Cyclical and constant strategies in renewable resources extraction

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Abstract
This paper is concerned with the classic topic of intertemporal resource economics: the optimal harvesting of renewable natural resources over time by one and several resource owners with conflicting interests. The traditional management model, dating back to Plourde (1970), is extended towards a two–state model in which harvesting equipment is treated as a stock variable. As a consequence of this extension, an equilibrium dynamics with bifurcations and limit cycles occur. Next we discuss conflicts as a game with two types of players involved: the traditional fishermen armed with the basic equipment and the heavy equipment users. Both players have a common depletion function, thought as harvesting, which is dependent both on personal effort and on intensity of equipment's usage.

Keywords: Renewable resources; exploitation of natural resources; differential games.

JEL classifications: C61, C62, Q32.

1. Introduction

Intertemporal economic problems can be formulated either as optimal management models or as dynamic games. A basic difference between the two types of formulation is that, in the former case there is only one decision maker, the regulator, i.e. there is only one strategically acting agent, while in the latter there are more than one strategically interacting agents, choosing their actions that determine the current and future levels of utility. Consider, for example a single stock of an exhaustible or reproductive resource that is simultaneously exploited by several agents that do not cooperate. Each agent chooses an extraction strategy to maximize the discounted stream of future utility.

Then, the actions taken determine not only their utility levels but also the level of the stock. There are several implications of the above formulation. First, the actions taken by agents determine the size of a single capital stock that fully describes the current state of the economic system. Second, if there is no mechanism that forces players to coordinate their actions, they will act strategically and play a non–cooperative game. Third, the equilibrium outcome will critically depend on the strategy spaces available to the agents.

There is a wide choice of possible actions (strategies) taken by the players. They may choose a simple time profile of actions and pre-commit themselves to these fixed actions over the entire planning horizon. Players then use open–loop strategies. Alternatively players might choose feedback or closed–loop or Markov strategies conditioning their actions on the current state of the system and reacting immediately every time the state variable changes, hence they are not required to precommit. Here we expose an example of several agents strategically exploiting the same renewable resource, like a fish stock, in order to expose the difference between open–loop and closed–loop strategies.

If fisheries use open–loop strategies they specify a time path of fishing effort in the beginning of the game and commit themselves to stick to these preannounced actions over the entire planning horizon. Alternatively, if they use feedback strategies they choose decision rules that determine current actions as a function of current stock of the resource. Feedback decision rules capture the strategic interactions present in a dynamic game. If a rival fishery makes a catch today that necessarily results in a lower level of the fish stock, the opponents react with actions that take this change in the stock into account. In that sense closed–loop strategies capture all the features of strategic interactions.

In these lines, the main contribution of this paper relies on the results obtained in the Nash equilibrium of the game for which the players compete having a common harvesting (depletion) function. In equilibrium terms we find the relation between the players' discount factors in order to ensure equilibrium with limit cycles.

The structure of the paper is the following. Section 2 reviews the existing relative literature. The next section concerns the conflicts as dynamic game with two players and with a common harvesting function. The last section concludes the paper.

2. Literature review

In environmental economics vast literature, one given important meaning is connected with the exploitation of natural resources. According to this approach a regeneration function is involved, which is necessary to model the interactions between the nature and the human activities. In an important model, Strobele (1988) considers the whole environment as renewable natural resource and the damage done to nature is described by a downward shift in the regeneration function due to the industrial waste emission. In the same, but more restrictive, way Hannesson (1982) compares the optimality of the monopolistic and social planning extractions, finding that the monopolistic standing optimal stock of the resource (say the nature) may either be larger or smaller than under the social planning.

Strobele and Wacker (1994) extend the one-specie exploitation to multiple species in a predator–prey model. They derive a modified golden rule of harvesting, applying optimal control theory. Their conclusions about the modified golden rule in the steady state, is related with the “additional productivity effects”. Farmer (2000), reconsidering Mourmouras’ type overlapping generations’ model with renewable natural resources, shows that there exists a non trivial stationary state which exhibits, by definition, intergenerational natural – capital equality.

Finally, natural resources harvesting differs from production. Renewable resources economic literature, based on the foundations of Gordon (1954), Scott (1955) and Smith (1969), suggests particular properties of the open access natural resources which requires tools of analysis beyond those supplied by elementary economic theory. Such an appropriate tool is the optimal control theory and the use of differential equations in dynamic systems (either in a continuous or a discrete framework), which are of common use in most models that explain the optimal management of natural resources extraction. These systems depend on more than one parameter that measures different economic and biological characteristics of the exploited resource. So the structural stability is a key point to study in order to explore whether the qualitative dynamical properties of the system persist when its structure is perturbed. In this context, the study of the structural stability is the first step to follow the analysis of the system.

On the other hand, it is reasonable to consider the stock of any renewable resource as a capital stock and treat the exploitation of that resource in much the same way as one would treat accumulation of a capital stock. This has been done to some extent by Clark (1973) and Clark and Munro (1975), whose papers contain a discussion of this point of view. However, the analysis is much simpler than it appears in the literature especially since the interaction between markets and the natural biology dynamics has not been made clear. Furthermore renewable resources are commonly analyzed in the context of models where the growth of the renewable resource examined is affected by two factors: the size of the resource itself and the harvesting rate. This specification does not take into account that human activities other than harvesting may have an impact on the growth of the natural resource (Levhari and Withagen, 1992).

Some externalities may arise in maximum sustained yield programs of replenishable natural resource exploitation followed by two fundamental problems. The first is that the existence of a social discount factor (or interest rate) may cause the maximum sustained yield program to be non-optimal (Plourde, 1970). The second problem relates to many externalities which may be present in harvesting resources. The most significant of these externalities is the stock externality in production. That is, there is a potential misallocation of inputs in the production of natural resource product due to the fact that one input, the natural resource, contributes to production but may not receive payment, as nobody owns the resource.

An analysis of the biomass harvesting (like fisheries) must take into account the biological nature of fundamental capital, the renewable resource, and must recognize the common property feature of land or sea, so it must allow that the fundamental capital is the subject of exploitation. The problem of fishing industry has been tackled by economists giving attention to the common property characteristics associated with both the open access and the lack of proper property rights to the fishery industry (Gordon, 1954; Bjøndal, 1992). A number of existing studies on fishery economics have paid attention to the form of properties: full rights or no rights at all (Smith, 1969; Plourde, 1971). Both cases lead to unique Nash non-cooperative outcomes with the social planner's outcome in the case of full rights and the open access in the case of no rights. The latter is the result of the tragedy of commons (for discussion see Clark and Munro, 1975).

3. A differential game with a common harvesting function

Let us denote by $x(t)$ the instantaneous renewable resource which is in common access at time t. Without any harvesting taking place the stock of resources grows according to the function $g(x)$, obviously dependent on the resource itself, satisfying the conditions $g(0)=0$, $g(x)>0$ for all $x\in(0,K)$, $g'(x)<0$ for all $x\in(K,\infty)$, $g''(x)\leq 0$. In the proposed game we assume that two types of players are involved. First it is the renewable resource extractors (players) acting with the traditional mode in the sense of Clark (Clark, 1990). The latter means that they are armed with the basic equipment, usually harvests only personally, but there is a crowd of this type of players. Next are the commercial heavy equipment users with a lot of vessels usually acting as factories. Carrying out harvesting is costly for the second type of players, e.g. damages in the available equipment, payroll for working men, also reducing its financial capital.

Considering now the depletion of the renewable resource stock (the harvesting function), one can thought that however, does not only depend on the intensive usage

$\nu(t)$ of the heavy equipped player, but is also influenced by the other players' overall effort $u(t)$ which act traditionally. We set as instrument variables the intensity of equipment and the personal harvesting effort respectively i.e. for the heavy equipped player (player type 2) the intensity of the harvesting equipment's usage $\nu(t)$, and for traditional fishermen (players of kind 1) its personal effort $u_i(t)$, both assumed non-negatives $\nu(t) \geq 0, \quad u_i(t) \geq 0$.

We denote the overall harvesting function by $\phi(u,\nu)$, also depending on overall effort $u(t) = \sum_i u_i(t)$ and on intensity as well. Combining the growth $g(x)$ with the harvesting function $\phi(u,\nu)$ the state dynamics can be written as

$$\dot{x} = g(x) - \phi(u,\nu), \qquad x(0) = x_0 > 0 \tag{1}$$

Along a trajectory the non-negativity constraint is imposed, that is

$$x(t) \geq 0 \quad \forall t \geq 0 \tag{2}$$

A higher intensity of harvesting equipment usage (for player 2) and also the effort of the crowd of traditionally acting fishermen (player 1) certainly leads to stronger depletion of the renewable resource, so it is enough reasonable to assume that the partial derivatives of the harvesting function to be positive with respect to the parameters, i.e. $\phi_u > 0, \ \phi_\nu > 0$. Moreover the law of diminishing returns is applied only for the type 1 player's effort undertaken, that is $\phi_{uu} < 0$ and for simplicity we assume $\phi_{\nu\nu} = 0$. Additionally, we assume that the Inada conditions, which guarantee that the optimal strategies are nonnegative, holds true, i.e.

$$\begin{aligned} &\lim_{u\to 0} \phi_u(u,\nu) = \infty, \qquad \lim_{u\to\infty} \phi_u(u,\nu) = 0 \\ &\lim_{\nu\to 0} \phi_\nu(u,\nu) = 0, \qquad \lim_{\nu\to\infty} \phi_\nu(u,\nu) = \infty \end{aligned} \tag{3}$$

The utility functions the two players want to maximize are defined as follows: Player 1, the representative traditional fishermen, derive instantaneous utility, on one hand from its own harvesting product, but their personal effort $u(t)$ gives rise to increasing and convex costs $a(u)$, and on the other hand from the high stock of renewable resource also denoted by the increasing function $\varphi(x)$.

After all the present value of payer's 1 utility is described by the following functional

$$J_1 = \int_0^\infty e^{-\rho_1 t} \left[\phi(u,\nu) + \varphi(x) - a(u) \right] dt \qquad (4)$$

Player 2, the heavy equipped, enjoys utility $\upsilon(x)$ from the renewable resource stock $x(t)$, but also from the equipment's intensity of use ν, which is described by the function $\beta(\nu)$. Utilities $\upsilon(x)$ and $\beta(\nu)$ are assumed monotonically increasing functions with decreasing marginal returns, that is $\upsilon'(x) > 0,\ \beta'(\nu) > 0$ and $\upsilon''(x) < 0,\ \beta''(\nu) < 0$. We also assume that the individually acting players' overall effort u has no impact on player's 2 utility. So, player's 2 utility function is defined, in additively separable form, as:

$$J_2 = \int_0^\infty e^{-\rho_2 t} \left[\upsilon(x) + \beta(\nu) \right] dt \qquad (5)$$

3.1. Periodic Solutions

In this subsection we explore whether periodic solutions are possible, starting with steady state and stability analysis of necessary conditions. As it is clear the problem can be treated as a differential game with two controls and one state. Corresponding Hamiltonians, optimality conditions and adjoint variables for the problem under consideration are respectively:

$$H_1 = \phi(u,\nu) + \varphi(x) - a(u) + \lambda_1\left(g(x) - \phi(u,\nu)\right)$$

$$H_2 = \upsilon(x) + \beta(\nu) + \lambda_2\left(g(x) - \phi(u,\nu)\right)$$

$$\frac{\partial H_1}{\partial u} = (1-\lambda_1)\phi_u(u,\nu) - a'(u) = 0 \tag{6}$$

$$\frac{\partial H_2}{\partial \nu} = \beta'(\nu) - \lambda_2\phi_\nu(u,\nu) = 0 \tag{7}$$

$$\dot{\lambda}_1 = \rho_1\lambda_1 - \frac{\partial H_1}{\partial x} = \lambda_1\left[\rho_1 - g'(x)\right] - \varphi'(x) \tag{8}$$

$$\dot{\lambda}_2 = \rho_2\lambda_2 - \frac{\partial H_2}{\partial x} = \lambda_2\left[\rho_2 - g'(x)\right] - \upsilon'(x) \tag{9}$$

where subscripts denote player 1 and player 2 respectively for Hamiltonias H_i and adjoints λ_i $i = 1,2$.

Steady state solutions for the state, adjoints and controls are solutions of the system of equations:

$g(x) = \phi(u,\nu)$, $\lambda_1\left[\rho_1 - g'(x)\right] - \varphi'(x) = 0$, $\lambda_2\left[\rho_2 - g'(x)\right] - \upsilon'(x) = 0$

$(1-\lambda)\phi_u(u,\nu) - a'(u) = 0$, $\beta'(\nu) - \mu\phi_\nu(u,\nu) = 0$.

The Jacobian matrix of the system of optimality conditions is the following

$$J = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \lambda_1} & \frac{\partial \dot{x}}{\partial \lambda_2} \\ \frac{\partial \dot{\lambda}_1}{\partial x} & \frac{\partial \dot{\lambda}_1}{\partial \lambda_1} & \frac{\partial \dot{\lambda}_1}{\partial \lambda_2} \\ \frac{\partial \dot{\lambda}_2}{\partial x} & \frac{\partial \dot{\lambda}_2}{\partial \lambda_1} & \frac{\partial \dot{\lambda}_2}{\partial \lambda_2} \end{pmatrix} = \begin{pmatrix} g'(x) & -\frac{\partial \phi(u,\nu)}{\partial \lambda_1} & -\frac{\partial \phi(u,\nu)}{\partial \lambda_2} \\ -\lambda_1 g''(x) - \varphi''(x) & \rho_1 - g'(x) & 0 \\ -\lambda_2 g''(x) - \upsilon''(x) & 0 & \rho_2 - g'(x) \end{pmatrix}$$

This also gives:

$$\text{tr}\,(J) = \rho_1 + \rho_2 - g'(x) \quad \text{and}$$

$$\det(J) = g'(x)(\rho_1 - g'(x))(\rho_2 - g'(x)) - \frac{\partial\phi(u,\nu)}{\partial\lambda_1}(\lambda_1 g''(x) + \varphi''(x))(\rho_2 - g'(x)) -$$
$$- \frac{\partial\phi(u,\nu)}{\partial\lambda_2}(\lambda_2 g''(x) + \upsilon''(x))(\rho_1 - g'(x))$$

According to Wirl (1997) the existence of a pair of purely imaginary eigenvalues requires that the following conditions are satisfied:

$$\text{tr}(J) > 0, \quad \det(J) > 0, \quad w > 0, \quad \det(J) = w \ \text{tr}(J)$$

where coefficient w is the result of the sum of the following determinants

$$w = \begin{vmatrix} g'(x) & -\frac{\partial\phi(u,\nu)}{\partial\lambda_1} \\ -\lambda_1 g''(x) - \varphi''(x) & \rho_1 - g'(x) \end{vmatrix} + \begin{vmatrix} \rho_1 - g'(x) & 0 \\ 0 & \rho_2 - g'(x) \end{vmatrix} +$$

$$+ \begin{vmatrix} g'(x) & -\frac{\partial\phi(u,\nu)}{\partial\lambda_2} \\ -\lambda_2 g''(x) - \upsilon''(x) & \rho_2 - g'(x) \end{vmatrix} =$$

$$= \rho_1\rho_2 - [g'(x)]^2 - \frac{\partial\phi(u,\nu)}{\partial\lambda_1}[\lambda_1 g''(x) + \varphi''(x)] - \frac{\partial\phi(u,\nu)}{\partial\lambda_2}[\lambda_2 g''(x) + \upsilon''(x)]$$

From now on the crucial condition for cyclical strategies (precisely for Hopf bifurcations to occur) is that

$$w > 0, \quad w = \frac{\det(J)}{\text{tr}(J)}$$

This after simple algebraic calculations reduces to

$$\rho_1\rho_2[\rho_1 + \rho_2 - 2g'(x)] =$$
$$= \frac{\partial\phi(u,\nu)}{\partial\lambda_1}[\lambda_1 g''(x) + \varphi''(x)]\rho_1 + \frac{\partial\phi(u,\nu)}{\partial\lambda_2}[\lambda_2 g''(x) + \upsilon''(x)]\rho_2 \qquad (10)$$

3.2. Specifications for the game

We specify the functions of the game as follows:

- A diffusion process for the renewable resource growth function, that is

$g(x) = rx(1-x)$,

- A Cobb–Douglas type function for the harvesting $\phi(u,\nu) = u^{\gamma}\nu$ and
- The utility function stemming from equipment's intensive use of player 2 in the form $\beta(\nu) = A - \nu^{(\xi-1)}/(1-\xi)$.

Note that the utility function $\beta(\nu)$ with $A > 0$ and $\xi \in (0,1)$ exhibits constant relative risk aversion in the sense of Arrow–Pratt measure of risk aversion. All the other functions are left in a linear form, i.e. both utilities stemming from the existing renewable resource stock are for player 1 $\varphi(x) = \varphi x$ and for player 2 $\upsilon(x) = \upsilon x$, while player's 1 effort cost in the linear fashion $a(u) = au$, as well. Note that all the involved coefficients, i.e. the intrinsic growth rate r and the slopes φ, υ and a are positive real numbers, but $\gamma \in (0,1)$ and $A > 0$ and $\xi \in (0,1)$, as already mentioned.

With the above specifications the following result holds true.

Proposition 1

A necessary condition for cyclical strategies in the game between traditionally acting and heavy equipped players, as described above, is the heavy equipped players are more impatient than the simple traditionally acting.

Proof

In Appendix

The intuition behind proposition 1 is straightforward. We start with a rather low and increasing intensity of equipment usage on behalf of the heavy equipped players. The traditionally acting players operate at a low effort, as well, because the increasing effort incurs costs, but they are worrying about the renewable resource level, consequently for their jobs, by the reason of player's 2 presences. Now suppose that the heavy equipped react as a farsighted, he would increase the equipment's intensity only moderately and the dynamical system would approach a stable steady state. But, due to their impatience they behave myopically and react by strongly increasing the intensity of their machines.

At this time the crowd of the traditionally acting players, has only two choices: to loose their jobs or to increase their overall effort. Suppose that they stay in the harvesting increasing their overall effort. But the latter means that the combination of high intensity on behalf of the heavy equipped and the higher effort on behalf the crowd leads to a strong reduction of the renewable resource stock. But the low level of the resource stock is unprofitable for the heavy equipped to work at a high intensity, therefore they have to decrease intensity and the cycle is closed. A new cycle starts again, possibly in another place because of the stock's reduction, but with the same results also described.

In our opinion the crucial point of this intuitive explanation is that player's 1 strategic variable u lags behind player's 2 strategic variable ν and both are lagged behind the state variable, the renewable resource's stock x.

3.3 The linear example

Let us now calculate the Nash equilibrium of the harvesting differential game. The concept of open-loop Nash equilibrium is based on the fact that every player's strategy is the best reply to the opponent's exogenously given strategy. Obviously, equilibrium holds if both strategies are simultaneously best replies.

Following Dockner *et al* (2000), we formulate the current value Hamiltonians for both players, as follows

$$H_1 = \phi(u,\nu) + \varphi(x) - a(u) + \lambda_1\left(g(x) - \phi(u,\nu)\right)$$

$$H_2 = \upsilon(x) + \beta(\nu) + \lambda_2\left(g(x) - \phi(u,\nu)\right)$$

The first order conditions, for the maximization problem, are the following system of differential equations for both players:

First, the maximized Hamiltonians are

$$\frac{\partial H_1}{\partial u} = (1-\lambda_1)\phi_u(u,\nu) - a'(u) = 0 \tag{11}$$

$$\frac{\partial H_2}{\partial \nu} = \beta'(\nu) - \lambda_2 \phi_\nu(u,\nu) = 0 \tag{12}$$

and second the costate variables are defined by the equations

$$\dot{\lambda}_1 = \rho_1\lambda_1 - \frac{\partial H_1}{\partial x} = \lambda_1\left[\rho_1 - g'(x)\right] + \varphi'(x) \tag{13}$$

$$\dot{\lambda}_1 = \rho_2\lambda_1 - \frac{\partial H_2}{\partial x} = \lambda_2\left[\rho_2 - g'(x)\right] + \upsilon'(x) \tag{14}$$

The Hamiltonian of player 1, H_1, is concave in the control u as far as $\lambda_1 < 1$ and is guaranteed by the assumptions on the signs of the derivatives, i.e. $\phi_{uu} < 0$, $\phi_{\nu\nu} = 0$ and from the decreasing marginal returns on the player's 2 utilities, i.e. $\upsilon''(x) < 0$, $\beta''(\nu) < 0$. Moreover, optimality condition (54) implies that the adjoint variable λ_1 is positive only if player's 1 marginal utility ϕ_u exceeds the marginal costs, since $\lambda_1 = \left(\phi_u(u,\nu) - a'(u)\right) / \phi_u(u,\nu)$.

We also assume linearity of the model. A linear population growth function, despite the critique as a fairly unrealistic model, is a good approximation for the exponential growth of the human population since 1900 (Murray, 2002). To be more precise we specify the following functions of the game to be in linear form:

i. the renewable resource's growth function in the form $g(x) = \omega \cdot x$, where ω is the growth rate,

ii. the utility function, $\varphi(x)$, which stems from the high stock of the renewable resource, in the form $\varphi(x) = \varphi \cdot x$

iii. the function that measures player's 1 effort cost in the form $u(t) = a \cdot u$

All the constants involved are positive numbers, that is $\omega, \varphi,\ a > 0$. From the second player's side, the functions maximized are specified as linear, i.e. the utilities arisen from the resource stock and high intensity realizations are written as $\upsilon(x) = \upsilon \cdot x(t)$ and $\beta(\nu) = \beta \cdot \nu(t)$ respectively.

After the above simplified specifications the canonical system of equations (11) - (12) can be rewritten as follows

$$\frac{\partial H_1}{\partial u} = (1 - \lambda_1)\phi_u(u,\nu) - a = 0 \tag{15}$$

$$\frac{\partial H_2}{\partial \nu} = \beta - \lambda_2 \phi_\nu(u,\nu) = 0 \tag{16}$$

$$\dot{\lambda}_1 = \rho_1 \lambda_1 - \frac{\partial H_1}{\partial x} = \lambda_1[\rho_1 - \omega] - \varphi \tag{17}$$

$$\dot{\lambda}_2 = \rho_2 \lambda_2 - \frac{\partial H_2}{\partial x} = \lambda_2[\rho_2 - \omega] - \upsilon \tag{18}$$

and the limiting transversality conditions has to hold

$$\lim_{t\to\infty} e^{-\rho_1 t} x(t)\lambda_1(t) = 0, \quad \lim_{t\to\infty} e^{-\rho_2 t} x(t)\lambda_2(t) = 0 \tag{19}$$

The analytical expressions of the adjoint variables (λ_1, λ_2), solving equations (18)-(19), are respectively:

$$\lambda_1(t) = \frac{\varphi}{\rho_1 - \omega} + e^{(\rho_1-\omega)t} C_1 \tag{20}$$

$$\lambda_2(t) = \frac{\upsilon}{\rho_2 - \omega} + e^{(\rho_2-\omega)t} C_2 \tag{21}$$

In order the transversality conditions to be satisfied it is convenient to choose the constant steady state values, and therefore the adjoint variables collapses to the following constants

$$\lambda_1 = \frac{\varphi}{\rho_1 - \omega}, \quad \lambda_2 = \frac{\upsilon}{\rho_2 - \omega} \tag{22}$$

To ensure certain signs for the adjoints (22) we impose another condition on the discount rates, which claim that discount rates are greater than the resource's growth, i.e. we impose the condition

$$\rho_i > \omega, \quad i = 1,2$$

thus, the constant adjoint variables have both positive signs.

The above condition seems to be restrictive but can be justified as otherwise optimal solutions do not exist. Indeed, choosing $\rho_2 < \omega$, player's 2 discount rate to be lower than the resource's growth rate, their objective functional becomes unbounded in the case they choose to carry out no harvesting. Similarly, choosing player's 1 discount rate lower than the growth rate the associated adjoint variable λ becomes a positive quantity in the long run. As a shadow price is implausible to be positive for optimal solutions, the above reasoning is sufficient for the assumption $\rho_i > \omega, \quad i = 1,2$.

Once the concavity of the Hamiltonians, with respect to the strategies, for both players is satisfied the first order conditions guarantee its maximization. Now, we choose the harvesting function's $\phi(u,\nu)$ specification, i.e. the specification of the function that reduces the renewable resource. This function is depending on both effort and intensity. We choose a similar to Cobb–Douglas production function specification, which is characterized by constant elasticities in the following form:

$$\phi(u,\nu) = u^{\sigma}\nu^{\zeta} \qquad 0 < \sigma < 1 < \zeta$$

The rest of this section is devoted to the calculations of the explicit formulas at the Nash equilibrium.

3.4. Optimal Nash Strategies

Applying first order conditions for the chosen specification function gives us

$$\phi_u(u,\nu) = \frac{a}{1-\lambda_1} \quad \Leftrightarrow \quad \sigma u^{\sigma-1}\nu^{\zeta} = \frac{a}{1-\lambda_1} \tag{23}$$

$$\phi_\nu(u,\nu) = \frac{\beta}{\lambda_2} \quad \Leftrightarrow \quad \zeta u^{\sigma}\nu^{\zeta-1} = \frac{\beta}{\lambda_2} \tag{24}$$

The combination of (23) and (24), using the Cobb–Douglas type of specification, reveals an existing interrelationship between strategies, that is

$$\phi(u^*,\nu^*) = (u^*)^{\sigma}(\nu^*)^{\zeta} \quad \Leftrightarrow \quad \frac{au^*}{\sigma(1-\lambda_1)} = \frac{\beta\nu^*}{\zeta\lambda_2} \quad \Leftrightarrow \quad \nu^* = u^*\frac{a\zeta\lambda_2}{\sigma(1-\lambda_1)\beta} \tag{25}$$

Expression (25) now predicts the interrelationship between the players' Nash strategies, for which the result of comparison between them is dependent on the constant parameters and on the constant adjoint variables, as well.

Substituting (25) back into (24) we are able to find the analytical expressions of the strategies, after the following algebraic calculations. Expression (24) now becomes:

$$\left(u^*\right)^{\sigma+\zeta-1} = \left[\frac{a}{\sigma\left(1-\lambda_1\right)}\right]^{1-\zeta}\left(\frac{\zeta\lambda_2}{\beta}\right)^{1-\zeta}\left(\frac{\lambda_2\zeta}{\beta}\right)^{-1} = \left[\frac{a}{\sigma\left(1-\lambda_1\right)}\right]^{1-\zeta}\left(\frac{\lambda_2\zeta}{\beta}\right)^{-\zeta}$$

and from the latter the analytical expressions for the equilibrium strategies are derived in a more comparable form now:

$$u^* = \left[\frac{a}{\sigma\left(1-\lambda_1\right)}\right]^{\frac{1-\zeta}{\sigma+\zeta-1}}\left(\frac{\zeta\lambda_2}{\beta}\right)^{\frac{-\zeta}{\sigma+\zeta-1}} \qquad (26)$$

$$\nu^* = \left[\frac{a}{\sigma\left(1-\lambda_1\right)}\right]^{\frac{\sigma}{\sigma+\zeta-1}}\left(\frac{\zeta\lambda_2}{\beta}\right)^{\frac{\sigma-1}{\sigma+\zeta-1}} \qquad (27)$$

Further substitutions in the equation of the resource's accumulation, $\dot{x} = \omega x - u^{\sigma}\nu^{\zeta}$, yield the following steady state value of the stock

$$x_{SS} = \frac{1}{\omega}\left[\frac{a}{\left(1-\lambda_1\right)\sigma}\right]^{\frac{\sigma}{\sigma+\zeta-1}}\left(\frac{\zeta\lambda_2}{\beta}\right)^{\frac{-\zeta}{\sigma+\zeta-1}} \qquad (28)$$

We summarize the above discussion in a proposition.

Proposition 2.

Assuming the harvesting function to exhibit constant elasticity and all the other functions to be linear, then the harvesting game yields constant optimal Nash strategies. The analytical expressions of the strategies are given by (26) and (27) for the traditional fishermen and the heavy equipped respectively. The steady state value of the resources' stock is given by expression (28).

Proposition 2 seems to be with a little economic meaning caused by the linearity of the paradigm. But the constancy of the resulting strategies can be seen in connection with the concept of time consistency, a central property in economic theory. Time consistency is a minimal requirement for a strategy's credibility, but in general open-loop strategies haven't the time consistency property by default, since these strategies are time, and not state dependent functions. Nevertheless, a constant strategy may be a time consistent one, since the crucial characteristic for time consistency, i.e. the independency of any initial state x_0, is met for the above constant strategies.

5. Conclusions

In environmental economics the exploitation of renewable resources it is a well overlooked field since the original model, dated back to Schaffer (1994). As it is well known the analysis concentrates on the two basic factors that affect the fishing industry, namely the size of the resource itself and the rate of human harvesting. The above specification does not take into account any other human activities which affect biomass, for example coastlines pollution.

Concerning long–run equilibrium, as it is well known, the simplest case of the saddle–point type stability requires only one characteristic of the renewable resource's growth function, i.e. the negative growth. But even the supposition of negative growth is sufficient for the saddle-point stability, the local monotonicity is not implied i.e. transient cycles may occur.

On the other hand harvesting management is not restricted in the traditional way of the renewable resource extraction in the sense of one man show. Commercial harvesting often requires investment and disinvestment in equipment, and the undertaken decision to expand or to reduce equipment obeys onto the state variable which is the existing renewable resource stock. Therefore, concerning harvesting, as a stock variable, equilibrium dynamics becomes more complex, and therefore much richer, also including saddle–point stability. In the discussion, in the main paper maid, the dynamics of such equilibrium dynamics reveal cyclical policies as optimal strategies, but from the above discussion only some conclusions has been drawn.

The emphasis given in the paper is restricted on the stability properties of the induced nonzero sum game between two types of players, which share a common depletion function thought as a harvesting. Precisely, the game set up between a crowd of weakly armed and a strongly armed player with a common depletion function yields an economic result, for which the discount rate plays the crucial role for periodic solutions. That is, the condition for periodic solutions is that the strongly equipped player to be more impatient than the weakly. Finally, for the supplement linear example of the same game we compute the optimal Nash strategies for both players, which are constant expressions.

Appendix

Proof of proposition 1

With the specifications, given in subsection 4.2, one can compute

$g'(x)=r(1-2x)$, $g''(x)=-2r$, $\phi_u(u,\nu)=\gamma u^{\gamma-1}$, $\phi_\nu(u,\nu)=u^\gamma$, $a'(u)=a$, $\beta'(\nu)=\nu^{\xi-2}$, $\varphi'(x)=\varphi$, $\upsilon'(x)=\upsilon$

$$\frac{\partial H_1}{\partial u}=0 \Leftrightarrow (1-\lambda_1)\phi_u(u,\nu)=a'(u) \Leftrightarrow (1-\lambda_1)\gamma u^{\gamma-1}\nu=a \tag{A.1}$$

$$\frac{\partial H_2}{\partial \nu}=0 \Leftrightarrow \beta'(\nu)=\lambda_2\phi_\nu(u,\nu) \Leftrightarrow \lambda_2 u^\gamma=\nu^{\xi-2} \tag{A.2}$$

Combining $(A.1)$ and $(A.2)$ the optimal strategies take the following forms

$$u^*=\lambda_2^{-1/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{(\xi-2)/[1+(1-\xi)(1-\gamma)]} \quad (A.3),$$

$$\nu^*=\lambda_2^{(\gamma-1)/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{\gamma/[1+(1-\gamma)(1-\xi)]} \tag{A.4}$$

and the optimal harvesting becomes

$$\phi(u^*,\nu^*)=\lambda_2^{-1/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{\gamma(\xi-1)/[1+(1-\gamma)(1-\xi)]} \tag{A.5}$$

with the following partial derivatives

$$\frac{\partial\phi}{\partial\lambda_1}=\frac{\lambda_2^{-1/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{\gamma(\xi-1)/[1+(1-\gamma)(1-\xi)]}}{(1-\lambda_1)}\frac{\gamma(\xi-1)}{1+(1-\xi)(1-\gamma)}= \tag{A.6}$$
$$=\frac{\phi(u^*,\nu^*)}{(1-\lambda_1)}\frac{\gamma(\xi-1)}{1+(1-\xi)(1-\gamma)}$$

$$\frac{\partial\phi}{\partial\lambda_2}=\frac{\lambda_2^{-1/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{\gamma(\xi-1)/[1+(1-\gamma)(1-\xi)]}}{\lambda_2}\frac{-1}{1+(1-\xi)(1-\gamma)}= \tag{A.7}$$
$$=\frac{\phi(u^*,\nu^*)}{\lambda_2}\frac{-1}{1+(1-\xi)(1-\gamma)}$$

Both derivatives (A.6), (A.7) are negatives due to the assumptions on the parameters $\gamma,\ \xi \in (0,1)$ and on the signs of derivates, that is $\phi_u > 0,\ \phi_\nu > 0,\ \upsilon'(x) > 0,\ \varphi'(x) > 0$, which ensures the positive sign of the adjoints $\lambda_1,\ \lambda_2$.

Condition $w\ = \dfrac{\det(J)}{\operatorname{tr}(J)}$ now becomes

$\rho_1\rho_2\left[\rho_1+\rho_2-2g'(x)\right]=\lambda_1\rho_1 g''(x)\dfrac{\partial\phi}{\partial\lambda_1}+\lambda_2\rho_2 g''(x)\dfrac{\partial\phi}{\partial\lambda_2}$, which after substituting the values from (A.6), (A.7) and making the rest of algebraic manipulations, finally yields (at the steady states)

$$\frac{\phi(u_\infty,\nu_\infty)g''(x)}{1+(1-\xi)(1-\gamma)}\left[\rho_1\gamma(1-\xi)\frac{\varphi}{\varphi+g'(x)-\rho_1}-\rho_2\right]-\rho_1\rho_2\left[\rho_1+\rho_2-2g'(x)\right]=0 \qquad (\text{A.8})$$

Where we have set $\dfrac{\lambda_1}{1-\lambda_1}=\dfrac{\varphi}{\rho_1-g'(x)-\varphi}$ stemming from the adjoint equation $\dot{\lambda}_1=\lambda_1\left(\rho_1-g'(x)\right)-\varphi'(x)$, which at the steady states reduces into $\lambda_1=\varphi'(x)/\left(\rho_1-g'(x)\right)$.

Condition $w>0$ after substitution the values from (A.6), (A.7) becomes

$$w=\rho_1\rho_2-\left[g'(x)\right]^2+\frac{\phi(u,\nu)g''(x)}{1+(1-\xi)(1-\gamma)}\left[\gamma(1-\xi)\frac{-\varphi}{g'(x)+\varphi-\rho_1}+1\right]>0 \qquad (\text{A.9})$$

The division (A.8) by ρ_1 yields

$$\frac{\phi(u_\infty,\nu_\infty)g''(x)}{1+(1-\xi)(1-\gamma)}\left[\gamma(1-\xi)\frac{\varphi}{\varphi+g'(x)-\rho_1}-\frac{\rho_2}{\rho_1}\right]-\rho_2\left[\rho_1+\rho_2-2g'(x)\right]=0 \quad (\text{A.10})$$

The sum $(\text{A.9})+(\text{A.10})$ must be positive, thus after simplifications and taking into account that $\phi(u_\infty,\nu_\infty)=g(x)$, we have:

$g(x)g''(x)\dfrac{\rho_1-\rho_2}{\rho_1\left[1+(1-\xi)(1-\gamma)\right]}>\left[\rho_2-g'(x)\right]^2$ and the result $\rho_2>\rho_1$ follows from the strict concavity of the logistic growth $g''<0$.

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