UNIVERSIDAD CARLOS III DE MADRID

working papers

PRICING STRATEGIES IN SOFTWARE PLATFORMS: VIDEO CONSOLES VS. OPERATING SYSTEMS

María Fernanda Viecens

Universidad Carlos III de Madrid[*]

Abstract

We study software platforms for which the total amount that users spend depends on the two-sided pricing strategy of the platform firm, and on the pricing strategy of application developers. When setting prices, developers may be constrained by one of two margins: the demand margin and the competition margin. By analyzing how these margins affect pricing strategies we find some conditions which explain features of the market of operating systems and its differences with the one corresponding to the video consoles. The problem that arises when the platform does not set prices (as an open platform) is considered. We show that policy makers should promote open source in operating systems platforms but not necessarily in video consoles. We also analyze the incentives for a platform to integrate with applications as a function of the extent of substitutability among them and provide a possible explanation for the observed fact of vertical disintegration in these industries.

Keywords: two-sided markets, technology platforms, complements, vertical disintegration, competition policy.

JEL Classification: L10, L12, D40

1 Introduction

Many modern industries work around software platforms. Typical examples are operating systems for computers, personal digital assistants, smart mobile phones or videogame consoles. The usual feature is that they connect or attend different types of customers that benefit from the interaction among them, characterizing what is known in the literature as two (multi)-sided platforms. On the one side, developers write the applications or software that improve the value of the platform for the users. On the other side, users derive utility from consuming the system (the platform and the applications). Because of this, users are concerned about the system price, i.e., the total amount spent in the platform and the software. The system price will hence depend on the two-sided pricing strategy of the platform firm which in turn affects the market of complementary applications, and on the pricing strategy in the developers' market. This paper offers a model of a monopolist two-sided platform that allows us to analyze the pricing strategies it will adopt, the level of entry it will induce in the applications' market and the welfare it will generate. Furthermore, by considering that it can become either an open platform or a proprietary one, we will study the implications of having one or the other. Finally, issues related to the vertical structure of the platform and to the role of outside options will also be analyzed.

Two well known and widely used sofware platforms are video consoles and computer operating systems. In both, users care for the total charge of the system (platform and applications). Nevertheless they have followed quite different pricing strategies. Operating system platforms charge high prices to the users and subsidize developers. However, video console firms charge low prices to users and make profits on the developers' side.[1] We provide here a possible explanation for the difference based on the margin at which developers compete. When setting prices, developers may be constrained by one of two margins, the demand margin and the competition margin. As long as the demand margin binds, prices of developers affect the overall demand of the system and they set the price that maximizes their profits, a price that is lower than their marginal contribution to the users utility. In contrast, if competition margin binds, developers can not affect overall demand of the system and they are forced to set a price equal to their contribution to the users surplus.[2] What margin is binding depends on the number of applications in the market and on the level of substitutability among them. In particular, the competition margin is more likely to bind as long as users prefer a system with many applications and these are near substitutes. In the market of video console gamers state that price is very important in deciding what game to buy. Some of them report having a huge number of games and, for instance, among the ten top rated PlayStation 2 games, 3 of them belong to the adventure genre and 3 to the role-playing

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[1] This issue is largely analyzed by Hagiu (2005).

[2] Lerner and Tirole (2004) introduce the two margins to analyze pricing startegies in patent pools.

genre.[3] These facts allow us to presume that developers writing for the video console are constrained by the competition margin. However, users of operating systems need a lower number of applications that indeed are far substitutes, like a text processor, a spreadsheet or a browser, so that we suspect the developers in this market are constrained by the demand margin. By analizing how these margins affect the pricing strategies and the profits of the platform, we find some conditions that may help to explain features of the market of operating systems and its differences with that corresponding to the video consoles, and shed some light on the different pricing routes they have followed. We observe that the platform price for users is higher when demand margin binds than when competition margin binds, and this is consistent with the observed fact that operating systems charge high prices to users, whereas video console firms charge low prices to them.

When considering the problem that arises if the platform does not set prices (as an open platform), our model allows us to contribute to the current enthusiastic discussion on whether governments should promote (as some of them do) open source platforms. Nowadays, 50% of European public administrations declare that they use some open source software and the figure is 35% for the USA. In addition, some large companies are also using open source programs.[4] The literature is not conclusive about recommendations. Hagiu (2005) shows that there is a tradeoff between the extent to which proprietary platforms internalize indirect network effects through profit-maximizing pricing and the two-sided deadweight loss they create. He shows that a proprietary platform may generate a higher level of product variety and welfare than an open platform. In contrast, Economides and Katsamakas (2006a) find that the variety of applications and social welfare is always larger when the platform is open source. We here show that outcomes may depend on the margin that binds. We find some results that suggest that policy makers should promote open source platforms where demand margin binds (as operating systems) but not necessarily in platforms where competition margin binds (as video consoles). In particular, we prove that if demand margin binds, a proprietary platform and an open platform will provide the same level of applications, so that the latter will generate more welfare for users. However, if competition margin binds a proprietary platform may generate a larger number of applications and higher welfare to users than an open platform.

In a book about empirical business and economics aspects of software based platforms, Evans, et. al. (2006) document that almost all the successful firms in these industries started being one-sided, producing applications at home, and later they disintegrated becoming in firms producing only the platform

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[3]See *Game Daily: June 22, 2004* at http://www.gamedaily.com, *Video Game Culture: Leisure and Play Preferences of B.C. Teens - Summary of Findings* at http://www.media-awareness.ca, and www.gamespot.com, September 2006.

[4]In a sample of 600 large companies in USA, 35% use one or more "free" softwares and 39% of 300 European large firms do so. Forrester Consulting, in El Mundo Digital 22/11/2006. In Spain, for instance, some "Comunidades Autónomas" are supporting open source. In 2007, the public administration of Extremadura will start to work with Linux. Andalucía and the Basque Country are also heading in the same direction (El País Digital, 16/11/2006).

and supported by independent developers.[5] We here try to provide a possible explanation for this observed fact based again on the margin that binds for developers. We analyze the incentives of a platform to integrate with applications (becoming one-sided) as a function of the extent of substitutability among them. We derive some conditions about the relationship between the welfare effects of a merger and the degree of substitution of the applications. We also offer an explanation for partial integration and we show that in the long run the platform will be partially integrated with the killer applications for which demand margin will bind and will allow free entry for developers of other applications.[6]

Finally, we study the effects on incumbent platform strategies for facing the threat of an outside option that offers a surplus for developers or users. Examples of outside options for users of the video game consoles are those games that can be played in the computer or online in the internet.[7] Writing these games is the outside option that developers have to the video console. Outside options for a proprietary operating system are the open platforms such as Linux. It is developing quickly in terms of number, variety and quality of applications and availability of support and other complementary services. In this sense, Linux is now an outside option to Windows and nowadays it is considered a serious threat to the latter.[8] Thus, we can interpret the analyses as an option that competes or threatens the incumbent platform. Questions we try to answer with this analyses are, for instance, given Windows being the incumbent firm, is it the grow importance of Linux in the users' benefit? What about developers of softwares?. If Linux becomes more important so that the value of writing applications for it increases, is this profitable for them? We find that it would not be in the interest of the users to promote the outside options (i.e., online games or computer games) to the video game console since, whenever competition margin binds, a higher outside option value for the users may lead to a decrease in their surplus. However, an increase in the value for developers of writing for an open platform such as Linux or Google has a positive impact in the users' surplus.This is the case because if demand margin binds, an increase in the outside option of the developers will always increase the users surplus.[9]

The structure of the paper is as follows. We present the model of a monopoly platform in section 2, and in section 3 we analyze the developers problem. In section 4 we solve the problem of a profit platform and compare its performance

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[5] Several facts that we cite along the article are documented by Evans, et. al. (2006).

[6] For instance, Microsoft produces operating system Windows and Office package. Nintendo wrote Mario Brothers, its killer game.

[7] Gamers report an average of 6,65 of hours spent per week on online-games and the home PC use of time explains 25% of children's and adult's games. http://www.cybersurvey.com/reports

[8] See www.cnn.com, World Business, "Reclusive Linux founder opens up", 19/05/2006, and "Microsoft vs. Open Source: Who Will Win?- HBS Working Knowledge, June 2005.

[9] In November 2006 Microsoft and Novell have signed a deal so that Linux programs can operate with Windows. Rivals will collaborate on technical development and marketing programs (The New York Times, 3/11/2006). A priori it seems the deal would benefit users and developers, but it warrants further analyses.

to that of an open platform in section 5. In section 6 we analyze incentives for integration and partial integration of the platform with applications. In section 7 we introduce outside options to the monopoly platform for developers and users. Finally, section 8 concludes.

2   A monopoly platform model

We assume that there is a monopoly platform and preferences of users are defined over the platform, its applications and an outside good. There is a measure one of users and their tastes for the platform are uniformly distributed along the unit interval. The utility of a user located at distance t from the platform is

$$U = V(M) + x - kt,$$

where M is the number of software varieties or applications, x is the numeraire good and k measures the degree of platform differentiation. It is further assumed that $V(M)$ is concave and increasing in M.[10]

Every user who purchases the platform consumes at most one unit of each application and maximizes her utility by choosing applications and consumption of the outside good subject to the constraint

$$\Sigma_{j=1}^{M} p_j + x + P^U = y,$$

where p_j is the price of a unit of application variety j, P^U is the charge that platform sets to the users and y is their income. A user's decision can be decomposed into two decision problems. First, the user sets her optimal basket of applications among the total number in the market,

$$G\left(M, \Sigma_{j=1}^{M} p_j, P^U\right) = \max_{M \leq N} \{V(M) - \left(\Sigma_{j=1}^{M} p_j\right)\} - P^U, \qquad (1)$$

where N is the number of applications in the market, then the user buys the platform if and only if

$$G\left(M, \Sigma_{j=1}^{M} p_j, P^U\right) - kt \geq 0.$$

The users demand for the system (size of the network) is hence determined by

$$t^d = \frac{G\left(M, \Sigma_{j=1}^{M} p_j, P^U\right)}{k} \epsilon [0, 1].$$

Note that demand depends on the price that platform sets for the users, but also on the number and prices of applications.

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[10]Similar utility functions are used by Church and Gandal, (1992, 1993, 2000) and Church et.al. (2003).

On the other side there are $\bar{N}$ potential developers of applications, each of them providing a single different application. Profits of developer of application i are given by

$$\pi_i = p_i t^d - F - P^D,$$

where F is a fixed cost of production, and P^D is the price that platform charges developers to allow them to write platform compatible applications.

Costs of the platform are assumed zero, so that platform profits are given by,

$$\Pi = P^U t^d + P^D N.$$

In this set-up we study the pricing strategies of the platform and developers. To do so we consider a game whose timing is as follows: in the first stage, the platform sets the charge to developers and these decide upon entry. In the second stage, the platform sets the price to the buyers. In the third stage, developers compete and set the prices for the applications to the buyers, then finally buyers decide if they buy the platform and the number of applications.

3 Application prices, users payments and System effects

When a user considers buying the platform, her decision will depend upon the prices set by developers. No user will purchase a video console without buying some video games, nor an operating system without buying the application softwares. Because of this we first study how developers set prices which will be a key point in our analysis. We then solve the second stage of the game at which the platform sets the price for users, taking N as given. Before that let us define two elasticities that will be used throughout the paper.

Ignoring the integer problem we define the elasticity of $V(N)$, a measure of the degree of substitutability of applications for the users,[11] as follows,

$$e_v(N) = \frac{V'(N)N}{V(N)}.$$

Since $V(N)$ is increasing and concave, it lies in the interval $(0,1)$. For a given N, we consider that applications to be near substitutes if $e_v(N)$ is sufficiently low.[12]

Similarly, let us define the elasticity of $V'(N)$,

$$\varepsilon_v(N) = \frac{V''(N)N}{V'(N)}.$$

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[11] It has also been interpreted as a measure of "degree of preference for variety" (see Kühn and Vives (1999) and Hagiu (2005)).

[12] Our interpretation here is similar to the one in Lerner and Tirole (2004): given N patents and two surplus functions $V_1(\cdot)$ and $V_2(\cdot)$, such that $V_1(N) = V_2(N)$, applications are more substitutable for surplus function $V_1(\cdot)$ than for $V_2(\cdot)$ if $V_1'(\cdot) < V_2'(\cdot)$.

Given that $V(N)$ is concave, it follows that $\varepsilon_v(N)$ is negative. The relationship between these two elasticities is the content of next lemma

Lemma 1 $e_v(N)$ is increasing in N as long as

$$e_v(N) < 1 + \varepsilon_v(N), \tag{2}$$

and is decreasing if the other inequality holds.[13]

3.1 Equilibrium application prices

The problem faced by developers is similar to the problem faced by a licensor in a patent pool. In the context of patents, the licensor problem has been studied by Lerner and Tirole (2004). In their model, the surplus derived from using N patents is also a function $V(N)$, strictly increasing in N. They show that, when setting a licensing fee, an individual licensor may be constrained by either of two margins that they call the *competition margin* and the *demand margin*. In our context, developers are constrained in a similar way. If the developer can not increase her price without, because of this, being excluded from the set of applications selected by the users, (in user's problem (1)) then the competition margin binds. In contrast, demand margin is said to bind for developer i, if she can individually raise her price without being excluded but leading to a reduction in the overall demand for the system (effect on t^d). In particular, if the demand margin binds, a developer chooses a price $p_i = \widehat{p}$ such that

$$\widehat{p} = \arg\max_{p_i} \ \{p_i \frac{V(N) - P^U - (N-1)\widehat{p} - p_i}{k}\}. \tag{3}$$

Consequently,

$$\widehat{p} = \frac{V(N) - P^U}{(N+1)}.$$

In contrast, if the competition margin binds, the price that a developer sets is its marginal contribution to the users utility, i.e.,

$$\widetilde{p} = V(N) - V(N-1).$$

Note that $\widetilde{p}$ depends on $V(N)$ but neither on the demand of the system t^d nor on P^U.[14] Besides, $\widetilde{p}$ is always positive, whereas $\widehat{p}$ is not necessarily so, as it will depend on the value of P^U.

Next lemma follows immediately from propositions 1 and 4 in Lerner and Tirole (2004).

Lemma 2 *There exists a unique and symmetric equilibrium such that, if $\widetilde{p} < \widehat{p}$, developers are constrained by the competition margin and charge equilibrium*

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[13]For instance, functions $V(N) = \log(1 + N)$ and $V(N) = (1 - \exp(-N))$ have $e_v(N)$ decreasing for all $N > 0$ and it is easy to show that they satisfy the reverse of (2) in all the relevant range of N. Function $V(N) = N^\beta$, with $\beta < 1$, presents constant elasticities, $e_v(N) = \beta$ and $\varepsilon_v(N) = \beta - 1$, then $e_v(N) = 1 + \varepsilon_v(N)$.

[14]If we ignore the integer problem, $\widetilde{p} = V'(N)$. Then, $\varepsilon_v(N)$ also represents the applications price elasticity to N when competition margin binds.

price $\widetilde{p}$, whereas if $\widetilde{p} > \widehat{p}$, developers are constrained by the demand margin and charge equilibrium price $\widehat{p}$.

As long as demand margin binds, developers set the price that maximizes their profits and this price is lower than their marginal contribution to the users' utility. In contrast, if the competition margin binds, the price that maximizes profits, as defined in (3), is higher than the marginal contribution to users surplus and then developers are forced to set a price equal to this contribution.

The consideration of both scenarios allows us to include in the analysis situations where the developers set the price that maximizes their profits and consider the reduction in the overall demand for the system when contemplating an application price increase (i.e. when demand margin is binding). Other papers in the literature, such as Hagiu (2005) and Church et. al. (2003), implicitly restrict their analyses to an scenario where the competition margin is always binding. In particular, Hagiu (2005) assumes that developers set prices for applications once users have bought the platform. Similarly, Church et. al. (2003) derive the equilibrium prices set by developers under the proviso that platform sales are invariant to application pricing.[15] Our contribution here will not only be to study the case in which the demand margin binds, but also the comparisons that will follow. Clearly, some of our results when the competition margin is the one that binds are similar to those found in these previous papers.

3.2   What is the binding margin?

We now try to establish what the conditions are that determine the margin that will bind, by using lemma 3.1, and the equilibrium values of prices $\widehat{p}$ and $\widetilde{p}$.

Lemma 3   *Developers are constrained by the competition margin if the platform sets a price to the buyers such that*

$$P^{U} < V(N) - \widetilde{p}(N+1). \tag{4}$$

If the opposite inequality holds, developers are constrained by the demand margin.

A closer look at (4) allows us to determine the binding margin as a function of the primitives in the model.

Proposition 1 *If*

$$e_{v}(N) < \left[1 - \frac{1}{\sqrt{N+1}}\right], \tag{5}$$

the competition margin will bind. If the opposite inequality holds, the demand margin will bind.

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[15] In Church et. al. (2003), $V(N) = N^{\beta}$. For this utility function they show that the Nash equilibrium in developers' prices is given by $p(N) = V'(N)$ when $N > 1$ and $\beta \leq \frac{1}{2}$, so that, in our terminology, the competition margin binds.

Proof: See Appendix A.

The proposition above shows that the degree of substitution among applications and the number of developers determine the margin that binds. As long as applications are near substitutes the competition margin is more likely to bind. The same occurs when N is large, as the following corollary shows.

Corollary 1 *If $e_v(N)$ is non-increasing there exists N^* such that if $N < N^*$ the demand margin binds and if $N > N^*$ the competition margin binds. However, if $e_v(N)$ is striclty increasing, N^* may fail to exist, so that the demand margin always binds.*[16]

Proof See Appendix A.

From proposition 1 we deduce that those developers that write applications which are not near substitutes or are indeed complements will tend to compete in the demand margin. Similarly, those systems composed by a very high number of applications are more likely to have developers competing in the competition margin.

Using the results above, if one looks at the observed facts in the video game industry discussed in the introduction,

1) 76% of gamers state that price is very/somewhat important in deciding what game to buy,

2) From a survey of over 1,000 game consumers it is known that around 19.10% of them purchase 1 or 2 games per month, 26.50% purchase 1 every two month and 6.90% 3 or more per month,[17]

3) Some players report having more than 50 games,

4) Among the ten top rated PlayStation 2 games, 3 of them belong to the adventure genre and 3 to the role-playing genre. Among the ten top rated Xbox 360 games, 2 of them belong to the Ice Hockey genre.

Facts 1 and 4 suggest that there exists a near substitution between the games. Facts 2 and 3 show that consumers usually own a system of console and video games composed of many applications.

If we compare these facts with those observed for systems of operating systems and applications (i.e. Windows) we find that it is not easy to find a consumer using a huge number of applications.[18] Moreover, applications are far substitutes (and sometimes complements). A user may need a text processor and a spreadsheet and also a browser. Then, we presume that developers writing for an operating system are constrained by the demand margin whereas those writing for the video console are constrained by the competition margin.

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[16] This is the case for instance for $V(N) = N + \sqrt{N}$ for which demand margin always binds. Note that if N^* exists, it is defined by $N^* = \left(\frac{1}{1-e_v(N^*)}\right)^2 - 1$.

[17] Zelos Group Survey: What Do Gamers Want? Everything. Electronic Gaming Business, Nov 19, 2003. http://www.findarticles.com

[18] Evans et.al. (2006) point out that, as opposed to the case of video consoles, "there's probably not much correlation between the number of applications that someone uses on a computer and the value that person places on that computer".

3.3 Users prices and system effects

In the second stage of the game, the platform sets the price for users, taking N as given. When the demand margin binds, the platform will set a price to the users as[19]

$$\widehat{P}^U = \frac{V(N)}{2}.\tag{6}$$

It then follows that the price set by developers will be $\widehat{p} = \frac{V(N)}{2(N+1)}$.

Meanwhile, if the competition margin binds, the optimal price that platform chooses for the users is

$$\widetilde{P}^U = \frac{V(N) - \widetilde{p}N}{2}.\tag{7}$$

Equations (6) and (7) put in evidence the existence of "system effects" in the industry. These effects arise when the value of one component depends on complementary components in the system.[20] The presence of system effects is reflected in the price that the platform sets to users which increases with the number of applications. In addition, when competition margin binds $\widetilde{p}$ affects $\widetilde{P}^U$ because of the complementarity between the applications and the platform. In particular, for a given N, when the price of the applications increases, the benefit that the platform makes per user decreases.[21]

When the competition margin binds, the relative charges paid by users can be expressed as a function of $e_v(N)$,

$$\frac{\widetilde{P}^U}{\widetilde{P}^U + \widetilde{p}N} = \frac{1 - e_v(N)}{1 + e_v(N)}.$$

Lemma 4 *As long as applications are more substitutes, applications will be relatively less expensive, and the platform can charge users more.*

When substitution is strong on the developers' side, prices in this market are very low and the platform takes advantage of this situation setting a higher price for the platform. Lemma 4 implies that it is profitable for the firm selling the console to accept games that compete among them or are near substitutes, which is consistent with the observed practice in the video game industry as stated in fact 4.

The relative charge paid by users for the platform when demand margin binds is given by

$$\frac{\widehat{P}^U}{\widehat{P}^U + \widehat{p}N} = \frac{N+1}{2N+1}.$$

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[19] See proof of proposition 1.

[20] See Evans and Schmalensee (2001). System effects are a clear feature of software platforms where the user buys a system (platform and applications) and cares for the total charge of the system.

[21] Note also that for the same N, $\widehat{P}^U > \widetilde{P}^U$, it is consistent with the observed fact that operating systems charge high prices to users whereas video console firms charge low prices to them.

Note that when demand margin binds, relative charges depend only on N whereas it depends on both, N and $V(N)$, when competition margin binds. The next proposition presents how relative charges vary with N.

Proposition 2 *If demand margin binds, the relative payment made by users to the platform is decreasing in N. However, if the competition margin binds, the relative payment is increasing in N whenever $e_v(N)$ is decreasing in N.*

As N increases, users tend to spend more on the bulk of applications when demand margin is binding. The same occurs, whenever the competition margin binds provided that $e_v(N)$ is increasing in N. However, $\frac{N+1}{2N+1} > \frac{1}{2}$, meaning that more than one half of the money that users spend in the system goes to the platform when demand margin binds. Meanwhile, it may occur that $\frac{1-e_v(N)}{1+e_v(N)} < \frac{1}{2}$ if $e_v(N) > \frac{1}{3}$.

When setting the price to users, the platform should optimally preserve this ratio, if not, a competitor with a better pricing strategy may easily overcome the incumbent's advantages.[22]

Let the users demand elasticity with respect to the price by the platform be $E_p = \frac{\partial t^D}{\partial P^U}\frac{P^U}{t^D} = -1$ and the elasticity of demand with respect to the number of applications be $E_s = \frac{\partial t^D}{\partial N}\frac{N}{t^D} = -\frac{\epsilon_v\widetilde{\varepsilon}_v}{1-\epsilon_v}$. The ratio $-\frac{E_s}{E_p}$ measures the effect of platform price equivalent to a 1% increase in N.[23] In the users' interest, a 1% increase in the number of applications is equivalent to a $\frac{\epsilon_v\widetilde{\varepsilon}_v}{1-\epsilon_v}\%$ price cut.[24] This ratio is increasing in ϵ_v and $\widetilde{\varepsilon}_v$. That is to say that an increase in N is more valued as long as it conveys a reduction in developers applications prices and applications are near complements.

4 Developers Entry and Welfare: Profit Platform vs Open Platform

In the first stage a proprietary platform sets a price to the developers that then decide upon entering the market. If the platform is open, this price is zero.[25] One could think that the platform, through the choice of prices for developers, determines the number of applications. However, this assertion may not always

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[22] For instance, in the market for video players, VHS overcame Beta after six years of higher installed base by Beta. The strategy of the winner was a widespread licensing of VHS and a low- priced VHS player, compared with a high-priced Beta player and restricted licensing (See Economides 2006).

[23] Note that if $V(N) = N^\beta$ the ratio is $\frac{-E_s}{E_p} = 1$.

[24] Clements and Ohashi (2005) have computed this ratio for the USA video game industry. They find that a 1% increase in game titles is equivalent (in average) to a 2.3% price cut of the console price.

[25] An open platform will charge zero to both users and developers. Nevertheless, we will assume that developers set positive prices to users for their applications. Applications for open platforms like Linux are often free for consumers. However, there are also several applications that are not free that are offered for Linux operating system (Economides and Katsamakas (2005a)).

be true. In particular, if developers' gross profits (i.e., $p(N)\,t^d(N)$) are increasing in the number of applications, then the platform can not affect entry which will equal $\bar{N}$. This is the case when the positive indirect network effect more than compensates the direct negative effect of competition. An additional developer exerts a positive effect on other developer's profits, explained by the fact that more participation by one side (i.e., developers) induces more participation by the other side (i.e., users), which benefits customers and makes them more willing to participate.[26] Consequently, whenever developers' gross profits are increasing at $\bar{N}$, the platform would charge a price $P^D\left(\bar{N}\right) = p\left(\bar{N}\right)t^d\left(\bar{N}\right) - F$ and its profits will be

$$\Pi = P^U\left(\bar{N}\right)t^d\left(\bar{N}\right) + \left(p\left(\bar{N}\right)t^d\left(\bar{N}\right) - F\right)\bar{N}.$$

In contrast, if developers' gross profits are decreasing at $\bar{N}$, because the positive effect on the demand is compensated by the negative effect on the price, then there is a one-to-one relation between N and P^D, so that the platform rather than maximizing profits over P^D can do so directly over N. The platform will hence optimally choose N to maximize its profits given by

$$\Pi = P^U(N)\,t^d(N) + \left(p(N)\,t^d(N) - F\right)N. \tag{8}$$

From the expression above it is clear that an increase in N affects the profits of the platform in two ways, through the profits made on users (first term in (8)) and through the profits made on the developers (second term in (8)). How these effects depend on the degree of substitution between the applications that developers offer is quite clear when looking at the profits made on the developers' side. If substitution is strong, their profits, gross of P^D, are lower, then the surplus that the platform may extract from them is also lower (or even negative if it is optimal for the platform to subsidize the developers, i.e., $P^D < 0$). Regarding the profits made on the users' side, recall that both $\tilde{P}^U$ and $\hat{P}^U$ are increasing in N. In addition, the positive effect of entry on $\tilde{P}^U$ and $\tilde{t}^d = \frac{\tilde{P}^U}{k}$ is higher when substitution between developers is higher (whenever $\frac{\partial \tilde{p}}{\partial N} = V''(N)$ is high). When N increases $\tilde{p}$ decreases, and this additional effect is taken into account by the platform when allowing access to the developers, becoming an additional incentive to promote entry. The optimal level of entry will depend on the margin that binds.

If demand margin binds, the platform will optimally choose $\hat{N}$ such that it solves

$$\frac{V(N)\,V'(N)\,(2N+1) - (V(N))^2\,\frac{N}{N+1}}{2k\,(N+1)^2} = F, \tag{9}$$

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[26] Farrell and Klemperer (2004) state that an indirect network effect arises whenever the indirect benefit outweighs any direct loss from more participation by one's own side. Thus, following this definition, there is an indirect network effect among developers as long as profits are increasing in N.

whereas if competition margin binds, it will choose $\tilde{N}$ such that

$$\frac{V\left(\tilde{N}\right)V'\left(\tilde{N}\right) - \left(V'\left(\tilde{N}\right)\right)^2 \tilde{N}\left[1 + \varepsilon_v\right]}{2k} = F. \tag{10}$$

The discussion above is the content of next lemma.

Lemma 5 *Let $G\left(N\right)$ $\left(H\left(N\right)\right)$ stand for the developers' gross profits when demand (competition) margin binds, and let N^* be such that if $N < N^*$ the demand margin binds and if $N > N^*$ the competition margin binds. Assume $H\left(N^*\right) > G\left(N\right)$ for all N.[27] The patterns of equilibrium entry in a proprietary platform will depend on the binding margin and the size of $\bar{N}$. In particular:*

i) If $H\left(N^*\right) > G\left(N\right)$ for all N and $\bar{N} > N^*$, in any stable equilibrium of developers' entry the competition margin will always bind and the level of entry will be

$$N = \left\{ \begin{array}{ll} \bar{N} & if \quad H'\left(\bar{N}\right) > 0 \\ \min\left(\tilde{N}, \bar{N}\right) & if \quad H'\left(\bar{N}\right) < 0, \end{array} \right.$$

where $\tilde{N}$ solves (10).

ii) If $\bar{N} < N^*$, the level of entry will be

$$N = \left\{ \begin{array}{ll} \bar{N} & if \quad G'\left(\bar{N}\right) > 0 \\ \min\left(\hat{N}, \bar{N}\right) & if \quad G'\left(\bar{N}\right) < 0, \end{array} \right.$$

where $\hat{N}$ solves (9).

Proof: See Appendix A.

When the platform is open there are no platform prices to affect agents decisions (recall that now $P^U = P^D = 0$), so that developers will enter until their profits are zero, i.e.,

$$p\left(N\right)t^d\left(N\right) - F = 0.$$

Lemma 6 *Let $G^o\left(N\right)$ $\left(H^o\left(N\right)\right)$ stand for the developers' gross profits when demand (competition) margin binds in an open platform and let N^{o*} be the N that determines the binding margin. Then,*

i) $G'^o\left(\hat{N}^*\right) = G'\left(\hat{N}^*\right) = 0$

ii) $\hat{N}^* = N^{o*} < N^*$

iii) $G^o\left(\hat{N}^*\right) = H^o\left(\hat{N}^*\right)$

Proof: See Appendix A.

Point *i)* implies that the maximum in gross profits when demand margin binds occurs at the same N in both types of platforms. Point *ii)* implies that

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[27]This is not a restrictive assumption, all the surplus functions that we are considering here satisfy it.

if demand margin binds, gross developers profits are increasing. Whereas, if competition margin binds, profits may be increasing or not. Note that a comparison of outcomes under open and proprietary platforms is not direct for the range of $N \epsilon \left(\hat{N}^*, N^* \right)$ as competition margin will bind under an open platform whereas the demand margin binds under a proprietary platform. Finally, point *iii)* shows that developers' profits are continuous at the point where the change from a margin to the other occurs.

If gross profits are increasing at $\bar{N}$, then $\bar{N}$ developers will entry. If not, the number of developers is determined by

$$\frac{V\left(\tilde{N}^o\right) V'\left(\tilde{N}^o\right) - \left[V'\left(\tilde{N}^o\right)\right]^2 \tilde{N}^o}{k} = F. \tag{11}$$

The next proposition compares the levels of entry that occur in each case and the effect on users' welfare.

Proposition 3 *i) If demand margin binds, a proprietary platform and an open platform will provide the same level of N, so that the latter will generate more welfare for users.*

ii) If competition margin binds a proprietary platform may generate a larger number of applications and higher welfare to users than an open platform.

Proof See Appendix A.

For comparison purposes, consider now the problem solved by a benevolent social planner. She would choose the optimal number of applications, N^{FB}, to maximize social welfare given by

$$W^* = \int_0^t V(N) \, dz - \int_0^t kz dz - FN,$$

where $t^{FB} = \frac{V\left(N^{FB}\right)}{k}$.

The first order necessary condition yields the first best allocation,

$$\frac{V\left(N^{FB}\right) V'\left(N^{FB}\right)}{k} = F. \tag{12}$$

Condition (12) that determines the first best level of N equalizes the marginal benefit with the marginal cost of an additional application. The former is the marginal utility enjoyed by users ($V'\left(N^{FB}\right)$ times the size of the market t^{FB}), whereas the latter is the fixed cost of producing one more application. Then if $\bar{N} < N^{FB}$ social planner chooses $\bar{N}$ and chooses N^{FB} otherwise.

As long as $\bar{N} < \hat{N}$ entry is $\bar{N}$ and equals N^{FB}. The same occurs when competition margin binds and $\bar{N} < \tilde{N}$. Then, when the effect of N on platform profits is strong (and this is more likely when e_v is high) the platform will tend to generate the same level of entry as the social planner.

Proposition 4 *Assume* $\bar{N} > \max\left(N^{FB}, \ \hat{N}, \ \tilde{N}\right)$. *If demand margin binds, a proprietary platform chooses a level of N smaller than the first best. However, if competition margin binds the comparison is not conclusive.*

Proof: See Appendix A.

Proposition 3 and 4 yield some insights into policies regarding the emergence of open source platforms competing with platforms such as Windows (Linux is the classic one, but there are also some others like Google which offer programs for free). In contrast, we do not observe the emergence of open platforms in the market of video consoles. The propositions above suggest that policy makers should promote open source in platforms like operating systems but not necessarily in those like video consoles.

5 Integration and the margin

Assume now that the platform firm can also develop its applications at zero marginal cost and at a fixed cost F per application. Then, if the platform is integrated, meaning that one firm produces the platform and the N applications, its system price will be

$$P^I = \frac{V(N)}{2}$$

and profits will be

$$\Pi^I = \left(\frac{V(N)}{2}\right)^2 \frac{1}{k} - FN.$$

We have shown that when integration is absent and demand margin binds, the resulting system price is $\frac{V(N)}{2}\left(1 + \frac{N}{N+1}\right)$ which is larger than P^I. The rational behind the result is clear: under separation there is a double marginalization as neither the platform nor the developers take into account the reduction of sales of the others when raising the price so that an inefficiently large price arises.

However, if integration is absent and competition margin binds, the resulting system price is $\frac{V(N)+V'(N)N}{2}$ which gets close to P^I as $V'(N)$ gets close to zero, which is the case when applications are very substitutes.

Proposition 5 *Inefficiencies of disintegration tend to disappear as long as competition margin binds and applications are near substitutes.*

Consider total profits of the firm. If demand margin binds, these are

$$\Pi^{DMB} = \left(\frac{V(N)}{2}\right)^2 \frac{1}{k}\left(\frac{2N+1}{N^2+2N+1}\right) - FN < \Pi^I,$$

so that the platform will always prefer being integrated in order to get developers to aware of the impact of their pricing strategies on the other developers and on the platform profits. Note that under separation even if the platform can control N through P^D, it can not control the price developers set.

If competition margin binds, profits are

$$\Pi^{CMB} = \left(\frac{V(N) - \widetilde{p}N}{2}\right)^2 \frac{1}{k} + \widetilde{p}\left(\frac{V(N) - \widetilde{p}N}{2}\right)N\frac{1}{k} - FN.$$

Again, as long as $\widetilde{p} = V'(N)$ tends to zero (because the extent of substitutability among applications is great or N is very high), profits tend to Π^I.[28]

The results above are consistent with the observed phenomena that initially platforms are vertically integrated and later disintegrate. Recall from corollary 1 that there exists N^* which determines the margin that is going to be binding. When the industry is less developed (initial steps of the industry with N low) the platform strictly prefers being integrated. As the industry evolves and the number of developers available in the market increases, the competition margin is likely to bind, prices of applications will be $V'(N)$, decreasing in N, and at this stage of the industry, the platform will be more willing to disintegrate.[29] As the market of developers matures and becomes more competitive, the firm can concentrate on producing only the platform. Note that other alternative explanations are offered in the literature for the phenomena of vertical disintegration that not can be explained within this model. For instance, Stigler notes that firms need to arise vertically integrated since technology is not familiar in the market. When the industry grows, production process are well known and scale of the market allows specialization, such that disintegrating is profitable. [30] Another different explanation for no integration is given by Gawer and Henderson (2005), when discussing Intel's strategy. They suggest that managers were aware of how important the generation of complements was to the success of Intel's business; however, although it is in the interest of the platform to enter complementary markets, the platform knows that this could discourage entry by new firms.[31] A more trivial explanation comes from the fact that the platform does not always possess the requisite capabilities to produce some of the complementary goods.[32]

5.1 Partial Integration

A widely observed fact in software industries is that some computer software are clearly more useful or more commonly used than others, Office software and

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[28] In particular, the necessary condition is that $V'(N)N$ be decreasing, i.e., $\varepsilon_v(N) > 1$.

[29] PDA's were born as "smart agendas" offering a limited number of applications. Then, they evolved to become "small computers". Something similar has ocurred in the mobile phone industry. In addition to the traditional communication service, today they allow for hundreds of applications. See "What is a Window Mobile" in www.microsoft.com.

[30] George Stigler, "The division of labor is limited by the extent of the market", Journal of Political Economy 59 (June 1951), quoted by Evans, et. al.(2006).

[31] Dave Johnson, a director of Intel, explained: "The market segment gets hurt if third parties think: "Intel, the big guys, are there, so I do not want to be there..."... it is not what we want, because we are trying to encourage people to do these complementary things". Gawer and Henderson (2005), pp. 18.

[32] Claude Leglise, director of the Developer Relation Group, responded: "Intel has no corporate competence in entertainment software. We do not know how to do video games, so forget it". Gawer and Henderson (2005), pp. 13.

Messenger are illustrative examples. At the same time, some video games are the most popular (killer games) in the market, so that applications' contributions to total surplus may be different. To incorporate this feature into our model, in what follows we allow applications to be heterogeneous.

Assume that each application i has a contribution $N_i \in [0, N]$, with the normalization

$$\sum_{i=1}^{N} N_i = N.$$

Note that $N_i = 1$ will bring back the homogeneity we have considered so far. Let us further assume that $\frac{\partial N_i}{\partial i} > 0$ and let us define $V(\cdot)$ by $V\left(\sum_{i=1}^{N} x_i N_i\right),$ where $x_i = 1$ if user buys application i and $x_i = 0$ otherwise. The next lemma is inspired in proposition 6 in Lerner and Tirole (2004).

Lemma 8 *Assume that gross surplus of users by applications is $V\left(\sum_{i=1}^{N} x_i N_i\right),$ where $x_i = 1$ if users buy application i and $x_i = 0$ otherwise, with $\frac{\partial N_i}{\partial i} > 0$. Then, there is a mass $0 \leq n \leq N$ of developers that are constrained by the competition margin and charge a price $\widetilde{p}_i = V'_i$, their marginal contribution to the total surplus. The rest of the developers are constrained by the demand margin and all of them set the same price $\widehat{p} = \frac{V(N) - P^U - \int_0^n \widetilde{p}_i d_i}{N - n + 1}$. Finally, the platform sets $P^U = \frac{V(N) - \int_0^n \widetilde{p}_i d_i}{2}$.*

When the platform decides P^U, it defines the value n, i.e., the mass of developers that will be constrained by the competition margin. For every $i \in [0, n]$ it must hold that $\widetilde{p}_i = V'_i < \widehat{p}$ and that V'_i is increasing in i. If $n = 0$, we have that every developer is constrained by the demand margin. Analogously, if $n = N$ every developer is constrained by the competition margin.

Proposition 6 *In the long run the platform will be partially integrated with the killer applications for which demand margin will bind, and will allow free entry for developers of other applications.*

This proposition may help us to explain why platforms are often partially integrated, most of them with the core application. Microsoft produces operating systems and some of the applications (i.e. Office package). Nintendo wrote Mario Brothers, the killer game of one of its consoles. In the US the proportion of games developed in house is about 10% for GambeCube and 8% for PlayStation and Xbox.

6 Platform Competition: the role of outside options

Up to now we have assumed that a monopolist platform (either proprietary or open) provides a good with no competition at all. Nevertheless in many

industries, either open and proprietary platforms coexist, or there are several for-profit platforms competing to attract both users and developers. We now extend our basic framework by assuming that a proprietary and an open platform operate in the same industry.[33] Our aim here is to analyse how a firm that offers a proprietary solution will respond to changes in an outside option that provides a positive surplus or profit to their clients (i.e., to users and developers). We analyze how the monopoly reacts in terms of prices and we abstract from other strategies such as investment.[34]

A user who purchases the open platform gets a net surplus $v = V(Z) - h$, where $V(Z)$ measures the utility users derive given the applications written for the open platform and h is an exogenous cost (interpreted as a transportation cost or a cost of learning to use this outside good). Consequently, users will purchase the proprietary platform as long as

$$V(\mathcal{N}) - kt \geq v > 0.^{35}$$

The game is solved as in previous sections but considering v. In what follows we provide some comparative statics analyses to changes in v in order to study its impact on users welfare. We start assuming that the competition margin binds. Then, we move to an scenario where the demand binds. We restrict the analyses to values of N for which developers' profits are decreasing so that the proprietary platform can affect entry.

Consider the impact of a change in v on developers' profits and on the number of applications. The condition that arises when the platform at the first stage maximizes with respect to N is

$$\frac{V(N)V'(N) - (V'(N))^2 N \ [1 + \ \varepsilon_v]}{2k} - \frac{V'(N)v}{2k} = F. \qquad (13)$$

and from the comparison with equation (10) it follows that the monopolist will reduce enty due to the term $\frac{V'(N)v}{2k}$. This term is decreasing in N and smaller as long as applications are very substitutes.

It means that developers of video consoles may not have incentives to increase the value of v (i.e., writing applications for computers or online games) because the monopolist may react reducing the level of entry and thus the incentives for them. However, this response will not be important whenever the games are near substitutes.

By taking into account its impact on entry, the next proposition provides results on the impact of outside options on users surplus.

Proposition 7 *Whenever the competition margin binds, a higher outside option value for the users may lead to a decrease in their surplus. In contrast,*

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[33]Since the open platform is considered non-profit, we will assume that it behaves myopically and hence does not play a best response against the pricing strategies by the proprietary platform. In contrast, the proprietary platform will take into account the presence of the open platform when deciding upon its pricing strategies.

[34]Economides and Katsamakas (2006b) study investment incentives of platforms and developers in a proprietary system and in an open source one.

[35]Note that v is used to proxy for the extent of product market competition.

if the demand margin binds, the impact on users' surplus of a higher outside option will generally be positive.

Proof. See Appendix A.

Regarding the other side of the market, we now assume that developers can obtain a profit of w when writing applications for the open platform. Note that nothing changes if developers are allowed to write for both platforms (i.e., to multihome). In that case developers get a higher total profit but the strategies of the proprietary platform do not change. Results are different if we assume that developers are forced to choose one of the platforms (i.e., to singlehome) due, for instance, to contractual arragements. Thus, developers will enter the market of the proprietary platform as long as

$$\pi_i = p_i t^d - F - P^D \geq w.$$

The effect of an increase in w is analogous to an increase in the fixed cost, so it clearly leads to a reduction in the level of N.

Proposition 8 *If the competition margin binds, an increase in the outside option of the developers will always reduce the users' surplus. However, if the demand margin binds, an increase in the outside option of the developers will always increase the users' surplus.*

Proof. See Appendix A.

We have shown that reinforcing competition pressure for developers when competition margin binds leads to a reduction in the users welfare. Results are quite different when demand margin binds. Promoting the benefits that writing for Linux has for the developers (sometimes interpreted as a "reputation effect"[36]) would be in favour of the users.

Let us provide an illustrative example. Consider $V(N) = N^\beta$ where $\beta = 0.45$ and a fixed cost $F = 0.14$. A value $\beta = 0.45$ determines that competition margin is binding as long as $N > 2.3$ and we restrict the analyses to this range of N. For a value $v = 0.1$, the surplus of the users is 0.71 whereas for an increase $\Delta v = 0.05$, the new users surplus is 0.59. It represents in terms of elasticities that a 1% increase in the users outside option implies a 19% decrease in the users surplus.[37]

To compare the effects of w and v, consider now $\beta = 0.25$ (so that competition margin binds as long as $N > 0.8$) and a fixed cost $F = 0.075$. Given the initial values $w = v = 0.1$, we find that a change in v (i.e., $\Delta v = 0.05$) exerts a direct impact on P^U equal to $\frac{\partial P^U}{\partial v} = -\frac{1}{2}$, whereas there is no direct impact when w changes (i.e., $\Delta w = 0.05$). However, when we compute the total effect, considering the indirect one by the effect on N, we find that $\frac{\partial \tilde{P}^U}{\partial v} \frac{v}{\tilde{P}^U} = -0.01$ and $\frac{\partial \tilde{P}^U}{\partial w} \frac{w}{\tilde{P}^U} = -0.03$, meaning that, under these parameters, the monopolist decides to reduce the price more for users when there is an outside option for the developers than when there is one for the users themselves.

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[36] Economides and Katsamakas (2005.b). Other motivations are explained in "Microsoft vs. Open Source: Who Will Win?- HBS Working Knowledge, June 2005.

[37] The exercise has been computed assuming $k = 1$.

7 Conclusions

We have solved a model that provides some results for a better understanding of the two-sided pricing strategies of a platform that sells a good whose value depends on the applications sold in a market of developers. We note that when setting prices the developers are constrained by two margins: the demand margin and the competition margin. What margin is binding depends on the number of applications in the market and on the level of substitutability among them.

We find that if the demand margin binds, policy makers should promote open source platforms. However this is not necessarily the case when competition margin binds.

We consider the case where applications are asymmetric in the users' surplus and we find that in the long run the platform will remain integrated with the applications for which demand margin binds and will leave for third-party developers the production of applications for which competition margin binds.

Finally, we find that it would not be in the interest of the users to promote the value of outside options for the platform when competition margin binds. However, an increase in the value of the outside option for developers would have a positive impact on the users surplus if demand margin binds.

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Appendix A

Proof of proposition 1

To show the result we compute the profits that each situation generates for the platform, then we compare them and deduce the optimal strategy for the platform. If the platform sets a price that satisfies $P^U < V(N) - \widetilde{p}(N+1)$, then the competition margin will bind for the developers and platform profits will be

$$\widetilde{\Pi}^{PU} = P^U \left[\frac{V(N) - \widetilde{p}N - P^U}{k} \right].$$

The price that maximizes profits, given the constraint, is

$$P^U = \frac{V(N) - \widetilde{p}N}{2} \text{ if } \widetilde{p} < \frac{V(N)}{N+2}, \text{ and}$$

$$P^U = V(N) - \widetilde{p}(N+1) \text{ if } \widetilde{p} > \frac{V(N)}{N+2}.$$

If the platform sets a price such that $P^U > V(N) - \widetilde{p}(N+1)$, so that demand margin will bind for the developers, platform profits will be

$$\widehat{\Pi}^{PU} = P^U \left[\frac{V(N) - P^U}{k(N+1)} \right].$$

The price that maximizes profits, given the constraint, is

$$P^U = \frac{V(N)}{2} \text{ if } \widetilde{p} > \frac{V(N)}{2(N+1)}, \text{ and}$$

$$P^U = V(N) - \widetilde{p}(N+1) \text{ if } \widetilde{p} < \frac{V(N)}{2(N+1)}.$$

Comparing above the profits we observe that if $\widetilde{p} < \frac{V(N)}{2(N+1)}$ the price that generates highest profits for the platform is $\widetilde{P}^U = \frac{V(N) - \widetilde{p}N}{2}$. If $\widetilde{p} > \frac{V(N)}{N+2}$, the platform will optimally choose $\widehat{P}^U = \frac{V(N)}{2}$. Finally, whenever the relevant interval is $\frac{V(N)}{2(N+1)} < \widetilde{p} < \frac{V(N)}{N+2}$, if $\widetilde{p} < \frac{V(N)}{N}\left[1 - \frac{1}{\sqrt{N+1}}\right]$ the platform will set $\widetilde{P}^U = \frac{V(N) - \widetilde{p}N}{2}$ and will set $\widehat{P}^U = \frac{V(N)}{2}$ otherwise. It follows that the competition margin will bind if $\widetilde{p} < \frac{V(N)}{N}\left[1 - \frac{1}{\sqrt{N+1}}\right]$, and this occurs whenever $e_v(N) < \left[1 - \frac{1}{\sqrt{N+1}}\right]$, as claimed.

Proof of corollary 1

Note that the function $\left[1 - \frac{1}{\sqrt{N+1}}\right]$ is increasing in N, equals zero at $N = 0$, and goes to one as N goes to infinity. Since $e_v(N) \epsilon (0,1)$, if $e_v(N)$ is a non increasing function, it will necessarily cross $\left[1 - \frac{1}{\sqrt{N+1}}\right]$. However, if $e_v(N)$ is an increasing function, a crossing point may not exist.

Proof of lemma 5

When the demand margin binds, developers will enter until profits are zero so that it is satisfied

$$\left(\frac{V(N)}{2(N+1)}\right)^2 \frac{1}{k} - F - P^D = 0.$$

If competition margin binds, the developers zero profit condition will be

$$V'(N)\left(\frac{V(N) - V'(N)N}{2k}\right) - F - P^D = 0.$$

Consequently, let $G(N) = \left(\frac{V(N)}{2(N+1)}\right)^2 \frac{1}{k}$ and $H(N) = V'(N)\left(\frac{V(N) - V'(N)N}{2k}\right)$.

i) In a stable equilibrium, profits are zero and decreasing. Consider now an equilibrium such that $G(N) = F + P^D$ (so that demand margin binds). Since $G(N) < H(N^*)$, when $\bar{N}$ is sufficiently large a coalition of developers will enter to obtain (at least) profits $H(N^*)$, and the result follows.

Then,

1) if $H'(\bar{N}) > 0$ gross developers profits are strictly increasing so that entry is $\bar{N}$.

2) if $H'(\bar{N}) < 0$ gross developers profits are strictly decreasing so that the platform will choose $N = \min\left(\tilde{N}, \bar{N}\right)$, and the result follows.

ii) We must distinguish two cases. 1) If $G'(\bar{N}) > 0$ gross developers profits are increasing and entry is $\bar{N}$.

2) If $G'(\bar{N}) < 0$ gross developers profits are decreasing so that the platform will choose $N = \min\left(\hat{N}, \bar{N}\right)$, and the result follows.

Proof of lemma 6

Note that $G^o(N) = \left(\frac{V(N)}{(N+1)}\right)^2 \frac{1}{k}$ and $H^o(N) = V'(N)\left(\frac{V(N) - V'(N)N}{k}\right)$. Result i) follows trivially. Note that the concavity of V ensures that $\hat{N}^*$ always exists. ii) Note that $\hat{N}^*$ solves $V'\left(\hat{N}^*\right) = \frac{V(\hat{N}^*)}{\hat{N}^*+1}$. The equality $\hat{N}^* = N^{o*}$ follows from the fact that in an open platform competition margin binds as long as $V'(N) < \frac{V(N)}{(N+1)}$. To prove that $\hat{N}^* < N^*$ recall from corollary 1 that N^* satisfies $V'(N^*) = \frac{V(N^*)}{N^*}\left(1 - \frac{1}{\sqrt{(N^*+1)}}\right)$. Since $V'(N)$ is decreasing and $\frac{V(N)}{(N+1)} > \frac{V(N)}{N}\left(1 - \frac{1}{\sqrt{(N+1)}}\right)$ for all N, it follows that $\hat{N}^* < N^*$. Part iii) follows from straigthforward computations.

Proof of Proposition 3

i) The first statement follows from point i) in lemma 7 (profits of developers are increasing under both regimes for the same range of N) then in both cases entry will equal $\overline{N}$. If demand margin binds, with a proprietary platform the system price is $P^U + pN = \frac{V(N)}{2} + \frac{V(N)}{2(N+1)}N$, that is higher than $\frac{V(N)}{N+1}N$, the system price with an open platform, so that the second statement follows.

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ii) From the comparison between (10) and (11), it follows that as long as $e_v > \frac{1}{1-\varepsilon_v}$ (i.e. $V'(N) - V''(N)N > \frac{V(N)}{N}$), a profit platform yields a higher N than the open platform. The second statement is proven by the fact that when competition margin binds, the users' surplus (net of kt) is increasing in N. The condition $e_v > \frac{1}{1-\varepsilon_v}$ imposes that $\varepsilon_v < -1$ since $e_v < 1$. An example for which a proprietary platform yields a higher N than an open platform is given by

$$V(N) = \begin{cases} (1 - \exp(-0.05N)) & if \quad N \le 7 \\ (0.8 - \exp(-0.1N)) & if \quad N > 7, \end{cases}$$

with $F = 0.0045$.[38] The proprietary platform chooses $N \simeq 25$ whereas the open chooses $N \simeq 24$. The competition margin binds for all $N > 13$.

Proof of Proposition 4

The first statement follows from the comparison between (9) and (12). The second statement follows from the comparison between (10) and (12) and the fact that as long as $-e_v(1 + \varepsilon_v) > 1$ (i.e. $-V''(N)N > \frac{V(N)}{N} + V'(N)$) the proprietary platform may generate excess of entry. As in the previous proof, the condition $-e_v(1 + \varepsilon_v) > 1$ requires $\varepsilon_v < -1$ and it is more stringent than the condition in the proof of proposition 3. It does not contradict the condition to be in the competition margin $e_v < 1 - \frac{1}{\sqrt{N+1}}$, nor the condition for a maximum in the social planner problem, $e_v < |\varepsilon_v|$, nor the fact that $V(N)$ is concave.

Proof proposition 7

Given a user t, if competition margin binds, her surplus gross of kt is equal to $V(N) - pN - P^U$. We observe that this surplus will be increasing (decreasing) in v as long as $\left[1 - V''(N)N\frac{\partial N}{\partial v}\right] \lessgtr 0$, and the first statement follows. To prove the second statement note that if demand margin binds, users' prices are: $\widehat{p} = \frac{V(N)-v}{2(N+1)}$ and $\widehat{P}^U = \frac{V(N)-v}{2}$. The platform optimally chooses the N that maximizes profits

$$\frac{1}{k(N+1)}\left(\frac{V(N)-v}{2}\right)^2 + \left(\left(\frac{V(N)-v}{2(N+1)}\right)^2\frac{1}{k} - F\right)N.$$

Note that expression (9) can also be written as

$$\frac{V(N)}{2k(N+1)^2}\left[V'(N)(2N+1) - V(N)\frac{N}{N+1}\right] = F,$$

and when the outside option appears it transforms in

$$\frac{V(N)}{2k(N+1)^2}\left[V'(N)(2N+1) - V(N)\frac{N}{N+1}\right] -$$

$$\frac{v}{2k(N+1)^2}\left[V'(N)(2N+1) - (2V(N)-v)\frac{N}{N+1}\right] = F.$$

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[38] The equilibrium occurs at $N > 7$ so that $V(N) = (0.8 - \exp(-0.1N))$. Note that $V(N) = (1 - \exp(-0.05N))$ if $N \le 7$ ensusres that $V(0) = 0$.

So, the effect on N of v will depend on the second term. If this is positive, the monopolist will reduce N whereas if this is negative the impact on N will be positive. Both situations may occur; however since platform profits are lower for each N, the most likely case is that the monopolist will reduce N.

Now, note that whenever demand margin binds and there is an outside option v, the users surplus, gross of the cost kt, equals

$$V(N) - \widehat{p}N - \widehat{P}^U = V(N) - \frac{[V(N) - v]N}{2(N+1)} - \frac{[V(N) - v]}{2}$$

The first derivative of this surplus with respect to v is going to be positive as long as $\frac{\partial N}{\partial v}\left[\frac{V'(N)}{N+1} - \frac{V(N)}{(N+1)^2} + \frac{v}{(N+1)^2}\right] + \frac{2N+1}{N+1} > 0$. The second term of the left hand side of the inequality is always positive. However, the term in brackets is negative as long as $\epsilon_V < \left[1 - \frac{v}{V(N)}\right]\frac{N}{N+1}$, and this is the case along the relevant range of N (when gross developers profits are decreasing). The first term will be positive if $\frac{\partial N}{\partial v} < 0$ (the most likely case) and negative otherwise, so that the result follows.

Proof proposition 8

If the competition margin binds the effect of an increase in w on users surplus is equal to $-\frac{V'''(N)N\frac{\partial N}{\partial W}}{2} < 0$ and the first statement follows. To prove the second statement, note that the surplus is decreasing in N if $e_v < \frac{N}{N+1}$ and this occurs for the relevant range of N. Given that $\frac{\partial N}{\partial w} < 0$, the second statement follows.