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Reallocation gains in a specific factors model with firm heterogeneity

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Working Paper No. 1115
October 2011

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ABSTRACT: We study firm heterogeneity in a specific factors model to address the effect of factor mobility on reallocation gains from trade. A model is proposed with Melitz type firm heterogeneity with two sectors, two countries and two fixed factors and one factor mobile across sectors. Equilibrium in each sector can be concisely represented by a demand and supply equation and a FE and ZCP condition. Varying the substitution elasticity between the fixed and mobile factor we show that the welfare gains from trade liberalization are larger in countries with a lower substitution elasticity. Furthermore, it is shown that the immobile production factor in the comparative disadvantage sector can still gain from trade liberalization due to the reallocation effect.

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printdate: October 28, 2011

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1 Introduction

Welfare gains from trade in models with heterogeneous firms are driven by a reallocation of resources from less productive to more productive firms additional to sectoral specialization (Melitz, 2003; Bernard, et al., 2003; Melitz and Ottaviano, 2008; Bernard, et al., 2007, among others). In this paper we address the impact of factor mobility across sectors on the reallocation gains from trade by incorporating Melitz type firm heterogeneity into the traditional specific factors model. In a one sector setting basic intuition suggests that higher factor mobility increases profitability of entry inducing larger reallocation effects. We show that in a multisector setting this basic intuition does not hold, as within-sector reallocations interact with between-sector reallocations.

We work with a specific factors model in a setting with two countries and two sectors and factor immobility across countries. There are three production factors: labor is mobile across sectors and the two types of capital are sector specific. We first start with a model using a Cobb-Douglas production function of input bundles in each sector. This implies that the elasticity of transformation between the two sectors is a function of the Cobb-Douglas parameter on the mobile production factor. Varying the Cobb-Douglas parameter to mimic for variation in factor mobility also picks up factor biased technical change. In this case the gains from liberalization become a function of the size of the different production factors.

We then generalize the specification to a CES production function of input bundles and vary the substitution elasticity between production factors. This exercise shows that the welfare gains from trade liberalization are smaller when the substitution elasticity is larger as a larger substitution elasticity implies less specialization across countries. Less labor is allocated to the comparative advantage sector, as there is less pressure to have equal factor proportions in production. With less specialization the gains from trade liberalization are smaller. We observe that intersectoral reallocation is the determining factor in this model, instead of intrasectoral reallocation as in one sector firm heterogeneity models. The policy implication is that in countries with more rigid factor markets, specialization is larger and the gains from trade liberalization are larger.

In the simulations we also show that the scarce production factor can still gain from trade liberalization under firm heterogeneity due to lower prices. This result is related to the result found by Bernard, et al. (2007) that in a Heckscher-Ohlin model the relatively scarce production factor might still gain from trade liberalization, a generalized Stolper Samuelson theorem. Similarly, our result implies that the - traditional - political economy implications of the specific factors model should be interpreted with caution, as it is very well possible that none of the production factors loses with trade liberalization. The real loss might be incurred not by production factors (and capital owners), but by firm owners, with sunk investments in specific varieties. Bombardini (2008) focuses on the gains and losses of firm owners with trade liberalization, showing that in more concentrated industries there is more lobbying for protection, as larger firms can coordinate their lobbying efforts more easily. Another result related to what Bernard, et al. (2007) find in the Heckscher-Ohlin setting is that the selection effect of trade liberalization is larger in comparative advantage sectors. Moving from autarky to costly trade the cutoff productivity goes up quicker and the exporting cutoff productivity is closer to the domestic cutoff in the comparative advantage sector.

A methodological contribution of the paper is to show how in multisectoral models equilibrium in each sector can be represented by four equations, a demand and supply equation and a free entry (FE) and zero cutoff profit (ZCP) condition.

The literature on the specific factors model starts with contributions by Viner (1931), Haberler (1950) (other contributions include Jones (1971) and Neary (1978)). On the interaction between the gains from trade and factor mobility, Artuc, et al. (2010) estimate in a structural model switching costs of workers between sectors. They show that from a lifetime perspective all factors gain from trade shocks, i.e. also the scarce factor, as the flows between sectors are large. Using a different model we arrive at the same finding. Balistreri, et al. (2009) show in a model with endogenous labor supply that a larger factor supply elasticity (of labor) leads to a bigger increase in the welfare gains from trade. Still, this model is not suitable to examine the effect of factor mobility on the (reallocation) welfare gains from trade,[1] because the larger welfare gains with a larger labor supply elasticity are driven to a large extent by the fact that welfare can simply increase more when consumption and leisure are more substitutable.

2 Model

We model a two sector, two country and two factors of production model with Melitz (2003) firm heterogeneity in both sectors. Upper nest utility and the production of input bundles are both CES.[2] With this general setup we nest two models. First, when both factors of production are fully mobile between sectors, we end up with a Heckscher-Ohlin model with firm heterogeneity like in Bernard, et al. (2007). The difference is that upper nest utility and the production of input bundles in our model are CES instead of Cobb-Douglas. Second, with labor being fully mobile and capital fully immobile, we get a specific factors model like in Jones (1971) and Neary (1978). In the special case that the production of input bundles in this model is Cobb-Douglas and the Cobb-Douglas parameters are equal in the two sectors, the elasticity of transformation between input bundles in the two sectors is a monotone (positive) function of the flexible factor (labor) Cobb-Douglas parameter. Before discussing this in detail, we show in general how equilibrium conditions in each sector can be expressed with four equations: a demand and supply curve and a free entry and zero cutoff profit condition.

2.1 Demand

There are two countries indexed by subscripts $i = H, F$. There are two composite goods, X_1 and X_2. All consumers have the following identical utility function

$$U_i = \left(\alpha_1 X_{i1}^{\frac{\rho-1}{\rho}} + \alpha_2 X_{i2}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{1}$$

[1] The authors do not claim this, but want to make a different point with their simulations, i.e. that in multisector simulations one cannot summarize the welfare gains from trade by single statistic measuring the trade openness of a country as Arkolakis, et al. (2008) claim in single sector models.

[2] The upper nest utility could be further generalized to allow for example for non-homothetic preferences.

X_{i1} and X_{i2} are both a function of a continuum of differentiated consumption goods $x_{im}(\omega)$:[3]

$$X_{im} = \left(\int\limits_{\omega \in \Omega_{im}} x_{im}(\omega)^{\frac{\sigma_m - 1}{\sigma_m}} d\omega \right)^{\frac{\sigma_m}{\sigma_m - 1}} \tag{2}$$

Ω_{im} is the set of available varieties in country i in sector m. Demand for a variety $x_{im}(\omega)$ is equal to:

$$x_{im}(\omega) = \frac{\alpha_m^{\rho} P_{im}^{\sigma_m - \rho} P_{U_i}^{\rho - 1} I_i}{p_{im}(\omega)^{\sigma}} \tag{3}$$

P_{im} is the price index corresponding to the aggregate X_m and defined as:

$$P_{im} = \left(\int\limits_{\omega \in \Omega_{im}} p_{im}(\omega)^{1 - \sigma_m} \right)^{\frac{1}{1 - \sigma_m}} \tag{4}$$

I_i is total income in country i and will be defined later. P_{U_i} is the price index corresponding to utility and defined as:

$$P_{U_i} = \left(\alpha_1^{\rho} P_{i1}^{1-\rho} + \alpha_2^{\rho} P_{i2}^{1-\rho} \right)^{\frac{1}{1-\rho}} \tag{5}$$

2.2 Production

There is a mass of producers of varieties in each sector differing in productivity φ. Firms face an iceberg trade cost τ_{ijm} for exports from i to j in sector m and a fixed export cost f_{ijm} for producing in country i and selling in country j. Assuming $f_{ijm}\tau_{ijm}^{\sigma_m - 1} > f_{iim} > 0$, only a subset of domestic producing firms can export. We assume $\tau_{iim} = 1$. Firms in sector m use homogeneous bundles Z_{im} as inputs with price $p_{Z_{im}}$. The cost function of a firm producing in country i and selling an amount x_{ijm} in country j having productivity φ is therefore:

$$C(x_{ijm}, \varphi) = \left(\frac{\tau_{ijm} x_{ijm}}{\varphi} + f_{ijm} \right) p_{Z_{im}} \tag{6}$$

Each firm produces a unique variety, so we can identify demand for variety ω by the productivity of the firm producing this variety. Demand $x_{ijm}(\varphi)$ and revenue $r_{ijm}(\varphi)$ of a firm with productivity φ

[3] Alternatively, we can interpret X_1 and X_2 as composite goods produced by CRS final goods producers using a continuum of intermediates $x_{im}(\omega)$.

producing in i and selling in j (in sector m) are equal to:

$$x_{ijm}(\varphi) = \frac{\alpha_m^{\rho} P_{jm}^{\sigma-\rho} P_{U_j}^{\rho-1} I_j}{p_{ijm}(\varphi)^{\sigma}} \tag{7}$$

$$r_{ijm}(\varphi) = \frac{\alpha_m^{\rho} P_{jm}^{\sigma-\rho} P_{U_j}^{\rho-1} I_j}{p_{ijm}(\varphi)^{\sigma-1}} \tag{8}$$

Maximizing profits using (6) and (8) generates the following markup pricing rule of a firm with productivity φ producing in country i and selling in country j:

$$p_{ijm}(\varphi) = \frac{\sigma_m}{\sigma_m - 1} \frac{\tau_{ijm} p_{Z_{im}}}{\varphi} \tag{9}$$

Entry and exit are like in Melitz (2003), i.e. potential firms can draw a productivity parameter φ from a distribution $F(\varphi)$ after paying a sunk entry cost $f_e p_{Z_{im}}$. Hence we assume that the same input bundles are used for development of new varieties as for production. Entering firms either decide to start producing for one or two markets or leave the market immediately. Firms face a fixed death probability δ each period. A cutoff productivity parameter φ_{ijm}^{*} for production in country i and sales in country j can be defined. Firms drawing a $\varphi \geq \varphi_{iim}^{*}$ enter the market in country i and all other firms leave immediately.

2.3 Sectoral Equilibrium

We can characterize equilibrium in sector m by a supply equation (its dual representation given by the definition of the price index), a demand equation, a free entry condition (FE) and a relation between the domestic and exporting zero cutoff profit condition (ZCP). We start with the price index, which consists of domestic and imported goods into country i:

$$P_{im} = \left(N_{iim} p_{iim} \left(\widetilde{\varphi}_{iim}\right)^{1-\sigma_m} + N_{jim} p_{jim} \left(\widetilde{\varphi}_{jim}\right)^{1-\sigma_m} \right)^{\frac{1}{1-\sigma_m}} \tag{10}$$

With N_{ijm} the mass of firms producing in i and selling in j. $\widetilde{\varphi}_{ijm}$ is a measure for average productivity of firms producing in i and selling in j:

$$\widetilde{\varphi}_{ijm} = \left(\frac{1}{1 - F\left(\varphi_{ijm}^{*}\right)} \int_{\varphi_{ijm}^{*}}^{\infty} \varphi^{\sigma_m - 1} f(\varphi)\, d\varphi \right)^{\frac{1}{\sigma_m - 1}} \tag{11}$$

Demand can be expressed as total spending on sector m goods produced in country i, hence goods produced for the domestic and exporting market:

$$R_{im} = N_{iim} \frac{\alpha_m^\rho P_{im}^{\sigma_m - \rho} P_{U_i}^{\rho - 1} I_i}{p_{iim} \left(\widetilde{\varphi}_{iim}\right)^{\sigma_m - 1}} + N_{ijm} \frac{\alpha_m^\rho P_{jm}^{\sigma_m - \rho} P_{U_j}^{\rho - 1} I_j}{p_{ijm} \left(\widetilde{\varphi}_{ijm}\right)^{\sigma_m - 1}} \tag{12}$$

Revenue R_{im} in country i in sector m is equal to total cost:[4]

$$R_{im} = P_{Z_{im}} Z_{im} \tag{13}$$

The ZCP dictates that a firm with cutoff productivity φ_{ijm}^* can just make zero profit. It can be written as:

$$\varphi_{ijm}^{*1-\sigma} = \alpha_m^\rho \left(\frac{\sigma_m}{\sigma_m - 1} \tau_{ijm} p_{Z_{im}} \right)^{1-\sigma_m} \frac{P_{jm}^{\sigma_m - \rho} P_{U_j}^{\rho - 1} I_j}{\sigma_m P_{Z_{im}} f_{ijm}} \tag{14}$$

Dividing the exporting by the domestic ZCP gives:

$$\varphi_{ijm}^* = \varphi_{iim}^* \tau_{ijm} \left(\frac{P_{im}}{P_{jm}} \right)^{\frac{\sigma_m - \rho}{\sigma_m - 1}} \left(\frac{I_i f_{ijm}}{I_j f_{iim}} \right)^{\frac{1}{\sigma_m - 1}} \left(\frac{P_{U_i}}{P_{U_j}} \right)^{\frac{\rho - 1}{\sigma_m - 1}} \tag{15}$$

The FE requires ex ante zero expected profit:

$$\sum_{j=1}^{2} \left(1 - F\left(\varphi_{ijm}^*\right)\right) \left(\frac{r_{ijm}\left(\widetilde{\varphi}_{ijm}\right)}{\sigma_m} - f_{ij} p_{z_i} \right) = \delta f_e p_{z_i} \tag{16}$$

For further discussion we introduce a Pareto distribution of productivities. The distribution of initial productivities from which entering firms draw is defined as follows:

$$F_{im}\left(\varphi\right) = 1 - \frac{\kappa_{im}^{\theta_{im}}}{\varphi^{\theta_{im}}}$$

with θ_{im} the shape parameter and κ_{im} the size parameter in this case. The FE and expressions for demand and supply can be reformulated, respectively, as follows (see for derivation Appendix A):

$$\frac{\left(\sigma_m - 1\right) \kappa_{im}^{\theta_{im}}}{\theta_{im} - \left(\sigma_m - 1\right)} \left(\frac{f_{iim}}{\varphi_{iim}^{*\theta_{im}}} + \frac{f_{ijm}}{\varphi_{ijm}^{*\theta_{im}}} \right) = \delta f_e \tag{17}$$

[4] This can be shown using the steady state entry/exit conditions together with the free entry condition, implying that all revenues are spent on factor bundles, directly through production or indirectly through development of new varieties.

$$p_{Z_{im}}^{\sigma_m} = \alpha_m^{\rho} A_{im} \kappa_i^{\theta} \left(\frac{P_{im}^{\sigma_m-\rho} P_{U_i}^{\rho-1} I_i}{\varphi_{iim}^{*\theta_{im}-\sigma_m+1}} + \tau_{ijm}^{1-\sigma} \frac{P_{jm}^{\sigma_m-\rho} P_{U_j}^{\rho-1} I_j}{\varphi_{ijm}^{*\theta_{im}-\sigma_m+1}} \right) \tag{18}$$

$$P_{im}^{1-\sigma_m} = \frac{A_{im}\kappa_{im}^{\theta_{im}} Z_{im} p_{Z_{im}}^{1-\sigma_m}}{\varphi_{iim}^{*\theta_{im}-\sigma_m+1}} + \tau_{jim}^{1-\sigma_m} \frac{A_{jm}\kappa_{jm}^{\theta_{jm}} Z_{jm} p_{Z_{jm}}^{1-\sigma_m}}{\varphi_{jim}^{*\theta_{jm}-\sigma_m+1}} \tag{19}$$

with $A_{im} = \frac{1}{\delta f_e(\theta_{im}-\sigma_m+1)} \left(\frac{\sigma_m-1}{\sigma_m}\right)^{\sigma_m}$. Equilibrium in sector m is now defined by equations for demand (18), supply (19), the free entry condition (17) and the relation between domestic and exporting ZCP (15).[5] These are functions of the endogenous variables the amount of input bundles, Z_{im}, the price index of outputs, P_{im}, the price of inputs $P_{Z_{im}}$, the domestic and exporting cutoffs φ_{iim}^{*} and φ_{ijm}^{*} and income I_i and the price index P_{U_i}.

2.4 Supply of Input Bundles

To close the model, we have to specify expressions for the supply of input bundles Z_{im} and income I_i. There are two factors of production, labor L_{im} and capital K_{im}. Input bundles are a CES function of the factors of production:

$$Z_{im} = \left(\beta_{L_{im}} L_{im}^{\frac{\eta-1}{\eta}} + \beta_{K_{im}} K_{im}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \qquad \text{with } 0 < \beta_{L_{im}}, \beta_{K_{im}} < 1; \;\; \eta > 0 \tag{20}$$

With mobile factors of production as in Bernard, et al. (2007), we get the following factor market equilibrium conditions:

$$\overline{K}_i = \left(\frac{\beta_{K_{i1}} p_{Z_{i1}}}{r_i} \right)^{\eta} Z_{i1} + \left(\frac{\beta_{K_{i2}} p_{Z_{i2}}}{r_i} \right)^{\eta} Z_{i2} \tag{21}$$

$$\overline{L}_i = \left(\frac{\beta_{L_{i1}} p_{Z_{i1}}}{w_i} \right)^{\eta} Z_{i1} + \left(\frac{\beta_{L_{i2}} p_{Z_{i2}}}{w_i} \right)^{\eta} Z_{i2} \tag{22}$$

$\overline{K}_i$ and $\overline{L}_i$ are the amount of capital and labor available and r_i and w_i are the rental rate and wage. The price index of input bundles dual to the production function in (20) is given by:

$$p_{Z_{im}} = \left(\beta_{L_{im}}^{\eta} w_i^{1-\eta} + \beta_{K_{im}}^{\eta} r_i^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{23}$$

In the specific factors model, labor is mobile across sectors and capital is immobile. The factor market equilibrium condition for labor is given by equation (22). The equilibrium condition for m sector specific

[5] The domestic ZCP is substituted into the FE, therefore we do not have to include it and inclusion of the ratio of ZCPs is sufficient.

capital is equal to:

$$\overline{K}_{im} = \left(\frac{\beta_{K_{im}} p_{Z_{im}}}{r_{im}}\right)^{\eta} Z_{im} \tag{24}$$

2.5 Closing the Model

To define equilibrium, an expression is needed for total income. Total income in country i is equal to payments of factor bundles:

$$I_i = p_{Z_{i1}} Z_{i1} + p_{Z_{i2}} Z_{i2} \tag{25}$$

Equilibrium is defined by supply, demand, the FE and the ZCP ratio and the definition of the price of input bundles for each country i and each sector m, respectively, equations (18), (19), (17), (15) and (23), the price index and total income for each country i, respectively equations (5) and (25). For the Bernard, et al. (2007) model we add the factor market equilibrium equations (21), (22) for each country i. For the specific factors model we add the following factor market equilibrium equations: (22) for each country i and (24) for each country i and each sector m.

As a special case, in the specific factors model we can express the elasticity of transformation between input bundles in the two sectors as a function of the parameters of the model only. If we set the substitution elasticity η in the production function of input bundles at 1 (more exactly $\eta \to 1$) and equalize the Cobb-Douglas parameters β_{im} for labor and capital in the two sectors, then we get the following expression for the transformation curve:

$$L_i = \gamma_{i1} Z_{i1}^{\frac{1}{\beta_i}} + \gamma_{i2} Z_{i2}^{\frac{1}{\beta_i}}; \qquad \gamma_{i1} = \overline{K}_{i1}^{\frac{\beta_i - 1}{\beta_i}} \qquad \gamma_{i2} = \overline{K}_{i2}^{\frac{\beta_i - 1}{\beta_i}} \tag{26}$$

Equation (26) can be derived as follows. The production function of input bundles with substitution elasticity η equal to 1 becomes:

$$Z_{im} = L_{im}^{\beta_i} K_{im}^{1-\beta_i} \qquad 0 < \beta_i < 1 \tag{27}$$

Solving equation (27) in each sector for labor L_{im} and substituting those expressions in the labor market equilibrium equation, $\overline{L}_i^M = L_{i1}^M + L_{i2}^M$, gives equation (26).The elasticity of transformation between X_{i1} and X_{i2} is equal to $\omega_i = \frac{\beta_i}{1-\beta_i}$ and is thus rising in the Cobb-Douglas parameter β_i of the mobile factor of production, labor. To close the model, we add that the marginal rate of transformation between Z_{i1}

and Z_{i2} has to be equal to the price ratio, $p_{Z_{i1}}/p_{Z_{i2}}$:

$$\frac{p_{Z_{i1}}}{p_{Z_{i2}}} = \frac{\gamma_{i1} Z_{i1}^{\frac{1-\beta_i}{\beta_i}}}{\gamma_{i2} Z_{i2}^{\frac{1-\beta_i}{\beta_i}}} \tag{28}$$

The specific factors model with Cobb-Douglas production of input bundles solves with demand, supply, FE and ZCP, respectively equations (18), (19), (17), (15), for each country i and each sector m and the price index, total income, the transformation curve and optimality along the transformation curve for each country i, respectively equations (5), (25), (26) and (28).

3 Analysis

Using the model of the previous section, we address various questions on the effect of factor mobility on the reallocation gains from trade. Because of the size and complexity of the model we cannot derive analytical results. Therefore, we use simulations.[6] The parameters used for the simulations are discussed and motivated in Appendix B.

3.1 Factor Mobility with Cobb-Douglas Production Function of Input Bundles

Let us start with the special case discussed above, production is Cobb-Douglas and there is mobile labor and immobile capital, to determine the effect of factor mobility on the reallocation gains from trade. As pointed out above, the elasticity of transformation between different goods, ω_i, in this case becomes an explicit function of the Cobb-Douglas parameter β_i of the mobile production factor. Thus, there is an explicit expression for the transformation curve between production in the two sectors.

To address the effect of factor mobility on trade related reallocation effects, trade costs are reduced for various values of the transformation elasticity ω, i.e. the Cobb-Douglas parameter β on the mobile production factor. Before turning to the differential impact on (immobile) capital and (mobile) labor, we use as welfare measure simply real total income, i.e. nominal total income divided by the price index.[7]

Table 1 shows the results for welfare defined as nominal total income divided by the price index and for the domestic cutoff parameter, both in country 1. The sum of sector-specific capital is equal to the

[6] We use GAMS to run the simulations. The code of the different simulation exercises is available upon request.

[7] This welfare measure reflects utility of any agent in our model, assuming that all agents get equal rewards from labor and the two types of capital. Alternatively, it is a measure for social welfare with the different factor owners getting equal weight in the social welfare function.

amount of labor and the two countries have equal endowments, i.e. $L_{11} = L_{21} = 3$, $K_{11} = K_{21} = 1$, $K_{12} = K_{22} = 2$. The other parameters are as discussed in Appendix B. Though we start from a symmetric equilibrium there is still trade due to firm heterogeneity and CES preferences. The simulation performed is a variation in trade costs for various values of the elasticity of transformation. With this assumption the cutoff productivities do not vary with the elasticity of transformation (φ^*_{11} is displayed in Table 1, the other results are available upon request).

Therefore, we undertake two other simulations, where the amount of labor is larger respectively smaller than the sum of sector specific capital. The results are shown in Tables 2 and 3. In the former we specifically assume that $L_1 = 5$, $L_2 = 3$, $K_{11} = K_{21} = K_{12} = K_{22} = 1$, whereas in the latter the endowment structures is $L_1 = 1$, $L_2 = 2$, $K_{11} = K_{12} = 2$, $K_{21} = K_{22} = 3$. The other parameters are as discussed in Appendix B. In Tables 2 and 3 the cutoff values do vary with the elasticity of transformation.

Table 1 Cobb-Douglas model with balanced endowments

	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$
			Welfare $W = I_1/P_{U_1}$			
$\tau = 1.0$	2.254	2.257	2.260	2.264	2.268	2.274
$\tau = 1.2$	2.082	2.085	2.087	2.091	2.095	2.101
$\tau = 1.4$	1.981	1.983	1.986	1.989	1.993	1.998
$\tau = 1.6$	1.918	1.920	1.923	1.926	1.930	1.935
$\tau = 1.8$	1.878	1.880	1.882	1.886	1.889	1.894
			Cutoff productivity φ^*_{111}			
$\tau = 1.0$	1.279	1.279	1.279	1.279	1.279	1.279
$\tau = 1.2$	1.181	1.181	1.181	1.181	1.181	1.181
$\tau = 1.4$	1.123	1.123	1.123	1.123	1.123	1.123
$\tau = 1.6$	1.088	1.088	1.088	1.088	1.088	1.088
$\tau = 1.8$	1.065	1.065	1.065	1.065	1.065	1.065

When there is more labor than capital as in Table 2, the cutoff productivity goes down as the elasticity of transformation rises. This is contrary to what was expected interpreting β as a measure for the elasticity of transformation, as more factor mobility should make the reallocation effect larger and thus the cutoff productivity larger. The reason is that a a higher parameter β implies that the abundant and thus cheaper production factor, labor, becomes more important in production. Therefore, input costs become cheaper, it is easier to survive and the cutoff productivity will go down. When there is more capital than labor as in Table 3, the reallocation effect is as expected: the cutoff productivity rises.

Welfare rises with an increase in the labor Cobb-Douglas parameter (higher elasticity of transformation) when there is more labor than capital and welfare declines when there is more capital than labor.

An increase in the Cobb-Douglas parameter of labor leads to a higher productivity in the production of input bundles when a country has more labor and decreases the productivity when the country has more capital. Changes in the Cobb-Douglas parameters reflect not only a change in the elasticity of transformation between sectors, but also factor biased technical change. Welfare will go up with an increase in the labor Cobb-Douglas parameter if a country has more labor than capital. Therefore, the sector specific model with Cobb-Douglas production of input bundles and Cobb-Douglas parameters that can be easily interpreted as a proxy for the elasticity of transformation is not an accurate model to evaluate the impact of more factor mobility on the reallocation gains from trade. We will evaluate this question again in a model with CES production functions of input bundles, varying the elasticity of substitution between the factors of production. Before, we however turn to the question whether the reallocation gains from trade can dominate the welfare losses of the scarce factors of production in the specific factors model.

Table 2 Cobb-Douglas model with labor biased endowments

	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$
			Welfare $W = I_1/P_{U_1}$			
$\tau = 1.0$	1.677	1.884	2.116	2.377	2.671	3.002
$\tau = 1.2$	1.559	1.757	1.979	2.230	2.513	2.833
$\tau = 1.4$	1.489	1.680	1.896	2.141	2.416	2.728
$\tau = 1.6$	1.445	1.632	1.844	2.084	2.355	2.661
$\tau = 1.8$	1.417	1.601	1.810	2.047	2.314	2.616
			Cutoff productivity φ^*_{111}			
$\tau = 1.0$	1.256	1.246	1.236	1.226	1.217	1.208
$\tau = 1.2$	1.168	1.162	1.156	1.150	1.145	1.140
$\tau = 1.4$	1.115	1.112	1.108	1.104	1.101	1.097
$\tau = 1.6$	1.082	1.080	1.077	1.075	1.073	1.070
$\tau = 1.8$	1.061	1.059	1.057	1.056	1.054	1.052

3.2 Reallocation Gains for the Scarce Production Factors and Across Sectors

In the specific factors model with perfect competition in product and factor markets, trade liberalization has a negative effect on the real remuneration of the sector specific production factor that is relatively scarce (see any textbook treatment of the specific factors model, for example Bowen, et al. , 1997). In our model this means that if country 1 is relatively abundant in sector 1 specific capital, trade liberalization implies a real loss for sector 2 specific capital.

Table 3 Cobb-Douglas model with capital biased endowments

	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$
			Welfare $W = I_1/P_{U_1}$			
$\tau = 1.0$	2.562	2.136	2.116	1.781	1.485	1.032
$\tau = 1.2$	2.289	1.904	1.979	1.584	1.318	0.912
$\tau = 1.4$	2.137	1.775	1.896	1.475	1.225	0.846
$\tau = 1.6$	2.045	1.698	1.844	1.409	1.170	0.806
$\tau = 1.8$	1.987	1.649	1.810	1.368	1.135	0.781
			Cutoff productivity φ^*_{111}			
$\tau = 1.0$	1.399	1.408	1.417	1.426	1.435	1.445
$\tau = 1.2$	1.251	1.256	1.261	1.266	1.271	1.277
$\tau = 1.4$	1.167	1.170	1.174	1.177	1.180	1.184
$\tau = 1.6$	1.117	1.119	1.122	1.124	1.126	1.128
$\tau = 1.8$	1.086	1.087	1.089	1.090	1.092	1.093

Bernard, et al. (2007) show in a Heckscher-Ohlin setting with firm heterogeneity that the scarce factor of production might still gain from trade liberalization, i.e. the Stolper-Samuelson theorem does not always hold. We will check this in our specific factor model setting, i.e. whether relatively scarce sector specific capital might also gain from trade liberalization, because of the beneficial reallocation effect implying a drop in the price index. To address this question, we conduct a simulation of a decline in trade costs between countries 1 and 2, with country 1 being relatively abundant in sector 1 specific capital. In particular, we assume that $L_{11} = L_{21} = 1$, $K_{11} = 2$, $K_{12} = 1$, $K_{21} = 1$, $K_{22} = 2$. The Cobb-Douglas parameters for labor and capital are equal to 1/2, i.e. $\beta_{L_{im}} = \beta_{K_{im}} = 1/2$. Table 4 shows the results of the simulation for the variables P_{U_1}, w_1, w_1/P_{U_1}, r_{11}, , r_{11}/P_{U_1}, r_{12}, r_{12}/P_{U_1}, φ^*_{111} and φ^*_{112}.

Table 4 Effect of trade liberalization on factor rewards

τ	2.0	1.8	1.6	1.4	1.2	1.0
P_{U_1}	1.848	1.837	1.816	1.777	1.706	1.582
w_1	0.823	0.832	0.842	0.853	0.862	0.866
w_1/P_{U_1}	0.445	0.453	0.464	0.480	0.505	0.547
r_{11}	0.260	0.266	0.273	0.280	0.286	0.289
r_{11}/P_{U_1}	0.141	0.145	0.150	0.157	0.168	0.182
r_{12}	0.304	0.300	0.297	0.293	0.290	0.289
r_{12}/P_{U_1}	0.164	0.164	0.163	0.165	0.170	0.182
φ^*_{111}	1.071	1.089	1.113	1.148	1.199	1.279
φ^*_{112}	1.035	1.047	1.068	1.103	1.164	1.279

The results confirm our expectations. As in the perfect competition case, lower trade costs lead to a lower price index, raise the real rewards for mobile labor and the nominal and real rewards of the abundant sector specific capital.[8] The scarce sector specific capital loses in nominal terms, but gains in real terms, due to the reallocation effect. The reallocation effect is illustrated in the last two rows of Table 4, showing an increasing domestic cutoff productivity as trade costs go down.

Thus when allowing for firm heterogeneity, the political economy implications of the basic specific factors model result does not necessarily apply. Scarce sector specific capital loses in that model which might explain why trade reforms are difficult. Contrary, in our model with imperfect competition and firm heterogeneity the scarce sector specific factors can gain from trade liberalization. Still, in both the declining and expanding sector firm owners with relatively low productivity might lobby against liberalization, (see Bombardini , 2008, for a discussion of this point). Furthermore it is important to note that the real income is generally increasing with a higher η, i.e. a larger substitution effect.

Another important result in Bernard, et al. (2007) using a Heckscher-Ohlin setting is that the increase in domestic cutoff productivity as trade costs are reduced is larger in the comparative advantage sector and that the exporting cutoff productivity is closer to the domestic cutoff productivity in the comparative advantage industry. The intuition is that profitability from exporting is larger in the comparative advantage sector. This implies that the exporting cutoff is lower and thus closer to the domestic cutoff. Moreover, trade liberalization will drive up demand for scarce resources more in the comparative advantage sector, squeezing more domestic firms out of the market.

Table 5 Effect of trade liberalization on cutoff productivities in comparative advantage and disadvantage Sectors

τ	2.0	1.8	1.6	1.4	1.2	1.0
φ^*_{111}	1.073	1.091	1.115	1.148	1.199	1.279
φ^*_{121}	1.843	1.724	1.607	1.494	1.383	1.279
φ^*_{112}	1.034	1.046	1.067	1.102	1.164	1.279
φ^*_{122}	2.408	2.145	1.895	1.662	1.453	1.279

In Table 5 we report the results from an exercise examining the change in cutoff levels in comparative advantage sectors relative to comparative disadvantage sectors. We use the same parameter values as in the previous exercise. Hence, country 1 has a comparative advantage in sector 1. In line with Bernard, et al. (2007) we find that domestic and exporting cutoff levels are closer to each other in the comparative

[8] The nominal rewards for labor go up in this simulation. In the perfect competition specific factors model the nominal rewards for labor can either go up or down, depending upon parameter configurations.

advantage sector. We can see this by comparing the rows for φ^*_{121} and φ^*_{122}, the exporting cutoffs in respectively the comparative advantage and disadvantage sectors of country 1.

The increase in the domestic cutoff productivity is smaller in the comparative advantage sector moving to free trade. At first sight this seems contrary to the findings in Bernard, et al. (2007), but the results are actually in line with what these authors have found. Moving from autarky to costly trade, the domestic cutoff productivity first goes down quicker in the comparative advantage sector and later on slower than in the comparative disadvantage sector. The reason is that the cutoff levels have to converge to the same values with free trade, because of equal prices and factor costs in the two sectors.

3.3 Factor Mobility with CES Production Function of Input Bundles

In subsection 3.1 we concluded that the specific factors model with Cobb-Douglas production of input bundles is not the proper model to explore the effect of factor mobility on the reallocation gains from trade. Varying the elasticity of transformation between sectors implies that also the Cobb-Douglas parameter on labor and capital varies, implying that the amount of labor and capital plays a role in the welfare effects. Therefore, we now turn to a generalized specific factors model with CES production of input bundles. The effect of more factor mobility is studied by varying the elasticity of substitution between the production factors of input bundles. We work with a clear comparative advantage of country 1 in sector 1 in a symmetric setup: $L_1 = L_2 = 10$, $K_{11} = K_{22} = 9$, $K_{12} = K_{21} = 1$. Table 6 presents the results of this exercise for the variables welfare defined as total income divided by the price index, the domestic cutoff productivity in the comparative advantage sector in country 1, φ^*_{111}, the exporting productivity in both the comparative advantage and disadvantage sector in country 1, φ^*_{121} and φ^*_{122} and the amount of labor used in the comparative advantage sector in country 1, L_{11}.

The last rows in Table 6 indicate that there is less specialization as trade costs and the substitution elasticity become larger. Higher barriers to trade make it more difficult for countries to specialize according to their comparative advantage. A higher substitution elasticity implies that there is less need to have equal factor proportions. Therefore, countries allocate less labor to their comparative advantage sector and there is henceforth less specialization.

Less specialization when the substitution elasticity is larger implies that the welfare gains from lower trade costs are smaller when the substitution elasticity is larger. With less specialization there is less to gain from lowering trade barriers and hence the welfare gains are smaller. The policy implication is that countries with less flexibility on their factor markets specialize more and can gain more from trade liberalization.

Table 6 Specific factors model with CES production function of input bundles

	$\eta = 1.2$	$\eta = 1.6$	$\eta = 2$	$\eta = 2.4$	$\eta = 2.8$
			Welfare $W = I_1/P_{U_1}$		
$\tau = 1.0$	4.374	4.374	4.374	4.374	4.374
$\tau = 1.4$	3.796	3.796	3.797	3.797	3.798
$\tau = 1.8$	3.522	3.525	3.529	3.533	3.537
$\tau = 2.2$	3.371	3.380	3.390	3.399	3.409
$\tau = 2.6$	3.282	3.297	3.312	3.328	3.343
			Cutoff CA domestic (φ^*_{111})		
$\tau = 1.0$	0.473	0.473	0.473	0.473	0.473
$\tau = 1.4$	0.443	0.442	0.442	0.442	0.442
$\tau = 1.8$	0.426	0.425	0.424	0.423	0.422
$\tau = 2.2$	0.415	0.413	0.412	0.411	0.409
$\tau = 2.6$	0.407	0.405	0.404	0.402	0.401
			Cutoff CA export (φ^*_{121})		
$\tau = 1.0$	0.473	0.473	0.473	0.473	0.473
$\tau = 1.4$	0.515	0.516	0.516	0.517	0.517
$\tau = 1.8$	0.558	0.561	0.564	0.567	0.570
$\tau = 2.2$	0.603	0.611	0.619	0.628	0.637
$\tau = 2.6$	0.653	0.667	0.682	0.698	0.714
			Cutoff CD export (φ^*_{122})		
$\tau = 1.0$	0.473	0.473	0.473	0.473	0.473
$\tau = 1.4$	0.680	0.680	0.679	0.678	0.677
$\tau = 1.8$	0.967	0.961	0.954	0.947	0.939
$\tau = 2.2$	1.290	1.271	1.250	1.229	1.207
$\tau = 2.6$	1.630	1.590	1.548	1.506	1.468
			Labor CA country 1 (L_{11})		
$\tau = 1.0$	9.000	9.000	9.000	9.000	9.000
$\tau = 1.4$	8.841	8.782	8.720	8.655	8.588
$\tau = 1.8$	8.559	8.374	8.171	7.951	7.720
$\tau = 2.2$	8.283	7.962	7.612	7.249	6.892
$\tau = 2.6$	8.046	7.614	7.161	6.722	6.316

CA comparative advantage sector; CD comparative disadvantage sector

The changes in cutoff levels in relation to trade costs are as expected: lower trade costs reduce the exporting cutoff productivities and raise the domestic cutoff productivities. The effect of changes in the substitution elasticity on the cutoff productivities are somewhat difficult to interpret and can be explained with the changes in the degree of specialization. We observed before that there is less specialization when the substitution elasticity increases. Therefore, country 1 faces more competition on the export market from domestic firms in its comparative advantage sector 1. This implies that the exporting cutoff productivity in its comparative advantage sector, φ^*_{121}, goes up with a larger substitution elasticity. The domestic cutoff productivity instead goes down: it is easier to survive domestically, as there is less entry to make profits on the exporting market. In the comparative disadvantage sector instead, exporting becomes easier, as there is less competition from domestic firms in the export market. Therefore the cutoff productivity, φ^*_{122}, goes down with a higher substitution elasticity.

4 Concluding Remarks

We introduced firm heterogeneity in the traditional specific factors model to study the effect of factor mobility. We showed that the welfare gains from trade liberalization are smaller with a higher substitution elasticity between the mobile and fixed factor, as this leads to less specialization and less scope for gains from lower trade costs. We also found that the scarce sector specific factor might gain from freer trade when firms are heterogeneous because of productivity gains due to firm selection within the sector. Finally, it is shown that the selection effect is stronger in the comparative advantage sector. A possible extension of the current research with policy implications could be to model one of the production factors as partly mobile and partly immobile and to study the effect of a larger mobile share on the factor rewards of the 'immobile' factor.

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Appendix A Concise Equilibrium

We start with a reformulation of the FE. Using $\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$ and the ZCP, the FE in equation (16) can be written as:

$$\sum_{j=1}^{2} \left(1 - F_{im}\left(\varphi^*_{ijm}\right)\right) f_{ijm} \left(\left(\frac{\widetilde{\varphi}_{ijm}}{\varphi^*_{ijm}}\right)^{\sigma_m - 1} - 1\right) = \delta f_e \tag{A.1}$$

We impose $\theta_{im} > \sigma_m - 1$ to guarantee that expected revenues are finite. The implication of assuming a Pareto distribution is that $\widetilde{\varphi}$ is proportional to φ^*:

$$\widetilde{\varphi}_{ijm} = \left(\frac{\theta_{im}}{\theta_{im} - (\sigma_m - 1)}\right)^{\frac{1}{\sigma_m - 1}} \varphi^*_{ijm} \tag{A.2}$$

Under a Pareto distribution, the FE, equation (16) becomes:

$$\sum_{k=1}^{2} \left(1 - F_{im}\left(\varphi^*_{ikm}\right)\right) \frac{\sigma_m - 1}{\theta_{im} - (\sigma_m - 1)} f_{ikm} p_{Z_{im}} = \delta f_e p_{Z_{im}} \tag{A.3}$$

Substituting for the cdf, the FE in equation (A.3) becomes with a Pareto distribution:

$$\frac{(\sigma_m - 1)\, \kappa_{im}^{\theta_{im}}}{\theta_{im} - (\sigma_m - 1)} \left(\frac{f_{iim}}{\varphi_{iim}^{*\theta_{im}}} + \frac{f_{ijm}}{\varphi_{ijm}^{*\theta_{im}}}\right) = \delta f_e \tag{A.4}$$

Next, we use the Pareto distribution and the FE to write the mass of firms as a function of the cutoff productivity. Total revenues in country i in sector m should be equal to the total value of input bundles:

$$\sum_{k=1}^{2} N_{ikm} r_{ikm}\left(\widetilde{\varphi}_{ikm}\right) = p_{Z_{im}} Z_{im} \tag{A.5}$$

Also, the relative mass of firms is a function of the shares of the densities of productivities that can produce profitably in the market:

$$N_{ikm} = \frac{1 - F_{im}\left(\varphi^*_{ikm}\right)}{1 - F_{im}\left(\varphi^*_{ijm}\right)} N_{ijm} \tag{A.6}$$

Substituting equation (A.6) into (A.5) gives:

$$\frac{N_{ijm}}{1 - F\left(\varphi^*_{ijm}\right)} \sum_{k=1}^{2} \left(1 - F_{im}\left(\varphi^*_{ikm}\right)\right) r_{ikm}\left(\widetilde{\varphi}_{ikm}\right) = p_{Z_{im}} Z_{im} \tag{A.7}$$

Using the FE in equation (16) this can be written as:

$$N_{ijm} = \left(1 - F_{im}\left(\varphi^*_{ijm}\right)\right) \frac{Z_{im}}{\sigma_m \delta f_e + \sum\limits_{k=1}^{2} \left(1 - F_{im}\left(\varphi^*_{ikm}\right)\right) \sigma f_{ikm}} \tag{A.8}$$

As a next step, substituting equation (A.3) into equation (A.8), we can write the number of firms N_{ijm} as a function of the cutoff level φ^*_{ijm} and input bundles Z_i:

$$N_{ijm} = \left(\frac{\kappa_{im}}{\varphi^*_{ijm}}\right)^{\theta_{im}} \frac{\sigma_m - 1}{\sigma_m \theta_{im} \delta f_e} Z_{im} \tag{A.9}$$

Next, we rewrite the expression for demand, equation (12), as follows. First, substitute equation (A.9) into equation (12) to find:

$$\begin{aligned} p_{Z_{im}} Z_{im} \quad = \quad & \left(\frac{\kappa_{im}}{\varphi^*_{iim}}\right)^{\theta_{im}} \frac{\sigma_m - 1}{\sigma_m \theta_{im} \delta f_e} Z_{im} \alpha^\rho_m \frac{P^{\sigma_m - \rho}_{im} P^{\rho-1}_{U_i} I_i}{p_{iim}\left(\widetilde{\varphi}_{iim}\right)^{\sigma_m - 1}} \\ & + \left(\frac{\kappa_{im}}{\varphi^*_{ijm}}\right)^{\theta_{im}} \frac{\sigma_m - 1}{\sigma_m \theta_{im} \delta f_e} Z_{im} \alpha^\rho_m \frac{P^{\sigma_m - \rho}_{jm} P^{\rho-1}_{U_j} I_j}{p_{ijm}\left(\widetilde{\varphi}_{ijm}\right)^{\sigma_m - 1}} \end{aligned} \tag{A.10}$$

Second, substitute the pricing equation (9) into equation (A.10) using equation (A.2):

$$\begin{aligned} p_{Z_{im}} \quad = \quad & \left(\frac{\kappa_{im}}{\varphi^*_{iim}}\right)^{\theta_{im}} \frac{\sigma_m - 1}{\sigma_m \theta_{im} \delta f_e} \alpha^\rho_m \frac{P^{\sigma_m - \rho}_{im} P^{\rho-1}_{U_i} I_i}{\left(\frac{\sigma_m}{\sigma_m - 1} \frac{p_{Z_{im}}}{\left(\frac{\theta_{im}}{\theta_{im} - (\sigma_m - 1)}\right)^{\frac{1}{\sigma_m - 1}} \varphi^*_{iim}}\right)^{\sigma_m - 1}} \\ & + \left(\frac{\kappa_{im}}{\varphi^*_{ijm}}\right)^{\theta_{im}} \frac{\sigma_m - 1}{\sigma_m \theta_{im} \delta f_e} \alpha^\rho_m \frac{P^{\sigma_m - \rho}_{jm} P^{\rho-1}_{U_j} I_j}{\left(\frac{\sigma_m}{\sigma_m - 1} \frac{\tau_{ijm} p_{Z_{im}}}{\left(\frac{\theta_{im}}{\theta_{im} - (\sigma_m - 1)}\right)^{\frac{1}{\sigma_m - 1}} \varphi^*_{ijm}}\right)^{\sigma_m - 1}} \\ = \quad & \alpha^\rho_m A_{im} \left(\frac{\kappa_{im}}{\varphi^*_{iim}}\right)^{\theta_{im}} \left(\frac{\varphi^*_{iim}}{p_{Z_{im}}}\right)^{\sigma_m - 1} P^{\sigma_m - \rho}_{im} P^{\rho-1}_{U_i} I_i \\ & + \alpha^\rho_m A_{im} \left(\frac{\kappa_{im}}{\varphi^*_{ijm}}\right)^{\theta_{im}} \left(\frac{\varphi^*_{ijm}}{\tau_{ijm} p_{Z_{im}}}\right)^{\sigma_m - 1} P^{\sigma_m - \rho}_{jm} P^{\rho-1}_{U_j} I_j \end{aligned} \tag{A.11}$$

$$p^{\sigma_m}_{Z_{im}} \quad = \quad \alpha^\rho_m A_{im} \kappa^{\theta_{im}}_{im} \left(\frac{P^{\sigma_m - \rho}_{im} P^{\rho-1}_{U_i} I_i}{\varphi^{*\theta_{im} - \sigma_m + 1}_{iim}} + \frac{\tau^{1-\sigma_m}_{ijm} P^{\sigma_m - \rho}_{jm} P^{\rho-1}_{U_j} I_j}{\varphi^{*\theta_{jm} - \sigma_m + 1}_{ijm}}\right) \tag{A.12}$$

with $A_{im} = \frac{1}{\delta f_e(\theta_{im}-\sigma_m+1)}\left(\frac{\sigma_m-1}{\sigma_m}\right)^{\sigma_m}$. To derive supply, equation (19), as a first step we substitute equation (A.9) into equation (10):

$$\begin{aligned} P_{im}^{1-\sigma_m} &= \left(\frac{\kappa_{im}}{\varphi_{iim}^*}\right)^{\theta_{im}} \frac{\sigma_m - 1}{\sigma_m\theta_{im}\delta f_e} Z_{im} p_{iim} \left(\widetilde{\varphi}_{iim}\right)^{1-\sigma_m} \\ &+ \left(\frac{\kappa_{jm}}{\varphi_{jim}^*}\right)^{\theta_{jm}} \frac{\sigma_m - 1}{\sigma_m\theta_{jm}\delta f_e} Z_{jm} p_{jim} \left(\widetilde{\varphi}_{jim}\right)^{1-\sigma_m} \end{aligned} \tag{A.13}$$

Second, equations (9) and (A.2) are substituted to find equation (19):

$$\begin{aligned} P_{im}^{1-\sigma_m} &= \left(\frac{\kappa_{im}}{\varphi_{iim}^*}\right)^{\theta_{im}} \frac{\sigma_m - 1}{\sigma_m\theta_{im}\delta f_e} Z_{im} \left(\frac{\sigma_m}{\sigma_m - 1}\frac{p_{Z_{im}}}{\left(\frac{\theta_{im}}{\theta_{im}-(\sigma_m-1)}\right)^{\frac{1}{\sigma_m-1}}\varphi_{iim}^*}\right)^{1-\sigma} \\ &+ \left(\frac{\kappa_{jm}}{\varphi_{jim}^*}\right)^{\theta_{jm}} \frac{\sigma_m - 1}{\sigma_m\theta_{jm}\delta f_e} Z_{jm}^{1-\sigma_m} \left(\frac{\sigma_m}{\sigma_m - 1}\frac{\tau_{jim}p_{Z_{jm}}}{\left(\frac{\theta_{jm}}{\theta_{jm}-(\sigma_m-1)}\right)^{\frac{1}{\sigma_m-1}}\varphi_{jim}^*}\right)^{1-\sigma} \\ &= \frac{A_{im}\kappa_{im}^{\theta_{im}} Z_{im} p_{Z_{im}}^{1-\sigma_m}}{\varphi_{iim}^{*\theta_{im}-\sigma_m+1}} + \frac{A_{jm}\kappa_{jm}^{\theta_{jm}} Z_{jm} \tau_{jim}^{1-\sigma_m} p_{Z_{jm}}^{1-\sigma_m}}{\varphi_{jim}^{*\theta_{jm}-\sigma_m+1}} \end{aligned} \tag{A.14}$$

Appendix B Simulations

The baseline parameter values are set in accordance with the parameters in the numerical analysis of Bernard, et al. (2007). The baseline parameters can be found in Table 7.

Without loss of generality the substitition elasticity between sectors, ρ, is set at 2 and the shift parameters of both sectors α_m are equal to each other, equal to 1/2. For the elasticity of substitution within sectors, σ_m, we work with a value of 3.8 and for the Pareto shape parameter θ_m and size parameter κ_m with values of 3.4 and 0.2, respectively, following estimates using plant-level US manufacturing data in Bernard, et al. (2003). The parameter value of the sunk entry cost f_e scales the mass of firms and without loss of generality f_e is set at 1. Fixed production costs, domestically and in the foreign market, f and f_x respectively, are 10% of the sunk entry cost, 0.1. The domestic and exporting fixed costs are equal in the baseline, implying equal domestic and exporting cutoff productivity when iceberg trade costs are 1. The death probability δ rescales the mass of entrants relative to the mass of producing firms and without loss of generality, a value of 0.025 is chosen. In the Heckscher-Ohlin model we assume that the shift parameters of the two sectors $\beta_{L_{im}}$ and $\beta_{K_{im}}$ are equal to 1/2. The substitution elasticity between labor and capital is subject to variation to mimic variation in factor mobility across sectors. In the specific factors model the substitution elasticity is equal to 1 and the Cobb-Douglas (shift) parameters are subject to variation. The choice of the size of the labor force and the amount of capital is arbitrary and motivated in the main text.

Table 7 Baseline simulation: Parameters and results for endogenous variables

Substitution elasticity sectors	ρ	2
Shift parameter sector m	α_m	0.5
Substitution elasticity sectors	σ_m	3.8
Pareto shape parameter	θ_m	3.4
Pareto shift parameter	κ_m	0.2
Sunk entry costs	f_e	1
Fixed costs	f	0.1
Fixed export costs	f_x	0.1
Death probability	δ	0.025
Shift parameter capital and labor	β_m	0.5