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Abstract.   The paper is an eclectic study of the uses of the Kalman filter in existing econometric literature. An effort is made to introduce the various extensions to the linear filter first developed by Kalman(1960) through examples of their uses in economics. The basic filter is first derived and then some applications are reviewed.

Keywords.   Kalman Filter; Time-varying Parameters; Stochastic Volatility; Markov Switching

1   Introduction

In statistics and economics, a filter is simply a term used to describe an algorithm that allows recursive estimation of unobserved, time varying parameters, or variables in the system. It is different from forecasting because forecasts are made for future, whereas filtering obtains estimates of unobservables for the same time period as the information set. The Kalman filter is a recursive linear filter, first developed as a discrete filter for use in engineering applications and subsequently adopted by statisticians and econometricians. The basic idea behind the filter is simple - to arrive at a conditional density function of the unobservables using Bayes' Theorem, the functional form of relationship with observables, an equation of motion and assumptions regarding the distribution of error terms. The filter uses the current observation to predict the next period's value of unobservable and then uses the realization next period to update that forecast. The linear Kalman filter is optimal, i.e. Minimum Mean Squared Error estimator if the observed variable and the noise are jointly Gaussian. Else, it is best among

the class of linear filters.

The paper discusses the linear Kalman filter, its derivation and some applications in economics. The basic linear filter with Gaussian, uncorrelated error terms is often inadequate for economic applications. Several extensions have been developed for adapting the algorithm to handle non-linear measurement equations, non-gaussian or correlated error terms. These and their related economic applications are discussed in Section 3. Section 4 concludes.

2   The Kalman Filter

Let $Z_t \in \Re^n$, be the observed values for variable(s) Z and let $X_t \in \Re^m$ be the vector of unobserved variable(s) of interest (also called the state(s) of the nature)[1]. The relationship between Z and X is assumed known and described by the measurement equation:

$$Z_t = H_t' X_t + v_t \tag{1}$$

where H_t is known, and v_t is Gaussian white noise with $E[v_t.v_s'] \equiv R_t \delta_{ts}$ where δ_{ts} is Kronecker delta, which is 1 for $t = s$ and 0 otherwise. X_t is assumed to evolve according to the equation of motion:

$$X_{t+1} = F_t X_t + w_t \tag{2}$$

where w_t is Gaussian white noise, with $E([w_t.w_s'] = Q_t \delta_{ts}$

Additional assumptions are that v_t and w_t are independent, initial state X_0 is a Gaussian random variable with mean $E[X_0|Z_{-1}] = E[X_0] = \bar{X}_0$ and $Var(X_0|Z_{-1}) = \Sigma_0$, independent of w_t and v_t. The Kalman filter gives an algorithm to determine the estimates $\hat{X}_{t|t-1} \equiv E[X_t|Z_{t-1}]$ and $\hat{X}_{t|t} \equiv E[X_t|Z_t]$ the corresponding covariance matrices $\Sigma_{t|t-1}$ and $\Sigma_{t|t}$. It comprises of the following equations:

$$\hat{X}_{t+1|t} = [F_t - K_t H_t']\hat{X}_{t|t-1} + K_t Z_t \tag{3}$$

$$\hat{X}_{0|-1} = \bar{X}_0 \tag{4}$$

_____

[1]The discussion in this section is based on Anderson and Moore (1979) and Meinhold and Singpurwalla(1983)

$$K_t = F_t \Sigma_{t|t-1} H_t [H_t' \Sigma_{t|t-1} H_t + R_t]^{-1} \tag{5}$$

$$\Sigma_{t+1|t} = F_t [\Sigma_{t|t-1} - \Sigma_{t|t-1} H_t (H_t' \Sigma_{t|t-1} H_t + R_t)^{-1} H_t' \Sigma_{t|t-1}] F_t' + G_t Q_t G_t' \tag{6}$$

$$\Sigma_{0|-1} = \Sigma_0 \tag{7}$$

$$\hat{X}_{t|t} = \hat{X}_{t|t-1} + \Sigma_{t|t-1} H_t (H_t' \Sigma_{t|t-1} H_t + R_t)^{-1} (Z_t - H_t' X_{t|t-1}) \tag{8}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1} H_t (H_t' \Sigma_{t|t-1} H_t + R_t)^{-1} H_t' \Sigma_{t|t-1}] \tag{9}$$

Notice that (4) and (8) imply:

$$F_t \hat{X}_{t|t} = K_t Z_t + (F_t - K_t H_t') \hat{X}_t | t - 1 \tag{10}$$

so (3) is equivalent to

$$\hat{X}_{t+1|t} = F_t \hat{X}_{t|t} \tag{11}$$

The matrix K_t is called the gain matrix and equation (6), which determines recursively the conditional error covariance matrix, is called the Riccati equation. The equations can be understood better when the system is viewed as unidimensional. Equations (3) and (6) are the prediction equations, which give the optimal estimates of future values based on current information set and equations (8) and (9) are updation equations that update the previous period's forecast based on the current realization of the observable. Notice that the gain matrix K_t depends inversely on R_t - the larger the variance of the measurement error, the lower the weight given to the measurement in making the forecast for the next period, given today's information set. A similar relationship holds when predicting the value of $X_{t|t}$ (8) - the forecast made with the previous period's information set is updated by the difference between the current measurement and the previous period's forecast of that measurement (i.e. $Z_t - H_t' \hat{X}_{t|t-1}$), but the weight attached to this error depends inversely on the variance of v_t.

When X_t is non-stationary, the algorithm can be initialized with arbitrary values for $X_{0|-1}$ and $\Sigma_{0|1}$, but with large diagonal elements for the latter to reflect the uncertainty about $X_{0|-1}$. Most of the weight would then be given to the new information in the second round of iteration. Also, the filter assumes that F_t, H_t, R_t and Q_t are known. When these are unknown, they can be estimated using Maximum Likelihood Estimation (MLE). For given values of parameters, the Kalman Filter gives $\eta_{t|t-1} = Z_t - Z_{t|t-1}$ and the conditional variance of the forecast error, $D_{t|t-1} \equiv E[\eta_{t|t-1}^2] = H_t' \Sigma_{t|t-1} H_t + R_t$. If X_0, v_t and w_t are Gaussian, then the conditional distribution of Z_t is also normal and MLE can be used to estimate unknown

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parameters.

One of the main strength of the algorithm comes from its recursive nature. The filter has consecutive prediction and updation cycles, whereby an estimate of X_t is first obtained based on information at t-1 and the new observation Z_t is used to update and improve the prediction. This means that the filter automatically utilises all information contained in previous forecasts and information sets, without having to store and process the entire historical data at every step.

Now we derive equations (3) - (9) from first principles. The random variable $[X_0' \ Z_0']'$ has mean $[\overline{X}_0' \ \overline{X}_0' H_0']'$ and covariance:

$$\begin{bmatrix} P_0 & P_0 H_0 \\ H_0' P_0 & H_0' P_0 H_0 + R_0 \end{bmatrix}$$

Since X_0 and Z_0 are jointly gaussian, X_0 conditioned on Z_0 has mean

$$\hat{X}_{0|0} = \overline{X}_0 + P_0 H_0 (H_0' P_0 H_0 + R_0)^{-1} (Z_0 - H_0' \overline{X}_0)$$

and covariance

$$\Sigma_{0|0} = P_0 - P_0 H_0 (H_0' P_0 H_0 + R_0)^{-1} H_0' P_0$$

The independence assumptions (1) then imply that $X_0|Z_0$ is normally distributed with mean

$$\hat{X}_{1|0} = F_0 \hat{X}_{0|0}$$

and covariance $\Sigma_{1|0} = F_0 \Sigma_{0|0} F_0' + G_0 Q_0 G_0'$
These and (2) imply that $Z_1|Z_0$ is normally distributed with mean and covariance

$$\hat{Z}_{1|0} = H_1' \hat{X}_{1|0} \text{ and } H_1' \Sigma_{1|0} H_1 + R_1$$

This implies that $E[(X_{1|0} - \hat{X}_{1|0})(Z_{1|0} - \hat{Z}_{1|0})|Z_0] = \Sigma_{1|0} H_1$ This implies that $[X_1' \ Z_1']'$ conditioned on Z_0 has mean $[\hat{X}_{1|0}' \ H_1' \hat{X}_{1|0}]'$ and covariance:

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$$\begin{bmatrix} \Sigma_{1|0} & \Sigma_{1|0}H_1 \\ H_1'\Sigma_{1|0} & H_1'\Sigma_{1|0}H_1 + R_1 \end{bmatrix}$$

Using this, we deduce that $X_1|(Z_0, Z_1)$ has mean

$$\hat{X}_{1|1} = \hat{X}_{1|0} + \Sigma_{1|0}H_1(H_1'\Sigma_{1|0}H_1 + R_1)^{-1}(Z_1 - H_1'\hat{X}_{1|0}$$

and covariance

$$\Sigma_{1|1} = \Sigma_{1|0} - \Sigma_{1|0}H_1(H_1'\Sigma_{1|0}H_1 + R_1)^{-1}H_1'\Sigma_{1|0}$$

Iterating the above steps, we get Equations (3) through (9).

3 Economic Applications of Kalman Filter

All ARMA models can be written in the state-space forms, and the Kalman filter used to estimate the parameters. It can also be used to estimate time-varying parameters in a linear regression and to obtain Maximum likelihood estimates of a state-space model. Another application of the filter is to obtain GLS estimates for the model $y_t = \beta' x_t + u_t$, where the error term u_t is Gaussian ARMA(p,q) with known parameters. This section discusses some economic models that have been estimated using either the linear Kalman filter described above, or its extensions.

3.1 Time Varying Parameters in a Linear Regression: Demand for International Reserves

The classical regression model, $y_t = \beta' x_t + u_t$ where u_t is white noise, assumes that the relationship between the explanatory and explained variables remains constant through the estimation period. When this assumption is an unreasonable one (for example, while studying macroeconomic relationships for countries that have undergone structural reforms during the sample period, for example, India in 1991 and the erstwhile Socialist Republics), and the model is specified as one with $\beta_t's$, the Kalman filter can be used to estimate the parameters. An example of this approach is the study by Bahmani-Oskooee and Brown (2004) that postulates structural changes in demand for international reserves during the 1970's. The reserve demand (R_t) of a country is specified as a function of its real imports (M_t), a variablility measure of balance of payments (VR_t), and its average propensity

to import (m_t). i.e.

$$logR_t = \beta_0 + \beta_1 logM_t + \beta_2 logVR_t + \beta_3 logm_t + \epsilon_t \qquad (12)$$

The βs are assumed to follow a random walk. The instability of βs is first demonstrated (and then estimates of time-varying parameters obtained using the Kalman Filter) by estimating rolling regressions. For the same sample size, the beginning of the sample period is shifted by one to repeatedly estimate $y_t = \beta' x_t + u_t$, correcting for serial correlation in errors. Quarterly data for 19 OECD countries is used, for the period 1959-94. The problem with this specification is that it ignores the supply side and takes the equilibrium quantities as realised demands. Another issue here (and with all time-varying parameter models) is that in order for the system to be identified, the βs are assumed to be a random walk. This would, without further restrictions, mean that the dependent variable is non-stationary (since it is a linear combination of the $\beta's$) and invalidate the usual t and F tests.

3.2 Modeling Regime Changes: Markov Switching Models

A number of macroeconomic and financial variables can plausibly be modeled to have different statistical and dynamic properties depending on the state of the nature and for the probabilities of moving from one state of nature to another to be well defined and constant. For example, the persistence of shocks to stock returns may be different during boom times than during recessions. These can be modeled using Markov Switching model if we assume that the switch between the boom and recession is governed by a Markov chain (and could alternatively be modeled using the Stochastic Volatility models discussed in Section 3.5 below).

Markov Switching approach can also be applied to extend or complement a number of other models. For example, in the time- varying parameters models discussed above, one could add a Markov structure to the variability of the parameters or add Markov Switching heteroskedasticity in the error term, to incorporate changing uncertainty due to future random shocks. In the the unobserved components models (see Section 3.4 below), for example where GDP is decomposed into trend and cyclical components, the trend component of the GDP may be modeled as a random walk with drift, where the latter evolves according to a Markov chain. Models of Markov Switching that can be put in state-space form can be estimated us-

ing the Kalman Filter. Such models may be written as:

$$Z_t = H_{S_t} X_t + A_{S_t} Y_t + v_t \qquad (13)$$

$$X_t = \widetilde{\mu} S_t + F_{S_t} X_{t-1} + G_{S_t} w_t \qquad (14)$$

$$\begin{pmatrix} v_t \\ w_t \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} R_{S_t} & 0 \\ 0 & Q_{S_t} \end{pmatrix} \right] \qquad (15)$$

where the subscripts S_t indicate that some elements of the concerned matrices may be state-dependent. The state, $S_t = 1, 2, ..., M$ is an unobserved, discrete-valued markov variable, with probabilities given by:

$$p = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{M1} \\ p_{12} & p_{22} & \cdots & p_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1M} & p_{2M} & \cdots & p_{MM} \end{pmatrix}$$

where $p_{ij} = Pr[S_T = j | S_t - 1 = i]$ with $\Sigma_{j=1}^M p_{ij} = 1$ for all i. The purpose here is to calculate estimates of X_t based on the information set at t-1, Ψ_{t-1}, conditional on S_t taking value j and S_{t-1} taking on value i. When the parameters of the model are known, the Kalman filter modifies as follows:

$$\hat{X}_{t|t-1}^{ij} = \widetilde{\mu_j} + F_j \hat{X}_{t-1|t-1}^{ij} \qquad (16)$$

$$\Sigma_{t|t-1}^{ij} = F_j [\Sigma_{t-1|t-1}^i F_j' + G_i Q_j G_j' \qquad (17)$$

$$\eta_{t|t-1}^{ij} = Z_t - H_j X_{t|t-1}^{ij} - A_i Y_t \qquad (18)$$

$$D_{t|t-1}^{ij} = H_j \Sigma_{t|t-1}^{ij} H_j' + R_j \qquad (19)$$

$$\hat{X}_{t|t}^{ij} = \hat{X}_{t|t-1}^{ij} + \Sigma_{t|t-1}^{ij} H_j' [D_{t|t-1}^{ij}]^{-1} \eta_{t|t-1}^{ij} \qquad (20)$$

where $\hat{X}_{t-1|t-1}^{ij}$ is the prediction of X_{t-1} based on information available at time t-1, and given state $S_{t-1} = i$, etc, $\eta_{t|t-1}^{ij} = Z_t^j - Z_{t|t-1}^{ij}$ and $D_{t|t-1}^{ij}$ is the conditional variance of the forecast error, $\eta_{t|t-1}^{ij}$. The above procedure, however is almost unimplementable as the number of cases would multiply M-fold with each iteration. To handle this, Kim and Nelson(1999) use the following procedure, which is a modification to the one suggested by Harrison and Stevens (1976). The idea is to collapse M x M posteriors into M posteriors at each stage. Although the resulting posteriors are approximations, they are crucial to making the procedure of any practical use.

$$\hat{X}_{t|t}^j = \frac{\Sigma_{i=1}^M Pr(S_{t-1}=i, S_t=j|Psi_t)X_{t|t}^{ij}}{Pr(S_t=j|\Psi_t)} \tag{21}$$

$$\Sigma_{t|t}^j = \frac{\Sigma_{i=1}^M Pr(S_{t-1}=i, S_t=j|Psi_t)[\Sigma_{t|t}^{ij} + (X_{t|t}^j X_{t|t}^{ij})(X_{t|t}^j X_{t|t}^{ij})']}{Pr(S_t=j|\Psi_t)} \tag{22}$$

The probabilities in the above equations are obtained through the Hamilton filter which essentially involves the prediction and updation rules used also in the Kalman filter.

The Hamilton filter gives conditional density of Z_t and $f(Z_t|\Psi_{t-1})$ for all t. These can be used to optimize the approximate log-likelihood function:

$$L = \Sigma_{t=1}^T ln(f(Z_t|\Psi_{t-1})) \tag{23}$$

with respect to underlying parameters using a non-linear optimizing procedure, which completes the description of the estimation procedure for the case where the parameters are not known.

3.3 Kalman Filter with Correlated Error Terms: Exchange Rate Risk Premia

The Kalman filter described in Section 2 assumes that the errors in the measurement and transition equations are uncorrelated. This assumption would fail in situations where shocks to a third factor cause movements in both the observed variable and the unobserved variable under consideration. An example of this can be found in the market for exchange rates, where new information that causes the spot rate to jump may also cause the risk premium to change. Examples of such new information include shocks to money supply and interest rates, a switch in currency regime, a repudiation of debt by the country or announced change in currency's convertibility. Cheung (1993) uses the Kalman filter algorithm for the state space model given by:

$$D_t = P_t + v_{t+1} \tag{24}$$

$$P_t = \phi P_{t-1} + a_t \tag{25}$$

$$\begin{pmatrix} a_t \\ v_t \end{pmatrix} \sim iidN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q^2 & C \\ C & R^2 \end{pmatrix}\right] \tag{26}$$

Also,

$$D_t \equiv F_t - S_{t+1} \tag{27}$$

$$P_t \equiv F_t - E_t S_{t+1} \tag{28}$$

$$v_{t+1} \equiv E_t S_{t+1} - S_{t+1} \tag{29}$$

where P_t is the unobservable risk premium, D_t is the prediction error from using forward rate as a one-period ahead forecast of the spot rate, F_t and S_t are one period ahead forward and spot exchange rates respectively. All variables are in natural logs. The filtering algorithm for this problem takes the following form:

$$\hat{P}_{t+1|t} = \phi \hat{P}_{t|t} + C(\Sigma_{t|t-1} + R^2)^{-1}(D_t - \hat{P}_{t|t-1}) \tag{30}$$

$$\Sigma_{t+1|t} = \phi^2 \Sigma_{t|t} + Q^2 - C^2(\Sigma_{t|t-1} + R^2)^{-1} - 2\phi C K_t \tag{31}$$

$$K_t = \Sigma_{t|t-1}(\Sigma_{t|t-1} + R^2)^{-1} \tag{32}$$

$$\hat{P}_{t|t} = \hat{P}_{t|t-1} + K_t(D_t - \hat{P}_{t|t-1} \tag{33}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1}[1 - K_t] \tag{34}$$

The filter is initialized using the unconditional mean and variance of risk premium. Maximum likelihood estimates of the parameters (ϕ, R^2, Q^2, and C) are obtained by first fitting an ARMA model to the prediction error, D_t. The risk premium series so obtained is used to test the validity of three theoretical formulations of risk premia based on Lucas (1982) asset pricing model.

3.4   Extended Kalman filter: Unobserved Components Model

Extended Kalman filter is simply the standard Kalman filter applied to a first order Taylor's approximation of a non-linear state-space model around its last estimate. This technique can be used, for example, to decompose the trend and cyclical components of the GDP when the parameters are also allowed to be time-varying. Ozbek and Ozale (2005) estimate the decompositions for Turkish GDP between 1988 and 2003. The model is as follows:
The GDP at time t, Y_t is postulated to be composed of the trend component, T_t and the cyclical component, C_t, where the latter becomes a measure of the output gap. The cyclical component is assumed to follow an AR(2) process whose parameters themselves are independent random walks. The trend component is modeled as a random walk with drift, which captures the impact of (often) extreme policy changes in the transition economies on

the steady state growth path. i.e.

$$C_t = \gamma_{1,t}C_{t-1} + \gamma_{2,t}C_{t-2} + \epsilon_t \tag{35}$$

$$\gamma_{1,t} = \gamma_{1,t-1} + \zeta_{\gamma 2,t} \tag{36}$$

$$\gamma_{2,t} = \gamma_{2,t-1} + \zeta_{\gamma 2,t} \tag{37}$$

$$T_t = \mu_t + T_{t-1} + z_t \tag{38}$$

$$\mu_t = \mu_{t-1} + \zeta a, k \tag{39}$$

where the error terms are assumed iid with zero means and constant variances. The presence of time-varying parameters along with unknown state variables introduces linearities in the model which can be handled using the extended Kalman filter.

3.5 Kalman filter in Financial Econometrics: Stochastic Volatility Models

Financial data have been observed to have certain regularities in statistical properties, including leptokurtic distributions, volatility clustering (clustering of high and low volatility episodes), leverage effects (higher volatility during falling prices and lower volatility during stock market booms) and persistence of volatility. The financial econometrics literature spawns econometric models that seek to capture many of these stylized facts of the data. The most popular approach uses GARCH models, where the variance is postulated to be a linear function of squared past observations and variances. Another approach is Stochastic Volatility (SV) models, first proposed by Taylor(1986), where log of the volatility is modeled as a linear, unobserved stochastic AR process. An ARSV(1) model models asset returns for $t = 1, 2, ..., T$ as:

$$y_t = \sigma_* \sigma_t \epsilon_t \tag{40}$$

$$h_{t+1} = \phi h_t + \eta_t \tag{41}$$

$$\eta_t \sim iid(0, \sigma_\eta^2), \quad |\phi| < 1 \tag{42}$$

where y_t is the return observed at time t, σ_t is the corresponding volatility, $h_t = log(\sigma_t^2)$, ϵ_t are iid random with 0 mean and a known variance, σ_ϵ^2 and σ_* is a scale parameter introduced to keep (25) constant-free. Equation (25) captures volatility clustering and if ϵ_t and η_{t+1} are allowed to be negatively

correlated, then the model can capture the leverage effect. The model is not identified if the variance of (log of) future volatility, σ_η^2 is 0. The process y_t is a martingale difference and is stationary when $|\phi| < 1$. Several ways of estimating the parameters of the model have been proposed. One is to linearize (23) by squaring it and taking logs and obtain estimators based on $log(y_t^2)$. This method is called the Quasi-Maximum Likelihood (QML) and was proposed independently by Nelson(1988) and Harvey et al. (1994). Linearizing (23), we obtain

$$log(y_t^2) = \mu + h_t + \xi_t \tag{43}$$

where $\mu = log(\sigma_*^2) + E(log(\epsilon_t^2))$, $h_t = log(\sigma_t^2)$ and $\xi_t = log(\epsilon_t^2) - E(log(\epsilon_t^2))$. Here, h_T is the unobserved stochastic process. This, along with (41) are in the familiar state-space form of the Kalman filter. However, using the filter directly here would yield only the Minimum Mean Squared Linear estimators, rather than the minimum mean squared estimators. Harvey et al. (1994) proposed treating ξ_t as if it were iid Gaussian and estimating the QML function of $log(y_t^2)$ given by (ignoring constants):

$$logL[log(y^2)|\theta] = -\frac{1}{2}\sum_{t=1}^{T} log\Omega_t - \frac{1}{2}\sum_{t=1}^{T} \frac{v_t^2}{\Omega_t} \tag{44}$$

where $v_t = log(y_t^2) - log\hat{(y_t^2)}$ is the one-step ahead prediction error of $log(y_t^2)$ and Ω_t is the corresponding mean-squared error. Note that the Kalman filter gives estimates of v_t and Ω_t, i.e., provides an algorithm for computing the maximum likelihood function [In the model given by (3) to (8), $v_t = X_t - \hat{X}_{t|t-1}$ and $\Omega_t = (H_t'\Sigma_{t|t-1}H_t + R_t)^{-1}$. Correspondingly, we can get equations defining v_t and Ω_t in the context of the current model]. The likelihood function is maximized using numerical methods to obtain estimates of $\theta = [\phi \ \sigma_\eta^2 \ \sigma_*^2]$. This procedure gives estimators of h_t that are consistent and asymptotically normal, but still inefficient as the density function used is an approximation.

While the QML method discussed above was based on $log(y_t^2)$, there are other methods of estimation of an ARSV(1) model that are based on the statistical properties of y_t itself. The most frequently used are the Generalized Method of Moments (GMM) estimator, Maximum Likelihood (ML) estimators and estimators based on an auxiliary model. The ML estimators use techniques in importance sampling and the Monte Carlo Markov Chain (MCMC) procedures and do not make use of the Kalman filter in their im-

plementation. The GMM methods don't yield estimates of the underlying volatilities σ_t^2 and these can be obtained using the Kalman Filter.

4 Summary

The Kalman Filter is a powerful tool and has been adapted for a wide variety of economic applications. It is essentially a least squares (Gauss Markov) procedure and therefore gives Minimum Mean Square Estimators, with the normality assumption. Even where the normality assumption is dropped, the Kalman filter minimizes any symmetric loss function, including one with kinks. Not only is it used directly in economic problems that can be represented in state-space forms, it is used in the background as part of several other estimation techniques, like the Quasi-Maximum Likelihood estimation procedure and estimation of Markov Switching models.

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