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# The External Effects of Public Sector Deficits

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This two-equation model measures how public sector deficits — and the way they are financed — affect the real exchange rate, the trade balance, the current account, and the level of external indebtedness.

Rodriguez developed a two-equation model for measuring how public sector deficits — and the way they are financed — affect the real exchange rate, the trade balance, the current account, and the level of external indebtedness. He concludes that:

The level and composition of government spending affects the real exchange rate because of the effect of spending on nontraded goods.

However, whether the government deficit affects the external sector depends on whether the proposition of Ricardian equivalence holds. The general thrust of that proposition is that a tax reduction financed by debt will have no real effect on the economy if the public discounts future taxes to service the debt and therefore increases savings by the exact amount of taxes reduced.

If the Ricardian equivalence does not hold — and empirical evidence is inconclusive — government deficits will directly affect the

excess of spending over income in the economy and therefore the trade balance.

Changes in the trade balance are bound to affect the real exchange rate. How much depends on how much expenditure must be switched to make the trade balance compatible with the change in aggregate spending. The dynamic effects are the result of induced changes in the rate of private accumulation of foreign assets.

A two-equation model is necessary: one equation relating the real exchange rate to the trade surplus and another describing the trade surplus as a function of structural parameters, the fiscal deficit, and the stock of foreign assets.

To make the model dynamic, one must allow for the fact that the level of foreign assets — one determinant of the trade surplus and current account — changes over time. The trade surplus, plus foreign interest earned, determines the evolution over time of the stock of foreign assets.

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by  
**Carlos Alfredo Rodriguez**

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## THE EXTERNAL EFFECTS OF PUBLIC SECTOR DEFICITS

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### INTRODUCTION

This paper is concerned with the analysis of the effects of public sector deficits, and the ways of financing them, on a specific set of macroeconomic variables related to the external sector, namely: the real exchange rate, the Trade Balance, the Current Account and the level of external indebtedness.

The deficit of the public sector, as measured by the Public Sector Borrowing Requirements, is the result of the difference between government spending and government tax revenues. It is therefore imperative in describing the effects of a given deficit to separate the effects of the financing of the deficit from those derived from the given levels of government spending or taxation. In order to do so we have to design a conceptual experiment. In our case we shall assume that there is available a neutral tax, e.g. a value added tax or a consumption tax such that changes in the level of this tax do not affect the relative structure of demand for goods or assets. The deficit is then generated by reducing this neutral tax and increasing accordingly the level of debt financing, either external or internal. From this perspective, what we will be analyzing is the effects of tax vs. debt financing in the context of an open economy. In the case of internal debt financing the government may resort to issuing interest bearing debt (bonds) or non- interest bearing debt (money).

The issue of tax vs. debt financing has received a lot of attention in the literature in reference to the well known Ricardian equivalence proposition. The general thrust of the Ricardian proposition is that a tax reduction financed with debt will have no real effects on the economy if the public discounts the future taxes to service the debt and therefore increases savings by the exact amount of taxes reduced. The empirical validity of the Ricardian equivalence is, however, quite inconclusive.\1/  
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1/For a survey on issues related to the Ricardian Equivalence see Leiderman and Blejer(1988).  
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In the context of an open economy, the real exchange rate is a crucial relative price for the allocation of resources in the external sector. This relative price will certainly be affected by the composition of government spending and may also be affected, depending on the validity of the Ricardian equivalence proposition,

by the way of financing of such spending. In Section I we shall discuss the general issues involved in the analysis of the Ricardian equivalence proposition in relation to the external effects of debt vs. tax financing. In Sections II and III we shall assume that the Ricardian equivalence proposition does not hold and concentrate on the short and long run analysis of government spending and financing on the set of variables related to the external sector.

## I. DEFICIT FINANCING AND THE TRADE BALANCE

We are concerned here with the short run effects of deficit financing on the levels of the real exchange rate, The Trade and Current Account, the levels of domestic and foreign indebtedness and, finally, the inflation rate to the extent that the deficit is financed with money creation.

Define the following variables:

(1)  $Y = \text{GDP}$

(2)  $F_{pg} = \text{Net financing from private sector to government: Taxes plus acquisition of domestic paper (debt or currency minus interest collected on domestic debt).}$

$F_{pg} = T + dC/dt + dD/dt - i \cdot D$  (C: Money, D: Internal Gov. Debt)

(3)  $F_{ep} = \text{Net Financing from foreign to private sector: Gross borrowing minus interest paid on foreign private debt.}$

$F_{ep} = E \cdot dD^*p/dt - i \cdot E \cdot D^*p$  ( $D^*p$ : external private debt, E: exchange rate)

(4)  $G = \text{Government spending on goods}$

(5)  $F_{eg} = \text{Net financing from foreign to government sector.}$

$F_{eg} = E \cdot d(D^*g)/dt - i \cdot E \cdot D^*g$  ( $D^*g$ : external government debt)

Private Sector Budget Constraint

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(6)  $G_p = Y + F_{ep} - F_{pg} = \text{Private spending on goods.}$

Government Budget constraint

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(7)  $G_g = F_{pg} + F_{eg} = \text{Government spending on goods}$

**Total Spending on goods**

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$$(8) \quad GT = G_p + G_g = Y + F_{ep} + F_{eg}$$

Starting from (8) we can derive a set of propositions that will be the basis for the subsequent analysis.

**PROPOSITION (1) : TOTAL SPENDING ON GOODS CAN EXCEED TOTAL OUTPUT  
===== ONLY IF IT IS EXTERNALLY FINANCED. Follows from (8)**

**PROPOSITION (2) : FOR A GIVEN COMPOSITION OF TOTAL SPENDING ON  
===== GOODS BETWEEN TRADED AND NON-TRADED, THE REAL EXCHANGE RATE DEPENDS ON THE DIFFERENCE BETWEEN TOTAL SPENDING AND TOTAL OUTPUT OF GOODS I.E. ON THE TRADE BALANCE DEFICIT THAT IS EQUAL TO THE AMOUNT OF EXTERNAL FINANCING. To be proved later.**

**PROPOSITION (3) : GOVERNMENT FINANCING STRATEGIES WILL AFFECT THE  
===== REAL EXCHANGE RATE ONLY IF THEY AFFECT THE TRADE BALANCE. Follows from P2.**

**PROPOSITION (4) : GOVERNMENT FINANCING STRATEGIES WILL AFFECT THE  
===== TRADE BALANCE ONLY IF THE RICARDIAN EQUIVALENCE PROPOSITION DOES NOT HOLD. IF THIS IS THE CASE A TAX REDUCTION FINANCED THROUGH INCREASED DEBT (INTERNAL OR EXTERNAL) WILL RESULT IN SOME INCREASE IN PRIVATE SPENDING. IN CONSEQUENCE THE TRADE SURPLUS WILL DETERIORATE AND THE REAL EXCHANGE RATE SHOULD FALL. WE WOULD THEREFORE OBSERVE THAT A FISCAL DEFICIT GENERATES A REAL APPRECIATION.**

Proposition (4) is our starting point of analysis. The relevant question is whether the government financing strategies can affect the level of private spending : i.e. the issue of the crowding out, in this case referring also to external borrowing. In order to discuss the effects of deficit financing on the real exchange rate we have to define a neutral experiment through which the deficit increase does not affect the composition of total spending which, of course, would be a very obvious way to affect the real exchange rate. The experiment will be a tax reduction coupled by an equivalent increase in government indebtedness (internal or external). In this way, we are assuming that a deficit is generated without a corresponding increase in the rate of government spending.

There are three ways to finance such a deficit: increase domestic debt, increase external debt or increased rate of money creation. In what follows we shall discuss each case separately.

(a) Tax Reduction financed by external government borrowing

Consider a situation where the government switches from tax financing to external financing. If the private sector reacts by investing the tax savings in foreign assets, there will be no effect on total spending or in the trade surplus. The real exchange rate will not be affected because government borrowing was unable to affect the Trade Balance. In terms of Eqn(8), the increase in  $F_{eg}$  is matched exactly by a decrease in  $F_{ep}$ , so that their sum remains unchanged.

The above conclusion follows from a straight generalization of the Ricardian Equivalence Theorem for foreign borrowing. This issue was analyzed in the context of an optimal model by Aurnheimer(1987), Leiderman and Blejer(1988), and Frenkel and Razin(JPE, June 1986), and has some empirical confirmation in the Argentine experience during 1978-81.

During 1978-81, the Argentine government acquired a substantial external debt that was to a great extent matched by private capital outflows. The private capital outflows, however, took place later in time when it was already perceived that the government's borrowing and exchange rate policy was doomed to failure. There was a transitional period, however, when the government debt was building up, during which the trade deficit deteriorated substantially (although part of it may have been due to the trade liberalization that took place coupled with the quasi-fixed exchange rate policy being followed). It is therefore not clear whether the private capital outflow observed was a private compensation for the increased government debt or a simple speculative movement induced by expectations of a large devaluation.

As mentioned in Leiderman and Blejer(op.cit.) there is a wide variety of reasons why the Ricardian equivalence proposition may not hold to its full extent, even in the open economy. Among these reasons they mention the existence of borrowing constraints, distortive taxation, uncertainty about the imposition of the required future taxes, differences in planning horizons for the private and public sectors, and we might add risk induced differentials in rates of interest at home and abroad and differences in spending propensities among taxpayers and bondholders.

(b) Tax reduction financed by internal borrowing

A similar result regarding substitutability can be described if the government deficit is financed with internal debt. If Ricardian equivalence holds, the lower taxes will be used by the private sector to acquire the increased internal issue of debt so that total private spending will not be increased. There might be, however, indirect

effects due to portfolio composition effects that may affect the composition of spending between consumption and investment goods.

However, if the private sector purchases the internal debt with increased foreign indebtedness, we will observe an increase in external financing and therefore the trade balance and the real exchange rate will be affected. In this case the Ricardian proposition would not hold since private spending has increased to the exact amount of the tax reduction. Here again, the issue should be subject to empirical verification: is government borrowing intermediated externally by the private sector or not?. This case corresponds to the standard version of the open economy with perfect international capital mobility, as presented by Mundell or Fleming in models in which Ricardian equivalence does not hold. In this context, any increased domestic borrowing by the government will tend to raise the domestic interest rate and induce private capital inflows in the exact amount of the government borrowing so that the interest rate remains unchanged.

#### (c) Tax reduction financed through inflation tax

This is the most obvious example of neutrality since it amounts to the substitution of a tax by another so that we should not expect any direct effect on the rate of private spending. However, a differential tax has been instrumented on a single financial asset, money, and this may have short and long run effects on the desired rates of acquisition of the other assets, in particular external assets. The higher inflation rate may stimulate larger desired holdings of external assets by the private sector. In the short run this implies larger capital outflows and therefore, through the reduced rate of private spending, a larger Trade Surplus (and higher real exchange rate). In the long run, as foreign private assets are larger, the interest income will be larger. This means that the Trade Surplus must be lower than otherwise since the interest earned must be spent on foreign goods. The long run effect should therefore be to lower the real exchange rate. The dynamic aspects of deficit financing through the inflation tax are analyzed in detail later in this paper.

The above analysis suggests that the non-neutrality of the deficit in the case of the inflation tax is due to the use of a non-neutral tax on one domestic asset, namely money, and not to the validity or lack of validity of the Ricardian equivalence proposition.

#### General Conclusions



A deficit financed with debt, be it domestic or foreign, is bound to affect the Trade Surplus only if the reduced taxes do affect the rate of private spending. If the private sector uses the reduced taxes to acquire the new issues of internal debt (if the deficit is internally financed) or to acquire foreign assets (if the deficit is externally financed), there will be no effects on the rate of private spending and therefore there will be no relation between the deficit and the Trade Balance or the real exchange rate. In this case, the Ricardian equivalence proposition will be valid, and the choice of tax or debt financing will be totally neutral, also in the case of an open economy.

Inflationary financing of the deficit will affect the external sector through the portfolio induced effects on desired private holdings of foreign assets. We expect totally opposite effects of a higher inflation rate on the Trade Balance in the short run and in the long run. In the short run higher inflation should improve the Trade Balance while the opposite should be valid in the long run.

In the next two Sections we shall proceed to describe in detail the relation between the real exchange rate and the levels of government spending and ways of financing under the assumption that Ricardian equivalence does not hold, e.g. that government deficits do have an impact on trade deficits and therefore on the real exchange rate. The analysis will focus on both the short run and dynamic response of the real exchange rate to changes in policy parameters related to the government sector: the composition of government expenditure and ways of financing of the deficit. The general purpose will be to develop a set of basic structural relationships that could eventually be subject to empirical estimation.

The second step of the analysis is to assess the relationship between the Trade Balance, the Fiscal Deficit and the alternative financing means. At this stage we develop a dynamic portfolio model with which to analyse the effects of government deficits on the desired rate of accumulation of foreign assets by the private sector, and therefore on the Trade Balance.

## II. THE SHORT RUN PROCESS OF DETERMINATION OF THE REAL EXCHANGE RATE

Consider an economy producing two types of goods, Traded (T) and Non-Traded (N), with prices  $P_T$  and  $P_N$ . We define the real exchange rate,  $e$ , as the relative price of Traded vs. Non-Traded goods:  $e = P_T/P_N$ . \1/

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\1/ The analysis in this Section draws and extends on the results presented in Rodriguez (1982).  
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Private sector nominal spending on goods is denoted by  $G_p$  and Government spending on goods is denoted by  $G_g$ . Total spending on goods (absorption) is the sum of private and government spending:

$$(1) \quad G = G_p + G_g$$

Nominal GDP is denoted by  $Y$  and the difference between GDP and Nominal absorption is the Trade Surplus (TS) :

$$(2) \quad TS = Y - G$$

On the demand side, assume the private sector spends a fraction  $b(e)$  of total private spending on Non-Traded goods :

$$(3) \quad G_{pn} = b(e) \cdot G_p.$$

Similarly, the government spends a fraction  $b_g$  on Non-Traded goods:

$$(4) \quad G_{gn} = b_g \cdot G_g.$$

Total nominal spending on Non-Traded goods is therefore:

$$(5) \quad G_n = G_{pn} + G_{gn} = b(e) \cdot G_p + b_g \cdot G_g$$

Define the ratio of Government spending to GDP as the policy parameter:

$$(6) \quad g = G_g/Y .$$

On the supply side, the nominal value of output of Non-Traded goods is represented as proportional to nominal GDP:

$$(7) \quad Y_n = a(e) \cdot Y$$

Equilibrium in the market for Non-Traded goods requires:

$$(8) \quad G_n = Y_n.$$

Substituting (5) and (7) into (8) we obtain:

$$(9) \quad b(e) \cdot G_p + b_g \cdot G_g = a(e) \cdot Y$$

Substituting  $G_p = G - G_g$  and  $G_g = g \cdot Y$ , we can express (9) as:

$$(10) \quad b(e) \{ G - g \cdot Y \} + b_g \cdot g \cdot Y = a(e) \cdot Y$$

Collecting terms, we can express the above as the condition for the Excess Demand for Non-Traded Goods (EDNT) to be equal to zero:

$$(11) \quad EDNT = b(e) \cdot G - \{ a(e) + g \cdot [b(e) - b_g] \} \cdot Y = 0$$

Finally, defining  $ts = 1 - (G/Y)$ ,

as the ratio of the Trade Surplus to GDP and substituting into (11) we obtain:

$$(12) \quad EDNT = b(e) \cdot (1 - ts) - a(e) + g \cdot (b(e) - b_g) = 0 = E(e, ts, g, b_g)$$

Walrasian stability requires that  $dE/de > 0$ . The other derivatives are:

$$dE/dts < 0$$

$$dE/db_g < 0$$

$$dE/dg >< 0 \text{ depending on } b(e) >< b_g.$$

Given the above derivatives we can solve explicitly for the real exchange rate (the market clearing relative price) as function of the other determinants:

$$(13) \quad e = F( ts, g, b_g )$$

+ ? - , where the signs under the variables indicate the expected sign of the partial derivatives.

According to the above equilibrium condition, the real exchange rate should depreciate as the Trade Surplus increases. The reason is simple: a larger trade Surplus means a reduction in spending relative to income. Part of the reduction in spending falls on Non-Traded goods so their price must fall (the real exchange rate raises). An increase in government spending on Non-Traded goods,  $g_n$ , should raise their price so the real exchange rate must fall. An increase in overall government spending for a given Trade Surplus must imply that government share in total spending has increased so that it has displaced private spending. In this case the demand for Non-Traded goods will raise or fall depending on who has a larger propensity to spend on this type of goods; this accounts for the ambiguity in the sign of the partial derivative with respect to  $g$ .

In the above analysis we have assumed the constancy of the terms of trade and therefore we have used an aggregate of Traded Goods. A more general analysis would account for at least the existence of Exportables and Import competing sectors. In that case, the Real Exchange rate would measure the relative price of some aggregate of both Traded goods prices. The equilibrium value of the real exchange rate in this context should also depend on the relative price of both traded goods, i.e the terms of trade, as well as on trade taxes and subsidies that create a differential between the internal and external terms of trade. The interrelation between commercial policy instruments and the equilibrium level of the real exchange rate has been addressed, among others, by Dornbusch(1974), Sjaastad(1979), Harberger(1988) and Rodriguez(1988).

Assume there are two traded goods, exportables and importables, with domestic prices determined by the following arbitrage conditions:

$$(14) \quad \begin{aligned} P_x &= P^*x \cdot (1 - T_x) \\ P_m &= P^*m \cdot (1 + T_m), \end{aligned}$$

where the starred variables refer to the (constant) foreign currency prices and  $T_x$  and  $T_m$  are ad-valorem trade taxes.

There are now two relative prices in this economy that we may denominate the export and the import real exchange rates:

$$(15) \quad \begin{aligned} e_x &= P_x/P_n \\ e_m &= P_m/P_n \end{aligned}$$

Since there are now three goods in the economy, the shares of expenditure and output of Non-Traded goods should now depend on both relative prices:

$$(16) \quad \begin{aligned} a &= a(ex, em) \\ b &= b(ex, em) \end{aligned}$$

Substituting (16) into (12), it is clear that the equilibrium condition in the market for Non-Traded goods (13) is therefore now changed to:

$$(17) \quad ex = ex(em, ts, g, bg)$$

Define the internal terms of trade as

$$(18) \quad TT = ex/em = (P^*x/P^*m) \cdot (1-Tx)/(1+Tm) = TT^* \cdot (1-Tx)/(1+Tm)$$

The above expression allows us to replace  $em$  in (17) by its equivalent in terms of  $ex$ ,  $TT^*$  and Trade taxes, so that we end up with the following reduced form equation:

$$(19) \quad ex = F(TT^*, Tx, Tm, ts, g, bg)$$

Since  $em$  is a function of  $ex$ ,  $TT^*$  and Trade taxes, we could also represent Non-Traded goods market equilibrium by the equivalent condition;

$$(20) \quad em = G(TT^*, Tx, Tm, ts, g, bg)$$

Finally, assuming that we still want to refer to a single concept of the real exchange rate, we can define it as an average of the two real exchange rates:

$$(21)$$

$$e = z \cdot ex + (1-z) \cdot em = z \cdot F(.) + (1-z) \cdot G(.) = H(TT^*, Tx, Tm, ts, g, bg).$$

As shown in Rodriguez(1988), the average Real exchange rate will still present a positive correlation with the Trade surplus, but the relation with the terms of trade will become ambiguous, depending on the weights used to construct it.

It follows from this section that government actions affect the real exchange rate at three different levels: Total Expenditures, the Composition of Expenditures and the External Financing of the Deficit only to the extent that it affects the Trade Balance (therefore proving Proposition 2 of Section I)

As previously discussed, the contribution of the government to the Trade Surplus is directly measured by its ability to obtain

foreign financing of its deficit, this adjusted by whichever compensating capital flows are generated from the private sector. We still have to determine, however, the process of determination of the Trade Surplus of the private sector in relation, not only to government determined parameters, but to the private sector desired rate of accumulation of domestic and foreign assets. To this subject we turn in the next section.

### III.SHORT AND LONG RUN INTERRELATIONS BETWEEN THE ASSETS MARKETS, THE TRADE SURPLUS AND THE REAL EXCHANGE RATE

In the previous section we derived the relationship between the real exchange rate, Terms of Trade, Trade Taxes, the Trade Surplus and the level and composition of government spending. It was also mentioned that the Trade Surplus, in turn, depends on foreign financing (or lending) from the private and public sectors. While the public sector capital flows can be considered a policy variable related to the deficit financing strategy, private capital flows have still to be explained as they are an endogenous variable (except in the limiting case in which there is no capital mobility).

In this section we extend the general equilibrium model of the previous section in order to incorporate the assets markets and to determine the equilibrium level of the Trade Surplus. For further discussion on the interaction between the Trade Balance, the real exchange rate and the assets markets see Dornbusch(1973), Rodriguez(1978), Calvo(1981) and Frenkel and Rodriguez(1982).

The interrelation between the assets markets, the Trade Balance and the real exchange rate becomes evident when analysing the effects of the imposition of the inflation tax.

As mentioned in the first section, deficit financing through the inflation tax is bound not to be neutral regarding its effects on the external sector and the real exchange rate. The reason is that the inflation tax is a non-neutral tax that falls on one particular domestic asset, namely money, and therefore sets the incentive for a portfolio shift in favor of foreign assets. Before going into the formal derivation of all the general equilibrium relationships, we shall provide some intuitive explanation of the most basic interrelation using the example of the inflation tax.

#### The Inflation Tax and the Assets Markets

Consider an economy producing and consuming both Traded and Non-Traded goods. Individuals hold domestic money and interest bearing foreign assets. The differential rate of return between both types of assets is the foreign interest rate plus the expected rate of devaluation. In long run equilibrium, the expected rate of

devaluation is assumed equal to the rate of inflation. An increased rate of monetary expansion generates the expectation of higher devaluation, and inflation, and a process of substitution of foreign assets for domestic money starts. For analytical simplicity we shall assume that there is a freely floating exchange rate (in any event, a fixed exchange rate would be inconsistent with the discretionary use of the inflation tax).

In the process of running down cash balances and acquiring foreign assets, the nominal exchange rate is expected to raise, as both the stock of money and foreign exchange are fixed at a moment in time. The rise in the nominal exchange rate (the price of Traded Goods) also induces (by substitution) some increase in the price of Non-Traded goods, although in a smaller magnitude as it shall be shown later.

The short run adjustment is therefore obtained through a raise in prices and the exchange rate that reduces the real value of total asset holdings of the private sector. The reduction in real asset holdings reduces the demand for Non-Traded goods and this is what allows for the increase in the real exchange rate and the improvement in the Trade Surplus.

The improvement in the Trade surplus starts a process of accumulation of foreign assets. As foreign asset holdings accumulate, the pressure from portfolio balance on the exchange rate is reduced and the Real Exchange rate starts falling back to its original level. However, since foreign assets are larger than before, the service account shows a larger surplus. In consequence, in the new long run equilibrium the Trade Balance must show a larger deficit since the Current Account must be balanced. In conclusion, the imposition of the inflation tax raises the real exchange rate during a transitional period and lowers it in the new long run equilibrium.

The above discussion suggests that the real stocks of assets and the inflation rate should also be as explanatory variables in the equation of determination of the Trade Balance, as they are linked to the desired rate of accumulation of foreign assets.

We now proceed to a formal demonstration of the above points in the context of a model that also incorporates domestic issues of public debt.

#### A Dynamic General Equilibrium Model of Determination of the Real Exchange Rate

The model we shall develop has the purpose of describing the dynamic effects on the external accounts and the real exchange rate of changes in the inflation tax, the foreign interest rate, or the stock of internal public debt.

We shall proceed in the context of an economy where individuals hold three types of assets: domestic Money (M), a domestic bond denominated in foreign exchange issued by the government (b), and a foreign asset (D). The three assets are assumed to be imperfect substitutes, and the relative demands for the assets depend on the differential rates of return offered.

Since we shall be analyzing the effects of the inflation tax, derived from the continuous issuance of money, we have assumed that the government bond is indexed to the exchange rate. If it were fixed in nominal terms, as money grows the relative amount of this bond would approach zero. The alternative would be that the government issues money and nominal bonds in order to keep constant the ratio between them. Assuming the bond is already indexed to the price level, or some of its components like the exchange rate, simplifies the analysis without loss of relevance.

The economy produces and consumes both Traded and Non-Traded goods. The excess supply of Traded Goods is the Trade surplus. The Trade Surplus plus the interest earnings on foreign asset holdings (the Current Account) equals the change in the stock of these assets.

Demands for goods depend on the two nominal prices ( $P_t$  and  $P_n$ ) and nominal expenditure on goods ( $G$ ). Demands are assumed to be homogeneous of degree zero in all nominal variables. The variable  $E$  represents the nominal exchange rate, that is assumed to equal the nominal price of Traded Goods:  $E = P_t$ . For simplicity we shall assume that the revenues of the inflation tax are neutrally redistributed to the public and that the interest on the internal public debt is also financed with a neutral tax. Supplies of both goods depend on the relative price,  $e = P_t/P_n = E/P_n$ , and on factor endowments, that we assume fixed (we abstract here from growth considerations). Given those assumptions, the supply and demand functions take the following form:

$$(1) \quad C_n = C_n(E, P_n, G) = C_n(e, G/E)$$

$$C_t = C_t(E, P_n, G) = C_t(e, G/E)$$

$$(2) \quad Q_n = Q_n(e)$$

$$Q_t = Q_t(e)$$

Define GDP, measured in terms of Traded Goods as:

$$(3) \quad y(e) = Q_t(e) + Q_n(e)/e = \text{GDP}$$



For later purposes, define the derivative of  $y(e)$  with respect to  $e$  as:

$$(4) \quad y'(e) = \{QT'(e) + (1/e) \cdot Qn'(e)\} - Qn(e)/e^2 = -Qn(e)/e^2 < 0 ,$$

since the term in brackets is identically equal to zero by the envelope theorem.

The Trade Surplus, measured in foreign exchange, equals the difference between GDP and expenditure:

$$(5) \quad TS = y(e) - G/E$$

Define  $ts = TS/y(e)$  as the ratio of the Trade Surplus to GDP. Substituting  $ts$  into the demand for  $Cn$ , we can express it as:

$$(6) \quad Cn = Cn(e, (1-ts) \cdot y(e)) = Cn(e, ts)$$

If the Trade Surplus were to be zero, the demand for  $Cn$  would unambiguously depend positively on  $e$  (this follows from the Slutsky expansion of the price effect on the demand for  $Cn$ ). If  $ts < 0$ , however, an income effect operating in the wrong direction appears. We shall assume that the substitution effect dominates, so that the demand for Non-Traded goods depends negatively on its relative price. We therefore assume the following signs for the partial derivatives of  $Cn$ :

$$(7) \quad d(Cn)/d(ts) < 0 , \text{ and} \\ d(Cn)/d(e) > 0$$

Equilibrium in the market for Non-Traded goods requires that the relative price,  $e$ , adjust to equal supply and demand:

$$(8) \quad Qn(e) = Cn(e, ts)$$

It is clear from (8) that an increase in the Trade Surplus is associated with a lower level of expenditure and therefore with a higher real exchange rate (as expenditure falls, demand for Non-Traded goods falls, so its relative price is reduced) :

$$(9) \quad e = e(ts) , \quad e' > 0$$

Equation (9) determines the real exchange rate that equilibrates the market for Non-Traded Goods as a function of the proportional excess of expenditure over GDP ( $ts$ ). Remember, however, that  $ts$  is also an endogenous variable to whose determination we now turn.

Since the Trade Surplus is directly associated with the desired rate of accumulation of foreign assets, we must turn to the description of the assets markets in order to determine the equilibrium level of the Trade Surplus.

Define the level of nominal assets,  $A$  as:

$$(10) \quad A = M + E.b + E.D = E.(m + b + D)$$

We shall assume that there is a long run desired level of real assets ( $a^*$ ) and that people adjust their expenditures in order to gradually reach it. Such desired level of real assets could be defined as a proportion of income or in terms of either commodity. To simplify the analysis, it is convenient to define the desired level of real assets as constant in terms of foreign exchange :

$$(11) \quad A^* = a^*.E$$

The level of nominal expenditures on goods equals the sum of nominal income ( $Y = E.y(e)$ ) plus foreign interest earned ( $r^*.E.D$ ) plus a fraction of the excess of actual asset holdings over the long run desired level:

$$(12) \quad G = Y + E.r^*.D + z.(A - A^*)$$

Expenditure functions similar to (12) can be readily derived from an intertemporal optimization model where both consumption and assets enter into the utility function. In this context,  $a^*$  would correspond to the steady state level of real assets. If the utility function is of the Cobb-Douglas type, the expenditure function will be linear in the relevant arguments as depicted in (12).

The Trade Surplus equals  $(Y-G)/E$ , therefore, using (12) and (10)

$$(13) \quad TS = z.(a^* - m - b - D) - r^*.D$$

Equation (13) describes the determination of the equilibrium Trade Surplus. As seen, it is directly related to the desired rate of accumulation of assets and also to the interest earned on foreign assets. If there were a fiscal deficit financed abroad, it should be subtracted from (13) in which case the Trade Surplus would become:

(13')  $TS = z.(A^* - b - m - D) - r^*.D - feg$ , where  $feg$  is the amount of external net government financing.

What we have determined here is the structural form for the desired rate of private foreign savings.

Eqn.(13') still has endogenous variables into the explanation of the Trade Surplus to the extent that  $m$  can change at any instant though jumps in the exchange rate. In order to determine the equilibrium value of  $m$  we have to describe the portfolio balance equilibrium conditions.

The rate of return for holding domestic money is  $-I$ , where  $I$  is the expected inflation rate. The rate of return on the domestic indexed bond is  $d+i-I$ , where  $d$  is the expected rate of devaluation and  $i$  is the dollar rate paid by the bond. Finally, the rate of return for holding the foreign asset is  $r^* + d - I$ . Since there are three assets, there should be two portfolio preference functions that we assume to depend on the difference between the rates of return of the two assets involved in each case:

$$(14) \quad m/D = L( r^* + d ) , L' < 0 \text{ and}$$

$$(15) \quad b/D = H( i - r^* ) , H' > 0.$$

The stock of the domestic indexed bond is a variable subject to government control. It is clear that the government cannot resort to bond financing as a permanent source of revenue in the absence of growth. We shall therefore consider  $b$  as a policy parameter that takes a fixed value and analyze the effects of changes in its level.

For the moment we shall assume that the expected rate of devaluation is a constant parameter. Substituting (14) into (13) we obtain:

$$(16) \quad TS = z.( a^* - b - (1+L)D) - r^*.D - feg$$

According to (16), the Trade Surplus depends on the stocks of domestic and foreign assets held (that are constant at a point in time), the foreign interest rate, the amount of net government foreign financing and inflationary expectations. We can now normalize  $TS$  by  $y(e)$  to obtain the variable  $ts$ :

$$(17) \quad ts = z.( a^* - b - (1+L).D - r^*.D/z - feg/z )/y(e) , L = L(r^*+d)$$

Notice that in (17), the real exchange rate enters into the determination of the Trade Surplus to GDP ratio not because it affects the Trade Surplus but because real GDP depends on it.

Short run equilibrium is determined by home goods market equilibrium (9) and when the  $ts$  equals the desired rate of assets accumulation (17).

Around the steady state equilibrium, assets equal the desired level so that  $a^* = b + (1+L).D$ . Since we are abstracting from growth, we shall assume the net foreign financing to the government is zero in the long run (otherwise government external debt would accumulate forever). We now proceed to evaluate the short run response of the Trade Surplus to changes in the different parameters, when those changes take place in the vicinity of the steady state equilibrium. These changes are obtained from differentiation of the short run equilibrium conditions (9) and (17):

$$(9) \quad e = e(ts)$$

$$(17) \quad ts = z \cdot \{ a^* - b - (1+L).D - r^*.D/z - feg \} / y(e), \quad L = L(r^*+d)$$

After differentiation, the changes in the Trade Surplus to GDP ratio become:

$$(18) \quad \begin{aligned} d(ts)/d(D)sr &= -z \cdot (1+L) + r^* / [y \cdot (1-J)] < 0 \\ d(ts)/d(b)sr &= -z / [y \cdot (1-J)] < 0 \\ d(ts)/d(d)sr &= -z \cdot L' \cdot D / [y \cdot (1-J)] > 0 \\ d(ts)/d(r^*)sr &= -[z \cdot L' \cdot D + D] / [y \cdot (1-J)] > 0 \\ d(ts)/d(a^*)sr &= z / [y \cdot (1-J)] > 0 \\ d(ts)/d(feg)sr &= -1 / [y \cdot (1-J)] < 0 \end{aligned}$$

$$\text{where :} \quad J = e' \cdot r^* \cdot D \cdot y' / y < 0$$

Since by (9) the real exchange rate depends (positively) only on the Trade Surplus (the effect of other parameters like  $TT$ , Trade Taxes and level or composition of government spending were already analyzed in the previous section and are assumed constant here), the partial derivatives in (18) also give the sign of the short run response of the real exchange rate to changes in the different parameters or in the state variable ( $D$ ).

In particular, it follows that an instantaneous depreciation of the real exchange rate takes place whenever expected devaluation or desired assets are raised, while an appreciation follows from increases in the stocks of domestic or foreign assets held by the private sector (b or D). Algebraically, these short run derivatives are:

$$\begin{aligned}
 (19) \quad d(e)/d(D)sr &= -e' \cdot [z(1+L) + r^*] / [y \cdot (1-J)] < 0 \\
 d(e)/d(b)sr &= -z \cdot e' / [y \cdot (1-J)] < 0 \\
 d(e)/d(d)sr &= -z \cdot L' \cdot e' \cdot D / [y \cdot (1-J)] > 0 \\
 d(e)/d(r^*)sr &= -D \cdot e' \cdot [z \cdot L' + 1] / [y \cdot (1-J)] < 0 \\
 d(e)/d(a^*)sr &= z \cdot e' / [y(1-J)] > 0 , \\
 d(e)/d(feg)sr &= -e' / [y \cdot (1-J)] < 0
 \end{aligned}$$

In order to close the model we have to describe the process of formation of the expected rate of devaluation. The model described here is similar in reduced form to the one of Calvo and Rodriguez (1979). There we closed the model using rational expectations and also showed that a quasi-rational rule of assuming that  $d$  equals the rate of monetary expansion yields identical qualitative results.

For simplicity of exposition, therefore, we shall assume that expectations of devaluation are equal to the constant rate of monetary expansion,  $\mu = d = (1/M) \cdot (dM/dt)$ . \1/

-----  
 1/ If  $d$  is not a constant, the derivation should proceed from the differentiation of the portfolio balance relation (14);

$$\mu - \hat{E} = -(L'/L) \cdot (\hat{dE}/dt) + (1/D) \cdot (dD/dt)$$

In the above expression, the  $\hat{\phantom{x}}$  over a variable indicates the proportional rate of change. If there are rational expectations, the expected change in  $E$  should equal the actual change (abstracting from uncertainty). Otherwise, it can also be assumed that the expected rate of devaluation is formed according to a process of adaptive expectations. In any event, the above expression is the basis for the endogenous determination of the expected rate of devaluation.  
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At any instant of time,  $t_s$  and  $e$  are jointly determined by the values of the state variable  $D$  and the parameters  $r^*, d = \mu, feg$  and  $b$ .

The dynamic behavior of foreign assets requires the specification

of the Current Account, CA. Since net government external borrowing is not sustainable in the long run, we shall assume that  $e$  takes a positive value only for a limited time and will therefore ignore it in the dynamic analysis that follows. The CA equals the Trade Surplus--from (16)-- plus foreign interest earnings:  
 (20)  $CA = d(D)/dt = z \cdot \{ a^* - b - [1+L(r^*+\mu)] \cdot D \}$

Clearly, the differential equation (20) describing the trajectory of foreign assets is stable. The stock of foreign assets converges asymptotically to the long run desired level

$$(21) \quad D_{ss} = \{ a^* - b \} / (1+L)$$

According to (21), the long run stock of foreign assets depends on  $a^*$ , the stock of the indexed government bond ( $b$ ), the foreign interest rate ( $r^*$ ) and the inflation tax rate ( $d = \mu$ ). Algebraically, these changes are;

(22)

$$d(D_{ss})/d(a^*) = 1/(1+L) > 0$$

$$d(D_{ss})/d(b) = -1/(1+L) < 0$$

$$d(D_{ss})/d(\mu) = -(a^* - b) \cdot L' / (1+L)^2 > 0$$

$$d(D_{ss})/d(r^*) = -(a^* - b) \cdot L' / (1+L)^2 > 0$$

We can now proceed to compute the long run effects on the real exchange rate of changes in the different parameters. The difference between the short run effects presented in (19) and the long run effects is that account must be taken of the adjustment of  $D$  to its long run value  $D_{ss}$ .

For example, the long run change in  $e$  in response to a change in  $\mu$  is computed as:

$$(23) \quad d(e)/d(\mu)_{ss} = d(e)/d(\mu)_{sr} + d(e)/d(D)_{sr} \cdot d(D)/d(\mu)_{ss}$$

Equations (24) summarize the long run effects of parameter changes on the real exchange rate:

(24)

$$d(e)/d(\mu)_{ss} = e' \cdot r^* \cdot D \cdot L' / [(1+L) \cdot y \cdot (1-J)] < 0$$

$$d(e)/d(a^*)_{ss} = -e' \cdot r^* / [(1+L) \cdot y \cdot (1-J)] < 0$$

$$d(e)/d(b)_{ss} = e' \cdot r^* / [(1+L) \cdot y \cdot (1-J)] > 0$$

$$d(e)/d(r^*)_{ss} = e' \cdot D \cdot [r^* \cdot L' - (1+L)] / [(1+L) \cdot y \cdot (1-J)] < 0$$

It is of interest to compare the difference between the short and the long run response of the Real Exchange rate to changes in the different parameters. Table 1 presents those differences.

TABLE 1

QUALITATIVE EFFECTS ON THE REAL EXCHANGE RATE OF CHANGES IN:

	$\mu$	$a^*$	$b$	$r^*$	$feg$
SHORT RUN	+	+	-	?	+
LONG RUN	-	-	+	+	---

The most significant feature of Table 1 is that in all cases the direction of the short run impact of a parameter change on the real exchange rate is the opposite of the direction of the long run change (except for  $r^*$  that has an ambiguous short run effect). An increase in the inflation tax rate depreciates  $e$  in the short run and appreciates it in the long run. The same qualitative effects take place when the desired level of assets is increased. An increase in the stock of government debt appreciates  $e$  in the short run and depreciates it in the long run. The short run impact of a higher foreign interest rate is ambiguous in the short run but it unambiguously induces an exchange rate depreciation in the long run.

## CONCLUSIONS

In this paper we have developed a methodology for the analysis of the effects of government spending and the ways of its financing on variables related to the external sector.

The level and the composition of government spending are bound to affect the real exchange rate on account of different spending propensities with the private sector. The government deficit, however, may or may not affect the external sector depending on the validity of the Ricardian equivalence proposition. If such equivalence does not hold we expect direct effects of government deficits on the economy's overall rate of spending, and therefore on the Trade Balance. Changes in the Trade Balance are bound to have both impact and dynamic effects on the real exchange rate. The impact effects are derived from the required expenditure switching necessary to convalidate the new level of the Trade Balance compatible with the change in aggregate spending. The dynamic effects are the result of induced changes in the private rate of accumulation of foreign assets.

It follows from our dynamic analysis that it will not be possible to find a stable static relationship between the real exchange rate and the structural parameters determining its dynamic behavior. Proper identification of the relevant relation requires making allowance for the fact that the level of foreign assets is a determinant of the Trade Surplus and the Current Account and that it changes through time. It is therefore necessary to estimate a two equation model: one equation relating the real exchange rate to the Trade Surplus and another describing the Trade Surplus as a function of structural parameters, the fiscal deficit and the stock of foreign assets. Finally, the Trade Surplus plus foreign interest earned (the CA) would determine the evolution over time of the stock of foreign assets.



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