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Working Paper 13703
http://www.nber.org/papers/w13703

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2007

Prepared for the CEPR conference "Globalization and the Organization of Firms and Markets," Munich, February 10-11, 2007. We thank Gene Grossman, Elhanan Helpman, Volcker Nocke and conference participants for very helpful comments. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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December 2007
JEL No. F12,L1

ABSTRACT

In this paper we develop a monopolistic competition model where firms exercise their market power across multiple products. Even with CES preferences, markups are endogenous. Firms choose their optimal product scope by balancing the net profits from a new variety against the costs of "cannibalizing" their own sales. With identical costs across firms, opening trade leads to fewer firms surviving in each country but more varieties produced by each of those firms. With heterogeneous costs, the number of firms surviving in equilibrium is quite insensitive to the market size. When trade is opened, more firms initially enter, but the larger market size reduces the cannibalization effect and expands the optimal scope of products. As a result, the less efficient firms exit, and the larger market is accommodated by more efficient firms that produce more varieties per firm on average.

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1. Introduction

Recent literature in trade has begun to explore multiproduct firms. As documented by Bernard, Redding and Schott (2006a), 41 percent of U.S. manufacturing firms produce in multiple 5-digit SIC industries, accounting for 91 percent of total sales. Using a large and unique dataset that contains product UPC barcodes, Broda and Weinstein (2007) also show that the majority of product creation and destruction happens within the boundaries of the firm. The concentration of sales in very large, multiproduct firms is even more apparent when we look at their exports sales. Bernard, Jensen and Schott (2007) show that the top one percent of U.S. trading firms account for over 80 percent of total trade in 2000. Over 10 percent of exporters and 20 percent of importers are trading 10 or more harmonized system (HS) products, and these firms account for about 90 percent of export and import value. Furthermore, these authors show that the variation in the number of exporters and exported products per firm explain most of the "gravity" results in trade: these two variables decline sharply with distance, and increase with importer income, whereas the average export value (per product and per firm) is actually increasing in distance and decreasing in importer income. Thus, the extensive margin of exporting firms is explaining much of the variation in the value of exports.

On the theoretical side, multiproduct firms have received attention for some years in the industrial organization literature (e.g., Brander and Eaton, 1984). The earliest paper we are aware of in the trade literature is Helpman (1985), who analyzes how a multinational will expand over multiple product lines. Like later authors, Helpman does not take into account the implied effect on the markups of the firms, but uses the constant-elasticity CES assumption with constant markups. Specifically, he assumed that the demand facing a firm producing product j is:

$$y_j = k\, p_j^{-\eta} \;,$$

where η is the elasticity of demand. The constant k would normally depend on the CES price index for the market, which leads to higher markups for multiproduct firms and also a "cannibalization" effect of increasing product varieties. By holding k constant, multiproduct firms charge the same markup as single product firms and do not consider the "cannibalization" effect. Instead, Helpman relies on diseconomies of scope to limit firms' expansion into new product lines.

The simplification of Helpman (1985) also occurs in more recent literature dealing with CES preferences. Brambilla (2006) presents an application of the Melitz (2003) model to investigate the introduction of new varieties by multiproduct firms, with empirical application to multinationals in China. She ignores the interaction in demand of the products produced by a firm, using constant markups like Helpman. Similarly, Bernard, Redding and Schott (2006b) theoretically investigate multiple products using a Melitz approach, but assume that a firm's products are in different categories of goods, so there is no "cannibalization" effect in demand and markups are constant. Allanson and Montagna (2005) propose a multiproduct version of the standard Dixit-Stiglitz model using nested two-tier CES preferences, but again ignore the interaction of multiple products in demand.

Departing from CES preferences, Nocke and Yeaple (2006) use a partial equilibrium inverse demand curve P(q) for every product produced by a firm. They likewise do not take into account the effect of increases in a firm's varieties on the demand for its existing products. They also assume decreasing returns to the range of products: the marginal cost of each variety is increasing in the number of varieties managed by one firm. That approach allows for a solution for the range of production for each firm, even without any interaction in demand.

From this brief summary, it is fair to say that there has been a reluctance in the trade literature to allow for "cannibalization" effects with multiple products per firm, at least in a CES setting. Endogenous markups have been introduced using alternative preferences: in particular, the linear-quadratic utility function from Melitz and Ottaviano (2005). Eckel and Neary (2006) use that approach and no longer treat the aggregate output (or price) index as exogenous. Instead, markups are endogenous and a cannibalization effect operates since a larger output of one variety tends to lower the demand for all other varieties. This gives multiproduct firms an incentive to restrict its range of varieties. Eckel and Neary (2006) also introduce an incentive on the cost side, whereby each firm has a *core competence* in one particular variety, and the marginal cost of a new variety is greater, the more distant it deviates from the "core competence."

In comparison, our model returns to the conventional CES preferences but relaxes the constant aggregate price index assumption. We will show that it is both tractable and interesting to remove this assumption. Initially, we solve for the equilibrium in a model where firms have identical costs, in the spirit of the early work of Krugman (1979, 1980), but allowing for multiple products per firms. That identical-cost model is closest to the business group model of Feenstra, Huang and Hamilton (2003) and Feenstra and Hamilton (2006). In that work, a group of firms (or equivalently, a firm with multiple products) jointly maximized profits in upstream and downstream markets, choosing optimal prices and product scope in each, and free entry of groups was assumed. It turns out the multiple equilibria are present in the model, with different organizations of the business groups.[1] That result followed from the simultaneous upstream and

[1] An economy with given parameters can permit a strongly vertically-integrated equilibrium, with a small number of very large groups, charging high prices for their sale of intermediate inputs; or a less-integrated equilibrium, with a large number of smaller groups, charging lower prices for their intermediate inputs. The former high-concentration equilibrium was compared to the large business groups known as *chaebol* in South Korea, while the latter low-concentration equilibrium was compared to the groups in Taiwan.

downstream competition between groups. The model presented in this paper departs from Feenstra and Hamilton by assuming that firms produce and sell in only the downstream sector, and by focusing on the comparison of autarky to international trade.

After analyzing the model with identical costs, we turn to a version of the model where firms have heterogeneous costs, as in Melitz (2003). Firms still choose their range of products optimally, and for that reason, cannot be treated as "small" relative to the market. The solution to the model cannot use the law of large numbers as applied by Melitz (2003); instead, we analyze the equilibrium with a combination of analytical and numerical results. We show analytically that the optimal number of varieties takes an inverted U-shape with respect to their market shares: the greatest range of product varieties is produced by firms with a mid-level of market share. When trade is opened, the larger market size reduces the cannibalization effect and expands the optimal scope of products. We show numerically that the number of firms in equilibrium is rather insensitive to the market size, but less efficient firms are forced to exit due to trade, so the larger market is accommodated by more efficient firms that produce more varieties per firm on average.

2. Preferences and Demand

There are L consumers (workers) in the economy, each endowed with one unit of labor. The utility function is:

$$U = y_0 + \rho \ln(Y), \quad \rho < 1, \tag{1}$$

where y_0 is the consumption of an outside good which is treated as numeraire. Y represents the consumption index for the horizontally differentiated products. As often used in the monopolistic competition model, the consumption index Y takes a CES form over a continuum of varieties:

$$Y = \left[\int_o^N y(i)^{\frac{\eta-1}{\eta}} \, di \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $y(i)$ is the quantity consumed of variety i, $\eta > 1$ is the elasticity of substitution between output varieties, and N is the total number of varieties.

We will assume that each firm j=1,…,M produces a positive mass of products $N_j > 0$. Without loss of generality, we arrange the order of products so that firm 1 produces the first N_1 varieties, firm 2 produces the next N_2 varieties, etc. Letting $N \equiv \sum_{j=1}^{M} N_j$ denote the total mass of product varieties, the consumption index Y becomes:

$$Y = \left[\int_0^{N_1} y(i)^{\frac{\eta-1}{\eta}} \, di + \int_{N_1}^{N_1+N_2} y(i)^{\frac{\eta-1}{\eta}} \, di + \ldots \int_{N-N_M}^{N} y(i)^{\frac{\eta-1}{\eta}} \, di \right]^{\frac{\eta}{\eta-1}}. \tag{2'}$$

The market for the numeraire good y_0 is competitive, and production requires one unit of labor for each unit of output, implying that the wage rate $w = 1$. Utility maximization under the typical budget constraint gives the familiar form of aggregate demand for each variety within the differentiated good sector,[2]

$$y(i) = \frac{R}{P^{1-\eta}} p(i)^{-\eta}, \tag{3}$$

where $R = \rho L$ denotes the aggregate expenditure on this sector, and the price index P is,

$$P = \left[\int_0^{N_1} p(i)^{1-\eta} \, di + \int_{N_1}^{N_1+N_2} p(i)^{1-\eta} \, di + \ldots \int_{N-N_M}^{N} p(i)^{1-\eta} \, di \right]^{\frac{1}{1-\eta}}. \tag{4}$$

[2] Aggregate demand is just the number of consumers L times the individual demand of each consumer.

Each firm j chooses the continuum of prices $p(i)$ for its product varieties of mass N_j. We simplify this optimal control problem by assuming that each firm has the same marginal cost for all its varieties, so it charges the same price for them. Letting $p_j = p(i)$ for firm j's mass of N_j varieties, then the aggregate price index in (4) can be written as,

$$P = \left[\int_0^{N_1} p_1^{1-\eta} di + \int_{N_1}^{N_1+N_2} p_2^{1-\eta} di + \ldots \int_{N-N_M}^{N} p_M^{1-\eta}\, di\right]^{\frac{1}{1-\eta}} = \left(\sum_{j=1}^{M} N_j p_j^{1-\eta}\right)^{\frac{1}{1-\eta}}. \quad (4')$$

Notice that a change in p_j for firm j will affect the aggregate price index P, provided that the mass of products N_j produced by firm j is strictly positive. Differentiating (3), the elasticity of demand for each variety $y_j = y(i)$ produced by firm j, with respect to the price p_j, is:

$$\frac{\partial y_j}{\partial p_j}\frac{p_j}{y_j} = -[\eta(1-s_j) + s_j], \quad (5)$$

where s_j denotes the market share of its products:

$$s_j = \frac{N_j y_j p_j}{\sum_{i=1}^{M} N_i y_i p_i} = \frac{N_j p_j^{1-\eta}}{\sum_{i=1}^{M} N_i p_i^{1-\eta}} \; . \quad (6)$$

We see from (5) and (6) that the elasticity of demand differs from η whenever $N_j > 0$. Moreover, larger firms face lower demand elasticity.

3. Production with Identical Costs

Firms maximize profits by choosing optimal prices p_j and product range N_j:

$$\max_{(p_j, N_j) \geq 0} \Pi_j = N_j y_j (p_j - \phi) - (k_0 + k_1 N_j), \quad (7)$$

where k_0 is a fixed “headquarters cost”, k_1 is the fixed cost of adding a marginal product into the product line, and y_j is the demand for the variety, with marginal cost ϕ and price p_j. In this section the costs are identical across firms, while in sections 4 and 5 we will introduce heterogeneity in marginal costs, like Melitz (2003).

To choose its optimal price, the firm takes the aggregate price index into consideration when making pricing decision, so its perceived price elasticity of demand is no longer a constant but equals that shown in (5) and (6). Then the optimal choice of p_j is:

$$p_j = \left[\frac{1}{(\eta - 1)(1 - s_j)} + 1 \right] \phi \ . \tag{8}$$

Thus, the demand elasticity and the markup are both *endogenously* determined. As implied by (5) and (8), a firm with higher market share would face less elastic demand and therefore be able to set higher markups.

Optimization over the product range N_j gives:

$$y_j(p_j - \phi) - s_j y_j (p_j - \phi) = k_1 \ . \tag{9}$$

The first term on the left of (9) gives the marginal benefit of adding a marginal variety. But adding this variety will reduce the demand for other varieties produced by the same firm, which is shown by the second term on the left. The larger the market share of the firm, the more severe is this “cannibalization effect”.

We can use (8) in (9) to obtain:

$$y_j \phi = (\eta - 1) k_1 \ . \tag{10}$$

Condition (10) can be recognized as the zero-profit condition in a CES model with single product firms, when the markup is $\eta/(\eta-1)$ and the fixed costs are k_1. In our model, it follows from the first-order conditions for price and the scope of varieties.

The optimum number of varieties is obtained by writing firm j's market share as:

$$\begin{aligned} s_j &= \frac{N_j y_j p_j}{R} = \left[1+\frac{1}{(\eta-1)(1-s_j)}\right]\frac{N_j y_j \phi}{R} \\ &= \left[1+\frac{1}{(\eta-1)(1-s_j)}\right]\frac{(\eta-1)N_j k_1}{R}, \end{aligned} \tag{11}$$

where the second equality follows from (8) and the third equality from (10). Firm j's optimal scope is then given by:

$$N_j = \left[\frac{s_j(1-s_j)}{\eta-(\eta-1)s_j}\right]\frac{R}{k_1} \quad , \tag{12}$$

where R is the total market expenditure with $R=\rho L$.

We will use the subscript "0" to denote the free entry equilibrium, which satisfies:

$$\Pi_0 = N_0 y_0 (p_0-\phi)-(k_0+k_1 N_0)=0. \tag{13}$$

Multiplying the equality in (9) by N_0, we get:

$$N_0 y_0 (p_0-\phi)(1-s_0)-k_1 N_0 = 0. \tag{14}$$

Then comparing (13) and (14), we obtain:

$$1-s_0 = \frac{k_1 N_0}{k_0+k_1 N_0} > 0, \tag{15}$$

from which we obtain another (implicit) solution for the equilibrium number of varieties for each firm:

$$N_0 = \frac{k_0}{k_1}\left(\frac{1}{s_0}-1\right) > 0. \tag{16}$$

We still need to solve for the market share in the zero-profit equilibrium. Notice that with identical firms, all active firms will end up with the same market share. Using (8) and (12), firm j's optimized profits can be rewritten as:

$$\Pi_j = \left[\frac{s_j^{\,2}}{\eta-(\eta-1)s_j}\right]R - k_0 \ . \tag{17}$$

In the zero-profit equilibrium, (17) becomes:

$$\Pi_0 = \left[\frac{s_0^{\,2}}{\eta-(\eta-1)s_0}\right]R - k_0 = 0\,, \tag{18}$$

which gives an expression for s_0:

$$s_0 = \frac{\sqrt{(\eta-1)^2 k_0^2 + 4\eta k_0 R} - (\eta-1)k_0}{2R} \tag{19}$$

The firms' market share, s_0, is a function of the market size R = ρL and the headquarters cost k_0. An increase in the country size L leads to a *lower* level of zero-profit market share s_0. On the other hand, a larger headquarter cost requires a larger cutoff market share and thus an upward impact on active firms' market shares (recall all firms have the same market share). That is:[3]

$$\frac{\partial s_0}{\partial L} < 0 \ \text{ and } \ \frac{\partial s_0}{\partial k_0} > 0 \quad . \tag{20}$$

Taking in to account the endogeneity of the market shares s_0, using (16) we can also obtain the impact of the market size L and the headquarters investment k_0 on the range of products for each firm:

[3] See the Appendix for a proof.

$$\frac{\partial N_0}{\partial L} > 0 \text{ and } \frac{\partial N_0}{\partial k_0} \gtrless 0 . \tag{21}$$

The headquarters fixed costs leads to a higher market share for the zero-profit firm in (20): it must be selling a positive mass of product $N_0 > 0$ with market share $s_0 > 0$ in order to cover the fixed costs $k_0 > 0$. As market size grows, however, the fixed costs are then relatively less important, and the borderline market share s_0 falls. But the range of products N_0 produced by the borderline firm continues to grow. The indeterminacy of the impact of the headquarters investment k_0 on the range of products N_0 comes from the indeterminacy of the relationship between N_0 and s_0: when k_0 is low, relatively more firms could stay in the market (each takes a small share, see (22) below), so an increase in k_0 would require a rise in the market share of each firm, resulting in variety expansion of each active firm; but when only a few firms are in the market (each takes a substantial share), an increase in k_0 would again raise the market share, but this time leading to an reduction in variety range. This corresponds to the "cannibalization effect" of the with-firm brand competition: when the currently existing brands already take a large share in the market, it may not be wise to add another brand since it subtracts demand from other brands of the same firm more than from other competitors.

The number of firms active in equilibrium is readily solved from,

$$Ms_0 = 1 , \tag{22}$$

so that $M = 1/s_0$. It follows from (20) that:

$$\frac{\partial M}{\partial L} > 0 \text{ and } \frac{\partial M}{\partial k_0} < 0 \quad . \tag{23}$$

An increase in country size (due to the opening of trade) leads to lower market share s_0 and as a result, the total number of firms M in the equilibrium is increasing in market size. The intuition for this result is similar to that discussed just above: as market size grows, the fixed costs become relatively less important, so the market share s_0 of the zero-profit firms falls. As a result, there is room for more firms in the market.

We can follow Krugman (1979) to obtain a solution for the number of firms *in each country* from the full-employment condition. The total labor input required for each firm j is:

$$L_j = k_0 + N_j(k_1 + \phi y_j). \tag{24}$$

Notice that the total labor utilized in this industry is ρL, so the total number of firms in each country is:

$$M = \frac{\rho L}{L_j} = \frac{\rho L}{k_0 + N_j(k_1 + \phi y_j)} = \frac{\rho L}{k_0 + N_0 \eta k_1}, \tag{25}$$

where the first equality follows the full employment condition, the second equality applies the definition of labor usage, while the last equality is from (10) with all firms have the same product range $N_j = N_0$ in equilibrium.

Consider, now, the thought-experiment in Krugman (1979) where trade is opened between two identical countries. That leads to a doubling of the market size (i.e. from L to 2L), so from (23), the total number of firms selling to consumers increases compared to the number of firms in each country in autarky. Since an increase in market size stimulates firms to expand their varieties (from (21)), then from (25) we see the number of firms producing in each country necessarily falls with the opening of trade. This effect is also observed in Krugman (1979), but in his work, the existence of this effect is due to the more general specification of the utility (the demand curve should be less convex than a constant-elasticity curve). Using CES utility with single-product firms would make that effect disappear. In contrast, the CES utility function is still utilized in our model, while the endogeneity of the demand elasticity comes from our multiple-products-per-firm assumption.

These results are summarized by:

Proposition 1

With identical marginal costs and positive headquarters fixed costs, increasing the market size through international trade leads to:

(a) the world number of surviving firms exceeds the number of firms in either country in autarky, while the number of producing firms in each country falls after trade relative to the number in autarky;

(b) an expansion in the range of varieties produced by each firm;

(c) no change in the quantity supplied of each variety (from (10));

(d) an improvement in consumer welfare due to both (a) and (b).

It also worth noting that the welfare gains of consumers in part (d) come from the increases in total available varieties due to (a) and (b), which result in a fall in the price index defined in (4'). Besides this "love of variety" gain, each firm also charge lower markups due to the diminishing market share after trade (as L rises s_0 falls in (20), which reduces prices), which is another source of gains from trade that is absent from conventional CES trade models.

Interestingly, after trade opening, while the number of local producers is reduced, they supply a larger number of varieties in total. That is:

Corollary 1

In the setting specified in Proposition 1, an increase in market size due to trade leads to an expansion in the varieties produced by surviving firms that dominates the decrease in the number of local firms, so that the varieties produced in each country expands.

Proof:

From (25), the total number of varieties supplied by local producers is:

$$MN_0 = \frac{\rho L}{k_0 + N_0 \eta k_1} N_0 = \frac{\rho L}{(k_0 / N_0) + \eta k_1},$$

where M is the number of local producers, and N_0 is the number of varieties produced by each firm. Opening trade leads to a larger N_0, and hence a larger MN_0. QED

4. Production with Heterogeneous Costs

We now assume that firms are heterogeneous in their marginal cost of production ϕ, as in Melitz (2003). For simplicity, we also assume no headquarters costs. We suppose that firms' productivity $1/\phi$ is drawn from a Pareto distribution with support $[1/\phi^{max}, \infty)$ and with shape parameter γ. Thus the marginal cost ϕ follows the distribution specified as $F(\phi) = (\phi / \phi^{max})^{\gamma}$, where ϕ^{max} is the highest possible marginal costs. Not all entrants will stay in the market and produce: firms with a high draw of marginal costs might not be profitable. In order to describe the equilibrium of the model, we need to be careful of the information that firms have and the timing of moves.

Specifically, in a *short-run* equilibrium we take as given the number of entering firms M_e, which is a non-stochastic variable. Given M_e, there will be a simultaneous and independent draw of marginal costs $\phi(M_e) = (\phi_1, \phi_2, ..., \phi_{M_e})$ for the M_e firms. These marginal costs are common knowledge of the firms who have entered the market, and they will each choose their optimal prices according to the same first-order condition that we had earlier:

$$p_j = \left[\frac{1}{(\eta - 1)(1 - s_j)} + 1 \right] \phi_j \ . \tag{26}$$

Optimization over the product range N_j also gives a similar equation as before:

$$y_j(p_j - \phi_j) - s_j y_j(p_j - \phi_j) \begin{cases} = k_1 \ \text{if} \ N_j > 0 \\ \leq k_1 \ \text{if} \ N_j = 0 \end{cases} \ . \tag{27}$$

The first term on the left of (27) gives the marginal benefit of adding a marginal variety. But adding this variety will reduce the demand for other varieties produced by the same firm, which is shown by the second term on the left. The larger the market share of the firm, the more severe is this "cannibalization effect". The net benefit should be balanced by the fixed costs of adding a marginal variety. But for very inefficient firms, their net benefit from adding an additional variety can never cover the corresponding fixed costs, hence we have the inequality specified in the second line on the right of (27).

Subject to these choices of price and product range, we will check whether the profits of all firms are non-negative. If not, then we assume that the highest-cost firm exits the market, and we check whether the profits for the remaining firms are all non-negative. If not, we assume that the next-highest cost firm exits the market, etc. This procedure is repeated until enough high-cost firms have exited so that the remaining firms have non-negative profits. We summarize this equilibrium procedure with the definition of the short-run equilibrium:

Definition 1

Given the number of entrants M_e, a short-run equilibrium has prices and product ranges chosen according to (26) and (27), and exit of enough of the highest-cost firms so that the profits of the remaining firms are non-negative.

To see how the short-run equilibrium is computed in practice, suppose that firms j = 1,2,…,M have positive profits with a positive choice of product range, so the equality in (27) holds. Then we can use that equality with (26) to obtain:

$$y_j \phi_j = (\eta - 1)k_1 \quad . \qquad (28)$$

From (28), and applying (3) and (26), we can readily solve for:

$$\phi_j = P\left(1 + \frac{1}{(\eta-1)(1-s_j)}\right)^{-\eta/(\eta-1)} \left[\frac{R}{(\eta-1)k_1}\right]^{1/(\eta-1)} \quad . \qquad (29)$$

Suppose that firm j=0 earns zero profits and has $N_0 = 0$ and $s_0 = 0$, so (29) becomes:

$$\phi_0 = P\left(1 + \frac{1}{(\eta-1)}\right)^{-\eta/(\eta-1)} \left[\frac{R}{(\eta-1)k_1}\right]^{1/(\eta-1)} \quad . \qquad (29')$$

Condition (29') gives the marginal cost of the borderline firm that is just able to produce a single (or negligible share) of products, and still breaks even. We can compare (29) to (29') to get the relative costs of firms as related to their market shares:

$$\frac{\phi_j}{\phi_0} = \left(\frac{1 + \frac{1}{(\eta-1)(1-s_j)}}{1 + \frac{1}{(\eta-1)}}\right)^{-\eta/(\eta-1)} \quad . \qquad (30)$$

Let $\tau_j = \phi_j / \phi_0$ denote the relative cost ratio, so lower τ represents higher productivity, from (30) we can express firm j's market size s_j as a function:

$$s(\tau_j) = \left[1 - \frac{1}{1 + \eta(\tau_j^{-(\eta-1)/\eta} - 1)}\right] \text{ with } \tau_j \le 1. \tag{31}$$

Obviously $s'(\tau_j) < 0$, so more productive firm has larger market share.

Given the number of entrants M_e, and the productivity draw $\phi(M_e) = (\phi_1, \phi_2, ..., \phi_{M_e})$, we can easily check whether these firms all earn non-negative profits by checking whether the markets shares implied by (31) add up to less than unity. That is, take the highest-cost firm among the j = 1,2,…,M_e draws of productivity, labeled as j=0, and hypothesize that it earns zero profits with $N_0 = 0$ and $s_0 = 0$. Expressing all other relative marginal costs as $\tau_j = \phi_j / \phi_0$, then the market share for the remaining firms are computed from (31). If market shares sum to exactly unity, then we have found a short-run equilibrium. If the market shares sum to less than unity, then evidently all firms can stay in the market, but our initial hypothesis that profits are zero and $s_0 = 0$ for the highest-cost firm is not true. Instead, that firm can have a positive market share $s_0 >$ 0, in which case the market shares of the other firms are computed as:

$$s(\tau_j) = 1 - \frac{1}{\left(\eta - 1 + \frac{1}{(1 - s_0)}\right)\tau_j^{-(\eta-1)/\eta} - \eta + 1} \quad , \tag{32}$$

which is derived in the same manner as (31) but allowing for a positive market share $s_0 > 0$ for the highest-cost firm. Then the equilibrium is determined by solving for s_0 from the equation:

$$\sum_{j=1}^{M_e} s(\tau_j) = \sum_{j=1}^{M_e}\left(1 - \frac{1}{\left(\eta - 1 + \frac{1}{(1 - s_0)}\right)\tau_j^{-(\eta-1)/\eta} - (\eta - 1)}\right) = 1 \quad , \tag{33}$$

where $\tau_j = 1$ for the highest-cost firm.

On the other hand, if the initial hypothesis that profits are zero with $s_0 = 0$ for the highest-cost firms results in a sum of market shares from (31) that *exceeds* unity, then the highest-cost firm cannot earn non-negative profits. So that the least efficient firm is dropped, and we take the firm with second-highest marginal costs, and hypothesize that it has zero profits with $s_0 = 0$. We then check whether the sum of market shares in (31) of the remaining firms is less than or equal to unity. If the sum of market shares equals unity then we have found a short-run equilibrium; if it is less than unity then we use (33), but without the highest-cost firm, to determine the equilibrium; and if the sum is more than unity then we also need to drop the firm with second-highest marginal costs. This procedure is repeated until we arrive at the number of firms $M \le M_e$ that survive in the short-run equilibrium for that particular draw of marginal costs $\phi(M_e) = (\phi_1, \phi_2, ..., \phi_{M_e})$.[4] In practice we will always have $M < M_e$ in equilibrium, so the market shares are determined by:

$$\sum_{j=1}^{M} s(\tau_j) = \sum_{j=1}^{M} \left(1 - \frac{1}{\left(\eta - 1 + \dfrac{1}{(1-s_0)} \right) \tau_j^{-(\eta-1)/\eta} - (\eta - 1)} \right) = 1 \quad , \tag{33'}$$

where $\tau_j = 1$ for the highest-cost firm that survives in equilibrium, with market share s_0.

We are now in a position to identify the conditions that must be satisfied in the *long-run* equilibrium, where the expected profits of firms are non-positive. Using (12) (which still holds with heterogeneous costs) and (26), firm j's optimized operating profit can be rewritten as:

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[4] Notice that the number of surviving firms is a random variable, depending on the draw of marginal costs, whereas the number of entering firms is a non-stochastic parameter for the short-run equilibrium.

$$\prod(\tau_j) = \left[\frac{s(\tau_j)^2}{1+(\eta-1)(1-s(\tau_j))}\right]R, \tag{34}$$

for any relative costs τ_j. We have already seen how the market shares determined in an short-run equilibrium, and these can be used in (34) to determine profits for the surviving firms. Then the expected profits are computed by taking the average of (34) over firms and then taking the expected value over all draws of $\phi(M_e) = (\phi_1, \phi_2, ..., \phi_{M_e})$:

$$\text{Expected profits} = E\left\{\frac{1}{M_e}\sum_{j=1}^{M_e} \prod(\tau_j)\right\}, \tag{35}$$

where we adopt the notation that $\prod(\tau_j) = 0$ for all firms j = M+1,… M_e that that do not survive in each short-run equilibrium. Notice that expected profits in (35) depend on the number of entrants M_e in several ways: that number is the dimension of the productivity vector $\phi(M_e) = (\phi_1, \phi_2, ..., \phi_{M_e})$ which is drawn for each short-run equilibrium; and it is also used to form the average of profits in (35). For the second reason, we expect that (35) is declining in the number of entering firms, at least as M_e gets sufficiently large: too many entrants will lower the probability that any firm survives, and lower expected profits. Then the long-run equilibrium is determined by:

Definition 2

A long-run equilibrium is the smallest number of entering firm M_e such that the expected profits in (35) is greater than or equal to the fixed cost of entry k_e , but adding one more entrant, then the expected profits would become strictly less than k_e.

In practice, the long-run equilibrium is determined by taking repeated draws of the productivity vector $\phi(M_e) = (\phi_1, \phi_2, ..., \phi_{M_e})$, for given M_e. For each draw, we compute an short-run equilibrium, and its associated profits from (34). Averaged these over many draws, we obtain expected profits in (35). In order to determine the long-run equilibrium number of entrants, we would need to repeat this procedure for various choices of M_e, until Definition 2 is satisfied. In practice, the numerical problem is simplified because for any choice of M_e, the short-run equilibrium does not depend on market size, which does not enter equations (31)-(33'). The market size enters profits in (34) in a linear fashion. This means that after taking many draws of the productivity vector and computing expected profits in (34) for one market size, the solution for another market size is obtained in direct proportion. So for any choice of M_e, it is fairly easy to find a market size R and fixed entry cost k_e that result in M_e being a long-run equilibrium.

Before turning to the computation of the equilibria, we can identify several analytical properties. The first property follows from our description above:

Lemma 1

The number of surviving firms M in the short-run equilibrium depends on the number of entering firms M_e and their particular draws of marginal cost, but does not depend on the market size L.

Notice that the market size does not enter into formulas (31)-(33'), which proves this Lemma. As we discuss in the next section, the number of entering firms in the long-run equilibrium certainly does depend on market size. With the doubling of market size, for example, then we will find numerically that the equilibrium number of entrants M_e also doubles. But we will further find that many of these firms do not survive for any draw of productivities, so that larger markets do not actually support more firms; instead, they will support more varieties per firm.

The result that larger markets support more varieties per firm can be seen from the optimal product range in (12) (which still holds with heterogeneous costs). Given a draw of productivities $\phi(M_e) = (\phi_1, \phi_2, ..., \phi_{M_e})$ and a corresponding short-run equilibrium, Lemma 1 tells us that changing the market size R =ρL will have no impact on the equilibrium market shares. Therefore, from (12) it is immediate that the product range for each market share will expand in direct proportion to the market size:

Lemma 2

Given the number of entering firms M_e and their particular draws of marginal cost in a short-run equilibrium, the range of varieties produced by each firm is in proportion to the market size L.

What about the relationship between the product range and firms' productivities? Differentiating (12), given a fixed market size R, shows that there is an inverted U-shaped relationship between market share and the number of products, where N_j reaches it maximum when the market share is:

$$s = \frac{\sqrt{\eta}}{\sqrt{\eta} + 1} \tag{36}$$

Lemma 3

There is an inverted U-shaped relationship between firms' productivities and the range of varieties they choose to produce.

To explain this result: more productive firms always have higher market share, as shown by (31); however, there is no monotonic relationship between firms' productivity level and their choices on product range. From (27), having a higher market share means that firms are hurt more from the "cannibalization effect." For this reason, the incentive to expand product lines weakens as productivity rises. Thus, the relationship between productivity and the range of products is non-monotonic: firms at an intermediate level of productivity develop the largest range of products, while the most productive and least productive firms have smaller ranges.

We now turn to the numerical calculation of the long-run equilibrium to describe further properties of the model with heterogeneous costs.

5. Numerical Solution with Heterogeneous Costs

One key deviation of our model from the Melitz (2003) model with heterogeneous productivities is that in our case firms are no longer small relative the market. Dropping the assumption that firms are small enough relative to the market means that we can no longer use the law of large numbers to get closed-form solutions in the long-run equilibrium for the market aggregates such as the average productivity and the like. Nonetheless, the properties of the long-run equilibrium can still be derived using a simple numerical experiment.

We follow steps described in last section in this numerical application. Specifically, we first choose the short-run number of entrants M_e which is non-stochastic. Each of those M_e entrants will draw a marginal cost parameter ϕ from a cost distribution with Pareto density $F(\phi) = (\phi / \phi^{max})^{\gamma}$, where we will arbitrarily let the upper bound of marginal cost $\phi^{max} = 5$. We also choose the shape parameter $\gamma = 5$ for now. Recall a larger shape parameter implies less dispersed cost distribution, we will also experiment with different values of γ later in this section.

The other key parameter is the elasticity of substitution between varieties η, we first set η = 6 and will experiment with other values. For other parameters, we set the fixed costs of introducing each variety at $k_1 = 5$, and the sunk costs of entry at $k_e = 10$.

Table 1 gives an example of the short-run equilibrium where the non-stochastic number of entrants is fixed at $M_e = 7$, given parameters as specified above. With 7 firms entering the market they are randomly assigned marginal cost parameters like those in the second column of Table 1. Then the remaining columns show the outcome of market competition. In this example, it turns out that only 5 firms survive, with the two highest-cost firms dropping out. It is clear from this example that market shares do not depend on market size, since from (31) to (33), firms market shares only depend on their relative marginal costs to the cutoff firm and the cutoff firm's market share. It is also clear that the profits and varieties are in direct proportion to the market size, which is exactly what is implied from (12) and (34).

TABLE 1: A SAMPLE SHORT-RUN EQUILIBRIUM:
PROFITS, VARIETY NUMBERS AND MARKET SIZE

	Firm Number	Marginal cost	Market share	R=1000		R=2000		R=4000	
				Profit	Varieties	Profit	Varieties	Profit	Varieties
Short-run, M_e=7	1	3.580	0.489	67.085	14.048	134.169	28.095	268.338	56.191
	2	3.746	0.411	42.871	12.278	85.741	24.555	171.483	49.110
	3	4.214	0.067	0.797	2.213	1.594	4.426	3.188	8.853
	4	4.256	0.021	0.074	0.695	0.149	1.390	0.297	2.781
	5	4.264	0.012	0.025	0.405	0.050	0.809	0.100	1.618
	6	4.465	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	7	4.953	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: Parameters take the values $k_1 = 5$, $k_e = 10$, $\gamma = 5$ and $\eta = 6$.

The short-run equilibrium in Table 1 is not necessarily a long-run equilibrium, because potential entrants may still see net benefits from entry. As long as the expected profits in the market is greater than the entry costs, firms will keep on entering the market. This process continues until M_e reaches the point where adding one more entrants leads expected profits less than the entry costs. We illustrate the long-run equilibria with different market sizes in Table 2.

TABLE 2: MARKET SIZE AND LONG-RUN EQUILIBRIUM NUMBER OF ENTRANTS

Market size	Long-run number of entrants	Number of surviving firms				Expected profits Sunk cost k_e=10
R	Long-run M_e	M_{min}	M_{max}	M_{mean}	M_{median}	
4,000	64	2	6	3.00	3	10.63
2,000	32	2	6	2.90	3	10.26
1,000	16	2	7	2.97	3	10.32
500	8	2	8	3.08	3	10.09

Note: Parameters take the values $k_1=5$, $k_e=10$, $\gamma=5$ and $\eta=6$, with 500 draws of marginal costs.

It is worth stressing the difference between Tables 1 and 2: while the short-run equilibrium in Table 1 is computed for a *particular* draw of marginal costs for the 7 firms, the long-run equilibria in Table 2 are computed for 500 draws of marginal cost for all entering firms. Then expected profits are computed as in (35), by averaging over firms and the draws of marginal cost. Of course, the number of surviving firms M can vary, depending on marginal costs, and that is why we report the minimum, maximum, mean and median of the distribution of M in Table 2. Indeed, we don't take a stand on the question of which particular draw of marginal costs holds in "the" long-run equilibrium: if we extended our model by introducing dynamics with a death rate of firms, along with the entry of new firms, then the marginal costs would fluctuate over time due to entry and exit and there is no stationary, long-run equilibrium (in contrast to Melitz, 2003). We avoid these dynamic issues, however, and simply solve for a long-run equilibria by determining the (non-stochastic) number of entering firms, as in Definition 2.

Table 2 shows that the long-run equilibrium number of entrants increases as the market size expands, and in fact, it increases in direct proportion. One reason for this result is that each surviving firm's profits, as shown in (34), is directly proportional to the market size. To ensure that the free entry condition holds, the number of entrants must increase with the market size so that the expected net benefits from entry are close to zero. However, profits in (34) also depend on the market shares of firms, which differ with the marginal costs in each short-run equilibrium, so it is surprising that the increase in the number of entrants is in *direct proportion* to the market size. Interestingly, the equilibrium number of surviving firms in Table 2 is quite insensitive to the market size. This result is related to Lemma 1, that the number of surviving firms in a short-run equilibrium does not depend on the size of the market, but we now find numerically that the same result holds when comparing the distribution of M across long-run equilibria.

With roughly the same distribution of the number of surviving firms M in markets of different size, but more entry in larger markets, it follows that more less-efficient firms must exit in larger market. Therefore, the surviving firms are expected to be of higher productivity. This result in confirmed in Table 3. In the upper panel of Table 3, we give the descriptive statistics (maximum, minimum, mean, median) of the highest marginal costs and highest prices for firms surviving in a long-run equilibrium. The lower panel of the same table gives the analogous descriptive statistics for the lowest marginal costs and lowest prices charged by firms surviving in a long-run equilibrium. It is clear from both panels that as market size expands, selection happens and the least efficient firms (with highest costs) drop out the market. The distribution of marginal costs and also prices is shifted to the left as the market size expands.

TABLE 3: MARKET SIZE, MARGINAL COSTS AND PRICES

Market size	Highest marginal cost				Highest prices			
R	ϕ_{min}	ϕ_{max}	ϕ_{mean}	ϕ_{median}	p_{min}	p_{max}	P_{mean}	p_{median}
4,000	1.30	3.44	2.45	2.58	1.65	4.17	3.11	3.14
2,000	1.42	3.94	2.80	2.82	1.89	4.74	3.49	3.50
1,000	1.84	4.65	3.30	3.33	2.51	5.60	4.12	4.17
500	1.62	4.94	3.79	3.86	2.21	5.94	4.71	4.77

Market size	Lowest marginal cost				Lowest prices			
R	ϕ_{min}	ϕ_{max}	ϕ_{mean}	ϕ_{median}	p_{min}	p_{max}	P_{mean}	p_{median}
4,000	0.70	3.02	1.96	1.99	1.55	4.04	2.98	3.01
2,000	0.69	3.40	2.22	2.23	1.85	4.62	3.36	3.36
1,000	0.92	4.24	2.62	2.64	2.48	5.49	3.96	4.01
500	1.02	4.43	3.00	3.07	2.18	5.83	4.52	4.60

Note: Parameters take the values $k_1=5$, $k_e=10$, $\gamma=5$ and $\eta=6$, with 500 draws of marginal costs.

With the selection effect shown in Table 3 operating, we expect that consumers are better off in larger countries. Reinforcing the selection effect is the impact of market size on the varieties produced and available to consumers. Table 4 gives statistics for the total number of varieties produced by all surviving firms and the aggregate market price index (as in (4')) in a long-run equilibrium. From the left columns, we see that the total number of varieties available roughly doubles as the market size doubles. From Table 2 we show that the distribution of the number of surviving firms remains roughly the same when market size changes, so each surviving firm produces more varieties on average to accommodate the integrated market while the highest-cost firms exit. Consumers gain from larger markets: they buy products at lower prices (Table 3), and they can also consume more varieties (Table 4, left columns). This leads to a drop in the aggregate price index, which is shown in the right-most columns of Table 4.

TABLE 4: EQUILIBRIUM VARIETIES AND AGGREGATE PRICE INDEX

Market size	Total Varieties				Aggregate price index			
R	min	max	mean	median	P_{min}	P_{max}	P_{mean}	P_{median}
4,000	57.3	127.0	110.7	113.5	0.63	1.57	1.18	1.19
2,000	28.2	63.6	55.7	56.8	0.83	2.05	1.52	1.53
1,000	15.2	32.3	27.8	28.5	1.28	2.78	2.06	2.09
500	6.0	15.9	14.0	14.3	1.29	3.38	2.70	2.74

Note: Parameters take the values $k_1=5$, $k_e=10$, $\gamma=5$ and $\eta=6$, with 500 draws of marginal costs.

We can think of the increase in market size as due to the integration of two economies through trade. In contrast to Melitz (2003), we have not introduced iceberg trade costs between the economies, and also do not have additional fixed costs of exporting. Without such costs, and assuming single-product firms with CES preferences, the integration of economies will have no impact on entry or exit in either country, even with heterogeneous firms (as shown by Bernard, Redding and Schott, 2007). But in our multi-product model, we have shown numerically that increasing the market size due to opening trade has several important impacts. These numerical results are summarized by:

Proposition 2

With a Pareto distribution of productivities and no fixed or iceberg costs of exporting, then increasing the market size through international trade leads to:

(a) the world number of surviving firms roughly equal to the number of surviving firms in either country in autarky;

(b) exit of the least efficient firms;

(c) an increase in the range of varieties produced by each firm on average;

(d) an increase in expected consumer welfare due to both (b) and (c).

This Proposition is proved by the numerical calculations of long-run equilibria we have just reviewed. We have shown in Table 2 that the distribution of M is quite insensitive to the market size, which is the basis for part (a) of Proposition 2: each country will have roughly the same number of surviving firms in autarky, regardless of their respective markets sizes, and likewise for the world economy after integration. Part (b) follows from our numerical finding that the number of entering firms M_e increases in proportion to the market size, but that the distribution of surviving firms M is quite insensitive to the market size: the larger number of entering firms means that the surviving firms are more efficient than otherwise, as shown in Table 3. Part (c) follow that insensitivity of the distribution of M to market size together with Table 4: since the total number of varieties produced in a country (or the world) increases in proportion to the market size, while the distribution of M is roughly the same, we expect more varieties per firm on average. Consumers gain from the expansion of product varieties and the fall in prices due to having more efficient firms.

We further demonstrate the numerical properties of our model by changing some of the key parameters. In each case, we will report results for two different market sizes, R = 1000 and R = 2000, to illustrate the effect of opening trade between two identical countries. This sensitivity analysis further confirms our numerical finding that the number of entering firms increased in direct proportion to the market size, and that the distribution of surviving firms is quite insensitive to the market size.

TABLE 5: EQUILIBRIUM NUMBER OF ENTRANTS AND THE SHAPE PARAMETER

γ	R=1000 Long-run M_e	Number of surviving firms M_{min}	M_{max}	M_{ave}	M_{median}	R=2000 Long-run M_e	Number of surviving firms M_{min}	M_{max}	M_{ave}	M_{median}
1	44	2	4	2.11	2	88	2	4	2.10	2
2	30	2	5	2.30	2	56	2	6	2.30	2
4	18	2	7	2.80	3	37	2	6	2.87	3
5	16	2	7	2.97	3	32	2	6	2.90	3
10	10	2	9	4.17	4	20	2	10	4.14	4
15	7	2	7	5.12	5	14	2	11	5.35	5
20	6	2	6	5.45	6	11	3	11	6.30	6

Note: Parameters take the values $k_1=5$, $k_e=10$, and $\eta=6$, with 500 draws of marginal costs.

We begin with the shape parameter of the Pareto distribution, γ, as shown in Table 5. Larger γ implies more firms are concentrated in the left of the marginal cost distribution, or the density has a thinner right tail. When $\gamma = 1$, the cost parameter follows a uniform distribution. As γ rises, fewer firms enter the market, while the average number of surviving firms increases. When γ is quite large (for example, $\gamma = 20$), most of firms have high marginal costs and most of entrants will finally break even and stay in the market. When γ is large, potential firms face a larger probability of drawing high marginal costs, and so realized entry falls; on the other hand, for those who do enter the market, since the competitors are more likely similar to themselves in marginal cost, more firms could survive in the end. Thus, the number of entering firms falls with γ in Table 5, but the median number of surviving firms rises.

TABLE 6: NUMBER OF ENTRANTS AND THE ELASTICITY PARAMETER

η	R=1000	Number of surviving firms				R=2000	Number of surviving firms			
	Long-run M_e	M_{min}	M_{max}	M_{ave}	M_{median}	Long-run M_e	M_{min}	M_{max}	M_{ave}	M_{median}
1.5	11	5	11	10.21	11	21	5	21	12.07	12
2.0	14	3	14	7.2	7	28	3	15	7.00	7
4.0	16	2	9	3.73	4	32	2	9	3.57	3
6.0	16	2	7	2.97	3	32	2	6	2.90	3
10	15	2	5	2.46	2	30	2	6	2.49	2
15	15	2	5	2.27	2	30	2	4	2.27	2
100	11	2	3	2.01	2	19	2	3	2.01	2

Note: Parameters take the values $k_1 = 5$, $k_e = 10$, and $\gamma = 5$, with 500 draws of marginal costs.

Another important parameter, the elasticity of substitution, η, exerts its impact through affecting the intensity of competition and the price elasticity of demand. When η is low, varieties are less substitutable for each other, within a firm or across firms. Consequently, firms are able to charge higher markups and get higher profits. Larger η implies lower profits since it is easier for consumers to substitute among varieties, and so the median number of surviving firms is falling with η, as shown in Table 6. On the other hand, the movement of the number of entering firms is ambiguous. As η gets larger, the number of entrants first rises and then falls. One explanation for this is that when η is low, the turning point for the most productive firm to restrict its range of products, shown in (36), is relatively low. This gives low-productivity firms more chance to survive in equilibrium. While on the other hand, if η is getting relatively large, the inverted U-shape between varieties and market shares leans to the right. In this case, even the largest firm won't restrict its variety expansion since the "cannibalization" effect does not hurt as much. Therefore, little room is left for the high-cost firms to survive, so for high values of η the number of entering firms fall as η grows further.

6. Conclusions

Recent literature has introduced multiproduct firms into monopolistic competition model with international trade. When CES preferences have been used, there has been a reluctance to recognize that firms with a positive mass of products will command extra market power: their elasticity of demand with respect to the common price of their varieties is less than the elasticity of substitution. As a firm's range of varieties expands, the elasticity of demand falls further. The goal of this paper has been to show that it is both tractable and interesting to take into account these demand effects. We show that the optimal markup for a firm is endogenous, and that it faces a "cannibalization effect" in demand as more product varieties are introduced. The optimal range of varieties is obtained by balancing the profits earned from each (using the optimal markups) with the "cannibalization effect" of shifting demand away from its own products.

We model international trade as expansion of market size, as two formerly separate economies integrate frictionlessly into a larger, global economy. With multi-product firms, we find interesting effects of trade even in the absence of trade costs.

We first take our benchmark model with firms with identical marginal costs. This corresponds to the model proposed by Krugman (1979, 1980). With identical costs and multiple varieties, firms respond to free trade by expanding their product range of varieties. This occurs because as the market size grows due to trade, each firm's market share decreases, which alleviates the cannibalization effect. Thus, firms tend to produce more varieties than they do in autarky. But not all firms can stay in the market, and there is exit in each country. Importantly, the expansion in varieties within the boundary of the firm dominates the fall in the number of local producing firms, so more varieties provided by (fewer) local producers. Consumers enjoy welfare gains because they not only have more domestic varieties, they also have more variety from imports, and because prices fall due to reduced market shares of each firm.

We then extend our benchmark model to incorporate heterogeneity in costs across firms. We show that in the short-run equilibrium, firms' market shares do not depend on the size of the market, while firms' variety ranges and profits are in direct proportion to the market size. Our numerical calculations confirm these predictions and also enable us to compute the long-run equilibrium. With heterogeneous costs, the number of firms surviving in equilibrium is quite insensitive to the market size. When trade is opened, more firms initially enter, but the larger market size reduces the cannibalization effect and expands the optimal scope of products. As a result, the less efficient firms exit, and the larger market is accommodated by more efficient firms that produce more varieties per firm on average.

Appendix:

Proof of equation (20):

Using the fact that $R = \rho L$, (19) can be rewritten as:

$$s_0(k_0/\rho L) = \frac{\sqrt{(\eta-1)^2(\frac{k_0}{\rho L})^2 + 4\eta\frac{k_0}{\rho L}} - (\eta-1)\frac{k_0}{\rho L}}{2}$$

Taking the derivative:

$$\frac{\partial s_0(k_0/\rho L)}{\partial L} = \frac{1}{2}\left[\frac{(\eta-1)^2\left(\frac{k_0}{\rho L}\right)+2\eta}{\sqrt{(\eta-1)^2(\frac{k_0}{\rho L})^2+4\eta\left(\frac{k_0}{\rho L}\right)}} - (\eta-1)\right].$$

We prove $\frac{\partial s_0}{\partial L} < 0$ by proving the term in the square bracket is positive, because:

$$\left[\frac{(\eta-1)^2\left(\frac{k_0}{\rho L}\right)+2\eta}{\sqrt{(\eta-1)^2(\frac{k_0}{\rho L})^2+4\eta\left(\frac{k_0}{\rho L}\right)}} - (\eta-1)\right] = (\eta-1)\left[\frac{(\eta-1)\left(\frac{k_0}{\rho L}\right)+2\frac{\eta}{(\eta-1)}}{\sqrt{(\eta-1)^2(\frac{k_0}{\rho L})^2+4\eta\left(\frac{k_0}{\rho L}\right)}} - 1\right]$$

$$= (\eta-1)\left[\frac{\sqrt{(\eta-1)^2\left(\frac{k_0}{\rho L}\right)^2+4\eta\left(\frac{k_0}{\rho L}\right)+4\frac{\eta^2}{(\eta-1)^2}}}{\sqrt{(\eta-1)^2(\frac{k_0}{\rho L})^2+4\eta\left(\frac{k_0}{\rho L}\right)}} - 1\right] > 0$$

Then $\frac{\partial s_0}{\partial k_0} > 0$ provided that L is fixed follows directly from the above proof.

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