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POLICY RESEARCH WORKING PAPER

Equity Markets, Transaction Costs, and Capital Accumulation

An Illustration

How does an economy's efficiency in financial transactions affect its efficiency in physical production? And how does the volume of financial transactions relate to the level of real activity?

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Summary findings

There is a close, if imperfect, relationship between the effectiveness of an economy's capital markets and its level (or rate of growth) of real development. This may be because financial markets provide liquidity, promote the sharing of information, or permit agents to specialize.

There is literature about how these functions help increase real activity, but surprisingly little literature predicting how the volume of activity in financial markets relates to the level or efficiency of an economy's productive activity.

Bencivenga, Smith, and Starr address this question: How does the efficiency of an economy's equity market — as measured by transaction costs — affect its efficiency in producing physical capital and, through this channel, final goods and services?

The answer: As the efficiency of an economy's capital markets increases (that is, as the transaction costs fall), the general effect is to cause agents to make longer-term — hence, more transaction-intensive — investments. The result is a higher rate of return on savings and a change in its composition.

These general equilibrium effects on the composition of savings cause agents to hold more of their wealth in the form of existing equity claims and to invest less in the initiation of new capital investments. As a result, a reduction in transaction costs can cause the capital stock either to rise or fall (under scenarios described in the paper).

Further, a reduction in transaction costs will typically alter the composition of savings and investment, and any analysis of the consequences of such changes must take those effects into account.

Equity Markets, Transactions Costs, and Capital Accumulation: An Illustration

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Introduction

It is now beyond dispute that there is a close -- if albeit imperfect -- relationship between the effectiveness of an economy's capital markets and its level (or rate of growth) of real development.[1] This may be because financial markets provide liquidity [Bencivenga and Smith (1991), Levine (1991)], promote the acquisition and dissemination of information [Diamond (1984), Boyd and Prescott (1986), Williamson (1986), Greenwood and Jovanovic (1990)], or permit agents to increase specialization [Cooley and Smith (1992)]. And yet, while there exists literature pursuing how each of these functions contributes to increasing real activity, surprisingly little of that literature provides predictions about how the volume of activity in financial markets is related to the level or efficiency of an economy's productive activity. Indeed, there is surprisingly little existing investigation of the following question: how are an economy's efficiency in performing financial transactions and its efficiency in performing physical production related? It is our purpose to pursue the answer to that question in some detail. In the process, we will also be able to discuss how an economy's volume of financial transactions and its level of real activity are related.

In addition, we would like to be able to say something about why the connections between the development of an economy's financial markets and its level of real development -- while close -- are not perfect. Many prominent growth successes -- for example Korea and Taiwan -- have experienced their success despite heavily regulated financial systems. And, all too often, attempts by governments to stimulate the development of financial markets in LDC's have been (apparently) counterproductive.[2] Why should this be the case if financial market development is, typically, conducive to real development? This is an issue we also wish to investigate.

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In attempting to answer the questions we have posed, we will draw on two fairly fundamental insights. One is that the most productive capital investments will often require that large amounts of funds be committed for substantial periods, with investors facing relatively long times to payout [Bohm-Bawerk (1891)]. The other is that investors are unlikely to commit funds to such investments in the absence of well-functioning capital markets that can provide them with liquidity. The second point was, of course, made quite forcefully by Hicks (1969) in the context of the question, what made the industrial revolution revolutionary? Hicks argued that the industrial revolution was not the consequence (or, at least, the immediate consequence) of a set of new technological innovations, as most of the innovations that were exploited in the early phases of the industrial revolution had occurred some time earlier. Rather, according to Hicks, what was new was that the implementation of these particular innovations on an economical scale required that investments of large magnitude be made in highly illiquid and activity specific capital for long periods. This would not have been possible in the absence of financial markets to provide liquidity. Thus technological innovation by itself was insufficient to stimulate growth; another precondition for the implementation of new technologies was the existence of liquid capital markets. The industrial revolution therefore had to wait for the financial revolution[3] before it could occur: according to Hicks

> What happened in the Industrial Revolution ... is that the range of fixed
> capital goods that were used in production ... began noticeably to increase ...
> But fixed capital is sunk; it is embodied in a particular form, from which it
> it can only gradually ... be released. In order that people should be willing ...
> to sink large amounts of capital, ... it is the availability of liquid funds which
> is crucial. This condition was satisfied in England ... by the first half of the
> eighteenth century ... The liquid asset was there, as it would not have been
> even a few years earlier. [Hicks (1969), pp. 143-145].

We view Hicks as asserting that individual investors face two important timing decisions with respect to their capital investments: the time to payout (or maturity) of the investment, and its holding period. With poorly developed equity markets the transfer of capital ownership is inhibited, and an individual investor will face a time to payout and a holding period that are identical. As Hicks argued, this will prevent an array of investments from being undertaken. However, once equity markets allow the ownership of capital to be transferred economically, individuals can separate decisions involving the maturity of an investment from the length of time they will hold it themselves. Hence such markets permit investors to choose a maturity of investment that maximizes yield, while also choosing a holding period to satisfy the desired timing of their own transactions. The maturity of an investment is no longer held hostage to the desired liquidation dates of wealth-holders.

This set of observations, of course, implies that the costs of transacting in equity markets are of great importance for affecting not just the *level* of investment, but its *composition*. Which kinds of investments appear economical will depend not just on their productivities, but on the cost of transferring ownership of them, if necessary. Thus the efficiency of an economy's financial system (measured by the costs of transacting in equity markets) has implications for which investments are undertaken. And which investments are undertaken, in equilibrium, is an issue that affects the composition of savings. The relationship between the costs of transacting in capital markets, the choice of investment, and the composition of wealth-holding between existing equity claims and the initiation of new capital investment is one that has far-reaching implications. These implications permit us to make some observations about why the association between financial market development and real development is not a perfect one.

In order to discuss the relationship between capital market efficiency (transactions costs), productive efficiency, and the composition of savings, we employ essentially the simplest model we can imagine.[4] Savings, investment, and consumption decisions are undertaken in an overlapping generations model with two period lived agents. In order to trivialize savings, labor supply, and production decisions, both preferences and the technology for producing final goods are assumed to be linear. The innovations of the analysis -- which we highlight by heavily simplifying all other aspects of the model -- are that there exist several technologies for converting current output into future capital, and that ownership of capital is costly to transfer. The first feature allows us to address the issue of the equilibrium choice of investment technology emphasized by Bohm-Bawerk and Hicks, the second allows us to index the effectiveness of an economy's capital markets by the costs of transacting in them.

More specifically, then, we assume that there are $J > 1$ technologies available for converting current output into future capital. These technologies are indexed by $j = 1, ..., J$, and technologies differ as follows. One unit of current output invested in technology j yields R_j units of capital -- gross of transactions costs -- after j periods. Thus investment technologies vary by maturity (j) and productivity (R_j).

Since agents are two period lived, the use of technologies with $j > 1$ requires the ownership of immature capital to be transferred in capital resale -- or equity -- markets. For simplicity we assume a proportional transactions cost structure in these markets. Agents, then, will decide which investment technologies to use based on their yields, *net* of transactions costs. Since transactions costs have a greater effect on the net of transactions cost yields of longer maturity investments (these investments are resold more times, and hence are more "transactions

intensive"), high transactions costs imply that the equilibrium maturity of investments will be relatively short (in order to economize on capital resale). As argued by Bohm-Bawerk and Hicks, this is likely to imply that the investments made are relatively unproductive.

We then investigate the consequences of (exogenous) reductions in transactions costs for steady state equilibria. For the reasons we have described, transactions cost reductions tend to favor the use of longer maturity investments, and hence are conducive to observing certain kinds of increases in productive efficiency. Reductions in transactions costs also necessarily raise the net of transactions costs rates of return on (all) investments, and they therefore raise the equilibrium rate of return on savings. However, transactions cost reductions have potentially complicated consequences for capital accumulation and steady state output.

Why should this be the case? If transactions costs represent real resource costs, their reduction has two effects. First, such a reduction raises the -- net of transactions cost -- productivity of all investment technologies. If the composition of wealth-holding remained constant, this effect would necessarily increase capital formation and steady state output. However, the composition of saving -- much less the equilibrium choice of investment technology -- will not typically remain constant with an increase in the efficiency of equity markets. Such changes make the ownership of already existing equity more attractive, ceteris paribus, and they can cause some fraction of agents' wealth to be transferred away from the initiation of new capital investment, and into the ownership of existing equity. This effect is detrimental to capital formation and production. Moreover, as it turns out, either effect can dominate. We describe when each situation does dominate, and hence when increasing the efficiency of equity markets is and is not conducive to capital accumulation. Thus we can not only describe why equity market

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conditions are important determinants of both productive efficiency and real activity, but also why the relationship between equity market conditions and real development is an imperfect one. In particular, in analyzing the effects of an improvement in the functioning of equity markets, it is necessary to be fully cognizant of how this will affect the composition of wealth-holding.

We are also able to examine how the level of transactions costs impacts on the volume of financial market activity, and on steady state welfare. We consider two polar cases: (i) transactions costs represent real resource costs, and (ii) transactions costs are pure transfers (such as fees or rents to brokers or market makers, or possibly taxes paid to the government). The latter possibility allows us to analyze the consequences of attempts to subsidize or tax various financial market activities by varying these fees.

In either case, a reduction in transactions costs increases the volume of equity market activity. However, since a transactions cost reduction may or may not lead to increased output levels, an increase in the volume of financial market activity can, but need not be, associated with an increase in the level of real activity. Also, when transactions costs represent genuine resource losses, their reduction leads to higher steady state welfare.[5]

The latter conclusion must be substantially modified, however, if transactions costs simply represent transfers. Here transaction cost reductions may either raise or lower steady state welfare. In particular, it is possible that an economy can undertake a socially excessive volume of financial market transactions. In this case, it will be desirable to raise the fees associated with equity market activity. This situation is particularly likely to occur in economies that (with zero fees) have low real interest rates but large transactions volumes.

The remainder of the paper proceeds as follows. Section I lays out the model economy we employ, while Section II describes the nature of trade and transactions costs, and sets out the conditions of a (steady state) competitive equilibrium. Section III examines how the level of transactions costs affects the choice of investment technology, the rate of return on savings, capital accumulation and output, steady state welfare, and the volume of equity market activity when transactions costs represent true resource costs. Section IV reconsiders these issues when transactions costs represent pure transfers. Section V concludes, and comments on some issues that can be addressed in more complicated versions of this framework.

I. The Model

A. The Environment

In this section we describe what we believe is the simplest model that can be used to illustrate the issues we have just discussed. This model confronts both households and producers with what are essentially trivial decisions. In doing so, we are able to focus on what seems to us the most central issue; how transactions costs in equity markets affect the composition of savings and investment, and — through those channels — capital accumulation.

To that end, we consider a two-period-lived, overlapping generations model with production. Time is indexed by t = 1,2, ..., and in each period a new young generation is born with N identical members.[6] Each agent is endowed with one unit of labor when young, which is supplied inelastically, and all agents are retired when old. No agents other than the initial old are endowed with capital or consumption goods at any date.

In each period there is a single consumption good produced, which can either be eaten or converted into capital. We assume that all agents care only about old period consumption, which we denote simply by c.[7] Thus each agent will save his entire young period income at each date.

The consumption good is produced according to a constant returns to scale — in fact a linear — technology using capital and labor as inputs. Thus a firm employing K_t units of capital and L_t units of labor at t can produce

$$F(K_t, L_t) = aK_t + bL_t$$

units of the final good.[8]

Capital is also produced from the final good using a set of linear capital investment technologies. We assume that there are J such technologies, indexed by j = 1,..., J. These

technologies differ along two dimensions; productivity and gestation period (or time to maturity). In particular, one unit of the final good invested in technology j at t yields $R_j > 0$ units of capital (gross of transactions costs) at $t + j$. Thus j represents the gestation length of capital investments in technology j, while R_j represents the (gross) productivity of that technology.

We assume further that if K_t denotes the total capital stock available at t, K_t is simply the sum of maturing capital investments produced through all technologies. Thus, more specifically, all capital -- produced by any investment technology -- is perfectly substitutable as an input in final goods production.[9,10]

Since agents are two period lived, the use of any investment technology with $j > 1$ requires owners of "capital in process" (henceforth CIP) to transfer ownership of it in equity markets. This is true of CIP in all periods prior to maturity, so that ownership of CIP is transferred through a sequence of holders in equity -- or capital resale -- markets. Our interest is in considering how the costs of transacting in these markets affects capital accumulation and per capita income, the equilibrium return on savings, the equilibrium choice of capital production technologies, and welfare in a steady state equilibrium.

For simplicity we assume a proportional transactions costs structure confronting agents who operate in equity markets. Our specific assumption is that transferring ownership of one unit of technology j CIP, that has been in process for h periods (is j-h periods from maturity), consumes $\alpha^{j,h}$ units of CIP. Thus, after a sale of one unit of type (j,h) CIP, $1 - \alpha^{j,h}$ units remain.[11]

Finally, we assume that when CIP matures it is used in the production process, and then depreciates completely. This assumption allows us to abstract from the existence of resale markets for mature -- as opposed to maturing -- capital.

B. Trade

Three kinds of transactions occur in this economy: capital and labor are rented in competitive factor markets, final output is bought and sold, and agents trade ownership of CIP in competitive equity markets. In order to focus on transactions costs in equity markets, and to otherwise keep the model as close to standard as possible,[12] we assume that there are no costs associated with transactions in output or factor markets. We also focus throughout on steady state equilibria. We therefore omit time subscripts wherever possible.

Let w denote the (steady state value of) the real wage rate, and let r denote the rental rate on capital. Equality between the appropriate factor prices and their corresponding marginal products requires that

(1) $r = a$

(2) $w = b$

Each young agent earns the wage income w, all of which is saved. Let S denote savings by a representative young agent, measured in units of CIP. The only decision confronting such an agent is how to allocate his savings among various alternative assets; the available assets are type j CIP (j = 1,..., J) of vintage h (h = 1,..., j − 1). Mature capital is simply rented to firms.[13]

Let $S^{j,h}$ denote the amount of type j CIP that is h periods old acquired by a representative agent. Then, for example, $S^{j,0}$ represents the amount of newly initiated investment in technology j, while $S^{j,j-1}$ is the amount of type j CIP acquired that will mature in one period. Similarly, let $P^{j,h}$ be the price -- in units of current consumption -- of one unit of technology j CIP that is h periods old. Since one unit of the final good invested in technology j at any date becomes one unit of

technology j CIP (by choice of units), $P^{j,0} = 1$. Moreover, mature CIP is simply capital, which is rented to firms. As one unit of technology j CIP yields R_j units of rentable capital on maturity, $P^{j,j} = rR_j$ must hold.[14] For $j > 1$ and $0 < h < j$, $P^{j,h}$ must be determined.

Without loss of generality, we can assume that transactions costs are born by sellers of CIP. Then, since each agent consumes only when old, the budget constraints confronting an individual agent are

(3) $$\sum_{j=1}^{J} \sum_{h=0}^{j-1} P^{j,h} S^{j,h} \leq w$$

(4) $$c \leq \sum_{j=1}^{J} \sum_{h=0}^{j-1} P^{j,h+1} S^{j,h} (1-\alpha^{j,h+1}).$$

We also impose that $S^{j,h} \geq 0$, $\forall (j,h)$.

It is easy to see that agents will purchase type (j,h) CIP only if type (j,h) CIP bears as high a rate of return (net of transactions costs) as any other available investment opportunity. Then if $S^{j,h} > 0$ holds for any agent,

(5) $$(1-\alpha^{j,h+1}) P^{j,h+1}/P^{j,h} \geq (1-\alpha^{\ell,m+1}) P^{\ell,m+1}/P^{\ell,m}$$

must hold for all ℓ, and for all $m \leq \ell - 1$.

If $S^{j,h} > 0$ for some pair (j,h), for some agent in a steady state equilibrium, then $S^{j,h} > 0$ must hold for all $h = 0, ..., j - 1$, and for some agent in that same equilibrium. This obviously requires that the return to holding technology j CIP is the same for all possible times to maturity; that is

(6) $$(1-\alpha^{j,h+1}) P^{j,h+1}/P^{j,h} = (1-\alpha^{j,h}) P^{j,h}/P^{j,h-1}$$

is satisfied for all $h = 0,..., j - 1$. Similarly, if technologies j and n are in use at all dates, then

(7) $\quad\quad\quad (1-\alpha^{j,h+1})\, P^{j,h+1}/P^{j,h} = (1-\alpha^{n,m+1})\, P^{n,m+1}/P^{n,m}$

must hold, for all $m = 0,\ldots, n-1$. Of course, if technology j is employed in a steady state

equilibrium, equation (5) may hold as a strict inequality for some $\ell \neq j$. In this case technology ℓ

is not in use in the equilibrium in question.

All capital investment technologies that are utilized in equilibrium, then, bear a common

(gross) rate of return, net of transactions costs. We denote this return by γ. Then, if technology j

is active in a steady state equilibrium,

(8) $\quad\quad\quad \gamma = (1-\alpha^{j,h+1})\, P^{j,h+1}/P^{j,h}$

for all $h = 0,\ldots, j-1$.

When rates of return on all capital investments in use are equated, clearly each young

agent is indifferent regarding the composition of his portfolio. The aggregate composition of

investment will, nevertheless, be determinate, as we will see shortly.

II. Steady State Equilibrium

In order to describe the steady state equilibrium capital stock, output level, and rate of

return on savings, it is necessary to know two things. First, we need to know which capital

production technology (or technologies) will be in use in such an equilibrium and, in addition, we

must know how savings will be divided among CIP of different times to maturity in this

technology. We now turn our attention to these issues.

A. The Equilibrium Choice of Investment Technology

Recall that $P^{j,0} = 1$ and $P^{j,j} = rR_j$ hold, for all j. We now note that, again for all j,

$$(9) \qquad P^{j,j} = P^{j,0} \prod_{h=0}^{j-1} (P^{j,h+1}/P^{j,h}).$$

If technology j is in use in a steady state equilibrium then equation (8) holds as well. Substituting (8) into (9) yields

$$(10) \qquad rR_j = (\gamma)^j / \prod_{h=0}^{j-1} (1-\alpha^{j,h+1}),$$

where $(\gamma)^j$ is simply γ raised to the jth power. We now define $\tilde{R}_j$ to be the productivity of technology j, net of transactions costs. Then

$$(11) \qquad \tilde{R}_j \equiv R_j \prod_{h=0}^{j-1} (1-\alpha^{j,h+1}).$$

Equations (10) and (11) imply that, if technology j is in use,

$$(12) \qquad \gamma = (r\tilde{R}_j)^{1/j},$$

or in other words, that the rate of return on savings is simply the internal rate of return on any capital production technology employed in equilibrium.

If technology j is active in equilibrium, it is also the case that (5) holds, $\forall \ell \neq j$, $\forall m = 0,..., \ell - 1$. Equations (5), (8), and (9) then imply that

$$(13) \qquad (r\tilde{R}_\ell)^{1/\ell} \leq \gamma$$

is satisfied for all $\ell \neq j$. Thus the capital production technologies employed in equilibrium are those that maximize the internal rate of return on capital investments, net of transactions costs.

Let j^* denote an equilibrium choice of capital production technology. Then

(14) $\qquad j^* = \underset{j}{\mathrm{argmax}}\ [(a\tilde{R}_j)^{1/j}].$

For the present we assume that j^* is unique, which will generically be the case.[15] It follows that the equilibrium rate of return on savings is

(15) $\qquad \gamma = (a\tilde{R}_{j^*})^{1/j^*}.$

To summarize, in choosing which technology to utilize, agents care only about the internal rate of return on investments, net of transactions costs. The costs of transacting in equity markets influence the equilibrium capital production technology through their influence on this rate of return. After characterizing the remaining aspects of an equilibrium, we will pursue the implications of this observation.

B. The Capital Stock, and the Composition of Savings

Let $\theta^{j,h}$ denote the fraction of per capita savings invested in the ownership of technology j CIP of vintage h.[16] Then for $j \neq j^*$, for all $h = 0, ..., j - 1$, $\theta^{j,h} = 0$. For $h = 0,..., j^* - 1$, we now describe the determination of $\theta^{j^*,h}$.

Since the values $\theta^{j,h}$ are (aggregate) portfolio weights, clearly

(16) $\qquad \displaystyle\sum_{j=1}^{J} \sum_{h=0}^{j-1} \theta^{j,h} = \sum_{h=0}^{j^*-1} \theta^{j^*,h} = 1$

must hold. In addition, the market for type (j^*,h) CIP must clear at each date: The demand for such CIP is, of course, given by $\theta^{j^*,h}\, w/P^{j^*,h}$.[17] The supply of type (j^*,h) CIP is the amount of new capital investments in technology j^* initiated h periods ago, less the amount of CIP consumed by the transactions technology in the interim. Thus the supply of type (j^*,h) CIP equals

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$$\theta^{j^*,0} \, w \prod_{\ell=0}^{h-1} (1-\alpha^{j^*,\ell+1})/P^{j^*,0} = \theta^{j^*,0} \, w \prod_{\ell=0}^{h-1} (1-\alpha^{j^*,\ell+1}),$$

since $1 - \prod_{\ell=0}^{h-1} (1-\alpha^{j^*,\ell+1})$ of the initial CIP created has been lost in the transactions process. The market for type (j^*,h) CIP clears, then, if

(17) $$\theta^{j^*,h} \, w = P^{j^*,h} \, \theta^{j^*,0} \, w \prod_{\ell=0}^{h-1} (1-\alpha^{j^*,\ell+1}).$$

We now observe that

(18) $$P^{j^*,h} = P^{j^*,0} \, (P^{j^*,h}/P^{j^*,h-1})(P^{j^*,h-1}/P^{j^*,h-2}) \ldots (P^{j^*,1}/P^{j^*,0}) = (\gamma)^h / \prod_{\ell=0}^{h-1} (1-\alpha^{j^*,\ell+1}).$$

Substituting (18) into (17), we obtain

(19) $$\theta^{j^*,h} = (\gamma)^h \, \theta^{j^*,0}.$$

Equations (16) and (19) then imply that

(20) $$\sum_{h=0}^{j^*-1} \theta^{j^*,h} = \theta^{j^*,0} \sum_{h=0}^{j^*-1} \gamma^h = \theta^{j^*,0} \, [1 - (\gamma)^{j^*}]/(1 - \gamma) = 1,$$

or equivalently, that

(21) $$\theta^{j^*,0} = (1 - \gamma)/[1 - (\gamma)^{j^*}].$$

In view of equation (15), equations (21) and (19) assert that the composition of savings is determined entirely by the internal rate of return on investments in the equilibrium capital production technology. In particular, equation (21) describes how this rate of return determines the amount of new capital investment, while (19) then governs how the remainder of agents' savings are allocated to the purchase of already existing CIP in equity markets.

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The internal rate of return on savings, of course, depends on two factors: the marginal product of capital (a), and the net of transactions cost productivity of the equilibrium investment technology ($\tilde{R}_{j*}$). We now investigate how changes in $\tilde{R}_{j*}$ influence capital accumulation.

Let k denote the per capita capital stock in a steady state equilibrium Then

(22) $k = \tilde{R}_{j*} \, \theta^{j*,0} \, w.$

Equation (22) simply notes that the steady state equilibrium capital stock (per capita), equals the per capita initiation of new capital investments $j*$ periods early ($\theta^{j*,0} w$), times the amount of capital produced, per unit invested, net of transactions costs ($\tilde{R}_{j*}$). Using equations (2) and (21) in (22) gives us the alternative equilibrium condition

(23) $k = (b\tilde{R}_{j*}) \, (1 - \gamma)/[1 - (\gamma)^{j*}] = (b/a) \, (a\tilde{R}_{j*}) \, (1 - \gamma)/[1 - (\gamma)^{j*}] =$

$(b/a) \, (\gamma)^{j*} \, (1 - \gamma)/[1 - (\gamma)^{j*}].$

If we now define the functions $H_j(x)$; $j = 1,..., J$, by

(24) $H_j(x) \equiv x^j(1 - x)/(1 - x^j),$

then we can rewrite (23) more compactly as

(25) $k = (b/a) \, H_{j*} \, (\gamma).$

Equation (25) indicates that the per capita capital stock is determined entirely by the relative productivity of labor and capital (b/a), and by the internal rate of return on capital investments, net of transactions costs. Since per capita output is simply b + ak, once k has been determined, so has the steady state level of per capita income.

C. Equity Market Activity

The real value of equity market transactions in each period -- in per capita terms -- is simply per capita saving, less the real value of new capital investments initiated. In particular, all savings -- other than what goes into new capital investments -- is used to purchase existing CIP in equity markets. Thus the real value of purchases in equity markets is

$$w(1 - \theta^{j^*,0}) = b \{1 - [(1 - \gamma)/[1 - (\gamma)^{j^*}]]\} = b \{[\gamma - (\gamma)^{j^*}]/[1 - (\gamma)^{j^*}]\},$$

where the second equality follows from (2) and (21). If we consider equity market activity as a fraction of total saving, this in turn is given by $[\gamma - (\gamma)^{j^*}]/[1 - (\gamma)^{j^*}]$. If we define the functions G_j; $j = 1,..., J$, by

$$(26) \qquad G_j(x) \equiv (x - x^j)/(1 - x^j),$$

then $G_{j^*}(\gamma)$ gives the fraction of savings that is used to purchase existing CIP in equity markets.

D. Some Results

We now wish to examine how all aspects of a steady state equilibrium -- the equilibrium choice of capital investment technology, the rate of return on savings, the per capita capital stock, and the volume of activity in equity markets -- depend on the underlying parameters of the economy and, in particular, on the costs of transacting in equity markets. This issue is the topic of Section III but, in order to pursue it, it will be useful to collect some properties of the functions H_j and G_j. These properties are stated in the following two lemmas (which are proved in the appendix).

Lemma 1. The functions $H_j(x)$ satisfy the following conditions, $\forall j = 1,..., J$.

(a) $\qquad H_j(x) \geq 0, \forall x \geq 0$

(b) $\lim_{x \to 0} H_j(x) = 0$

(c) $\lim_{x \to \infty} H_j(x) = \infty$

(d) $H_j(1) = 1/j$

(e) $H_j'(x) > 0, \ \forall \ x > 0.$

(f) $H_{j+1}(x) \le H_j(x)$ holds, $\forall \ x \ge 0.$

Lemma 2.   The functions $G_j(x)$ satisfy the following conditions, $\forall \ j = 1,..., J.$

(a) $0 \le G_j(x) \le 1, \ \forall \ x \ge 0.$

(b) $G_{j+1}(x) \ge G_j(x), \ \forall \ x \ge 0.$

(c) $G_j'(x) > 0, \ \forall \ x > 0.$

III.   The Effects of Changes in Transactions Costs

A.   A Representation of Transactions Costs

We now investigate how changes in the level of transactions costs affect all aspects of a steady state equilibrium. In order to do so, it will be convenient to be able to represent transactions costs as depending on a single scalar parameter, which we denote by z. Our specific technical assumption is that

(27) $\tilde{R}_j = \tilde{R}_j(z); \, j = 1, ..., J.$

Thus, in other words, the net of transactions cost productivity of each capital investment technology is a function of the transactions cost parameter z. We assume that $\tilde{R}_j'(z) \ge 0$, for all j, so that increases in z represent reductions in transactions costs.

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1. An Example

Suppose that $\alpha^{j,0} = \alpha^{j,j} = 0$, and that $\alpha^{j,h} = \alpha \in (0,1)$ for all $h \neq 0,j$. (This is simply the case of constant proportional transactions costs.) Then $\tilde{R}_j \equiv R_j(1-\alpha)^{j-1}$. If we let $z \equiv 1 - \alpha$, then

$$\tilde{R}_j(z) \equiv R_j z^{j-1}.$$

2. The Structure of Transactions Costs

In order to obtain definitive results on the consequences of a change in z, it will be necessary to place some structure on the functions $\tilde{R}_j(z)$. We now make the following assumptions. (i) Since there are no transactions costs associated with one period investments, $\tilde{R}_1 = R_1$. Therefore

$$(28) \qquad \tilde{R}_1'(z) \equiv 0.$$

(ii) Since long-maturity projects are resold more times than short-maturity projects (that is, they are more "transactions intensive"), we assume that a reduction in transactions costs has a larger proportional effect — averaged over the life of an asset — on longer than on shorter-gestation investments.[18] Our specific technical assumption is that

$$(29) \qquad \tilde{R}_j'(z)/j\,\tilde{R}_j(z) > \tilde{R}_\ell'(z)/\ell\,\tilde{R}_\ell(z)$$

whenever $j > \ell$. It is easy to verify that (29) implies that a reduction in transactions costs has a larger proportional effect on the internal rate of return on long than on short-gestation capital investments. It is also easy to verify that (28) and (29) are satisfied by some obvious transactions cost structures; for instance they are satisfied by our previous example.

B. The Dependence of Equilibrium Values on Transactions Costs

It will now prove useful to have a notation for the dependence of various equilibrium outcomes on z. The equilibrium rate of return on savings, of course, is simply

$$(30) \qquad \gamma(z) = \max \, [\tilde{R}_1(z), (\tilde{R}_2(z))^{1/2} ,..., (\tilde{R}_J(z))^{1/J}].$$

In addition, suppose we define j(z) by

$$(31) \qquad j(z) \equiv \arg\max_j \{[\tilde{R}_j(z)]^{1/j}\},$$

so that j(z) is simply the choice of capital production technology that maximizes the internal rate of return on investments, given the transactions cost parameter z. Then, defining the function H(z) by[19]

$$(32) \qquad H(z) \equiv H_{j(z)}[\gamma(z)],$$

the steady state equilibrium capital stock (per capita) -- k(z) -- is given by [see equation (25)]

$$(33) \qquad k(z) = (b/a) \, H(z),$$

if j(z) is unique. Similarly, if we let

$$(34) \qquad G(z) \equiv G_{j(z)}[\gamma(z)],$$

then the fraction of total savings consumed by purchases of existing CIP is nothing more than G(z) [again, if j(z) is unique]. Thus G(z) gives the fraction of wealth held in the form of equity for each value of z.

It is now straightforward to show how the capital stock and the value of equity market transactions depend on z. To do so, it will be useful to have the following preliminary result.

Proposition 1. (a) $z' > z$ implies that $j(z') \geq j(z)$. (b) $z' > z$ implies that $\gamma(z') \geq \gamma(z)$, and the inequality is strict if $j(z') > 1$.

The proof of proposition 1 appears in the appendix. The proposition asserts that the equilibrium

maturity length of capital investments rises as transactions costs decline, as does the rate of return

on savings. The first result reflects the fact that - under the assumption of equation (29) -

transactions cost reductions have a larger proportional impact on the net of transactions cost rates

of return for longer than for shorter maturity investments. As a consequence, one effect of a

decline in transactions costs is to increase the relative attractiveness of longer gestation

investment technologies. The second result follows from the observation that transactions cost

reductions raise the internal rate of return on all investment technologies (with $j > 1$), and hence

necessarily increase the rate of return on savings if $j(z) > 1$.

Figure 1 plots $k(z)$ as a "function" of z. Proposition 1 establishes that $j(z)$ is non-

decreasing in z. Thus, if $j(z) = 1$ holds for any values of z, it necessarily holds only for "small"

values. As long as $j(z) = 1$ does hold, $k(z) = (b/a) H_1(aR_1)$, since aR_1 is the internal rate of return

on technology 1. Thus, for "small z", the per capita capital stock is independent of z.

As z continues to increase, at some point technology 1 ceases to maximize the internal

rate of return on savings, as lower transactions costs increase the return on longer-maturity capital

investments. If technology ℓ maximizes the internal rate of return on investment for moderate

values of z, then $k(z) = (b/a) H_\ell\{[a\tilde{R}_\ell(z)]^{1/\ell}\}$ gives the equilibrium level of the per capita capital

stock. By similar reasoning, continued increases in z can eventually cause some even longer-

maturity capital investments to be brought into use, so that technology n, $n > \ell$, maximizes

internal rates of return on investment. Here $k(z) = (b/a) H_n\{[a\tilde{R}_n(z)]^{1/n}\}$. Of course, once $j(z) =$

J, no further increases in the maturity of capital investments are possible, and $k(z) =$

$(b/a)H_J\{[a\tilde{R}_J(z)]^{1/J}\}$.

The solid locus in figure 1 represents (b/a)H(z). When j(z) is unique, H(z) is simply a function given by equation (32). However, there will be a finite set of values of z where j(z) is *not* unique, and where two or more capital production technologies yield the same internal rate of return. For example, technologies ℓ and $n > \ell$ yield the same internal rate of return iff

$$[a\tilde{R}_\ell(z)]^{1/\ell} = [a\tilde{R}_n(z)]^{1/n}.$$

The assumption of equation (29) implies that this condition can hold at *at most* one value of z. When it does hold, investors are indifferent between employing either technology.

If technologies ℓ and n both maximize the internal rate of return on capital investments, investors might utilize technology ℓ exclusively, or they might utilize technology n exclusively, or they might utilize convex combinations of the two technologies. The exclusive use of technology ℓ results in a (steady state) per capita capital stock of $(b/a)H_\ell\{a\tilde{R}_\ell(z)]^{1/\ell}\} = (b/a)H_\ell\{[a\tilde{R}_n(z)]^{1/n}\}$. The exclusive use of technology n results in a (steady state) per capita capital stock of $(b/a)H_n\{[a\tilde{R}_n(z)]^{1/n}\}$. Convex combinations of the two technologies result in (steady state) per capita capital stocks which are convex combinations of these values.[20]

Lemma 1 establishes that, if $n > \ell$, $H_n\{[a\tilde{R}_n(z)]^{1/n}\} < H_\ell\{[a\tilde{R}_n(z)]^{1/n}\}$ holds. As a result, the steady state per capita capital stock must be a non-monotonic function of z. In particular, proposition 1 establishes that if $z' > z$ holds, then $\gamma(z') > \gamma(z)$, so long as $j(z') > 1$. Since $H'_j(\gamma) > 0\ \forall\ j$, if $j(z') = j(z) > 1$, a reduction in transactions costs must result in increased (long-run) capital accumulation. However, if $j(z') > j(z)$, and if z' and z are sufficiently close, $H_{j(z)}[\gamma(z)] > H_{j(z')}[\gamma(z')]$ must hold. Thus small increases in transactions costs that increase the equilibrium choice of investment maturity are actually detrimental to capital formation.

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Why does this occur? The answer has to do with the level of activity in equity markets, which is depicted in figure 2. For "small" z, $j(z) = 1$ holds, and so does $G(z) = G_1[\gamma(z)] = 0$ [see equation (26) with $j = 1$]. This simply reflects the fact that the use of one period gestation production technologies necessitates no capital resale, and hence involves no financial transactions. However, as z increases so does $j(z)$, and when $j(z) > 1$ holds, so does $G(z) > 0$. Moreover, since $G'_j(\gamma) > 0$ holds for all j, as does $G_{j+1}(\gamma) \geq G_j(\gamma)$, $G(z)$ is unambiguously increasing in z. Thus a reduction in transactions costs necessarily increases the fraction of wealth held in the form of equity (ownership of CIP).[21]

This observation implies that a reduction in transactions costs has two consequences that work in opposite directions from the standpoint of capital formation. First, a reduction in transactions costs increases $\tilde{R}_j(z)$ for all $j > 1$, and this effect acts to increase the capital stock, ceteris paribus. However, reductions in transactions costs also increase the fraction of savings held in the form of equity. As a result, less new capital investment is initiated -- a consequence that acts to reduce the capital stock. If $j(z') = j(z)$, the former effect necessarily dominates. However, if $j(z') > j(z)$ holds, and if $z' - z$ is sufficiently small, the second effect necessarily dominates. It follows that an increase in the efficiency of equity markets need not imply capital deepening.

Notice that this last observation depends on two factors. One is that the choice of capital production technology in use depends on the level of transactions costs, and that there is such a choice to be made. The other is that the choice of capital production technology affects the composition of savings between equity-holdings, and the initiation of new capital investment. It is these channels by which transactions cost reductions can be detrimental to capital accumulation.

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Analyses of the role of transactions costs that ignore these compositional effects, then, can easily give highly misleading answers about the consequences of the increased efficiency of equity markets.

To summarize the results of this section, then, a reduction in transactions costs (a) increases the rate of return on savings (and does so strictly if $j^* > 1$), (b) increases the fraction of savings held in the form of equity, and (c) may either increase or reduce the steady state per capita capital stock.

— ———

1. Example 1.

We now produce an example illustrating the equilibrium choice of j. The example is identical to that of section II.A-1, and in addition we set $J = 3$. Thus $\tilde{R}_1(z) = R_1$, $\tilde{R}_2(z) = R_2 z$, and $\tilde{R}_3(z) = R_3 z^2$, with $z \in [0,1)$.

For this example, $j(z) = 1$ iff $R_1 \geq R_2 z$ and $R_1 \geq R_3 z^2$ hold. Thus $j(z) = 1$ iff

$$(34) \qquad z \leq \min[R_1/R_2, (R_1/R_3)^{0.5}].$$

Similarly, $j(z) = 2$ iff $R_2 z \geq R_1$ and $R_2 z \geq R_3 z^2$ are both satisfied. In particular, then, $j(z) = 2$ iff

$$(35) \qquad R_1/R_2 \leq z \leq (R_2/R_3)^{0.5}.$$

$j(z) = 3$ holds if (34) and (35) are violated.

Evidently, for this example $j(z) = 1$ for "small enough" z, while $j(z) = J = 3$ for "large enough" z. $j(z) = 2$ holds for some z iff $R_1/R_2 \leq (R_2/R_3)^{0.5}$ holds.

2. Example 2.

This example explores the configuration of the relationship between transactions costs and the steady state per capita capital stock in some detail for the case $J = 2$. Since here $\tilde{R}_1 = R_1$, we can summarize transactions costs entirely by the magnitude of $\tilde{R}_2$.

Technology 1 maximizes the internal rate of return on capital investments iff

$$(36) \qquad aR_1 \geq (a\tilde{R}_2)^{0.5}$$

holds. Equation (36), of course, is equivalent to

$$(36') \qquad \tilde{R}_2 \leq aR_1^2.$$

For values of $\tilde{R}_2$ satisfying (36') as a strict inequality, $j^* = 1$, and $k = (b/a)H_1(aR_1)$. Here k is independent of $\tilde{R}_2$, and hence independent of the costs of transacting in equity markets.

When $\tilde{R}_2 = aR_1^2$, $j^* \in \{1,2\}$, and either technology can be in use. Moreover, lemma 1 establishes that $H_1(aR_1) > H_2(aR_1) = H_2[(a\tilde{R}_2)^{0.5}]$ holds. Thus k lies in the interval $[H_2(aR_1), H_1(aR_1)]$. And finally, when (36') is violated (that is, when $\tilde{R}_2$ is large enough, or transactions costs are low enough), $j^* = 2$. Then $k = (b/a) H_2[(a\tilde{R}_2)^{0.5}]$ holds. Since $H_2' > 0$, the steady state capital stock increases with increases in $\tilde{R}_2$ (reductions in transactions costs) beyond this point.

Of course, $H_1(aR_1) = aR_1$ and $H_2[(a\tilde{R}_2)^{0.5}] = a\tilde{R}_2/[1 + (a\tilde{R}_2)^{0.5}]$ both hold [see equation (24)]. Then $H_2[(a\tilde{R}_2)^{0.5}] \geq H_1(aR_1)$ holds iff

$$(37) \quad (\tilde{R}_2)^{0.5} \geq [R_1(a)^{0.5} + (aR_1^2 + 4R_1)^{0.5}]/2 > R_1(a)^{0.5}.$$

For values of $(\tilde{R}_2)^{0.5}$ between $R_1(a)^{0.5}$ and the right-hand side of (37), $H_2[(a\tilde{R}_2)^{0.5}]$ lies below

$aR_1 = H_1(aR_1)$. In this range, moderate levels of transactions costs induce the use of technology 2. However, more capital would be produced if technology 1 were utilized instead. Figure 3 summarizes the relationship between k and $\tilde{R}_2$.

C. Steady State Welfare

We now describe the effects of reductions in transactions costs for steady state welfare. To do so, recall that all young agents earn the real wage income b, and save all of this income for old period consumption at the gross rate of return $\gamma(z)$. Thus steady state welfare is simply $\gamma(z)b$. Therefore, by proposition 1.b, a reduction in transactions costs cannot reduce steady state welfare, and a reduction in transactions costs must actually raise steady state welfare if $j^* > 1$.

The simplicity of this result depends heavily, however, on the fact that the real wage rate here is independent of the capital stock, so that all welfare effects occur through changes in the rate of return on savings. If the real wage did depend on k, then reductions in transactions costs that reduce k would also reduce w. The effect of transactions cost reductions would then depend on the relative magnitudes of the changes in $\gamma(z)$ and k(z). If $z' > z$ and $k(z') < k(z)$ hold, and if the real wage is a function of the capital-labor ratio, then it can easily occur that transactions cost reductions do reduce steady state welfare. Examples illustrating this possibility appear in Bencivenga, Smith, and Starr (1994 a,c).

IV. Transactions Costs as Pure Rents

In this section we analyze the same set of issues as before under the assumption that transactions costs represent pure fees (and rents) paid to a broker or market maker. Thus, while

representing costs to equity market participants, these fees associated with transactions no longer represent a social resource loss. For simplicity we assume that the fees collected by brokers/market makers are simply rebated to old agents as a lump-sum; we can think of this as corresponding to a situation where all young agents are given equal shares in a brokerage firm. In addition, to enforce the notion that there are no social resource losses associated with the transactions process, we assume that none of the time of a broker/market maker is diverted from labor supply when young. An alternative interpretation of the model of this section, of course, is simply that the government taxes or subsidizes equity market transactions, and rebates the proceeds to old agents as a lump-sum.[22] Many developing country governments do act to subsidize the formation of equity markets [Fry (1988)]; the formulation of this section allows us to analyze the consequences of this activity.

To further simplify matters, we assume that[23]

(38) $\qquad \alpha^{j,0} = \alpha^{j,j} = 0; j = 1, ..., J \qquad \alpha^{j,h} = \alpha \leq 1; \forall j, \forall h = 1, ..., j - 1.$

Thus there are no fees associated with the initiation of new capital investments, or with transactions in factor markets. On all transactions transferring the ownership of CIP (equity), however, equity market participants face a fee that is proportional to the real value of their transactions.

A. Steady State Equilibrium Conditions

The same reasoning as in section II establishes that the equilibrium choice of capital production technology, j^*, is that which maximizes the internal rate of return on capital investments, net of the transactions costs perceived by an owner of CIP. Thus

$$j^* = \arg\max_j [(a\tilde{R}_j)^{1/j}]$$

holds, where $\tilde{R}_j$ continues to be given by (11). It is also the case that the steady state equilibrium rate of return perceived by young agents is simply the internal rate of return on technology j^*, net of the transactions costs they face. Thus

$$\gamma = (a\tilde{R}_{j^*})^{1/j^*},$$

as before. In addition the aggregate portfolio weights $\theta^{j,h}$ are defined as previously, and equation (16) continues to hold.

The equilibrium conditions that do require modification are those describing capital formation and market clearing in CIP. In the former case transactions costs here no longer consume CIP, and hence $\tilde{R}_{j^*}$ is no longer relevant to the amount of capital received per unit invested j^* periods earlier. Rather all capital investment ultimately translates into usable physical capital, and

(39) $k = R_{j^*} \theta^{j^*,0} b.$

Equation (39) replaces equation (22). Similarly, no CIP is ever used in the transactions process, and hence the supply of type (j^*,h) CIP in a steady state equilibrium is simply $w\theta^{j^*,0}/P^{j^*,0} = b\theta^{j^*,0}$, while the demand for type (j^*,h) CIP is $b\theta^{j^*,h}/P^{j^*,h}$. Thus market clearing in type (j^*,h) CIP requires

(40) $b\theta^{j^*,h}/P^{j^*,h} = b\theta^{j^*,0}; h = 0, ..., j^* - 1.$

Equation (40) replaces equation (17).

In order characterize a steady state equilibrium, we note that (18) continues to hold. Moreover, from (38),

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$$\prod_{\ell=0}^{h-1} (1 - \alpha^{j^*,\ell+1}) = (1 - \alpha)^h; \ h = 1, ..., j^* - 1.$$

It follows that

(41) $\qquad P^{j^*,h} = [\gamma/(1 - \alpha)]^h; \ h = 0, ..., j^* - 1.$

Substituting (41) into (40) and rearranging terms gives

(42) $\qquad \theta^{j^*,h} = \theta^{j^*,0} [\gamma/(1 - \alpha)]^h; \ h = 0, ..., j^* - 1.$

Therefore, from (16) and (42),

(43) $\qquad \displaystyle\sum_{h=0}^{j^*-1} \theta^{j^*,h} = \theta^{j^*,0} \sum_{h=0}^{j^*-1} [\gamma/(1 - \alpha)]^h = \theta^{j^*,0} \{1 - [\gamma/(1 - \alpha)]^{j^*}/\{1 - [\gamma/(1 - \alpha)]\} = 1$

must be satisfied. Thus

(44) $\qquad \theta^{j^*,0} = \{1 - [\gamma/(1 - \alpha)]\}/\{1 - [\gamma/(1 - \alpha)]^{j^*}\}.$

As before, transactions cost and technological parameters entirely determine the equilibrium

choice of capital production technology, the rate of return on savings, and the composition of

savings across different vintages of type j* CIP.

In order to determine the steady state equilibrium value of the per capita capital stock,

substitute equation (44) into equation (29) to get

(45) $\qquad k = bR_{j^*} \{1 - [\gamma/(1 - \alpha)]\}/\{1 - [\gamma/(1 - \alpha)]^{j^*}\}.$

However, since $\gamma = (a\tilde{R}_{j^*})^{1/j^*} = [aR_{j^*}(1 - \alpha)^{j^*-1}]^{1/j^*}$, $\gamma/(1 - \alpha) = [a\tilde{R}_{j^*}/(1 - \alpha)]^{1/j^*}$ holds.

Therefore, equation (45) reduces to

(46) $\qquad k = bR_{j^*} \{1 - [aR_{j^*}/(1 - \alpha)]^{1/j^*}\}/\{1 - [aR_{j^*}/(1 - \alpha)]\}.$

Thus

$$j^* = \underset{j}{\text{argmax}} [(a\tilde{R}_j)^{1/j}] = \underset{j}{\text{argmax}} [aR_j(1-\alpha)^{j-1}]^{1/j}$$

and equation (46) give the steady state equilibrium level of the per capita capital stock as a function of the transactions fee α.

We now wish to analyze how the steady state equilibrium value of k varies with α. In order to simplify the exposition, we focus our attention on the case $J = 2$.

B. An Example: $J = 2$.

When there are only two technologies for producing capital, $j^* = 1$ holds iff

(47) $\qquad aR_1 \geq (a\tilde{R}_2)^{1/2}$.

Since $\tilde{R}_2 = R_2(1-\alpha)$, equation (47) is equivalent to

(48) $\qquad \alpha \geq 1 - (aR_1^2/R_2)$.

If α satisfies (48) as a strict inequality, $j^* = 1$, while if α satisfies (48) at equality, $j^* \in \{1,2\}$. Violation of equation (48) implies that $j^* = 2$.

When $j^* = 1$, equation (46) implies that $k = bR_1$. Alternatively, when $j^* = 2$, equation (46) implies that

$$k = bR_2\{1 - [aR_2/(1-\alpha)]^{0.5}\}/\{1 - [aR_2/(1-\alpha)]\} = bR_2/\{1 + [aR_2/(1-\alpha)]^{0.5}\}.$$

Thus the steady state equilibrium capital stock per capita, as a function of α, is given by

$$k = bR_1 \quad ; \quad \alpha > 1 - (aR_1^2/R_2)$$

(49) $\qquad k \in [bR_1/[(R_1/R_2) + 1], bR_1]; \alpha = 1 - (aR_1^2/R_2)$

$$k = bR_2/\{1 + [aR_2/(1-\alpha)]^{0.5}\}; \alpha < 1 - (aR_1^2/R_2)$$

Equation (49) is depicted in figure 4.

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As shown in figure 4, for high values of α (high transactions costs), the necessity of transferring the ownership of CIP makes technology 2 prohibitively expensive to use. Hence technology 1 is the equilibrium choice of capital production technology, equity markets are inactive, and the value of α is irrelevant to the capital stock. However, once α is low enough [and specifically, no greater than $1 - (aR_1^2/R_2)$], transactions costs are sufficiently small to allow technology 2 to be competitive. Once this occurs, further reductions in transactions costs raise the internal rate of return on technology 2, and result in a higher level of the steady state per capita capital stock. In contrast to what happens in section III, however, this does *not* transpire because technology 2 becomes more productive. (Recall that R_2 is fixed.) Rather the capital stock increases because reductions in transactions costs raise $\theta^{2,0}$, and thus alter the composition of savings in a way that is *favorable* to capital formation. Thus again the consequences of changes in transactions costs for the composition of savings are an essential part of the story.

Of course $\theta^{1,0} = 1$, while $\theta^{2,0} < 1$ holds. The consequence of this observation is that as α transits from being just below $1 - (aR_1^2/R_2)$ to being just above $1 - (aR_1^2/R_2)$, an increased fraction of savings is used to initiate new capital investment. For this reason, local reductions in transactions costs that cause j* to increase also necessarily cause the capital stock to decline. Indeed, it is easy to show that the steady state equilibrium capital stock with j* = 2 is no less than that with j* = 1 iff

(50) $\alpha \leq 1 - (aR_1^2/R_2) [R_2/(R_2 - R_1)]^2 < 1 - (aR_1^2/R_2)$.

Equation (50) describes how low α must be in order for the use of technology 2 *not* to be associated with a reduction in the steady state equilibrium capital stock.

C. Steady State Welfare   (J = 2)

We now examine how steady state welfare varies with α. The answer to this question involves the consideration of two factors. First, since the real wage rate is just b and since all young period income is saved, one component of old period consumption is simply γb. Moreover, since

$$\gamma = \max \{aR_1, [aR_2(1 - \alpha)]^{0.5}\},$$

clearly the choice of a transactions fee can affect γ. Second, the choice of α can affect the lump-sum transfer received by old agents. Recall that the real value of financial transactions per capita is given by $b(1 - \theta^{j^*,0})$. In addition, all agents pay a fee of α per transaction, with transactions measured in real terms. Thus the transfer received by an old agent in real terms is given by $\alpha b(1 - \theta^{j^*,0})$, and steady state welfare is given by

$$U = b\max \{aR_1, [aR_2(1 - \alpha)]^{0.5}\} + \alpha b(1 - \theta^{j^*,0}).$$

If $j^* = 1$ [that is, if $\alpha > 1 - (aR_1^2/R_2)$], then $\theta^{j^*,0} = \theta^{1.0} = 1$, and steady state utility is just $U_1 = baR_1$. On the other hand, if $j^* = 2$, then

$$\theta^{j^*,0} = \theta^{2.0} = \{1 - [\gamma/(1 - \alpha)]\}/\{1 - [\gamma/(1 - \alpha)]^2\} =$$

$$\{1 + [\gamma/(1 - \alpha)]\}^{-1} = 1/\{1 + [aR_2/(1 - \alpha)]^{0.5}\}.$$

As a result, steady state welfare is given by

(51) $U_2(\alpha) \equiv b[aR_2(1 - \alpha)]^{0.5} + \alpha b [1 - 1/\{1 + [aR_2/(1 - \alpha)]^{0.5}\}] = b[aR_2(1 - \alpha)]^{0.5} +$

$\alpha b\{[aR_2/(1 - \alpha)]^{0.5}/\{1 + [aR_2/(1 - \alpha)]^{0.5}\}\} = b[aR_2(1 - \alpha)]^{0.5} \{1 +$

$\alpha/(1 - \alpha) \{1 + [aR_2/(1 - \alpha)]^{0.5}\}\} = b[aR_2(1 - \alpha)]^{0.5} \{1 + [aR_2(1 - \alpha)]^{0.5}\}/\{1 -$

$\alpha + [aR_2(1 - \alpha)]^{0.5}\} = b(aR_2)^{0.5} \{1 + [aR_2(1 - \alpha)]^{0.5}\}/\{(1 - \alpha)^{0.5} + (aR_2)^{0.5}\}.$

It is straightforward but tedious to show that $U_2'(\alpha) \geq (\leq)0$ iff $aR_2 \leq (\geq)1$. Thus if $aR_2 < 1$ holds, steady state welfare is increasing in the transactions fee for all $\alpha < 1 - (aR_1^2/R_2)$. Once $\alpha > 1 - (aR_1^2/R_2)$, steady state welfare is constant at U_1. If $aR_2 > 1$, then steady state welfare is declining in α for all $\alpha < 1 - (aR_1^2/R_2)$. We now consider each case separately.

1. **Case 1: $aR_2 < 1$.**

In this case, $U_2(\alpha)$ is maximized by setting $\alpha = 1 - (aR_1^2/R_2)$; the largest value of α consistent with $j^* = 2$. Evaluating (51) at this value of α yields

$$(52) \qquad U_2[1 - (aR_1^2/R_2)] = b(1 + aR_1)/[1 + (R_1/R_2)].$$

Then $U_2[1 - (aR_1^2/R_2)] \geq U_1 = baR_1$ holds iff

$$(53) \qquad 1 \geq aR_1^2/R_2,$$

or that is, iff the value of α that maximizes steady state welfare with $j^* = 2$ is positive. Thus, to summarize, if $aR_2 < 1$ *and* (53) hold, the welfare maximizing value of α is $1 - (aR_1^2/R_2) \geq 0$. If, on the other hand, $aR_2 < 1$ holds and (53) fails, $U_2(\alpha) < U_1$ for all α. Then welfare maximization dictates setting $\alpha > 1 - (aR_1^2/R_2)$. In either case, it is undesirable to make efforts to reduce transactions costs below $1 - (aR_1^2/R_2)$, and it is certainly undesirable to subsidize equity market transactions.

Of course if $aR_2 < 1$ holds, then the internal rate of return on technology 2 — gross of transactions costs — is less than the steady state growth rate of the economy. Here capital investments in technology 2 are socially unproductive. If (53) holds as a strict inequality as well, then this same statement is also true of investments in technology 1. As a consequence, steady

state welfare is maximized by setting the transactions fee in a way that *minimizes* the steady state equilibrium capital stock.

2. Case 2: $aR_2 > 1$.

In this case $U_2(\alpha)$ is decreasing in α, and is therefore maximized by setting α arbitrarily small. From (51),

$$\lim_{\alpha \to \infty} U_2(\alpha) = baR_2.$$

Thus if $aR_2 > 1$ and $R_2 > R_1$ both hold, it is optimal to maximally subsidize equity market transactions. If $aR_2 > 1$ and $R_2 < R_1$ hold, then it is welfare maximizing to set $\alpha > 1 - (aR_1^2/R_2)$, and to have $j^* = 1$. When $aR_2 > 1$ holds, the internal rate of return on investments in technology 2 (gross of transactions costs) exceeds the rate of growth of the economy, so that such investments are socially productive. When $R_2 > R_1$ also holds, technology 2 should be utilized, and steady state welfare maximization involves *maximizing* the steady state equilibrium capital stock.

3. Summary.

As the examples just given indicate, it can either be desirable to subsidize, or to heavily charge agents transacting in equity markets. It will be optimal to confront agents undertaking such transactions with relatively heavy fees when $aR_2 < 1$ or, in other words, when the internal rate of return on technology 2 (at a zero transactions cost level) is less than the real growth rate of the economy. It will also be optimal to impose high fees in these markets when $aR_2 > 1$ and $R_1 > R_2$ both hold. Thus, even if the internal rate of return on technology 2 exceeds the rate of growth (at a level of zero transactions costs), it is undesirable to use technology 2 if it is less

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productive than technology 1. However, when $aR_2 > 1$ (the internal rate of return on technology 2 exceeds the growth rate) and $R_2 > R_1$ (technology 2 is more productive than technology 1), there is good reason to subsidize equity market activity. However, as should be apparent from this discussion, an evaluation of the desirability of subsidizing equity market activity -- even in this simple example -- requires a good deal of knowledge about the internal rates of return on investments available to an economy.

These observations do suggest a criterion for determining when it is desirable to tax (or raise the costs faced by) equity market participants. A socially excessive volume of financial market transactions is undertaken in economies with relatively high levels of equity market activity ($j^* = 2$, so that $\theta^{2.0} < 1$), and with real interest rates (gross of transactions costs) less than the long-run real rate of growth of the economy ($aR_2 < 1$). In this situation, it is desirable to take actions to reduce the attractiveness of participating in equity markets.

V.   Some Final Thoughts

We have posed for ourselves the following question: how does the efficiency of an economy's capital resale, or equity markets -- as measured by the costs of transacting in them -- affect its efficiency in producing physical capital and, through this channel, final goods and services? In order to propose an answer to this question, we have followed Hicks (1969) in emphasizing the role of equity markets in providing liquidity to holders of long-lived and inherently illiquid capital. As the efficiency of an economy's capital markets increases (that is, as transactions costs fall), the general effect is to cause agents to make longer term, and hence more transactions intensive investments. The result is a higher rate of return on savings, as well as a

change in its composition. These general equilibrium effects on the composition of savings cause

agents to hold more of their wealth in the form of existing equity claims, and to invest less in the

initiation of new capital investments. As a result, a reduction in the resouce losses suffered in the

transactions process can cause the capital stock either to rise or to fall, and we have described

conditions under which each situation will obtain. However, a general point that bears emphasis

is that a reduction in transactions costs will typically alter the composition of savings and

investment, and that any analysis of the consequences of such changes must take these effects into

account.

As a practical matter, we would expect the costs of transacting in secondary capital

markets to be reflected in the term structure of asset yields observed in an economy. The yield to

maturity (gross of transactions costs) on investments in assets of type j^* is given by

$$\delta_{j^*} \equiv (P^{j^* j^*})^{1/j^*} \equiv \gamma / [\prod_{h=0}^{j^*-1} (1-\alpha^{j^*,h+1})]^{1/j^*}.$$

Similarly, the yield to maturity on assets of type j $(j \neq j^*)$[24] is given by $\delta_j \equiv (P^{j,j})^{1/j}$. If agents can

engage in short sales -- incurring normal transactions costs as they do so -- then it is easy to show

that δ_j $(j \neq j^*)$ satisifies

$$\gamma / [\prod_{h=0}^{j-1} (1-\alpha^{j,h+1})]^{1/j} \geq \delta_j \geq \gamma [\prod_{h=0}^{j-1} (1-\alpha^{j,h+1})]^{1/j}.$$

Then term premia (or yield spreads between maturies), which are simply given by $\delta_j - \delta_\ell$, reflect

the relative costs of transacting in different assets. As transactions costs increase we would

typically expect term premia to increase as well, so that the efficiency of an economy's financial

system would generally be reflected in the slope of its yield curve. Improvements in the

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functioning of financial markets should be expected to flatten the term structure of returns,[25] and

to have the other consequences we have noted.

It is also the case that governments and central banks often contemplate interventions

designed to affect the slope of the term structure. The consequences of such interventions -- or at

least of ones that work by affecting the transactions costs agents perceive -- can be analyzed via

the methods discussed in section IV. As suggested there, one possible outcome is that the

reduction of unnecessary transactions costs flattens the term structure, and allows socially more

productive investments to be undertaken.

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Footnotes

1. For documentation of this claim in historical or modern development contexts, see Cameron (1967) and McKinnon (1973) and Shaw (1973), respectively. For quantitative analyses of the experiences of a variety of economies, see Goldsmith (1969), Antje and Jovanovic (1992), or King and Levine (1993a, b).

2. For a discussion of various examples, see Galbis (1979), van Wijnbergen (1983, 1985), Diaz-Alejandro (1985), Khatkhate (1988), or Fry (1988).

3. The financial revolution is a term applied by Dickson (1967) to the rapid development of English financial markets in the first half of the eighteenth century.

4. Many details of this model are generalized in Bencivenga, Smith, and Starr (1994a,b,c).

5. This result would need to be qualified in more general models, like those of Bencivenga, Smith and Starr (1994a,c).

6. Note that we are abstracting both from heterogeneity, and population growth. Both are inessential simplifications that reduce notational requirements.

7. We thus abstract from any interesting labor supply or savings decisions on the part of households.

8. The use of a linear technology confronts the firm with an essentially trivial decision regarding the choice of factor inputs.

9. The assumption that all capital, however produced, is perfectly substitutable in production is relaxed by Bencivenga, Smith, and Starr (1994c).

10. Our assumptions on capital production technologies imply that capital investments are completely unproductive until they mature. This can be thought of as an "Austrian" model of investment. It is possible to alter the analysis to allow all capital investments to mature in one period, but to have capital produced via different technologies having different productive lifetimes. This, however, is a more complicated model, and we do not pursue it here.

11. Under this specification, transactions costs represent a pure resource loss. We consider below the alternative case in which transactions costs represent a pure transfer to market makers (or a tax paid to the government).

12. For example, as close to Diamond (1965) as possible. See also Azariadis (1992), chapter 13, or Galor and Ryder (1989).

13. Since there are no transactions costs in factor markets, this amounts to assuming that $\alpha^{j,j} = 0$, $\forall\, j = 1,..., J$.

14. That is, the price of mature CIP is simply the rental value of the associated capital.

15. We consider the possibility of multiple equilibrium capital production technologies in section III.B.

16. If all households were behaving identically, $\theta^{j,h} \equiv P^{j,h}\, S^{j,h}/w$ would hold.

17. Notice that $\theta^{j^*,h}\, w$ gives the value, in real terms, of the demand for type (j^*,h) CIP. Division by $P^{j^*,h}$ converts this demand into units of CIP.

18. The notion that transactions costs are larger for long maturity assets is certainly consistent with casual observation. For instance, the *Wall Street Journal* of July 23, 1993, reported a bid/ask spread on three month treasury bills of the previous day equal to 0.005 percent of price. For a thirty year treasury bond, this spread was 0.062 percent of price, while for a thirty year treasury strip (a pure discount instrument, equivalent to a long-term bill) the spread was 0.7 percent of price. Thus transactions costs vary by a factor of 100 with maturity alone, despite the fact that these observations ignore the obvious likelihood that a long-term instrument will be rolled-over many more times during its lifetime than a short-term instrument.

19. $H(z)$ is a function iff $j(z)$ is unique. We discuss below what happens when two or more technologies maximize the internal rate of return on investments.

20. See Bencivenga, Smith, and Starr (1994a) for a formal proof of this assertion.

21. When $j(z)$ is not unique, $G(z)$ consists of a vertical segment for the same reasons as before. In particular, if technologies ℓ and $n > \ell$ both maximize the internal rate of return on capital invesments, then agents can invest exclusively in technology ℓ, exclusively in technology n, or in convex combinations of the two technologies. As a result, the fraction of wealth held in the from of equity can lie anywhere in the interval $[G_{\ell}\{[a\,\tilde{R}_a(z)]^{1/a}\}, G_a\{[a\,\tilde{R}_a(z)]^{1/a}\}]$.

22. The assumption that resources collected in the form of fees or taxes are rebated to old agents prevents a transfer of the proceeds from those who bear these costs (by assumption sellers, or old agents) to those who do not (by assumption buyers, or young agents). A transfer of resources from old to young agents would, under our assumptions, raise the aggregate savings rate and, in and of itself, constitute a stimulus to capital formation. We, on the other hand, wish to isolate the effects of transactions costs alone. Therefore we rebate the proceeds of fee or tax collections to those who pay the fees or taxes. Given our preference assumptions, the result is that savings patterns are unaltered by the existence of fees/taxes.

23. We do not restrict α to be non-negative since, under the interpretation of α as a tax/subsidy, negative values of α correspond to the subsidization of equity market activity.

24. Of course these assets are held in zero net quantity in equilibrium.

25. For more detail on the term structure of asset yields in a model of this type, along with a precise statement of the claim just made, see Bencivenga, Smith, and Starr (1994c).

Appendix

A. Proof of Lemma 1.

Parts (a) – (c) follow immediately from the definition of H_j. Part (d) follows from using

L'Hospital's rule to evaluate $H_j(1)$. For part (e), straightforward differentiation yields

$$(A.1) \qquad H_j'(x) = x^{j-1} [j - (j + 1)x + x^{j+1}]/(1 - x^j)^2.$$

It is straightforward to show that, $\forall\, x \neq 1$, $j - (j + 1)\,x + x^{j+1} > 0$ holds. Thus $H_j'(x) > 0$, $\forall\, x > 0$

and $x \neq 1$. An application of L'Hospital's rule yields $H_j'(1) > 0$.

To establish part (f), note that $H_{j+1}(x) \leq H_j(x)$ is equivalent to

$$(A.2) \qquad (1 - x)x^{j+1}/(1 - x^{j+1}) \leq (1 - x)x^j/(1 - x^j).$$

Since $(1 - x)/(1 - x^{j+1}) > 0$ and $(1 - x)/(1 - x^j) > 0$ both hold, (A.2) is equivalent to

$$[x^{j+1}(1 - x^j) - x^j(1 - x^{j+1})]/(1 - x) = -x^j \leq 0,$$

which is obviously satisfied. $\square$

B. Proof of Lemma 2.

Part (a) is easily established directly using the definition of G_j. For part (c),

straightforward differentiation yields

$$(A.3) \qquad G_j'(x) = [1 - jx^{j-1} + (j - 1)x^j]/(1 - x^j)^2.$$

For $x \neq 1$, it can be shown that $1 - jx^{j-1} + (j - 1)x^j > 0$ holds; hence $G_j'(x) > 0$ $\forall\, x > 0$, $x \neq 1$.

For $x = 1$, repeated application of L'Hospital's rule yields $G_j'(1) > 0$.

For part (b), note that $G_{j+1}(x) \geq G_j(x)$ is equivalent to

$$(A.4) \qquad x(1 - x^{j-1})/(1 - x^j) \leq x(1 - x^j)/(1 - x^{j+1}).$$

Since $(1 - x^j)(1 - x^{j+1}) > 0$ holds, (A.4) is equivalent to the condition.

(A.4') $1 - x^{j+1} - x^{j-1} + x^{2j} \leq 1 - 2x^j + x^{2j}$.

(A.4'), in turn, reduces to

(A.4'') $x^{j-1}(x - 1)^2 \geq 0$,

which obviously holds. $\square$

C. Proof of Proposition 1.

(a) Suppose to the contrary that $z' > z$ holds, but that $j(z) > j(z')$. Let $j = j(z)$ and $j' = j(z')$. Then, by the definition of $j(z)$,

(A.5) $[a\tilde{R}_j(z)]^{1/j} \geq [a\tilde{R}_{j'}(z)]^{1/j'}$.

Moreover,

(A.6) $d/dz\,[a\tilde{R}_j(z)]^{1/j} = [a\tilde{R}_j(z)]^{1/j}\,[\tilde{R}'_j(z)/j\tilde{R}_j(z)] > d/dz\,[a\tilde{R}_{j'}(z)]^{1/j'} =$

$[a\tilde{R}_{j'}(z)]^{1/j'}\,[\tilde{R}'_{j'}(z)/j'\tilde{R}_{j'}(z)]$.

for all z, where the inequality follows from (A.5) and equation (29). But (A.5) and (A.6) imply that

(A.7) $[a\tilde{R}_j(z')]^{1/j} > [a\tilde{R}_{j'}(z')]^{1/j'}$.

However, (A.7) contradicts the definition of $j(z')$, establishing the result. $\square$

(b) Suppose that $j(z) = j(z')$. Then $\gamma(z') = [a\tilde{R}_{j(z)}(z')]^{1/j(z)} \geq [a\tilde{R}_{j(z)}(z)]^{1/j(z)} = \gamma(z)$, since $\tilde{R}'_{j(z)} \geq 0$. Moreover, if $j(z) > 1$, $\tilde{R}'_{j(z)} > 0$ holds, as does $\gamma(z') > \gamma(z)$.

If $j(z') > j(z)$, on the other hand, then

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$$\gamma(z') \equiv [a\tilde{R}_{j(z')}(z')]^{1/j(z')} > [a\tilde{R}_{j(z)}(z')]^{1/j(z)} \geq [a\tilde{R}_{j(z)}(z)]^{1/j(z)},$$

where the first inequality follows from the definition of j(z), and the second from $\tilde{R}'_{j(z)} \geq 0$. This

establishes the claim. $\square$

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Figure 1: Capital Accumulation and Transactions Costs
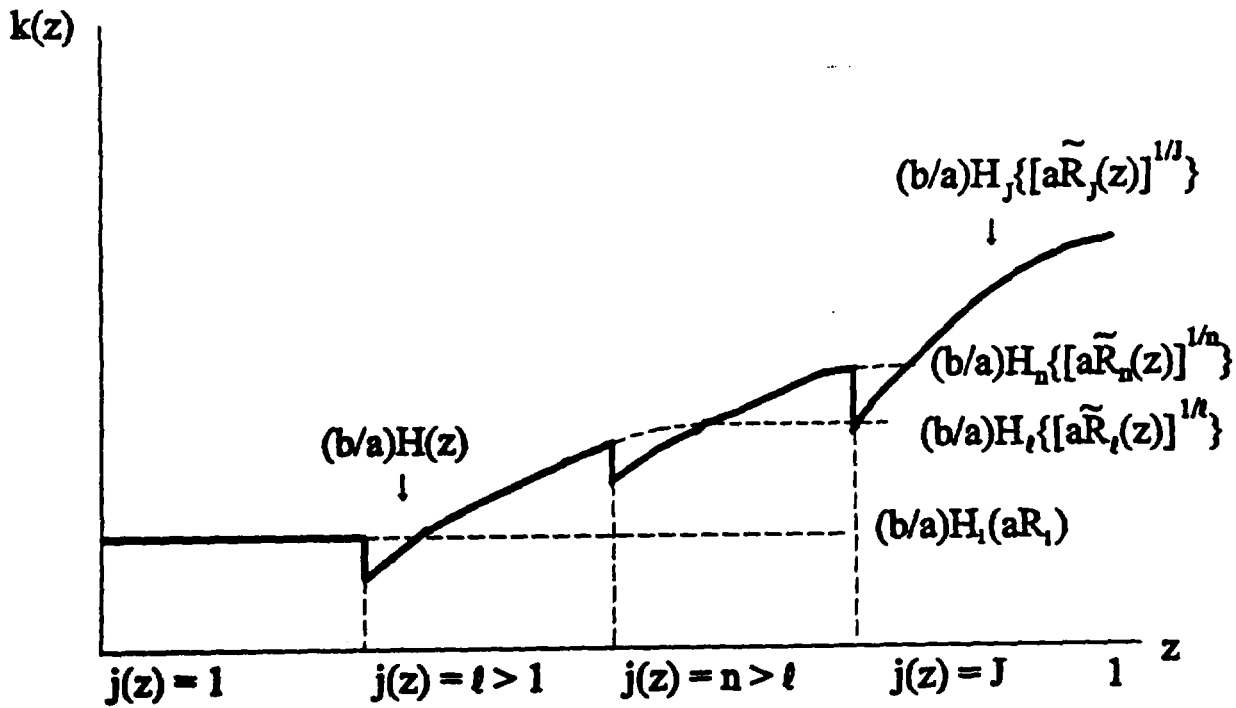

For small values of z, j(z) = 1. As z increases, the equilibrium maturity of capital investments also rises; first to ℓ, then to n > ℓ, and ultimately to J. (b/a)H(z) gives the steady state capital stock at each value of z.

Figure 2: The Fraction of Wealth Held in Equities and Transactions Costs

G(z)

1

$G_J[\gamma(z)]$

$G_n[\gamma(z)]$

G(z)

$G_\ell[\gamma(z)]$

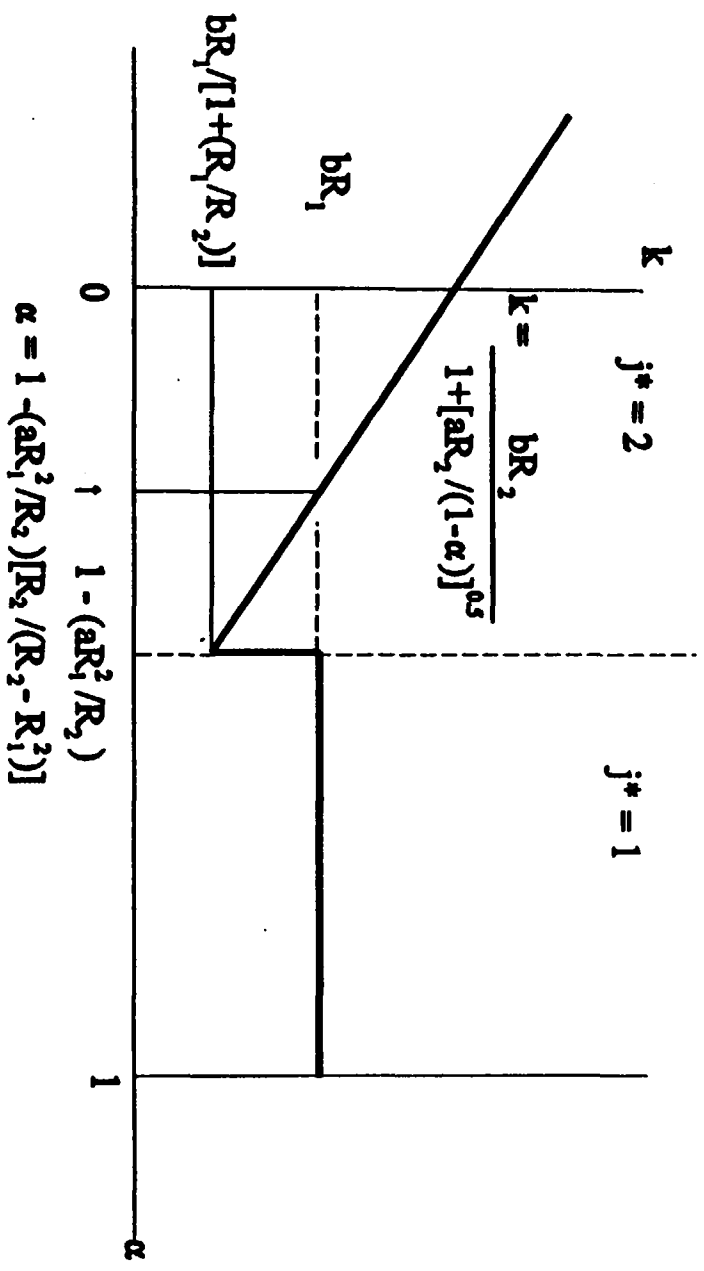j(z) = 1   j(z) = ℓ > 1   j(z) = n > ℓ j(z) = J   1

z
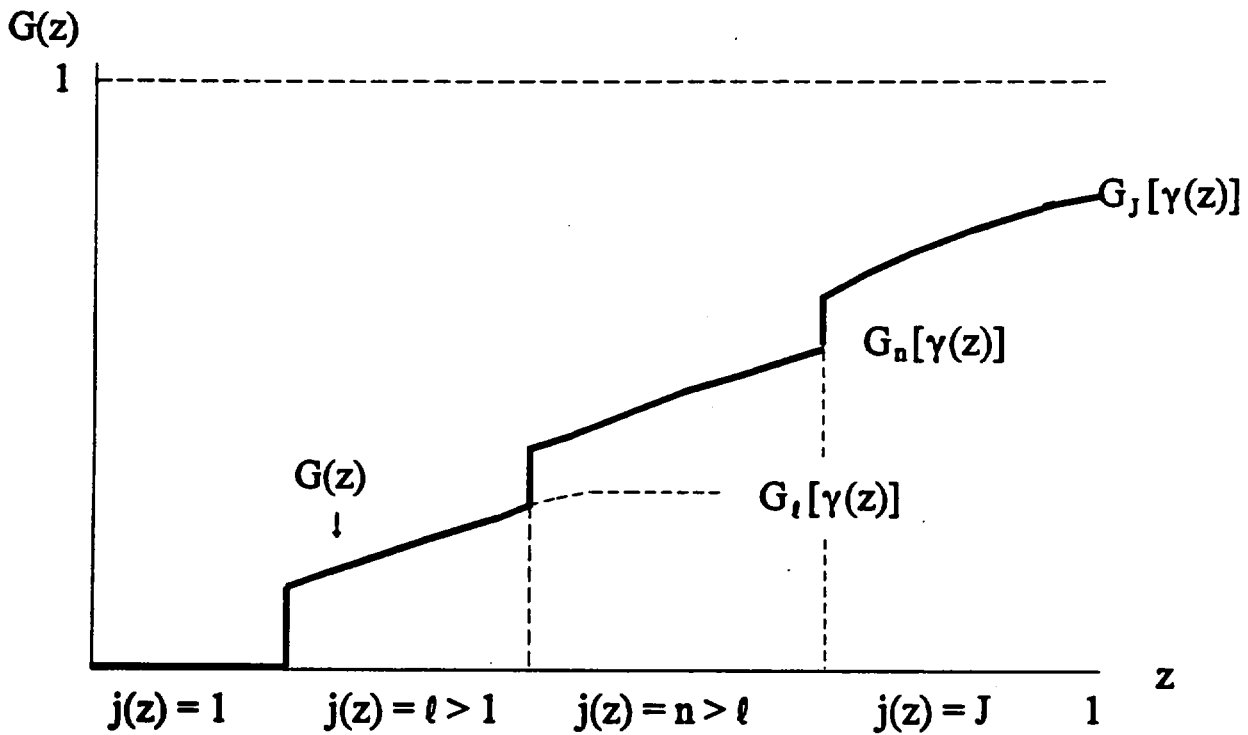
As in figure 1, at low values of z, j(z) = 1. As z increases the equilibrium maturity of capital investments will also increase; first to ℓ, then to n > ℓ, and ultimately to J. G(z) gives the fraction of wealth held in the form of CIP (or equity claims) at each value of z.

Figure 3: Example 2
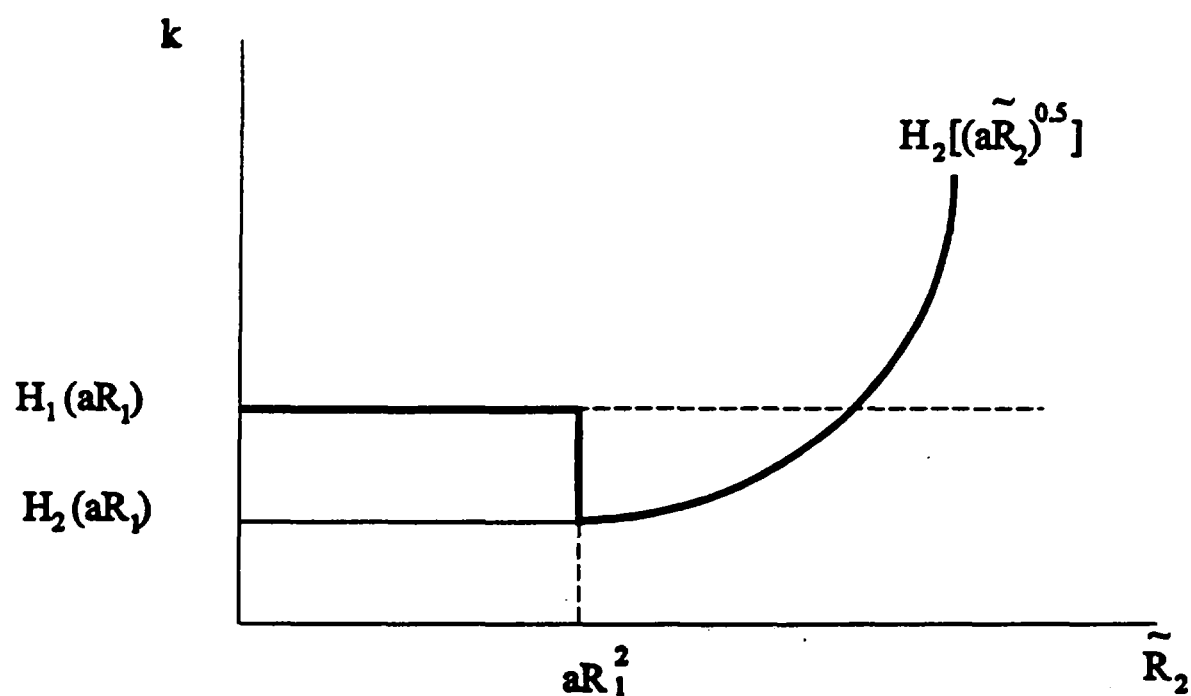
When $\tilde{R}_2 < aR_1^2$, $j^* = 1$ and k is independent of $\tilde{R}_2$. When $\tilde{R}_2 > aR_1^2$, $j^* = 2$ and increases in $\tilde{R}_2$ result in a higher steady state capital stock.

Figure 4: Transactions Costs as Fees

$bR_1/[1+(R_1/R_2)]$

bR_1

$k = \dfrac{bR_2}{1+[aR_2/(1-\alpha)]^{0.5}}$

k $j^* = 2$ $j^* = 1$

0   1 $1-(aR_1^2/R_2)$ 1

$\alpha = 1-(aR_1^2/R_2)[R_2/(R_2-R_1^2)]$

When $\alpha > 1 - (aR_1^2/R_2)$, $j^* = 1$. When $\alpha < 1 - (aR_1^2/R_2)$, $j^* = 2$. Here k is decreasing in α.

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