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Abstract

This paper revisits the empirical evidence of purchasing power parity under the current float by the recursive mean adjustment (RMA) method (So and Shin, 1999). We first demonstrate superior finite sample performance of the RMA-based unit root test over the augmented Dickey-Fuller test via Monte Carlo experiments for 18 linear and nonlinear autoregressive data generating processes. The RMA-based unit root test rejects the null hypothesis of unit root for 16 out of 20 current float real exchange rates relative to the US dollar.We also find that the computationally simple RMA-based asymptotic confidence interval can provide useful information regarding the half-life of the real exchange rate.

Keywords: Recursive Mean Adjustment, Finite Sample Performance, Purchasing Power Parity, Half-Life

JEL Classification: C12, C22, F31

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1 Introduction

Purchasing power parity (PPP) asserts that the real exchange rate is a mean reverting stochastic process around its long-run equilibrium level. PPP serves as a key building block for many open economy macro models. Despite its popularity and extensive research, empirical validity of PPP still remains inconclusive due to mixed empirical evidence.

Testing for long-run PPP is typically carried out by implementing unit root tests for real exchange rates. Studies employing conventional augmented Dickey-Fuller (ADF) tests find very little evidence of PPP with the current float (post Bretton Woods system) real exchange rates. It is well known that the ADF test has low power when time span of the data is relatively short. Indeed, empirical studies that use long-horizon data, rather than using the current float data, find more favorable evidence for PPP (A. Taylor, 2002, among others).[1] In an effort to overcome the power problem, an array of researches employed panel unit-root tests for the current float data and report evidence in favor of PPP. It should be noted, however, that (first-generation) panel unit-root tests may be oversized (Phillips and Sul, 2003).[2] Therefore, it is not clear that panel approaches with the current float data solve the power problem.

Another important issue we note is the following. It is a well-known statistical fact that the least squares (LS) estimator for autoregressive (AR) processes suffers from serious small-sample bias when the stochastic process includes a non-zero intercept and/or deterministic time trend. The bias can be substantial especially when the process is highly persistent (Andrews, 1993).

Since the pioneering work of Kendall (1954), many bias-correction methods have been developed. Andrews (1993) proposed a method to obtain the exactly median-unbiased estimator for AR(1) process with normal errors. Andrews and Chen (1994) extend the work of Andrews (1993) and develop approximately median-unbiased estimator for AR(p) processes. Hansen (1999) developed a nonparametric bias correction method of grid bootstrap that is robust to distributional assumptions.

[1] See Rogoff (1996) for a survey.

[2] Phillips and Sul (2003) show that conventional panel unit-root tests tend to reject the null of unit root too often in presence of cross-section dependence. O'Connell (1998) finds much weaker evidence for PPP controlling for cross-section dependence.

Murray and Papell (2002) employ methods proposed by Andrews(1993) and Andrews and Chen (1994) to correct for the downward median-bias in the persistent parameter estimates and find that confidence intervals for the half-lives of most current float real exchange rates extend to positive infinity. Based on this, they conclude that the univariate estimation methods provides no useful information on the real exchange rates dynamics. Similar evidence is reported by Rossi (2005).

We revisit these issues by employing an alternative method, recursive mean adjustment (RMA) by So and Shin (1999), that belongs to a class of (approximately) mean-unbiased estimators. The RMA estimator is computationally convenient to implement yet powerful and has been employed in various studies. For instance, Choi *et al.* (2009) develop an RMA-based bias-reduction method for dynamic panel data models, Sul *et al.* (2005) employ RMA to mitigate prewhitening bias in heteroskedasticity and autocorrelation consistent estimation. R. Taylor (2002) employs RMA for seasonal unit root test and found superior size and power properties. Cook (2002) applied RMA to correct a severe oversize problem of the Dicky-Fuller test in the presence of level break.

We first demonstrate superior finite sample performance of the RMA-based unit root test over the ADF test by Monte Carlo experiments for 18 linear and nonlinear autoregressive data generating processes. We also show that, unlike the LS-based methods, a simple RMA asymptotic confidence interval can provide good coverage properties.

To evaluate its practical usefulness, we test the null of unit root for 20 current float quarterly real exchange rates relative to the US dollar using a more powerful RMA-based unit root test (Shin and So, 2001). Surprisingly, the test rejects the null for 16 countries at the 10% significance level while the conventional ADF test rejects the null only for 5. Second, unlike Murray and Papell (2002) and Rossi (2005), we obtain compact confidence intervals for the half-lives of those countries that pass the RMA-based unit root test. To the best of our knowledge, our findings provide the strongest evidence for PPP over the current float.

The remainder of the paper is organized as follows. Section 2 describes So and Shin's (1999) RMA and three alternative methods to construct confidence intervals for the persistent parameter estimate. In Section 3, we present Monte Carlo simulation results to evaluate the finite sample performance of the unit root test with RMA. Section 4 reports our main empirical results with real data. Concluding remarks follow in the last section.

2 The Methodology

2.1 Recursive Mean Adjustment

Let p_t be the domestic price level, p_t^* be the foreign price level, and e_t be the nominal exchange rate as the unit price of the foreign currency in terms of the home currency. All variables are expressed in natural logarithms and are integrated processes of order 1. When PPP holds, there exists a cointegrating vector $[1 \quad 1 \quad -1]'$ for the vector $[p_t^* \quad e_t \quad p_t]'$, the log real exchange rate, $s_t = p_t^* + e_t - p_t$, can be represented by a stationary AR process such as,

$$\begin{aligned} s_t &= c + u_t, \\ u_t &= \sum_{j=1}^{p} \rho_j u_{t-j} + \varepsilon_t, \end{aligned} \tag{1}$$

where $\rho = \sum_{j=1}^{p} \rho_j$ is less than one in absolute value ($|\rho| < 1$) and ε_t is a mean-zero white noise process. Equivalently, the AR model (1) can be alternatively represented by,

$$s_t = c(1-\rho) + \sum_{j=1}^{p} \rho_j s_{t-j} + \varepsilon_t, \tag{2}$$

which implies the following augmented Dickey-Fuller form,

$$s_t = (1-\rho)c + \rho s_{t-1} + \sum_{j=1}^{k} \beta_j \Delta s_{t-j} + \varepsilon_t, \tag{3}$$

where $k = p - 1$, $\beta_j = -\sum_{s=j+1}^{p} \rho_s$, and $\rho = \sum_{j=1}^{p} \rho_j$ as previously defined.

Assuming that PPP holds, the persistence parameter ρ can be estimated by the conventional

LS estimator. When $p = 1$, (1) can be written as,

$$s_t = (1-\rho)c + \rho s_{t-1} + \varepsilon_t \tag{4}$$

By the Frisch-Waugh-Lovell theorem, (4) can be equivalently estimated by,

$$s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \eta_t, \tag{5}$$

where $\bar{s} = T^{-1}\sum_{i=1}^{T} s_i$ is a sample mean and $\eta_t = \varepsilon_t - (1-\rho)c - (1-\rho)\bar{s}$. Note that ε_t thus η_t is correlated with the demeaned regressor $(s_{t-1} - \bar{s})$ because ε_t is correlated with s_i for $i = t, t+1, \cdots, T$, which is embedded in the regressor $(s_{t-1} - \bar{s})$ through $\bar{s}$. Since the exogeneity assumption fails, the LS estimator, $\hat{\rho}_{LS}$, is biased. The bias has an analytical representation and one can obtain the exactly mean-unbiased estimate by using a formula by Kendall (1954).[3]

This paper corrects for the bias by employing an alternative method, the recursive mean adjustment (RMA), proposed by So and Shin (1999). The RMA method is computationally simple yet powerful and flexible enough to deal with higher order AR models. For this, rewrite (4) as,

$$s_t - \bar{s}_{t-1} = \rho(s_{t-1} - \bar{s}_{t-1}) + \xi_t, \tag{6}$$

where $\bar{s}_{t-1} = (t-1)^{-1}\sum_{i=1}^{t-1} s_i$ is the recursive mean and $\xi_t = \varepsilon_t - (1-\rho)c - (1-\rho)\bar{s}_{t-1}$. Since ε_t is orthogonal to the adjusted regressor $(s_{t-1} - \bar{s}_{t-1})$, the RMA estimator $\hat{\rho}_{RMA}$ substantially reduces the bias.

When $p = k+1 > 2$, we follow a single-equation version of Choi *et al.*'s (2009) method. That is, we first estimate (3) by the LS and construct the following.

$$s_t^{+} = (1-\rho)c + \rho s_{t-1} + \varepsilon_t^{+}, \tag{7}$$

[3] Tanaka (1984) and Shaman and Stine (1988) extend Kendall's exact mean-bias correction method to AR(p) models. However, their methods are computationally complicated when the lag order is large.

where $s_t^+ = s_t - \sum_{j=1}^{k} \hat{\rho}_{j,LS} \Delta s_{t-j}$ and $\varepsilon_t^+ = \varepsilon_t - \sum_{j=1}^{k} (\hat{\rho}_{j,LS} - \rho_j) \Delta s_{t-j}$. Then, we apply RMA to (7),

$$s_t^+ - \bar{s}_{t-1} = \rho(s_{t-1} - \bar{s}_{t-1}) + \nu_t, \tag{8}$$

where $\nu_t = \varepsilon_t^+ + (1-\rho)c - (1-\rho)\bar{s}_{t-1}$. Finally, the RMA estimator $\hat{\rho}_{RMA}$ is obtained by,

$$\hat{\rho}_{RMA} = \frac{\sum_{i=2}^{T} (s_{t-1} - \bar{s}_{t-1})(s_t^+ - \bar{s}_{t-1})}{\sum_{i=2}^{T} (s_{t-1} - \bar{s}_{t-1})^2} \tag{9}$$

After estimating $\hat{\rho}_{RMA}$ and its associated standard error, one can use the RMA-based ADF t-statistic to test the null hypothesis of a unit-root ($H_0 : \rho = 1$). As Shin and So (2001), the RMA-based unit root test possesses greater power than the LS-based ADF unit root test. Due to reduced-bias estimation, the left p^{th} percentile of the null distribution of the test statistic shifts to the right, while asymptotic distributions of the RMA and LS estimators are identical under the alternative. This leads to an improvement in power over the LS-based unit root test.

2.2 Constructing Confidence Intervals

Given the point estimate $\hat{\rho}_{RMA}$, it is important to obtain a reliable confidence interval for the estimate. We consider the following three methods to compute confidence intervals: the asymptotic confidence interval, the percentile bootstrap confidence interval, and the bootstrap-t confidence interval.

It is not advisable to use the asymptotic confidence interval for $\hat{\rho}_{LS}$ because its distribution is biased and non-normal. So and Shin (1999), however, show that the asymptotic confidence interval for the RMA estimator has a very good coverage property via Monte Carlo simulations. Instead of discussing details, we provide some illustrative explanations in Figures 1 and 2.

We first implement a small Monte Carlo simulation experiment to obtain 2.5%, 50%, and 97.5% quantile function estimates for the sample sizes (N) of 50 and 150 (Figure 1). It should be noted that unlike the LS estimator, the RMA-based t statistic quantile functions are very similar to those from normal approximation-based theoretical quantile functions for both cases of $N = 50, 150$. As we can see in Figure 2, empirical distributions of the RMA-based t statistic for an array of different persistent parameters are very similar to the standard normal distribution with negligible bias. Figures 1 and 2 jointly demonstrate that normal approximation-based confidence band can be used for the RMA method, but not for the LS estimator.

>>> **Figures 1 and 2** <<<

The 90% asymptotic confidence interval for $\hat{\rho}_{RMA}$ is,

$$[\hat{\rho}_{RMA} - 1.645 \cdot se(\hat{\rho}_{RMA}),\ \hat{\rho}_{RMA} - 1.645 \cdot se(\hat{\rho}_{RMA})]\,, \tag{10}$$

where $se(\hat{\rho}_{RMA}) = \hat{\sigma}(\hat{\rho}_{RMA}) / \left[\sum_{i=2}^{T}(s_{t-1} - \bar{s}_{t-1})^2\right]^{1/2}$ and $\hat{\sigma}(\hat{\rho}_{RMA})$ is the estimated standard error.

For the (nonparametric) percentile bootstrap confidence interval, let $\hat{F}$ be the empirical cumulative distribution function of $\hat{\rho}_{RMA}$ obtained from nonparametric bootstrap simulations. The 90% confidence interval is,

$$\left[\hat{F}^{-1}_{0.05},\ \hat{F}^{-1}_{0.95}\right] = \left[\hat{\rho}_{RMA,0.05},\ \hat{\rho}_{RMA,0.95}\right], \tag{11}$$

where $\hat{F}^{-1}_{\alpha}$ is the α percentile of the bootstrap distribution.

Finally, the bootstrap-t confidence interval (Efron and Tibshirani, 1993) is obtained as follows. Denote $\hat{Z}$ as the empirical cumulative distribution function of,

$$\hat{z}^i = \frac{\hat{\rho}^i_{RMA} - \hat{\rho}_{RMA}}{se(\hat{\rho}^i_{RMA})}, \tag{12}$$

where $\hat{\rho}^i_{RMA}$ and $se(\hat{\rho}^i_{RMA})$ are the RMA point estimate and the standard error from the i^{th} bootstrap sample. The 90% confidence interval is then obtained by,

$$[\hat{\rho}_{RMA} - \hat{z}_{0.95} \cdot se(\hat{\rho}_{RMA}),\ \hat{\rho}_{RMA} - \hat{z}_{0.05} \cdot se(\hat{\rho}_{RMA})]\,, \tag{13}$$

where $\hat{z}_\alpha$ is the α percentile of the bootstrap distribution $\hat{Z}$.

3 Finite Sample Performance

We conduct simulation experiments to explore finite sample performance of the unit-root test with RMA with 18 linear and nonlinear data generating processes (DGP) adopted by Choi and Moh (2007). The DGPs are summarized in Table 1.[4] The DGPs consist of various AR models (DGPs 1 to 7), the endogenous and exogenous regime switching models (DGPs 8 to 13) and the structural break models (DGPs 14 to 16). Also we consider two nonstationary processes (DGPs 17 and 18) to explore size of the test. We consider sample sizes of $T \in \{50, 100, 200, 500\}$ and each simulation run consists of 5,000 replications. Each replication generates $T+500$ observations then discards the first 500 observations to minimize start up effects. We set the values for the associated autoregressive parameter (ρ and ρ_1) identical for all the stationary DGPs and consider two different values for ρ (0.5 and 0.9) to gauge the effect of associated AR parameter on power of the test.

>>> **Table 1** <<<

Table 2 presents the rejection rates of the unit root tests at the 10% significance level. As Choi and Moh (2007) find that ADF tests show satisfactory power performance against unit root process with modest ρ or high ρ but T is large.

The Monte Carlo experiments show, however, that the ADF test with RMA (ADF^{RMA}) dominates the LS-based ADF test (ADF^{LS}). This superior discriminatory power of ADF^{RMA} against unit root process stands out for all sample sizes and values of ρ. Also the power improvement occurs more often as the sample size becomes larger. Through their simulation experiments, Choi and Moh (2007) find that the ADF^{LS} test stands out among five tests they compared when the sample size is relatively small for majority of DGPs.[5] However, our simulation experiments clearly

[4] See Choi and Moh (2007) for more detail.

[5] The five tests they compared are the ADF test, the momentum-threshold autoregressive (M-TAR) test, the sign test, the KSS test, and the inf-t test. See Choi and Moh (2007) for more detail.

demonstrate that the ADF^{LS} test cannot outperform the ADF^{RMA} for most cases. Armed with the superior performance of the RMA, we next carry out empirical application of the RMA to the real exchange rate dynamics.

>>> **Table 2** <<<

4 Empirical Application

We consider CPI based real exchange rates for 21 industrialized countries. Our data set consists of quarterly observations from 1974:Q1 to 1998:Q4 for Eurozone countries and from 1974:Q1 to 2005:Q4 for non-Eurozone countries. The USA is taken as the numeraire (home) country and nominal exchange rates and CPIs are from the *International Financial Statistics*.

We start with the conventional ADF test for the real exchange rates. We select the number of lags (k) by the General-to-Specific rule as recommended by Ng and Perron (2001) for the ADF test. The test results are presented in Table 3. As can be seen from the table, the ADF test with the LS estimator (without bias correction) rejects the null of unit root for only 5 out of 20 countries at the 10% significance level. However, when we apply RMA method to correct for the bias in the LS estimator the test results change dramatically. The null of unit root is now rejected for 16 out of 20 countries at the 10% significance level.[6]

>>> **Table 3** <<<

So and Shin (1999) and Shin and So (2001) show that the RMA estimator is powerful and presents excellent coverage property yet computationally simple to obtain. Table 4 reports our estimates for the persistent parameter. To examine the performance of the RMA estimator ($\hat{\rho}_{RMA}$),

[6] Higher power is obtained with RMA because the reduced-bias estimation right-shifts the critical values while the limiting distribution of ρ is not affected by RMA. See Shin and So (2001) for detailed explanations.

we also report the LS estimator ($\hat{\rho}_{LS}$). We first note from the Table 4 that the RMA estimator yields significant bias-corrections. For all real exchange rates, persistent parameter estimates become greater with RMA. For example, the persistent parameter estimate increases from 0.920 (LS) to 0.940 (RMA) for the UK. The RMA estimator delivers effective bias correction that ranges from 0.009 (Canada) to 0.02 (UK) which is far from negligible.

Given the point estimate $\hat{\rho}_{RMA}$, it is important to obtain a reliable confidence interval for the estimate. Particularly for the case of highly persistent parameter estimates, confidence intervals provide useful information in exploring dynamics of the time-series of interest. It is well-known that the asymptotic confidence interval for $\hat{\rho}_{LS}$ performs very poorly (see Hansen 1999, for example). So and Shin (1999), however, show that the asymptotic confidence interval for the RMA estimator exhibits a very good coverage property via Monte Carlo simulations. To gauge the effectiveness of bias correction attained by the RMA estimator, we consider three alternative ways to compute confidence intervals. They are asymptotic confidence interval (CI_A), the percentile bootstrap confidence interval (CI_ρ), and the bootstrap-t confidence interval (CI_t).

>>> **Table 4** <<<

The 90% confidence intervals we got from the percentile bootstrapping are narrow but upper bounds for the persistent parameter estimates are less than unity for all 21 countries. This does not conform to the results of unit-root tests with RMA since the upper bounds are too low with the percentile methods even for the countries where our unit-root test fails to reject the null. By contrast the bootstrap-t method returns higher lower bounds for the parameter estimates but now the upper bounds hit unity for almost all the countries. Only 2 out of 21 countries show less than unity upper bounds at the 90% confidence intervals with bootstrap-t method and the results are not consistent with the unit-root test results in Table 3.

However, we obtain the compact 90% *asymptotic* confidence intervals for $\hat{\rho}_{RMA}$ for 16 out of 20 countries. These confidence intervals are also consistent with the results of the unit-root tests with

RMA appeared in Table 3. It seems that the asymptotic confidence interval performs reasonably well in terms of both parsimony and efficiency even though it is computationally simple.

Murray and Papell (2002) claim that univariate approaches provide virtually no useful information on the size of real exchange rate half-lives since the confidence intervals for the point estimates are too wide and often the upper bounds are infinite. However, when we apply RMA to correct for the bias in the LS estimates in univariate ADF regressions, we obtain much tighter confidence intervals for the persistent parameter estimates with less than unity upper bounds. That is, by stark contrast to Murray and Papell (2002), our findings suggest that the univariate methods can provide useful information regarding the size of real exchange rate half-lives with more powerful but straightforward bias correction method of RMA.

5 Concluding Remarks

This paper revisits the empirical evidence on real exchange rate dynamics with recently developed RMA method. We demonstrate superior finite sample performance of the RMA-based unit root test via Monte Carlo simulations experiments for 18 linear and nonlinear autoregressive models. We also show that the normal approximation-based confidence interval can be used for the RMA method but not for the LS estimator. Using the current float quarterly real exchange rate data we find that the unit-root test with the RMA estimator rejects the null of unit root for 16 out of 20 industrialized countries while the conventional ADF tests rejects the null only for 5 countries. By stark contrast to Murray and Papell's (2002) and Rossi's (2005) results, we find that the simple asymptotic confidence interval can provide useful information regarding the half-life of the real exchange rate.

Reference

1. Donald W. K. Andrews (1993), "Exactly median-unbiased estimation of first order autoregressive/unit root models," *Econometrica* 61, 139-65.

2. Donald W. K. Andrews and Hong-Yuan Chen (1994), "Approximately median-unbiased estimation of autoregressive models," *Journal of Business and Economic Statistics* 12, 187-204.

3. Chi-Young Choi and Young-Kyu Moh (2007), "How useful are tests for unit-root in distinguishing unit-root processes from stationary but non-linesr processes?" *Econometrics Journal* 10, 82-112.

4. Chi-Young Choi, Nelson C. Mark, and Donggyu Sul (2009), "Bias reduction in dynamic panel data models by common recursive mean adjustment," *Oxford Bulletin of Economics & Statistics*, forthcoming.

5. Steven Cook (2002), "Correcting size distortion of the dickey-fuller test via recursive mean adjustment," *Statistics & Probability Letters* 60, 75-79.

6. Bradley Efron and Robert J. Tibshirani (1993), *An Introduction to the Bootstrap*, Chapman and Hall/CRC, London, UK.

7. Alastair Hall (1994), "Testing for a unit root in time series with pretest data-based model selection," *Journal of Business and Economic Statistics* 12, 461-470.

8. Bruce E. Hansen (1999), "The grid bootstrap and the autoregressive model," *Review of Economics and Statistics* 81, 594-607.

9. M. G. Kendall (1954), "Note on bias in the estimation of autocorrelation," *Biometrika* 41, 403-404.

10. Christian J. Murray and David H. Papell (2002), "The purchasing power parity persistence paradigm," *Journal of International Economics* 56, 1-19.

11. Serena Ng and Pierre Perron (2001), "Lag length selection and the construction of unit root tests with good size and power," *Econometrica* 69, 1519-1554.

12. Paul G. J. O'Connell (1998), "The overvaluation of purchasing power parity," *Journal of International Economics* 44, 1-19.

13. Peter C. B. Phillips and Donggyu Sul (2003), "Dynamic panel estimation and homogeneity testing under cross section dependence," *Econometrics Journal* 6, 217-260.

14. Kenneth Rogoff (1996), "The purchasing power parity puzzle," *Journal of Economic Literature* 34, 647-668.

15. Barbara Rossi (2005), "Confidence intervals for half-life deviations from purchasing power parity," *Journal of Business and Economic Statistics* 23, 432-442.

16. Paul Shaman and Robert A. Stine (1988), "The bias of autoregressive coefficient estimators," *Journal of the American Statistical Association* 83, 842-848.

17. Dong Wan Shin and Beong Soo So (2001), "Recursive mean adjustment for unit root tests," *Journal of Time Series Analysis* 22, 595-612.

18. Beong Soo So and Dong Wan Shin (1999), "Recursive mean adjustment in time-series inferences," *Statistics & Probability Letters* 43, 65-73.

19. Donggyu Sul, Peter C. B. Phillips, and Chi-Young Choi (2005), "Prewhitening bias in HAC estimation," *Oxford Bulletin of Economics & Statistics* 67, 517-546.

20. Katsuto Tanaka (1984), "An asymptotic expansion associated with the maximum likelihood estimators in ARMA models," *Journal of the Royal Statistical Society*, Ser. B 46, 58-67.

21. Alan M. Taylor (2002), "A century of purchasing power parity," *Review of Economics and Statistics* 84, 139–150.

22. Robert Taylor (2002), "Regression-based unit root tests with recursive mean adjustment for seasonal and nonseasonal time series," *Journal of Business and Economic Statistics* 20, 269-281.

Table 1. Summary of DGPs

DGP No.	Data Generating Process	Model
1	$y_t = \rho y_{t-1} + \varepsilon_t$	AR(1)
2	$y_t = \rho y_{t-1} + \phi y_{t-1}^2 + e_t, \quad e_t \sim iid(0, \sigma_1^2)$	Generalized AR(1)
3	$y_t = \rho y_{t-1} + \phi y_{t-1} e_{t-1} + e_t, \quad e_t \sim N(0, \sigma_1^2)$	Bilinear (BL)
4	$y_t = (\rho \lvert y_{t-1} \rvert)/(\lvert y_{t-1} \rvert + c) + \varepsilon_t$	Nonlinear AR
5	$y_t = x_t^2 + \epsilon_t,\ x_t = \rho x_{t-1} + \varepsilon_t,\ \epsilon_t \sim N(0,1)$	Squared relation (SR)
6	$y_t = exp(x_t) + \epsilon_t,\ x_t = \rho x_{t-1} + \varepsilon_t, \quad \epsilon_t \sim N(0,1)$	Exponential relation (ER)
7	$y_t = \alpha + [1 + e^{-\gamma(y_{t-1} - x_t)}]^{-1} + [1 + e^{-\gamma(y_{t-1} + x_t)}]^{-1} + v_t,$	Binary Neural
	$x_t = \rho x_{t-1} + e_t, \quad v_t \sim N(0, \sigma_1^2) \quad e_t \sim N(0, \sigma_2^2)$	Network (BNN)
8	$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-1} I(y_{t-1} \geq c) + \varepsilon_t$	SETAR(1)
9	$y_t = y_{t-1} + \varepsilon_t$, if $\lvert y_{t-1} \rvert \leq k$	EQ-TAR
	$y_t = \rho y_{t-1} + \varepsilon_t$, if $\lvert y_{t-1} \rvert > k$	
10	$y_t = k(1-\rho) + \rho y_{t-1} + \varepsilon_t$, if $y_{t-1} > k$	Band-TAR
	$y_t = y_{t-1} + \varepsilon_t$, if $\lvert y_{t-1} \rvert \leq k$	
	$y_t = -k(1-\rho) + \rho y_{t-1} + \varepsilon_t$, if $y_{t-1} < -k$	
11	$y_t = \alpha + \rho_1 y_{t-1} + \theta \cdot (\beta + \rho_2 y_{t-1}) + \varepsilon_t,$	ESTAR
	where $\theta = 1 - e^{-\gamma(y_{t-1} - c)^2}$	
12	$y_t = \alpha + \rho_1 y_{t-1} + \theta \cdot (\beta + \rho_2 y_{t-1}) + \varepsilon_t,$	LSTAR
	where $\theta = [1 + e^{-\gamma(y_{t-1} - c)}]^{-1}$	
13	$y_t = \rho_t y_{t-1} + e_t, \quad e_t \sim iid(0, 0.4)$	Markov-switching (MS) in
	$\rho_t = \rho_1 S_t + \rho_2(1 - S_t)$	AR coefficients
14	$y_t = \alpha_1 + \rho y_{t-1} + \varepsilon_t$, if $t \leq \lambda T$ where $0 < \lambda < 1$	Structural Change (SC)
	$y_t = \alpha_2 + \rho y_{t-1} + \varepsilon_t$, if $t > \lambda T$	in level
15	$y_t = \alpha_1 + \rho y_{t-1} + \varepsilon_t$, if $t \leq \lambda_1 T$ where $0 < \lambda_i < 1$	Multiple SCs
	$y_t = \alpha_2 + \rho y_{t-1} + \varepsilon_t$, if $\lambda_1 T < t \leq \lambda_2 T$	
	$y_t = \alpha_3 + \rho y_{t-1} + \varepsilon_t$, if $\lambda_2 T < t \leq T$	
16	$y_t = \alpha + \rho y_{t-1} + \sigma_1 \varepsilon_t$, if $t \leq \lambda T$ where $0 < \lambda < 1$	SC in innovation
	$y_t = \alpha + \rho y_{t-1} + \sigma_2 \varepsilon_t$, if $t > \lambda T$	
17	$y_t = \alpha + y_{t-1} + \varepsilon_t$	Unit-root process
18	$y_t = y_{t-1} + \sigma_t \varepsilon_t, \quad \sigma_t = \sigma_1 S_t + \sigma_2(1 - S_t)$	Regime switching
		with unit-root

Note: $I(s)$ denotes an indicator function which takes on the value of 1 if the argument is true and 0 otherwise. Parameter values in simulations are set to $k = 3$, $\phi = -0.1$, $\gamma = 100$, $\alpha = \alpha_1 = 0$, $\alpha_2 = -0.5$, $\alpha_3 = 1.5$, $\sigma_1 = 0.01$, $\sigma_2 = 0.05$, $\varepsilon_t \sim N(0,1)$, $P_{11} = \text{Prob}(S_t = 1 \mid S_{t-1} = 1) = 0,95$, $P_{22} = \text{Prob}(S_t = 2 \mid S_{t-1} = 2) = 0,9$ where S_t is a discrete, unobserved state variable that takes on the value of 1 or 2 in the regime switching models of DGPs 13 and 18.

Table 2. Rejection Rates at 10% Significance Level

DGP/	ADF^{LS}				ADF^{RMA}			
T/	50	100	200	500	50	100	200	500
$\rho = \rho_1 = 0.5;\ \rho_2 = 0.05$								
1	0.94	0.98	**1.00**	**1.00**	**0.99**	**1.00**	**1.00**	**1.00**
2	0.94	0.98	**1.00**	**1.00**	**0.99**	**1.00**	**1.00**	**1.00**
3	0.94	0.97	**1.00**	**1.00**	**0.99**	**1.00**	**1.00**	**1.00**
4	0.96	0.99	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
5	0.95	0.98	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
6	0.92	0.96	**1.00**	**1.00**	**0.99**	**1.00**	**1.00**	**1.00**
7	0.96	0.99	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
8	0.93	0.98	**1.00**	**1.00**	**0.99**	**1.00**	**1.00**	**1.00**
9	0.49	0.84	0.97	**1.00**	**0.63**	**0.94**	**1.00**	**1.00**
10	0.28	0.38	0.81	**1.00**	**0.38**	**0.55**	**0.95**	**1.00**
11	0.92	0.97	**1.00**	**1.00**	**0.98**	**1.00**	**1.00**	**1.00**
12	0.95	0.98	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
13	0.93	0.97	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
14	0.84	0.88	0.94	**1.00**	**0.93**	**0.95**	**1.00**	**1.00**
15	0.24	0.21	0.16	0.14	**0.29**	**0.54**	**0.92**	**1.00**
16	0.81	0.91	0.99	**1.00**	**0.98**	**1.00**	**1.00**	**1.00**
17	0.13	0.10	0.09	0.11	0.17	0.13	0.12	0.11
18	0.15	0.13	0.12	0.10	0.20	0.17	0.14	0.12
$\rho = \rho_1 = 0.9;\ \rho_2 = 0.05$								
1	0.29	0.52	0.88	**1.00**	**0.44**	**0.73**	**0.99**	**1.00**
2	0.29	0.52	0.88	**1.00**	**0.44**	**0.73**	**0.99**	**1.00**
3	0.29	0.56	0.86	**1.00**	**0.44**	**0.73**	**0.99**	**1.00**
4	0.95	0.99	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
5	0.69	0.86	0.96	**1.00**	**0.82**	**0.95**	**1.00**	**1.00**
6	0.76	0.82	0.94	0.99	**0.83**	**0.93**	**0.98**	**1.00**
7	0.95	0.99	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
8	0.22	0.36	0.67	0.99	**0.35**	**0.53**	**0.89**	**1.00**
9	0.22	0.33	0.71	0.99	**0.35**	**0.51**	**0.91**	**1.00**
10	0.19	0.21	0.32	0.90	**0.27**	**0.31**	**0.51**	**0.99**
11	0.19	0.25	0.51	0.97	**0.29**	**0.40**	**0.73**	**1.00**
12	0.50	0.84	0.97	**1.00**	**0.68**	**0.95**	**1.00**	**1.00**
13	0.46	0.77	0.96	**1.00**	**0.89**	**0.98**	**1.00**	**1.00**
14	0.08	0.08	0.21	0.76	**0.13**	**0.17**	**0.51**	**1.00**
15	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
16	0.30	0.47	0.73	0.99	**0.60**	**0.81**	**0.98**	**1.00**

Note: Entries represent the fraction of times when the null hypothesis is rejected out of 5,000 replications. Numbers in bold face indicate dominance.

Table 3. Unit Root Tests: Short-Horizon Quarterly Real Exchange Rates

Country	Lag	ADF^{LS}	ADF^{RMA}
Australia	3	-2.525	-1.672*
Austria	3	-2.273	-1.856*
Belgium	3	-2.312	-1.827*
Canada	3	-2.023	-1.333
Denmark	3	-2.582*	-2.191†
Finland	3	-2.740*	-2.502†
France	1	-2.329	-1.749*
Germany	4	-2.596*	-2.285†
Greece	4	-2.233	-1.753*
Ireland	1	-2.467	-2.016†
Italy	3	-2.467	-2.202†
Japan	3	-2.263	-1.496
Netherlands	3	-2.367	-1.955†
New Zealand	3	-3.154†	-2.829‡
Norway	1	-2.297	-1.819*
Portugal	3	-1.682	-1.137
Spain	1	-1.955	-1.304
Sweden	3	-2.292	-1.813*
Switzerland	3	-2.832*	-2.353†
UK	1	-2.382	-1.788*

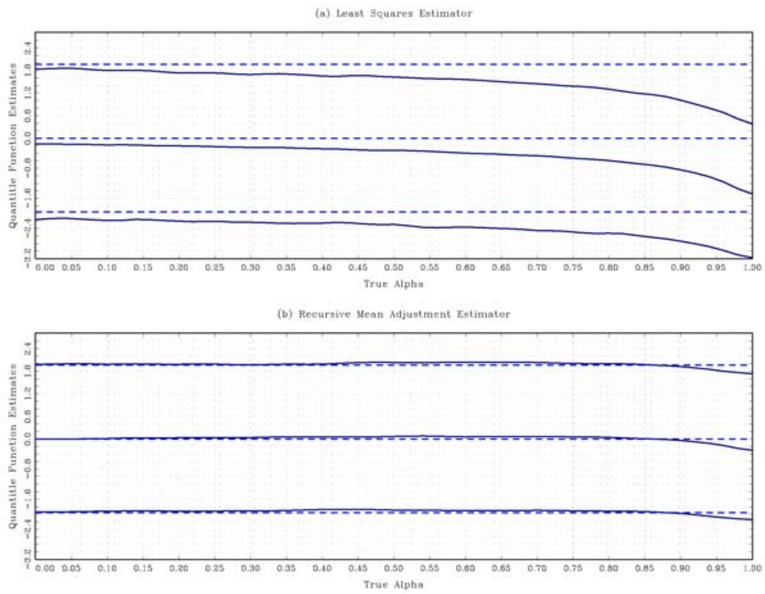
Note: i) Sample periods are 1973Q1-1998Q4 for the Euro-zone countries and 1973Q1-2004Q4 for the non Euro-zone countries. ii) ADF^{LS} and ADF^{RMA}refer to the augmented Dickey-Fuller unit root t-test with LS and RMA estimator, respectively, when an intercept is included. iii) The number of lags was chosen by the general-to-specific rule (Hall, 1994). iv) The asymptotic critical values for the ADF^{RMA} test were obtained from Shin and Soh (2001). v) $*$, †, and ‡ refer the cases that the null of unit root is rejected at the 10%, 5%, and 1% significance level.

Table 4. Recursive Mean Adjustment Estimates

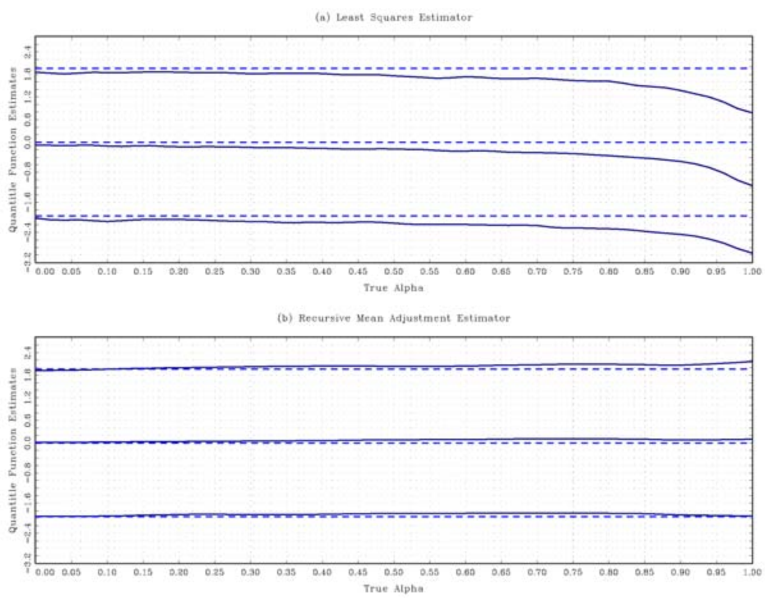
Country	Lag	$\hat{\rho}_{LS}$	$\hat{\rho}_{RMA}$	CI_A	CI_ρ	CI_t
Australia	3	0.945	0.964	[0.929,0.999]*	[0.877,0.979]*	[0.949,1.000]
Austria	3	0.933	0.947	[0.901,0.994]*	[0.852,0.975]*	[0.916,1.000]
Belgium	3	0.938	0.953	[0.911,0.995]*	[0.866,0.976]*	[0.926,1.000]
Canada	3	0.973	0.982	[0.960,1.000]	[0.925,0.992]*	[0.971,1.000]
Denmark	3	0.930	0.943	[0.901,0.986]*	[0.863,0.969]*	[0.913,1.000]
Finland	3	0.908	0.921	[0.869,0.973]*	[0.826,0.959]*	[0.878,0.991]*
France	1	0.930	0.947	[0.898,0.997]*	[0.847,0.972]*	[0.918,1.000]
Germany	4	0.900	0.918	[0.860,0.977]*	[0.798,0.958]*	[0.873,1.000]
Greece	4	0.932	0.949	[0.902,0.997]*	[0.843,0.977]*	[0.919,1.000]
Ireland	1	0.905	0.923	[0.860,0.986]*	[0.804,0.959]*	[0.881,1.000]
Italy	3	0.918	0.931	[0.880,0.983]*	[0.833,0.966]*	[0.892,1.000]
Japan	3	0.950	0.967	[0.932,1.000]	[0.881,0.983]*	[0.951,1.000]
Netherlands	3	0.924	0.940	[0.890,0.991]*	[0.839,0.970]*	[0.906,1.000]
New Zealand	3	0.913	0.927	[0.885,0.969]*	[0.852,0.955]*	[0.896,0.985]*
Norway	1	0.933	0.947	[0.899,0.995]*	[0.854,0.972]*	[0.917,1.000]
Portugal	3	0.958	0.972	[0.932,1.000]	[0.871,0.990]*	[0.952,1.000]
Spain	1	0.951	0.967	[0.926,1.000]	[0.872,0.986]*	[0.947,1.000]
Sweden	3	0.953	0.964	[0.931,0.997]*	[0.895,0.982]*	[0.943,1.000]
Switzerland	3	0.915	0.932	[0.885,0.980]*	[0.842,0.960]*	[0.901,1.000]
UK	1	0.920	0.940	[0.885,0.995]*	[0.838,0.964]*	[0.911,1.000]

Note: i) We use Taylor's (2002) over a century-long real exchange rate data extended through 1998 for the Euro-zone countries and 2004 for the non Euro-zone countries. ii) The number of lags is chosen by the general-to-specific rule (Hall, 1994). iii) α^{L} and α^{R} refer to the least squares α estimate and the recursive mean adjustment α estimate, respectively. iv) For each real exchange rate, the 95% nonparametric bootstrap confidence interval was obtained from 2.5% and 97.5% percentile estimates from 10,000 bootstrap replications from the empirical distribution at the least squares point estimates (Efron and Tibshirani, 1993).

Figure 1. Quantitle Function Estimates of t-Statistics by the LS and the RMA Methods



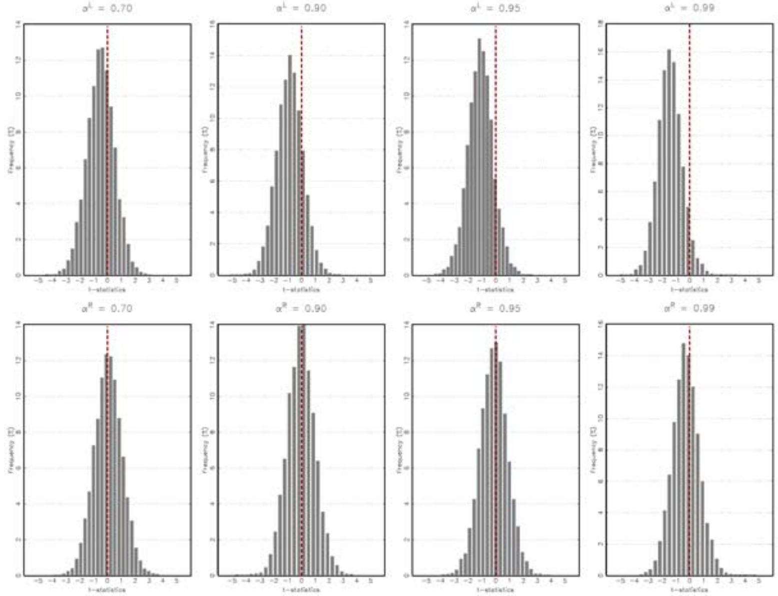
(a) Number of Observations = 50



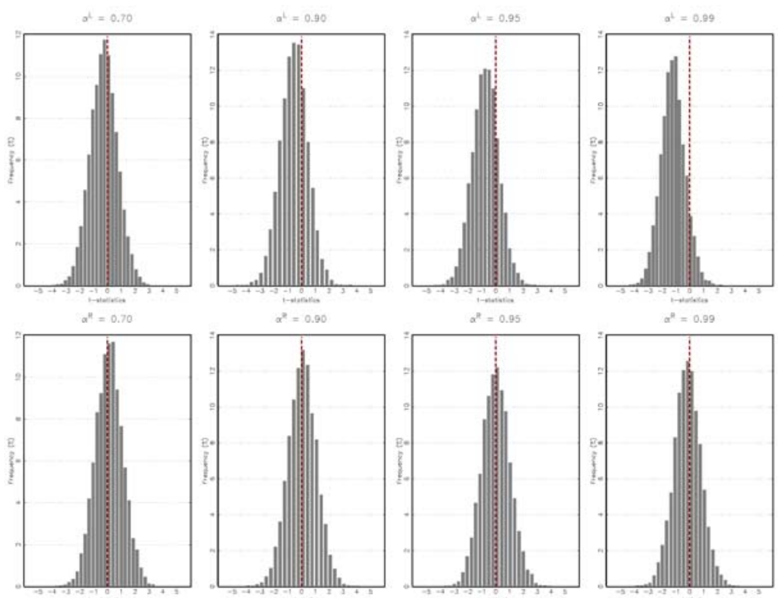
(b) Number of Observations = 150

Note: 2.5%, 50%, and 97.5% quantile function estimates of the t-statistics from 10,000 Monte Carlo simulations with Gaussian errors are reported.

Figure 2. Empirical Distributions of t-Statistics by the LS and the RMA Methods



(a) Number of Observations = 50



(b) Number of Observations = 150

Note: 2.5%, 50%, and 97.5% quantile function estimates of the t-statistics from 10,000 Monte Carlo simulations with Gaussian errors are reported.