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PROVIDING LONG-TERM CARE WITHOUT CROWDING-OUT FAMILY SUPPORT AND PRIVATE INSURANCE[*]

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Abstract

In this paper we are interested in the organization of long-term care within a given population. Three care financers are identified: the family, the government and the care receiver who can buy a dependency insurance. Our interest lies in the effect of governmental intervention on the demand/supply of these three forms of LTC i.e. how state intervention affects the provision of LTC by the market and the family. Knowing that, we search for an efficient organization of LTC.

For that, we consider a heterogeneous society composed of different pairs of parent/child. Each parent has a probability of becoming dependent and he/she must receive appropriate cares if this happens. Children are active on the labor market. Additionally, they may devote part of their income to help their dependent parents. The population is heterogeneous. Parents differ according to their income level; children are altruistic or not. These characteristics cannot be observed by the government.

Information asymmetry is a serious constraint on what can actually be implemented by the government. We show that rich parents may not always subscribe to a LTC insurance, even if it is socially optimal that they do so. This non-purchase of insurance is due to the high opportunity cost of insurances, even if they are supplied on the market at an actuary fair price i.e. insurance is crowded-out by other forms of LTC.

As a consequence, the government will reduce the opportunity cost of insurances by decreasing its support to other forms of LTC provided directly by the state or by the family. Alternatively, the government can reduce the share of market financed LTC within the economy.

Keywords: Long-term care, crowding-out, dependency insurance, altruism.

JEL Classification: D64, H55, I18

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1 Introduction

In OECD countries, the share of people over 65 and over 80 is increasing and demographic trends show that these proportions will continue to increase in the future. This demographic process is accompanied by an increase in the demand for long-term care (LTC) by elderly dependent at the end of their life.[1] A major challenge for an ageing of the society is therefore to finance the provision of appropriate long-term care to dependent people.

Facing the problem of an ageing population and the associated increase in demand for LTC, countries have chosen different institutional solutions to tackle this problem (see Karlson et al., 2004 for a detailed comparison between Germany, Japan, Sweden, United States and UK and OECD, 2005). In Germany, the government introduced in 1995 a mandatory long-term care insurance program that covers most of the population. The system is financed by a new tax on wages equals to 1.7% of the salary. The LTC insurance is a PAYG system and it is managed as a part of the social security system. An elderly dependent can apply for LTC benefits; his/her dependency level determines the level of help he/she receives. Benefits are of three kinds: professional care at home, institutional care and cash. The right to these forms of help is independent of the income level. Introducing cash payment is meant to support the provision of informal care by relatives. By doing so, informal helpers can receive a compensation for their LTC provision. The German mandatory insurance is an exception and most of the countries do not have universal LTC coverage. In many countries, an elderly dependent person does not necessarily receive help from the state. In the UK for example, the local authorities provide care in residential homes and public intervention in LTC financing is targeted to low income people. Service provision and financial intervention by local authorities are subject to means-testing and higher income individuals must self-finance their LTC needs with their own resources or must rely on informal help from their relatives.

Organizing LTC financing is a complex issue since many care providers and care financers are involved. Three different categories of long-term care can be distinguished: nursing home care, residential care provided informally by the relatives and paid residential care. Informal care is by large the most important source of LTC. For Sweden, Johansson (2000) estimated that two-thirds of the total volume of LTC is provided informally by relatives and friends. Bonsang (2007) documents on the basis of the SHARE-1 survey[2] that 30.9% of adult children aged between 50

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[1]Long term cares refer to the provision of help/care to a dependent person for his/her activities of daily living (ADL). LTC excludes medical cares that are usually financed by other means.

[2]The SHARE survey (wave 1) has been carried out in 2004 in 10 European Countries. It contains detailed information on a sample of individuals aged over 50.

and 69, with at least one living parent (not necessarily dependent) and not living with them, provide help in time to their parents, with an average of 25.1 hours per month. Help consists in personal care for 27.5% of the helping children. Direct financial help is much less common since it concerns only 2.6% of the children.

The share of private spending remains important in LTC financing. The OECD estimated that LTC spending accounts for 1.35% of GDP in Germany and for 1.37% in the UK. In both countries, the private spending represents 30% of the total expenses. Nursing home care is by large the most expensive form of LTC and it captures the largest fraction of LTC spending (more than 60% of the total public spending in most of the OECD countries). Because nursing home cares are costly for both the state and the individual and because elderly dependents prefer to stay at home (whenever it is possible), policies are settled to support informal care and paid residential care. These initiatives include in-kind benefit, budget for LTC care and financial support for informal helpers.

Private insurances could constitute an interesting alternative to public and private financing of LTC. But the market for dependency insurance is not very well developed. Several reasons may explain that. Pauly (1990) and Brown and Finkelstein (2007) show that there is an important crowding out of private insurances by the public financing of LTC, Medicaid in the US. Pestieau and Sato (2007) show that parents may prefer cares from their family to a private insurance, especially those who have a low income and those who anticipate an important help from their children. In light of that, a major problem for the organization of LTC by the state is that state intervention may seriously crowd-out LTC provided by the market and/or the family. This might be a serious concern for a financially constrained government facing an ageing population.

In this paper we are interested in the financing of LTC within a given population. Three sources of care financing are identified: family support provided by the relatives (the child in our model), private financed cares by the individuals either directly or through a private insurance[3] and government financed cares. Government intervention is either direct: The government provides nursing home care places or indirect: The government supports LTC provision by the family. Our interest lies in the effect of governmental intervention on the demand/supply of these three forms of LTC i.e. how state intervention affects the provision of LTC by the market and the family. Knowing that, we search for an efficient organization of LTC.

For that, we consider a society composed of different pairs of parent/child. Each parent has

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[3]Parents always prefer to buy a private insurance to insuring themselves through saving (at least for reasonable loading factor charged by the insurance company). Brown and Finkelstein (2007) document a load factor of 18% for LTC insurance.

a probability of becoming dependent and he/she must receive appropriate cares if this happens. Children are active on the labor market. Additionally, they may devote part of their income to help their dependent parents. The population is heterogeneous. Parents differ according to their income level. For simplicity, we consider two income levels i.e. we distinguish "rich" and "poor" parents. A child may or may not be concerned about the long-term care received by his/her dependent parent. That is, we distinguish altruistic and non-altruistic children. In our model, the government cannot observe the type of the parents (rich or poor) and of the children (altruistic or not). Information asymmetry is a serious constraint on what can actually be implemented by the government.

Parents have the option to buy a private insurance that finance LTC in case of dependency. A parent decides on the amount of insurance he/she subscribes. We consider that the insurance market is competitive, meaning that LTC insurances are offered for an actuary fair premium. If the insurance market is not competitive, the problems we enlighten in this paper are exacerbated.

Non-insured parents receive cares from their children if they agree to do so i.e. if the child is altruistic and if he/she prefers to help his/her parent to other forms of LTC. Non-insured parents of non-altruistic children have no other option than going to a public nursing home. We assume that these three forms of care are mutually exclusive. This means for example that an altruistic child does not help his/her parent if he/she is insured.

In this context, we search for the optimal policy mix, taking into account the reaction of private actors. Of course, the best policy depends on the **instruments** and the **information** available to the social planner. Consider that the LTC insurance is fair. If redistribution of the society's resources can be done at no cost through lump-sum taxes and subsidies, the best policy consists in delegating the LTC financing to the market. All parents are insured (at a fair price) and the government cancels out ex-ante differences in wealth with appropriate income redistribution.

If the government cannot distribute all the resources, the market solution may no longer be the most efficient one. A possible limit in the government's ability to redistribute resources is its inability to tax the wealth of the parents.[4] As a matter of fact, parent's wealth consists of assets that may not, for whatever reasons, be taxed. Moreover, the government may not have the ability to observe the wealth of the parents. This seriously limits the possibility of financing the insurance of the poor parents with a redistributive policy. And, financing a universal insurance with labor income taxes might be prohibitively costly mainly because rich parents will also be subsidized.

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[4]Another is distortionary taxation.

The government then adopts another financing scheme for LTC mixing market, state and family financed cares. Instead of redistributing income to finance private insurances, parents could subscribe to a LTC insurance if they have enough resources and if they agree to do so. Otherwise they can be helped by their family or directly by the state. For that, the government offers publicly financed nursing homes. Moreover, the government can support family financed cares by supporting altruistic children. Hence, without perfect redistribution, the market, the family and the state could all contribute to LTC financing.

But, information constraint limits what can be actually implemented by the state. The main problem is that rich parents may prefer to receive cares from their family or from the state to subscribe a private insurance, even if they would receive more cares in the latter case. The reason is that, even for a fair premium, the insurance cost might be considerable once opportunity costs are taken into account. If a parent is insured, he/she renounces to the other forms of LTC. Hence the LTC he/she might receive in the absence of insurance constitutes the opportunity cost of the insurance. We can then associate to this opportunity cost an implicit load factor for the insurance. This load factor might be considerable, discouraging the parents to subscribe to a private insurance. This rational non-purchase of LTC insurance, even for a fair premium, has been pointed first by Pauly (1990). We observe the same in our model. Because of the high implicit cost, rich parents may not subscribe to private LTC insurances.

The government cannot constraint the rich parents to be insured because wealth is unobservable. Hence, facing rich parents that do not have incentives to be insured, the government has two options. It can either reduce the share of the market in LTC financing and expend the family and the state financed support. The cost being that each dependent parent receives less for his/her LTC needs because resources must be shared by a larger number of claimants. This solution is adopted in Germany where dependent parents have an unconditional access to LTC support financed by labor income taxes. Or, it can decrease the opportunity cost of insurances by reducing its support to LTC financing by the state and the family. This must be done in a way that preserve the incentives for the altruistic children to help their dependent parents. The means-testing and the mandatory individual participation to LTC financing adopted in the UK are means to reduce the opportunity cost of private insurances for the richer individuals. In both cases, poor parents suffer from the non-purchase of insurance by the rich ones.

This paper is closely linked to Pestiau and Sato (2007) and Jousten et al. (2005). In their model, Pestiau and Sato (2007) consider a population of heterogeneous parent/child pairs. In particular, they focus on children with different labor productivities. This in turn affects the amount of help a child may provide to his/her dependent parent. And parents anticipating

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different levels of care by their families will have a different attitude towards other sources of LTC, provided by the state and the family. Optimal policies are derived in this context. In the current paper, we consider other sources of heterogeneity within the population: children differ with respect to their altruism; parents differ with respect to their initial wealth level. Jousten et al. (2005) develop a model where the only source of heterogeneity is the childrens altruism. There is no LTC insurance in this model and it focuses on the impact of the altruism on the supply of institutional care by the government. If the government does not observe the degree of altruism, a too generous provision of publicly financed nursing home crowds-out informal help. Welfare consequences on each category of the population are then evaluated. This paper adds another source of heterogeneity on the parents side of the population.

2   Model

We consider an heterogeneous population of N parent/child pairs. Parents differ according to their wealth endowment; children differ according to their degree of altruism. The population is divided into four groups. Groups 1 and 2 contain the rich parents (wealth level I^H) and their respectively non-altruistic and altruistic child. Poor parents are in group 3 (altruistic child) and 4 (non-altruistic child). n_i, $i = 1, 2, 3, 4$ is the proportion of each group in the total population N that we normalize to 1.

The parents have an initial wealth level $I \in \{I^H, I^L\}$, with $I^L < I^H$. Independently of his/her wealth, each parent faces a probability of dependency π. The utility (V) of a parent depends on his/her consumption level C^p and the help H he/she receives in case of dependency.

$$V = v(C^p) + \pi h(H)$$

Children are either altruistic or not. Both types of children have a utility level $u(C^c)$ when they consume C^c. In addition, altruistic children also care about the help H received by his/her parent in case of dependency (but not on his/her parent consumption if he/she remains in good health). The degree of altruism is measured by a parameter $\beta \in \{0, 1\}$. For simplicity, we will consider that children are either perfectly altruist ($\beta = 1$) or non-altruist ($\beta = 0$). Perfect altruism means that there is no divergence of interests between the state and the child on the level of care that must be offered to his/her parent. The utility levels (U) of altruistic and non-altruistic children are respectively:

$$U = u(C^c) + h(H)$$

$$U = u(C^c)$$

All the children have the same labor income w.

For closed form solutions, we will consider a logarithmic specification for the functions $v(.)$, $u(.)$ and $h(.)$.

The total welfare W is the sum of all utilities *excluding the altruistic component of the children's utility function* to avoid double counting.

$$W = \sum_{i=1}^{4} n_i (u(C_i^c) + v(C_i^p) + \pi h(H_i)))$$

Rich and poor parents are endowed with an initial wealth level of I^H and I^L; children have a labor income w. So that, the total resources of the economy are $(n_1 + n_2)I^H + (n_3 + n_4)I^L + w$.

A benevolent government maximizes the total welfare W. A major problem for the government comes from information asymmetries between the government and the population. In this paper, we consider that the government cannot observe the individual characteristics of the population. In particular, we consider that the government does not observe the wealth of the parents and the altruism of the children.[5] This means that the policy cannot be contingent on the wealth of the parents (they can always pretend that they are poor) nor on the degree of altruism of the children (they can always pretend to be non-altruist). These information asymmetries seriously constraint the intervention in LTC financing by the government.

3 Provision of long-term care

In case of dependency, the parents can benefit from *institutionalized* (or public) and/or *non institutionalized* (or private) assistance. This assistance consists in either in-house care (food, nursing assistance,...) or in a nursing home. We distinguish tree source of LTC care financing: *the market*, *the family* and *the state*. Market financing of LTC consists of private insurance subscribed by the parents before dependency occurs. Dependent parents receive a payment from their insurance company to finance their LTC needs. Family financing consists in financial transfers from children to their parents. Resources received from the family or the insurance company can be spent in LTC. The state intervenes directly and indirectly in the provision of LTC. It offers publicly financed cares for the persons in need. We will consider that this direct intervention consists of public nursing homes. In addition, it (may) subsidizes the provision of LTC by the family. For example, the state may offers financial help to the helping children.[6] We

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[5]There is potentially a third source of information asymmetry between the government and the population if the dependency status is not perfectly observable. We left aside this and consider that the parents cannot cheat on their dependency status. See Kuhn and Nuscheler (2007) for an analysis of this case.

[6]OECD (2005) lists the solution adopted by a sample of member countries to finance non-profesionnal LTC.

do not consider the possibility for the state to subsidy private insurances.[7] In the remaining, we will consider that these three forms of care are mutually exclusive.

3.1 The market: private insurance

A private dependency insurance is available on the market. The insurance taker must decide on the premium paid and the corresponding payment in case of dependency. If the insurance is actuary fair, the premium is equal to the expected insurance payment, that is for a premium of πa, the policy holder receives a payment of a in case he/she is dependent. The insurance is not actuary fair if for a repayment of a, the premium exceeds πa. We will assume that the market for LTC insurances is competitive. Hence LTC insurances are offered for a fair premium.

The insurance must be taken before dependency occurs. A parent, endowed with wealth level I, that decides to buy an insurance chooses the amount of insurance a in order to maximize his/her expected utility:

$$\max_a v(I - \pi a) + \pi h(a)$$

With the ln formulation, the solution to this problem is:

$$a^* = \frac{I}{(1 + \pi)}$$

A parent buys insurance if its expected utility with an amount of insurance a^* exceeds his/her expected utility with another type of LTC, provided by either the state or the family. We will show that renouncing to other forms of LTC is the opportunity cost of the insurance. This opportunity cost implies that, even at a fair price, there is a positive load factor for the LTC insurance i.e. the premium exceeds πa once opportunity costs are taken into account.

3.2 The family: informal care provided by altruistic children

Parents of altruistic children may rely on their help if they need LTC. Those who anticipate family help will not subscribe to an insurance. If dependency occurs, the child will decide on the amount of help he/she provides to his/her parent. A child endowed with resources y will devote a part s of his/her available income to help his/her parent.[8] For a child with an altruism parameter $\beta = 1$, the optimal amount of help is found by solving:

$$\max_s u(y - s) + h(s)$$

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[7]Pestieau and Sato (2007) introduce this possibility but in their model, the LTC insurances are not actuary fair.

[8]Pestieau and Sato (2007) consider that the children can devote part of their time or part of their income to provide cares to their parents.

Taking the ln formulation, s^* is equal to:

$$s^* = \frac{y}{2}$$

3.3 The state: public nursing homes

The government finances public nursing homes. Parents that decide to go to the public nursing home do not receive help from insurance companies and their family. We consider that the production technology for nursing homes is imperfect: for an investment of g, the corresponding quality of LTC receives in a public home is γg with $\gamma \leq 1$.

4 First best

In the first best, the government decides on the consumption levels of the children (C_i^c), of the parents (C_i^p) and on the level of LTC (H_i). The government faces the following budget constraint: $\sum_{i=1}^{4}(C_i^c + C_i^P + H_i) = (n_1 + n_2)I^H + (n_3 + n_4)I^L + w$. Welfare maximizing consumption and care levels are: $\forall i$,

$$u'(C_i^c) = v'(C_i^P) = h'(H_i)$$

In the first best situation, the government equates the marginal utility of consumption and of help for all individuals. The consumption and help levels are determined by the budget constraint.

4.1 Decentralization of the first best

Suppose that the government can make lump-sum transfers between all individuals. In this case, if the private insurance is fair, the first best can be decentralized with a generalized market financing of LTC. With fair insurance, we have $C^P = H$. More precisely, with an available income of y, we have $C^P = H = \frac{y}{1+\pi}$. Hence, to decentralize the first best, the available income of the parents must be π percent higher than the available income of the children. So that, the parents' consumption after buying the insurance is equal to those of the children. Hence, the government can decentralize the first best with appropriate lump-sump transfers.

As an alternative to a generalized market financing, the first best can also be decentralized with a mixed family/market mechanism. If altruistic children are appropriately compensated, the first best can be implemented. A necessary condition for that is perfect altruism ($\beta = 1$). Finally note that if providing public nursing home places is frictionless ($\gamma = 1$), the first best can be also be decentralized by that mean.

To summarize, the first best can be decentralized if (1) the government can make any kind of lump-sump transfers between individuals and (2) either the private insurance is fair or public home provision is efficient ($\gamma = 1$). If one of these conditions does not hold, the first best cannot be implemented.

4.2 Information constraint

In this paper, we assume that the government cannot observe the individual characteristics of the parents (their wealth level) and of the children (their altruism). These information constraints limit the possible actions of the government. Rich parents can always claim they are poor. The transfers needed to decentralize the first best are then unfeasible. Moreover, altruism being unobservable, altruistic children will help their parents only if they have an interest to do so.

5 Second best

5.1 Government intervention in LTC financing

Depending on the solution chosen for LTC financing, the amount of help received by a dependent parents is a^*, s^* or γg. The government intervenes in the financing of LTC but its action is constrained by the unobservability of the individuals' characteristic. In this paper, we consider two different interventions by the government: (1) a direct financing of public nursing home, (2) an intervention in the provision of informal LTC by the family. For that, the government pays a subsidy σ to the children that help their parents. Remember that even if altruism is not observable, the parent's dependency and the provision of informal help can be observed.

To finance these policies, the government imposes a flat tax t on labor income. This means that only the children contribute to the financing of the governmental intervention. The total resources available for LTC financing are thus tw. The government must keep the budget balanced and in the remaining, we denote by μ the Lagrange multiplier of the resource constraint.

To make thing simpler, we make the assumption that poor parents do not have access to private insurances. This assumption is not very demanding since, as we will see, rich parents may not subscribe to a LTC insurance.

5.2 Organizing LTC financing

There are many possible ways to finance and organize LTC in this economy. Parents of group 1 have two options to finance their LTC needs: They can go to the public nursing home or

they can buy a LTC insurance. Parents of group 2 have a third possibility: they can benefit from child support. Parents of group 3 can either benefit from child support or go to the public nursing home. Finally, parents of group 4 have no other option than going to the public home. This means that there are 12 different ways of financing LTC.

Let us denote by m^M the number of parents that buy a LTC insurance, by m^F, the number of parents that receive help from their family and by m^S the number of parents that go to the public nursing home, with $m^M + m^F + m^S = 1$, $m^F \leq n_2 + n_3$ and $m^M \leq n_1 + n_2$. The government must pay a subsidy σ to m^F children and finance public nursing homes for m^S parents. The budget constraint of the government writes as follow:

$$m^F \pi \sigma + m^S \pi g \leq wt$$

The objective of the government is to maximize the welfare W defined as follow:

$$
\begin{aligned}
W \;=\;& (n_1 + n_2)v(I^H) + (n_3 + n_4)v(I^L) \\
+\;& m^M[u(w(1-t)) + v(\frac{I^h}{1+\pi}) + \pi h(\frac{I^h}{1+\pi}) - v(I^H)] \\
+\;& m^F[(1-\pi)u(w(1-t)) + \pi u(\frac{w(1-t)+\sigma}{2}) + \pi h(\frac{w(1-t)+\sigma}{2})] \\
+\;& m^S[u(w(1-t)) + \pi h(\gamma g)]
\end{aligned}
$$

The first term is the utility parents derived from consuming their wealth endowment. The second term is the utility of the parents that buy a LTC insurance and the utility of their children. Notice that parents that are insured do not consume I^H but $\frac{I^h}{1+\pi}$ i.e. their wealth endowment minus the insurance premium. The third term is the utility of helping children and their parents and the last term is the utility of the parents that go to the public home and their children.

The problem that the government faces is the following: it must decide on which form of LTC financing for each group of parents (among the 12 available) and it must decides on the tax level t, on the subsidy level σ and on the amount of financing for the public homes g. The government faces two types of constraint. First, the budget must be balanced. Second, the individuals must prefer the proposed solution to any other available solution. These second set of constraints emerges from the fact that the government cannot observe the individual characteristics (wealth and altruism) and therefore, the proposed LTC financing must be incentive compatible.

5.3   The unconstrained problem

Let us ignore for a while (until next subsection) the incentive constraints. We split the government problem into two sub-problems. First, we search for the 12 possible organizations of LTC,

the optimal values of the t, σ and g. Second, we compare the welfare to determine the optimal LTC organization for the economy.

To determine the optimal values of t, σ and g, we maximize the welfare W subject to the government budget constraint. This constraint binds at the optimum. Hence, the Lagrangian of the problem can be expressed as:

$$L = W + \mu[m^F \pi \sigma + m^S \pi g - wt]$$

LEMMA 5.1 *For any m^M, m^F, m^S, the solution of the unconstrained problem is such that:* $w(1 - t^*) = \sigma^* = g^*$

PROOF: The first-order conditions of the maximization problem read as follow:

$$\frac{\partial L}{\partial t} = (1 - \pi m^F)\frac{1}{w(1-t)} + \pi m^F \frac{2}{w(1-t)+\sigma} + \mu = 0 \qquad (5.1)$$

$$\frac{\partial L}{\partial \sigma} = \frac{2}{w(1-t)+\sigma} + \mu = 0 \qquad (5.2)$$

$$\frac{\partial L}{\partial g} = \frac{1}{g} + \mu = 0 \qquad (5.3)$$

Solving, we have: $w(1 - t^*) = -\frac{1}{\mu} = \sigma^* = g^*$. ∎

Lemma 5.1 has two implications. First, altruistic children are perfectly compensated for the help they give to their parents and so their consumption is not altered when they help their parents. Second, the government spends the same amount for a dependent parent in a nursing home than for a dependent parent who receives care from his/her family. There are two differences with the first best. First, the marginal utility of care is not equalized for all the dependent parents due to the imperfect nursing home technology. Second, the parents' marginal utility of consumption is no longer equal because their wealth endowment is non observable. Hence, lemma 5.1 implies that whenever $\gamma < 1$, the parents of an altruistic child are better-off if they receive familial support than if they go to a public nursing home. Together, these imply that parents of groups 2 and 3 have a higher utility when they receive help from their family than when they go to the public home. Then, the number of possible organization of LTC reduces to four. In table 1, we note the type of help received by each group of parents in the four possible solutions.

Now, we move to the first part of the problem to see which of these four solution gives the highest welfare. To obtain the optimal tax and subsidy levels and the optimal spending in nursing home, we must solve the last first order condition (the derivative of L with respect to

	Solution 1			Solution 2		
Group #	Market	Family	State	Market	Family	State
1	x			x		
2	x				x	
3		x			x	
4			x			x

	Solution 3			Solution 4		
Group #	Market	Family	State	Market	Family	State
1			x			x
2	x				x	
3		x			x	
4			x			x

Table 1: The 4 possible organization of LTC

μ) which, after integrating the results of lemma 5.1 gives us:

$$\sigma^* = g^* = \frac{w}{1 + \pi(1 - m^M)}, \quad t^* = \frac{\pi(1 - m^M)}{1 + \pi(1 - m^M)} \tag{5.4}$$

The optimal values of t, σ and g depend on the number of elderly dependent that receive a financial assistance from the state, either directly through admission in a public home or indirectly through the compensation paid to their helping child. And, the highest the number of parents in the state financed system, the lowest is the public contribution per individual and the highest the tax rate. In other words, the largest $m^F + m^S$, the highest the tax rate and the lowest σ and g. Let us denote by t_i^*, σ_i^* and g_i^* the optimal values of t, σ and g in solution $i = 1, ..., 4$, we have: (i) if $n_1 \geq n_2$, $t_4^* > t_3^* > t_2^* \geq t_1^*$ and $\sigma_4^* = g_4^* < \sigma_3^* = g_3^* < \sigma_2^* = g_2^* < \sigma_1^* = g_1^*$ and (ii) if $n_2 > n_1$, the ordering between solution 2 and 3 is inverted.

For each solution i, let us denote by $q_i = 1 + \pi(1 - m^M)$. To keep the problem simpler, we will assume that $n_1 = n_2$. Denote by W_i the welfare level when solution i is applied. The comparison of the welfare levels gives the following:

LEMMA 5.2 *Define* $Z^{12} = \frac{1}{n_2\pi}(q_1 \ln q_1 - q_2 \ln q_2) + \frac{1+\pi}{\pi} \ln(1+\pi)$ *and* $Z^{24} = \frac{1}{n_1\pi}(q_2 \ln q_2 - (1+\pi) \ln(1+\pi)) + \frac{1+\pi}{\pi} \ln(1+\pi)$. *We have:*

1. $Z^{12} > Z^{24}$.

2. $W_1 \geq Max[W_2, W_3, W_4]$ *if* $\ln \frac{I^H}{w} \geq Z^{12}$.

3. $W_2 \geq Max[W_1, W_3, W_4]$ if $\ln \frac{I^H}{w} \in [Z^{24} + n_1 \ln \gamma, Z^{12}]$.

4. $W_4 \geq Max[W_1, W_2, W_3]$ if $\ln \frac{I^H}{w} \leq Z^{24} + n_1 \ln \gamma$.

PROOF: See Appendix. ∎

Lemma 5.2 reads as follow: if the wealth endowment of the rich parents is high enough compared to the labor income of the child, the welfare is maximized when the rich parents are left out of the state-financed LTC system and rely on private insurance schemes to finance their LTC needs. Leaving aside the rich parents form the publicly supported cares has two advantages: the tax rate is lower, which is beneficial to all the children, and the per-capita contribution of the state to dependent parents is higher, which obviously benefits to all the parents that received state-financed LTC.

When the wealth endowment of the rich parents declines relative to the labor income, the highest welfare is achieved in a generalized state-financed LTC system.[9] But this switch from private insurance to state financed LTC is organized in two steps. Because the public provision of nursing homes involves resource losses, there are intermediate values of $\frac{I^H}{w}$ for which rich parents behaves differently depending if their child is altruistic or not. For these intermediate values, the rich parents of non-altruistic child will continue to finance their LTC needs with private insurance while the rich parents of altruistic child will be helped by their family in case of dependency. It is only when the ratio $\frac{I^H}{w}$ declines further that all the parents will depend on state-financed LTC and that the private insurance will no longer be bought (even at an actuary fair price).[10]

Call $\tilde{Z}^k = e^{Z^k}$, lemma 5.2 can be restates as follow: solution 1 dominates for $\frac{I^H}{W} \geq \gamma \tilde{Z}^{12}$ and solution 2 dominates for $\frac{I^H}{W} \in [\gamma^{n_1} \tilde{Z}^{24}, \gamma \tilde{Z}^{12}]$. These conditions are represented on figure 1.

5.4 Incentive constraints

We now introduce the incentive constraints in the above problem. The government cannot observe the wealth of the parents nor the altruism of the children. Rich parents can then pretend that they are poor and altruistic children can pretend that they are not. Hence, the LTC financing must be such that each group of parents/children chooses the proposed solution rather than another possible way to finance LTC.

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[9]This solution is the one adopted in Germany where all the elderly dependent can benefit from the universal dependency insurance financed by labor taxes.

[10]For the state, instead of solution 2, it would be optimal to smoothly increase the number of parents in the state financed system. However, this turns out to be unfeasible because all the parents in group 2 will adopt the same behavior.
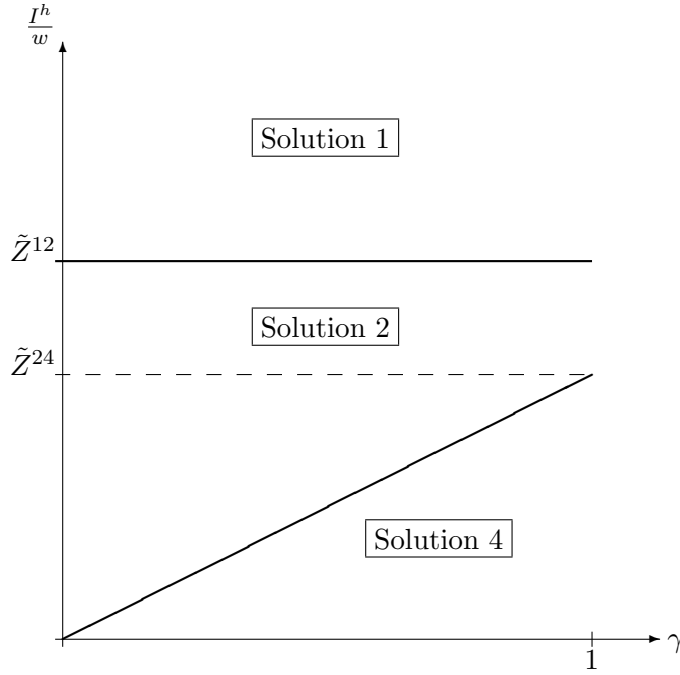
Figure 1: LTC financing in the unconstrained problem

Two sets of incentive constraints must be considered. First, if the organization of LTC prescribes that rich parents (or some of them) buy an insurance they must agree to do so. Rich parents may have incentives to mimic the behavior of the poor ones. By doing so, they save on private insurance and therefore enjoy a higher consumption and, in case of dependency, they do receive assistance from the state or from their family. Therefore, whenever the optimal organization of LTC calls for market mechanism for the rich, the planer must ensure that the rich parents indeed prefer the market solution to any other available one. This means that their utility with the LTC insurance must be higher than the other options they have for LTC financing: family support and public homes for parents of group 2 and public nursing home only for the parents of group 1. The corresponding incentive constraints write as follow:

$$v(\frac{I^H}{1+\pi}) + \pi h(\frac{I^H}{1+\pi}) \geq v(I^H) + \pi h(\gamma g) \qquad (IC1)$$

$$v(\frac{I^H}{1+\pi}) + \pi h(\frac{I^H}{1+\pi}) \geq v(I^H) + \pi h(\frac{w(1-t)+\sigma}{2}) \qquad (IC2)$$

Second, if the proposed LTC financing is such that altruistic children (or some of them) should help their dependent parents, they must agree to do so rather than mimicking the behavior

of non-altruistic children. The corresponding incentive constraint writes as follow:

$$U(\frac{w(1-t)+\sigma}{2}) + h(\frac{w(1-t)+\sigma}{2}) \geq U(w(1-t)) + h(\gamma g) \qquad (IC3)$$

We first check if and when the unconstrained solution, described in lemma 5.2, satisfies the corresponding incentive constraints.

Consider first solution 1. This solution is incentive compatible if for $t = t_1^*, \sigma = \sigma_1^*$ and $g = g_1^*$, the constraints (IC1), (IC2) and (IC3) are satisfied. By lemma 5.1, we know that (IC3) is satisfied for sure.

Let $\bar{Z} = \frac{1+\pi}{\pi}\ln(1+\pi) - \ln q_1$, the unconstrained solution is not incentive compatible when $\ln\frac{I^H}{w} < \bar{Z}$. More precisely, the constraint (IC2) is not satisfied for $\ln\frac{I^H}{w} < \bar{Z}$. Moreover, (IC1) is neither satisfied for $\ln\frac{I^H}{w} < \bar{Z} + \ln\gamma$. We can show that solution 1 is not always incentive compatible in the parameter space where it gives the highest welfare in the unconstrained problem. That is:

LEMMA 5.3 *There exists a parameter space where solution 1 is optimal but not incentive compatible: $\bar{Z} > Z^{12}$.*

PROOF: The inequality $\bar{Z} > Z^{12}$ can be simplified to $1 > -\frac{q_1}{n_2\pi}$. ∎

So for $\ln\frac{I^H}{w} \in [Z^{12}, \bar{Z}]$, the highest welfare would be achieved if rich parents are insured but some or all of them prefer to receive LTC from their family or from the state. Clearly, the budget will not be balanced if this happens.

Consider next solution 2. This solution is incentive compatible of for $t = t_2^*, \sigma = \sigma_2^*$ and $g = g_2^*$, the constraints (IC1) and (IC3) hold. Moreover, inequality (IC2) should be reversed. Define $\bar{\bar{Z}} = \frac{1+\pi}{\pi}\ln(1+\pi) - \ln q_2$, solution 2 does not satisfy (IC_2) if $\ln\frac{I^H}{W} < \bar{\bar{Z}} + \ln\gamma$. We can show that solution 2, when it dominates the other possible solutions, is not always incentive compatible. That is:

LEMMA 5.4 *There exists a parameter space where solution 2 is optimal but not incentive compatible: $\bar{Z} > \bar{\bar{Z}} > Z^{12}$.*

PROOF: The inequalities $\bar{\bar{Z}} > Z^{12}$ and $\bar{Z} > \bar{\bar{Z}}$ can be simplified to $n_2 > 0$. ∎

Finally solution 4 is always incentive compatible. In figure 2, we represent the parameter space where the unconstrained solution does not satisfy the corresponding incentive constraints. To construct the figure, let $\tilde{Z} = e^{\bar{Z}}$ and $\tilde{\tilde{Z}} = e^{\bar{\bar{Z}}}$.

Information asymmetry has for consequence that rich parents do not always subscribe to a private LTC insurance when it is optimal (for the society) that they do so. Pauly (1990)
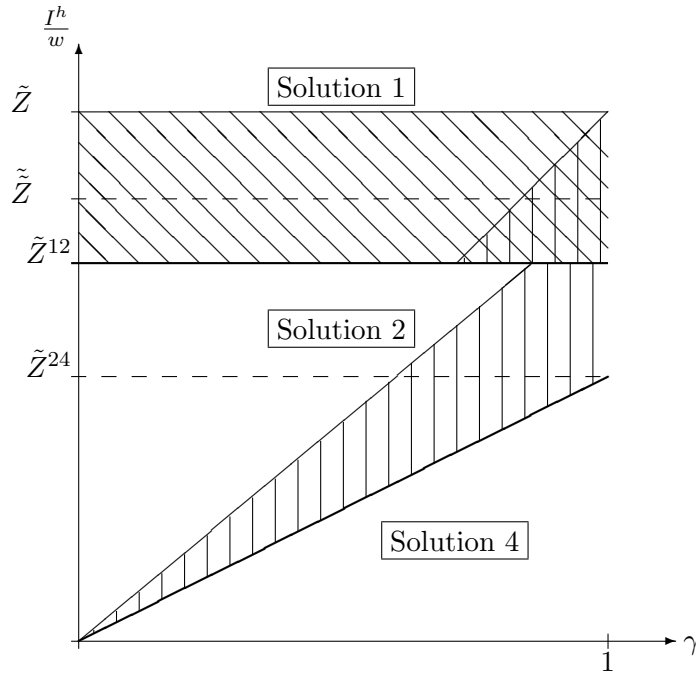
Figure 2: Incentive constraints in the unconstrained problem

explained this rational non-purchase of private LTC insurance and his explanation fits our model very well. For a rich parent, buying an insurance means that consumption if he/she remains in good health decreases while consumption in case of dependency does not necessarily increase. LTC support is higher only if the insurance repayment is higher than any other form of care available. But even if the rich parents receive more cares when they are insured, they do not necessarily buy an insurance because they trade-off the *additional* care benefit with the insurance premium.

Because the three forms of care are mutually exclusive, a parent that subscribe to an insurance renounces to the other forms of care. Hence, even if the insurance is offered at a fair premium, the cost of the insurance could be quite high once opportunity costs are incorporated. For parents of group 1, the opportunity cost of a LTC insurance is the level of LTC they can receive in a public nursing home. For parents of group 2, it is the LTC received from their child. Once opportunity costs are included, cost of insurance increases dramatically and this discourages insurance subscription.

Even if the insurance company does not charge a load factor and offers the insurance at a fair price, there is an implicit load factor because parents renounce to other forms of help. For a repayment of a, the parents pay πa and renounces to either γg^* (group 1) or s^* (group 2). So

the total cost of an insurance is $\pi a + \gamma g^*$ or $\pi a + s^*$. Differently, we can define a implicit load factor, $\tilde{\theta}_i$, for the parents of group $i = 1, 2$ equals to:

$$\tilde{\theta}_1 = 1 + \frac{\gamma g^*}{\pi a}, \tag{5.5}$$

$$\tilde{\theta}_2 = 1 + \frac{s^*}{\pi a} \tag{5.6}$$

This modified load factor is the additional cost per unit of insurance paid by the parents. As it is clear from these formulations, the higher the help received by the parents either from the state or from their child, the higher this implicit load factor. And obviously, a high load factor discourage insurance taking by the parents.

High insurance costs implies that rich parents buy it only if they could expect a much higher quality of care if they are insured. This is the case if I^H is high compared to g^* and/or s^*. In our solution 1 we have $\tilde{Z} > 1$. This means that if $\frac{I^H}{w} = 1$, the rich parents do not subscribe to a LTC insurance. It is only when the rich parents have a wealth level sufficiently higher than the children that they buy an insurance. This can be seen from expressions (5.5) and (5.6): When labor income increases, the implicit load factor of insurance increases. When the parent's wealth increase, they buy more insurance (if they buy an insurance), and the load factor is inversely proportional to the insurance level.

The other source of information asymmetry does not create problem. Because altruistic children are perfectly compensated for the help they give to their parents, they have incentives to do so. In Jousten *et al.* (2005), because of distortionary taxation, altruistic children are worse-off than non altruistic ones. Hence altruistic children have incentives to behaves like non-altruistic ones.

Note that, even if the incentive constraint (IC3) is never binding in the above problem, it does not mean that this constraint is irrelevant in the design of a LTC financing scheme. We will see that this constraint must be taken into account in the problem. More in particular, when the government distorts the LTC financing to constraint the rich parents (or some of them) to subscribe to a private insurance, it must check that altruistic children continues to have the right incentives to help their parents.

As shown on figure 2, the unconstrained solution can not be implemented for $\frac{I^H}{w} \in [\tilde{Z}^{12}, \tilde{Z}]$ $\cup [\tilde{Z}^{24}, Min[\tilde{Z}^{12}, \tilde{\tilde{Z}} + \ln \gamma]]$. In the parameter space where the unconstrained solution is not incentive compatible, the government has two options: it can either change t, σ and γ in order to make the proposed LTC financing system incentive compatible or it can switch to another LTC financing solution. We examine in turns these two alternatives.

5.5 The constrained problem

Suppose that the government wants to have all the rich parents insured (solution 1). For that, they must be prevented from relying on the help of their child (constraint (IC2)) and from applying to public homes (constraint (IC1)). The only way to do so is to reduce the state-financing of LTC, that is reducing the help to the altruistic children σ and the quality of public home g. But by doing so, the state must take into account that reducing σ may have an impact on the behavior of the altruistic children of poor parents. If they receive a lower compensation for helping their parents they may be tempted to mimic non-altruistic children. So we must maximize the welfare W_1 subject to the budget constraint, the incentive constraints (IC1), (IC2) and (IC3) and check which constraint is binding.

For $\frac{I^H}{w} \in [Z^{12}, \bar{Z}]$, the constraint (IC2) is binding. It results that altruistic children are less compensated for their financial support to their parents. But this has an impact on the other two incentive constraints because, with reduced child support, the government makes a surplus and this surplus is redistributed to public home financing and tax decreases. But this exacerbates incentive problems by making the nursing home care more attractive for both altruistic children and rich parents. So for high values of γ, the public support to nursing homes must be decreased too. These policy changes hurts the poor parents who receive less support from their child and lower quality nursing home.

Suppose that the government wants to have only the rich parents of non-altruistic children to be insured (solution 2). To prevent rich parents to apply to the public homes, the government lowers their quality. The complete solution to these problems is described in greater details in appendix B.

5.6 Optimal policy

We conclude our analysis by establishing the optimal LTC financing when incentive issues are taken into account. We establish that:

PROPOSITION 5.1 *There exists $\Delta_{12}(\gamma)$ and $\Delta_{24}(\gamma)$ such that,*

1. $\Delta_{12}(\gamma) \in [Z^{12}, \bar{Z}]$.

2. $\Delta_{24}(\gamma) \in [Z^{24} + n_1, \bar{\bar{Z}} + \ln \gamma]$.

3. *For $\ln \frac{I^h}{w} \geq \Delta_{12}(\gamma)$, solution 1 is optimal.*

4. *For $\ln \frac{I^h}{w} \in [\Delta_{24}(\gamma), \Delta_{12}(\gamma)]$, solution 2 is optimal.*

5. *For* $\ln \frac{I^h}{w} \le \Delta_{24}(\gamma)$, *solution 4 is optimal.*

PROOF: See appendix C.

Proposition 5.1 reads as follow: Under asymmetric information, the government continues to use the same policy mix than in the unconstrained problem. This means that only solutions 1,2 and 4 are considered as optimal solutions.

In those parameters space where the unconstrained solution is not incentive compatible the government has two options: either it keeps the same solution but it distorts the instrument to satisfy the incentive constraints or it switches to another policy. The costs of these two alternatives must be compared.

Distortions and the associated welfare losses are particularly important for high values of γ and low values of $\frac{I^H}{w}$. Consequently, in these situations, the government will switch to another policy mix.

Finally notice that incentive issues reduce the use of market mechanism to finance LTC needs. For incentive reasons, the parameter spaces where rich parents are insured are reduced. Rich parents switches from market financing to family financing or nursing homes for incentive purpose. Wealth unobservability therefore reduces the use of private insurances, even if they are offered for a fair price.

6 Conclusions

In this paper, we studied the LTC financing in a society composed of heterogeneous pairs of parent-child. In particular, we considered rich and poor parents with altruistic and non-altruistic children. We assumed that these individual characteristics cannot be observed and we studied the optimal LTC financing scheme in this context.

There are three potential care financers: the elderly himself who can subscribe to a LTC insurance to finance his LTC needs; the family, altruistic children may devote part of their income to finance the LTC needs of their parents and the government who intervenes directly to provide nursing homes spaces for those uninsured parents who do not receive help from their family, and indirectly to support altruistic children.

The main problem we identified is the rational non-purchase of private insurance by the rich parents (Pauly, 1990). We showed how non-purchase of LTC insurance might be rational because potential family care and/or public subsidies for poor create opportunity cost that may cause person to reject even actuarially fair LTC insurance. Hence, private insurances have a

high opportunity cost and consequently, private LTC insurances are crowded-out by family and government financed LTC. This leads to inefficient provision of care

Because of unobservable income, the government cannot constraint the parents to buy an insurance. Neither can it exclude them from family or government financed care. Hence, the government remains with two options: either, it diminishes the opportunity cost of private insurances by making the other forms of care less attractive or it renounces to market financed LTC. Both solutions have a negative impact on the LTC received by the poor parents. Rich parents who have multiple options for LTC financing, including the private insurance option, exert a negative externality on those who have less options.

In this model, we considered selfish parents that only care about their consumption and LTC levels. Altruism of parents toward children may be a reason to buy LTC insurance or to save to finance LTC. Parents that do not wish to be burden to children may self-finance their LTC needs. However, generous public support for altruistic children may still crowd out private insurances even if parents are altruistic though the effect could be of smaller amplitude.

Means-testing could, at least partially, overcome the information problem between the government and the parents. By making financial support conditional on resources, the government reduces the number of available options for rich parents and thereby reduces the opportunity cost of insurances. Means-testing is extensively used in the US and in the UK. In the US, participation in the *Medicaid* program is conditioned on resources. In the UK, all the individuals must contribute to their LTC financing in proportion of their own resources. In both cases, assets are included in the total resources. However, despite means-testing, the crowding-out of private insurances remains important (Brown and Finkelstein, 2007). By contrast, in Germany, with the introduction of an almost universal public insurance, public financing of LTC is not conditioned on income. With this solution, individually financed cares only complement the state financing and an individual who buys additional LTC provision does not renounce to his/her rights to publicly financed cares. The drawback is that the government finances (at least part of) the LTC needs of people that are wealthy enough to finance these needs with their own resources. Consequently, for a given budget (or tax rate), the individual's claim to LTC financing is smaller. And, as we have shown, the individual's benefit is smaller also for a higher tax rate.

In this paper, we considered that the three forms of care are mutually exclusive. Few evidences are available on the mix between the three forms of care both at the individual and the aggregate levels. Considering mixed LTC financing of LTC does not eliminate the high opportunity cost of insurances. If the government is the payer of last resort, like in the Medicaid

program, private insurances continue to have a high opportunity cost and crowding-out of private insurance remains important (Brown and Finkelstein, 2007) . If the private insurances finance additional cares, parents only complement state financed cares with private insurances. In such a system, most of the dependent parents receive state financed cares. Hence, the government must then finance an almost universal service for the dependent parents and the financial burden of such a policy can be considerable.

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A   Proof of proposition 5.2

Under the hypothesis that $n_1 = n_2$, we have $W_2 - W_3 = -n_1 \ln \gamma \geq 0$ and solution 3 is always weakly dominated by solution 2.

Solution 1 dominates if $W_1 \geq W_2$ and $W_1 \geq W_4$. Rewriting these two conditions, we have:

$$W_1 - W_2 \geq 0 \quad \Leftrightarrow \quad \ln \frac{I^H}{w} \geq Z^{12} \tag{A.7}$$

$$W_1 - W_4 \geq 0 \quad \Leftrightarrow \quad \ln \frac{I^H}{w} \geq Z^{14} + n_1 \ln \gamma, \tag{A.8}$$

where $Z^{14} = \frac{1}{(n_1+n_2)\pi}(q_1 \ln q_1 - (1 - n_1 - n_2)(1 + \pi) \ln(1 + \pi))$. We can show that $Z^{12} > Z^{14}$. This inequality is true if

$$n_1 q_1 \ln q_1 + n_2(1 + \pi) \ln(1 + \pi) > (n_1 + n_2)q_2 \ln q_2 \tag{A.9}$$

Taking $n_1 = n_2$ and defining $f(x) = (1 + \pi(1 - x)) \ln(1 + \pi(1 - x))$, (A.9) is equivalent to:

$$f(2n_1) + f(0) > 2f(n_1)$$

Since $f(x)$ is a convex function, this inequality holds true and $Z^{12} > Z^{14}$.

Solution 2 dominates if $\ln \frac{I^H}{w} \leq Z^{12}$ and $W_2 \geq W_4$. This condition can be expressed as:

$$W_2 - W_4 \geq 0 \Leftrightarrow \ln \frac{I^H}{w} \geq Z^{24} + n_1 \ln \gamma, \tag{A.10}$$

where $Z^{24} = \frac{1}{n_1\pi}(q_2 \ln q_2 - (1+\pi) \ln(1+\pi)) + \frac{1+\pi}{\pi} \ln(1+\pi)$. Taking $n_1 = n_2$, $Z^{12} > Z^{24}$ if (A.9) holds.

B   Constrained solution

Constrained solution 1   The objective of the government is to maximize L with $m^M = n_1 + n_2$ and $m^F = n_3$ subject to the incentive compatible constraints. Denote by the λ_i, the Lagrange multiplier of constraint (ICi), $i = 1, 2, 3$.

The solution to this problem reads as follow:

$$\tilde{t}_1 = \frac{\lambda_3 + (1 - n_3\pi)\pi(1 - n_1 - n_2 - \lambda_1 - \lambda_2)}{(1 - n_3\pi)(1 + \pi(1 - n_1 - n_2 - \lambda_1 - \lambda_2))} \tag{B.11}$$

$$\tilde{\sigma}_1 = \frac{w(\lambda_3(2 - n_3\pi) - \pi(\lambda_2 - n_3)(1 - n_3\pi))}{n_3\pi(1 - n_3\pi)(1 + \pi(1 - n_1 - n_2 - \lambda_1 - \lambda_2))} \tag{B.12}$$

$$\tilde{g}_1 = \frac{w(\pi(n_4 - \lambda_1) - \lambda_3)}{n_4\pi(1 + \pi(1 - n_1 - n_2 - \lambda_1 - \lambda_2))} \tag{B.13}$$

We must now check which of the incentive constraint is binding.

Given lemma 5.3, for all $\ln \frac{I^H}{w} < \bar{Z}$, λ_2 is positive. This implies that, in order to prevent rich parents of an altruistic child to rely on family support, the government reduces its subsidy to caregiving children: $\frac{\partial \tilde{\sigma}_1}{\partial \lambda_2} < 0$. Reducing support to altruist children save resources for the government who reduces the tax rate $\frac{\partial \tilde{t}_1}{\partial \lambda_2} > 0$ and increases the nursing home care spending $\frac{\partial \tilde{g}_1}{\partial \lambda_2} > 0$. Note that these distortions are inversely related to the ratio $\frac{I^h}{w}$: $\frac{\partial \lambda_2}{\partial I^H / w} \leq 0$.

A positive λ_2 makes then parents in nursing homes better-off. Clearly this is only feasible if parents of group 1 prefer to be insured and if children of group 3 continue to provide support to their parents. With $\lambda_2 > 0$ the incentive constraints (IC1) and (IC3) are satisfied if:

$$\ln \frac{I^H}{w} + (1+\pi)\ln(1+\pi) \quad - \quad \ln(1 + \pi(1 - n_1 - n_2 - \lambda_2)) \geq \ln \gamma \tag{B.14}$$

$$\left(\frac{2n_3 - \lambda_2}{2n_3}\right)^2 \quad \geq \quad \gamma \tag{B.15}$$

Because λ_2 is a function of $\frac{I^H}{w}$, these two conditions define the parameter spaces where the incentive constraints (IC1) and (IC3) are slack in the constrained problem. When one of these constraints is binding, the corresponding Lagrange multiplier must be positive. When it is the case, the nursing home quality is reduced.

Constrained solution 2 In solution 2, constraint (IC2) is irrelevant and only (IC1) must be considered. The constraint is binding for $\ln \frac{I^h}{w} < \bar{\bar{Z}} + \ln \gamma$ and the constrained solution reads as follow:

$$\tilde{t}_2^* = \frac{\pi(1 - n_1 - \lambda_1)}{1 + \pi(1 - n_1 - \lambda_1)}, \tilde{\sigma}_2^* = \frac{w}{1 + \pi(1 - n_1 - \lambda_1)}, \tilde{g}_2^* = \frac{(n_4 - \lambda_1)w}{n_4(1 + \pi(1 - n_1 - \lambda_1))}.$$

The quality of nursing home is reduced to prevent rich parents of group 1 to apply. This relaxes the resource constraint and the government increases its subsidy to helping children and reduces the tax rate.

C   Proof of proposition 5.1

By lemma 5.1, we need only to consider solutions 1, 2 and 4 among the 12 available.

- $\ln \frac{I^H}{w} = Z^{12}$ is the locus of $\frac{I^H}{w}$ and γ such that the welfare with the unconstrained solution 1 (W_1) is equal to the welfare with the unconstrained solution 2 (W_1) (lemma 5.2).

- $\tilde{W}_1 \leq W_1$ for all parameters such that $\ln \frac{I^H}{w} \leq \bar{Z}$ and $\tilde{W}_1 = W_1 > W_2$ for $\ln \frac{I^H}{w} = \bar{Z}$ (lemma 5.3).

- $\frac{\partial \tilde{W}_1}{\partial I^H/w} > 0$ since $\frac{\partial \tilde{W}_1}{\partial \lambda_1} < 0$, $\frac{\partial \tilde{W}_1}{\partial \lambda_2} < 0$, $\frac{\partial \tilde{W}_1}{\partial \lambda_3} < 0$ and $\frac{\partial \lambda_1}{\partial I^H/w} < 0$, $\frac{\partial \lambda_2}{\partial I^H/w} < 0$, $\frac{\partial \lambda_3}{\partial I^H/w} < 0$.

Combining these three facts, there exists a locus $\ln \frac{I^H}{w} = \Delta_{12}(\gamma)$ such that, on the locus $\tilde{W}_1 = W_2$ and $Z^{12} < \Delta_{12}(\gamma) < \bar{Z}$.

- $\ln \frac{I^H}{w} = Z^{24} + \ln \gamma$ is the locus of $\frac{I^H}{w}$ and γ such that the welfare with the unconstrained solution 2 (W_2) is equal to the welfare with the unconstrained solution 4 (W_4) (lemma 5.2).

- $\tilde{W}_2 \leq W_2$ for all parameters such that $\ln \frac{I^H}{w} \leq \bar{\bar{Z}} + \ln \gamma$ and $\tilde{W}_2 = W_2 > W_4$ for $\ln \frac{I^H}{w} = \bar{\bar{Z}} + \ln \gamma$ (lemma 5.4).

- $\frac{\partial \tilde{W}_2}{\partial \frac{I^H}{w}} > 0$.

Combining these three facts, there exists a locus $\ln \frac{I^H}{w} = \Delta_{24}(\gamma)$ such that, on the locus $\tilde{W}_2 = W_4$ and $Z^{24} + \ln \gamma < \Delta_{24}(\gamma) < \bar{\bar{Z}} + \ln \gamma$.