and

(5b) $Dp_2(t) = \lambda_2 p_2(t) + u_2\ C\ z(t)$,

where u_1 and u_2 are the first and second row of V^{-1}. Normalizing the second row of V to be equal to 1 we have

$$V^{-1} = \frac{1}{v_1 - v_2} \begin{bmatrix} 1 & -v_2 \\ -1 & v_1 \end{bmatrix} ,$$

where v_1 and v_2 are the elements of the first row of V. Postmultiplication by C yields

$$u_1 Cz(t) = \frac{1}{v_1 - v_2}\ z(t)\ , \text{ and}$$

$$u_2 Cz(t) = -\ \frac{1}{v_1 - v_2}\ z(t)\ .$$

Since $\lambda_2 > 0$, we solve (5b) forward to obtain

(6) $$p_2(t) = e^{\lambda_2 t}\ k_2 + \frac{1}{v_1 - v_2} \int_t^{\infty} e^{\lambda_2 (t-v)} z(t,v) dv\ ,$$

where $z(t,v)$ is the value of z expected for time v as of time t. For $p_2(t)$ to be bounded, it is required that $k_2 = 0$. (5a) is solved backward to obtain

(7) $$p_1(t) = e^{\lambda_1 (t-t_0)} k_1 + \frac{1}{v_1 - v_2} \int_{t_0}^{\infty} e^{\lambda_1 (t-v)} z(v) dv\ ,$$

where $z(t,v) = z(v)$ for $v \leq t$ by the assumption that the past and the present are known.

It is important to note that t_0 is the time when the last surprise occurred. This means that in (6) $z(t,v) = z(t_0,v)$ since $t > t_0$. The value of k_1 is derived from initial condition at $t = t_0$ for the predetermined variable W(t).

Evaluating the first row of (3) at $t = t_0$ yields

$$(8) \quad k_1 = p_1(t_0) = \frac{1}{v_1} W(t_0) - \frac{v_2}{v_1} p_2(t_0) .$$

Note that since W is predetermined, while A is not, k_1 jumps at t_0. (6), (7) and (8) are the solutions of the problem (x can be obtained postrecursively inverting (3)) when no surprises occur between t_0 and t.

If surprises occur and do so in continuous time, the system is still valid with the caveat that $t = t_0$; therefore k_1 will vary with t:

$$(8') \quad k_1(t) = \frac{1}{v_1} W(t) - \frac{v_2}{v_1} p_2(t) .$$

Setting $t = t_0$ in (7) and using (8') yields

$$(7') \quad p_1(t) = k_1(t) .$$

Given the normalization of V, x = Vp is written in scalar form as

$$(9a) \quad W(t) = v_1 p_1(t) + v_2 p_2(t) \text{ , and}$$

$$(9b) \quad A(t) = p_1(t) + p_2(t) .$$

Substituting (8') in (9b) yields

$$(10) \quad A(t) = \frac{1}{v_1} W(t) - \frac{v_2}{v_1} p_2(t) + p_2(t) = \frac{1}{v_1} W(t) + \frac{v_1 - v_2}{v_1} p_2(t) .$$

Using (6) in (10), yields

$$(11) \quad A(t) = \frac{1}{v_1} \left[W(t) + f(t) \right] \text{ , and}$$

$$(12) \quad f(t) = \int_t^{\infty} e^{\lambda_2 (t-v)} [G-T(t,v)] dv .$$

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