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February 2007

Online at <http://mpa.ub.uni-muenchen.de/22620/>  
MPRA Paper No. 22620, posted 10. May 2010 / 12:20

**Advertising and Entry Deterrence:  
How the Size of the Market Matters**

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**Abstract**

We analyze the relationship between market size and entry when an incumbent and potentially an entrant compete to gain market share and advertising is the only strategic variable. Entry occurs when the relative effectiveness of incumbent's advertising is smaller than a threshold level that depends on the size of the market. This threshold level is monotonically and positively related to market size. Consequently, equilibrium with entry is more likely the greater is the size of the market.

*Key words:* advertising; entry; market size

*JEL classification:* D43

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## 1. Introduction

In many cases, competition between firms takes the form of a contest, where firms choose their prices and advertising levels in order to gain market share. In this paper, we consider a standard entry model. Assume that one firm, the incumbent, irreversibly precommits himself to a certain strategy before the potential entrant considers entry. Firms compete to gain market share and advertising is the only strategic variable. Schmalensee (1974) who questions whether the presence of a dynamic effect of advertising may lead to a barrier to entry, shows that the incumbent needs to have an advantage for the advertising to be of deterrence entry. However, the equilibrium achieved, with or without entry, does not depend on the size of the market. We show that this result is due to the linearity of the advertising cost function. With a more general nonlinear advertising cost, the size of the market does matter.

We exclude the price as a decision variable. This assumption can be justified in many markets where there is little price competition. In the pharmaceutical industries, brands use advertising rather than price to influence the post-patent competition and then the entry decision of generic firms (Hellerstein (1998), Scott Morton (2000)).

The remainder of the paper is organized as follows: In section 2, we specify the model and notation. Section 3 derives the Stackelberg equilibrium and its properties. Section 4 presents a comparative statics analysis. Section 5 concludes the paper

## 2. The Model

Consider an incumbent firm  $X$  and a rival  $Y$  who considers the possibility to enter into the incumbent's market. Let  $x$  and  $y$  denote the advertising spending of the incumbent and the entrant, respectively. These quantities measure the number of ads that firms send to consumers. The size  $V$  of the market is fixed. We assume that firms have constant marginal production costs equal to  $c$  and no fixed costs, and the price is regulated at the level  $p > c$ . Let  $m = p - c$  the profit margin. The incumbent makes its advertising decision  $x$  and then the potential entrant chooses its advertising effort  $y$  under full knowledge of  $x$ . Let  $c_X(\cdot)$  be the advertising cost of firm  $X$ . We assume that  $c_X$  is convex and strictly increasing with  $c'_X(0) > 0$ . The advertising cost of firm  $Y$  is described by the convex and strictly increasing function  $c_Y(\cdot)$  with  $c_Y(0) = 0$ ,  $c'_Y(0) > 0$ ,  $c''_Y(y) \geq 0$ , and  $c'''_Y(y) \leq 0$ .

In this paper, we assume that the market share  $S$  of firm  $X$  has the following form:

$$S = \frac{1}{1 + \frac{y}{\theta x}} \quad (1)$$

where  $\theta$  is a positive parameter that indicates the effectiveness of incumbent's advertising against the entrant's advertising. This implies that, even if two firms expend the same amount of advertising effort, they may not have the same market share. If the incumbent's advertising effort is twice as effective as that of its competitor ( $\theta = 2$ ), and the two

firms choose the same advertising spending, then the incumbent will achieve a market share twice as large as the other's share.

The relationship given in equation (1) has been employed in a number of fields, including the economics of advertising (Schmalensee (1972)), rent-seeking (Nitzan (1994)) and the economics of conflict (Hirshleifer (1995)). A large empirical and theoretical literature, published in marketing journals (Cooper and Nakanishi (1988)), has also adopted this relationship, which is referred to as a market share attraction form. The market share received by the incumbent,  $S$ , is increasing in his effort  $x$  and decreasing in the effort of its competitor,  $y$ , as it obviously should.

The optimization problem of  $X$  is

$$\text{Maximize } \pi_x = SmV - c_x(x) \quad x \geq 0 \quad (2)$$

and the program of firm  $Y$  is

$$\text{Maximize } \pi_y = (1-S)mV - c_y(y) \quad y \geq 0 \quad (3)$$

### 3. Equilibrium

To analyze the allocation of the advertising spending in this scenario, we begin by considering the second-stage choice of firm  $Y$ . At this second stage, firm  $Y$  takes  $x$  as given and chooses  $y$  solution to the program (3). We are interested in deriving the conditions under which  $Y$  chooses not to enter, that is when  $y = 0$ .

Since  $\pi_y$  is strictly concave in the strategy  $y$ , the maximizer is unique. The solution is given in the following proposition.

**Proposition 1:** Let  $x^{\text{det}} = mV / \theta c'_y(0)$ . Given the advertising spending  $x$  of the incumbent, the optimal reaction  $y$  of the potential entrant is null for  $x \geq x^{\text{det}}$  and strictly positive for  $0 < x < x^{\text{det}}$  with  $x$  and  $y$  satisfying

$$\theta mVx = (\theta x + y)^2 c'_y(y) \quad (4)$$

In equation (4),  $x^{\text{det}}$  is the minimum spending of firm  $X$  to advertising necessary to prevent entry.  $x^{\text{det}}$  depends positively on the size  $V$  of the market and the profit margin  $m$  and depends negatively on the relative effectiveness  $\theta$  of the incumbent's advertising. The threshold  $x^{\text{det}}$  depends negatively on the value of  $c'_y(0)$ , which approximates the average cost to the entrant of small levels of advertising spending. Figure 1 depicts the best-reply  $y$  of the potential entrant to the advertising spending of the incumbent,  $x$ , for two different marginal cost functions  $c'_y(\cdot)$ . For  $x < x^{\text{det}}$ , there is an inverted-U shaped relationship between  $y$  and  $x$ . Otherwise,  $y$  equals 0.

### Insert Figure 1

When entry is allowed and the incumbent choice is fixed, there is a positive relationship between the advertising spending of the entrant and the attractiveness of the market, measured by  $mV$ , but the effect of an increase in the relative effectiveness of incumbent's advertising on the advertising spending of the entrant is ambiguous.

We consider now the choice of the first stage of the incumbent. At this first stage, the incumbent chooses its advertising spending  $x$  solution to the program (2). When choosing  $x$ , the incumbent firm takes into account the reaction of the potential entrant.

If  $x \geq x^{\text{det}}$ , then  $y = 0$  and  $S = 1$ . Equation (2) implies that the profit of the incumbent, for  $x \geq x^{\text{det}}$ , is a decreasing function of  $x$ . Consequently, the optimal choice of the incumbent is obtained at  $x$  not greater than  $x^{\text{det}}$ .

Let's begin by showing that the constraint  $x \geq 0$  is not binding. We have:

**Lemma 1:** When  $x \rightarrow 0$  then  $y \rightarrow 0$  and  $y/x \rightarrow +\infty$ .

**Proof:** See the Appendix.

Furthermore, we have:

**Lemma 2:** In the limit as  $x$  approaches zero, the derivative of  $\pi_x$  with respect to  $x$  becomes infinite.

**Proof:** See the Appendix.

It follows from Lemma 2 that the optimal choice of the incumbent is positive. We show next that the type of equilibrium, with or without entry, depends on the sign of the (left-hand) derivative of  $\pi_x$  at  $x = x^{\text{det}}$ .

Consider first the case in which the (left-hand) derivative  $d\pi_x/dx$  is negative for  $x = x^{\text{det}}$ . The profit function of the incumbent has an interior maximum at a value of  $x$  that satisfies

$$\left( \frac{\partial S}{\partial x} + \frac{dy}{dx} \frac{\partial S}{\partial y} \right) mV = c'_x(x) \quad \text{with } 0 < x < x^{\text{det}} \quad (5)$$

The first term in the left-hand side of equation (5) captures the direct effect of greater advertising effort  $x$  by the incumbent on its profit. The second term represents the strategic effect of  $x$  on the choice of the potential entrant. In this case, the incumbent maximizes its profit by choosing its advertising spending such that the marginal revenue equals the marginal cost. Given that this advertising level is smaller than  $x^{\text{det}}$ , the entry is then allowed.

Consider second the case in which the (left-hand) derivative  $d\pi_x/dx$  is not negative at  $x = x^{\text{det}}$ . We need the following Lemma:

**Lemma 3:** The marginal revenue of the incumbent for all  $x$  such that  $0 < x < x^{\text{det}}$  is greater than its marginal revenue at  $x = x^{\text{det}}$ .

**Proof:** See the Appendix.

Let  $R = \frac{\partial S}{\partial x} + \frac{dy}{dx} \frac{\partial S}{\partial y}$ . By Lemma 3 it comes that

$R(x)mV > R(x^{\text{det}})mV \geq c'_x(x^{\text{det}}) \geq c'_x(x)$  for all  $0 < x < x^{\text{det}}$ . Hence,  $d\pi_x/dx$  is positive for  $0 < x < x^{\text{det}}$  and the profit function of the incumbent is strictly increasing.



Consequently, the incumbent chooses an advertising spending equal to  $x^{\text{det}}$  and then entry is prevented. We summarize the preceding analysis with the following

**Lemma 4:** The type of equilibrium, with or without entry, depends on the sign of the (left-hand) derivative of  $\pi_x$  at  $x = x^{\text{det}}$ . The entry is prevented if and only if this derivative is not negative.

#### 4. Comparative Statics

Depending on the configuration of parameters  $\theta$  and  $V$ , the incumbent will or will not seek to deter entry. If the relative effectiveness of the incumbent's advertising,  $\theta$ , is smaller or equal than a critical level, then, entry is allowed for all values of the market size. Otherwise, the incumbent finds profitable to deter entry provided that the market size is not large.

We show easily that the (left-hand) derivative of  $\pi_x$  for  $x = x^{\text{det}}$  is given by

$$\frac{d\pi_x}{dx}(x = x^{\text{det}}) = \frac{\theta c'_Y(0)}{2 + mVc''_Y(0)/[c'_Y(0)]^2} - c'_X(mV / \theta c'_Y(0)) \quad (6)$$

Let's consider the special case in which  $c_X(x) = x$  and  $c_Y(y) = y$ . In this case, explicit solution for equation (4) and well-behaved reduced-form profit function are obtained. Equation (6) becomes:

$$\frac{d\pi_x}{dx}(x = x^{\text{det}}) = \frac{\theta}{2} - 1$$

and then we obtain the result derived by Schmalensee (1974):

**Proposition 2:** When marginal advertising costs are constant, with  $c'_X = c'_Y \equiv 1$ , the type of equilibrium does not depend on the size of the market. Furthermore, the entry is prevented only when the incumbent has an advantage for the effectiveness of its advertising, with  $\theta \geq 2$ .

When considering to change its advertising spending, the incumbent compares its marginal revenue with its constant marginal cost. One can show easily that the marginal revenue, for  $0 < x < x^{\text{det}}$ , is positive and decreasing. The equilibrium does not depend on the size of the market since marginal revenue at  $x = x^{\text{det}}$  depends only on the relative effectiveness  $\theta$  of the incumbent's advertising.

In contrast, when advertising costs are nonlinear, the size of the market does matter. Figures 2, 3 and 4 illustrate the following results.

**Proposition 3:** For a given value of  $\theta$ , there are two cases:

- (i) If  $\theta \leq 2c'_X(0)/c'_Y(0)$ , then entry is allowed for all values of  $V$ .
- (ii) if  $\theta > 2c'_X(0)/c'_Y(0)$ , there exists a threshold level  $V^*$  such that entry is prevented when the size of the market  $V$  is not larger than  $V^*$  and entry is allowed when  $V$  is larger than  $V^*$ .  $\theta$  and  $V^*$  are monotonically and positively related.

**Proposition 4:** For a given value  $V$ , there exists a threshold level  $\theta^*$  such that for  $\theta$  not smaller than  $\theta^*$  entry is prevented and for  $\theta$  smaller than  $\theta^*$  entry is allowed. Furthermore,  $V$  and  $\theta^*$  are monotonically and positively related.

**Insert Figures 2, 3 and 4**

Hence, the result of Proposition 4 shows that a larger size of the market implies a higher probability of the entry accommodating equilibrium. The intuition is the follows. When the size of the market is high, strong incentives exist for the potential entrant to advertise to gain market share. It follows that the incumbent needs to spend heavily on advertising to deter the entry of its competitor. But in this situation, since the advertising exhibits increasing marginal cost, the marginal revenue will be smaller than the marginal cost. Consequently, the incumbent prefers to moderate his spending in advertising and then enter occurs.

Considering more general specifications for advertising costs yields another striking result which contrasts with the result obtained in the Proposition 2. The incumbent may have incentives to deter entry even though it does not exist an advantage for the effectiveness of its advertising. For example, the entry deterring equilibrium may hold for  $\theta$  not larger than one. That's when  $2c'_x(0)/c'_y(0)$  is smaller than  $\theta$  and the size of the market is small enough ( $V < V^*$ ) (see figure 2).

## 5. Conclusion

We showed that the size of the market matters when analyzing a Stackelberg model in which an incumbent and a potential entrant compete to gain market share and advertising is the only strategic variable. Ellison and Ellison (2000) report evidence that drugs with higher revenues are most likely to attract generic entry. Scott Morton (2000) analyzes the U.S. pharmaceutical industry and question whether pre-expiration brand advertising deters generic entry. She shows that market attractiveness, measured by pre-expiration brand revenue, is the most important factor determining the number of entrants.

## Appendix

**Proof of Lemma 1:** Equation (4) yields  $\theta mVx \geq y^2 c'_y(y) \geq y^2 c'_y(0)$ . Then  $0 < y^2 \leq \theta mVx / c'_y(0)$ . It follows that  $y \rightarrow 0$  for  $x \rightarrow 0$ . Also, it comes from Equation (4) that  $y/\theta x = \sqrt{mV / (\theta x c'_y(y))} - 1$ . Since  $x c'_y(y) \rightarrow 0$  for  $x \rightarrow 0$ , hence  $y/x \rightarrow +\infty$  for  $x \rightarrow 0$ .

**Proof of Lemma 2:** Using Equation (4) and by applying the envelope theorem, we obtain the derivative  $dy/dx$  and then we have

$$\begin{aligned}
R &= \frac{\theta y}{(\theta x + y)^2} - \frac{c'_Y(y)}{mV} \left\{ \frac{\theta mV - 2\theta(\theta x + y)c'_Y(y)}{2(\theta x + y)c'_Y(y) + (\theta x + y)^2 c''_Y(y)} \right\} \\
&= \frac{y}{x} \frac{c'_Y(y)}{mV} - \frac{c'_Y(y)}{mV} \left\{ \frac{mV(\theta x + y) - 2\theta mVx}{2mVx + mVx(\theta x + y)c''_Y(y)/c'_Y(y)} \right\} \\
&= \frac{y}{x} \frac{c'_Y(y)}{mV} - \frac{c'_Y(y)}{mV} \left\{ \frac{mV(y/x - \theta)}{2mV + mV(\theta x + y)c''_Y(y)/c'_Y(y)} \right\}
\end{aligned}$$

Hence,  $R = A + B$ , with  $A = \frac{y}{x} \frac{c'_Y(y)}{mV} \left[ 1 - \frac{1}{2 + (\theta x + y)c''_Y(y)/c'_Y(y)} \right]$  and

$B = \frac{c'_Y(y)}{mV} \left[ \frac{\theta}{2 + (\theta x + y)c''_Y(y)/c'_Y(y)} \right]$ . By Lemma 1, it comes that  $B \rightarrow \frac{\theta c'_Y(0)}{2mV}$  and

$A \rightarrow +\infty$  for  $x \rightarrow 0$ .

**Proof of Lemma 3:** For  $x = x^{\det}$ ,  $A = 0$  and  $B = \frac{c'_Y(0)}{mV} \left[ \frac{\theta}{2 + \theta x^{\det} c''_Y(0)/c'_Y(0)} \right]$ .

For  $0 < x < x^{\det}$ ,  $A > 0$ . We then need to show that

$$\frac{c'_Y(y)}{mV} \left[ \frac{\theta}{2 + (\theta x + y)c''_Y(y)/c'_Y(y)} \right] \geq \frac{c'_Y(0)}{mV} \left[ \frac{\theta}{2 + \theta x^{\det} c''_Y(0)/c'_Y(0)} \right]$$

Since  $c'_Y(y) \geq c'_Y(0)$  and  $c''_Y(y) \leq c''_Y(0)$ , the proof is completed if we show that

$(\theta x + y) \leq \theta x^{\det}$  for every  $(x, y)$  verifying equation (4). But equation (4) yields

$\theta mVx = (\theta x + y)^2 c'_Y(y)$  and  $\theta mVx^{\det} = \theta^2 (x^{\det})^2 c'_Y(0)$ . Consequently, for every  $x$  such that

$0 < x < x^{\det}$  we have :

$$(\theta x + y)^2 c'_Y(y) \leq \theta mVx^{\det} = \theta^2 (x^{\det})^2 c'_Y(0) \leq \theta^2 (x^{\det})^2 c'_Y(y)$$

## References

- Cooper, L. G. and M. Nakanishi, (1988), *Market-Share Analysis*, Boston: Kluwer.
- Ellison, G., and S. F. Ellison, (2000), "Strategic Entry Deterrence and the Behavior of Pharmaceutical Incumbents Prior to Patent Expiration," *MIT Working Paper*.
- Hellerstein, J., (1998), "The Importance of the Physician in the Generic versus Trade-Name Prescription Decision," *the Rand Journal of Economics*, 29, 108-136.
- Hirshleifer, J., (1995), "Anarchy and its breakdown," *Journal of Political Economy*, 103, 26-52.
- Nitzan, S., (1994), "Modeling rent-seeking contests," *European Journal of Political Economy*, 10, 41-60.
- Schmalensee, R., (1972), *The Economics of Advertising*, Amsterdam: North Holland.
- Schmalensee, R., (1974), "Brand Loyalty and Barriers to entry," *Southern Economic Journal*, 40, 579-588.
- Scott Morton, F. M., (2000), "Barriers to Entry, Brand Advertising, and Generic Entry in the U. S. Pharmaceutical Industry," *International Journal of Industrial Organization*, 18, 1085-1104.

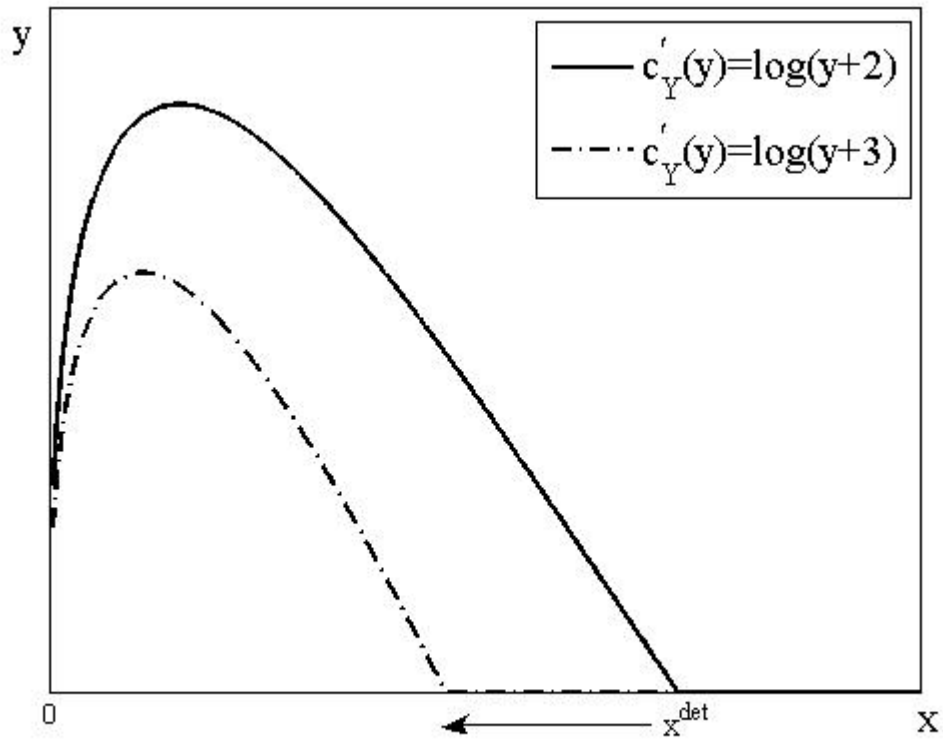


Figure 1: The best-reply function of Y

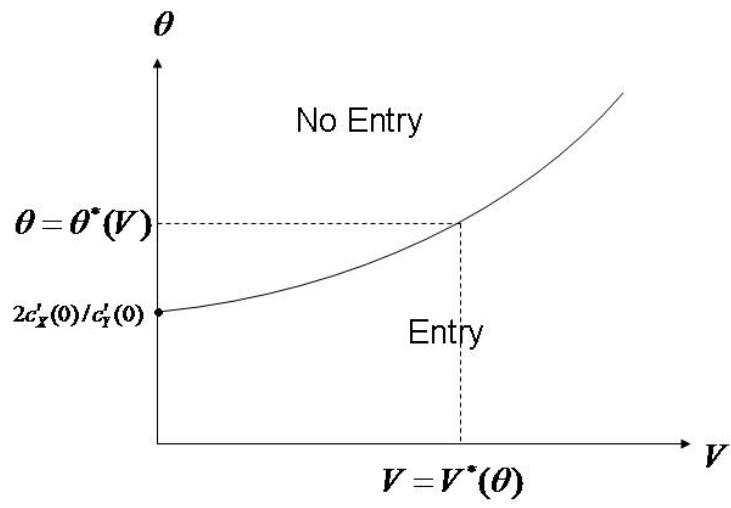


Figure 2: Entry or no entry



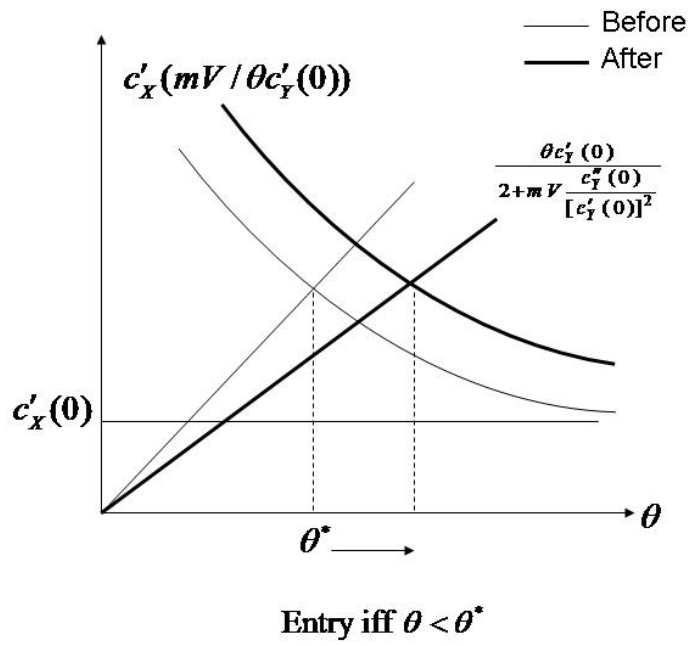


Figure 3:  $\theta^*$  increases when  $V$  increases

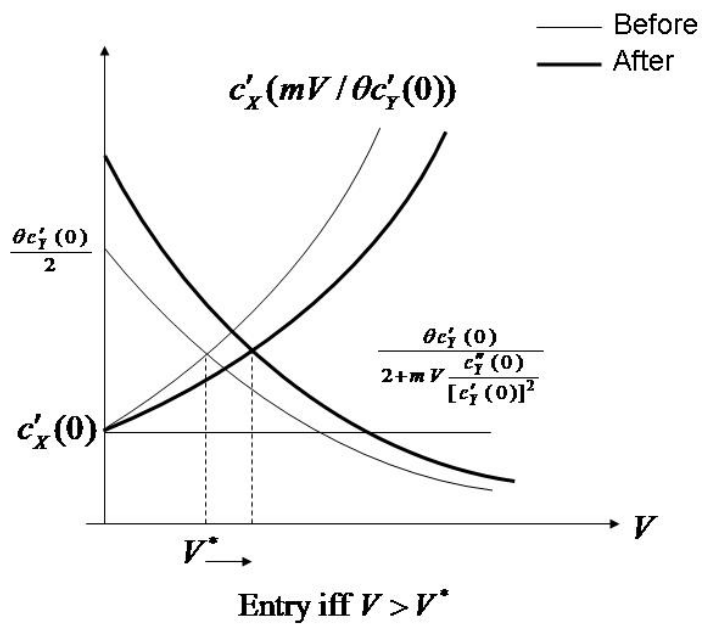


Figure 4:  $V^*$  increases when  $\theta$  increases