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INFLATION STABILIZATION VERSUS PRICE-LEVEL STABILIZATION

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ABSTRACT

This paper characterizes the properties of various interest-rate rules in a basic forward-looking model.
We compare simple Taylor rules and rules that respond to price-level fluctuations (called Wicksellian
rules). We argue that by introducing an appropriate amount of history dependence in policy, Wicksellian
rules perform better than optimal Taylor rules in terms of welfare, robustness to alternative shock processes,
and are less prone to equilibrium indeterminacy. A simple Wicksellian rule augmented with a high
degree of interest rate inertia resembles a robustly optimal rule, i.e., a monetary policy rule that implements
the optimal plan and that is also completely robust to the specification of exogenous shock processes.

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1   Introduction

Is the central bank's objective best achieved by a policy that responds to fluctuations in inflation or the price level? This remains an open question that has regained attention recently among central banks, such as the Bank of Canada which is about to renew its "inflation targeting" mandate. Several early studies have found that it is preferable for the interest rate to respond to inflation fluctuations than to price-level fluctuations in order to minimize the short-run variability of inflation and output (e.g., Lebow *et al.,* 1992; Haldane and Salmon, 1995). The intuition for this result is simple: in the face of an unexpected temporary rise in inflation, price-level targeting requires the policymaker to bring inflation below the target in subsequent periods. With nominal rigidities, fluctuations in inflation result in turn in fluctuations in output. In contrast, with inflation targeting, the drift in the price level is accepted: bygones are bygones. Price-level targeting is a "bad idea" according to this conventional view because it would "add unnecessary short term fluctuations to the economy" (Fischer, 1994, p. 282), while it would only provide a small gain in long-term price predictability in the US (McCallum, 1999).

However, when agents are forward-looking, it is highly desirable for policy to be history-dependent, as explained in Woodford (2003a,b). Committing to a monetary policy of this kind allows the central bank to affect the private sector's expectations appropriately, hence to improve the performance of monetary policy. This suggests that past deviations of the inflation rate should not be treated as bygones.

In this paper, we consider a basic forward-looking New Keynesian model in which the social welfare loss function depends on the variability of inflation, the output gap and the interest rate.[1] We seek to determine whether it is best for policy to respond to fluctuations in inflation or in the price level in this model, by comparing the properties of simple interest-rate rules. Simple monetary policy rules are often prescribed as useful guides for the conduct of monetary policy. Most prominently, a commitment to a Taylor rule (after Taylor, 1993) — according to which the short-term policy rate responds to fluctuations in inflation and some measure of the output gap — is known to yield a good welfare performance a large class of models (see, e.g., papers collected in Taylor, 1999a; Taylor and Williams, 2010). We thus compare the performance of such Taylor rules to that of so-called Wicksellian rules according to which the short-term policy rate depends

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[1]This objective function can be viewed as a quadratic approximation to the underlying representative agent's expected utility.

on deviations of the price level from a trend and the output gap.[2]

In our model, as in Woodford (2003a), Goodfriend and King (2001) and Kahn, King and Wolman (2003), optimal policy — i.e., the policy that minimizes the assumed social welfare loss function subject to the restrictions imposed by the modelled private sector behavior — involves strong price-level stabilization, though it requires some drift of the price level in the face of some shocks. Wicksellian rules perform however very well in terms of welfare by introducing a desirable amount of history dependence in policy. In fact, we show that Wicksellian rules perform better than optimal Taylor rules in our model. Under price-level stabilization, forward-looking agents expect relatively low inflation in subsequent periods in the face of a temporary increase in inflation, as they understand that the policymaker will have to bring inflation below trend. This in turn dampens the initial increase in inflation, lowers the variability of inflation and welfare losses.[3] While Williams (2003) finds a similar result by simulating the large-scale FRB/US model under alternative simple interest-rate rules when assuming rational expectations, our analysis of a simple macroeconomic model — yet a model that incorporates key tradeoffs faced by policymakers — allows us to derive a number of analytical results that provide a clear intuition about the welfare implications of simple Taylor rules and Wicksellian rules, and their sensitivity to various assumptions.[4]

In addition, while simple Taylor rules are often argued to be robust to various types of model

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[2]Wicksellian rules are named after Wicksell (1907) who argued that "price stability" could be obtained by letting the interest rate respond positively to fluctuations in the price level.

[3]It is important to note that the inflation rate used in much of John Taylor's work (e.g., in Taylor, 1993) is a moving average of past quarterly inflation rates, so that his proposed rule incorporates in fact some degree of history dependence. To understand the role of history dependence introduced by the price level, we consider here "Taylor rules" that involve only the contemporaneous inflation rate.

[4]We assume that the central bank is able to credibly commit to a policy rule for the entire future, so as to achieve a better performance of monetary policy. Another branch of the literature assumes instead that the policymaker cannot commit but that it acts under full discretion. These studies generally compare the effects of a regime in which the policymaker is assigned a loss function that involves inflation variability (called inflation targeting), to a regime in which the loss function involves price-level variability (price-level targeting). Svensson (1999), Dittmar *et al.* (1999), and Cecchetti and Kim (2004) show that when the perturbations to output are sufficiently persistent, price-level targeting results in lower inflation variability than inflation targeting. (However under commitment, Svensson (1999) obtains the conventional result that price-level targeting is responsible for a higher variability of inflation.) While these authors use a Neoclassical Phillips curve or a backward-looking model, Vestin (2006), and Dittmar and Gavin (2000) show that these results hold also in a simple "New Keynesian" model. Specifically, they show that when the central bank acts under discretion, price-level targeting results in a more favorable trade-off between inflation and output gap variability relative to inflation targeting, even when perturbations to output are not persistent.

misspecifications (Levin, Williams and Wieland, 1999; Levin and Williams, 2003), we show that their welfare performance can however be very sensitive to the particular assumptions made about the shock processes. Instead, Wicksellian rules are more robust to alternative shock processes. Specifically, we show that (i) optimized coefficients of simple Taylor rules depend critically on the assumed degree of persistence of exogenous disturbances; (ii) such optimized Taylor rules result in an indeterminate equilibrium for some parameter configurations; (iii) the welfare performance relative to the first best deteriorates sharply in the event that the economy is hit by shocks with a higher persistence than the typical historical shocks. In contrast, optimized Wicksellian rules (i) are less sensitive to the assumed shock persistence, (ii) generally result in a determinate equilibrium, and (iii) maintain a very good welfare performance in the face of changes in shock processes or in the face of misspecified shocks.

This sensitivity of optimized simple Taylor rules is arguably undesirable to the extent that in practice central banks may not want to commit to policy rules that perform well only in the face of a few typical shocks, as they may not be able to conceive at the time of commitment all possible shocks that will affect the economy in the future. Policymakers might thus be more inclined to commit to a rule that is robust to the statistical properties of the exogenous disturbances.

As shown in Giannoni and Woodford (2003a,b, 2010), it is possible under general conditions to derive a *robustly optimal rule* that implements the optimal equilibrium and that is completely independent of the specification of the exogenous shock processes. We report this rule here for the model considered and argue that it is a close cousin of the simple Wicksellian rule augmented with a large amount of interest-rate inertia. This latter rule remains extremely simple and introduces about the right amount of history dependence, regardless of the persistence of exogenous disturbances. Such a rule should thus be particularly appealing to policymakers who search for simple rules but worry about unforeseeable circumstances (shocks) affecting the economy in the future.

The rest of the paper is organized as follows. Section 2 reviews the model used in our analysis. Section 3 characterizes the optimal plan. Section 4 determines simple optimal Taylor rules and discusses their properties. Section 5 derives simple optimal Wicksellian rules and compare their implications to optimal Taylor rules and the optimal plan, in terms of their dynamic responses to disturbances, their welfare implications, the sensitivity of the optimal policy coefficients to the degree of persistence in the exogenous disturbances. Section 6 introduces interest-rate inertia. It first presents a simple rule that implements the optimal equilibrium and that is robust to the specification of the process of exogenous disturbances, and then argues that it resembles a Wicksellian

rules with a large degree of interest-rate inertia. Section 7 concludes.

2 A Simple Structural Model

We consider a variant of the simple New Keynesian model that has been widely used in recent studies of monetary policy, following Goodfriend and King (1997), Rotemberg and Woodford (1997), Clarida, Galí and Gertler (1999), and Woodford (2003a,b).

2.1 Structural equations

The behavior of the private sector is summarized by two structural equations, an intertemporal IS equation and a New Keynesian aggregate supply equation.[5] The intertemporal IS equation, which relates spending decisions to the interest rate, is given by

$$y_t - g_t = \mathrm{E}_t \left(y_{t+1} - g_{t+1} \right) - \sigma^{-1} \left(i_t - \mathrm{E}_t \pi_{t+1} \right), \tag{1}$$

where y_t denotes the log of (detrended) real output, π_t is the quarterly inflation rate, i_t is the nominal interest rate (all three variables expressed in deviations from their values in a steady-state with zero inflation and constant output growth), and g_t is an exogenous variable representing autonomous variation in spending such as government spending. This equation can be obtained by performing a log-linear approximation to the representative household's Euler equation for optimal timing of expenditures, using the market clearing condition on the goods market. The parameter $\sigma > 0$ represents the inverse of the intertemporal elasticity of substitution. As we will also be interested in describing the evolution of the log of the price level p_t, we note that by definition of inflation, we have

$$p_t = \pi_t + p_{t-1}. \tag{2}$$

It is assumed that prices are sticky, as in Calvo (1983), and that suppliers are in monopolistic competition. It follows that a log-linear approximation to the first-order condition for the suppliers' optimal price-setting decisions yields the familiar New Keynesian supply equation

$$\pi_t = \kappa \left(y_t - y_t^n \right) + \beta \mathrm{E}_t \pi_{t+1}, \tag{3}$$

where $\kappa > 0$ depends on the speed of price adjustment, $\beta \in (0, 1)$ denotes the discount factor of the representative household, and y_t^n represents the *natural* rate of output, i.e., the equilibrium

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[5]Derivations of the structural equations from first principles can be found, e.g., in Woodford (2003a, chap. 3).

rate of output under perfectly flexible prices. This natural rate of output is a composite exogenous variable that may depend on a variety of perturbations such as productivity shocks, shifts in labor supply, but also fluctuations in government expenditures and shifts in preferences. We also allow for exogenous time variation in the degree of inefficiency of the natural rate of output, $y_t^e - y_t^n$, where y_t^e is the rate of output that would maximize the representative household's welfare in the absence of distortions. Fluctuations in $y_t^e - y_t^n$ could be due, e.g., to exogenous variation in the degree of market power of firms or in distortionary taxation. As we will evaluate monetary policy in terms of deviations of output from the *efficient* rate, it will be convenient to define the "output gap" as

$$x_t \equiv y_t - y_t^e.$$

We can then rewrite the structural equations (1) and (3) as

$$
\begin{aligned}
x_t &= \mathrm{E}_t x_{t+1} - \sigma^{-1} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^e \right) & (4) \\
\pi_t &= \kappa x_t + \beta \mathrm{E}_t \pi_{t+1} + u_t, & (5)
\end{aligned}
$$

where we now have two composite exogenous variables

$$
\begin{aligned}
r_t^e &\equiv \sigma \mathrm{E}_t \left[\left(y_{t+1}^e - y_t^e \right) - \left(g_{t+1} - g_t \right) \right] \\
u_t &\equiv \kappa \left(y_t^e - y_t^n \right).
\end{aligned}
$$

In (4), r_t^e denotes the "efficient" rate of interest, i.e., the equilibrium real interest rate that would prevail in the absence of distortions.

2.2 Shock processes

We think of the composite shocks r_t^e and u_t as being functions of a potentially large number of underlying disturbances, with each of the underlying disturbance having a different degree of persistence. We assume that the central bank knows perfectly the shocks that have hit the economy until the present, but may not be able to assess the realization of all possible future shocks. Following Giannoni and Woodford (2003a), we let the shocks r_t^e and u_t be composites of an infinity of types of disturbances, each having a Wold representation

$$
\begin{aligned}
r_t^e &= \sum_{j=0}^{\infty} \rho_r^j \varepsilon_{r,t-j} + \sum_{m=0}^{\infty} \sum_{j=0}^{\infty} \hat{\rho}_m^j \hat{\varepsilon}_{r,m,t-j} & (6) \\
u_t &= \sum_{j=0}^{\infty} \rho_u^j \varepsilon_{u,t-j} + \sum_{m=0}^{\infty} \sum_{j=0}^{\infty} \hat{\rho}_m^j \hat{\varepsilon}_{u,m,t-j} & (7)
\end{aligned}
$$

5

where $\varepsilon_{k,t}$, and $\hat{\varepsilon}_{k,m,t}$ are iid, mean-zero random variables, for all $k \in \{r, u\}$ and $m, t \geq 0$, but where the innovations $\hat{\varepsilon}_{k,m,t}$ have a distribution with a large atom at zero, and the parameters $\rho_k, \hat{\rho}_k \in [0, 1)$ determine the persistence of each of these innovations. The composite shocks r_t^e and u_t are thus affected in each period by typical innovations $\varepsilon_{r,t}$ and $\varepsilon_{u,t}$, with a persistence given by ρ_r, ρ_u, and they may be infrequently affected by a large number of other types of unforecastable disturbances, $\hat{\varepsilon}_{r,m,t}$ and $\hat{\varepsilon}_{u,m,t}$ each of which may have a different degree of persistence. To simplify the analysis, we furthermore assume that such infrequent innovations have not been observed in the past, up to the date 0 at which the policymaker will set policy, so that the historical exogenous processes can be correctly characterized by stationary AR(1) processes

$$r_t^e = \rho_r r_{t-1}^e + \varepsilon_{rt} \tag{8}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{ut}. \tag{9}$$

up to date 0, and the conditional forecasts are given by

$$\mathrm{E}_0 r_t^e = \rho_r^t r_0^e \tag{10}$$

$$\mathrm{E}_0 u_t = \rho_u^t u_0 \tag{11}$$

for all $t \geq 0$. Since it is impractical for the central bank to catalog all of the possible disturbances $\hat{\varepsilon}_{r,m,t}$, $\hat{\varepsilon}_{r,m,t}$ before they are realized, and since the policymaker cannot reject the hypothesis that the past shocks and conditional forecasts of future shocks are described by (8)–(11), we assume that the central bank wants to choose at date 0 a rule that would be optimal (at least within the class of rules that it considers) under the assumption that the shock processes are given by (8)–(9).[6]

2.3 Policy objective

We assume that the policymaker seeks to minimize the expected loss criterion

$$\mathrm{E}[L] = \mathrm{E}\left\{ (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_x \left(x_t - x^* \right)^2 + \lambda_i \left(i_t - i^* \right)^2 \right] \right\}, \tag{12}$$

where $\lambda_x, \lambda_i > 0$ are weights placed on the stabilization of the output gap and the nominal interest rate, $\beta \in (0, 1)$ is the discount factor mentioned above, and where $x^* \geq 0$ and i^* represent some

---

[6]The central bank does not need to regard it as certain that (8)–(9) are correct. However, we assume that it will only consider rules that would be optimal in the case that (8)–(9) were correct. Subject to that requirement, we assume that it would also like its rule to be as robust as possible to alternative shock processes within the more general family (6)–(7).

optimal levels of the output gap and the nominal interest rate. The expectation E[·] is conditional on the state of the economy at the time that the policy is evaluated, which we assume takes place before the realization of the shocks at that date. This loss criterion can be viewed as a second-order Taylor approximation to the lifetime utility function of the representative household in the underlying model (see Woodford, 2003a, chap. 6). The concern for interest rate variability in (12) reflects both welfare costs of transactions and an approximation to the zero lower bound on nominal interest rates. The approximation of the utility function allows us furthermore to determine the relative weights λ_x, λ_i, and the parameters x^*, i^* in terms of the parameters of the underlying model.[7]

The inefficient supply shock is responsible for a trade-off between the stabilization of inflation on one hand, and the output gap on the other hand. Indeed, in the face of an increase in u_t, the policymaker could completely stabilize the output gap by letting inflation move appropriately, or he could stabilize inflation, by letting the output gap decrease by the right amount, but he could not keep both inflation and the output gap constant. By how much he will let inflation and the output gap vary depends ultimately on the weight λ_x. In the absence of inefficient supply shocks, however, both inflation and the output gap could be completely stabilized by letting the interest rate track the path of the efficient rate of interest, r_t^e (which incidentally is equal to the natural rate of interest in the absence of inefficient supply shocks, as $y_t^e = y_t^n$). But when $\lambda_i > 0$ in (12), welfare costs associated to fluctuations in the nominal interest rate introduce a tension between stabilization of inflation and the output gap on one hand and stabilization of the nominal interest rate on the other hand.

2.4   Calibration

In the rest of the paper, we characterize optimal monetary policy for arbitrary positive values of the parameters. At times however we focus on a particular parametrization of the model, using the parameter values estimated by Rotemberg and Woodford (1997) for the U.S. economy, and

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[7]Woodford (2003a, chap. 6)'s derivation of the loss criterion from first principles accounts for transaction frictions and the approximation of the lower bound on interest rates, but abstracts from inefficients supply shocks u_t. The welfare function (12) remains a valid approximation of the underlying utility in the presence of inefficient supply shocks to the extent that we consider only small deviations from the efficient steady state, and evaluate all derivatives at that steady state.

summarized in Table 1.[8] The weights λ_x and λ_i are calibrated as in Woodford (2003a), using the calibrated structural parameters and the underlying microeconomic model. Rotemberg and Woodford (1997) provide estimated time-series for the disturbances y_t^n and g_t. They do however not split the series for the natural rate of output in an efficient component y_t^e, and an inefficient component. For simplicity, we calibrate the variance of r_t^e by assuming that all shifts in the aggregate supply equation are efficient shifts, so that the variance of the efficient rate of interest is the same as the variance of the natural rate of interest reported in Woodford (2003a). In our benchmark calibration, we set $\mathrm{var}\,(u_t)$ to its upper bound $\kappa^2\,\mathrm{var}\,(y_t^n)$, assuming that all shifts in the aggregate supply equation are due to inefficient shocks. We however verify that our conclusions are not sensitive to alternative calibrations of $\mathrm{var}\,(u_t)$.

3   Optimal Plan

Before evaluating alternative policies below, it will be useful to consider as a benchmark the optimal state-contingent plan. This plan characterizes the optimal stochastic processes of endogenous variables $\{\pi_t, x_t, i_t\}$, i.e., those that minimize the unconditional expectation of the loss criterion (12) subject to the constraints (4) and (5) imposed by the private sector's behavior at all dates, assuming that the policymaker can commit to the plan for the entire future.[9] Following Currie and Levine (1993) and Woodford (2003b), we write the policymaker's Lagrangian as

$$
\begin{aligned}
\mathcal{L} \;=\; & \mathrm{E}\left\{\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{2}\left[\pi_t^2 + \lambda_x\left(x_t - x^*\right)^2 + \lambda_i\left(i_t - i^*\right)^2\right]\right.\right. \\
& \left.\left. + \phi_{1t}\left[x_t - x_{t+1} + \sigma^{-1}\left(i_t - \pi_{t+1} - r_t^e\right)\right] + \phi_{2t}\left[\pi_t - \kappa x_t - \beta\pi_{t+1} - u_t\right]\right)\right\}. \quad (13)
\end{aligned}
$$

The first-order necessary conditions with respect to π_t, x_t, and i_t are

$$
\pi_t - (\beta\sigma)^{-1}\,\phi_{1t-1} + \phi_{2t} - \phi_{2t-1} \;=\; 0 \quad (14)
$$

$$
\lambda_x\left(x_t - x^*\right) + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} \;=\; 0 \quad (15)
$$

$$
\lambda_i\left(i_t - i^*\right) + \sigma^{-1}\phi_{1t} \;=\; 0 \quad (16)
$$

---

[8] While the econometric model of Rotemberg and Woodford (1997) is more sophisticated than the present model, their structural equations correspond to (1) and (3) when conditioned upon information available two quarters earlier in their model.

[9] This section generalizes slightly the results of Clarida *et al.* (1999), and Woodford (2003a, 2003b) who consider the optimal plan either in the presence of inefficient supply shocks, or with a concern of interest-rate stabilization ($\lambda_i > 0$), but not both at the same time.

at each date $t \geq 0$, and for each possible state. In addition, we have the initial conditions

$$\phi_{1,-1} = \phi_{2,-1} = 0 \tag{17}$$

indicating that the policymaker has no previous commitment at time 0.

The optimal plan is a bounded solution $\{\pi_t, x_t, i_t, \phi_{1t}, \phi_{2t}\}_{t=0}^{\infty}$ to the system of equations (4), (5), (14) – (16) at each date $t \geq 0$, and for each possible state, together with the initial conditions (17). To characterize the optimal responses to perturbations, we rewrite the equations above in terms of deviations from the optimal steady-state: $\hat{\pi}_t \equiv \pi_t - \pi^{op}$, $\hat{x}_t \equiv x_t - x^{op}$, $\hat{\imath}_t \equiv i_t - i^{op}$, and $\hat{p}_t \equiv p_t - p_t^{op}$.[10] We note that the same equations (4), (5), (14) – (16), and (2) hold now in terms of the hatted variables, but without the constant terms. Using (16) to substitute for the interest rate, we can rewrite the dynamic system (4), (5), (14), and (15) in matrix form as

$$\mathrm{E}_t \begin{bmatrix} \hat{q}_{t+1} \\ \hat{\phi}_t \end{bmatrix} = M \begin{bmatrix} \hat{q}_t \\ \hat{\phi}_{t-1} \end{bmatrix} + m e_t, \tag{18}$$

where $\hat{q}_t \equiv [\hat{\pi}_t, \hat{x}_t]'$, $\hat{\phi}_t \equiv \left[\hat{\phi}_{1t}, \hat{\phi}_{2t}\right]'$, e_t is a vector of exogenous disturbances, and M and m are matrices of coefficients. Investigation of the matrix M reveals that if a bounded solution exists, it is unique.[11] In this case the solution for the endogenous variables can be expressed as

$$\hat{z}_t = D\hat{\phi}_{t-1} + \sum_{j=0}^{\infty} d_j \mathrm{E}_t e_{t+j}, \tag{19}$$

where $\hat{z}_t \equiv [\hat{\pi}_t, \hat{x}_t, \hat{\imath}_t, \hat{p}_t]'$, and the Lagrange multipliers follow the law of motion

$$\hat{\phi}_t = N\hat{\phi}_{t-1} + \sum_{j=0}^{\infty} n_j \mathrm{E}_t e_{t+j} \tag{20}$$

for some matrices D, N, d_j, n_j that depend upon the parameters of the model. Woodford (2003b) has emphasized that in the optimal plan, the endogenous variables should depend not only upon expected future values of the disturbances, but also upon the predetermined variables $\hat{\phi}_{t-1}$.

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[10] The steady-state values of the endogenous variables, which satisfy the previous equations at all dates in the absence of perturbations, are given by $i^{op} = \pi^{op} = \frac{\lambda_i i^*}{\lambda_i + \beta}$, and $x^{op} = \frac{1-\beta}{\kappa} \frac{\lambda_i i^*}{\lambda_i + \beta}$. The optimal steady-state inflation is independent of x^* though not of i^*. When $i^* \neq 0$, the log price level follows a deterministic trend $p_t^{op} = \pi^{op} + p_{t-1}^{op}$.

[11] This dynamic system has a unique bounded solution (given a bounded process $\{e_t\}$) if and only if the matrix M has exactly two eigenvalues outside the unit circle. The matrix M has two eigenvalues with modulus greater than $\beta^{-1/2}$ and two with modulus smaller than $\beta^{-1/2}$.

4 Commitment to an Optimal Taylor Rule

We next consider an optimal policy problem in the case that the policymaker restricts its policy by setting the interest rate according to the standard "Taylor rule"

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \psi_0, \tag{21}$$

at all dates $t \geq 0$, where ψ_π, ψ_x, and ψ_0 are policy coefficients. As mentioned in the introduction, such a simple rule while not fully optimal is known to perform well in a wide range of models. We focus here on the simplest Taylor rule without inertia, but we discuss an extension of this rule that includes the lagged interest rate in section 6.2.

The policymaker is assumed to commit to the rule (21), in which the coefficients ψ_π, ψ_x, and ψ_0 are chosen so as to maximize the expected welfare (12), subject to the structural equations (4) and (5), and assuming that the shock processes are given by (8)–(9). To determine the optimal policy coefficients, it is useful to proceed in two steps: we first characterize the optimal equilibrium that is consistent with the given rule, and second we determine policy coefficients that correspond to that equilibrium.

Using (21) to substitute for the interest rate in the structural equations (4) and (5), we observe that inflation and the output gap must satisfy the following system of difference equations

$$\mathrm{E}_t z_{t+1} = A z_t + a e_t, \tag{22}$$

where $z_t \equiv [\pi_t, x_t, 1]'$, and $e_t \equiv [r_t^e, u_t]'$ and A and a are matrices of coefficients. Given that z_t does not involve any predetermined variable, the resulting equilibrium, if it exists, must be non inertial. The evolution of the endogenous variables can then be described by

$$\pi_t = \pi^{ni} + \pi_r r_t^e + \pi_u u_t, \qquad x_t = x^{ni} + x_r r_t^e + x_u u_t, \qquad i_t = i^{ni} + i_r r_t^e + i_u u_t, \tag{23}$$

where π^{ni}, x^{ni}, i^{ni} are the steady-state values of the respective variables in this equilibrium, and π_r, π_u, and so on, are the equilibrium response coefficients to fluctuations in r_t^e and u_t.

As we show in Appendix A.1, both i_r and i_u are positive for any positive weights λ_i, λ_x. Thus the optimal non-inertial plan involves an adjustment of the nominal interest rate in the direction of the perturbations. Furthermore, the response coefficients π_r, x_r are positive if and only if

$$\frac{\sigma}{\kappa} > \frac{\rho_r}{(1 - \beta \rho_r)(1 - \rho_r)}, \tag{24}$$

10

that is, whenever the fluctuations in the efficient rate are not too persistent (relative to the ratio $\frac{\sigma}{\kappa}$). Thus when (24) holds, a positive shock to the efficient rate stimulates aggregate demand, so that both the output gap and inflation increase. In the limiting case that the interest rate does not enter the loss function ($\lambda_i \to 0$), or when the persistence of the perturbations is such that (24) holds with equality, we obtain $\pi_r = x_r = 0$ and $i_r = 1$. As a result, in the absence of inefficient supply shocks, the central bank optimally moves the interest rate by the same amount as the efficient rate in order to stabilize the output gap and inflation completely.

When the disturbances to the efficient rate are sufficiently persistent (ρ_r large enough but still smaller than 1) for the inequality (24) to be reversed, inflation and the output gap decrease in the face of an unexpected positive shock to the efficient rate in the optimal non-inertial plan. Even if the nominal interest rate increases less than the natural rate, optimal monetary policy is restrictive in this case, because the real interest rate ($i_t - \mathrm{E}_t \pi_{t+1}$) is higher than the efficient rate of interest r_t^e.

For the Taylor rule to be consistent with an optimal equilibrium of the form (23), we show in Appendix A.1 that the policy coefficients must satisfy

$$\psi_\pi = \frac{x_u i_r - i_u x_r}{x_u \pi_r - \pi_u x_r} \tag{25}$$

$$\psi_x = \frac{\pi_r i_u - i_r \pi_u}{x_u \pi_r - \pi_u x_r}, \tag{26}$$

Substituting the coefficients $\pi_r, x_r, \dots$ with their values characterizing the optimal non-inertial equilibrium, yields the coefficients of the optimal Taylor rule as functions of the underlying structural problem. However, for the optimal Taylor rule to implement the optimal non-inertial equilibrium, it must guarantee that the dynamic system (22) admits a unique bounded solution. Since both π_t and x_t are non-predetermined endogenous variables at date t, and $\{e_t\}$ is bounded, this is the case if and only if A has exactly two eigenvalues outside the unit circle. It is well known (see, e.g., Woodford, 2003a, chap. 4) that if we restrict our attention to the case in which $\psi_\pi, \psi_x \geq 0$, then the policy rule (21) results in a determinate equilibrium if and only if

$$\psi_\pi + \frac{1-\beta}{\kappa} \psi_x > 1. \tag{27}$$

4.1 Optimal Taylor rule and sensitivity to shock processes

To get some intuition about the optimal Taylor rule, let us consider the special case in which both perturbations have the same degree of persistence, i.e., $\rho_r = \rho_u \equiv \rho$. In this case, the optimal Taylor

rule coefficients reduce to

$$\psi_\pi = \frac{\kappa}{\lambda_i \left(\sigma \left(1 - \rho\right) \left(1 - \beta\rho\right) - \rho\kappa\right)} \qquad (28)$$

$$\psi_x = \frac{\lambda_x \left(1 - \beta\rho\right)}{\lambda_i \left(\sigma \left(1 - \rho\right) \left(1 - \beta\rho\right) - \rho\kappa\right)}. \qquad (29)$$

When (24) holds, both optimal Taylor-rule coefficients are positive. The optimal coefficient on inflation, ψ_π, increases with the slope to the aggregate supply, κ, to prevent a given output gap from creating more inflation. Similarly the optimal coefficient on output gap, ψ_x, increases when λ_x increases, as the policymaker is more willing to stabilize the output gap. In addition, the optimal Taylor rule becomes more responsive to both inflation and output gap fluctuations, when the weight λ_i decreases, as the policymaker is willing to let the interest rate vary more, and when the intertemporal IS curve becomes flatter (σ is smaller), as shocks to the efficient rate of interest have a larger impact on the output gap and inflation.

These expressions reveal that the optimal Taylor coefficients are particularly sensitive to the assumed degree of persistence of the shocks. As ρ increases to approach the bound (24), which corresponds to $\rho \simeq 0.68$ in our calibration, the optimal Taylor rule coefficients become in fact unboundedly large, and become negative when the inequality in (24) is reversed, i.e., when $\rho > 0.68$. Table 2 reports the optimal coefficients (given by (A.55) and (A.56) in Appendix A.1) for different degrees of persistence of the perturbations, using the calibration summarized in Table 1. As shown in Figure 1, these optimal Taylor coefficients may change substantially with different degrees of shock persistence. Again, optimal policy coefficients approach infinity for ρ around 0.68.

While the white region of Figure 1 indicates the set of Taylor rules that result in a unique bounded equilibrium, the gray region indicates combinations (ψ_π, ψ_x) that result in indeterminacy of the equilibrium. Figure 1 reveals for example that when both shocks are purely transitory $(\rho_r = \rho_u = 0)$, the "optimal" Taylor rule lies in the region of indeterminacy. In fact, the "optimal" coefficients ψ_π, ψ_x, while positive, are not large enough to satisfy (27). This means that for any bounded solution $\{z_t\}$ to the difference equation (22), there exists another bounded solution of the form

$$z_t' = z_t + v\xi_t$$

where v is an appropriately chosen (nonzero) vector, and the stochastic process $\{\xi_t\}$ may involve arbitrarily large fluctuations, which may or may not be correlated with the fundamental disturbances r_t^e and u_t. Committing to an "optimal" Taylor rule that lies in the region of indeterminacy

can thus result in a large set of bounded equilibria, including some that involve an arbitrarily large value of the loss criterion (12). Note from Figure 1 that the problem of indeterminacy arises not only when $\rho_r = \rho_u = 0$, but also in some cases when the disturbances are more persistent (e.g., when $\rho_r = 0.35$ and $\rho_u = 0$, or when $\rho_r = \rho_u = 0.9$).

4.2 Desirability of history dependence

Even if we abstract from equilibrium indeterminacy, the optimal Taylor rule may yield substantially higher welfare losses than the first best. Table 2 reports the policymaker's loss, $\mathrm{E}\,[L]$, in addition to the following measure of variability

$$V\,[z] \equiv \mathrm{E}\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t \hat{z}_t^2\right]$$

for the four endogenous variables, π, x, i, and p, so that $\mathrm{E}\,[L]$ is a weighted sum of $V\,[\pi]\,, V\,[x]\,,$ and $V\,[i]$ with weights being the ones of the loss function (12).[12] When $\rho_r = \rho_u = 0.35$, as in the baseline calibration, the loss is 1.28 in the optimal plan, while it is 2.63 when committing to an optimal Taylor rule. The welfare losses of generated by this simple policy rule stem primarily from a higher variability of inflation and of interest rates.

To understand better the source of the welfare losses under the simple Taylor rule, we show in Figure 2 the response of endogenous variables to an unexpected disturbance to the efficient rate of interest, using again calibration summarized in Table 1 and assuming for illustrative purposes no shock persistence ($\rho_r = 0$). Under the optimal Taylor rule (dashed lines), the nominal interest rate increases by less than the natural rate of interest, in order to dampen the variability of the nominal interest rate. Monetary policy is therefore relatively expansionary so that inflation and the output gap increase at the time of the shock. In later periods however, these variables return to their initial steady-state as the perturbation vanishes. In contrast, in the optimal plan (solid lines), the short-term interest rate is more inertial than the efficient rate. Inertia in monetary policy is especially desirable here because it induces the private sector to expect future restrictive monetary policy,

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[12]The table reports the statistics in the case in which $x^* = i^* = 0$, so that the steady state is the same for each plan (and is zero for each variable). The statistics measure therefore the variability of each variable around its steady state, and the column labeled with $\mathrm{E}\,[L]$ indicates the loss due to temporary disturbances in excess of the steady-state loss. All statistics in Table 2 are reported in annual terms. The statistics $V\,[\pi]\,,$ $V\,[i]\,,$ and $\mathrm{E}\,[L]$ are therefore multiplied by 16. Furthermore, the weight λ_x reported in Table 1 is also multiplied by 16 in order to represent the weight attributed to the output gap variability (in annual terms) relative to the variability of annualized inflation and of the annualized interest rate.

hence future negative output gaps which in turn have a disinflationary effect already when the shock hits the economy. Thus the expectation of an inertial policy response allows the policymaker to offset the inflationary impact of the shock by raising the short-term interest rate by less than with the simple Taylor rule.[13]

Similarly, in the face of an unexpected transitory inefficient supply shock u_t (with $\rho_u = 0$), Figure 3 shows that the optimal Taylor rule induces the nominal interest rate to increase, so as to reduce output (gap), and therefore to mitigate inflationary pressures. In the optimal plan (solid lines), however, it is optimal to maintain the output gap below steady state even after the shock has vanished. This generates the expectation of a slight deflation in later periods and thus helps dampening the initial increase in inflation. The last panel confirms that the price level initially rises with the adverse shock but then declines back to almost return to its initial steady-state level. In fact the new steady-state price level is slightly below the initial one. The optimal interest rate that is consistent with the paths for inflation and the output gap hardly deviates from the steady-state, but it remains above steady-state for several periods, so as to achieve the desired deflation in later periods.

Figures 2 and 3 reveal that with the optimal Taylor rule of the form (21), the policy response does not introduce any inertia so that the interest rate deviates from the steady state only as long as the shocks last. In contrast, in the optimal plan, the effects of disturbances are mitigated more effectively on impact by being spread out over a longer period of time, through an inertial policy.

4.3   Welfare implications of alternative shock processes

We have shown above that the optimal Taylor rule coefficients are sensitive to the degree of persistence of shocks. This does not imply however that this sensitivity has important welfare implications, as two simple Taylor rules with different coefficients may in principle result in similar outcomes. To evaluate the welfare implications of alternative shock processes, we suppose that the central bank has committed to a simple Taylor rule, optimized under a correct assumption about the past shocks — i.e., that their law of motion is given by (8)–(9), with degrees of series correlation

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[13]When shocks are more persistent, the nominal interest rate also increases modestly on impact, in the optimal plan, but is expected to be higher than the efficient rate in later periods, so that agents can expect a tight monetary policy in the future, with negative output gaps and a decline of price level then. However to achieve a similar future path of the output gap and the price level, the optimal Taylor rule needs raise the interest rate sufficiently on impact, so as to bring down inflation and the output gap already at the time of the shock. This is why the optimal Taylor rule coefficients become negative when ρ_r is sufficiently large.

$\rho_r = \rho_u = 0.35$ — but that it now faces new disturbances $\hat{\varepsilon}_{r,m,t}$ and $\hat{\varepsilon}_{u,m,t}$ which propagate through the economy with a different persistence $\hat{\rho}$.[14]

Figure 4 plots welfare losses $E[L]$ for various policy rules as a function of the degree of persistence of the shocks r_t^e and u_t. The dashed line represents the welfare losses implied by the commitment to the Taylor rule optimized in our benchmark calibration, i.e., with $\rho_r = \rho_u = 0.35$. Note that this policy rule which has coefficients $\psi_\pi = 1.72$, $\psi_x = 0.57$ is not too different from the policy rule initially proposed by Taylor (1993). The figure shows again that in the benchmark case, the welfare loss under the optimal Taylor rule (2.63) is about twice as large as in the optimal plan (1.28) denoted here by the solid line, as documented in Table 2. However, as the shock persistence ρ increases, the welfare performance of this simple Taylor rule deteriorates considerably with losses approaching 50, i.e., about 15 times the loss under optimal policy, as ρ approaches 1. Figure 5 shows that the welfare deterioration is due to dramatic increases in inflation and interest-rate volatility under the simple Taylor rule when the shocks become more persistent. While the Taylor rule is relatively successful at stabilizing the output gap, this does not contribute much to the overall welfare given the low value of λ_x. Figures 4 and 5 consider changes in the persistence of both shocks r_t^e and u_t. Similar figures emerge however when one considers changes in the persistence of one shock at a time.

5   Commitment to a Simple Wicksellian Rule

As emphasized in the previous section, simple Taylor rules of the form (21) lack history dependence, a key property of optimal policy in forward-looking models, and involve optimal policy coefficients that are sensitive to the degree of persistence of exogenous disturbances. We now turn to an alternative very simple rule that introduces a desirable amount of history dependence and that turns out to be less sensitive to shock persistence. It is given by

$$i_t = \psi_p \left(p_t - \bar{p}_t \right) + \psi_x x_t + \psi_0 \tag{30}$$

at all dates $t \geq 0$, where $\bar{p}_t$ is some deterministic trend for the (log of the) price-level satisfying

$$\bar{p}_t = \bar{p}_{t-1} + \bar{\pi}, \tag{31}$$

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[14] We focus here on the properties of policy rules that have been optimized in a particular model, but we do not evaluate policy rules that would be robust to uncertainty about the underlying model, or uncertainty about driving shock processes. For the characterization of policy rules that are robust to model uncertainty, see e.g., Giannoni (2002, 2007), Hansen and Sargent (2008).

and $\bar{\pi}$ is a constant. Following Woodford (2003a, chap. 2) we call such a rule a Wicksellian rule. The price level depends by definition not only on current inflation but also on all past rates of inflation. It follows that the rule (30) introduces history dependence in monetary policy, as it forces the policymaker to compensate any shock that might have affected inflation in the past. While rules of this form are as simple as standard Taylor rules, they have received less attention in recent studies of monetary policy. One reason may be because it is widely believed that such rules would result in a larger variability of inflation (and the output gap), as the policymaker would respond to an inflationary shock by generating inflation below target in subsequent periods. However, as we show below, this is not true when agents are forward-looking and they understand that the policymaker *commits* to a rule of the form (30). Although the policymaker and the private sector do not care about the price level *per se*, as the latter does not enter the loss criterion (12), we shall argue that a Wicksellian rule has desirable properties for the conduct of monetary policy.

To characterize the equilibrium that obtains if the policymaker commits to (30), we consider a steady state in which in which inflation, the output gap and the nominal interest rate take respectively the constant values π^{wr}, x^{wr}, i^{wr}, we define the deviations from the steady state as $\hat{\pi}_t \equiv \pi_t - \pi^{wr}$, $\hat{x}_t \equiv x_t - x^{wr}$, $\hat{\imath}_t \equiv i_t - i^{wr}$, and we let $\hat{p}_t \equiv p_t - \bar{p}_t$ be the (percentage) deviation of the price level from its trend. As discussed in Appendix A.2, using (30) to substitute for $\hat{\imath}_t$ in the intertemporal IS equation, we can rewrite (4), (5), and (2) in matrix form as

$$\mathrm{E}_t z_{t+1} = \hat{A} z_t + \hat{a} e_t, \tag{32}$$

where $z_t \equiv [\hat{\pi}_t, \hat{x}_t, \hat{p}_{t-1}]'$, $e_t \equiv [r_t^e, u_t]'$, and $\hat{A}$ and $\hat{a}$ are matrices of coefficients. Assuming again that the law of motion of the disturbances is given by (8) and (9), the resulting equilibrium is then of the form

$$\hat{z}_t = z_r r_t^e + z_u u_t + z_p \hat{p}_{t-1} \tag{33}$$

for any variable $\hat{z}_t \in \{\hat{\pi}_t, \hat{x}_t, \hat{\imath}_t, \hat{p}_t\}$, where z_r, z_u, z_p are equilibrium response coefficients to fluctuations in r_t^e, u_t, and p_{t-1}. As further shown in Appendix A.2, the policy coefficients ψ_p and ψ_x relate in turn to the equilibrium coefficients as follows

$$\psi_p = \frac{x_u i_r - i_u x_r}{x_u p_r - x_r p_u} \tag{34}$$

$$\psi_x = \frac{p_r i_u - i_r p_u}{x_u p_r - x_r p_u}. \tag{35}$$

The optimal equilibrium resulting from a Wicksellian rule (30) is therefore characterized by the optimal steady state and the optimal response coefficients z_r, z_u, z_p in (33) that minimize the loss

function (12) subject to the constraints (2), (4), (5), and (30), where ψ_p and ψ_x are given by (34) and (35). For the Wicksellian rule to implement the desired equilibrium, though, it must guarantee that the dynamic system (32) admits a unique bounded solution. This is the case if and only if $\hat{A}$ admits exactly two unstable eigenvalues. An analysis of the matrix $\hat{A}$ yields the following result.

Proposition 1 *In the model composed of (4), (5), (2), with σ, $\kappa > 0$ and $0 < \beta < 1$, a commitment to the Wicksellian policy rule (30) results in a unique bounded rational expectations equilibrium $\{\pi_t, x_t, i_t, p_t\}$, if*

$$\psi_p > 0 \quad and \quad \psi_x \geq 0. \tag{36}$$

Proof. See Appendix A.3. ∎

Hence any Wicksellian rule with positive coefficients implies a determinate equilibrium.[15] In general, the coefficients of optimal Wicksellian rule are complicated functions of the parameters of the model. Moreover, unlike those of the optimal Taylor rule, they are also function of the variance of the shocks. Rather than trying to characterize analytically the optimal Wicksellian rule, we proceed with a numerical investigation of its properties and its implications for equilibrium inflation, output gap and the nominal interest rate.

5.1 A comparison of Taylor rules and Wicksellian rules

Figure 6 reports optimal coefficients of the Wicksellian rule for different degrees of shock persistence $(\rho_r; \rho_u)$. It is noteworthy that all optimal policy coefficients are positive for this wide range of shock persistence. It follows from Proposition 1 that these policy rules result in a determinate equilibrium. This contrasts with the optimal Taylor rules presented in Figure 1, which for some combinations yield an indeterminate equilibrium. Furthermore, while the optimal Taylor rule coefficients vary importantly with different degrees of shock persistence, Figure 6 shows that the optimal coefficients of Wicksellian rules are concentrated in a narrower area that those of optimal Taylor rules (Figure 1). The optimal Wicksellian rules are thus less sensitive to the different assumptions about serial correlation of the disturbances.

In addition, optimal Wicksellian rules also introduce a kind of history dependence that is desirable for monetary policy. In fact, as shown in Figures 2 and 3, transitory increases in r_t^e and u_t do generate persistent deviations of the endogenous variables when policy is set according to the optimal Wicksellian rule (dashed-dotted lines). Commitment to an optimal Wicksellian policy

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[15]A similar result is mentioned in Kerr and King (1996), in the case that ψ_x is set to 0.

allows the policymaker to achieve a response of endogenous variables that is closer to the optimal plan than is the case with the optimal Taylor rule.

One particularity of the equilibrium resulting from a Wicksellian policy, of course, is that the price level is stationary, unlike the Taylor rule which does not offset shifts in the price level following exogenous disturbances. In the optimal plan, policy also eventually brings the price level to its original trend in the face of inefficient supply shocks u_t. However, when the economy is hit by exogenous fluctuations in r_t^e and there is a concern for interest-rate stabilization ($\lambda_i > 0$), it is not optimal for the price level to be stable. In fact, in the optimal plan, Figure 2 shows that the price level is expected to end up at a slightly lower level in the future. While not fully optimal, the Wicksellian policy reduces welfare losses considerably by introducing history dependence and offsetting fluctuations in the price level. Another notable feature of optimal Wicksellian policy in Figure 3 is that the interest rate rises importantly, so that the response of inflation remains close to the optimal response. While this of creates a significant drop in output (gap) in our calibration, the welfare loss is only moderately affected by the recession, given the low weight λ_x.

In fact a comparison of the welfare implications for both Taylor and Wicksellian rules suggests that Wicksellian rules result in general in a lower welfare loss, in the model considered here. We first show this analytically in a simple case and then proceed with a numerical investigation of the more general case.

A special case. To simplify the analysis, we consider the special case in which the short-term aggregate supply equation is perfectly flat so that $\kappa = 0$, and both shocks have the same degree of serial correlation ρ. In this case, we can solve for equilibrium inflation using (5), and we obtain

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + u_t = \sum_{j=0}^{\infty} \beta^j \mathrm{E}_t u_{t+j} = (1 - \beta\rho)^{-1} u_t.$$

Inflation is perfectly exogenous in this case. The best the policymaker can do is therefore to minimize the variability of the output gap and the interest rate.

Using (28) and (29), we note that the optimal Taylor rule reduces in this case to

$$\hat{\imath}_t = \frac{\lambda_x}{\lambda_i \sigma (1 - \rho)} \hat{x}_t,$$

and so involves no response to inflation. Since inflation is cannot be affected by monetary policy in this case, it would be desirable to respond to inflation only if this would help dampening fluctuations in the output gap and the interest rate. However, since the Taylor rule is non inertial, the

equilibrium endogenous variables depend only on contemporaneous shocks (see (23)). It follows that one cannot reduce the variability of future output gaps and interest rates by responding to current shocks in inflation. Responding to contemporaneous fluctuations in inflation would only make the interest rate and the output gap more volatile.

In contrast, with a Wicksellian rule, the policymaker's response to contemporaneous price-level fluctuations and the belief that he will respond in the same way to price-level fluctuations in the future have a desirable effect on the expected future path of the output gap and the interest rate. We can establish the following result.

Proposition 2 *When $\kappa = 0$ and $\rho_r = \rho_u \equiv \rho > 0$, there exists a Wicksellian rule of the form (30) that results in a unique bounded equilibrium, and that achieves a lower loss than the one resulting from the optimal Taylor rule.*

Proof. See Appendix A.4. ∎

General case: A numerical investigation. In the more general case in which $\kappa > 0$ and we allow for arbitrary degrees of serial correlation of the shocks, the analytical characterization is substantially more complicated. However a numerical investigation suggests again that appropriate Wicksellian rules perform better than the optimal Taylor rule in terms of the loss criterion (12). Using the calibration of Table 1, and for various degrees persistence of the disturbances, Table 2 reveals that the loss is systematically lower with the optimal Wicksellian rule than it is with the optimal Taylor rule. For instance, when $\rho_r = \rho_u = .35$, the loss is 1.67 with the Wicksellian rule, compared to 2.63 with the Taylor rule, and 1.28 with the fully optimal rule.[16] This relatively good performance of the Wicksellian rules is due to the low variability of inflation and the nominal interest rate. On the other hand, the output gap is in general more volatile under the optimal Wicksellian rule. Of course the variability of the price level is much higher for fully optimal rules and optimal Taylor rules, but this does not affect the loss criterion. While the results of Table 2 are based on our benchmark calibration, we still find, for alternative assumptions about the parameters λ_x, ρ_r, ρ_u, and the variances of the shocks, that the welfare loss $\mathrm{E}[L]$ implied by the optimal Wicksellian rule is lower than that implied by the optimal Taylor rule, and is only slightly higher than in the optimal plan.

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[16]Recall that Table 2 indicates the losses due to fluctuations around the steady state. However, since the steady states are the same for the optimal Taylor rule and the optimal Wicksellian rule, the comparison of statistics is also relevant for levels of the variables, for any values x^*, i^*.

In addition, the simple Wicksellian rules turn out to be very robust to alternative specifications of the shock processes. Looking again at Figures 4 and 5, we observe that an optimal Wicksellian rule — optimized under the assumption that the serial correlation of the shocks is $\rho_r = \rho_u = 0.35$ — performs again very well when the economy is hit by new disturbances $\hat{\varepsilon}_{r,m,t}$ and $\hat{\varepsilon}_{u,m,t}$ which propagate through the economy with a different persistence $\hat{\rho}$. The welfare losses under the Wicksellian rule (dashed-dotted line) remain only slightly above the losses in the first best, even for very high degrees of shock persistence. This is very different from the performance of simple Taylor rules which imply very high losses when ρ approaches 1, and suggests that the commitment to bringing the price level back to its original trend is an effective way of guarding against misspecifications or changes of the shock processes.

6 Introducing Interest-Rate Inertia

Wicksellian rules have the desirable feature of introducing history dependence while the simple Taylor rules considered so far are by assumption not inertial. One natural question is thus whether the performance of simple Taylor rules could not be dramatically improved by letting the interest rate respond also to past interest rates, as this would introduce at least some form of history dependence in policy. To answer this question, we first characterize a fully optimal rule.

6.1 A Robustly Optimal Policy Rule

As argued in Giannoni (2001, Chap. 1) and Giannoni and Woodford (2003a,b, 2010), it is possible under general conditions to find a policy rule that is optimal — i.e., that minimizes the welfare loss $\mathrm{E}[L]$ subject to the constraints (4) and (5) imposed by the private sector — and that is also robust to alternative specifications of the shock processes. This *robustly optimal policy rule* is obtained by combining the first-order necessary conditions (14)–(17) characterizing the optimal state-contingent plan. These first-order conditions involve only two types of variables: variables entering the policymakers' objective function (i.e., target variables) and Lagrange multipliers. Combining them to eliminate the Lagrange multipliers yields a single equation involving only target variables, which can be interpreted as an implicit policy rule. We thus solve (16) for ϕ_{1t} as a function of i_t, and (15) for ϕ_{2t} as a function of x_t, i_t, i_{t-1}, and use the resulting expressions to substitute for the Lagrange

multipliers in (14). This yields the instrument rule[17]

$$i_t = \psi_\pi \pi_t + \psi_x \Delta x_t + (1 + \psi_i) i_{t-1} + \psi_{\Delta i} \Delta i_{t-1} - \psi_i i^* \tag{37}$$

where $\Delta x_t \equiv x_t - x_{t-1}$ denotes first differences, and the policy coefficients are given by

$$\psi_\pi = \frac{\kappa}{\lambda_i \sigma} > 0, \qquad \psi_x = \frac{\lambda_x}{\lambda_i \sigma} > 0, \qquad \psi_i = \frac{\kappa}{\beta \sigma} > 0, \qquad \psi_{\Delta i} = \beta^{-1} > 1. \tag{38}$$

This rule necessarily holds in the optimal plan in all period $t \geq 2$, for it is consistent with the first-order conditions (14)–(16) at these dates. For this policy rule to *implement* the optimal equilibrium, it must not merely be consistent with with the optimal plan, it must also determine a *unique* bounded equilibrium. Remarkably, the rule (37) also has this very desirable property. In fact, a commitment to the policy rule (37) at all dates $t \geq 0$ implies a determinate rational-expectations equilibrium (see Giannoni and Woodford, 2003b, Proposition 1).[18]

The equilibrium implied by a commitment to the time-invariant policy rule (37) at all dates $t \geq 0$ is the unique bounded solution the structural equations (4)–(5), the first-order conditions (14)–(16) at all dates $t \geq 0$, where the initial Lagrange multipliers $\phi_{1,-1}$, $\phi_{2,-1}$ are not given by (17) but depend instead on the historical values x_{-1}, i_{-1}, and i_{-2}, through the equations (14)–(16). Such a policy involves the same response to random shocks in periods $t \geq 0$ as in the optimal (Ramsey) plan, and is a rule that is optimal from a *timeless perspective* (see, e.g., Woodford 1999).[19]

A further very interesting feature of this policy rule is that it does not involve any shock.[20] The optimal policy rule (37) has thus the very desirable property of being completely *robust* to the specification of the shock processes, even if the latter are of the form specified in (6)–(7), as long as they are bounded.

An implication of this is that a commitment to the optimal rule (37) with coefficients given by (38) does not only implement the optimal plan in the case of the assumed autocorrelation of the

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[17]This rule is analogous to what Svensson (2003) calls an optimal specific targeting rule.

[18]As further shown in Giannoni and Woodford (2010), a policy rule (or target criterion) constructed in this fashion from the first-order conditions associated with the optimal policy problem implies a locally determinate equilibrium under very general conditions, even in the context of large-scale nonlinear models.

[19]The optimal (Ramsey) plan is the bounded solution to the structural equations (4)–(5), the first-order conditions (14)–(16) at all dates $t \geq 0$, where the initial Lagrange multipliers $\phi_{1,-1}$, $\phi_{2,-1}$ are given by (17). Such a plan can be implemented by the time-varying rule given by $i_0 = \psi_\pi \pi_0 + \psi_x x_0$, in period 0, $i_1 = \psi_\pi \pi_1 + \psi_x (x_1 - x_0) + (1 + \psi_{i1}) i_0$ in period 1 and (37) at all dates $t \geq 2$.

[20]More generally, as long as the shocks enter in an additively separable fashion in the policymaker's objective function and in the constraints imposed by the private sector, the first-order conditions to the optimal policy problem don't involve any exogenous shocks or even any properties of their driving processes.

shocks, but also for any other degree of shock persistence. As a result, the policymaker would not need to reconsider its commitment and change the rule in the event that the economy would be hit by shocks that have different properties than then ones observed prior to the commitment. By keeping the policy rule unchanged, it would continue to achieve the optimal equilibrium, hence the lowest possible loss for any value of ρ. The welfare losses associated with the rule (37)–(38) are therefore the ones associated with the optimal plan, and displayed by the solid line in Figures 4 and 5, for different values of ρ.[21]

Equation (37) indicates that to implement the optimal plan, the central bank should relate the interest rate positively to fluctuations in current inflation, in *changes* of the output gap, and in lagged interest rates.[22] Note finally that the interest rate should not only be inertial in the sense of being positively related to past values of the interest rate, it should be *super-inertial*, as the lagged polynomial for the interest rate in (37)

$$1 - \left(1 + \frac{\kappa}{\beta\sigma} + \beta^{-1}\right)L + \beta^{-1}L^2 = (1 - z_1 L)(1 - z_2 L)$$

involves a root $z_1 > 1$ while the other root $z_2 \in (0, 1)$. A reaction greater than one of the interest rate to its lagged value has initially been found by Rotemberg and Woodford (1999) to be a desirable feature of a good policy rule in their econometric model with optimizing agents. As explained further in Woodford (2003b), it is precisely such a *super-inertial* rule that the policymaker should follow to bring about the optimal responses to shocks when economic agents are forward-looking. Optimal policy requires rapidly raising the interest rate to deviations of inflation and the output gap from the target (which is 0), if such deviations are not subsequently undone. But of course,

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[21] While assume here that the policymaker knows with certainty the model of the economy, though it may face uncertainty about the shock processes, Walsh (2004) has shown that the same rule turns out to be robust to mis-specifications of the structural model of the kind considered by Hansen and Sargent (2008). According to Hansen and Sargent's robust control approach, the policymaker views its model as an approximation to the true model, with the true model being in a neighborhood of the approximating model. Such a problem can be represented by a game between the policymaker who seeks to minimize the loss function (12) while a malevolent agent tries to maximize it. The central bank thus attempts to characterize a robust rule that performs as well as possible in this worst-case scenario. As Walsh (2004) has shown, such a robust rule in our model would take exactly the same form as (37)– (38). However, as Walsh (2004) emphasizes, while the rule is the same in the two approaches, different macroeconomic behavior would be observed, as expectations are formed differently in the two approaches.

[22] From a practical point of view, it might be an advantage to respond to changes in the output gap rather than the level as the *change* in the output gap may be known with greater precision. For example, Orphanides (2003) shows that subsequent revisions of U.S. output gap estimates have been quite large (sometimes as large as 5.6 percentage points), while revisions of estimates of the quarterly change in the output gap have been much smaller.

such a policy is perfectly consistent with a *stationary* rational expectations equilibrium, and in fact is the one generating the lowest overall welfare loss and a low variability of the interest rate *in equilibrium.* In fact, the interest rate does not explode in equilibrium because the current and expected future optimal levels of the interest rate counteract the effects of an initial deviation in inflation and the output gap by generating subsequent deviations with the opposite sign of these variables, as shown in Figures 2 and 3.[23]

The coefficients of the optimal policy rule are reported in the upper right panel of Table 2, for our benchmark calibration.[24] For comparison, the last panel of Table 2 reports the coefficients derived from Judd and Rudebusch's (1998) estimation of actual Fed reaction functions between 1987:3 and 1997:4, along with the statistics that such a policy would imply if the model provided a correct description of the actual economy.[25] As shown on Table 2, the estimated historical rule in the baseline case involves only slightly smaller responses to fluctuations in inflation and the output gap than the optimal rule. However the estimated response to lagged values of the interest rate is sensibly smaller that the optimal one.

6.2   Simple Rules and Interest Rate Inertia

The analysis of the optimal policy rule (37) suggests that it is desirable for the current interest rate to respond strongly to movements in the past interest rate. While the policy rule (37) achieves the lowest possible loss in the model considered and remains relatively simple, recent research has given considerable attention to even simpler policy rules (see, e.g., contributions collected in Taylor, 1999). As we now show, even if we allow for considerable inertia in interest rate in the policy rule, it remains preferable to respond to fluctuations in the price level than in the inflation rate. To see this, consider a minor departure from the optimal rule (37) with coefficients given by (38), neglecting the term $\psi_i \left(i_{t-1} - \bar{\imath} \right)$ and setting $\psi_{\Delta i}$ to 1 instead of β^{-1}.[26] After this simplification,

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[23]Optimal interest-rate rules are super-inertial under general conditions, as long as the private sector is sufficiently forward-looking (see, Giannoni and Woodford, 2003a). Some authors have however criticized such rules, on the grounds that they perform poorly in non-rational expectations, backward-looking, models (e.g., Taylor, 1999b). This should not be surprising since super-inertial rules rely precisely on the private sector's forward-looking behavior.

[24]The coefficients ψ_x reported here are multiplied by 4, so that the response coefficients to output gap, and to annualized inflation are expressed in the same units. (See footnote 12.)

[25]The estimated historical policy rule refers to regression A for the Greenspan period in Judd and Rudebusch (1998).

[26]Doing so prevents the rule from being super-inertial, a feature that has been criticized on the grounds that such rules lead to explosive behavior in models which involve no rational expectations and no forward-looking behavior

and using (2), the rule (37) reduces to

$$\Delta i_t = \psi_\pi \left(\Delta p_t - \bar{\pi} \right) + \psi_x \Delta x_t + \Delta i_{t-1} \tag{39}$$

where ψ_π and ψ_x are again given by (38), and the steady-state inflation rate is given by $\bar{\pi} = \lambda_i i^*/\beta$. Assuming furthermore that at some point $t_0 - 1$ in the past, the interest rate satisfied $i_{t_0-1} = \psi_\pi \left(p_{t_0-1} - \bar{p}_{t_0-1} \right) + \psi_x x_{t_0-1} + i_{t_0-2}$ and using (31) implies that a commitment to (39) at all dates $t \geq t_0$ is equivalent a commitment to the rule

$$i_t = \psi_\pi \left(p_t - \bar{p}_t \right) + \psi_x x_t + i_{t-1} \tag{40}$$

at all dates $t \geq t_0$. This of course is none else than a Wicksellian rule augmented with the lagged interest rate. Given that the coefficient on the lagged interest rate is 1, this *quasi-optimal rule* specifies how changes in the interest rate Δi_t should be set as a function of fluctuations in the price level (in log deviations from a trend) and the output gap. The rule (40) reveals that a desirable policy involves even more history dependence than we had considered in the case of simple Wicksellian rules.

Table 3 quantifies the welfare losses implied by a commitment to the quasi-optimal rule (40) for different degrees of shock persistence. Figures 4 and 5 also plot the welfare losses (with black dots) as a function the shocks' autocorrelation. Importantly, this very simple rule performs remarkably well, with welfare losses appear only marginally higher than in the fully optimal rule for a very wide range of shock persistence, given that the policy coefficients are totally invariant to the assumed properties of the shock process.

To contrast, we consider now an expanded version of the Taylor rule that allows for interest rate inertia

$$i_t = \psi_\pi \left(\pi_t - \bar{\pi} \right) + \psi_x x_t + i_{t-1}, \tag{41}$$

where ψ_π and ψ_x are again given by (38). Clearly, introducing a large amount of interest rate inertia contributes to reducing the welfare losses substantially: comparing Tables 2 and 3, we note that the welfare losses drop from 2.63 to 1.38 when introducing the lagged interest rate in the Taylor rule and $\rho_r = \rho_u = 0.35$. This is not surprising, in light of the discussion in sections 2 and 3, as the rule (41) resembles closely the simple Wicksellian rule, were it not for the response to the output gap.[27] In addition, Levin *et al.* (1999) show that rules that have a coefficient of one on the lagged

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(see, e.g., Taylor 1999b).

[27]Indeed, assuming that at some point $t_0 - 1$ in the past the interest rate satisfied $i_{t_0-1} = \psi_\pi \left(p_{t_0-1} - \bar{p}_{t_0-1} \right) +$

interest rate tend to perform well across models, and Orphanides and Williams (2007) show that such rules are robust to potential misspecification of private sector learning, in a model in which agents have imperfect knowledge about the the structure of the economy. However, as Figure 4 and 5 show, the performance of the rule (41) deteriorates also markedly as the shock persistence increases. So, while allowing for high degree of interest inertia in interest rates allows to improve the performance of simple policy rules, our results show that it remains preferable for the interest rate to respond to price-level fluctuations than to inflation fluctuations in the model considered, and the gains from price-level stabilization are larger in the face of misspecifications of the shock processes.

7   Conclusion

This paper has characterized the properties of various simple interest-rate rules in the context of a stylized structural forward-looking model of the economy. We have compared the performance of simple Taylor rules and simple Wicksellian rules — which determine the interest rate as a function of deviations of the price level from its trend and an output gap — to determine whether the central bank's objective function, which is assumed to depend on the volatility of inflation, output gap and interest rate, is best achieved by a policy that responds to fluctuations in inflation or the price level. We have shown that appropriate Wicksellian rules result systematically in a lower welfare loss, a lower variability of inflation and of the nominal interest rate than optimal Taylor rules, by introducing desirable history dependence in monetary policy. The coefficients of optimal Wicksellian rules have the further advantage of being less sensitive to alternative degrees of persistence in the shock processes. An implication of this is that Wicksellian rules perform better than simple Taylor rules in the face of changes in shock processes. This makes a commitment to simple Wicksellian rules more appealing as their robustness property provides little ground for a reconsideration of the commitment when the economy is affected by new kinds of disturbances. Moreover, Wicksellian rules are less prone to equilibrium indeterminacy than optimal Taylor rules.

The fact that simple Wicksellian rules perform so well in our model becomes clear when we observe a simple Wicksellian rule augmented with a large amount of interest-rate inertia (40) resembles a *robustly optimal rule* which, as argued in Giannoni and Woodford (2003a,b, 2010),

$\psi_x x_{t_0-1}$, a commitment to the "difference rule" $\Delta i_t = \psi_\pi (\pi_t - \bar{\pi}) + \psi_x \Delta x_t$ at all dates $t \geq t_0$ is equivalent a commitment to the Wicksellian rule $i_t = \psi_\pi (p_t - \bar{p}_t) + \psi_x x_t$ at all dates $t \geq t_0$.

implements the optimal plan and is also completely robust to the specification of exogenous shock processes. A Wicksellian rule of this form states that *changes* (and not the level) of the policy rates should depend positively on the deviations of the *price level* from trend, and the output gap. This rule remains very simple, is again fully robust to the specification of the shock processes, and introduces an even great amount of history dependence than simple Wicksellian rules, which yields a remarkable welfare performance in the model considered. Such a rule should thus be particularly appealing to policymakers who search for a simple rule, yet worry about unforeseeable circumstances (shocks) affecting the economy in the future.

Our results have been derived here in an arguably very stylized model, in which agents have full information about the current state of the economy, and are completely rational. This has allowed us to emphasize that the history dependence generated by the price-level stabilization results in important welfare gains and has good robustness properties to the assumed shock processes. Recent research suggests that the benefits from price-level stabilization hold in more general setups. In fact, Preston (2008) assumes in a similar model that private agents are non-rational and learn adaptively. He shows that a price-level target corrects past mistakes and yields better welfare results than inflation stabilization when the central bank cannot perfectly understand private agents' behavior. This is consistent with Orphanides and Williams (2007)'s conclusion on the desirability of "difference" interest-rate rules, which resemble our Wicksellian rules. Gaspar, Smets and Vestin (2007), using in a medium-scale model involving a number of rigidities and inertial behavior of the private sector find that the stabilization of the price-level path is a simple and effective way of implementing a desirable equilibrium. A concern raised by Mishkin and Schmidt-Hebbel (2001) is that a commitment to price-level stabilization may propagate iid measurement errors in inflation. However Gorodnichenko and Shapiro (2007) considering a forward-looking model with backward-looking features show that a rule akin to our Wicksellian rule can effectively stabilize the economy in the face of imperfectly observed shifts in potential output growth or surprises in the price level. Boivin (2009) and Woodford (2010) similarly argue that stabilizing the price level might be more desirable in the event that the price level is not perfectly observed, provided that the public is sufficiently forward looking and understands the policy regime. In addition, Eggertsson and Woodford (2003), Wolman (2005) and Billi (2008) have argued that a commitment to price-level stabilization (possibly around a drifting path), may be an effective way of preventing deflations, and exiting from deflationary traps. While these papers emphasize different desirable features of price-level stabilization, they all point to some robustness property of price-level stabilization.

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A Appendix

A.1 Optimal non-inertial plan and optimal Taylor rule

In a non-inertial equilibrium, the coefficients π^{ni}, π_r, π_u, and so on characterizing the solution (23) need to satisfy the structural equations (4) and (5) at each date, and for every possible realization of the shocks. These coefficients need therefore to satisfy the following feasibility restrictions, obtained by substituting (23) into the structural equations (4) and (5):

$$(1 - \beta)\,\pi^{ni} - \kappa x^{ni} = 0 \tag{A.42}$$

$$\pi^{ni} - i^{ni} = 0 \tag{A.43}$$

$$(1 - \rho_r)\,x_r + \sigma^{-1}\,(i_r - \rho_r \pi_r - 1) = 0 \tag{A.44}$$

$$(1 - \beta\rho_r)\,\pi_r - \kappa x_r = 0 \tag{A.45}$$

$$(1 - \rho_u)\,x_u + \sigma^{-1}\,(i_u - \rho_u \pi_u) = 0 \tag{A.46}$$

$$(1 - \beta\rho_u)\,\pi_u - \kappa x_u - 1 = 0. \tag{A.47}$$

Similarly, substituting (23) into (12), and using $\mathrm{E}\,(r_t^e u_t) = 0$, we can rewrite the loss function as

$$
\begin{aligned}
\mathrm{E}\,[L_0] &= \left[\left(\pi^{ni}\right)^2 + \lambda_x \left(x^{ni} - x^*\right)^2 + \lambda_i \left(i^{ni} - i^*\right)^2\right] + \left(\pi_r^2 + \lambda_x x_r^2 + \lambda_i i_r^2\right)\mathrm{var}\,(r_t^e) \\
&\quad + \left(\pi_u^2 + \lambda_x x_u^2 + \lambda_i i_u^2\right)\mathrm{var}\,(u_t).
\end{aligned}
$$

To determine the optimal non-inertial plan, we choose the equilibrium coefficients that minimize the loss $\mathrm{E}\,[L_0]$ subject to the restrictions (A.42) – (A.47). The steady-state of the optimal non-inertial equilibrium is then given by

$$i^{ni} = \pi^{ni} = \frac{(1 - \beta)\,\kappa^{-1}\lambda_x x^* + \lambda_i i^*}{1 + (1 - \beta)^2\,\kappa^{-2}\lambda_x + \lambda_i}, \qquad x^{ni} = \frac{1 - \beta}{\kappa}\frac{(1 - \beta)\,\kappa^{-1}\lambda_x x^* + \lambda_i i^*}{1 + (1 - \beta)^2\,\kappa^{-2}\lambda_x + \lambda_i}, \tag{A.48}$$

and the optimal response coefficients to fluctuations in r_t^e and u_t in the optimal non-inertial equilibrium are given by

$$\pi_r = \frac{\lambda_i\,(\sigma\gamma_r - \rho_r\kappa)\,\kappa}{h_r}, \qquad \pi_u = \frac{\lambda_i\sigma\,(\sigma\gamma_u - \rho_u\kappa)\,(1 - \rho_u) + \lambda_x\,(1 - \beta\rho_u)}{h_u} \tag{A.49}$$

$$x_r = \frac{\lambda_i\,(\sigma\gamma_r - \rho_r\kappa)\,(1 - \beta\rho_r)}{h_r}, \qquad x_u = -\frac{\kappa - \rho_u\lambda_i\,(\sigma\gamma_u - \rho_u\kappa)}{h_u} \tag{A.50}$$

$$i_r = \frac{\lambda_x\,(1 - \beta\rho_r)^2 + \kappa^2}{h_r} > 0, \qquad i_u = \frac{\sigma\kappa\,(1 - \rho_u) + \lambda_x\,(1 - \beta\rho_u)\,\rho_u}{h_u} > 0, \tag{A.51}$$

where

$$\gamma_j \equiv (1 - \rho_j)\,(1 - \beta\rho_j) > 0$$

$$h_j \equiv \lambda_i\,(\sigma\gamma_j - \rho_j\kappa)^2 + \lambda_x\,(1 - \beta\rho_j)^2 + \kappa^2 > 0,$$

and where $j \in \{r, u\}$.

For the Taylor rule to be consistent with an equilibrium of the form (23), the policy coefficients must satisfy the following restrictions

$$i_r = \psi_\pi \pi_r + \psi_x x_r \tag{A.52}$$

$$i_u = \psi_\pi \pi_u + \psi_x x_u \tag{A.53}$$

$$i^{ni} = \psi_\pi \pi^{ni} + \psi_x x^{ni} + \psi_0 \tag{A.54}$$

obtained by substituting the solutions (23) into (21). Solving (A.52)–(A.53) for the policy coefficients and substituting the coefficients $\pi_r, x_r, ...,$ with the expressions in (A.49)–(A.51) characterizing the optimal non-inertial equilibrium, yields the optimal Taylor coefficients

$$\psi_\pi = \frac{(\kappa - \rho_u \lambda_i (\sigma \gamma_u - \rho_u \kappa)) \left(\xi_r (1 - \beta \rho_r) + \kappa^2\right) + (\sigma \kappa (1 - \rho_u) + \rho_u \xi_u) \lambda_i (\sigma \gamma_r - \rho_r \kappa) (1 - \beta \rho_r)}{\lambda_i (\sigma \gamma_r - \rho_r \kappa) ((\kappa - \rho_u \lambda_i (\sigma \gamma_u - \rho_u \kappa)) \kappa + (\lambda_i \sigma (\sigma \gamma_u - \rho_u \kappa) (1 - \rho_u) + \xi_u) (1 - \beta \rho_r))} \tag{A.55}$$

$$\psi_x = \frac{(\lambda_i \sigma (\sigma \gamma_u - \rho_u \kappa) (1 - \rho_u) + \xi_u) \left(\xi_r (1 - \beta \rho_r) + \kappa^2\right) - \lambda_i (\sigma \gamma_r - \rho_r \kappa) \kappa (\sigma \kappa (1 - \rho_u) + \rho_u \xi_u)}{\lambda_i (\sigma \gamma_r - \rho_r \kappa) ((\kappa - \rho_u \lambda_i (\sigma \gamma_u - \rho_u \kappa)) \kappa + (\lambda_i \sigma (\sigma \gamma_u - \rho_u \kappa) (1 - \rho_u) + \xi_u) (1 - \beta \rho_r))} \tag{A.56}$$

where $\gamma_j \equiv (1 - \rho_j)(1 - \beta \rho_j) > 0$, $\xi_j \equiv \lambda_x (1 - \beta \rho_j) > 0$, and $j \in \{r, u\}$. Note that these expressions are well defined provided that $\sigma \gamma_r - \rho_r \kappa \neq 0$.

The constant ψ_0 is then obtained by solving (A.54), using the preceding expressions for ψ_π and ψ_x.

A.2 Optimal Wicksellian rule

We observe from (4) and (5) that in a steady state

$$i^{wr} = \pi^{wr}, \qquad x^{wr} = \frac{1 - \beta}{\kappa} \pi^{wr}. \tag{A.57}$$

The structural equations (4), (5) can then be expressed in terms of the hatted variables representing deviations from the steady state, and the policy rule (30) may be written as[28]

$$\hat{\imath}_t = \psi_p \hat{p}_t + \psi_x \hat{x}_t. \tag{A.58}$$

Using the latter equation to substitute for $\hat{\imath}_t$ in (4), we can write the resulting system in matrix form as:

$$\mathrm{E}_t \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{x}_{t+1} \\ \hat{p}_t \end{bmatrix} = \hat{A} \begin{bmatrix} \hat{\pi}_t \\ \hat{x}_t \\ \hat{p}_{t-1} \end{bmatrix} + \hat{a} \begin{bmatrix} r_t^e \\ u_t \end{bmatrix} \tag{A.59}$$

where the matrix $\hat{A}$ is given by

$$\hat{A} = \begin{bmatrix} \beta^{-1} & -\kappa \beta^{-1} & 0 \\ \sigma^{-1} (\psi_p - \beta^{-1}) & \left(1 + \sigma^{-1} \left(\frac{\kappa}{\beta} + \psi_x\right)\right) & \psi_p \sigma^{-1} \\ 1 & 0 & 1 \end{bmatrix}. \tag{A.60}$$

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[28] To obtain (A.58), we make an implicit assumption on the coefficient ψ_0 which has no effect on the welfare analysis that follows. First, note from (30) that $\hat{p}_t$ must be constant in the steady state. For convenience, we set this constant to zero. The optimal policy coefficient ψ_0 is the only coefficient affected by this normalization, but this has no effect on optimal monetary policy. Comparing (30) and (A.58) one can see that ψ_0 is implicitly given by $\psi_0 = i^{wr} - \psi_x x^{wr}$. Note also from the definition of inflation that $\pi_t = p_t - p_{t-1} = \hat{p}_t - \hat{p}_{t-1} + \bar{\pi}$. Hence, in the steady state, we have $\pi^{wr} = \bar{\pi}$.

Assuming again that the law of motion of the disturbances is given by (8) and (9), the resulting equilibrium is of the form (33).

Using the solution (33) and noting that $\mathrm{E}\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t \hat{z}_t\right] = 0$, we can write the loss criterion (12) as

$$\mathrm{E}\left[L\right] = \left[(\pi^{wr})^2 + \lambda_x\left(x^{wr} - x^*\right)^2 + \lambda_i\left(i^{wr} - i^*\right)^2\right] + \mathrm{E}\left[\hat{L}\right]. \tag{A.61}$$

where

$$\mathrm{E}[\hat{L}] \equiv \mathrm{E}\left\{(1-\beta)\sum_{t=0}^{\infty}\beta^t\left[\hat{\pi}_t^2 + \lambda_x\hat{x}_t^2 + \lambda_i\hat{i}_t^2\right]\right\}. \tag{A.62}$$

The optimal steady state is then found by minimizing the first term in brackets in (A.61) subject to (A.57). Since this is the same problem as the one encountered for the optimal non-inertial plan, we have

$$\pi^{wr} = \pi^{ni}, \qquad x^{wr} = x^{ni}, \qquad \text{and} \qquad i^{wr} = i^{ni}. \tag{A.63}$$

where π^{ni}, x^{ni}, and i^{ni} are given in (A.48).

To determine the optimal equilibrium responses to disturbances, we note, as in the optimal non-inertial plan, that the solution (33) may only describe an equilibrium if the coefficients z_r, z_u, z_p satisfy the structural equations (4) and (5) at each date, and for every possible realization of the shocks. These coefficients need therefore to satisfy the following feasibility restrictions, obtained by substituting (33) into the structural equations (4), (5), and using (2):

$$x_r\left(1 - \rho_r\right) - x_p p_r + \sigma^{-1}\left(i_r + (1 - p_p - \rho_r)p_r - 1\right) = 0 \tag{A.64}$$

$$x_u\left(1 - \rho_u\right) - x_p p_u + \sigma^{-1}\left(i_u + (1 - p_p - \rho_u)p_u\right) = 0 \tag{A.65}$$

$$x_p - x_p p_p + \sigma^{-1}\left(i_p + (1 - p_p)p_p\right) = 0 \tag{A.66}$$

$$\left(\beta\rho_r + \beta p_p - 1 - \beta\right)p_r + \kappa x_r = 0 \tag{A.67}$$

$$\left(\beta\rho_u + \beta p_p - 1 - \beta\right)p_u + \kappa x_u + 1 = 0 \tag{A.68}$$

$$\left(\beta p_p - 1 - \beta\right)p_p + \kappa x_p + 1 = 0. \tag{A.69}$$

Similarly, substituting the solution (33) into the policy rule (A.58) yields

$$i_r = \psi_p p_r + \psi_x x_r \tag{A.70}$$

$$i_u = \psi_p p_u + \psi_x x_u \tag{A.71}$$

$$i_p = \psi_p p_p + \psi_x x_p. \tag{A.72}$$

Using (A.70) and (A.71), we can then determine the policy coefficients ψ_p and ψ_x, to obtain

$$\psi_p = \frac{x_u i_r - i_u x_r}{x_u p_r - x_r p_u} \tag{A.73}$$

$$\psi_x = \frac{p_r i_u - i_r p_u}{x_u p_r - x_r p_u}, \tag{A.74}$$

provided that $x_u p_r - x_r p_u \neq 0$. Substituting (A.73) and (A.74) into (A.72), we obtain

$$i_p - \frac{x_u i_r - i_u x_r}{x_u p_r - x_r p_u}p_p - \frac{p_r i_u - i_r p_u}{x_u p_r - x_r p_u}x_p = 0, \tag{A.75}$$

which is an additional constraint that must be satisfied by the equilibrium coefficients, for the structural equations and the policy rule to be satisfied at each date and in every state.

Finally using (2), the solution (33), and the laws of motion (8) and (9), we can rewrite the loss (A.62) as

$$
\begin{aligned}
\mathrm{E}[\hat{L}] \ =\ & \mathrm{var}\,(r_t^e)\left(\left(p_r^2 + \lambda_x x_r^2 + \lambda_i i_r^2\right) + \left(p_r\,(p_p-1) + \lambda_x x_r x_p + \lambda_i i_r i_p\right)\frac{2\beta\rho_r p_r}{1-\beta\rho_r p_p}\right) \\
& + \mathrm{var}\,(u_t)\left(\left(p_u^2 + \lambda_x x_u^2 + \lambda_i i_u^2\right) + \left(p_u\,(p_p-1) + \lambda_x x_u x_p + \lambda_i i_u i_p\right)\frac{2\beta\rho_u p_u}{1-\beta\rho_u p_p}\right) \qquad (A.76) \\
& + \left((p_p-1)^2 + \lambda_x x_p^2 + \lambda_i i_p^2\right)\left(\mathrm{var}\,(r_t^e)\frac{\beta p_r^2}{1-\beta p_p^2}\frac{1+\beta\rho_r p_p}{1-\beta\rho_r p_p} + \mathrm{var}\,(u_t)\frac{\beta p_u^2}{1-\beta p_p^2}\frac{1+\beta\rho_u p_p}{1-\beta\rho_u p_p}\right).
\end{aligned}
$$

The optimal equilibrium resulting from a Wicksellian rule (30) is therefore characterized by the optimal steady state (A.63), and the optimal response coefficients p_r, p_u, and so on, that minimize the loss function (A.76) subject to the constraints (A.64) – (A.69) and (A.75). The coefficients of the optimal Wicksellian rule that are consistent with that equilibrium are in turn determined by (A.73) and (A.74).

A.3   Proof of Proposition 1

The model composed of (4), (5), (2), with a commitment to the Wicksellian policy rule (30) can be expressed in matrix form as the dynamic system (A.59) with transition matrix (A.60). The characteristic polynomial associated to $\hat{A}$ is

$$
P\,(X) = X^3 + A_2 X^2 + A_1 X + A_0
$$

where

$$
\begin{aligned}
A_0 \ &=\ -\frac{\sigma + \psi_x}{\beta\sigma} \\
A_1 \ &=\ \frac{\kappa + \sigma\beta + 2\sigma + \psi_x\beta + \kappa\psi_p + \psi_x}{\beta\sigma} \\
A_2 \ &=\ -\frac{2\sigma\beta + \sigma + \kappa + \psi_x\beta}{\beta\sigma}.
\end{aligned}
$$

The system (A.59) results in a determinate equilibrium if and only if the characteristic polynomial $P\,(X)$ admits two roots outside and one root inside the unit circle. Using Proposition C.2 of Woodford (2003a), $P\,(X)$ has one root inside the unit circle and two roots outside if

$$
P\,(1) > 0, \qquad P\,(-1) < 0, \qquad \text{and} \qquad |A_2| > 3.
$$

Assume that ψ_p and ψ_x satisfy (36). This implies

$$
\begin{aligned}
P\,(1) \ &=\ \frac{\kappa}{\beta\sigma}\psi_p > 0 \\
P\,(-1) \ &=\ -\frac{2\kappa + 4\sigma\,(1+\beta) + \kappa\psi_p + 2\,(1+\beta)\,\psi_x}{\beta\sigma} < 0 \\
|A_2| \ &=\ 2 + \beta^{-1} + \frac{\kappa + \psi_x\beta}{\beta\sigma} > 3.
\end{aligned}
$$

Hence $P\,(X)$ has exactly 2 roots outside the unit circle, and (A.59) results in a determinate equilibrium.

A.4 Proof of Proposition 2

In this proof, we show that the loss criterion (12) for the optimal Taylor rule is higher than the one for a particular Wicksellian rule. Since the optimal steady-state is the same for both families of rules, it is sufficient to compare the loss $\mathrm{E}[\hat{L}]$ resulting from deviations from the steady-state.

Loss for optimal Taylor rule. When $\kappa = 0$, and $\rho_r = \rho_u = \rho$ the equilibrium resulting from the optimal Taylor rule, i.e., the optimal non-inertial equilibrium characterized in (A.49) – (A.51), reduces to

$$
\begin{aligned}
\pi_r &= 0, && \pi_u = (1 - \beta\rho)^{-1} \\
x_r &= \frac{\lambda_i \sigma (1 - \rho)(1 - \beta\rho)^2}{h}, && x_u = \frac{\lambda_i \sigma (1 - \rho)(1 - \beta\rho)\rho}{h} \\
i_r &= \frac{\lambda_x (1 - \beta\rho)^2}{h}, && i_u = \frac{\lambda_x (1 - \beta\rho)\rho}{h}
\end{aligned}
$$

where $h \equiv \lambda_i \sigma^2 (1 - \rho)^2 (1 - \beta\rho)^2 + \lambda_x (1 - \beta\rho)^2 > 0$. It follows from (2) that $p_r = 0$, $p_u = (1 - \beta\rho)^{-1}$ and $p_p = 1$, in this equilibrium. Using these expressions to substitute for the equilibrium coefficient in the loss function (A.76), we obtain[29]

$$
\mathrm{E}[\hat{L}^{tr}] = \frac{\lambda_i \lambda_x}{\lambda_i \sigma^2 (1 - \rho)^2 + \lambda_x} \operatorname{var}(r_t^e) + \frac{\lambda_x \left(1 + \lambda_i \rho^2\right) + \lambda_i \sigma^2 (1 - \rho)^2}{\left(\lambda_i \sigma^2 (1 - \rho)^2 + \lambda_x\right)(1 - \beta\rho)^2} \operatorname{var}(u_t). \tag{A.77}
$$

Loss for some particular Wicksellian rule. In the case in which $\kappa = 0$ and $\rho_r = \rho_u = \rho > 0$, the restrictions (A.64) – (A.69) and (A.75) constraining the equilibrium resulting from any Wicksellian rule (A.58) can be solved in terms of x_r, x_u to yield:

$$
p_r = 0, \qquad p_u = \frac{1}{1 - \beta\rho}, \qquad p_p = 1 \tag{A.78}
$$

$$
x_p = \frac{x_u (1 - \beta\rho) - x_r \rho}{1 + x_r \sigma \rho} \tag{A.79}
$$

$$
i_r = 1 - x_r \sigma (1 - \rho), \qquad i_u = \rho \frac{(1 - x_r \sigma (1 - \rho))(1 + x_u \sigma (1 - \beta\rho))}{(1 + x_r \sigma \rho)(1 - \beta\rho)}, \qquad i_p = 0. \tag{A.80}
$$

There is also a second solution which is not admissible as it involves $p_p = \beta^{-1} > 1$, hence an explosive price level (in terms of deviations from a trend). Consider now an equilibrium in which x_r and x_u satisfy

$$
x_r = \frac{\lambda_i \sigma (1 - \rho)}{\lambda_i \sigma^2 (1 - \rho)^2 + \lambda_x} \tag{A.81}
$$

$$
x_u = \lambda_i \sigma \rho \frac{\left(\lambda_i \sigma^2 (1 - \rho)^2 + \lambda_x\right) \beta \left(1 - \beta\rho^2\right) - \lambda_x \rho (1 - \beta)}{(1 - \beta\rho)\Delta}, \tag{A.82}
$$

where

$$
\Delta \equiv \left(\lambda_i \sigma^2 (1 - \rho)^2 + \lambda_x\right)\left(\lambda_i \sigma^2 \left(1 - \beta\rho^2\right)(1 - \beta\rho) + \lambda_x (1 + \beta\rho)\right),
$$

---

[29] Note that the equilibrium characterized here is a special case of the equilibrium (33) used to derive the loss function (A.76).

34

and where (A.78) – (A.82) are used to compute the remaining coefficients. The latter are given by (A.78) and

$$x_p = -\lambda_i \sigma \rho \frac{\left(\lambda_i \sigma^2 (1-\rho)^2 + \lambda_x\right)(1-\beta\rho^2)(1-\beta)}{\Delta}$$

$$i_r = \frac{\lambda_x}{\lambda_i \sigma^2 (1-\rho)^2 + \lambda_x}$$

$$i_u = \rho \lambda_x \frac{\lambda_i \sigma^2 (1-\beta\rho^2)(1-\rho) + \lambda_x (1+\beta\rho)}{(1-\beta\rho)\Delta}$$

$$i_p = 0$$

Expressions (A.81) and (A.82 do in general not correspond to the values of x_r and x_u that would minimize the loss criterion (A.76). (In fact they would minimize (A.76) in the special case that $\mathrm{var}(u_t) = 0$.) As a result this equilibrium is not the optimal equilibrium that might be obtained with a Wicksellian rule. We focus here on a suboptimal equilibrium because it is easier to characterize analytically. We will call this equilibrium a "quasi-optimal equilibrium". One implication of course is that the resulting loss, $\mathrm{E}[\hat{L}^{qwr}]$, cannot be smaller than the one obtained in the optimal equilibrium, $\mathrm{E}[\hat{L}^{wr}]$, so that

$$\mathrm{E}[\hat{L}^{qwr}] \geq \mathrm{E}[\hat{L}^{wr}].$$

The Wicksellian rule of the form (A.58) that implements this quasi-optimal equilibrium is obtained by using (A.73) and (A.74). Substituting for the above equilibrium coefficients in (A.73) and (A.74) yields:

$$\psi_p = \frac{\rho \lambda_x (1-\beta)(1-\beta\rho^2)}{(1-\rho)(\lambda_i \sigma^2 (1-\beta\rho^2)(1-\beta\rho) + \lambda_x (1+\beta\rho))}, \qquad \psi_x = \frac{\lambda_x}{\lambda_i \sigma (1-\rho)}.$$

Since $\psi_p > 0$ and $\psi_x > 0$, it follows from proposition 1 (in Appendix A.3) that this rule results in a unique bounded equilibrium.

Next, substituting the above equilibrium coefficients in the loss criterion (A.76), we obtain

$$
\begin{aligned}
\mathrm{E}[\hat{L}^{qwr}] &= \left(\lambda_x x_r^2 + \lambda_i i_r^2\right)\mathrm{var}(r_t^e) \\
&\quad + \left(\frac{1}{(1-\beta\rho)^2} + \lambda_x\left(x_u^2 + 2x_u x_p \frac{\beta\rho}{(1-\beta\rho)^2} + x_p^2 \frac{\beta(1+\beta\rho)}{(1-\beta\rho)^3(1-\beta)}\right) + \lambda_i i_u^2\right)\mathrm{var}(u_t) \\
&= \frac{\lambda_i \lambda_x}{\lambda_i \sigma^2 (1-\rho)^2 + \lambda_x}\mathrm{var}(r_t^e) \\
&\quad + \left(\frac{1}{(1-\beta\rho)^2} + \lambda_i \lambda_x \rho^2 \frac{\lambda_i \sigma^2 \beta(1-\rho)^2(1-\beta\rho^2) + \lambda_x(1-\beta^2\rho^2)}{(1-\beta\rho)^3 \Delta}\right)\mathrm{var}(u_t) \quad \text{(A.83)}
\end{aligned}
$$

Comparing the losses. Comparing (A.77) and (A.83), we obtain after some algebraic manipulations: $\mathrm{E}[\hat{L}^{tr}] > \mathrm{E}[\hat{L}^{qwr}]$. Thus

$$\mathrm{E}[\hat{L}^{tr}] > \mathrm{E}[\hat{L}^{qwr}] \geq \mathrm{E}[\hat{L}^{wr}],$$

which completes the proof. ■

Table 1: "Calibrated" Parameter values

Structural parameters		
β	σ	κ
0.99	0.1571	0.0238

Shock processes			
ρ_r	ρ_u	$\mathrm{var}\left(r_t^e\right)$	$\mathrm{var}\left(u_t\right)$
0.35	0.35	13.8266	0.1665

Loss function	
λ_x	λ_i
0.048	0.236

Table 2: Statistics and Optimal Policy Rules

		Statistics					Coefficients of optimal policy rule					
		$V[\pi]$	$V[x]$	$V[i]$	$V[p]$	$E[L]$	p_t	π_t	x_t	x_{t-1}	i_{t-1}	i_{t-2}
Optimal Plan / Optimal Rule												
$\rho_r = 0$	$\rho_u = 0$	0.157	10.215	0.983	0.938	0.883	--	0.641	0.325	-0.325	2.163	-1.010
	$\rho_u = 0.35$	0.192	10.994	0.983	0.951	0.956	--	0.641	0.325	-0.325	2.163	-1.010
	$\rho_u = 0.9$	0.195	20.057	0.983	1.098	1.397	--	0.641	0.325	-0.325	2.163	-1.010
$\rho_r = 0.35$	$\rho_u = 0$	0.217	11.056	1.922	2.727	1.206	--	0.641	0.325	-0.325	2.163	-1.010
	$\rho_u = 0.35$	0.252	11.835	1.922	2.740	1.279	--	0.641	0.325	-0.325	2.163	-1.010
	$\rho_u = 0.9$	0.255	20.898	1.922	2.887	1.720	--	0.641	0.325	-0.325	2.163	-1.010
$\rho_r = 0.9$	$\rho_u = 0$	0.487	5.196	6.765	46.597	2.337	--	0.641	0.325	-0.325	2.163	-1.010
	$\rho_u = 0.35$	0.522	5.975	6.765	46.610	2.410	--	0.641	0.325	-0.325	2.163	-1.010
	$\rho_u = 0.9$	0.525	15.038	6.766	46.757	2.851	--	0.641	0.325	-0.325	2.163	-1.010
Optimal Non-Inertial Plan / Optimal Taylor Rules												
$\rho_r = 0$	$\rho_u = 0$	0.269	13.495	2.030	1.680	1.401	--	0.641	0.325	--	--	--
	$\rho_u = 0.35$	0.391	13.957	2.233	4.224	1.593	--	1.291	0.263	--	--	--
	$\rho_u = 0.9$	0.144	30.354	2.159	3.217	2.122	--	3.658	0.038	--	--	--
$\rho_r = 0.35$	$\rho_u = 0$	0.358	9.989	6.747	3.631	2.435	--	0.888	0.694	--	--	--
	$\rho_u = 0.35$	0.479	10.451	6.949	6.175	2.627	--	1.724	0.572	--	--	--
	$\rho_u = 0.9$	0.233	26.848	6.876	5.168	3.156	--	5.041	0.089	--	--	--
$\rho_r = 0.9$	$\rho_u = 0$	0.500	0.529	10.437	39.175	2.993	--	-1.743	-3.222	--	--	--
	$\rho_u = 0.35$	0.622	0.991	10.640	41.719	3.185	--	-2.575	-2.495	--	--	--
	$\rho_u = 0.9$	0.375	17.388	10.566	40.712	3.714	--	-5.108	-0.283	--	--	--
Optimal Wicksellian Rules												
$\rho_r = 0$	$\rho_u = 0$	0.122	13.923	1.237	0.008	1.087	1.997	--	0.182	--	--	--
	$\rho_u = 0.35$	0.146	15.191	1.294	0.015	1.186	1.383	--	0.228	--	--	--
	$\rho_u = 0.9$	0.142	24.466	1.391	0.090	1.653	0.853	--	0.274	--	--	--
$\rho_r = 0.35$	$\rho_u = 0$	0.149	15.898	2.646	0.013	1.543	2.872	--	0.139	--	--	--
	$\rho_u = 0.35$	0.161	17.597	2.778	0.018	1.669	2.338	--	0.201	--	--	--
	$\rho_u = 0.9$	0.096	28.790	2.761	0.012	2.140	3.323	--	0.140	--	--	--
$\rho_r = 0.9$	$\rho_u = 0$	0.158	7.072	10.459	0.089	2.973	2.613	--	0.134	--	--	--
	$\rho_u = 0.35$	0.166	8.239	10.699	0.102	3.093	2.426	--	0.241	--	--	--
	$\rho_u = 0.9$	0.110	18.822	10.779	0.094	3.567	2.626	--	0.259	--	--	--
Estimated Historical Rule												
$\rho_r = 0$	$\rho_u = 0$	0.178	10.531	1.313	0.833	0.997	--	0.424	0.297	-0.032	1.160	-0.430
	$\rho_u = 0.35$	0.330	10.639	1.351	3.586	1.164	--	0.424	0.297	-0.032	1.160	-0.430
	$\rho_u = 0.9$	5.182	13.019	5.078	555.455	7.012	--	0.424	0.297	-0.032	1.160	-0.430
$\rho_r = 0.35$	$\rho_u = 0$	0.215	11.941	2.961	1.218	1.492	--	0.424	0.297	-0.032	1.160	-0.430
	$\rho_u = 0.35$	0.367	12.049	2.999	3.972	1.659	--	0.424	0.297	-0.032	1.160	-0.430
	$\rho_u = 0.9$	5.219	14.429	6.726	555.841	7.507	--	0.424	0.297	-0.032	1.160	-0.430
$\rho_r = 0.9$	$\rho_u = 0$	3.080	9.485	21.954	317.172	8.728	--	0.424	0.297	-0.032	1.160	-0.430
	$\rho_u = 0.35$	3.232	9.592	21.993	319.926	8.895	--	0.424	0.297	-0.032	1.160	-0.430
	$\rho_u = 0.9$	8.084	11.973	25.719	871.794	14.743	--	0.424	0.297	-0.032	1.160	-0.430

Notes: The gray cases indicate that the policy rule results in an indeterminate equilibrium
The estimated historical rule refers to Judd and Rudebusch (1998).

Table 3: Statistics and Quasi-Optimal Policy Rules

|		Statistics					Coefficients of optimal policy rule					
		$V[\pi]$	$V[x]$	$V[i]$	$V[p]$	$E[L]$	p_t	π_t	x_t	x_{t-1}	i_{t-1}	i_{t-2}
Quasi-Optimal Rule (p)												
$\rho_r = 0$	$\rho_u = 0$	0.128	10.214	1.222	0.011	0.911	0.641	--	0.325	--	1.000	--
	$\rho_u = 0.35$	0.163	10.995	1.222	0.024	0.984	0.641	--	0.325	--	1.000	--
	$\rho_u = 0.9$	0.164	20.097	1.223	0.171	1.425	0.641	--	0.325	--	1.000	--
$\rho_r = 0.35$	$\rho_u = 0$	0.146	11.025	2.527	0.015	1.276	0.641	--	0.325	--	1.000	--
	$\rho_u = 0.35$	0.181	11.806	2.528	0.028	1.349	0.641	--	0.325	--	1.000	--
	$\rho_u = 0.9$	0.182	20.907	2.528	0.175	1.790	0.641	--	0.325	--	1.000	--
$\rho_r = 0.9$	$\rho_u = 0$	0.134	4.898	10.069	0.018	2.751	0.641	--	0.325	--	1.000	--
	$\rho_u = 0.35$	0.170	5.679	10.069	0.030	2.824	0.641	--	0.325	--	1.000	--
	$\rho_u = 0.9$	0.171	14.780	10.069	0.177	3.265	0.641	--	0.325	--	1.000	--
Quasi-Optimal Rule (π)												
$\rho_r = 0$	$\rho_u = 0$	0.179	9.923	1.089	1.138	0.916	--	0.641	0.325	--	1.000	--
	$\rho_u = 0.35$	0.293	10.151	1.123	2.987	1.049	--	0.641	0.325	--	1.000	--
	$\rho_u = 0.9$	1.848	15.403	2.346	181.241	3.147	--	0.641	0.325	--	1.000	--
$\rho_r = 0.35$	$\rho_u = 0$	0.215	10.964	2.137	1.979	1.250	--	0.641	0.325	--	1.000	--
	$\rho_u = 0.35$	0.329	11.192	2.171	3.828	1.383	--	0.641	0.325	--	1.000	--
	$\rho_u = 0.9$	1.885	16.444	3.394	182.082	3.482	--	0.641	0.325	--	1.000	--
$\rho_r = 0.9$	$\rho_u = 0$	0.234	5.354	8.871	6.154	2.590	--	0.641	0.325	--	1.000	--
	$\rho_u = 0.35$	0.348	5.582	8.905	8.003	2.723	--	0.641	0.325	--	1.000	--
	$\rho_u = 0.9$	1.903	10.835	10.129	186.257	4.821	--	0.641	0.325	--	1.000	--
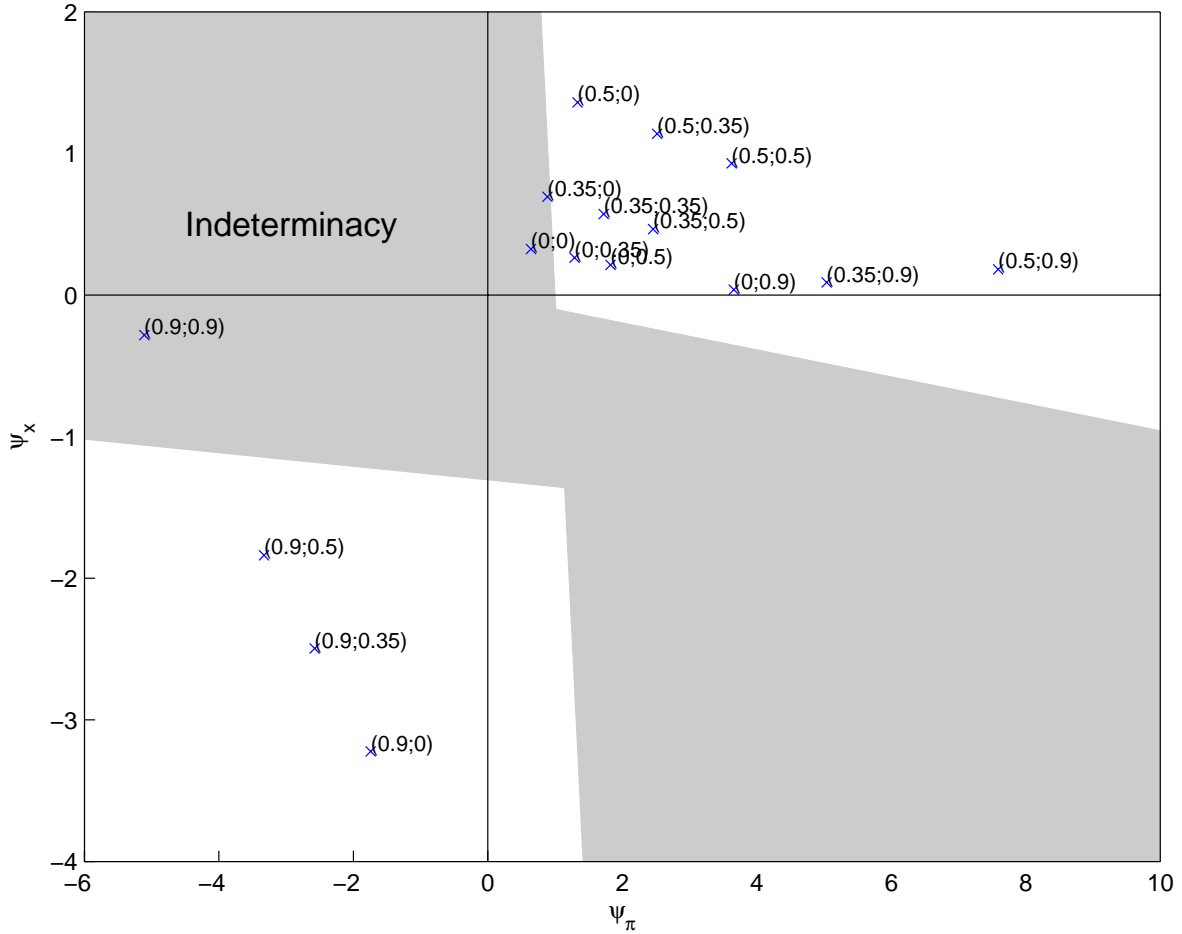
Figure 1: Optimal Taylor rules for different degrees of shock persistence $(\rho_r; \rho_u)$

Notes: Each $\times$ denotes an optimal pair of Taylor-rule coefficients (ψ_π, ψ_x) for degrees of serial correlation of the exogenous shocks given by $(\rho_r; \rho_u)$ and indicated in the figure. The optimal policy coefficients are computed using (A.55)–(A.56). Taylor rules in the gray region result in an indeterminate equilibrium.
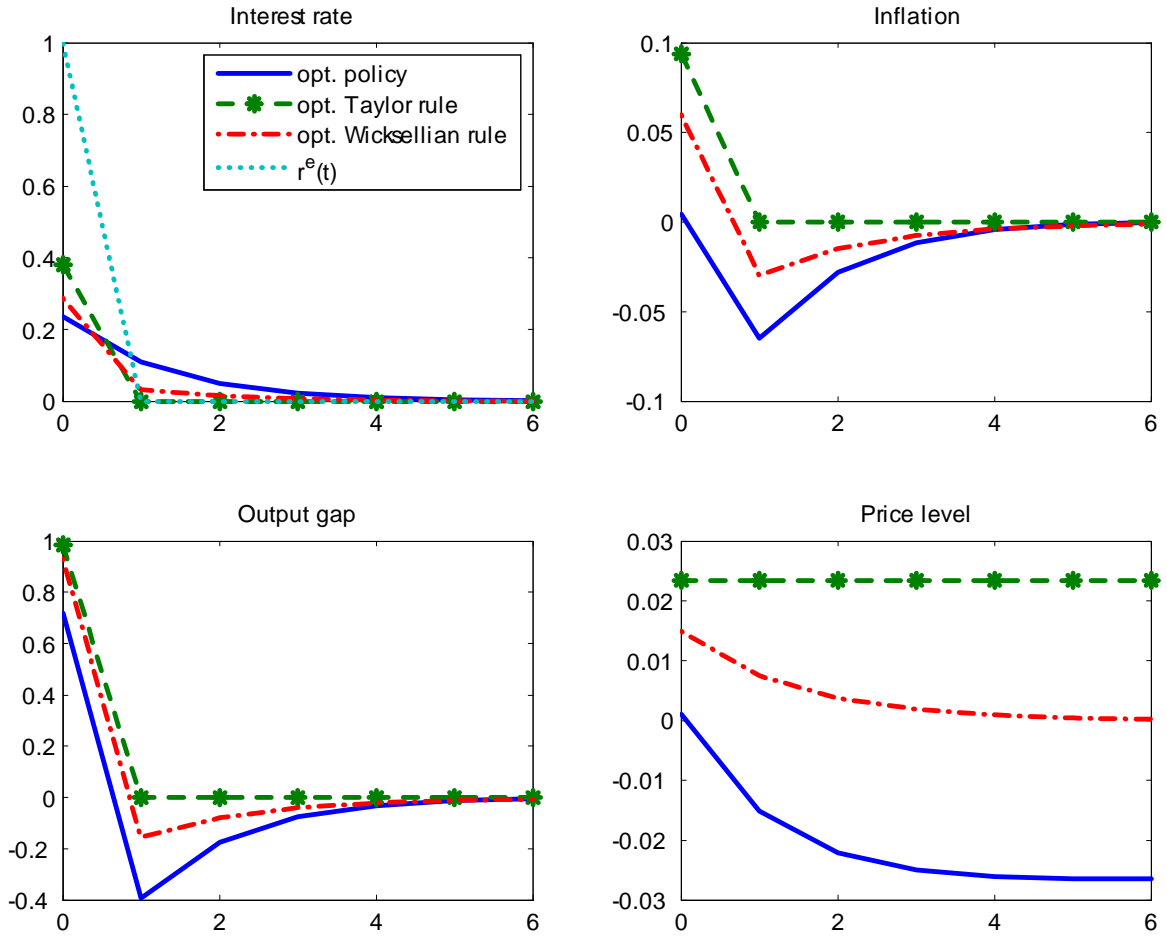
Figure 2: Impulse responses to an innovation in r^e with autocorrelation of $\rho_r = 0$.

Notes: The responses of $\hat{\imath}_t$ and $\hat{\pi}_t$ are multiplied by 4 so that the responses of all variables are reported in annual terms.
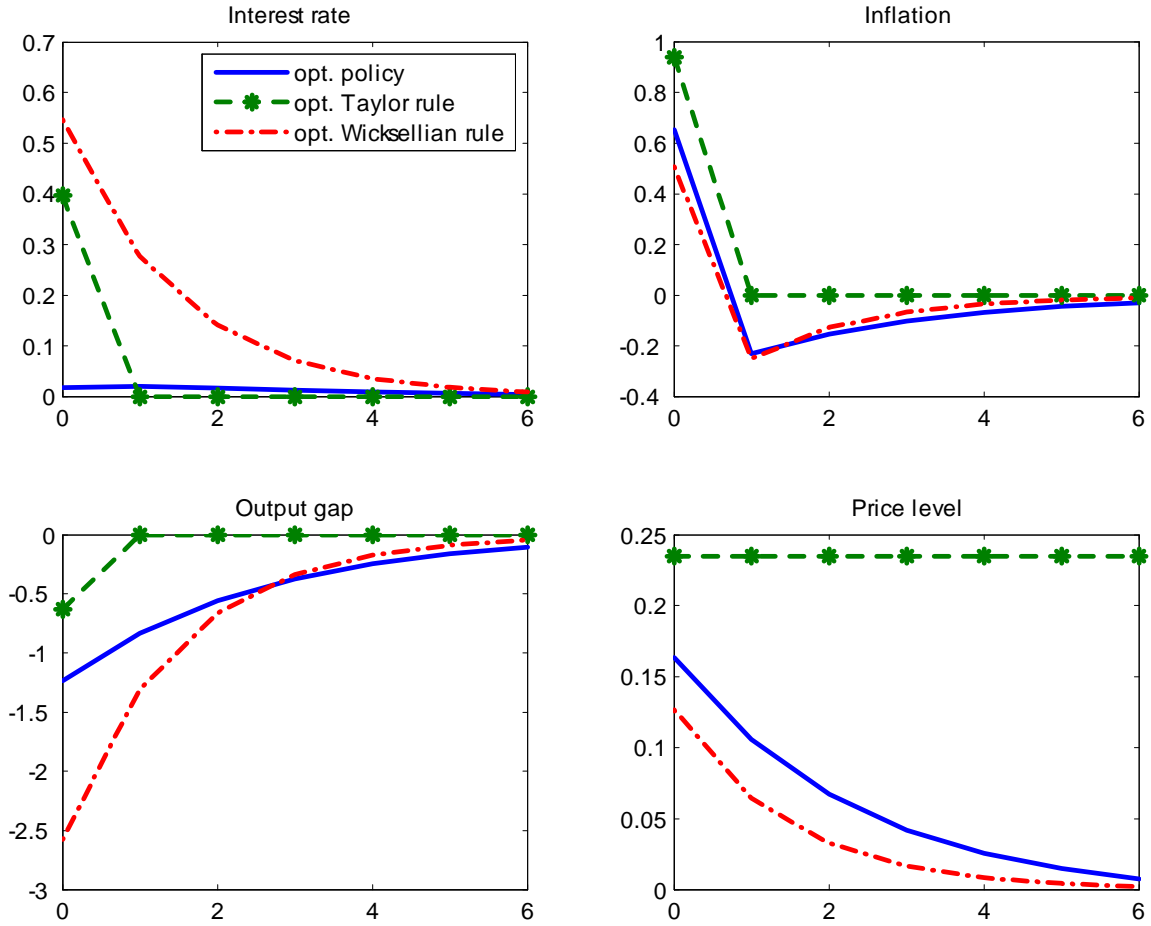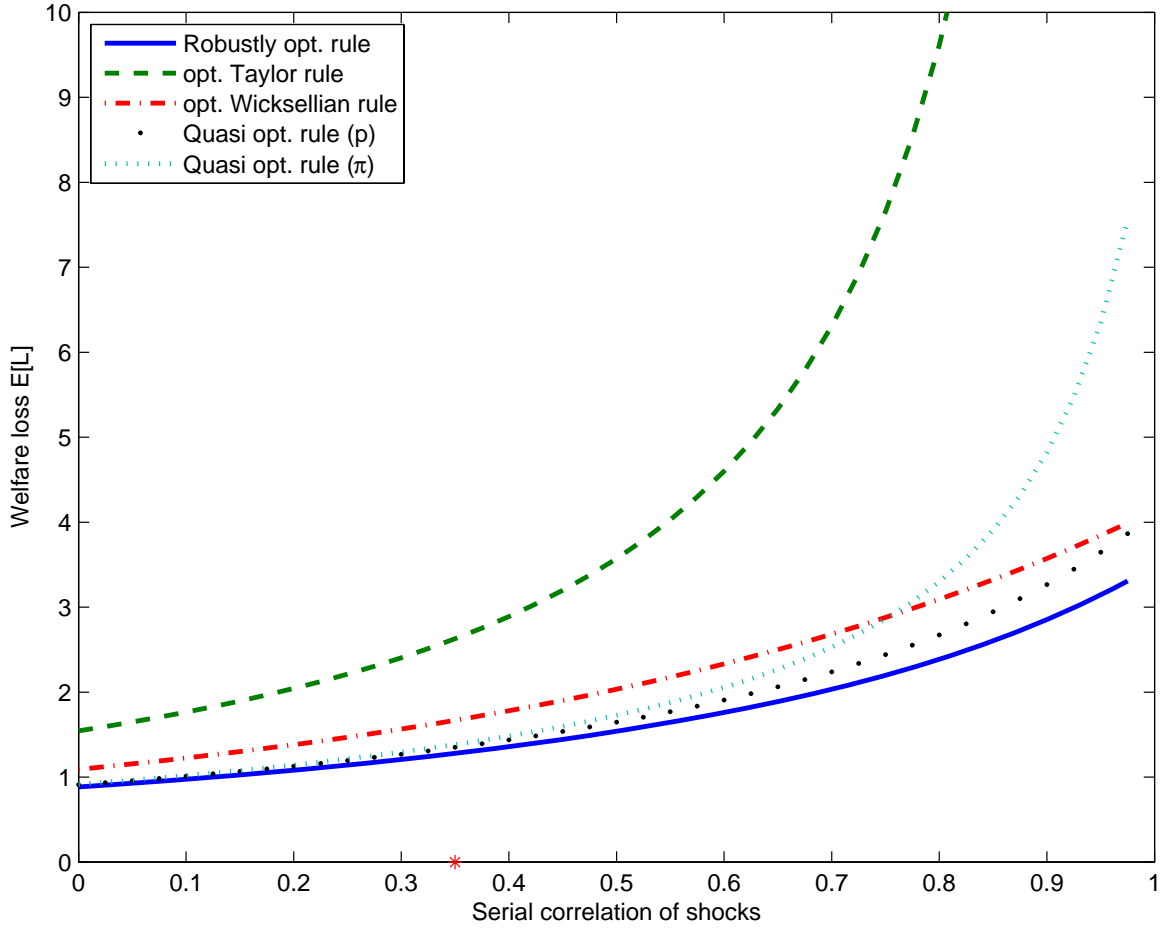
Figure 3: Impulse responses to an innovation in u with autocorrelation of $\rho_u = 0$.

Notes: The responses of $\hat{\imath}_t$ and $\hat{\pi}_t$ are multiplied by 4 so that the responses of all variables are reported in annual terms.

Figure 4: Welfare losses of alternative policy rules as a function of shock persistence.

Notes: Each curve plots the welfare losses E[L] implied by a particular policy rule for different degrees of serial correlation in the shock processes ($\rho_r = \rho_u = \rho$). The Taylor and Wicksellian rules are optimized assuming the benchmark shock persistence of $\rho = 0.35$. The quasi optimal rule (p) is given in (40) while the quasi optimal rule (π) is given in (41).
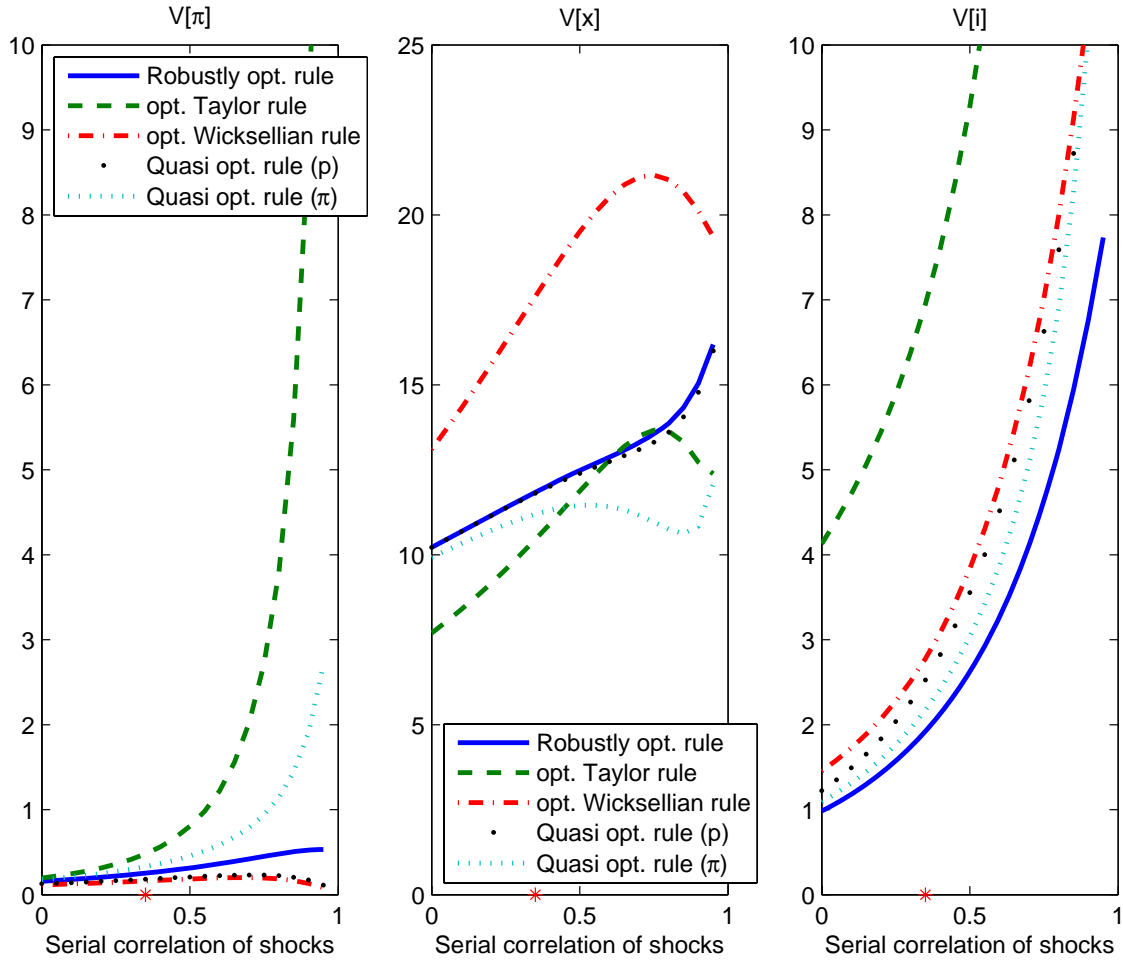
Figure 5: Volatility of key variables under alternative rules, as a function of shock persistence.

Notes: Each curve plots the volatility measures $V[\pi]$, $V[x]$, and $V[i]$ implied by a particular policy rule for different degrees of serial correlation in the shock processes ($\rho_r = \rho_u = \rho$). The Taylor and Wicksellian rules are optimized assuming the benchmark shock persistence of $\rho = 0.35$. The quasi optimal rule (p) is given in (40) while the quasi optimal rule (π) is given in (41).
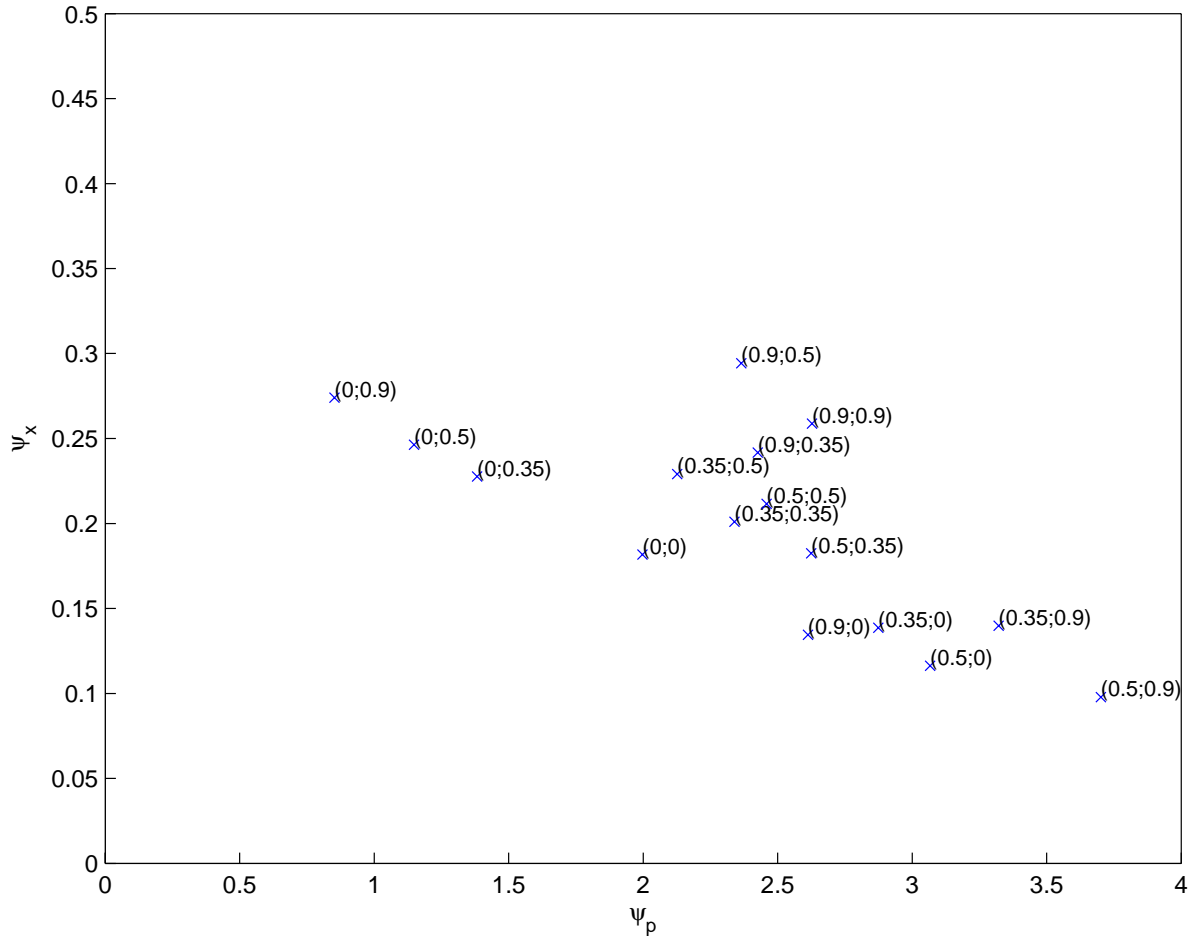
Figure 6: Optimal Wicksellian rules for different degrees of shock persistence $(\rho_r; \rho_u)$.

Notes: Each × denotes an optimal pair of Wicksellian-rule coefficients (ψ_p, ψ_x) for degrees of serial correlation of the exogenous shocks given by $(\rho_r; \rho_u)$ and indicated in the figure.