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The Precautionary Demand for Commodity Stocks

Boum-Jong Choe

Producers' stockholding and hedging decisions are a precautionary behavior against output and price risks. Commodity-exporting developing countries that face these risks should typically hold small stocks and hedge a large part of their expected supplies.

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This paper — a product of the International Trade Division, International Economics Department — is part of a larger effort in the department to explain commodity price behavior and model the global markets for primary commodities. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Sarah Lipscomb, room S7-062, extension 33718 (July 1992, 38 pages).

Choe shows producers' stockholding and hedging decisions as a precautionary behavior against output and price risks. The traditional view is that producer stocks are held for their *convenience yield*. Choe's approach explains recent just-in-time inventory management and allows unified treatment of the precautionary and speculative demands for stocks and the use of futures contracts.

Choe also assumes a more sensible preference function so that demand functions for stocks and futures are nonlinear. Stocking and hedging decisions, which are interdependent, are solved simultaneously. As a result of these refinements, the optimal decision rules are significantly different.

Several useful results emerge from Choe's analysis:

• When both output and price risks exist, stocks and futures can be combined to reduce the overall exposure to risks (measured by the precautionary premium or units of output the producer is willing to pay for eliminating risks).

In an unbiased futures market (futures price equals expected spot price at the maturity of the futures contract), commodity producers should short-hedge (sell futures contracts) less than the expected available supplies, if output and price risks are negatively correlated. And they should short-hedge more than the expected available supplies, if those risks are positively correlated. When the futures price deviates from the expected spot price (futures price bias), speculative trading dominates producers' futures positions. The demand for futures is highly sensitive to the futures price bias, while the demand for stocks is not.

• It is well-known that commodity-exporting developing countries face great price risk and — particularly with agricultural commodities — uncertain output as well. The optimal stocking and hedging rules Choe derives could have practical applications for these countries.

Earlier analyses that considered only the hedging problem typically suggest relatively low optimal hedge ratios (the proportion of expected available supplies that is short-hedged); this ratio was also insensitive to expected available supplies and to the degree of risk aversion.

The optimal decision rules Choe derives suggest that the optimal hedge ratio is likely to be much higher than ratios given in earlier studies. It depends on initial endowments, output and price expectations, and the degree of absolute risk aversion.

The Precautionary Demand for Commodity Stocks

by
Boum-Jong Choe

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THE PRECAUTIONARY DEMAND FOR COMMODITY STOCKS

I. Introduction

This paper develops a theory of the precautionary demand for commodity stocks. It posits that commodity stocks are held for precautionary purposes by producers, consumers, and intermediate processors, while speculators hold stocks on the expectation of capital gains from a subsequent price rise. Producer and consumer stocks usually account for the largest share of commercial stocks[1] held at any point in time. For example, at the end of 1990, stocks held by producers and consumers of copper were 72 percent of all commercial stocks of the market economy countries. Yet, the theory explaining the behavior of this class of stocks has not progressed much beyond the concept of *convenience yield*, first introduced by Kaldor (1939). This paper proposes an alternative theory. Holding of stocks by producers and consumers is viewed as precautionary behavior towards output and price risks. As a theory of behavior towards risks, the precautionary stock demand model encompasses speculative demand by both producers and consumers. Furthermore, both stocks and futures are treated as precautionary instruments, in contrast to the dichotomy that only stocks provide convenience yield while futures are hedging instruments.

Convenience yield has long been the standard explanation for positive stock holding by producers and consumers when prices are expected to fall. It has been broadly construed that stocks provide a service by just being there and therefore command a premium over their market value. The precise meaning of the concept, however, has not been made clear. According to Kaldor, "the yield of stocks of raw materials ... consists of '*convenience*', the possibility of making use of them the moment they are wanted." Working (1948, 1949) and Telser (1958)

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[1] Stocks held by governments (such as U.S. strategic stockpiles) or international organizations (such as the buffer stocks of the International Rubber Organization) are considered as noncommercial stocks.

advanced the view that the mere presence of adequate stocks could lower the costs of producing a given level of output. Telser further suggested that "holding stocks permits the rate of production or sales to be *varied* at lower cost than would be incurred if the firm attempted to purchase stocks as they were needed." On the other hand, Brennan (1958) and Cootner (1967) emphasized the marketing aspect; that larger stocks allow *flexibility* in adapting to changing market conditions and hence could result in greater revenue. The Working and Telser interpretation applies to stocks of inputs while that of Brennan and Cootner relates to stocks of outputs. Regardless, they share the view that stocks impart a service by just being there, and this service commands a premium.

The convenience yield theory has not advanced much beyond conjecture. Convenience yield has been presumed to be an increasing function of stock levels, with the marginal yield diminishing rapidly as the stock level increases. However, there has not been any attempt to offer a microeconomic foundation to the theory. As a result, it has not been clear what factors might determine the shape of the yield curve, nor in what units it might be measured. Further, should the yield vary between commodities and over time?

Newbery and Stiglitz (1981, p. 196) attempted to define convenience yield in terms of risk behavior, in essence, as the amount a risk-averse agent is willing to pay for the marginal unit of stock in excess of that of a risk-neutral agent. According to this definition, convenience yield can be either positive or negative, depending on the sign of the covariance between price and marginal utility. However, negative convenience yield runs counter to the concept espoused by earlier writers and also cannot explain stockholding at times of expected price declines.

In this paper, we begin by positing that producers hold stocks as a precaution against unexpected variations in the output of the commodity. The greater the quantity of stocks agents hold, the more secure they feel about future uncertainty and the less willing they are to pay for the risk. The reduction in the precautionary premium due to an additional unit of stocks (the marginal precautionary premium) is expected to decline as the stock level increases, in much the same way as the marginal convenience yield does. Both the marginal precautionary premium and the marginal convenience yield should approach zero as stocks increase to a sufficiently high level.

However, the recent advent of just-in-time inventory management practices suggests that as long as supplies are available with little chance of disruption, there is no need to keep a substantial quantity of stocks. The increased use of this practice suggests that stocks in and of themselves do not render productive services or cost savings as presumed by the convenience yield theorists -- at least not large enough to cover the costs of stock carryover. Stocks can be pared down to the bare minimum if the possibility of disruptions to the smooth operation of the firm can be essentially eliminated. The implication to be drawn from this example is that precautionary demand theory is likely to have more explanatory power for real world data than the convenience yield theory.

The precautionary stock demand model is a two-period expected utility maximization model under stochastic output and prices, with or without futures trading. The non-negativity constraint on stocks and the concavity of the utility function imply a convex demand function for stocks, very much in line with what the proponents of convenience yield had in mind.

The model has rich policy implications for the commodity-exporting developing countries in answering questions such as the following: What is the optimal level of physical stocks these countries should hold and how is this affected by expectations and risk factors? To what extent should these countries hedge their future output and stocks using futures contracts? Explicit algebraic solutions to these questions are provided; such solutions are not elsewhere available in the literature.

The theory also has implications for commodity price determination, including the relationship between expected spot and futures prices. Physical stocks provide the linkage between the current and expected spot prices of commodities, while the futures market determines the relationship between futures and expected spot prices. The model solutions suggest that normally futures markets will be biased when both output and price risks are present. The direction of the bias, however, is not certain; the precautionary stock demand theory does not necessarily support the theory of *normal backwardation* (the phenomenon that futures prices tend to be lower than the expected spot price) despite the presence of a positive precautionary premium.

This paper is divided into five sections. The next section considers the stockholding problem of a competitive producer that faces only output risk. Section III extends the analysis to the case where agents face both output and price risks. Section IV introduces the possibility of trading in futures contracts as well as in physical stocks. The last section concludes the paper.

II. Competitive Producer with Output Risk

Consider the case of a competitive commodity producer who has to decide how much of current output to carry over to the next period. The decision is complicated by the fact that the producer's expected output in the next period is subject to random variation. The uncertainty of output is particularly great for agricultural commodities, but the problem exists for most other products in one degree or other. For the sake of simplicity, we assume in this section that the producer knows with certainty the next-period spot price of the commodity. The competitiveness assumption allows us to derive the producer's storage rule taking prices as given.

Output uncertainty is assumed to arise from purely random factors, such as weather and accidents. It raises problems for producers in at least two ways. First, given the price output volatility implies revenue volatility. A large revenue shortfall, if unprepared for, can necessitate a producer having to make costly adjustments to planned operations. Second, the producer has to cope with the inability to satisfy customer demands or meet other contractual obligations. Possible consequences of failing to meet these demands include damage to the firm's reputation as a reliable supplier; alternatively, the cost of quickly finding substitutes could be high. One way of dealing with a potential output shortfall is to maintain a cushion of physical stocks. This is an age-old method. Grains have been routinely stored as a precaution against bad harvests (an old example is the story of Joseph in Egypt). A more recent example is the strategic stockpiling of petroleum in the wake of the oil price shocks. The critical question in all of these cases is what should be the optimal level of such stocks.

Consider a producer that operates for only two periods, the current period (period 0) and the next period (period 1). At the end of the current period, the producer knows the quantity produced during that period, Q_0, and the stocks carried over from the preceding period, I_0. Let $A_0 = Q_0 + I_0$, the total amount of the commodity available to the producer at the end of the current period. Before the beginning of the next period, the producer has to decide how much of A_0 to sell and how much to carry over to the next period. If the producer sells S_0, then stocks carried over to period 1 are $I_1 = A_0 - S_0$. The equality implies that commodities always have a positive value and therefore are not wasted.

As of period 0, the producer's output in period 1 is an unknown quantity, $\tilde{Q}_1$, where the tilde indicates that it is a random variable. We assume that $\tilde{Q}_1 = \overline{Q}_1 + \theta$, $E(\theta) = 0$, $E(\theta^2) = \sigma_\theta^2$. E(.) is the expectation operator. In period 1, the firm will have at its disposal the realized output during that period, plus stocks carried over from the previous period, minus storage costs and wastage. Thus, $\tilde{A}_1 = \tilde{Q}_1 + \delta I_1$, $0<\delta<1$, where $1-\delta$ is the proportion lost due to wastage and storage costs. Because of the two-period construction, we assume that the producer sells the entire quantity of commodities at hand before the end of period 1; i.e., $S_1 = A_1$.[2]

The producer's revenue in each period is equal to the quantity of sales times the price. We assume that the producer consumes all his revenues in each period. Let C_0 and C_1 denote the producer's consumption in the respective periods. Then,

$$\begin{aligned} C_0 &= P_0 S_0 = P_0 (A_0 - I_1), \\ \tilde{C}_1 &= P_1 \tilde{S}_1 = P_1 (\tilde{Q}_1 + \delta I_1). \end{aligned} \tag{1}$$

The producer chooses the level of I_1 so as to maximize

$$\begin{gathered} Max \;\; J = \mathrm{U}(C_0) + \beta \mathrm{E}_0 \, \mathrm{V}(\tilde{C}_1), \;\; 0<\beta<1, \\ s.t. \;\; (1), \; 0 \le I_1 \le A_0, \\ I_0, \, Q_0, \, \overline{Q}_1, \, P_0, \, P_1 \;\; given. \end{gathered} \tag{2}$$

U and V are period 0 and period 1 utility functions, respectively. E_0 denotes expectation based on information available in period 0, and β is the time rate of discount. The utility functions are assumed to be continuously differentiable and are strictly concave; i.e., $U', V' > 0$; $U'', V'' < 0$; $U''', V''' > 0$. It is assumed that the producer can neither borrow nor lend physical stocks.

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[2] If the producer remains in business after period 1, it would be necessary to allow for stock carryover to period 2. This essentially calls for an extension to the multi-period case. Such an extension, however, has been feasible only for a restricted class of utility functions that imply undesirable risk behavior. Kimball (1990b) has shown that solutions of a two-period optimal savings model closely mimic those of its multi-period counterpart. The two-period model examined in this paper can also be viewed as the optimization problem of the final two periods of a finite-horizon multi-period model. In fact, a numerical solution of the multi-period problem can be obtained by using the two-period stocking rule backwards iteratively, starting from the terminal year.

Differentiating (2) with respect to I_1 and setting the result equal to zero yields the first order condition for the maximum:

$$
\begin{aligned}
P_0 U'(C_0) &\geq \beta\delta P_1 \, E_0 \, V'(\tilde{C}_1), \\
&= \quad if \quad 0<I_1<A_0,
\end{aligned}
\tag{3}
$$

where A_0 is assumed to be a positive number. When an interior solution obtains, the marginal utility of consumption in the current period equals the expected marginal utility of consumption in the next period discounted by the discount factor, storage cost, and price differential.

When the expected next-period output is sufficiently large compared with the current availability, it is possible to have a corner solution at $I_1=0$. Actually, the producer may wish to carry negative physical stocks, if possible. Thus, at $I_1=0$, the non-negativity constraint would have a positive shadow price, equivalent to the net gain in utility by increasing current consumption beyond A_0 (by borrowing from next-period output). The other possible corner solution is $I_1=A_0$, but this can be shown to be suboptimal. Under the assumption that U(.)=V(.), if $I_1=A_0$, then,

$$
\begin{aligned}
J \,\big|_{I_1=A_0} &= U(0)+\beta E_0 V(\tilde{Q}_1+\delta A_0) \\
&\leq \beta E_0 V(\tilde{Q}_1)+\beta V(A_0) \\
&\leq U(A_0)+\beta E_0 V(\tilde{Q}_1) = J \,\big|_{I_1=0}.
\end{aligned}
$$

Thus, under the circumstances postulated, $I_1=A_0$ is suboptimal and therefore will not be attained.

A. *Precautionary Premium*

To focus on the producer's behavior towards risk, it is useful to introduce the concept of *precautionary premium*. The concept also helps us solve the stochastic equation in (3). Write out the expectations term of (3) and postulate that there is a premium, π, in units of next-period output, that the producer is willing to pay to remove the output uncertainty; i.e.,

$$V'[P_1(\bar{Q}_1-\pi+\delta I_1)] = E_0 V'[P_1(\bar{Q}_1+\tilde{\theta}+\delta I_1)]\,. \tag{4}$$

In the terminology of Kimball (1990), π is called the *equivalent precautionary premium*, in the sense that the *certainty equivalent* of $\tilde{Q}_1$ is $\bar{Q}_1-\pi$.

The above definition of precautionary premium is a direct extension of Arrow and Pratt's *equivalent risk premium*. An approximate expression for π can be found in much the same way Pratt (1964) used to derive an approximate formula for equivalent risk premium. Applying Taylor series expansion on both sides of (4) and rearranging terms yields

$$\pi(P_1,\bar{Q}_1,I_1,\sigma_\theta) \approx -\frac{1}{2}\sigma_\theta^2 P_1\frac{V'''(\bar{C}_1)}{V''(\bar{C}_1)}, \tag{5}$$

where $\bar{C}_1=(\bar{Q}_1+\delta I_1)P_1$. Note that $\pi>0$ as long as V is strictly concave.

The precautionary premium measures the sensitivity of the decision variable, I_1, to risk, whereas the risk premium measures the degree of risk aversion. Kimball defines $-V'''/V''$ as the index of *absolute prudence*, a measure of precautionary motive, in much the same way Arrow-Pratt defined $-V''/V'$ as the index of absolute risk aversion. The higher the absolute prudence, the higher the precautionary premium, and the higher the precautionary demand for stocks.

Kimball shows that $-V'''/V'' \geq -V''/V'$, if the index of absolute risk aversion declines as income increases. Thus, under the declining absolute risk aversion utility function, considered by many as more realistic than the case of constant absolute risk aversion, absolute prudence is greater than absolute risk aversion. Therefore, the precautionary motive to hold stocks could be quite strong.

Convenience yield represents cost savings or revenue gains due to the (intangible) services provided by stocks. As such, it is conceptually different from the precautionary premium which specifically limits the service provided by stocks to risk reduction. Since stockholding is a precautionary behavior toward risk, a risk-neutral agent will not hold precautionary stocks, nor will a risk-averse agent when there is no risk.[3] The convenience yield theory predicts stockholding in both situations. Convenience yield is a function of the current stock level while the precautionary premium is a function of the *next-period* output and price variability. As a result, the precautionary stock demand model has richer dynamic implications than the convenience yield model.

B. *Solution with the Constant Relative Risk Aversion Utility Function*

Substituting (4) into (3), the first-order condition becomes

[3] Newbery and Stiglitz (1981, p.196) show that a risk-neutral agent will hold stocks under output and price risks if

$$\xi = \frac{E\tilde{P}_1 V'}{P_1 EV'} - 1$$

is positive. They define ξ as the measure of convenience yield. However, according to this definition, a risk-neutral agent will not hold stocks if there is only output risk and no price risk, because ξ becomes zero. Clearly, this definition is not general enough to make convenience yield a theory of stock holding under uncertainty.

$$P_0\, U'[P_0(A_0-I_1)] = \beta\delta\, P_1\, V'[P_1(\overline{Q}_1+\delta I_1-\pi)]\,. \tag{6}$$

From here on, we ignore the fact that π will be approximated by (5). Note that (6) is a deterministic equation; the use of the precautionary premium allowed us to convert a stochastic equation into a deterministic one. This is, in fact, the standard method of solving stochastic dynamic optimization problems.

Equation (6) can be solved for I_1 under specific assumptions about the utility function. In this and subsequent sections we assume that the relevant utility function exhibits constant relative risk aversion (CRRA), which takes the form:

$$U(C) = \frac{C^{1-\lambda}}{1-\lambda}, \quad -\infty<\lambda<1. \tag{7}$$

The index of *relative* risk aversion is given by $-CU''/U'=\lambda$, which is constant. The index of *relative* prudence is given by $-CU'''/U''=1+\lambda$, which is also constant and greater than relative risk aversion. If $\lambda=0$, CRRA degenerates into a linear function, implying risk neutrality. If $\lambda=1$, CRRA becomes the logarithmic utility function that exhibits unitary relative risk aversion. If $\lambda<0$, the utility function becomes convex, implying that the agent is a risk lover. Henceforth, we limit our analysis to the risk-averse case, $0<\lambda<1$. Within this range, the higher the value of λ, the greater the degree of risk aversion and prudence. With CRRA, the indexes of absolute risk aversion (λ/C) and absolute prudence ($[1+\lambda]/C$) decline as consumption increases.

Two other important classes of utility functions widely used in the literature are the constant absolute risk aversion (CARA) utility function and the quadratic utility function. Most previous studies have used the CARA because it lends itself to mean-variance analysis and hence to explicit solutions for the demands for stocks and futures. However, because of the constant absolute risk aversion assumption, the resulting demand equations are linear in prices (see, for example, Newbery and Stiglitz). In the following sections, it will be seen that, with CRRA, demand equations are nonlinear in prices, which is intuitively more appealing than the linear version. The quadratic utility function is ruled out because it implies zero precautionary premium and therefore is unsuitable for modeling precautionary behavior.

For the sake of simplicity and without loss of generality, assume that U and V are both CRRA utility functions with the same λ. The precautionary premium in this case is approximated by

$$\pi \approx \frac{(1+\lambda)\,\sigma_\theta^2}{2(\bar{Q}_1+\delta I_1)}. \tag{8}$$

Given the index of absolute prudence, the precautionary premium is an increasing function of the variance of output and a declining function of the expected output in the next period and stock carryover.

In the case of CRRA, the first-order condition (6) for an interior solution is

$$P_0[P_0(A_0-I_1)]^{-\lambda} = \beta\delta P_1[P_1(\bar{Q}_1+\delta I_1-\pi)]^{-\lambda}. \tag{9}$$

Substitute (8) into (9) and rearrange the terms to get a quadratic equation in I_1, which has the following two real solutions:

$$I_1 = \frac{\delta A_0-(1+2\gamma\delta)\overline{Q}_1 \pm\sqrt{q}}{2\delta(1+\gamma\delta)}, \tag{10}$$

where

$$\gamma = (\beta\delta)^{-\frac{1}{\lambda}}\left(\frac{P_1}{P_0}\right)^{1-\frac{1}{\lambda}},$$

$$q = (\overline{Q}_1+\delta A_0)^2+2\delta\gamma(1+\delta\gamma)(1+\lambda)\sigma_\theta^2.$$

Non-negativity of I_1 requires that if a negative solution is obtained, then I_1=0. This constitutes a corner solution. Of the two solutions above, the one with the negative square root term produces a negative solution for I_1 for a wide range of reasonable values of variables and parameters. The other solution with the positive square root term mostly produces a positive solution but a negative solution is also possible for some extreme values of the variables and parameters. When two solutions of (10) are put together, the optimal solution will consist of a segment where I_1=0 (corner solution) and a segment where I_1>0 (interior solution). Henceforth, we focus on the interior solution.

A sufficient condition for obtaining an interior solution from (10) is $A_0>\beta\overline{Q}_1$. That is, producers will not carry stocks to the next period if the expected next-period output is sufficiently larger than the currently available supplies.

C. *Sensitivities*

What is the effect of output uncertainty on stockholding behavior? To answer this question, suppose that the producer has perfect foresight regarding next-period output. For such an agent, σ_θ=0 in equation (10) and it simplifies to

$$I_1^{pf} = \frac{A_0 - \gamma \bar{Q}_1}{1+\gamma\delta}, \tag{11}$$

where the superscript (pf) denotes the case of perfect foresight. Stock holding in this case is for the purpose of optimally allocating consumption over time in a certain world. The condition for zero stock carryover in a certain world is $A_0 = \gamma\bar{Q}_1$, or

$$\frac{P_1}{P_0} = \left(\frac{A_0}{\bar{Q}_1}\right)^{\frac{\lambda}{\lambda-1}} (\beta\delta)^{\frac{1}{\lambda-1}},$$

If $A_0 = \bar{Q}_1$ and the agent is risk neutral, then the above becomes

$$\frac{P_1}{P_0} = (\beta\delta)^{-1}. \tag{12}$$

Equation (12) is the intertemporal arbitrage condition for speculators in a certain world.

It can be easily verified that $I_1 - I_1^{pf} = \sqrt{G} - (\bar{Q}_1 + \delta A_0) > 0$. That is, the producer holds more stocks under output uncertainty than under output certainty. We call $I_1 - I_1^{pf}$ the precautionary stockholding, the size of which depends on $2\delta\gamma(1+\delta\gamma)(1+\lambda)\sigma_\theta^2$. Note that this term contains the numerator of the expression for the precautionary premium in (8). Thus, the larger σ_θ, the larger the precautionary premium and the larger the precautionary stock demand. This confirms the earlier statement that the precautionary premium measures the sensitivity of stockholding to output risk.

The relationship between optimal I_1 and A_0 implied by a complete model of a commodity market is known as the competitive storage equation, and has been the subject of investigation by Gustafson (1958), Deaton and Laroque (1990), and Williams and Wright (1991), among others. The precautionary stock demand model in (10) represents a competitive storage equation if prices in the equation clear the market. An important feature of the equation is that it is nonlinear in A_0. To see this more clearly, differentiate (10) twice with respect to A_0 to get

$$\begin{aligned} \text{MPS} = \frac{\partial I_1}{\partial A_0} &= \frac{1+(\overline{Q}_1+\delta A_0)\, q^{-\frac{1}{2}}}{2(1+\gamma\delta)} > 0, \\ \frac{\partial^2 I_1}{\partial A_0^2} &= \frac{\delta q^{-\frac{1}{2}}[1-(\overline{Q}_1+\delta A_0)^2 q^{-1}]}{2(1+\gamma\delta)} > 0, \end{aligned} \tag{13}$$

where the first partial derivative is recognized as the marginal propensity to store (MPS). MPS is positive and increasing as a function of A_0, confirming the nonlinearity.

An interesting question is how MPS changes in response to output risk. To find out, differentiate MPS with respect to σ_θ^2,

$$\frac{\partial^2 I_1}{\partial A_0 \partial \sigma_\theta^2} = -\frac{1}{2}\delta\gamma(1+\lambda)(\overline{Q}_1+\delta A_0)\, q^{-\frac{3}{2}} < 0, \tag{14}$$

where the negative value results from the declining absolute risk aversion property of CRRA. There are two different ways of interpreting (13). First, it says that MPS will be lower the higher the output risk. Second, the producer will be less sensitive to output risk the greater the initial endowment (or wealth). The second interpretation is intuitively easier to understand than the first, although the two are equivalent.

Analytic expressions for the sensitivity of stockholding with respect to the other variables and parameters are not simple enough to reveal their signs.

Partial derivatives of (10) with respect to $\bar{Q}_1, \delta, \beta, P_1$, for example, can be easily derived but their signs are not immediately clear. However, it should be expected that an increase in $\bar{Q}_1$ will reduce stock demand, while an increase in P_1 (a decrease in γ) will increase it. The higher the cost of storage (the smaller the δ) or the higher the time rate of discount (the smaller the β), the lower the demand for stocks. These expectations can be easily confirmed by numerical simulation for the appropriate range of values.

III. Competitive Producer under Output and Price Risks

In reality, producers usually have to deal with both output and price risks. When this is the case, the problem is complicated by the fact that the act of stock carryover in effect removes commodities from sale at a certain price in the current period for sale at an uncertain price in the next period. Stocks, therefore, increase exposure to the price risk, while reducing exposure to the output risk. The problem is to find the optimal combination of the two. For the moment, we ignore the possibility of hedging the price risk through futures trading.

Since Muth (1961), the competitive storage literature has focused on the speculative demand for stocks under price uncertainty. Little has been done about optimal stockholding under output and price risks, with or without futures trading. In this and the next section, we derive the optimal stockholding and hedging rules for risk-averse producers.

A. *The Precautionary Premium and the Optimal Stocking Rule*

For a price-taking competitive producer, the market price of a commodity may be taken as a random variable independent of his own sales or stock-holding decisions. We assume that $\tilde{P}_t = \overline{P}_t+\tilde{\varepsilon}$, where $E(\tilde{P}_1)=\overline{P}_1$, $E(\tilde{\varepsilon})=0$, $E(\tilde{\varepsilon}^2)=\sigma_\varepsilon^2$. We do, however, allow for certain covariance between output and price disturbances; i.e., $E(\tilde{\theta}\tilde{\varepsilon})=\rho\sigma_\theta\sigma_\varepsilon$. [4]

With output and price risks, the next-period consumption is expressed as

$$\tilde{C}_1 = \tilde{P}_1\tilde{S}_1 = \tilde{P}_1(\tilde{Q}_1+\delta I_1),$$

while the current-period consumption remains the same as in (1). The two-period optimization problem in this case can be stated exactly as in (2). The first-order condition for the optimum is

$$\begin{aligned} P_0\, U'(C_0) &\geq \beta\delta\, E_0\, \tilde{P}_1 V'(\tilde{C}_1), \\ &= \text{ if } \quad I_1 \neq 0. \end{aligned} \tag{15}$$

When both price and output risks are present, the equivalent precautionary premium is defined as the payment in units of output the agent is willing to make for having both $\tilde{\theta}$ *and* $\tilde{\varepsilon}$ eliminated from (15). Let π^* represent the equivalent precautionary premium defined by

$$\overline{P}_1 V'[\overline{P}_1(\overline{Q}_1-\pi^*+\delta I_1)] = E_0\ (\overline{P}_1+\tilde{\varepsilon})V'[(\overline{P}_1+\tilde{\varepsilon})(\overline{Q}_1+\tilde{\theta}+\delta I_1)], \tag{16}$$

where the supercript (*) stands for the case when output and price risks exist. In order to derive the approximate expression for π^*, apply the Taylor-series

[4] Even under competitive conditions, a producer's output level could show non-zero correlation with market prices if the random disturbance on output applies to a large segment of producers; bad weather is a good example. An extension of the model to the case where the market price depends on the producer's sales decision or actual output level in the second period will complicate the algebra but not the substantive part of the analysis.

expansion on both sides of (16) around $\pi^*=0$, $\tilde{\theta}=0$, $\tilde{\varepsilon}=0$ to get

$$\bar{P}_1 V'[\bar{P}_1(\bar{Q}_1+\delta I_1-\pi^*)] \approx \bar{P}_1 V'(\bar{C}_1)-\pi^*\bar{P}_1^2 V''(\bar{C}_1),$$

$$E_0(\bar{P}_1+\tilde{\varepsilon})V'[(\bar{P}_1+\tilde{\varepsilon})(\bar{Q}_1+\tilde{\theta}+\delta I_1)] \approx$$
$$\bar{P}_1 V'(\bar{C}_1)+E_0(\tilde{\theta}\tilde{\varepsilon})[2\bar{P}_1 V''(\bar{C}_1)+\bar{P}_1\bar{C}_1 V'''(\bar{C}_1)]$$
$$+\frac{1}{2}E_0(\tilde{\theta}^2)\bar{P}_1^3 V'''(\bar{C}_1)+\frac{1}{2}E_0(\tilde{\varepsilon}^2)(\bar{Q}_1+\delta I_1)[V''(\bar{C}_1)+\bar{C}_1 V'''(\bar{C}_1)],$$

and solve for π^*

$$\pi^* = -\left(\frac{2\rho\sigma_\theta\sigma_\varepsilon}{\bar{P}_1}+\frac{\sigma_\varepsilon^2\bar{C}_1}{\bar{P}^3}\right)-\left(\frac{\rho\sigma_\theta\sigma_\varepsilon\bar{C}_1}{\bar{P}_1}+\frac{\sigma_\varepsilon^2\bar{C}_1^2}{2\bar{P}_1^3}+\frac{\sigma_\theta^2\bar{P}_1}{2}\right)\frac{V'''(\bar{C}_1)}{V''(\bar{C}_1)}, \tag{17}$$

where $\bar{C}_1=\bar{P}_1(\bar{Q}_1+\delta I_1)$. The precautionary premium for output and price risks is larger (smaller) than the sum of precautionary premia for output and price risks separately, if the covariance between output and price risks is positive (negative).

To derive a closed-form solution for the optimal stocking rule, we assume a CRRA utility function. Using $-\bar{C}_1 V'''/V''=1+\lambda$ in (18), π^* for CRRA is

$$\pi^* = -\frac{\rho\sigma_\theta\sigma_\varepsilon(1-\lambda)}{\bar{P}_1}-\frac{\sigma_\varepsilon^2(1-\lambda)\bar{C}_1}{2\bar{P}_1^3}+\frac{\sigma_\theta^2(1+\lambda)\bar{P}_1}{2\bar{C}_1}. \tag{18}$$

The optimal solution for I_1 can be obtained from substitution of (18) into (16), and the resulting expression into (15). The result is again a quadratic equation in I_1, which has two real solutions

$$I_1^* = \frac{\delta A_0-(1+2a\gamma^*\delta)\bar{Q}_1-b\gamma^*\delta\pm\sqrt{Q^*}}{2\delta(1+a\gamma^*\delta)}, \tag{19}$$

where

$$Q^* = [\bar{Q}_1+\delta(A_0-b\gamma^*)]^2+4c\gamma^*\delta(1+a\gamma^*\delta),$$

$$\gamma^* = (\beta\delta)^{-\frac{1}{\lambda}}(\frac{\bar{P}_1}{P_0})^{1-\frac{1}{\lambda}},$$

$$\delta_s = \frac{\sigma_s}{\bar{P}_1},$$

$$a = 1+\frac{\delta_s^2(1-\lambda)}{2},$$

$$b = \rho\delta_s\sigma_\theta(1-\lambda),$$

$$c = \frac{\sigma_\theta^2(1+\lambda)}{2}.$$

By the same logic as in the case of equation (10), the optimal interior solution is likely to be given by the solution with a positive sign for the square root term in (19).[5] Note that if only the output risk is present, (19) collapses to (10).

B. *Speculative Demand for Stocks*

When both price and output are uncertain, the producer's demand for stocks consists of two components: the precautionary part and the speculative part. If the price is expected to rise, stocks provide the opportunity for profits as well as insurance against an output shortfall. In this case, both speculative and precautionary demands will be positive. If the price is expected to decline, the speculative demand will be negative while the precautionary demand will still be

[5] This statement requires the assumption that $0<\lambda<1$, $a>0$ and $b>0$. This shows it is unlikely that we can obtain a positive solution for I_1 with $-\sqrt{Q^*}$ in (19).

positive. To what extent will the speculative behavior affect the precautionary demand?

The terms in (19) do not easily separate into the two components. To measure each component, assume that the producer faces only the price risk and that the output level is known with certainty. Output certainty implies $\sigma_\theta=0$ and, hence, $b=c=0$. Then, (19) reduces to

$$I_1^{pr}=\frac{A_0-a\gamma^*\overline{Q}_1}{1+a\gamma^*\delta}, \tag{20}$$

where the superscript (pr) denotes the case of price risk only. Subtract (20) from (19) to get

$$I_1^{pc} = I_1^*-I_1^{pr}=\frac{-(\overline{Q}_1+\delta A_0+b\gamma^*\delta)+\sqrt{q^*}}{2\delta(1+a\gamma^*\delta)},$$

where the superscript (pc) denotes the precautionary component against the output risk.

Not all of I_1^{pr} is the speculative component of the stock demand because the agent would hold some stocks even at zero expected profits. To separate the speculative component from (20), suppose the arbitrage condition (12) holds under perfect foresight. Then, (20) reduces to

$$I_1^{ia}=\frac{A_0-(\beta\delta)^{-1}\overline{Q}_1}{1+\beta^{-1}}, \tag{21}$$

where the superscript (ia) denotes stock demand purely for the purpose of intertemporal allocation of consumption. Stock demand represented by (21) contains zero speculative element. Define

$$I_1^{sp} = I_1^{pr} - I_1^{ia} = \frac{(\overline{Q}_1+\delta A_0)\,[(\beta\delta)^{-1}-a\gamma^*]}{(1+a\gamma^*\delta)\,(1+\beta^{-1})}, \tag{22}$$

as the speculative component (sp) of the stock demand. Note that the speculative demand will be positive, negative, or equal to zero depending on whether $(\beta\delta)^{-1}$ is greater than, smaller than, or equal to $a\gamma^*$. The intertemporal arbitrage condition under *price uncertainty* is given by the zero speculative demand condition, $(\beta\delta)^{-1}=a\gamma^*$, which can be rewritten as

$$\frac{\overline{P}_1}{P_0} = (\beta\delta)^{-1}\left[1+\frac{1}{2}(1-\lambda)\,\sigma_s^2\right]^{\frac{\lambda}{1-\lambda}}. \tag{23}$$

One can immediately note that the term in the square bracket above represents the precautionary premium for price risk. Speculative demand for stocks will be positive only if the expected next-period price is greater than the current spot price by a sufficient margin to cover the precautionary premium for the price risk, in addition to the cost of storage and time rate of discount.

Collecting the components of the stock demand, we have

$$I_1^* = I_1^{pc} + I_1^{sp} + I_1^{ia}.$$

Note that total stock demand has to be non-negative, but not all individual components need to be positive.

C. *Effects of Price Risk*

What impact will price uncertainty have on stock demand? Will agents hold more or less stocks under price and output uncertainty than under output uncertainty alone? To answer these questions, one can compare (10) and (19) or differentiate (19) with respect to δ_s. Neither exercise, however, reveals a clear answer a priori. Under price risk, stocks reduce exposure to the output risk at the cost of increasing exposure to the price risk. Therefore, it could be expected that the presence of price risk generally will dampen the demand for stocks. However, the issue is more complicated than that because of the correlation between price and output risks.

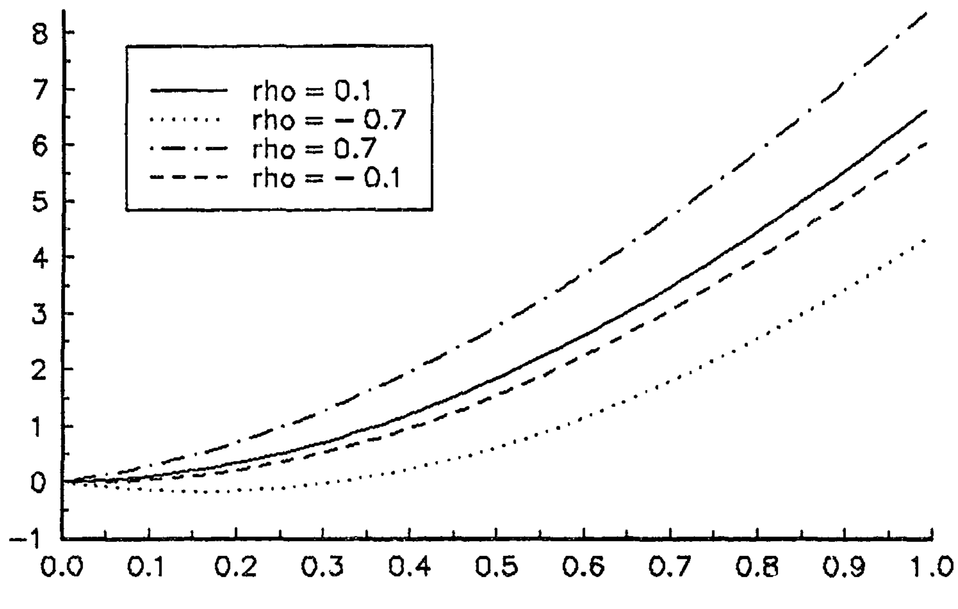
To find the impact of price risk, we use the numerical method with plausible ranges of values for the parameters and initial conditions in equations (10) and (19). Figure 1 shows the difference in stock demand, $I_1 - I_1^*$ for a range of price variability. It turns out that if the price and output risks are positively correlated, the additional price risk reduces the demand for stocks. If the two are negatively correlated, the demand for stocks first increases as the coefficient of variation of price risk increases and then declines after a certain point. This is because an increase in price risk increases the overall level of risk exposure if price and output risks are positively correlated, but if the two are negatively correlated there is a high enough price risk at which the overall level of risk exposure is smaller than at a low price risk.

As in the previous section, define MPS by

$$MPS = \frac{\partial I_1^*}{\partial A_0} = \frac{1 + [\bar{Q}_1 + \delta(A_0 - b\gamma^*)]/\sqrt{q^*}}{2(1 + a\gamma^*\delta)}. \tag{24}$$

A sufficient condition for a positive MPS is $A_0 > b\gamma^*$, which will hold for a reasonable level of σ_θ or if $\rho<0$. Thus, under both output and price risks, the

Figure 1: Effect of Price Risk on Storage



competitive storage equation retains the positive slope for most situations. Differentiating (24) again with respect to A_0 yields an expression similar to (13), which can be shown to be positive. Thus, the nonlinearity of MPS is also maintained under output and price risks.

To find the effect of the price risk on MPS, one can compare (13) and (24) or differentiate (24) with respect to δ_s. Neither, however, reveals the impact unequivocally. Again, numerical calculations show that, for all possible ranges of values for parameters and variables, MPS declines as δ_s increases, regardless of the value of ρ.

IV. Precautionary Stock Demand under Futures Trading

Since futures contracts are available for a wide range of commodities, it is important to investigate how precautionary behavior adapts to futures trading opportunities. Futures contracts allow agents to lock in the next-period price in the current period, thus eliminating the price risk. Implications of futures trading for commodity production and storage, and for market equilibrium in general, have long been the subject of investigation. Past research has focused on the effects of futures trading on production, market efficiency under futures trading, and the use of futures for risk reduction and revenue stabilization. In this section, we focus on derivation of the optimal storage and hedging rules under output and price risks. These rules allow us to better understand the nature of futures prices in relation to price expectations and risk behavior. We also show how these rules differ from those of previous studies derived under more restrictive assumptions.

Futures contracts have been considered primarily as an instrument for hedging against price risk. Their use, however, is not so limited. Futures are used extensively as a speculative instrument. For a producer facing output and price risks, short hedging with futures reduces price risk but increases exposure to output risk because the firm has to deliver the quantity hedged out of its stocks at the contract's maturity or buy out the contract at the prevailing market price. Physical stocks reduce exposure to output risk but increase exposure to price risk. Under futures trading, exposure to output risk can also be reduced by long hedging, by which means the agent can assure delivery of the commodity. However, a long futures position increases exposure to price risk because the quantity hedged may have to be sold at a yet uncertain price.

What is the optimal level of hedging for a producer facing both output and price risks? The problem is more complicated than the case of price risk only because the agent should take a short position to hedge against the price risk and a long position to hedge against the output risk. There is an apparent conflict. Furthermore, the agent can use physical stocks to hedge against the output risk. Optimal hedging rules for futures under output and price risks have been investigated by Rolfo (1980), Newbery and Stiglitz (1981), and Anderson and Danthine (1983), using the CARA utility function. These papers show that the optimal hedge consists of a speculative component that depends linearly on the futures price bias and a hedging component that depends on the correlation between revenue and price. Marcus and Modest (1984) derive a multi-period optimal production rule for risk-neutral agents under output and price risks and futures trading. They find that the optimal production rule does not differ much from that under price risk alone. This section provides explicit solutions for optimal futures and stock positions simultaneously, under both price and output risks.

A. *Optimal Hedging and Storage*

Without loss of generality, let us assume that futures contracts do not require margin deposits and transactions costs are zero. Then, a futures contract entered into in the current period does not affect current-period consumption. Unlike physical stocks, it cannot be used to allocate consumption over time. Any profit or loss from futures trading materializes in the next period and affects the next-period consumption by

$$\tilde{C}_1 = \tilde{P}_1(\tilde{Q}_1+\delta I_1+X_1)-P_f X_1, \tag{25}$$

where X_1 is the futures position taken in the current period, for contracts maturing in the next period. The agent is long on futures if $X_1>0$, or short if $X_1<0$. P_f is the futures price of the commodity.

The agent's optimization problem is to maximize J in (2) with respect to

I_1 and X_1, subject to (25) and initial endowments. The first order conditions for the maximum are

$$
\begin{aligned}
P_0 U'(C_0) &\geq \beta\delta E_0 \tilde{P}_1 V'(\tilde{C}_1), \\
&= \quad if \;\; I_1 \neq 0, \\
E_0 V'(\tilde{C}_1)(\tilde{P}_1 - P_f) &= 0.
\end{aligned} \tag{26}
$$

The second condition above should hold as an equality for all values of X_1 and non-negative values of I_1.

As before, the equivalent precautionary premium can be defined as the payment, π^f, in units of output that satisfy

$$\overline{P}_1 V'[\overline{P}_1(\overline{Q}_1+\delta I_1-\pi^f)+(\overline{P}_1-P_f)X_1] = E_0\tilde{P}_1 V'[\tilde{P}_1(\tilde{Q}_1+\delta I_1)+(\tilde{P}_1-P_f)X_1], \tag{27}$$

where the superscript (f) denotes the case of futures trading. Again, using the same steps as in (16), π^f can be approximated by

$$\pi^f \approx -\left(2\rho\sigma_\theta\breve{\sigma}_\varepsilon+\overline{A}_1\breve{\sigma}_\varepsilon^2\right) - \overline{P}_1\left(\rho\sigma_\theta\breve{\sigma}_\varepsilon\overline{A}_1+\frac{1}{2}\breve{\sigma}_\varepsilon^2\overline{A}_1^2+\frac{1}{2}\sigma_\theta^2\right)\frac{V'''(\overline{C}_1)}{V''(\overline{C}_1)}, \tag{28}$$

where $\overline{C}_1=\overline{P}_1(\overline{Q}_1+\delta I_1)+(\overline{P}_1-P_f)X_1$ *and* $\overline{A}_1=\overline{Q}_1+\delta I_1+X_1$. To solve (26), it is necessary to use an approximation of $P_f\, E_0 V'(\tilde{C}_1)$. Let

$$P_f\, V'[\overline{P}_1(\overline{Q}_1+\delta I_1-\pi^{ff})+(\overline{P}_1-P_f)X_1] = P_f\, E_0 V'[\tilde{P}_1(\tilde{Q}_1+\delta I_1)+(\tilde{P}_1-P_f)X_1], \tag{29}$$

then

$$\pi^{ff} \approx -\rho\sigma_\theta\breve{\sigma}_\varepsilon - \overline{P}_1\left(\rho\sigma_\theta\breve{\sigma}_\varepsilon\overline{A}_1+\frac{1}{2}\breve{\sigma}_\varepsilon^2\overline{A}_1^2+\frac{1}{2}\sigma_\theta^2\right)\frac{V'''(\overline{C}_1)}{V''(\overline{C}_1)}. \tag{30}$$

Now we can substitute (27) through (30) into (26) to express the first order conditions in terms of certainty equivalents. We again assume a CRRA utility function. The two first-order conditions can be combined to express X_1 in terms of I_1

$$X_1 = \phi(A_0-I_1)-(\overline{Q}_1+\delta I_1)-\frac{\rho\sigma_\theta}{\breve{\sigma}_\varepsilon}, \tag{31}$$

where $\phi=(\beta\delta)^{\frac{1}{\lambda}}\breve{\sigma}_\varepsilon^{-2}\left(\frac{\overline{P}_1}{P_0}\right)^{-1}\left[\left(\frac{\overline{P}_1}{P_0}\right)^{\frac{1}{\lambda}}-\left(\frac{P_f}{P_0}\right)^{\frac{1}{\lambda}}\right]$.

Equation (31) provides the optimal hedging rule, given I_1. It consists of three components. The first term in (31) is the speculative component that depends, among other things, on the spread between the expected spot and futures prices. The second term represents the pure hedging component that reflects the agent's precautionary behavior against the price risk. The third term is what Anderson and Danthine (1983) called the hedging adjustment term (HAT) to account for the correlation between output and price risks.

In an unbiased futures market, in the sense that $P_f=\overline{P}_1$, the expected capital gain from buying or selling a futures contract is zero. Therefore, there will be no speculative demand for futures. In (31), the speculative component is zero if and only if $P_f=\overline{P}_1$ (note that $\phi=0$ if $P_f=\overline{P}_1$). This result is in agreement with that of Anderson and Danthine (1983), among others, derived under the CARA assumption. In a biased futures market, the producer's speculative demand is positive or negative depending on whether the expected spot price is higher or lower than the futures price.

The second and third terms in (31) reflect the agent's precautionary behavior. If the futures market is unbiased and output risk does not exist, the first and third terms in (31) vanish and the producer's optimal futures position will be short hedging the entire expected output plus stocks carried over net of stocking costs. Under output uncertainty, however, the producer should short hedge more or less than completely, depending on whether HAT is positive or negative. The presence of HAT in (31) is the result of introducing output uncertainty and the covariance between output and price risks, and was recognized earlier by Newbery and Stiglitz (1981), Rolfo (1980), and Stiglitz (1983). The sign of HAT depends on that of ρ. If the price and output risks are negatively correlated and output falls below expectations, the producer will have to make up for the shortfall by buying at a higher-than-expected price. Thus, the producer needs to take precautionary measures by hedging less than completely. If the price and output risks are positively correlated and output falls short, price will also fall below expectations. The producer will be able to close the short futures position at a lower-than-expected price. Thus, the producer does not need to take a precautionary measure; instead, it is better to short more than completely in order to take advantage of the opportunity for capital gains.[6]

Using (31) in (26) yields an explicit solution for I_1 in terms of exogenous variables and parameters. Since the first-order conditions involve two nonlinear equations in two unknowns, the solution becomes very complicated. To simplify the expressions, let

Then,

---

[6] Anderson and Danthine noted that the presence of HAT alters some of the earlier results concerning producer behavior under output uncertainty. They showed that output uncertainty does not necessarily reduce the amount of hedging by producers or the level of expected out. Further, they argued that the availability of futures trading does not necessarily increase the planned output of producers, in contrast to what Holthausen (1979) and Feder, Just, and Schmitz (1980) showed for a producer facing price risk. Our model, with appropriate extension, could corroborate the Anderson and Danthine proposition.

$$\zeta=\frac{P_f}{\overline{P}_1}, \qquad \psi=(\beta\delta)^{\frac{1}{\lambda}}\left(\frac{P_f}{P_0}\right)^{\frac{1}{\lambda}}\left(\frac{\overline{P}_1}{P_0}\right)^{-1},$$

$$\omega=(1-\zeta)\phi-\delta\zeta, \qquad \omega^*=2(1-\zeta)\phi-\delta\zeta,$$

$$\omega^+=(1-\zeta)\phi+\delta\zeta, \qquad \tau=(1-\zeta)\phi-\psi,$$

$$\tau^*=2(1-\zeta)\phi-\psi, \qquad \kappa=\rho\sigma_\theta\gamma_s[1-2(1-\zeta)\gamma_s^{-2}].$$

$$I_1^f = \frac{-h+\sqrt{q^f}}{2d}, \tag{32}$$

where

$$d = \omega^2-\omega\psi-\frac{1}{2}(1+\lambda)(\gamma_s\phi)^2,$$

$$h = -2\omega[(1-\zeta)\phi A_0+\zeta\overline{Q}_1]+\psi[\omega^*A_0+\zeta\overline{Q}_1]$$
$$+(1+\lambda)A_0(\phi\gamma_s)^2+(1-\zeta)\rho\sigma_\theta\gamma_s^{-1}[2(1-\zeta)\phi-\phi\gamma_s^2-\psi]+\delta\zeta\kappa,$$

$$q^f = [\zeta\tau^*(\delta A_0+\overline{Q}_1)+\omega^+\kappa+(1-\zeta)\psi\rho\sigma_\theta\gamma_s^{-1}]^2-4[\zeta(\delta A_0+\overline{Q}_1)+\kappa]$$
$$[(\zeta(1-\zeta)\phi\tau-\frac{1}{2}(1+\lambda)\zeta(\phi\gamma_s)^2)(\delta A_0+\overline{Q}_1)-\delta\zeta(1-\zeta)((2(1-\zeta)-\gamma_s^2)\phi-\psi)\rho\sigma_\theta\gamma_s^{-1}]$$
$$-4d(\rho\sigma_\theta)^2[\zeta-\frac{1}{2}+\frac{1}{2}\lambda+(1-\zeta)^2\gamma_s^{-2}]+2d(1+\lambda)\sigma_\theta^2.$$

B. *Speculative Behavior and the Theory of Normal Backwardation*

When both futures contracts and physical stocks are available, an agent can speculate with either or both. It is immaterial which instrument is used as long as the expected returns are the same. However, since physical stocks incur storage costs while futures contracts, by assumption, do not imply transactions costs, there is little reason for agents to speculate with stocks. Normally, spot and futures prices move closely together, thus allowing speculators to focus on futures contracts as the main speculative vehicle. Indeed, the optimal stock demand given by (32) is insensitive to changes in price expectations in relation to spot and futures prices, while the demand for futures in (31) is highly sensitive. Figure 2 shows the optimal solutions for stockholding and the futures position for a hypothetical case, for a range of values for the futures price bias, defined as the expected spot price minus the futures price. A negative bias indicates contango, while a positive bias represents backwardation. It is shown that as the market shifts from contango to backwardation, the agent moves rapidly from a large short position to a large long position. Stock demand, however, remains relatively stable. The result confirms the conjecture that futures become the main speculative instrument when futures trading is available.

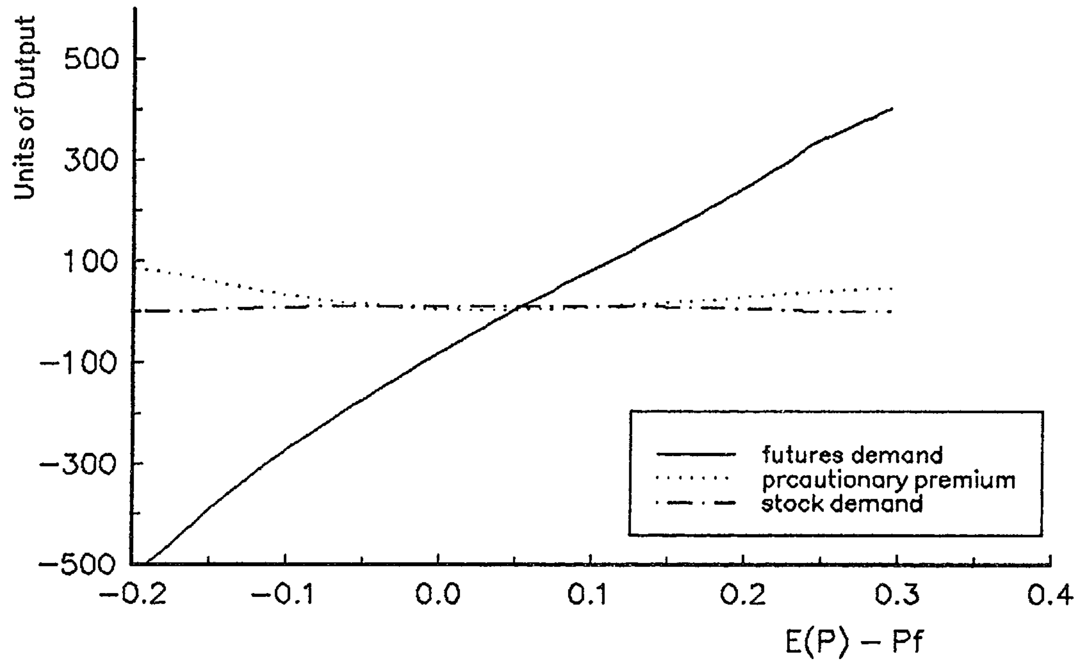
Figure 2 also plots the precautionary premium against futures price bias. The premium is large when the bias in absolute terms is large and reaches a minimum near zero bias. In fact, the precautionary premium would have been exactly zero at the point of zero futures price bias (unbiased futures market), had there not been output uncertainty. This is so because the producer faces zero risk in an unbiased futures market with output certainty; the producer can completely eliminate the price risk by using the futures price. It can be seen that the effect of HAT on the precautionary premium is probably relatively small. It can also be seen that the precautionary premium can be large when the futures market is in substantial backwardation or contango, resulting in sizable futures positions.

More formally, the precautionary demand model presented in this section implies a relationship between the precautionary premium and futures price bias. Equations (31), (32), (27), (28), and (26) imply

$$f(\pi^f, \overline{P}_1 - P_f \mid \overline{Q}_1, A_0, P_0, \beta, \delta, \lambda, \sigma_\theta, \sigma_\epsilon) = 0. \tag{33}$$

The implicit function (33) takes a complicated form and it is not feasible to show analytically the importance of the precautionary premium in explaining the

Figure 2: Futures Demand and Precautionary Premium as a Function of Futures Price Bias



futures price bias. However, judging from Figure 2, two features of (33) stand out. First, the precautionary premium could vary considerably across the spectrum of futures price bias. Second, most of the variation in the precautionary premium results from changes in the futures position.

When the market is in contango and the agent is short hedged, the amount of precautionary premium the agent is willing to pay increases as the short hedge position increases. The increase in the precautionary premium for an additional unit of short hedge position may be called the marginal precautionary premium

(MPP). Similarly, MPP can be defined for the region of backwardation. Figure 3 plots the MPP implied by Figure 2. MPP is negative when the agent is in a short hedge position, meaning that the precautionary premium increases as the short position increases. It is positive when the agent is long on futures. Note that MPP is a nonlinear function of the futures price bias but falls below the 45 degree line. When the market is in contango, the agent needs to be paid MPP for shorting an extra unit of futures, but the market compensates him by only as much as $P_f - \overline{P}_1$. In Figure 3, the latter is smaller than MPP in absolute terms for a wide range of futures price bias, meaning that the agent is willing to go short to a greater extent than the market compensates him for his risk taking. For a significant range of backwardation, MPP is smaller than $\overline{P}_1 - P_f$, meaning that the agent wants to be compensated by more than the precautionary premium to take up a long position.

In all, a substantial part of futures price bias is accounted for by the precautionary premium, although the proportion varies depending on the agent's futures position. The producer will be a short hedger for a wide range of futures price bias, but could become a long hedger if the expected spot price is sufficiently higher than the futures price.

C. *Effects of Futures Trading on Storage*

First, we ask the question whether agents will hold more or less stocks, speculative and precautionary, under futures trading than without it. *Prima facie*, it may be argued that since futures contracts offer costless means of reducing the price risk, the producer can reduce the overall level of risk exposure under futures trading compared to the situation where futures trading does not exist. The producer's precautionary premium under futures trading will be smaller than without it. To simplify, suppose the futures market is unbiased, then $\phi=0$, $\zeta=1$, $\psi=\frac{1}{\gamma^*}$, $-\omega=-\omega^*=\omega^+=\delta$, $\tau=\tau^*=-\frac{1}{\gamma^*}$, $\kappa=\rho\sigma_\theta\check{\sigma}_s$,

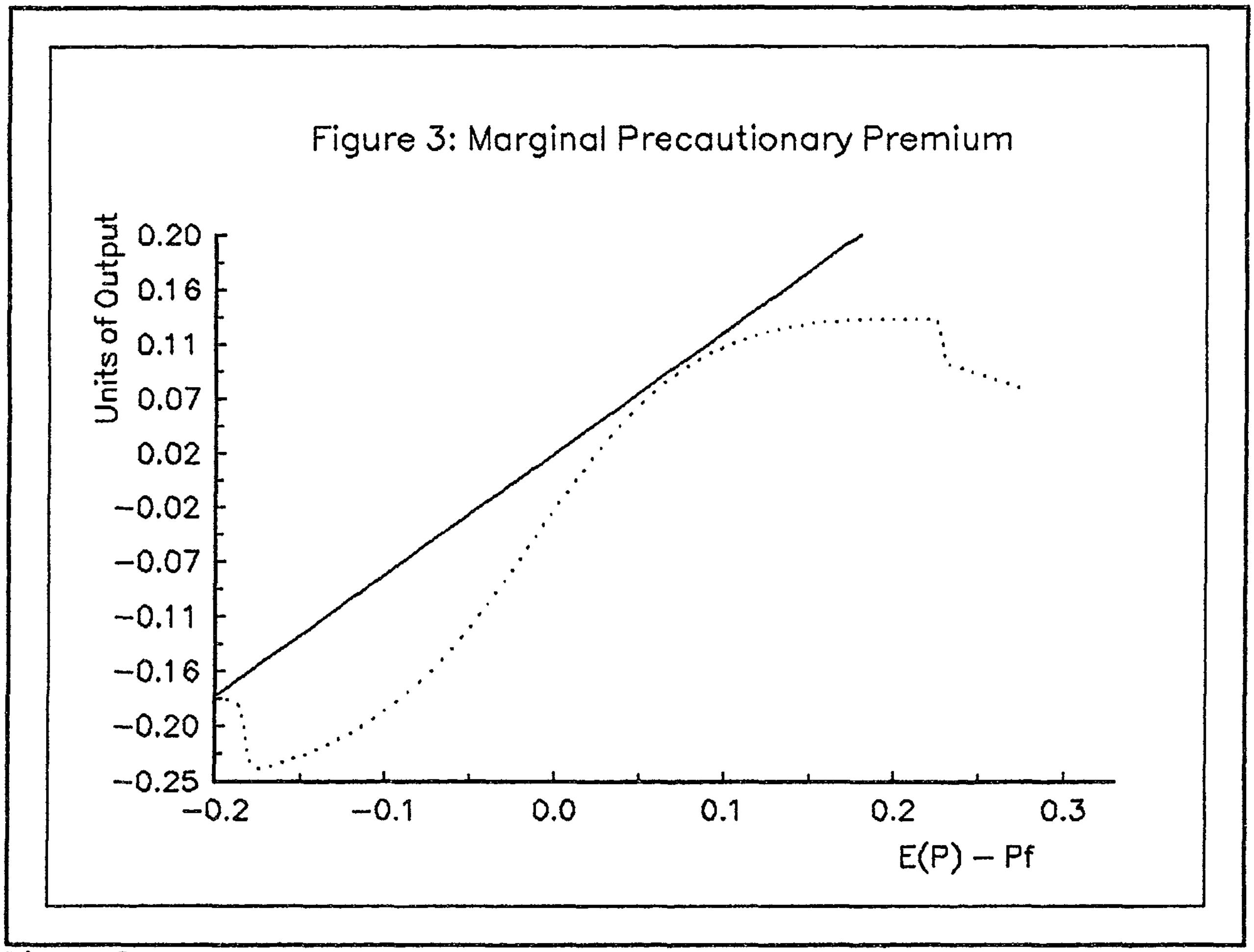


Figure 3

and (32) reduces to

$$I_1^{uf} = \frac{\delta A_0 - (1+2\delta\gamma^*)\bar{Q}_1 - \delta\gamma^*\rho\sigma_\theta\breve{\gamma}_s + \sqrt{Q^{uf}}}{2\delta(1+\delta\gamma^*)}, \tag{34}$$

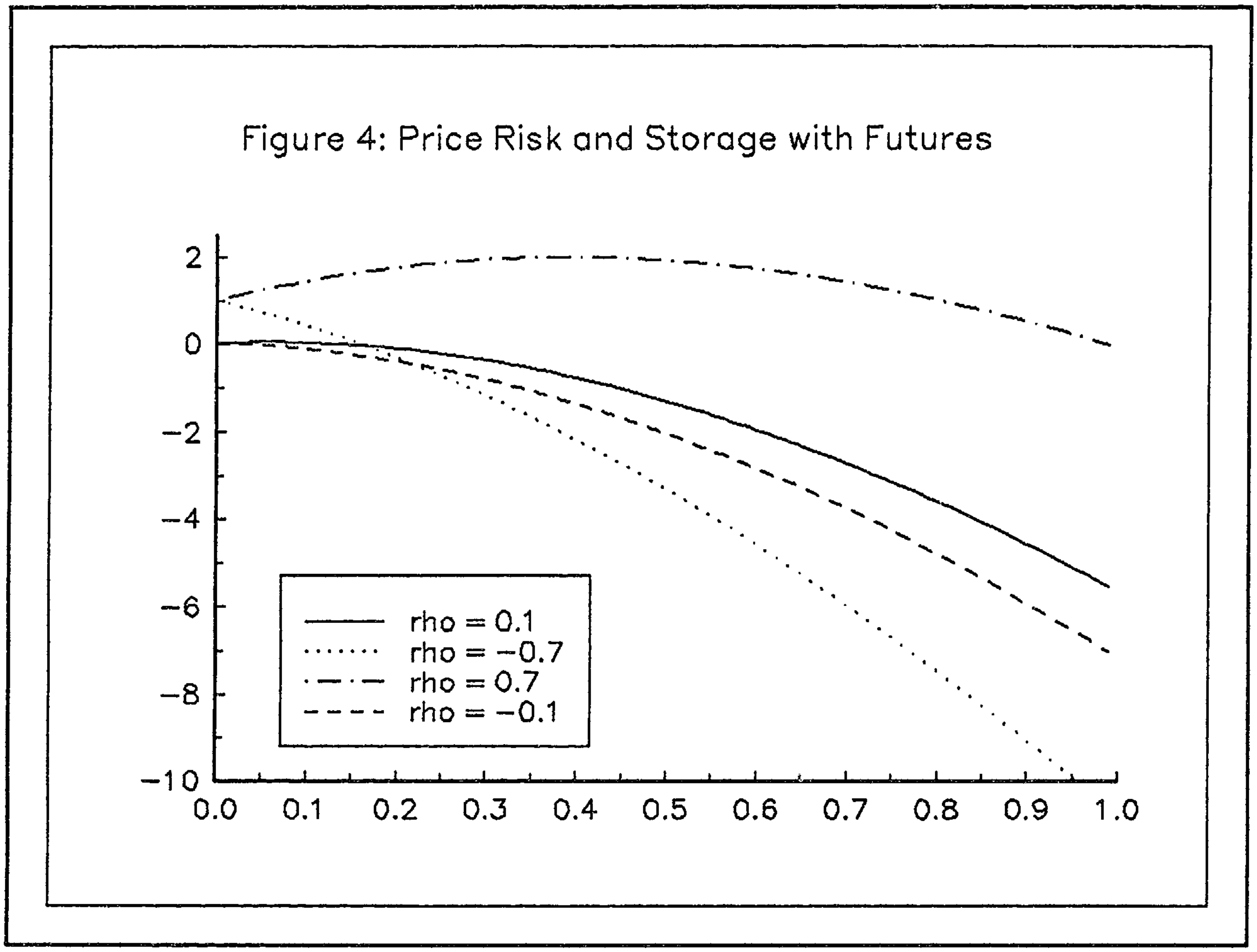
where

$$q^{uf} = [(\bar{Q}_1+\delta A_0)-\delta\gamma^*\rho\sigma_\theta\breve{\sigma}_\varepsilon]^2+2\delta\gamma^*(1+\delta\gamma^*)(1+\lambda)\sigma_\theta^2,$$

and the supercript (uf) denotes the case of an unbiased futures market.

First, let us compare (34) with (10) where there is no price risk. The two look strikingly similar in that the price risk factor appears only in conjunction with its correlation with the output risk and not as frequently as in (19). This result reflects the (price) risk-reducing role of an unbiased futures market. Comparing (34) with (10), it is immediately clear that $I_1^{uf}>I_1$ if $\rho<0$ and vice versa. That is, if $\rho<0$, the price increases when output falls short and the producer can sell stocks at a higher-than-expected price. Therefore, the producer would want to hold more stocks than in the case of price certainty. Recall that an unbiased futures market does not afford the producer complete elimination of the price risk if the output risk is present. On the other hand, if $\rho>0$, stocks will face lower-than-expected prices when output falls short, thus reducing the incentive to hold stocks. Thus, the optimal response is to reduce stockholding but increase the short futures position.

For the more general case when price risk is present, (32) should be compared with (19). However, the relative size of the two expressions is difficult to ascertain algebraically. Figure 4 depicts the differences in stock demand for a range of price risks. It is clear that the producer generally holds more stocks under futures trading than without it. Also, the greater the price risk, the more stocks the producer holds under futures trading. There is a minor exception to this when the covariance is highly positive. The reason for this is that the producer usually finds it desirable to short hedge expected output and stocks to reduce the price risk and cover the increased exposure to the output risk by increasing stockholding.



V. Conclusions

This paper presents a formulation of the producer demand for physical stocks and futures as a form of precautionary behavior. This formulation overcomes the difficulties in the traditional explanation that involves speculative demand on the one hand and convenience yield on the other. The model provides simultaneous solutions for optimal stock carryover and futures positions for a producer facing output and price risks. As such, it could be useful in practical applications, including developing countries that rely on exports of primary commodities.

When both output and price risks exist, stocks reduce exposure to the output risk but increase exposure to the price risk. The resulting stock demand equation explains a combination of speculative and precautionary demands for stocks, overcoming the shortcoming of earlier models that failed to recognize the simultaneous existence of output and price risks. Stock demand under output and price risks could be larger or smaller than under output or price risk alone, depending on the sign of the correlation between the two risks. The precautionary premium also depends on this correlation and drives a wedge between current and expected spot prices.

Futures contracts are a useful hedging and speculative instrument for commodity producers and its availability lowers the precautionary premium agents are willing to pay. In an unbiased futures market, commodity producers should short hedge more or less than completely, depending on whether output and price risks are positively or negatively correlated. Since producers usually should short hedge, and this increases exposure to output risk, it is necessary to increase stockholding under futures trading. The precautionary premium is non-zero in an unbiased futures market. The demand for futures is extremely sensitive to futures price bias, but the demand for stocks is not. In a biased futures market, the precautionary premium could be large and be the main component of the bias.

The optimal hedging rule derived in this paper could have useful practical applications for developing countries heavily dependent on exports of primary commodities. It incorporates improvements on earlier models in at least two areas. First, it assumes a sensible preference function that yields nonlinear demand functions for stocks and futures. Second, it solves stock-holding and hedging decisions simultaneously. These refinements lead to optimal hedging rules significantly different from those in earlier studies. For example, Rolfo's

(1980) results for cocoa-producing developing countries suggest relatively low optimal hedge ratios that are insensitive to output and stock levels, and to the degree of absolute risk aversion. Increased exposure to output risk is the main reason cited by Rolfo for the reluctance of these countries to hedge. This paper suggests that, when output risk can be countered by stockholding, producers will be more willing to hedge. The fact that cocoa producers have not been significant users of futures trading may be because they normally ship out entire harvests to industrial countries for processing and therefore are not adequately prepared to deal with output risk. The optimal policy for these countries is to keep some stocks and hedge a substantial portion of their expected output and stocks.

The model presented here requires further investigation and extension. It awaits practical application to real world situations. Generalization to multi-period stochastic optimization is an obvious extension.

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