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# EFFICIENT PARTNERSHIP DISSOLUTION UNDER BUY-SELL CLAUSES

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**Abstract**

When a partnership comes to an end, partners have to determine the terms of the dissolution. A well known way to do so is by enforcing a buy-sell clause. Under its rules one party offers a price for the partnership and the other party chooses whether to sell her share or buy her partner´s share at this price. It is well known that in a model with private valuations this dissolution rule may generate inefficient allocations. However, we show that if partners negotiate for the advantage of being chooser, then buy-sell clauses result in an ex-post efficient outcome. We argue that this endogenous selection of the proposer is consistent with how buy-sell clauses are drafted in practice. For an example with interdependent valuations, we further show that the buy-sell clause can perform better than an auction.

***Keywords:*** partnership dissolution, buy-sell clause, shootout mechanism
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# 1 Introduction

Partners in commercial relationships have to think about the possibility that the partnership might end. A management deadlock, e.g. a fundamental disagreement about the future strategy of the commonly owned firm, might lead to the inevitability of splitting up. Due to market inefficiencies and frictions, it is sometimes more sensible for one partner to buy out the other and realize the profits than to liquidate and sell the company to a third party.[1] Being aware of this, business attorneys recommend including a buy-sell clause (commonly referred to as shoot-out clauses) in the initial partnership agreement to govern the dissolution process. In brief, a buy-sell clause is a deadlock resolution that works as follows: one party proposes a price, and the other decides whether to buy or sell at that price. Lawyers consider the buy-sell clause as an essential part of partnership agreements,[2] and thus not surprisingly real life examples of partnerships that include buy-sell clauses in their partnership agreements abound.[3]

Buy-sell clauses have also caught the attention of economic theorists. When partners' valuations are private information, reflecting what they know about their ability to run the business, buy-sell clauses are not recommended by economists to solve a deadlock (see McAfee [1992]). Contrary to legal advice, they have stressed that buy-sell clauses could result in inefficient dissolutions, i.e., the business may not end up in the hands of the partner who values it most. When partners only know their own valuation, the

[1] Empirical evidence confirms that buy-outs take place very often. Hauswald and Hege [2003], from a sample of 193 two-parent US joint ventures, report that in 32% of them there was a buy-out and in 20% there was either a liquidation or sale to a third party, during the time period from 1985 to 2000.

[2] The importance of buy-sell agreements is now so broadly recognized that a lawyer's failure to recommend or include them in modern joint venture agreements is considered *malpractice* among legal scholars and practitioners (see Brooks and Spier [2004]). E.g., the Guide to US Real State Investing issued by the Association of Foreign Investors in Real Estate (AFIRE), refers to the buy-sell clause and states that "such a clause is usually thought of as the ultimate mechanism for resolving disputes".

[3] The following partnerships all included buy-sell clauses: the partnership for the German TV-channel VOX between the media companies CLT-UFA and News Corporation; the home shopping partnership between Liberty and Comcast; the joint venture to run the New York-New York Hotel between Primadonna Resorts Inc. and MGM Grand Inc.; and the joint venture to own and operate "Original Levi's Stores" between Levi Strauss & Co. and Designs Inc.

proposer has to set a price based on her estimate of her partner's valuation. She offers a price above (below) her valuation if she believes that her partner's valuation is likely to be higher (lower) than her own.[4] Consequently, whenever the other party's valuation lies between hers and the price, she becomes a buyer, whereas efficiency requires her to sell. Similarly, when the other party's valuation lies between the price and hers, she inefficiently becomes the seller. Nevertheless, this inefficiency does not have bite if the *right* party is called to name the price. When both partners have low valuations (and would hence propose a price above their valuation) an efficient dissolution is guaranteed if the partner with the higher valuation proposes the price. Similarly, if both partners have high valuations, efficiency is guaranteed if the partner with the lower valuation proposes. If one partner's valuation is below the median and the other's is above then either party proposing the price ensures efficiency. The determination of the proposer is then crucial for efficiency.

From a legal point of view a buy-sell clause (as other dissolution clauses) consists of two parts: a specification of the circumstances under which it applies and a description of what the partners are obliged to do in case it applies. The first part defines when a management deadlock (MD) is deemed to occur. The second part prescribes a set of actions in a chronological order. It states that (possibly after a cooling-off period), either party can serve a so-called Deadlock Option Notice, which is irrevocable, and specifies a single price at which the party (the proposer) giving notice offers to either sell all her shares at the specified price, or purchase all shares held by the other party at this specified price. After receiving the Deadlock Option Notice the other party shall, at its sole option, elect either: (a) to purchase all of the shares at the price stated in the Deadlock Option Notice, or (b) to require the proposer to purchase all of the shares held by that other party at the price stated in the Deadlock Option Notice.[5] The formulation of the buy-sell clause

[4]If a proposer perceives herself being more likely the seller, she is interested in a high price and would therefore optimally increase her price above her valuation. By contrast, if she expects to be buyer she optimally decreases her offer below her valuation.

[5]A blueprint of a buy-sell clause can be found in Precedent 10, page 253 in Hewitt [2005]. The descriptions of the clause in J. Cadman [2004] or G. Stedman and J. Jones [1990], are also consistent with the one reported here.

reveals that it only applies to two-parent partnerships. Furthermore, most corporate lawyers recommend its use only if both partners own (roughly) the same share in the firm.[6] It is worth noting that the buy-sell clause neither covers the contingency that no party proposes a price nor the contingency where both propose a price. Moreover, it does not resolve the question of who has to propose. Parties who disagree on this issue might ask a court to dissolve the partnership as the legal case *Hotoyan v Jansezian* illustrates. Two partners who had signed an agreement that included a buy-sell clause, went to the Ontario Court as they disputed over the matter of who had to go first.[7]

An alternative way of settling the issue on who has to propose was taken by Comcast UK and Telewest, each 50% owners of the Cable London franchise. After 16 years of partnership, in February 1998 Comcast announced its intention to sell its cable interest to the NTL group. This announcement resulted in negotiations between Comcast UK and Telewest for the dissolution of their partnership. In August 1998 the Telewest spokesman announced that they had "solved an ambiguity in the original ownership agreement". They agreed that by no later than 30 September 1999, Comcast (or NTL after the amalgamation with Comcast) would notify Telewest of a price at which Telewest would be required either to purchase or sell. The buy-out was completed in August 1999 with Comcast/NTL proposing a price of approx. £428 million to Telewest, who decided to buy.[8] The agreement between Telewest and Comcast suggests that a buy-sell clause may be used after a negotiation stage to identify the proposer.[9]

Since the buy-sell clause (as formulated in partnership agreements) does not pre-specify a certain partner as proposer but it allows for the proposer to be selected endoge-

[6]Hewitt writes ([2005], p. 248): "[The buy-sell clause] works reasonably well only if the proportionate shareholdings of the parties are roughly equal".

[7]See [1999] O.J. No. 4486, Court File No. 99-CL-3263.

[8]Further details about this case can be found in the press releases of Telewest at www.telewst.co.uk, in particular in the press releases from 17 August 1998 and 25 July 1999. In these press releases , they also state that Telewest agreed to acquire Birmingham Cable for £125 million. We perceive that these transfers incorporate the payment Telewest had to make for being chooser.

[9]Further evidence that parties negotiate over which partner has to propose comes from the cases *Damerow Ford Co. v. Bradshaw,* 876 P.2d 788 (1994), and *Rochez Bros., Inc. v. Rhoades,* 491 F.2d 402 (1974).

nously by the partners if a deadlock arises, we here model the dissolution game induced by a buy-sell clause. In accordance with contractual formulations in partnership agreements, we study the performance of buy-sell clauses taking into account that parties may negotiate over the identity of the proposer, may just wait for the other party to propose, or may end up in court. Our main analysis covers the classical private values environment as introduced by Cramton et al. [1987] and McAfee [1992]. We show that if partners negotiate the role of proposer (and these negotiations start without delay after the deadlock occurs) then the partnership is dissolved efficiently, i.e., the partner with the higher valuation buys all shares.

We model the negotiations as an ascending auction with parties bidding for the right to choose. It can be thought of as a simplified model of a negotiation procedure in which parties make alternating offers for the right to choose. We show that when partners take into account the information that will be revealed through their negotiations, the party with the valuation closer to the median valuation will propose and hence that an efficient dissolution will take place. If partners engage in costly waiting for the other to propose the price (which we model as a war of attrition), the partner with the higher valuation finally will buy. As in this environment there are inefficiencies due to costly waiting partners would agree to engage in negotiations rather than to wait. Since partnership agreements allow partners to negotiate as well as to strategically wait, we also analyze a framework, where partners are free to negotiate, delay or name a price at any time. We show that in a sequential equilibrium they will negotiate immediately and dissolve the partnership efficiently.

To investigate the robustness of the efficiency results we also analyze the buy-sell clause in an interdependent values model with one-sided private information (based on Jehiel and Pauzner [2006]). We show that the buy-sell clause, though it does not achieve full efficiency, outperforms other possible dissolution mechanism; in particular, it is better than an auction.

Our results may matter to academics as well as practitioners. From a normative point of view, we argue that practitioners need to make partners aware of the option to negotiate the right to choose, as this might not be explicitly stated in the partnership agreement and may avoid a costly war of attrition and/or costly and inefficient court rulings. From an economic point of view our analysis shows that the buy-sell clause is capable of rendering efficient dissolutions. In a private values model they perform as well as auctions which have been suggested as an efficient solution to the dissolution problem, see Cramton et al. [1987] and McAfee [1992].[10] In our interdependent values model we argue that other dissolution mechanisms like auctions (Cramton et al. [1987]) may perform worse.

This paper is organized as follows. In Section 2 we introduce the private values model and briefly discuss the buy-sell clause with a pre-defined proposer (as analyzed in McAfee [1992]). In Section 3 we first analyze the performance of buy-sell clauses if partners negotiate who proposes, we then compare this to the situation where partners wait for the other to propose, and finally, present a unified framework that incorporates negotiations and delay. In Section 4 we analyze the clause for an interdependent values model. Section 5 contains our conclusions. Finally, proofs are relegated to the Appendix.

## 2 The model

We study partnership dissolution in a symmetric independent private values framework.[11] Two risk-neutral partners, 1 and 2, have valuations $v_1$ and $v_2$ respectively, for the sole ownership of the business. Valuations are independently and identically distributed according to a cdf $F$ with continuous density $f > 0$ and support $[0, 1]$. Even though the valuation $v_i$ is private to partner $i$, reflecting what she knows about her ability to run the business, the model does not preclude partners from sharing information on a common value component of the business (as employee structure, assets, financial situation, etc.). Each

[10] Surprisingly, auctions are rarely considered as an alternative to buy-sell clauses in the literature on corporate law. Hoberman [2001], when discussing alternatives to the buy-sell clause, does not even mention auctions. The same is true for books devoted to the drafting of shareholders' agreements (see e.g. Cadman [2004] and Stedman and Jones [1990]).

[11] Throughout the paper we use the term *partnership,* but it should be clear that the model also applies to other legal entities. In particular to unincorporated joint ventures (partnerships), corporate joint ventures, strategic alliances, Societas Europaea and to some "dual-headed" structures.

partner owns an equal share of the partnership.[12] An agent's utility is linear in money and share, i.e., the utility of partner $i$ who holds a share of $\alpha$ in the partnership and receives a payment $m$ is given by $U_i = \alpha v_i + m$.

A desirable property for a dissolution mechanism (such as the buy-sell clause) to have is that it allocates the (shares of the) partnership efficiently. An allocation is said to be efficient if the partner with the highest valuation receives the entire partnership. A dissolution mechanism is (ex-post) efficient if there exists an equilibrium in which the partner with the higher valuation gets the entire partnership and no money is burnt.

In the buy-sell clause one party specifies a price (the *proposer*), and the other party decides whether to buy or sell at that price (the *chooser*). If $p$ is the price specified by the proposer, the chooser selects either $p/2$ or the business, in which case she pays the proposer $p/2$. It can easily be verified that the chooser decides to take the money as long as the price $p$ is larger than her valuation; otherwise, she decides to buy her partner's share.

If one of the partners proposes a price $p$ (immediately after the deadlock occurs), this proposer's expected utility (or expected pauoff) is:

$$\begin{aligned} U^P(v_P, p) &= (v_P - p/2)\Pr(v_C \leq p) + (p/2)\Pr(v_C > p) \\ &= (v_P - p)F(p) + p/2, \end{aligned}$$

where subscripts $P$ and $C$ stand for proposer and chooser, respectively. In what follows, we assume that the standard hazard rate conditions are satisfied:[13]

$$\frac{d}{dx}\left(x + \frac{F(x)}{f(x)}\right) \geq 0 \qquad \text{and} \qquad \frac{d}{dx}\left(x - \frac{1-F(x)}{f(x)}\right) \geq 0. \tag{1}$$

[12] These assumptions are true for most partnerships. In Hauswald and Hege's [2003] sample of joint ventures, the data show that about 80% are two-partner joint ventures. Further, about two-thirds of two-partner joint ventures have 50-50 equity allocations. Similarly, from a sample of 668 worldwide alliances, Veugelers and Kesteloot [1996] report that more than 90% of the alliances only involve two parties. Even though the data cover partnerships between unevenly sized firms, more than 50% exhibit 50-50 ownership.

[13] Under these conditions, the virtual valuation for a type who is a net seller, $\left(x + \frac{F(x)}{f(x)}\right)$, is increasing in her valuation. Similarly for the virtual valuation of a type who is a net buyer, $\left(x - \frac{1-F(x)}{f(x)}\right)\cdot$ Increasing hazard rates ensure that expected payments are increasing in valuations.

Let us define the revenue maximizing price for the proposer by $p^*(v_P) = \arg\max_p U^P(v_P, p)$, and the derivative of $U^P$ with respect to its second argument by $U_2^P$. It is important to note that the proposer's optimal strategy depends on the distribution of the chooser's valuation, whereas the chooser's optimal strategy depends only on the proposed price $p$ and her own valuation $v_C$ (it is therefore independent of any distributional assumptions).

The next proposition characterizes the equilibrium price set by the proposer and some of its properties. Let $v^{med}$ be the median valuation.

**Proposition** ***(McAfee [1992]).*** *The optimal price $p^*(v_P)$ is the unique solution for $p$ to $U_2^P(v_P, p) = 0$. It is non-decreasing and satisfies $p^*(v_P) = v_P$ if $v_P = v^{med}$, $p^*(v_P) < v_P$ if $v_P > v^{med}$, and $p^*(v_P) > v_P$ if $v_P < v^{med}$.*

The rationale behind the properties of the equilibrium price is as follows. If a partner with a valuation above the median sets a price equal to her own valuation, she will more likely end up buying the business. She would hence improve her payoff by reducing the buying price.[14] Similarly, if her valuation is below the median she is better off setting a price above her valuation, as she is more likely the selling partner.

McAfee [1992] shows that using a buy-sell clause to dissolve a partnership may lead to inefficient allocations. The inefficiency might arise when partners' valuations are either both below or both above the median valuation. However, inefficiencies only arise when the wrong partner is proposing. To make this point clear, consider first that partners' valuations are both below the median. As either partner will name a price larger than her valuation, efficiency requires that the partner with the largest valuation proposes. Similarly, if valuations are both above the median then an efficient allocation emerges whenever the partner with the smallest valuation proposes. This suggest that the partner with the lowest valuation should choose if both valuations are below the median, and propose if they are above the median. A natural question to ask is then whether an endogenous determination of the proposer can render efficient allocations and if the answer is affirmative, whether the framing of the clause as it can be found in partnership

[14] By lowering the price, she would also sell to partners with a valuation slightly below her own, therefore making a loss on these trades. This loss is of second order whereas the gain because of buying at a lower price is of first order.

agreements does allow for this to happen.

# 3 Endogenous proposer selection

When a management deadlock comes forth, parties must decide on who proposes. Legal cases, as those referred to in the introduction, show that parties stuck in a deadlock have followed different routes, from waiting for the other to come forward with a price (and finally going to Court), to reaching agreement through negotiations on who will propose. We here study the performance of these exit strategies. As the dissolution games we consider are sequential games with incomplete information, our equilibrium concept is Perfect Bayesian Equilibrium.

## 3.1 Negotiating the right to choose

We first examine the outcome of a dissolution when partners must abide by the buy-sell clause in their shareholder agreement, and they negotiate to determine the party entitled to choose. We consider a dissolution procedure which consists of two stages. In the first stage, *the negotiation stage,* partners determine who becomes chooser and proposer. In the second stage, *the pricing stage,* they dissolve the partnership according to the rules of the buy-sell clause. We will refer to this sequential game as the *dissolution game.* The negotiation stage, is modelled as an ascending auction with a fast clock on bids. Both parties raise their bids continuously, and either party can drop out of the auction at any time. The party who drops out becomes proposer and receives a payment equal to the bid at which the auction ends. The ascending auction can be seen as a continuous version of an alternating-offer negotiation game.[15]

We solve the game assuming that negotiation strategies are U-Shaped. We then show that the purported negotiation strategies are consistent with equilibrium behavior. In what follows we also provide a rough intuition for why partners negotiate in accordance

[15]The assumption of a continuously increasing price is a simplification and describes the limiting case of an offer game where partners in an alternating order increase their offers on a discrete price grid until one of them drops out.

with a U-shaped function.

### 3.1.1 The Pricing Stage

An important aspect of the dissolution game is that information about partners' valuations is revealed by the strategies played in the negotiation stage. The proposing party hence updates her beliefs about the distribution of her partner's valuation. Information is only important for the proposer as the chooser's decision is belief independent. He only needs to compare his valuation with the proposed price.

As a starting point we assume that there is an equilibrium in which bidding strategies are silent, such that no inference about valuations can be made. If this were the case partners would always prefer to choose rather than propose and their willingness to pay for being entitled to choose would be U-shaped.[16] Or more formally, if we denote the interim utility of the proposer and the chooser with valuation $v$ by $U^P(v)$ and $U^C(v)$ respectively, the difference $U^C(v) - U^P(v)$ is strictly decreasing for $v < v^{med}$ and strictly increasing for $v > v^{med}$. The intuition for this result is that a chooser benefits from the fact that the proposer is uncertain about whether she will sell or buy. E.g., consider the type $v^{med}$. If this type proposes, she optimally names her valuation as she is equally likely to be buyer or seller. Her expected utility as proposer is hence half her valuation. As chooser she faces prices close to her valuation. If they are above she sells and gets an expected utility slightly larger than half her valuation. If they are below, she buys which also gives her an expected utility slightly above half her valuation. The difference in expected payoffs is hence strictly positive.[17] This difference is even larger for "more extreme" valuations: as prices are set around $v^{med}$ (recall that prices are set between $v^{med}$ and the proposer's valuation) the difference between valuation and price is larger for a chooser with an "extreme" valuation. In addition, when these "extreme" types propose, they cannot set a price too close to $v^{med}$, as it would result in unprofitable trade with high probability. Consequently, they cannot take much advantage of their "extreme"

[16] See the proof of Theorem 9 in McAfee [1992].

[17] The preference to be the chooser also holds in a common values environment with incomplete information (see Morgan [2004] and Brooks and Spier [2004]). In a model with complete information Crawford [1977] shows that partners prefer to propose.

valuations as proposer. This is why these extreme types have an even higher willingness to pay than types close to $v^{med}$.

To solve the pricing stage, we assume that bidding strategies are indeed U-shaped. As such strategies reveal information, the behavior of the losing party at the pricing stage takes into account what she learns from the negotiation stage. In particular, a losing party who bid $b(v)$ knows that the other party was willing to bid higher. Because of the U-shaped form of the bidding function, she further knows that there may exist another valuation $\widetilde{v}_P$ that would have dropped out at the same bid, i.e. $b(v) = b(\widetilde{v}_P)$. Assuming that $\widetilde{v}_P > v_P$ the losing party concludes her partner's valuation must be either above $\widetilde{v}_P$ or below $v_P$. Figure 1, illustrates this argument. The precise updating is the content of the next lemma.

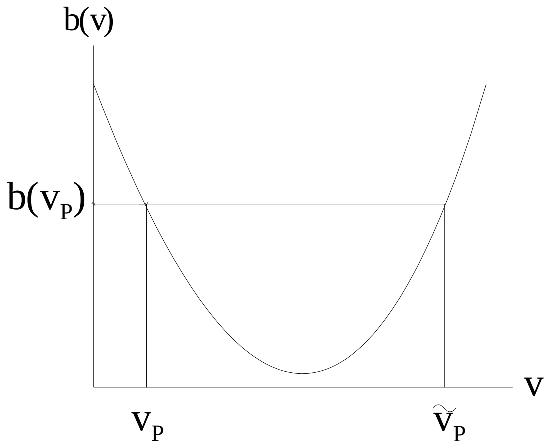


Figure 1: The proposer's inference after the negotiation stage.

**Lemma 1** *If the bidding strategies at the negotiation stage are U-shaped, a proposer who bids $\widehat{b}$ has updated beliefs given by*

$$\begin{aligned} F^C(x) &= \Pr\left(v_C \leq x \mid v_C \in [0, v^*) \cup (v^{**}, 1]\right) \\ &= \begin{cases} \frac{F(x)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in [0, v^*) \\ \frac{F(v^*)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in [v^*, v^{**}] \\ \frac{F(x)-F(v^{**})+F(v^*)}{F(v^*)+1-F(v^{**})} & \text{if} \quad x \in (v^{**}, 1], \end{cases} \end{aligned} \quad (2)$$

*where $b(v^*) = b(v^{**}) = \widehat{b}$ and $v^* < v^{**}$. If $b(v^*) = b(v^{**})$ does not hold for two different types, then the updated distribution is given by the formula above with $b(v^*) = \widehat{b}$.*

Furthermore, as $v^{med}$ is a natural candidate type for having the minimal willingness to pay we assume (and confirm later) that the equilibrium bids are symmetric around this type. In particular we assume that types $v$ and $s(v)$, $v < v^{med} < s(v)$, bid the same amount $b(v)$, where $s(v) := F^{-1}(1 - F(v))$ is the complementary quantile of $v$. An immediate consequence of this symmetry property is that in the pricing stage it is optimal for the proposer to set a price equal to her valuation.

**Lemma 2** *Assume partners bid according to a bidding function $b(v)$ in the negotiation stage that is strictly decreasing for $v < v^{med}$ and has the symmetry property $b(v) = b(s(v))$. Then a proposer with valuation $v_P$ optimally sets a price $p = v_P$ in the pricing stage and a chooser with valuation $v_C \neq v_P$* optimally *buys if and only if $v_C > v_P$.*

Note that what is needed for efficiency is the symmetry of the U-shaped bidding functions. This guarantees that the losing party assigns the same probability to her partner having a valuation larger than hers as she does to her partner having a valuation smaller than hers.

### 3.1.2 The Negotiation Stage

We now focus on the bidding functions that will be optimal for the partners. We first note that the overall utility of a partner in the dissolution game can be decomposed into her expected payoff in the pricing stage, plus the payments she expects to receive/pay from the negotiations. Equilibrium bidding strategies must reflect parties' willingness to pay to become chooser, which in our two-stage game is given by the expected payoff from the pricing stage and the expected transfers from the negotiation stage. As argued in the last subsection, a plausible conjecture is that bidding functions are U-shaped and symmetric.

To investigate whether symmetry is consistent with equilibrium behavior consider the effects of a small (marginal) change in the bids of types $v$ and $s(v)$. If this deviation had the same (marginal) effect on their expected payoff in the pricing stage, they would have the same (local) incentives to bid $b(v)$. To see that these incentives are indeed equal consider a type $v$ partner who marginally increases her bid to $b(v)$. We just have to look at the change in expected payoff that results from trades with the marginal types $v$ and $s(v)$. She now becomes chooser with respect to these types (her marginal gain), whereas before was proposing to those types (her marginal loss).

As chooser from a partner with valuation $v$ who proposes a price of $v$ she buys and hence gets $v - \frac{v}{2} = \frac{v}{2}$. As chooser from a partner with valuation $s(v)$ she sells and gets $\frac{s(v)}{2}$. Since both events happen with "probability" $f(v)$,[18] her marginal gain equals $\frac{v+s(v)}{2} f(v)$.

As proposer she would set a price of $v$. She would hence buy from another partner with valuation $v$ getting $\frac{v}{2}$ with probability $f(v)$. She would sell to $s(v)$, receiving $\frac{v}{2}$ with probability $f(v)$. Therefore, her marginal loss is $vf(v)$.

Subtracting $\frac{v+s(v)}{2} f(v)$ from $vf(v)$ results in an overall marginal change in expected payoff equal to $\frac{s(v)-v}{2} f(v)$.

Consider now a deviation by a type $s(v)$ who marginally increases her bid to $b(s(v-dv))$.[19] Her marginal gain from being chooser is $\left(\frac{3}{2}s(v) - \frac{v}{2}\right) f(v)$. Note that as chooser she buys from $v$ and pays $\frac{v}{2}$ thus getting $s(v) - \frac{v}{2}$, and she sells to $s(v)$ receiving $\frac{s(v)}{2}$. Her marginal loss from deviating is $s(v) f(v)$. The difference in expected payoff from being chooser instead of proposer (with respect to the marginal types $v$ and $s(v)$) is $\frac{s(v)-v}{2} f(v)$. Thus, the marginal change in expected payoff from the pricing stage are the same for $v$ and $s(v)$.

In equilibrium this change in expected payoff has to equal the change in transfers in the auction. The latter includes the direct effect of an increase in $b(v)$ on payments and on winning probabilities. By increasing the bid type $v$ is more likely to win, she will hence have to pay the bid of the marginal type $b(v)$ whereas before she was receiving this amount as payment. This happens with probability $2f(v)$ which gives a marginal loss of $-4b(v)f(v)$. But with probability $2F(v)$ she remains loser and now gets a larger payment. Her marginal gain is then $2F(v) \frac{d}{dv} b(v)$. The difference in expected payoff in the auction adds up to $-4b(v)f(v) - 2F(v) \frac{d}{dv} b(v)$. In equilibrium, the marginal loss and the marginal gain from deviating must cancel out. Therefore the following differential

[18] Note that a decrease in $v$ increases $s(v)$ so that the probability of meeting a type $s(v)$ partner is given by $-\frac{d}{dv} F(s(v)) = -\frac{d}{dv}(1 - F(v)) = f(v)$.

[19] Deviations by decreasing the bid are computationally more involved and require an Envelope-Theorem argument. See the proof of Theorem 1 (in the Appendix) for details.

equation has to be fulfilled:

$$2F(v)\frac{d}{dv}b(v) + 4b(v)f(v) = \frac{s(v) - v}{2}f(v).$$

The next theorem provides an equilibrium bidding function $b_N(v)$ that solves this differential equation and is indeed U-shaped.

**Theorem 1** *The following strategies constitute an equilibrium of the dissolution game:*

*- In the first stage, both partners bid according to the following bidding function*

$$b_N(v) = \begin{cases} \frac{\frac{1}{2}\int_0^v (s(t)-t)F(t)f(t)dt}{2F^2(v)} & if \quad v \leq v^{med} \\ \frac{\frac{1}{2}\int_v^1 (t-s(t))(1-F(t))f(t)dt}{2(1-F(v))^2} & if \quad v > v^{med}, \end{cases} \tag{3}$$

*where*

$$s(v) := F^{-1}(1 - F(v)).$$

*- In the pricing stage, the proposer sets a price equal to her valuation.*

The equilibrium bidding functions are strictly decreasing for valuations below the median $v^{med}$ and strictly increasing for $v > v^{med}$. In addition we have that $b_N(v) = b_N(s(v))$, i.e., for any $v$ we have that the mass of valuations that submit a higher bid is equally distributed on valuations smaller than $v$ and valuations that are larger than $v$. Note that the result does not require the cdf to be symmetric. The most important property of the equilibrium in Theorem 1 is that it renders an efficient dissolution of the partnership.

**Corollary 1** *The equilibrium is ex-post efficient.*

## 3.2 Waiting for the other to propose

If partners are ignorant about the possibility of negotiations, (e.g. because they are not explicitly stated in the partnership agreement), either party may end up waiting for the other to name a price, as neither wants to propose. The time it takes to reach an agreement might impose costs on the parties as it may prevent them from either getting involved in other ventures or from running the business. This costly and time-consuming route can be modelled as a war of attrition.

Let $b_W(v)$ denote the time at which a party with valuation $v$ quits this war of attrition (and makes a price offer), given that the other party has not quitted yet. Note that $b_W(v)$ reflects the cost incurred by both parties when the first to quit has valuation $v$. As partners with extreme valuations are more reluctant to name a price, they will wait longer. Relying on arguments similar to the ones given in Section 3.1 we show that $b_W(v)$ being a symmetric U-shaped function is consistent with equilibrium behavior.

**Theorem 2** *The following strategies constitute an equilibrium of the war of attrition game:*

*- In the war of attrition both partners quit according to the following function*

$$b_W(v) = \begin{cases} \int_v^{v^{med}} \frac{(s(t)-t)f(t)}{4F(t)} dt & if \quad v \leq v^{med} \\ \int_{v^{med}}^{v} \frac{(t-s(t))f(t)}{4(1-F(t))} dt & if \quad v > v^{med}, \end{cases}$$

*where*

$$s(v) := F^{-1}(1 - F(v)).$$

*- In the pricing stage, the proposer sets a price equal to her valuation.*

Note that in the war of attrition both parties incur the cost of waiting, whereas in the negotiations any payment is a transfer from one party to the other: the war of attrition achieves allocative efficiency but it generates inefficient costs of waiting. A consequence of these efficiency losses is that partners, independently of their valuation, prefer to negotiate immediately rather than play the war of attrition game described in this section.

**Corollary 2** *A partner's expected payoff in the dissolution game with immediate negotiations is always strictly higher than her payoff in the war of attrition.*

## 3.3 A unified framework

As argued in the introduction, both negotiations and waiting for the other to propose, are consistent with the rules of the buy-sell clause. Corollary 2 suggest that it is preferable that partners negotiate (with negotiations starting immediately when the deadlock occurs) rather than enter into a war of attrition. But as the buy-sell clause does not dictate this to happen, an important question is whether partners (who are aware of all possibilities) agree to negotiate immediately after the deadlock occurs (thus minimizing the cost of waiting). To analyze this question we extend our model and allow for the possibility of strategic delay of negotiations.

The *unified dissolution game* begins at $t = 0$ when the deadlock occurs. We assume that it is costly for agents to stay in a deadlocked partnership, and, as in Section 3.2 we normalize the marginal cost of staying in the partnership to one. At any point in time $t$, each partner can either propose a price, offer to negotiate or stay in the deadlocked partnership and wait for the other partner to propose a price.[20] Whenever one partner makes a price offer the other partner has to decide whether to buy or sell at that price and the game ends. If both offer a price at the same time, the proposer is determined randomly by the flip of a fair coin. All actions following the price offer take place instantaneously. Whenever a partner offers to negotiate (and the other does not make a price offer at the same time), the other partner can then decide to either accept or reject that offer. If it is accepted or if both offer negotiations at the same time, these commence. As before, negotiations are modelled as an ascending auction with a fast clock on bids where the winner pays the loser.[21] The loser proposes a price and the other partner decides on whether to buy or sell. If it is rejected, the game continues.

The following theorem shows that the unified dissolution game is efficient, i.e., it has an equilibrium where negotiations start without delay and the resulting allocation is ex-post efficient.

[20] It is worth noting that going to court is also a feasible action according to the rules of the buy-sell clause. We will not consider this possibility here as by proposing a price equal to her own valuation a partner can always guarantee herself a payoff of at least $v/2$. This is the payoff a partner can obtain by going to court, if one models the court as a fair but uninformed body that does not manage to elicit private information from partners. The court resolves the dispute by allocating the partnership to either partner with equal probability, setting a price that does only depend on common value components. Our main results do not depend on this implicit assumption and can also be derived for more "sophisticated" court decisions.

[21] It should be noted that we do not consider negotiating to be a time-consuming (and thus costly) activity. Whereas in the war of attrition costly delay is used to screen types, in negotiations payments to the other partner serve as the screening device. Therefore we assume that negotiations take place with a fast clock on bids.

**Theorem 3** *The unified dissolution game is efficient.*

The proof of Theorem 3 constructs an equilibrium at which partners negotiate and dissolve the partnership efficiently at $t = 0$. As at any time negotiations are always better than the war of attrition (starting at that time, see Corollary 2), in the constructed equilibrium any partner always offers to negotiate, always accepts negotiations and never proposes a price. The latter is optimal given that both partners believe that an offer to negotiate will come immediately after.

Lets us finally stress that other dissolution mechanisms, in particular auctions as defined in Cramton et al [1987], can be embedded in this framework as well. If a partnership agreement specifies that partners must dissolve by using an auction, then auctions will not give raise to delay either. Both partners will optimally start the auction as soon as the deadlock occurs as they can not gain by delaying it since no new information can be gained from waiting. Consequently, imposing a fast clock on bids, an auction will be efficient as well.

# 4 Interdependent values

If one (or both) partner's valuation for the partnership depends on private information held by the other partner, the analysis of the buy-sell clause becomes significantly more complicated. This is because a proposer might have private information that affects her partner's valuation and thus is cautious to reveal this information. Furthermore, adverse selection makes the dissolution process more complicated.[22]

In this section we consider an interdependent-values model with one-sided information adapted from Jehiel and Pauzner [2006] (JP for short). Two risk-neutral partners, labelled $I$ (for informed) and $U$ (for uninformed) jointly own a partnership. The informed partner's value is denoted by $v_I$ and the uninformed partner's value is given by $v_U(v_I)$, a strictly increasing function with $v_U(0) > 0$, $v_U(1) < 1$ that intersects with the 45 degree line exactly once at $v^{eff}$, i.e., $v_U(v_I) = v_I$ iff $v_I = v^{eff}$. JP show that in this model: 1)

[22]See Fieseler et al [2003] for a study on the negative effects of adverse selection for the efficiency of dissolution mechanisms.

the efficient dissolution can be achieved if the informed party can choose on whether to buy or sell her shares at a price of $p = v^{eff}$, and 2) the welfare maximising, individually rational mechanism (i.e., the second best mechanism) is to offer two different prices to the informed partner: one at which she can buy and one at which she can sell her shares.[23]

Though similar, the efficient dissolution procedure in 1), where price and identity of chooser are pre-determined, should not be confused with a buy-sell clause. The following lemma shows that if the uninformed were to name the buy-sell price (to the informed) this would in general differ from $p = v^{eff}$, i.e., this could result in an inefficient dissolution. If the proposer is the uninformed partner, he faces a two-sided winner's curse. If he ends up as the sole owner of the partnership this is because his valuation is (likely to be) low (otherwise the informed partner would have bought). Similarly, if he is selling, this is because valuations are high.

**Lemma 3** *Consider the pricing stage with interdependent valuations*
*If the uninformed partner proposes, he will set a price $p^*$ that satisfies*

$$2\left(v_U\left(p^*\right) - p^*\right) f(p^*) = 2F(p^*) - 1. \tag{4}$$

*The informed partner will buy if the price is below her valuation and sell otherwise. In addition, $p^* = v^{eff}$ if and only if $v^{med} = v^{eff}$.*

Lemma 3 shows that the efficient mechanism in JP requires the buy-sell price to be set by a third party, at least we cannot implement it by simply assigning the right to choose to the uninformed partner. In what follows we analyze how endogenous proposer selection (which is not considered in JP) affects the performance of the buy-sell clause in the model with interdependent values.

To simplify the analysis of the buy-sell clause and the comparison of results with the private values baseline model, we assume that both partners are equally likely to have the higher valuation, i.e., we assume that $v^{med} = v^{eff}$. Note that this is also fulfilled in the

[23] An efficient mechanism exists if agents can observe their payoffs before monetary transfers are made, see Mezzetti [2004], and the discussion in Jehiel and Pauzner [2006] on this assumption. In Mezzetti's two-stage mechanism, the partnership is allocated in the first stage, then in the second stage, the sole owner experiences and subsequently reports the value of the partnership. Final transfers are contingents on the reports in the two stages.

model with private values and guarantees that we are "sufficiently far" from a situation with complete information.[24] For tractability reasons we will also assume a linear model, i.e., the distribution of $v_I$ and $v_U(v_I)$ is uniform. We will comment on how our results depend on these assumptions. We first consider a situation where the identity of the proposer is given. We compare its outcome with the second-best mechanism (as derived in JP), argue that the latter can be achieved in the unified dissolution game and finally compare with the outcome of an auction.

Different to the private values model (in Sections 2 and 3) the outside option of going to court is important here. Adverse selection problems might make the uninformed partner worse off than after a court settlement. If the uninformed were pre-determined to propose, Lemma 3 shows that he would offer a price $p = p^{eff}$, we next show that his expected payoff will be below $\frac{E[v_U(v_I)]}{2}$, which is what he could get from a court settlement.[25] Thus the uninformed would never name a price but rather go to court if allowed to do so (in this sense proposing the price is not individually rational for the uninformed as his outside option would give him $\frac{E[v_U(v_I)]}{2}$).

**Lemma 4** *In the linear model with $v^{med} = v^{eff}$, if the uninformed proposes her expected payoff is smaller than $\frac{E[v_U(v_I)]}{2}$.*

In contrast if the informed were pre-selected as proposer, her dilemma would be how to conceal information. When proposing, she might want to adhere to a strategy that does not fully reveal her valuation. The next lemma shows how she accounts for this.

**Lemma 5** *If the informed partner proposes, she will follow a pricing strategy $p(v_I)$ such that*[26]

$$p(v_I) = \begin{cases} v^* & 0 \leq v_I \leq v^* \\ v_U(v_I) & v^* \leq v_I \leq v^{**} \\ v^{**} & v^{**} \leq v_I \leq 1 \end{cases} \tag{5}$$

[24] If e.g. $v^{eff} = 0.1$ and the distribution of $v_I$ is concentrated around 0.5, then both partners know that the informed is much more likely to have a higher valuation. In such situations partners might prefer to propose as the model is "close enough" to a complete information framework (see Crawford [1977]).

[25] See also Footnote 14 on our modelling of Court.

[26] Note that the strategies described here are also equilibrium strategies for non-uniform distributions and/or non-linear valuation functions.

*where* $v^* = E[v_U(v_I)/p(v_I) = v^*]$ *and* $v^{**} = E[v_U(v_I)/p(v_I) = v^{**}]$, *with* $v^* < v^{eff} < v^{**}$.[27] *The uninformed will buy if offered* $v^*$, *will sell if offered* $v^{**}$, *and will buy and sell with equal probability if offered any other price.*

When the informed partner proposes individual rationality holds for both players but the partnership is dissolved efficiently only if types are "extreme", i.e., in $[0, v^*] \cup [v^{**}, 1]$. Thus in this case a comparison of the equilibrium allocation with that of the second-best mechanism is interesting. JP show that the second-best mechanism gives the informed a menu of three options: sell shares at a price $p^{\text{sell}}$, buy shares at a price $p^{\text{buy}}$, or not trade. Thus in both mechanisms, the partnership is allocated efficiently whenever the informed (and thus also the uninformed) has an extreme valuation and it is allocated randomly otherwise.

**Corollary 3** *If the informed partner proposes, the buy-sell clause implements the second-best allocation.*

Note that the buy-sell clause is not the decentralized second-best procedure given by JP. That requires two different prices $p^{\text{sell}}$ and $p^{\text{buy}}$ to be offered to the informed which have to be determined by either a third party or by the partners before any information is learned. In contrast the game induced by the buy-sell clause achieves the second best outcome because of the way the informed proposes.

As in the private values environment, it can be shown that both partners prefer the other to propose: the uninformed to avoid a winner's curse and the informed to hide her information and get informational rents, so that there is a conflict on who should propose. But contrary to that environment, now the uninformed partner prefers to settle in court to ensure his status quo rather than to propose a price. For the informed party this option is not attractive as he is better off proposing. Under the (credible) threat of ending up in court, the informed party will name the price immediately. Thus with endogenous proposer selection the buy-sell clause results in the second best outcome.[28]

[27] Existence of $v^*$ follows from the intermediate value theorem as $E[v_U(v_I)/p(v_I) = 0] > 0$ and $E\left[v_U(v_I)/p(v_I) = v^{eff}\right] < v^{eff}$ (and similarly for $v^{**}$).

[28] Negotiations will now be rejected by the uninformed as they are weakly dominated by going to court. Note that in any equilibrium of the buy-sell clause with negotiations in which the uninformed becomes

As already mentioned in Section 2, another interesting dissolution mechanism is an auction mechanism which works as follows: both partners submit a sealed bid (at $t = 0$). The partner with the higher bid receives all shares and pays (to her partner) an amount that equals half of her bid (Winner's Bid Auction).[29] Different to the private values model in Sections 2 and 3, where the auction can be shown to be efficient (and thus performing as well as the buy-sell clause) the following example shows that the auction might not only generate less total welfare but might also violate the individual rationality constraint of the uninformed partner.

**Example 1** *Let $v_U(v_I) = \frac{1}{3} + \frac{1}{3}v_I$. Total welfare in the first best is given by 0.583 and a dissolution by flipping a coin gives 0.5.*

*i). In the dissolution game, the following constitutes the equilibrium pricing strategy of the informed*

$$p(v_I) = \begin{cases} \frac{1}{3} + \frac{1}{3}\left(\frac{1}{5}\right) & 0 \leq v_I \leq 2/5 \\ \frac{1}{3} + \frac{1}{3}v_I & 2/5 \leq v_I \leq 3/5 \\ \frac{1}{3} + \frac{1}{3}\left(\frac{4}{5}\right) & 3/5 \leq v_I \leq 1. \end{cases}$$

*The allocation induced by this equilibrium coincides with the second-best allocation. It gives a total welfare of 0.580.*

*ii). In the Winner's Bid Auction, the informed bidder bids according to*

$$\beta_I(v_I) = \begin{cases} \frac{2}{9}v_I + \frac{1}{3} & v_I \in [3/7, 1] \\ 3/7 & v_I \in [0, 3/7]. \end{cases}$$

*The uninformed randomizes on $[3/7, 5/9]$ according to the cdf $J_U(b) = \sqrt[7]{\frac{63}{8}\left(b - \frac{3}{7}\right)}$. He makes profits equal to $\frac{7}{36}$ smaller that his status-quo payoff, $\frac{1}{4}$. Total welfare equals 0.526.*[30]

We have shown that in linear-uniform environments the dissolution game induced by buy-sell clauses can achieve the second-best outcome whereas an auction-like mechanism cannot. In other environments, the equilibrium allocation of the buy-sell clause resembles

chooser he receives $\frac{E[v_U(v_I)]}{2}$ but has to pay a positive amount. A rigorous proof of this statement has been omitted to shorten the paper but can be obtained from the authors.

[29] The Winner's Bid Auction belongs to the class of $k+1-$auctions described in CGK [1987]. Properties of other $k+1-$auctions are similar.

[30] The calculations to derive these results are available from the authors upon request.

the second-best allocation but it may not always coincide with it. It is not surprising that the optimality result for the buy-sell clause does not extend to more general functions, as the optimal dissolution mechanism is not detail-free, i.e., it depends on prior beliefs (see Proposition 5 in JP). This is not different to the bilateral trade model with private values, for which Leininger et al. [1989] and Satterthwaite and Williams [1989] show that a double auction (e.g. a detail-free mechanism) only achieves the second-best if valuations are distributed uniformly. Our results here are within the same spirit. On the positive side, buy-sell clauses are simple mechanisms whose rules that do not depend on particularities of the underlying model.[31]

# 5 Conclusion

In this paper we have shown that widely used buy-sell clauses can dissolve partnerships efficiently. We have argued that the possibility of an efficient dissolution is related to how it is decided on which partner proposes, a relationship that has not attracted sufficient attention, neither in the economics nor in the legal literature. Economic models so far have assumed the identity of the proposer as given whereas the legal literature is ambiguous and imprecise with respect to this point. Our analysis suggests that lawyers, who advise partners about the practical use of the clause, should be aware of the broad 'road map' that partners may follow when they must abide by the provisions laid down in the clause and of their consequences. In particular, if partners' valuations are private, negotiations about the right to choose can render efficient dissolutions whereas partners who simply wait for the other partner to name the price will incur in costly delay.

Our main model considers two-parent partnerships with 50-50 ownership structure and independent private values. The first two restrictions do not limit the applicability of our findings much. On one hand, many partnerships do indeed have this 50-50 structure. On the other hand, buy-sell clauses can only be found in two-partner partnership agreements for which their underlying triggering event, a management deadlock, is most likely to occur

[31] *The Wilson doctrine* recognizes the advantages of detail-free mechanisms for practical implementation and advocates for its use (Wilson [1987]).

if neither of the partners has exclusive control rights. Furthermore, lawyers recommend them for (roughly) equal shareholdings.[32] The assumption on values being private may seem inappropriate for some partnerships as those between an inventor and a venture-capital fund. Because of this, we have studied the robustness of our main results to modelling values as interdependent. We again find that the buy-sell clause performs well and in addition show that it can strictly outperform auctions. Whereas we believe that our results provide strong arguments in favour of using a buy-sell clause for deadlock resolution, other contractual arrengements may perform better in other setups.[33]

Partnership agreements usually contain several termination provisions and each of them applies to a specific, pre-defined triggering event (like e.g. deadlock, end of purpose, a partner wishes to exit the venture, etc). Put- or call options, which allow one party to either sell or buy the other partner's shares at a pre-specified price, are also frequently discussed as a way to dissolve partnerships, though they are rarely used in case of a deadlock. Options perform badly in allocating ownership rights efficiently, but are useful to mitigate the double-moral hazard problem in partners' investment decisions.[34]

# 6 Appendix

**Proof of Lemma 1:**

It is straightforward and it is hence omitted. ■

**Proof of Lemma 2:**

If the bidding strategies in the negotiation stage are U-shaped, a proposer who bid $\widehat{b}$ has updated beliefs given by (2). It is then easy to verify that $F^C(x)$ satisfies the standard

[32] As our model is continuous in shares we conjecture that our results will not change much if we slightly depart from the 50-50 assumption.

[33] Arguments outside our formal model might be in favour or against the buy-sell clause. For example, extensive case law might lead partners to prefer the buy-sell clause to an auction. If valuations are asymmetrically distributed, efficiency will not be guaranteed either with a buy-sell clause or with an auction. It is not clear to us which mechanism will perform better in this environment. Any comparison must rely on particular distributional specifications, see de Frutos [2000].

[34] Nöldeke and Schmidt [1998] show that an option contract might lead to efficient investment prior to the dissolution.

hazard rate conditions (1) for any $x \in [0, v^*] \cup [v^{**}, 1]$.

Since for any price $p$ the chooser optimally buys if and only if $v_C > p$, the proposer's utility when setting a price of $p$ is given by

$$U^P(v_P, p) = \left(v_P - \frac{p}{2}\right) F^C(p) + \frac{p}{2}\left(1 - F^C(p)\right).$$

For any price $p \in [v^*, v^{**}]$, the proposer is equally likely buyer and seller as

$$F^C(p) = \frac{F(v^*)}{F(v^*) + 1 - F(v^{**})} = \left(\frac{1 - F(v^{**})}{F(v^*) + 1 - F(v^{**})}\right) = \left(1 - F^C(p)\right).$$

Consequently, setting $p = v_P$ in the pricing stage is optimal. ■

**Proof of Theorem 1:**

Consider a bidder with valuation $v$ smaller than $v^{med}$ who bids $b(\widehat{v})$ when the other bidder bids according to (3). Let $U(v, \widehat{v})$ denote the interim utility (in the dissolution game) of this bidder. By imitating a bidder of type $\widehat{v}$ she will be a chooser in the pricing stage if the other agent's valuation is within the interval $[\widehat{v}, s(\widehat{v})]$ and a proposer otherwise. $U(v, \widehat{v})$ can be decomposed in the expected payoff from being chooser (denoted by $U^C(v, \widehat{v})$) and proposer (denoted by $U^P(v, \widehat{v})$) in the pricing stage, and the payments she expects to receive/ pay in the auction. Her expected utility is then

$$U(v, \widehat{v}) = U^P(v, \widehat{v}) + U^C(v, \widehat{v}) - \int_{\widehat{v}}^{s(\widehat{v})} b(x) f(x)\, dx + 2F(\widehat{v})\, b(\widehat{v}). \tag{6}$$

Differentiating this overall expected utility with respect to its second argument we have

$$\begin{aligned} U_2(v, \widehat{v}) &= U_2^P(v, \widehat{v}) + U_2^C(v, \widehat{v}) - b(s(\widehat{v})) f(s(\widehat{v})) \frac{ds(\widehat{v})}{d\hat{v}} \\ &\quad + 3b(\widehat{v}) f(\widehat{v}) + 2F(\widehat{v}) \frac{db(\widehat{v})}{d\hat{v}} \\ &= U_2^P(v, \widehat{v}) + U_2^C(v, \widehat{v}) + 4b(\widehat{v}) f(\widehat{v}) + 2F(\widehat{v}) \frac{db(\widehat{v})}{d\hat{v}}, \end{aligned}$$

where the second equality follows from the symmetry of the bidding function with $b(\hat{v}) = b(s(\hat{v}))$ and from the definition of $s(\cdot)$ which gives $f(s(\widehat{v}))\frac{ds(\widehat{v})}{d\hat{v}} = -f(\widehat{v})$.

For $b(\cdot)$ to be an equilibrium strategy, it must be optimal for type $v$ to bid $b(v)$, which provides the following necessary condition

$$U_2(v, \hat{v}) \mid_{\hat{v}=v} = 0 \text{ for all } v \in [0, v^{med}]. \tag{7}$$

We must show that $\max_{\widehat{v}} U(v, \widehat{v}) = U(v, v)$. Note that we only need to show this for $\widehat{v} \leq v^{med}$ by the symmetry of $b(\cdot)$. Since the probability of winning, the payments and the information revealed is exactly the same when bidding $b(\hat{v})$ and $b(s(\hat{v}))$, a deviation to a bid $b(\widehat{v})$ with $\widehat{v} > v^{med}$ is equivalent to deviate to a bid $b(s(\tilde{v}))$ for some $\tilde{v} \leq v^{med}$.

In order to derive $U_2(v, \hat{v})$ we first compute $U_2^P(v, \hat{v})$. A losing bidder who bid $b(\widehat{v}) \in [b(0), b(1)]$ correctly infers that the other partner's valuation is either smaller than $\hat{v}$ or larger than $s(\hat{v})$. She uses this information to update her beliefs. Consequently, she proposes a price $p$ which maximizes

$$U_{\widehat{v}}(v,p) = \left(v - \frac{p}{2}\right) F_{\widehat{v}}^C(p) + \frac{p}{2}\left(1 - F_{\widehat{v}}^C(p)\right),$$

where

$$F_{\widehat{v}}^C(x) = \begin{cases} \frac{F(x)}{2F(\widehat{v})} & \text{if} \quad x \in [0, \widehat{v}] \\ \frac{F(\widehat{v})}{2F(\widehat{v})} & \text{if} \quad x \in [\widehat{v}, s(\widehat{v})] \\ \frac{F(x) - F(s(\widehat{v})) + F(\widehat{v})}{2F(\widehat{v})} & \text{if} \quad x \in [s(\widehat{v}), 1]. \end{cases}$$

Differentiating proposer's utility gives

$$\frac{d}{dp} U_{\widehat{v}}(v,p) = \begin{cases} (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)}{2F(\widehat{v})} + \frac{1}{2} & \text{if} \quad p \leq \widehat{v}, \\ 0 & \text{if} \quad p \in (\widehat{v}, s(\widehat{v})) \\ (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p) - 1 + 2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} & \text{if} \quad p \geq s(\widehat{v}). \end{cases}$$

It is easy to see from this expression that the optimal price depends on $\hat{v}$.

Two cases have to be distinguished. If $\widehat{v} \leq v$ then the following two inequalities hold:

$$\begin{array}{rl} (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)}{2F(\widehat{v})} + \frac{1}{2} > 0 & \text{for} \quad p \leq \widehat{v}, \text{ and} \\ (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)-1+2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} < 0 & \text{for} \quad p \geq s(\widehat{v}). \end{array}$$

Hence, setting a price in the interval $[\widehat{v}, s(\widehat{v})]$ is optimal, resulting in a utility as proposer equal to

$$U^P(v, \widehat{v}) = (v - \frac{1}{2}p^{opt})F(\widehat{v}) + \frac{1}{2}p^{opt}(1 - F(s(\widehat{v}))) = vF(\widehat{v}).$$

Since $\widehat{v} \leq v$, we further obtain $\lim_{\widehat{v} \nearrow v} U_2^P(v, \widehat{v}) = vf(v)$.

Consider now that $v^{med} \geq \widehat{v} \geq v$. In this case we have

$$\begin{array}{c} (v-\widehat{v})\frac{f(\widehat{v})}{2F(\widehat{v})} - \frac{F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} \leq 0 \text{ and} \\ (v-p)\frac{f(p)}{2F(\widehat{v})} - \frac{F(p)-1+2F(\widehat{v})}{2F(\widehat{v})} + \frac{1}{2} < 0 \text{ for all } \ p \geq s(\widehat{v}). \end{array}$$

The optimal price must hence satisfy $p^{opt} \leq \widehat{v}$. Consequently, the utility as proposer is now equal to

$$\begin{aligned} U^P(v,\widehat{v}) &= (v-\frac{1}{2}p^{opt})F\left(p^{opt}\right)+\frac{1}{2}p^{opt}\left[F(\widehat{v})-F\left(p^{opt}\right)+1-F(s(\widetilde{v}))\right] \\ &= p^{opt}\left(F(\widehat{v})-F\left(p^{opt}\right)\right)+vF\left(p^{opt}\right), \end{aligned}$$

where $p^{opt}$ satisfies the following FOC

$$\left(v-p^{opt}\right)f\left(p^{opt}\right)-F\left(p^{opt}\right)+F(\widehat{v})=0. \tag{8}$$

Since $\lim_{\widehat{v}\searrow v} p^{opt}=v$, we obtain[35]

$$\begin{aligned} \lim_{\widehat{v}\searrow v} U_2^P(v,\widehat{v}) &= \lim_{\widehat{v}\searrow v}\left[F(\widehat{v})-F\left(p^{opt}\right)-\left(p^{opt}-v\right)f\left(p^{opt}\right)\right]\frac{d}{d\widehat{v}}p^{opt}+p^{opt}f(\widehat{v}) \\ &= \lim_{\widehat{v}\searrow v} p^{opt}f(\widehat{v})=vf(v). \end{aligned}$$

Analysis above ensures that $U_2^P(v,\hat{v})\mid_{\hat{v}=v}=vf(v)$ for all $\hat{v}\leq v^{med}$.

We now compute $U_2^C(v,\hat{v})$. As the proposer always sets a price equal to her valuation, the expected utility as chooser will be

$$U^C(v,\widehat{v})=\begin{cases} \int_{\widehat{v}}^{v}\left(v-\frac{x}{2}\right)f(x)\,dx+\int_{v}^{s(\widehat{v})}\frac{x}{2}f(x)\,dx & \text{if } \widehat{v}\leq v \\ \int_{\widehat{v}}^{s(\widehat{v})}\frac{x}{2}f(x)\,dx & \text{if } \widehat{v}\geq v. \end{cases}$$

Differentiating the chooser's utility with respect to $\hat{v}$ yields

$$U_2^C(v,\widehat{v})=\begin{cases} \left(\frac{\widehat{v}-s(\widehat{v})-2v}{2}\right)f(\widehat{v}) & \text{if } \widehat{v}\leq v \\ -\left(\frac{\widehat{v}+s(\widehat{v})}{2}\right)f(\widehat{v}) & \text{if } \widehat{v}\geq v, \end{cases}$$

Evaluating $U_2^C(v,\widehat{v})$ at $v=\widehat{v}$ gives $U_2^C(v,v)=-\left(\frac{s(v)+v}{2}\right)f(v)$.

Putting these results together the first order condition (7) becomes

$$-\left(\frac{s(v)-v}{2}\right)f(v)+2F(v)\frac{d}{dv}b(v)+4b(v)f(v)=0. \tag{9}$$

For (7) to hold at $v=0$ we need that $b(0)=\frac{1}{8}$. Thus a differentiable equilibrium has to be a solution of the boundary value problem determined by (9) and the terminal condition $b(0)=\frac{1}{8}$. Note that the differential equation (9) can be written as

$$\frac{s(v)-v}{2}F(v)f(v)=\frac{d}{dv}\left(2b(v)F^2(v)\right),$$

[35] To make sure that $p^{opt}$ is uniquely defined (given $v$ and $\widehat{v}$) we need the hazard rate condition to hold. For the complete argument see McAfee [1992].

and then integrated to obtain (3).

We next show that bidding $b(v)$ indeed does not result in a lower payoff (for a bidder with valuation $v$) than bidding $b(\widehat{v})$ with $\widehat{v} \leq v^{med}$. Observe first that for $\widehat{v} \leq v$ we have that

$$U_2(v, \widehat{v}) = -\frac{s(\widehat{v}) - \widehat{v}}{2} f(\widehat{v}) + 2F(\widehat{v}) \frac{d}{dv} b(\widehat{v}) + 4f(\widehat{v}) b(\widehat{v})$$

with $U_{2,1}(v, \widehat{v}) = 0$, and therefore a bid of $b(\widehat{v})$ does not give a larger payoffs than a bid of $b(v)$. Assume next that $\widehat{v} \geq v$. To show the optimality of $b(v)$ in this case, it is sufficient to show that $U_{2,1}(v, \widehat{v}) \geq 0$ for all $v \leq \widehat{v} \leq v^{med}$ (see McAfee [1992]). Using the abbreviation $p_v := \frac{\partial}{\partial v} p^{opt}$ we have that $U_{2,1}(v, \widehat{v}) = U^P_{2,1}(v, \widehat{v}) = p_v f(\widehat{v})$.

Using (8) we obtain that

$$p_v = -\frac{f(p^{opt})}{(v - p^{opt}) f'(p^{opt}) - 2f(p^{opt})},$$

which shows that

$$\begin{aligned} U_{2,1}(v, \widehat{v}) &= \frac{f(p^{opt}) f(\widehat{v})}{2f(p^{opt}) - (v - p^{opt}) f'(p^{opt})} = \frac{f(p^{opt}) f(\widehat{v})}{2f(p^{opt}) - \frac{F(p^{opt}) - F(\widehat{v})}{f(p^{opt})} f'(p^{opt})} \\ &= \frac{f(\widehat{v})}{\left.\frac{d}{dp}\right|_{p=p^{opt}} \left(p + \frac{F(p) - F(\widehat{v})}{f(p)}\right)} \geq 0, \end{aligned}$$

where the second equality follows from the optimality of $p^{opt}$ (recall (8)) and the last inequality from the hazard rate conditions (1).

We can hence conclude that the candidate equilibrium bid maximizes the expected utility in (6). The case $v > v^{med}$ can be shown similarly and it is hence omitted. Finally, partners' updated beliefs following a bid below $b_N(0)$ (above $b_N(1)$) coincide with those when the bid was $b_N(0)$ ($b_N(1)$). Bids outside the set of equilibrium bids are always dominated by either the lowest or highest bid in the range of equilibrium bids. ■

**Proof of Theorem 2:**

Arguments are similar to those in the proof of Theorem 1 and it is hence omitted. ■

**Proof of Corollary 2:**

Since in both dissolution games the final allocation is the same, they only differ in the payments that partners make/receive, which in the war of attrition are given by the waiting costs. As partners' valuations are iid in any ex-post efficient equilibrium the difference in utilities between the two games is given by the difference in utilities for the "worst-off" types (e.g. $v^{med}$), as implied by the Revenue Equivalence Theorem. The statement then follows from the fact that the utility of a partner with valuation $v^{med}$ is higher in the dissolution game with negotiations than in the war of attrition as

$$U_N(v^{med}) = \frac{v^{med}}{2} + \int_0^{v^{med}} (s(z) - z)\, F(z) f(z) dz > \frac{v^{med}}{2} = U_W(v^{med}). \blacksquare$$

**Proof of Theorem 3:**

Consider the following strategy: Any type of any partner will always offer negotiations, always accept negotiations and never name a price directly. After negotiations have been accepted, partners play according to the strategies described in Theorem 1. If a partner names a price the other has a dominant strategy (to buy iff her valuation is above this price). If negotiations are accepted or rejected at $t$ partners do not update beliefs (this is because even if the worst-off partner expects to receive a negative utility if he names a price and/or offers negotiations, he prefers to offer negotiations as he expects these to be accepted). Clearly it is neither profitable to reject negotiations nor just wait as this just increases the time costs. Furthermore, to name a price is always worse for any type of partner than to offer negotiations. This immediately follows from Corollary 2: as naming a price straight away is (weakly) worse than the war-of-attrition (otherwise any partner would name a price immediately in the war of attrition) and (according to Corollary 2) this is (strictly) worse than negotiations. $\blacksquare$

**Proof of Lemma 3:**

If the Uninformed proposes she will set a price $p^*$ to maximize:

$$U_U^P(p) = \int_0^p \left( v_U(v) - \frac{p}{2} \right) f(v)\, dv + \int_p^1 \frac{p}{2} f(v)\, dv$$

The optimal price will hence satisfy the foc

$$2\left(v_U(p^*) - p^*\right) f(p^*) = 2F(p^*) - 1,$$

which provides (4). From the equation is trivial to see that $p^* = v^{eff}$ if and only if $v^{eff} = v^{med}$. $\blacksquare$

**Proof of Lemma 4:**

Uniformity of valuations requires $f(v_I) = 1$ (on $[0,1]$) and $v_U(v_I) = av_I + b$ with $0 < a < 1$ and $b = \frac{1}{2}(1-a)$ so that $v^{eff} = v^{med}$. We first note that $U_U^P(p^*) < \frac{E[v_U(v)]}{2}$ holds iff $G(p) = \int_0^p (v_U(v) - p) f(v)\, dv < H(p) = \int_p^1 (v_U(v) - p) f(v)\, dv$ holds at $p = p^*$. Since $p^* = v^{eff} = 1/2$, straigthforward computations show that $G(1/2) = -\frac{1}{8}a < H(1/2) = \frac{1}{8}a$, so that the uninformed gets less than $\frac{E[v_U(v)]}{2}$. ■

**Proof of Lemma 5:**

The Uninformed's strategy is optimal for any beliefs system consistent with equilibrium play. Note that when he faces $p = v^*$, since $v^* = E[v_U(v_I)/p(v_I) = v^*]$ he should buy if he believes that this price offer comes from types of the Informed in $[0, v^*]$, which must be the case on the equilibrium path. The same reasoning applies to any other equilibrium price. Consider now the Informed problem. At the purported equilibrium strategies his expected utility is given by

$$U_p^I(v_I, p(v_I)) = \begin{cases} \frac{v^*}{2} & 0 \leq v_I \leq v^* \\ \frac{v_I}{2} & v^* \leq v_I \leq v^{**} \\ v_I - \frac{v^{**}}{2} & v^{**} \leq v_I \leq 1 \end{cases}$$

To show that $p(v_I)$ constitute an equilibrium we need to define out-off-equilibrium beliefs for the Uninformed partner that support this pricing strategy. Consider first prices below $v^*$.To set a lower price is strictly dominated for types in $[0, v^*]$, so the uninformed assigns these deviations to types in $[v^*, 1]$ and he optimally buys. Thus a deviation to lower prices is not profitable. Types below $v^*$ will be selling but for a lower price, for types in $[v^*, v^{**}]$ we have $\frac{v_I}{2} \geq \frac{v^*}{2}$ so that they will also end-up worse-off, and finally types above $v^{**}$ will sell when they prefer to buy. Consider next prices above $v^{**}$. The uninformed will believe they come from types in $[0, v^{**}]$ and will sell. Deviation to these prices are not profitable either. Consider finally prices in the gaps, i.e., in $[v^*, v_U(v^*)] \cup [v_U(v^{**}), v^{**}]$. These deviations will to be profitable as the Uninformed will buy and sell with the same probability when facing any such price. This is the case as the uninformed will believe that any such price comes from $v_U^{-1}(p)$ and he will buy and sell with equal probability. Deviations to these prices are not profitable as they yield the status-quo utility. Since no deviation is profitable the purported equilibrium prices constitute an equilibrium. ■

**Proof of Corollary 3:**

Uniformity of valuations requires $f(v_I) = 1$ (on $[0,1]$) and $v_U(v_I) = av_I + b$ with $0 < a < 1$, $0 < b < 1$. In the second best assignment function we have that (see JP, Proposition 5) the Uninformed receives all shares if $v_I \leq v_{SB}^*$, the Informed receives all shares if $v_I \geq v_{SB}^{**}$, and the partnership is given to either party with equal probability if $v_I \in [v_{SB}^*, v_{SB}^{**}]$ where $(v_{SB}^*, v_{SB}^{**})$ is the smallest interval such that $0 < v_{SB}^* < \frac{b}{1-a}$ and $\frac{b}{1-a} < v_{SB}^{**} < 1$ satisfy

$$\begin{aligned}
\int_0^{v_{SB}^*} (av_I + b - v_{SB}^*)\, dv_I &= \int_{v_{SB}^{**}}^1 (av_I + b - v_{SB}^{**})\, dv_I, \\
(av_{SB}^* + b - v_{SB}^*) \frac{1}{v_{SB}^*} &= (v_{SB}^{**} - av_{SB}^{**} - b) \frac{1}{1 - v_{SB}^{**}}.
\end{aligned}$$

Solving this system gives as a unique solution: $v_{SB}^* = \frac{2b}{2-a}$ and $v_{SB}^{**} = \frac{2b+a}{2-a}$.

Note that this allocate is identical to that of the buy-sell clause described in Lemma 5, as

$$v^* = E[av_I + b \; / p(v_I) = v^*] \Leftrightarrow \frac{1}{F(v^*)} \int_0^{v^*} (av_I + b - v^*) f(v_I)\, dv_I = 0$$

This is solved by $v^* = \frac{2b}{2-a}$. Similarly one can show that $v^{**} = \frac{2b+a}{2-a}$ which shows that $v^* = v_{SB}^*$ and $v^{**} = v_{SB}^{**}$. ■

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