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INTERDEPENDENT PREFERENCES AND SEGREGATING EQUILIBRIA*

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Abstract

This paper shows that models where preferences of individuals depend not only on their allocations, but also on the well-being of other persons, can produce both large and testable effects. We study the allocation of workers with heterogeneous productivities to firms. We show that even small deviations from purely “selfish” preferences leads to widespread workplace skill segregation. This result holds for a broad class and distribution of social preferences. That is, workers of different abilities tend to work in different firms, as long as they care somewhat more about the utilities of workers who are “close”.

Keywords: contract theory, mechanism design, envy, social preferences, skill segregation.
JEL Classification: A13, C72, D64, D80, J41.

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1 Introduction

We have by now ample evidence that preferences of individuals between allocations do not depend only on their own material well-being. Rather, the actions and material allocations of other individuals impact directly a person's utility, and are thus taken into account when making a decision. But the research in models of "social preferences," as they are sometimes called, has not delivered empirical implications which change qualitatively our view of economic behavior. We show, however, that these models produce both large and testable effects. We study worker allocation to firms in a contract-theoretic framework, where agents differ in their productivity. We show that even small deviations from purely "selfish" preferences leads to widespread workplace skill segregation.

The current interest in social preferences' models arises in a large part to explain "anomalous" results from experimental economics. The papers in the area typically devote entire sections to show that their models can robustly account for the data generated by many different experiments. In doing so, they often estimate coefficients for the models. The coefficients estimated are, however, typically small, even for the relatively small stakes games played in the laboratory. The approach is, then, subject to the criticism that social preferences will lead only to small scale effects in the real world. Therefore, it could be argued that it is not useful to incorporate them into mainstream models of labor markets, consumer behavior, and so on. Our aim is to show that this view is incorrect.

We study a labor market in which firms compete for workers of heterogeneous (and unobservable) quality by offering (menus of) contracts. Social preferences' models involve interpersonal comparisons of utility across agents. It is natural to assume that these comparisons do not necessarily span the whole population, but only individuals who are "close." This is implicitly acknowledged by current research on social preferences, as, in the typical application, the comparisons are only among agents playing a particular game. However, the range of interpersonal comparisons has been a generally neglected issue. To make the notion of closeness precise, we introduce a spatial

structure in the model. Firms choose locations in a ring, and workers compare their material payoffs to those of workers in their same firm and in other firms located within a certain distance in the ring.

The efficiency units of workers' labor are perfect substitutes but the individual endowments of efficiency units are the private information of each worker. That is, some workers are more productive/skilled than others, but workers of different skills are perfectly substitutable in some fixed proportions. With this structure, and the traditional "selfish" preferences, the equilibria would not make a prediction on the distribution of skill levels by firm or location. Any distribution would be consistent with equilibrium. With the introduction of social preferences, of however small strength, the equilibrium becomes both skill and spatially segregated, that is, firms hire only from one skill pool and firms employing workers of a given skill level form spatial clusters.¹ This equilibrium selection holds with social preferences of quite general form, and possibly with a heterogeneous distribution in the population.

The segregation and clustering results would also hold in a model with complete information. We introduce incomplete information for a few reasons. First of all, the incomplete information makes it more evident that the externality driving segregation is different than the one in models of say, racial segregation. We deal here with a pecuniarity externality, that is, high-skilled types do not separate from low-skill types because they intrinsically dislike them. They do it, rather, because the market tends to produce different material payoffs for both. Second, the standard screening model implies that when workers have private information about their productivity, firms should offer a menu of contracts to workers, who would self-select into the appropriate category. This is not how firms normally behave. Instead it seems like the "market" itself offers a "menu of firms" with different working conditions, into which the workers self-select. We offer a parsimonious explanation for this observation. Finally, having a model that is robust to incomplete information is an obvious strength that is introduced

¹In a sense we can argue that social preferences operate here as a kind of "equilibrium-refinement." The advantage of this way of refining equilibria is that the payoff perturbation is economically and empirically well-motivated.

at a relatively low complexity cost.

2 Background and related work

We bring together several strands of the economics literature.

Literature on labor market segregation In the last years, empirical analysis of labor markets identifies an increasing trend towards homogeneous composition of firms in terms of pay and skill. For instance, Acemoglu (1999) shows, with US data, that the sorting of workers across occupations increased between 1983 and 1993.² Davis and Haltinwanger (1991) had already observed that the rise of wage inequality in America is imputable in part to differently abled workers sorting themselves across firms: “The tremendous magnitude of the rise in the size-wage gap indicates that sorting by worker ability across plants of different sizes probably increased over time (page 156-7).” In turn, Burgess, Lane and McKinney (2004), with UK data, find that within-group wage inequality can be explained by the dynamic of worker-job allocation in the period 1986-1998.

A different approach to the study of this problem is taken by Kramarz, Lollivier and Pelé (1996), who compute a measure a specialization for different professional categories proposed by Kremer and Maskin (1996). They find that specialization increased enormously in France between 1986 and 1992: “Blue collar unskilled workers are more and more separated from other types of workers, and therefore, tend to work together in the same firms. This is true for each of the six categories of skills. The number even doubled for clerks.” (page 375). A final kind of evidence for segregation is given by Brown and Medoff (1991), who investigate wage-size differentials (that is, difference in wages across firms of different sizes). They only find evidence for explanations of these differentials based on sorting by the level of skill into firms of different sizes.

²The evidence is from the Current Population Survey, and occupations are ranked according to the wage residuals, after controlling for worker observables such as education, sex, experience, and location (metropolitan dummy).

The available explanations for this evidence typically rely on complementarities between similarly skilled individuals. Kremer and Maskin (1996) and Saint-Paul (2001) are good examples of these explanations.³ Our model, on the other hand, does not impose any form of production complementarities between workers. We propose a form of pecuniary externality. In our model, market outcomes favor more productive workers, and individuals dislike inequalities in their own neighborhood.⁴ Although the two kinds of model can explain the recent rise in skill segregation, the explanations are empirically distinguishable. For example, the empirical evidence in Kremer and Maskin (1996) deals mainly with ex-ante observable skill differences. Our model, however, makes predictions even about ex-ante unobservable skill heterogeneity. That is, we can explain increasing wage differentials even after controlling for observables. This is a definite advantage of our explanation, as a large part of the recent increase in wage inequality cannot be attributed to observables.⁵

Literature on social preferences The implications of our model seem, thus, consistent with the available empirical evidence. What about our assumptions? There is direct evidence for the kind of externality we assume in Bewley (1999).⁶ About 78% of the businesspeople whom he interviewed say that internal equity is important for internal harmony and morale.⁷ Morale here means “cooperativeness, happiness or

³De Bartolomé (1990) and Bénabou (1993) are also related, but they focus more strongly on human capital acquisition and residential segregation.

⁴Other models of segregation rely on group externalities, like Becker (1957) and Schelling (1971). Unlike in our paper, those models assume that individuals intrinsically like or dislike members of other groups. We have a spillover related only to the market outcome. High and low types would coexist happily if wages were equal.

⁵Juhn et al. (1993) and Katz and Autor (1999), Section 2.4, quantify the contribution of residual inequality to the total increase in wage dispersion over the last 25 years. They place it between two thirds and three fourths of the total amount.

⁶Other papers which offer survey evidence on the importance of equity concerns in organizations are: Blinder and Choi (1990), Bewley (1995), Agell and Lundborg (1995, 2003), or Campbell and Kamlani (1997).

⁷Bewley (1999), table 6.5.

tolerance of unpleasantness, and zest for the job.”⁸ One can find in Section 6.5 of Bewley (1999) many revealing quotes from managers about the disruptive effects of lack of equity on the job.⁹ He finds as well that internal inequity in firms leads to higher turnover,¹⁰ as our model predicts.

The current wave of work on social preferences in economics was a result of the large experimental evidence that conflicted with the hypothesis of selfishness. For example, in the experimental lab there is more contribution to public goods than purely selfish maximization could lead us to expect.¹¹ Perhaps more relevant for this paper, experimental subjects often reject unequal offers in ultimatum bargaining games (Güth, Schmittberger and Schwarze 1982).¹² A variety of models have been devised to explain these observations.¹³ It would be too difficult to discuss all those models in detail, so we refer to the excellent surveys of Sobel (2005) and Fehr and Schmidt (2000b). A common feature in many of these models is the assumption that individuals dislike payoff inequality.¹⁴ Our innovation with respect to this literature is that we think explicitly about the set of individuals to which the utility comparisons apply. We also provide further testable implications for the model (and implicitly relevant economic applications) and we work with very general social preferences.

Literature on economic implications of social preferences There are few papers which study the labor market implications of social preferences. The seminal contribution by Frank (1982) showed that wages may depart from the value of marginal productivity if workers cared sufficiently strongly about relative payoffs. He assumes

⁸Bewley (1999) p. 42.

⁹From “Internal equity is very important,” to “Inequity causes disharmony” and even “Unfairness can cause upheaval within an organization and lead to disfunctional activities.”

¹⁰Bewley (1999), table 6.5.

¹¹See Ledyard’s (1995) survey on public goods in the *Handbook of Experimental Economics*.

¹²See also Roth’s (1995) survey on bargaining in the *Handbook of Experimental Economics*.

¹³Bolton (1991), Rabin (1993), Levine (1998), Bolton and Ockenfels (1999), Fehr and Schmidt (2000a), Charness and Rabin (2002).

¹⁴The analysis of extended social preferences is typically done directly at the level of utilities rather than preferences. A notable exception is Segal and Sobel (2006) who provide an axiomatization of when preferences over strategies and outcomes in a game can be represented as a weighted average of players’ utilities.

people like to be better paid than others, and dislike to be paid worse. Under these conditions, the more productive are paid less than the value of their marginal product as they obtain the “pleasure” of earning more than others. The less productive, on the other hand, are paid more than their marginal productivity to compensate for their “suffering” caused by an inferior wage.¹⁵ Fershtman, Hvide and Weiss (2005), in a similar framework, explore the effects of status on effort, and show that firms with workers of heterogeneous productivities may form, wages may differ across the economy for equally productive workers, and the quest for status may increase total output. Both of these works assume that people actually “like” to be better paid than others. This seems to go against the experimental evidence that motivates the social preferences models with which we work.

Cabrales, Calvó-Armengol and Pavoni (2006) study long-term contracts in a dynamic learning model in the style of Harris and Hölmström (1982) where agents have social preferences (of the difference-aversion type) and there are moving costs between firms. The equilibrium of the model displays both between and within-firm wage dispersion. An increase in moving costs reduces the amount of segregation by skill level, thus increasing within-firm wage dispersion. Also, long terms contracts introduce novel internal labor market features such as a dynamic form of wage compression, gradual promotions, and wage non-monotonicity. The study of these dynamic features comes at the cost (with respect to this paper) of a less general structure for social preferences. For example, the span in social preferences is restricted to workers in the same firm, which makes it easy to abstract from firm location in social space, a major issue in the present work. Also, they assume a simple linear difference aversion form of social preferences, which is also homogeneous across workers. Here, on the other hand, heterogeneity in preferences is allowed, and workers may feel social concerns only when located at some places in the wage distribution. Finally, in this paper we allow for private information.

¹⁵Frank (1985) discusses the implications of this framework. For example, the economically puzzling presence of minimum wages, safety regulations, forced savings and other regulations. He shows they may arise to compensate for the externality that is generated by social preferences.

Fehr, Klein and Schmidt (2007), and Rey-Biel (2005) use preferences with difference-aversion, which is one type of social preferences compatible with our more general setup. Both papers explore the effects of social preferences on incentive contracts under moral hazard. Fehr, Klein and Schmidt (2007) show theoretically and experimentally that the presence of even a minority of people with concerns for fairness can alter in an important way the kind of contracts that are efficient. Rey-Biel (2005) shows theoretically that the threat of inequity in pay after bad performance can actually induce effort at a lower cost to the principal than without social preferences.

3 The model

There are N workers, with two types, L and H , which are their private information. The productivity of a worker of type $t \in \{L, H\}$ is θ_t . We assume that $\theta_H > \theta_L$. The prior probability of an H type is $1 > p > 0$. The material payoff function of a worker i who receives a wage w , and exerts effort e , is:

$$u_i(w, e|t) = w - c_t(e)$$

The function $c_t(e)$ represents the disutility experienced by a worker of type t when exerting effort e . For a given effort level, $e \neq 0$, the cost of effort of an L type is higher than that of an H type, that is, $c_L(e) > c_H(e)$. We also assume that $c_{t,e}(e, \theta) > 0$ and $c_{t,ee}(e, \theta) > 0$, for all $t \in \{L, H\}$.¹⁶ Effort levels are verifiable.

Beyond their own material payoffs, individuals care about the material payoffs of others:

$$U_i(\mathbf{w}, e_i|t) = u_i(w_i, e_i|t) - V_i(\mathbf{w})$$

where $\mathbf{w} = (w_1, \dots, w_n)$ is the wage profile in the population. The function $V_i(\cdot)$ is a player-specific function summarizing each player's social concerns. It takes nonnegative values and is equal to zero when all players receive the same wage.

We now make three additional assumption on this function

¹⁶In fact, we need to ensure that indifference curves are non-thick and generate strictly convex upper contour sets.

- A1** For all player i , there is a finite set N_i of other individuals such that $V_i(\mathbf{w}_{N_i}, \mathbf{w}_{N \setminus N_i}) = V_i(\mathbf{w}_{N_i}, \mathbf{w}'_{N \setminus N_i})$ for all \mathbf{w} and $\mathbf{w}'_{N \setminus N_i}$. The set N_i has a symmetry property, that is, $j \in N_i$ implies that $i \in N_j$, for all i, j .
- A2** For any i , if $N_i \neq \emptyset$, there exists a wage profile \mathbf{w} such that \mathbf{w}_{N_i} is uneven and $V_i(\mathbf{w}) > 0$.
- A3** For all player i , and for all two wage profiles \mathbf{w} and \mathbf{w}' , we have $|w_i - w'_i| > |V_i(\mathbf{w}) - V_i(\mathbf{w}')|$.

Assumption A1 captures the fact that individuals are embedded in a network of social relationships, and that social preferences display a limited span of influence. That is, in addition to the utility they obtain from their own wage and effort, they also experience utility (or disutility) from the material payoffs of close neighbors in their network, and only them.

Assumption A2 states that every player suffers from some uneven wage profiles in his neighborhood. This formulation is very general, as it includes difference aversion, inequity aversion, and social concerns that depends on the position in the distribution of earnings (i.e. workers who care about inequality only when they are at the low end of the distribution).¹⁷ Besides, this assumption is compatible with heterogeneity in social concerns across individuals, as long as this social concerns, of whichever form, are present for every individual for at least some wage profile.

Assumption A3 states that social concerns are of a secondary order compared with the importance of material payoffs.

The number of firms is endogenously determined in equilibrium. These firms are located in nodes which are distributed in a ring. There is a countably infinite number of such nodes. We allow for more than one firm to occupy the same location. Each firm can employ any number of workers, and technology is constant returns to scale. Net profit for each worker is equal to his productivity θ , minus the wage w he receives. Firms' profits are determined by the sum of profits per worker. If the firm does not employ any worker, it makes zero profits.

¹⁷We thank a referee for pointing this out.

The game proceeds in three stages. First, each firm decides whether to enter, and if so, chooses a location in the ring. Second, each firm offers a menu of contracts to some workers which specifies the wage and effort required of different worker types. Recall that types are private information of the workers, but effort levels are verifiable, thus contractible. Third, each worker i specifies the menus acceptable to him, and the contracts within this menu that he would take. A worker who does not accept any contract obtains a reservation payoff of zero.

An employed worker gets the material payoffs derived by the implemented contract in the firm for which he works. The neighborhood N_i of some employed worker i is composed by those workers (if any) employed by firms located in i 's employer node, and in a finite string of adjacent nodes. This neighborhood is the one that enters in the determination of the final social payoffs. Remark 6 below states that our main result is robust to variations of this neighborhood structure, such that involving an arbitrary finite string of adjacent nodes rather than the two closest ones.

4 Results

In this section we show that, for the game we just described, in all the subgame perfect equilibria where agents do not use dominated strategies, different types of workers earn a wage equal to their productivity, but they work in different locations. Workers earn their productivity for the usual reasons in a model with competitive wage-setters. The intuition for the spatial segregation result is simple. Since wages equal productivities, and those differ across workers, a low type working in an environment with high types suffers because of his aversion to inequality. A competitor firm, possibly a currently inactive entrant, which is making zero profits in that environment can profitably deviate. He can do so by moving to an empty location and offering a wage slightly below his productivity to the low type that works around high types. Provided this wage is close enough to the productivity, the worker will accept and the firm makes strictly positive profits.

Given the simplicity of the intuitions involved, it may come as a bit of a surprise that

we need to resort to undominated subgame perfect equilibrium as a solution concept. The reason becomes more apparent once we look at the following example, which we have stripped down to the essentials to be easier to follow. In particular we have even dispensed with the incomplete information and the cost of effort.

Example 1 *Let two workers, L and H , whose respective productivities, θ_L and θ_H , are common knowledge. They have no cost of effort. The following actions form part of a subgame perfect equilibrium outcome. Exactly 4 firms decide to enter. Firm 1 locates on node 1 and offers worker L a wage equal to θ_L and worker H a wage equal to θ_H , firm 2 locates on node 1 and offers worker L a wage equal to θ_L , firm 3 locates on node 5 and offers worker L a wage $w_L^3 = \theta_L - V_L(\theta_H, \theta_L)$, and worker H a wage equal to θ_H , firm 4 locates on node 5 and offers worker H a wage equal to θ_H . Worker H accepts the offer of firm 1 and worker L accepts the offer of firm 3.*

The use of dominated strategies by both the firms and the workers is crucial in the construction of the example. In the example, firms make many offers of wages equal to productivity that are not used in the equilibrium path. Those unused offers, which are weakly dominated, are what (out of equilibrium) supports the equilibrium outcome we postulate. Even more importantly, the responses of the players are also (almost) dominated. Take, for example, a deviation by firm 2 to location 3 that offers the L worker a salary w_L^2 higher than the one he obtains in equilibrium. If L accepts this offer, he is sure to obtain a utility equal to w_L^2 , as he is sure not to experience disutility from inequality. In the proof we assume, instead, that he accepts the standing offer of firm 1. This is because he believes that, after this offer of w_L^2 , worker H will decide to accept the standing offer of firm 4, so that the L worker will not experience disutility from inequality by moving to firm 1. But notice that, for w_L^2 arbitrarily close to θ_L , he has to be arbitrarily sure that H will indeed move. We find this rather unsatisfactory because of its probable unrealism.

There is one problem that arises if we choose to eliminate dominated strategies. When wages can be chosen from the real numbers, the set of undominated strategies is open. Any wage that is strictly smaller than the productivity of a worker is undominated, but a wage equal to productivity is weakly dominated. So we cannot construct

Nash equilibria in undominated strategies, as any wage offer different from the productivity can always be defeated by a nearby proposal. To get rid of this difficulty, we discretize the wage space. We consider a family of discrete wage spaces with increasingly fine grids that approaches the continuum when the grids becomes infinitely fine.

More precisely, let n^0, n^1, n^2, \dots be an increasing sequence of integers such that $n^k \rightarrow +\infty$. For each $k \in \mathbb{N}$, let

$$\Theta^k = \left\{ \frac{a}{n^k} \mid a \in \mathbb{N} \right\}.$$

We assume that $\theta_t \notin \Theta^k$, for all $k \in \mathbb{N}$ and $t \in \{L, H\}$.¹⁸ For all $k \in \mathbb{N}$, let $\varepsilon^k = 1/n^k$, and for all $t \in \{L, H\}$, let $\theta_t^k = \arg \max \{x \leq \theta_t \mid x \in \Theta^k\}$. By definition, θ_t^k is the highest element in the discrete wage space Θ^k smaller than type t 's productivity. We have, $\varepsilon^k > \theta_t - \theta_t^k > 0$, for all $t \in \{L, H\}$.

The location and contracting game where firms chose wages in Θ^k is denoted by \mathcal{G}^k .

Proposition 2 *There exists an integer K such that, for all $k \geq K$, $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of \mathcal{G}^k , contracts accepted with positive probability are different across types, and pay t type employees a wage θ_t^k , $t \in \{L, H\}$.*

Corollary 3 *When $k \rightarrow +\infty$, contracts accepted with positive probability pay employees exactly their productivity.*

The presence of social preferences does not change the contracts observed in equilibrium, with respect to the equilibrium contracts when agents do not have extended preferences. The proof is very similar as the one for the standard model. One needs to be a bit careful with the deviations that defeat non-equilibrium outcomes. The problem is that those deviations could increase inequality, so either they would not be followed, or they would be too expensive to be profitable. However, we have assumed that a marginal increase in inequality (even considering the whole group) is not more valuable than an increase in material payoff of the same size (assumption A3). The

¹⁸Precisely, to avoid including a weakly dominated strategy in the wage space.

assumption of a countably infinite number of locations allows any firm, incumbent or entrant, to move at an empty location with no firms close by. In combination with the assumption of a finite span of social concerns (assumption *A1*) and the sensitivity of social payoffs to the local wage profiles (assumption *A3*) allows to construct deviations that are just like the ones in the standard proofs, adjusted for the potential increase in the inequality. Example 7 at the end of this section shows that without assuming a high enough number of location, our segregation result would not hold, even if *A1*, *A2* and *A3* are satisfied.

The main difference between the equilibria in our model and the ones in the standard model is that firms, here, do not employ workers of different types. Otherwise some firm would have a deviation that would allow it to earn strictly positive profits by attracting workers of just one type with a lower salary. Their decrease in material payoffs is compensated by a decrease in disutility due to a more egalitarian work environment. So in any equilibrium, types are geographically separated. One consequence of this segregation is that, at equilibrium, contracts accepted with positive probability are identical within types, irrespective of employee's location.

Proposition 4 *There exists an integer K such that, for all $k \geq K$, $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of \mathcal{G}^k , firms are spatially segregated by types separated by empty locations.*

Social preferences thus predict both skill and spatial workplace segregation as, at equilibrium, firms hire only from one skill pool and firms employing workers of a given skill level form spatial clusters.

Remark 5 *All previous results hold when individuals are averse to inequality in material utilities (that is, wage minus cost of contracted effort), rather than inequality in wages, that is, when extended social payoffs are of the form:*¹⁹

$$U_i(\mathbf{u}, e_i|t) = u_i(w_i, e_i|t) - V_i(\mathbf{u})$$

¹⁹See Bramoullé (2005) for a critical account of different structures of social preferences: (i) concern for others' allocations, (ii) concern for others' material payoffs, and (iii) concern for others' extended social payoffs.

Remark 6 *All previous results hold when the neighborhoods N_i are composed by those workers (if any) employed by firms located in i 's employer node, and in an arbitrary finite string of adjacent nodes encompassing i 's location.*

We have assumed that there is a countably infinite number of possible locations. The following example shows that firms may not be spatially segregated by types (separated by empty locations) when this assumption does not hold.

Example 7 *There are exactly 4 different nodes on the ring, 2 workers of type L and 2 workers of type H. Individual productivities are common knowledge and workers have no cost of effort. Extended preferences are of the form*

$$U_i = u_i - \frac{1}{\#N_i} \sum_{j \in N_i} \alpha |w_j - w_i|, \quad 0 \leq \alpha < 1. \quad (1)$$

Then, there exists a non-segregated equilibrium where exactly 4 firms enter, one at each node, with one H type worker at nodes 1 and 2, and one L type worker at nodes 3 and 4. Each worker is employed by one firm and wages are equal to productivities.

5 Conclusion

This paper shows that small deviations from “selfish” preferences leads to sorting of workers into firms by abilities. This coincides with empirically observed sorting patterns. A natural question is whether our explanation is more important than others for explaining the observation. One competing hypothesis, which would lead to similar results in our context, is that workers of the same type have complementary sets of skills. The two hypothesis are observationally distinguishable for several reasons. First, as mentioned in the discussion of the literature, our model makes predictions about segregation also when the type differences are not ex-ante observable.

Second, in our model, the pecuniary externality is driven by the fact that firms compete between themselves. In the absence of that externality there would be no reason for separation. So if a firm had market power in the labor market, and the outside option of workers was not related to their type (say, the skills were highly

job-specific), all workers would be paid the same. Thus, our model would not predict sorting, whereas the model with complementarities would still predict them. While it is not easy to think of markets that precisely fit those conditions, there are many markets for qualified workers in Europe, like those of physicians and teachers, where the public sector has strong market power. If the amount of sorting in those markets were somewhat smaller than in others for workers of similar characteristics, our hypothesis would clearly have explanatory power. More empirical field work seems like a good avenue for further research.

Third, the pecuniary externality rooted in social preferences has a wide span of testable implications for labor market dynamics, as we analyze fully in Cabrales, Calvó-Armengol and Pavoni (2006).

On the other hand, experimental work appears to be more challenging for this topic than for others that have to do with social preferences. It will be difficult to control in the lab the network structure of preferences. By choosing subjects from physically distant places, and running the experiment on the Internet, one could emulate the social structure of the model. In any case, we believe that a contribution of this paper is that it confronts the field with the important issue of who is included in the interpersonal comparisons and how much. A better understanding of this issue could also contribute to clarify the other important (at least from an evolutionary point of view) question of why agents care about payoff differences.

One other observation on empirical testing arises from the fact that individuals may not be averse to inequality when the output measure of others is very objective. It may be debatable who is the best economist in a certain department (the current fashion for ranking individuals notwithstanding), but it is less controversial who is the top scorer in a soccer team. If indeed aversion to inequality depends on the objectivity of the output measure, then one would expect less sorting by skill-type (thus more within-firm inequality) in soccer teams than in universities.

References

- [1] Acemoglu, D. (1999): "Change in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *American Economic Review* 89(5), 1259-1278.
- [2] Agell, J. and P. Lundborg, P. (1995), "Theories of Pay and Unemployment: Survey Evidence from Swedish Manufacturing Firms," *Scandinavian Journal of Economics* 97, 15-29.
- [3] Agell, J. and P. Lundborg, P. (2003), "Survey Evidence on Wage Rigidity and Unemployment: Sweden in the 1990s," *Scandinavian Journal of Economics* 105, 295-307.
- [4] Becker, G. (1957): *The Economics of Discrimination*, Chicago: University of Chicago Press.
- [5] Bewley, T.F. (1995), "A Depressed Labor Market as Explained by Participants," *American Economic Review Papers and Proceedings* 85, 250-254
- [6] Bewley, T.F. (1999): *Why Wages don't Fall during a Recession*, Harvard: Harvard University Press.
- [7] Bénabou, R. (1993): "Workings of a City: Location, Education, and Production," *Quarterly Journal of Economics* 108, 619-652.
- [8] Blinder, A. and D. Choi, D. (1990), "A Shred of Evidence on Theories of Wage Stickiness," *Quarterly Journal of Economics* 105, 1003-1015.
- [9] Bolton, G. (1991): "A Comparative Model of Bargaining: Theory and Evidence," *American Economic Review* 81, 1096-1135.
- [10] Bolton, G. and A. Ockenfels (2000): "ERC: A Theory of Equity, Reciprocity and Competition," *American Economic Review* 90, 166-193.
- [11] Brown, C. and J. Medoff (1989): "The Employer Size Wage Effect," *Journal of Political Economy* 97, 1027-1059.
- [12] Bramoullé, Y. (2005): "Mutual Altruism and Misery Traps," mimeo, Université de Laval.

- [13] Burgess, S., J. Lane and K. McKinney (2004), "Matching, Reallocation and Changes in Earnings Dispersion," mimeo, Bristol University.
- [14] Cabrales, A., A. Calvó-Armengol and N. Pavoni (2006), "Social Preferences, Skill Segregation and Wage Dynamics," *Review of Economic Studies* (forthcoming).
- [15] Campbell III, C.M. and K.S. Kamlani (1997), "The Reasons for Wage Rigidity: Evidence from a Survey of Firms," *Quarterly Journal of Economics* 112, 759-789.
- [16] Charness, G. and M. Rabin (2002): "Some Simple Tests of Social Preferences," *Quarterly Journal of Economics* 117, 817-869.
- [17] Davis, S. and J. Haltwinger (1991): "Wage Dispersion Between and Within US Manufacturing Plants, 1963-1986," *Brookings Paper on Economic Activity: Microeconomics* 115-200.
- [18] de Bartolomé, Charles (1990): "Equilibrium and Inefficiency in a Community Model with Peer Group Effects," *Journal of Political Economy* 98, 110-133.
- [19] Fehr, E. and K. Schmidt (2000a): "A Theory of Fairness, Competition and Cooperation," *Quarterly Journal of Economics* 114, 817-868.
- [20] Fehr, E. and K. Schmidt (2000b): "Theories of Fairness and Reciprocity: Evidence and Economic Applications," forthcoming in: M. Dewatripont, L.P. Hansen, and S. Turnovski, *Advances in Economic Theory. Eight World Congress of the Econometric Society*, Cambridge: Cambridge University Press.
- [21] Fehr, E., Klein, A. and K. Schmidt (2007): "Fairness and Contract Design," *Econometrica* 75, 121-154.
- [22] Fershtman, C., H.K. Hvide and Y. Weiss (2005): "Cultural Diversity, Status Concerns and the Organization of Work," *Research in Labor Economics*.
- [23] Frank, R.H. (1984): "Are Workers Paid their Marginal Product?," *American Economic Review* 74, 549-571.
- [24] Frank, R.H. (1985): *Choosing the Right Pond: Human Behavior and the Quest for Status*, Oxford: Oxford University Press.

- [25] Güth, W., Schmittberger, R. and B. Schwarze (1982): “An Experimental Analysis of Ultimatum Bargaining,” *Journal of Economic Behavior and Organization* 3, 367-388.
- [26] Harris, M. and B. Hölmström (1982): “A Theory of Wage Dynamics,” *Review of Economic Studies* 49, 315-333.
- [27] Katz, L. E., and D. H. Autor (1999), “Changes in Wage Structure and Earnings Inequality,” Ch. 26, in O. Ashenfelter and D. Card eds., *Handbook of Labor Economics*, Elsevier.
- [28] Kramarz, F., Lollivier, S. and L.-P. Pelé (1996): “Wage Inequalities and Firm-Specific Compensation Policies in France,” *Annales d’Economie et de Statistiques* 41/42, 369-386.
- [29] Kremer, M. and E. Maskin (1996): “Wage Inequality and Segregation by Skill,” NBER Working Paper 5718.
- [30] Juhn, C., K. M. Murphy and B. Pierce (1993), “Wage Inequality and the Rise in Returns to Skill,” *Journal of Political Economy* 101, 410-442.
- [31] Ledyard, J.O. (1995): “Public Goods: A Survey of Experimental Research,” in J.H. Kagel and A.E. Roth eds., *Handbook of Experimental Economics*, Princeton NJ: Princeton University Press.
- [32] Levine, D. (1998): “Modelling Altruism and Spitefulness in Game Experiments,” *Review of Economic Dynamics* 1, 593-622.
- [33] Mas-Colell, A., Whinston, M.D. and J.R. Green (1995): *Microeconomic Theory*, Oxford University Press.
- [34] Rabin, M. (1993): “Incorporating Fairness into Game Theory,” *American Economic Review* 83, 1281-1302.
- [35] Rey-Biel, P. (2005): “Inequity Aversion and Team Incentives,” mimo, Universitat Autònoma de Barcelona.
- [36] Rotemberg, J. (1994): “Human Relations in the Workplace,” *Journal of Political Economy* 102, 684-717.

- [37] Roth, A.E.(1995): “Bargaining Experiments,” in J.H. Kagel and A.E. Roth eds., *Handbook of Experimental Economics*, Princeton NJ: Princeton University Press.
- [38] Segal, U. and J. Sobel (2006): “Tit for Tat: Foundations of Preferences for Reciprocity in Strategic Settings,” *Journal of Economic Theory* (forthcoming).
- [39] Sobel, J. (2005): “Interdependent Preferences and Reciprocity,” *Journal of Economic Literature* 43, 392-436.
- [40] Saint-Paul, G. (2001): “On the Distribution of Income and Worker Assignment Under Intrafirm Spillovers, with an Application to Ideas and Networks,” *Journal of Political Economy* 109, 1-37.
- [41] Schelling, T. (1971): “Dynamic Models of Segregation,” *Journal of Mathematical Sociology* 1, 143-186.

Let $k \in \mathbb{N}$ and \mathcal{G}^k the corresponding game. We denote by $m_{f,i}^k = \langle w_{f,i,L}^k, e_{f,i,L}^k; w_{f,i,H}^k, e_{f,i,H}^k \rangle$ the menu of contracts offered by firm f to player i . For all $i \in N$, let $M_i^k = \{m_{f,i}^k\}_{f \in F}$ denote the set of contracts offered to player i by all firms. A pure strategy Nash equilibrium of \mathcal{G}^k 's second stage (acceptance) game is a profile of accepted menus $\times_{i \in N} M_i^k$.

Proof of Proposition 2. We decompose it into the following lemmata.

Lemma 8 *For all $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of \mathcal{G}^k , firms ex ante profits are nonnegative and strictly smaller than ε^k .*

Proof. Suppose not. Let $k \in \mathbb{N}$ and \mathcal{G}^k the corresponding game. Then there exists some subgame perfect Nash equilibrium (SPNE) of \mathcal{G}^k where some firms ex ante profits are higher or equal than ε^k . Consider such a SPNE, denoted by *SPNE.

Let \bar{m}^{k*} be the menu that makes the highest expected profit at *SPNE. This menu is offered by some firm f to some player i , that is, $\bar{m}^{k*} = m_{f,i}^{k*} = \langle w_{f,i,L}^{k*}, e_{f,i,L}^{k*}; w_{f,i,H}^{k*}, e_{f,i,H}^{k*} \rangle$, and player i accepts it. Let $t_i \in \{L, H\}$ denote player i 's type. Given that f 's ex ante profits are higher or equal than ε^k , necessarily $\theta_{t_i} - w_{f,i,t_i}^{k*} \geq \varepsilon^k$. We distinguish two cases.

Case 1: $\theta_L - w_{f,i,L}^{k*} \geq \varepsilon^k$. Consider some firm $g \neq f$ making zero profits at *SPNE, possibly a new entrant. Let g deviate by locating at an empty location surrounded by two empty adjacent locations. Let g offer player i the menu of contracts $m_{g,i}^{k\circ} = \langle \theta_L^k, e_{f,i,L}^{k*}; w_{f,i,H}^{k*}, e_{f,i,H}^{k*} \rangle$ at this location. We have $\theta_L - w_{f,i,L}^{k*} \geq \varepsilon^k > \theta_L - \theta_L^k$, implying in particular that $\theta_L^k > w_{f,i,L}^{k*}$. Player i may be simultaneously receiving offers from other firms (besides from g) which are equivalent, in terms of material payoffs, to $m_{g,i}^{k\circ}$. But, if player i didn't accept those offers at the *SPNE, it is because player i would have faced a strict disutility due to inequality in case of accepting them. At g 's new location, there is certainly no inequality (by A1). At any other location, though, the extended utility accruing from any menu equivalent to $m_{g,i}^{k\circ}$ in terms of material payoffs depends, in general, on the reactions of other players. Therefore, by A2, it is a weakly dominant strategy for player i to accept $m_{g,i}^{k\circ}$, and g 's deviation is profitable in expected terms.

Case 2: $\theta_L - w_{f,i,L}^{k*} < \varepsilon^k$ but $\theta_H - w_{f,i,H}^{k*} \geq \varepsilon^k$. In particular, Let $g \neq f$ making zero profits at *SPNE, deviating by locating at an empty location surrounded by two empty adjacent locations, and offering player i the menu of contracts $m_{g,i}^{k\circ} = \langle w_{f,i,L}^{k*}, e_{f,i,L}^{k*}; \theta_H^k, e_{f,i,H}^{k*} \rangle$ at this location. Again, by A2, it is a weakly dominant strategy for player i to accept g 's offer given that it increases his material payoffs, and that there is no disutility due to inequality at g 's new location by A1 (and g 's deviation is profitable). Indeed, switching contracts modifies both the material payoffs and the inequality payoffs accruing to some individual. By A3, variations in inequality induced by unilateral switching of contracts do never offset the corresponding variations in material payoffs, and unilateral decisions to pick up a contract out of an array of alternatives are governed solely by material payoff concerns. Therefore, no L type worker accepts $(\theta_H^k, e_{f,i,H}^{k*})$ because the corresponding material payoffs are strictly lower than those obtained with some alternative offered contract. ■

Lemma 9 *There exists an integer K such that, for all $k \geq K$, $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of \mathcal{G}^k , contracts of different types accepted with positive probability are different.*

Proof. Suppose not. Let $k \in \mathbb{N}$ and \mathcal{G}^k the corresponding game. We distinguish two cases.

Case 1. There exists one firm \bar{f} that offers a menu $\bar{m}^k = \langle \bar{w}^k, \bar{e}^k; \bar{w}^k, \bar{e}^k \rangle$ with identical wage \bar{w}^k and effort level \bar{e}^k to both workers' types. In the effort-wage space, denote by \mathcal{U}_H° the strict upper contour set corresponding to the material payoffs of an H type worker applying for firm \bar{f} at its location. Similarly, denote by \mathcal{U}_L the upper contour set of the material payoffs of an L type worker applying for firm \bar{f} at its location. Consider some firm g making zero profits, possibly a new entrant. Suppose that g deviates to an empty location and offers a menu $\langle \bar{w}^k, \bar{e}^k; \tilde{w}^k, \tilde{e}^k \rangle$ to some of \bar{f} 's current workers, where $(\tilde{w}^k, \tilde{e}^k)$ is chosen in $\Psi^k = (\mathcal{U}_H^\circ \setminus \mathcal{U}_L) \cap \{w < \theta_H \mid w \in \Theta^k\}$. We show that for k high enough, $\Psi^k \neq \emptyset$. By assumption, for all $e \in \mathbb{R}_+$, $c_L(e) > c_H(e)$. Therefore, for k high enough, $\mathcal{U}_H^\circ \setminus \mathcal{U}_L \neq \emptyset$. We are left to prove that $(\mathcal{U}_H^\circ \setminus \mathcal{U}_L) \cap \{w < \theta_H \mid w \in \Theta^k\} \neq \emptyset$. It suffices to show that, for k high enough, $\bar{w}^k < \theta_H^k$. Suppose

on the contrary that, for all $k \in \mathbb{N}$, $\bar{w}^k \geq \theta_H^k$. For k high enough, $\theta_H^k > \theta_L$. For such values of k , \bar{f} 's *ex post* profits made with H type workers are smaller or equal than ε^k , whereas \bar{f} 's *ex post* profits made with L type workers are strictly negative. There is a positive probability that L type workers accept menu \bar{m}^k . Therefore, given that $\varepsilon^k \downarrow 0$, when $k \rightarrow +\infty$, there exists an integer K such that, for all $k \geq K$, \bar{f} 's *ex ante* profits are negative, which violates Lemma 8. Therefore, for all $k \geq K$, we have $\bar{w}^k < \theta_H^k$. With such menu of contracts, by A2 and A3, it is a weakly dominant strategy for all H type workers in \bar{f} 's workforce to accept g 's offer given that it increases their material payoffs, and there is no disutility due to inequality at g 's new location. This deviation is profitable to g .

Case 2. There exists one firm \bar{f}_1 who offers a menu \bar{m}_1^k including contract (\bar{w}^k, \bar{e}^k) only accepted by L type workers and a firm \bar{f}_2 who offers a menu \bar{m}_2^k including contract (\bar{w}^k, \bar{e}^k) only accepted by H type workers. But then, by Lemma 8, all *ex post* profits of firm \bar{f}_1 with L type workers are nonnegative and smaller or equal than ε^k , implying that $\bar{w}^k = \theta_L^k$. Similarly, all *ex post* profits of firm \bar{f}_2 with H type workers are nonnegative and smaller or equal than ε^k , implying that $\bar{w}^k = \theta_H^k$, which is impossible as, for high enough values of k , we have $\theta_L^k \neq \theta_H^k$. ■

Lemma 10 *There exists an integer K such that, for all $k \geq K$, $k \in \mathbb{N}$, at every subgame perfect Nash equilibrium of \mathcal{G}^k , contracts accepted with positive probability by L type workers (resp. H type workers) offer wage θ_L^k (resp. wage θ_H^k), that is, contracts accepted with positive probability make *ex post* profits which are nonnegative and strictly smaller than ε^k .*

Proof. Let $k \in \mathbb{N}$ and \mathcal{G}^k the corresponding game. We first show that for any firm f and independently of its location, the wage $w_{f,i,L}^k$ proposed by f to some player i , and accepted by i whenever $t_i = L$, is such that $w_{f,i,L}^k \geq \theta_L^k$. Suppose on the contrary that some firm f offers at some location a wage $w_{f,i,L}^k < \theta_L^k$ which is part of a contract accepted with positive probability. Consider some firm g making zero profits, possibly a new entrant. Suppose that g deviates to an empty location and offers the contract $(\theta_L^k, e_{f,i,L}^k)$ to some of f 's current workers. Then, g makes *ex post* profits which are

higher or equal than ε^k with any worker eager to accept such wage offer, whatever his type. Therefore, g makes *ex ante* profits which are higher or equal than ε^k , which is impossible by Lemma 8.

We now show that the wage $w_{f,i,H}^k$ proposed by any firm f to some player i , and accepted by i whenever $t_i = H$, is such that $w_{f,i,H}^k \geq \theta_H^k$. Suppose not. Then, there exists some firm f offering a contract $(w_{f,i,H}^k, e_{f,i,H}^k)$ accepted with positive probability by some H type workers, where $w_{f,i,H}^k < \theta_H^k$. Lemma 9 implies that, for k high enough, no L type worker accepts this contract. In other words, for k high enough, the extended social payoffs of any L type worker accepting $(w_{f,i,H}^k, e_{f,i,H}^k)$ are strictly lower than the extended utility obtained with some alternative contract. Switching contracts modifies both the material payoffs and the inequality payoffs accruing to some individual. Given A3, variations in inequality induced by unilateral switching of contracts do never offset the corresponding variations in material payoffs, and unilateral decisions to pick up a contract out of an array of alternatives are governed solely by material payoff concerns. Therefore, for k high enough, no L type worker accepts $(w_{f,i,H}^k, e_{f,i,H}^k)$ because the corresponding material payoffs are strictly lower than those obtained with some alternative offered contract. Consider some firm g making zero profits. Suppose that g deviates to an empty location and offers the contract $(\theta_H^k, e_{f,i,H}^k)$ to some of f 's current workers. By A2 it is a weakly dominant strategy for all H type workers in f 's workforce to accept g 's offer given that it increases their material payoffs, and there is no disutility due to inequality at g 's new location. The increase in material payoffs is $\theta_H^k - w_{f,i,H}^k = q\varepsilon^k$, for some $q \neq 1$. We know that, for k high enough, no L type worker accepts f 's original contract $(w_{f,i,H}^k, e_{f,i,H}^k)$, and this decision is taken by comparing only material payoffs from different contracts. Also, $\varepsilon^k \downarrow 0$, when $k \rightarrow +\infty$. Therefore, there exists an integer K such that, for all $k \geq K$, no L type worker accepts g 's contract offer. When $k \geq K$, only H type workers accept firm g 's offer, and g 's *ex post* profits with all of them are strictly higher than ε^k , which is impossible by Lemma 8.

Therefore, for all $k \geq K$, $k \in \mathbb{N}$, $f \in F$ and $i \in N$, we have $w_{f,i,L}^k \geq \theta_L^k$ and $w_{f,i,H}^k \geq \theta_H^k$. By Lemma 8, firms make *ex ante* profits which are nonnegative and

smaller or equal than ε^k . Therefore, $w_{f,i,L}^k = \theta_L^k$ and $w_{f,i,H}^k = \theta_H^k$. ■

Proof of Proposition 4. Let $k \in \mathbb{N}$ and \mathcal{G}^k the corresponding game. Consider a subgame perfect Nash equilibrium of \mathcal{G}^k , denoted by *SPNE. Given a location ℓ , denote by n_ℓ the number of workers employed at ℓ and at its two adjacent nodes at *SPNE. We have $n_\ell = n_{\ell,L} + n_{\ell,H}$, where $n_{\ell,t}$ denotes the number of t type workers employed at ℓ and at its two adjacent nodes, $t \in \{L, H\}$. For all $t \in \{L, H\}$, let

$$q_{\ell,t} = \begin{cases} \frac{n_{\ell,t}}{n_\ell}, & \text{if } n_\ell \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

We prove that $q_{\ell,t} \in \{0, 1\}$, for all $t \in \{L, H\}$. Suppose not. Let ℓ such that $0 < q_{\ell,L} < 1$.²⁰ Let ℓ' be an empty location surrounded by two empty locations.

We now prove that workers employed at ℓ experience a nonzero disutility due to inequality at *SPNE. Suppose not. Denote by u_i^* the material payoffs of player i at *SPNE and by U_i^* its extended social payoffs. Then, for all i, j employed at ℓ and its two adjacent nodes, $U_i^* = u_i^* = u_j^* = U_j^*$. Given that $0 < q_{\ell,L} < 1$, there exists at least two workers of different types employed at ℓ or its vicinity which are in the direct neighborhood of each other. We denote those workers by i_L and i_H , where $t_{i_L} = L$ and $t_{i_H} = H$. In the effort-wage space, denote by \mathcal{U}_H° the strict upper contour set corresponding to the material payoffs of i_H , and by \mathcal{U}_L the upper contour set corresponding to the material payoffs of i_L . Let $\Phi^k = (\mathcal{U}_H^\circ \setminus \mathcal{U}_L) \cap \{w < \theta_H^k \mid w \in \Theta^k\}$. We show that for k high enough, $\Phi^k \neq \emptyset$. Indeed, denote by $(w_{\ell,i_H}^*, e_{\ell,i_H}^*)$ the contract accepted by i_H at location ℓ at *SPNE, where $w_{\ell,i_H}^* \in \Theta^k$. Let (w, e_{ℓ,i_H}^*) , $w \in \Theta^k$, such that $u_{i_L}(w, e_{\ell,i_H}^*) = u_{i_H}(w_{\ell,i_H}^*, e_{\ell,i_H}^*)$. Given that, for all $e \in \mathbb{R}_+$, $c_L(e) > c_H(e)$, necessarily $w > w_{\ell,i_H}^*$. For k high enough, there exists some $w' \in \Theta^k$ such that $w > w' > w_{\ell,i_H}^*$, implying that $\mathcal{U}_H^\circ \setminus \mathcal{U}_L \neq \emptyset$. If k is high enough, we also have $\Phi^k \neq \emptyset$. Consider some firm g making zero profits at *SPNE, possibly an entrant. Suppose that g deviates to ℓ' and offers a contract $(\tilde{w}, \tilde{e}) \in \Phi^k$. We know from Lemma 9 that, at equilibrium, when k is high enough, no L type worker accepts the contract with which i_H obtains $U_{i_H}^* = u_{i_H}^*$ at ℓ . Recall also from the proof of Lemma 10, that uses A3, that

²⁰Note that $q_{\ell,L} = 1 - q_{\ell,H}$, and $0 < q_{\ell,H} < 1$ is equivalent to $0 < q_{\ell,L} < 1$.

unilateral deviations to pick up a contract out of an array of alternatives are governed solely by material payoffs concerns. Therefore, for high enough values of k , $(\tilde{w}, \tilde{e}) \in \Phi^k$ can be chosen so as not to be accepted by any L type worker. Then, g only attracts H type workers to ℓ' (those initially employed at ℓ , and possibly some others). We deduce from Lemma 10 that H type workers are paid θ_H^k at equilibrium. By construction of Φ^k , $\tilde{w} < \theta_H^k$. Therefore, g makes *ex ante* profits which are higher or equal than ε^k , which is impossible by Lemma 8.

Therefore, at ℓ , employed workers face a strictly positive disutility due to inequality. Any L type worker employed at ℓ would be strictly better off at ℓ' with the same contract because he would face a smaller disutility due to inequality. Therefore, any firm making zero profits at the current equilibrium moving to ℓ' and offering a contract $\theta_L^k - \varepsilon^k$, where k is high enough, could attract such L type workers (and possibly some H type workers too) and make *ex ante* profits strictly higher than ε^k , thus violating Lemma 4. ■

Proof of Example 1. To show that this is indeed part of a subgame perfect equilibrium, we need to specify the responses of the workers to deviations by the firms. In fact we do not need to specify responses to all possible deviations, but only to unilateral deviations of one firm. Worker H is already obtaining a salary equal to productivity, so no deviation that intends to attract H can ever be profitable. Thus, the only possibly profitable deviations are those that affect worker L . Clearly, firm 3 is already making the maximum possible profit in this environment, so only deviations by firms 1, 2 and 4 need to be considered:

- (a) Suppose that firm 1 deviates by offering L , at some location, the wage w_L^1 , with $\theta_L > w_L^1 > w_L^3$. If worker H responds to this deviation by choosing to work for firm 4, and worker L responds by choosing to work for firm 2, then the deviation by 1 is not profitable.
- (b) Suppose that firm 2 deviates by offering L , at some location, the wage w_L^2 , with $\theta_L > w_L^2 > w_L^3$. If worker H responds to this deviation by choosing to work for firm 4, and worker L responds by choosing to work for firm 1, then the deviation

by 2 is not profitable.

- (c) Suppose that firm 4 deviates by offering L , at some location, the wage w_L^4 , with $\theta_L > w_L^4 > w_L^3$. If worker H responds to this deviation by choosing to work for firm 3, and worker L responds by choosing to work for firm 2, then the deviation by 4 is not profitable. ■

Proof of Example 7. It is readily checked that this game has two subgame perfect Nash equilibria (modulo a relabelling of nodes). In both cases, workers are paid exactly their productivity at equilibrium:

- (a) a *segregated* equilibrium, where both H type workers are located at node 1, and both L type workers are located at node 2, and individual extended payoffs at equilibrium are $U_i = \theta_{t_i}$, $i \in \{1, 2, 3, 4\}$.
- (b) a *non-segregated* equilibrium, where H type workers are located at nodes 1 and 2, and L type workers at nodes 3 and 4, and extended payoffs are $U_i = \theta_{t_i} - \alpha(\theta_H - \theta_L)/2$, $i \in \{1, 2, 3, 4\}$. ■