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Abstract

We develop a dynamic duopoly model of R&D competition to improve the quality of a final good. The innovation process is sequential and cumulative, and takes place alongside production in an infinite-horizon setting. In this context we study the R&D incentive impacts resulting from a "research exemption" or "experimental use" provision. We specify and solve the innovation and production model under two distinct intellectual property right (IPR) regimes, essentially a patent system with and without a research exemption. The model applies closely to the question of the optimal mode of IPR protection for plants, where traditional plant breeder's rights allow for a well-defined research exemption, whereas standard utility patents do not. We characterize the properties of the relevant Markov perfect equilibria and investigate the profit and welfare effects of the research exemption. We find that firms, *ex ante*, always prefer full patent protection. The welfare ranking of the two IPR regimes, on the other hand, depends on the relative magnitudes of the costs of initial innovation and improvements. In particular, a research exemption is most likely to provide inadequate R&D incentives when there is a large cost to establish the initial research program.

Keywords: Cumulative innovation; Experimental use exemption; Intellectual property rights; Markov perfect equilibrium; Patents; Stochastic games.

JEL Classification: L00, O31, O34, C73

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1. Introduction

The economic analysis of intellectual property rights (IPRs) has long emphasized their ability to provide a solution to the appropriability and free-rider problems that beset the competitive provision of innovations (see Scotchmer, 2004, for an overview). But whereas there is agreement that legally provided rights and institutions are necessary to offer suitable incentives for inventive and creative activities, it is less clear what the extent of such rights should be. The predicament here is very much related to the second-best nature of the proposed solution to the market failures that arise in this context (Arrow, 1962). Because they work by creating a degree of monopoly power, IPRs introduce a novel source of distortions. Whereas the prospect of monopoly profits can be a powerful *ex ante* incentive for the would-be innovator, and can bring about innovations that would not otherwise take place, the monopoly position granted by the exclusivity of IPRs is inefficient from an *ex post* point of view (the innovation is underutilized). This is the essential economic trade-off of most IPR systems: there are dynamic gains due to more powerful innovation incentives, but there are static losses because of a restricted use of innovations (Nordhaus, 1969).

The trade-off of IPR systems is more acute when one considers that new products and processes are themselves the natural springboard for more innovations and discoveries (Scotchmer, 1991). When innovation is cumulative, the first inventor is not necessarily compensated for her contribution to the social value created by subsequent inventions. This problem is particularly evident when the first invention constitutes basic research (perhaps leading to so-called research tools) that is not directly of interest to final users. To address this intertemporal externality requires the transfer of profits from successful applications of a given patented innovation to the original inventor(s). What the features of an IPR system should be to achieve that has been addressed in a number of studies. Green and Scotchmer (1995) consider how patent breadth and patent length should be set in order to allow the first inventor to cover her cost, subject to the constraint that the second-generation innovation is profitable, and highlight the critical role of licensing. This and related studies, including Scotchmer (1996), and Matutes, Regibeau, and Rockett (1996), can be viewed as supporting strong patent protection for the initial innovations. Somewhat different conclusions can emerge, however, when the two innovation stages are modeled as research and development (R&D) races (Denicolò, 2000).

A critical issue, in this setting, relates to how one models the features of an IPR system, and the foregoing studies emphasize the usefulness of the concepts of "patentability" and "infringement." For instance, in the two-period model of Green and Scotchmer (1995), both innovations are presumed patentable, and the question is whether or not the second innovation should be considered as infringing on the original discovery. The notion of patentability refers broadly to the "novelty" and "nonobviousness" requirements of the patents statute (so that, as in O'Donoghue (1998) and Hunt (2004), one can define the minimum innovation size required to get a patent). On the other hand, the context for infringement is defined by the "breadth" of patent rights. This property can be made especially clear in quality ladder models of sequential innovation through the notion of "leading breadth"—the minimum size of quality improvement that makes a follow-on innovation non-infringing (O'Donoghue, Scotchmer, and Thisse, 1998; Denicolò and Zanchettin, 2002).

By contrast, in this paper we study how the IPR system affects incentives in a sequential innovation setting by focusing on the "research exemption" or "experimental use" doctrine. When a research exemption exists, proprietary knowledge and technology can be used freely in others' research programs aimed at developing a new product or process (which, if achieved, would in principle still be subject to patentability and infringement standards). On the other hand, if a research exemption is not envisioned, the mere act of trying to improve on an existing product may be infringing (regardless of success and/or commercialization of the second-generation product). In the U.S. patent system there is no general statutory research exemption, and, as clarified by the 2002 *Madey v. Duke University* decision by the Court of Appeals for the Federal Circuit (CAFC), the experimental use defense against infringement based on case law precedents can only be construed as extremely narrow (Eisenberg, 2003). On the other hand, a special research exemption is contemplated for pharmaceutical drugs as part of the provisions of the Hatch-Waxman Act of 1984, whereby firms intending to market generic pharmaceuticals are exempted from patent infringement for the purpose of developing information necessary to gain federal regulatory approval.[1] Furthermore, a few specialized intellectual property statutes—including the 1970 Plant Variety Protection Act and the 1984 Semiconductor Chip Protection Act—contemplate a well-defined research exemption. Indeed, the innovation environment and the intellectual property context for plants offer perhaps the sharpest characterization of the possible implications of a research exemption in a sequential setting, and we will consider them in more detail in what follows.

The intense debate that followed the CAFC ruling in *Madey* has renewed interest in the desirability of a research exemption in patent law (Thomas, 2004). Quite clearly, a broad research exemption may have serious consequences for the profitability of innovations from basic research, thereby adversely affecting the incentives for R&D in some industries that rely extensively on research tools (e.g., biotechnology). On the other hand, there is the concern that

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1 The recent decision of the U.S. Supreme Court, in *Merck v. Integra*, appears not only to uphold but also to extend the scope of the Hatch-Waxman experimental use defense (Feit, 2005).

limiting the experimental use of proprietary knowledge in research may have a negative effect on the resulting flow of innovations. Explicit economic modeling of the research exemption, however, appears to be lacking. In this paper we propose to contribute to the economic analysis of the research exemption in IPR systems by focusing on the case of strictly sequential and cumulative innovations.

The quality ladder model developed in this paper draws upon the modeling approach of Bessen and Maskin (2002), while conceptually it belongs to the line of research on the optimal patent breadth discussed earlier. Bessen and Maskin find that it might be optimal, both from the social and individual firm's point of view, to have weak patent protection when innovation is cumulative. This result is driven by a critical complementarity assumption, in particular that the improvement possibilities on the quality ladder are exhausted if all firms fail to innovate in any given period (implying that having rivals engaged in R&D might, in principle, be beneficial). We depart from the Bessen and Maskin setup by formulating a fully dynamic model of an infinite-horizon stochastic innovation race suitable for an explicit characterization of equilibrium. To do so, we find it desirable to formulate the "complementarities" between firms somewhat differently. Specifically, in our formulation the quest for the next innovation step does not end when both firms are unsuccessful (both can try again).

Related literature includes formal models of dynamic R&D competition between firms engaged in "patent races."[2] As with most contributions in this setting, we postulate a memoryless stochastic arrival of innovation; to keep a closer connection with the setup of Bessen and Maskin (2002), we model that process by means of a geometric distribution, rather than with exponential distribution typically used when modeling R&D races (e.g., Reinganum, 1989). More importantly, in our model we delineate precisely the differences between the two IPR modes of interest (i.e., patents with and without the research exemption). In most R&D dynamic competition models, on the other hand, the nature of the underlying intellectual property regime is not addressed explicitly and IPR effects are often captured by a generic winner-takes-all condition. In addition, in our model both the incumbent and challenger can perform R&D, production takes place alongside R&D, and the stage payoffs are state-dependent (an attractive feature, in a quality ladder setting, under typical market structures). Conversely, to keep the analysis tractable, here we consider a fixed number of firms (two) and thus we neglect the issue of entry in the R&D contest that has been prominent in many previous studies. We also assume away the inefficiency of the static patent-monopoly case, as in other studies in

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2 We cannot begin to do justice to this copious literature. See Tirole (1988, chapter 10) for an introduction.

this area, but still allow for dynamic welfare spillovers to consumers via a Bertrand competition assumption.

In what follows we first discuss in some detail the intellectual property environment for plants, a context that provides perhaps the sharpest example of the possible implications of a research exemption. We then develop a new game-theoretic model of sequential innovation that captures the stylized features of the problem at hand. The model is solved by relying on the notion of Markov perfect equilibrium under the two distinct intellectual property regimes of interest. The results permit a first investigation of the dynamic incentive issues entailed by the existence of a research exemption provision in intellectual property law. First, we find that the firms themselves always prefer (*ex ante*) the full patent protection regime (unlike what happens in Bessen and Maskin, 2002). The social ranking of the two intellectual property regimes, on the other hand, depends on the relative magnitudes of the costs of initial innovation and improvements. It must be noted, as will become apparent below, that our model makes a stark assumption about the nature of the IP regime without a research exemption provision (i.e., the winner of the first innovation race faces no further R&D competition), which in principle should bias our results in favor of the research exemption. But even within this stylized framework, we still find that the research exemption need not result in higher social welfare. In particular, the research exemption is most likely to provide inadequate incentives when there is a large cost to establish a research program (as is arguably the case for the plant breeding industry where developing a new variety typically takes several years). On the other hand, when both initial and improvement costs are small relative to the expected profits (perhaps the case of the software industry noted by Bessen and Maskin, 2002), the weaker incentive to innovate is immaterial (firms engage in R&D anyway) and the research exemption regime dominates.

2. A Model of Sequential and Cumulative Innovation

We develop an infinite-horizon production and R&D contest between two firms under two possible IPR regimes—that is, with and without the research exemption. The model that we construct is sequential and cumulative and reflects closely the stylized features of plant breeding. This industry is also of interest because, as mentioned, it has access to a *sui generis* IPR system that contemplates a well-defined research exemption.

2.1. Plant Variety Protection, Patents, and the "Research Exemption"

The Plant Variety Protection (PVP) Act of 1970 introduced a form of IPR protection for sexually reproducible plants that complemented that for asexually reproduced plants of the 1930 Plant Patent Act and represented the culmination of a quest to provide IPRs for innovations thought to lie outside the statutory subject matter of utility patents (Bugos and Kevles, 1992). PVP certificates, issued by the U.S. Department of Agriculture, afford exclusive rights to the varieties' owners that are broadly similar to those provided by patents, including the standard 20-year term, with two major qualifications: there is a "farmer's privilege," that is, seed of protected varieties can be saved by farmers for their own replanting; and, more interestingly for our purposes, there is a "research exemption," meaning that protected varieties may be used by other breeders for research purposes (Roberts, 2002). In addition to PVP certificates, to assert their intellectual property, plant innovators can rely on trade secrets, the use of hybrids, and specific contractual arrangements (such as bag-label contracts). More importantly, in the United States plant breeders can now also rely on utility patents. The landmark 1980 U.S. Supreme Court decision in *Diamond v. Chakrabarty* opened the door for patent rights for virtually any biologically based invention and, in its 2001 *J.E.M. v. Pioneer* decision, the U.S. Supreme Court held that plant seeds and plants themselves (both traditionally bred or produced by genetic engineering) are patentable under U.S. law (Janis and Kesan, 2002).

As noted earlier, the U.S. patent law does not have a statutory research exemption (apart from the provisions of the Hatch-Waxman Act discussed earlier). Hence, a plant breeder who elects to rely on patents can prevent others from using the protected germplasm in rivals' breeding programs. That is not possible when the protection is afforded by PVP certificates. The question then arises as to which IPR system is best for plant innovation, and whether the recently granted access to utility patents significantly changed the innovation incentives for U.S. plant breeders. Alternatively, one can consider the differences in the degrees of protection conferred by patents and PVPs in an international context. Rights similar to those granted by PVP certificates, known generically as "plant breeder's rights" (PBRs), are available for plant innovations in most other countries, but patents are not (Le Buanec, 2004). Indeed, under the TRIPS (trade-related aspects of intellectual property rights) agreement of the World Trade Organization, it is not mandatory for a signatory country to offer patent protection for plant and animal innovations, as long as a *sui generis* system (such as that of PBRs) is available (Moschini, 2004). Thus, in many countries (including most developing countries), PBRs are the only available intellectual property protection for plant varieties.[3]

Given the structural differences between patents and PBRs, the notion of a research exemption is clearly central to this intellectual property context. Furthermore, it is interesting to note that the prototypical sequential and cumulative nature of R&D in plant breeding can be

3 Even in European countries, where plant innovations are included in the patentable subject matter, somewhat anachronistically, plant varieties *per se* are explicitly not patentable by the statute of the European Patent Office (Fleck and Baldock, 2003).

closely represented by a quality ladder model. Plant breeding is a lengthy and risky endeavor that has been defined as consisting of developing new genetic diversity (e.g., new varieties) by the reassembling of existing diversity. Thus, the process is both sequential and cumulative, because new varieties would seek to maintain the desirable features of the ones they are based on while adding new attributes. As such, a critical input in this process is the starting germplasm (whole genome), and that in turn is critically affected by whether or not one has access to existing successful varieties, which in turn is directly affected by a research exemption. In a dynamic context, of course, the quality of the existing germplasm is itself the result of (previous) breeding decisions, and so it is directly affected by the features of the IPR regime in place. Industry views on the matter highlight the possibility that freer access to others' germplasm will erode the incentive for critical pre-breeding activities aimed at widening the germplasm diversity base (Donnenwirth, Grace, and Smith, 2004).

2.2. Model Outline

We consider two firms that are competing to develop a new product variety along a particular development trajectory. At time zero both firms have access to the same germplasm and, upon investing an amount c_0, achieve success with probability p (each firm's outcome is independent of the other's). We refer to the pursuit of the first innovation as the "**Initial Game**." Note that in this model the R&D process is costly and risky, and that the two firms are identical *ex ante* (i.e., the game is symmetric). If at least one firm is successful, the initial game terminates and a patent is awarded. When only one firm is successful, that firm gets the patent. When both firms are successful, the patent is randomly awarded (with equal probability) to one of them. If neither firm is successful, they have the option of trying again, which would require a new investment of c_0.

Given at least one success, the contest moves to the production and improvement stage, which we call the "**Improvement Game**." At the start of this game, firms are asymmetric: one of them, referred to as the "Leader," has been successful (and holds the patent) whereas the other firm, referred to as the "Follower," has not (does not). There are two relevant activities that characterize the improvement game: rent extraction through production, and further R&D efforts. Rent extraction is the prerogative of the Leader: specifically, the leading firm captures a return of Δ in the first period of the improvement game. What happens to the distribution of rent after the first period may depend on possible R&D undertaken in the improvement game, and that, in turn, depends on the property rights conveyed by the patent awarded at the end of the initial game. For the latter, we distinguish between two prototypical IPR regimes that differ according to the treatment reserved for the research exemption. The R&D structure of the improvement game is similar to that of the initial game: upon an initial investment, a firm achieves the next improvement with probability p. But to recognize that the initial innovation is "more important" in some well-defined sense, we assume that the per-period cost of R&D in the improvement game is $c \leq c_0$.

Whether or not both firms can participate in the improvement game depends on the nature of IPRs, specifically on whether or not a "research exemption" is contemplated. The first regime that we consider, which we refer to as "**Full Patent**" (FP), presumes that the patent awards an exclusive right to the patent holder, such that further innovations can be pursued only by the patent holder (or upon a license by the patent holder). Thus, the FP regime characterizes the environment of U.S. utility patents which, as discussed earlier, envisions an extremely limited role for a research exemption. The second regime, which we refer to as the "**Research Exemption**" (RE), allows any firm (i.e., including the Follower) to pursue the next innovation, although the patent gives the right of rent extraction (i.e., collecting Δ in the current period) to the holder of the patent. Hence, the RE regime reflects the attributes of a PBR system, such as the one implemented in the United States under the PVP Act. We should note that both patents and PBRs confer rights that are limited in time (20 years). But because we are characterizing the differences between the two regimes, without much loss of generality we ignore this feature and model both rights as having, in principle, infinite duration.

Under the FP regime, therefore, only the patent holder can pursue further innovations. Ignoring the possibility of licensing (we will return to this issue later), we model the improvement game under the FP regime as a monopoly undertaking by the firm that won the initial game. Under the RE regime, on the other hand, both firms are allowed to participate in the follow-up R&D. Because under the RE both firms can use the same starting point, upon a success in the first improvement game we either have the Leader owning two consecutive innovations or the Follower being the successful firm and thereby becoming the Leader. We emphasize again that the foregoing structure reflects the strict sequential and cumulative nature of the innovation process that we wish to model: the current quality level is, in effect, an essential input into the production of the next quality level.

Each additional innovation is worth an additional Δ, per period, to society.[4] What a success is worth to the innovator, however, depends on the IPR regime and on the possible constraining effects of competition among innovators. We make the simplifying assumption that only the best product is sold in this market, but what the owner can charge is the marginal

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4 Because in our model we capture the asymmetry between initial innovation and follow-up improvements by postulating different R&D costs (c_0 and c), we assume that the value of each successive quality improvement is the same.

value over what the competitor can offer (i.e., we assume Bertrand competition). For example, if two firms have achieved n and m innovation steps, respectively, with $m > n$, the firm with m steps will be the one selling any product and will make an *ex post* per-period profit of $(m-n)\Delta$.

To summarize, we consider an infinite-horizon R&D contest between two firms under two possible IPR regimes. Under the FP regime, both firms can participate in the initial game, but only the successful firm can be engaged in the improvement games. Under the RE regime, both firms can participate in both the initial game and the improvement games.

2.3. The Stochastic Game

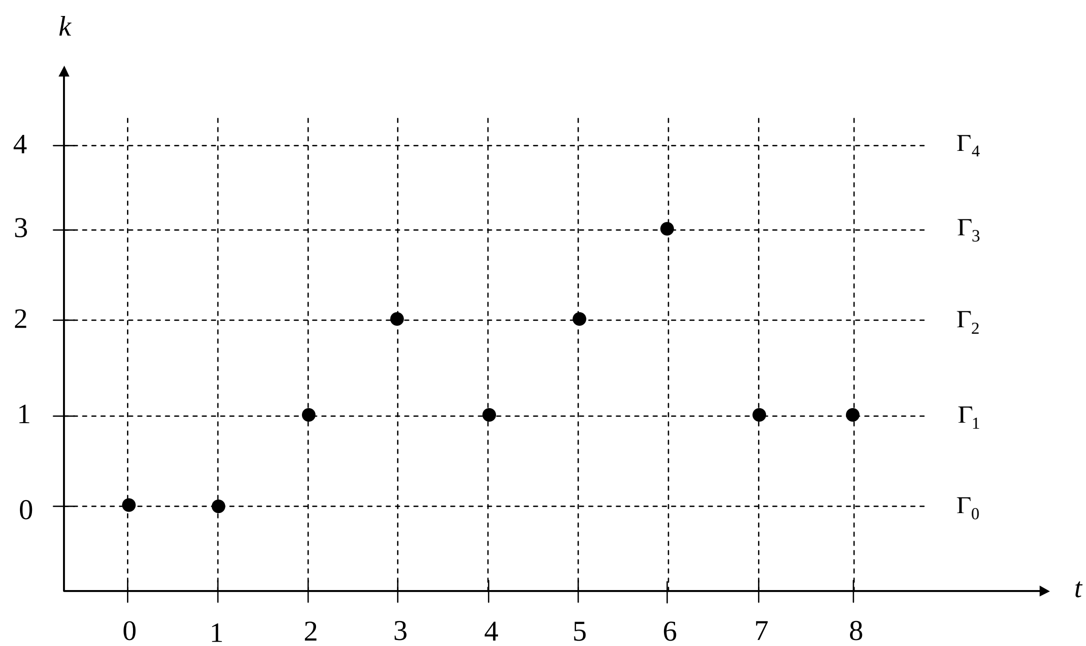
To formalize the model outlined in the foregoing as an infinite-horizon R&D stochastic game, the set of players (the two firms) is $G \equiv \{1,2\}$. At each stage $t = \{0,1,2,....\}$ of the initial game, labeled Γ_0, the two firms simultaneously choose an action a_t^i from the history-invariant action set $A \equiv \{I, N\}$, where I = invest and N = no investment. Action I entails a cost to the firm of $c_0 > 0$ and brings success with probability $p \in (0,1)$ if the other firm does not invest, whereas it brings success with probability $q \in (0, p)$ if the other firm also invests. Specifically, when both firms invest, and firms' outcomes are independent, the probability of at least one success is $1-(1-p)^2$, and thus $q \equiv p(2-p)/2$. At the beginning of the initial game, firms are identical and the game is symmetric. After a single "success," the firms will be asymmetric for the rest of the game. Under the FP regime, the loser of the initial game drops out and the winner becomes a monopolist in both the exploitation of the innovation and in further R&D activities. Under the RE regime, on the other hand, both firms can participate in the improvement game. If a firm chooses to invest in any period of the improvement game, the required cost is $c \in [0, c_0]$, and the success probabilities are just as in the initial game (i.e., a single firm innovates with probability p, and when both firms invest, each wins the contest with probability q).

The improvement game under the FP regimes is technically not a game because there are no strategic interactions (the winner of the initial game is a monopolist). Under the RE regime, on the other hand, we actually have a family of improvement games, which we label as Γ_k, with each distinguished by the number $k = 1,2,3,...$ of successive innovation steps held by the Leader. Thus, after the first innovation we have $k = 1$. If the Leader is the firm that innovates again then we have $k = 2$ and the status of each firm does not change. Whenever the Follower wins the stage game, however, then firms swap their roles (e.g., the Follower becomes the

Leader) and the number of steps ahead that determines the payoff drops back to $k = 1$. Hence, $k = 1, 2, 3, \ldots$ represents one of the "state" variables of the game. Figure 1 provides an illustration. Note that, in this setup, the RE regime ensures that "leapfrogging" is possible, although the Leader's advantage can also accumulate and persist, whereas with the FP regime there is "persistence" of the monopoly position provided by the initial innovation.[5]

Stage payoffs are determined under a Bertrand competition assumption. Specifically, under either regime, in each period the last firm to be successful (the Leader) collects an amount $k\Delta$, where $\Delta > c$ measures the per-period value of a stage innovation, and $k \in \{1, 2, 3, \ldots\}$ denotes the total number of innovation steps that the leading firm has over the competitor. The value of the entire game to the firms, from the perspective of the initial period and under the two IPR regimes of interest, is derived in what follows. Throughout, $\delta \in (0,1)$ denotes the discount factor.

Figure 1. Stages, states, and implied games under the "research exemption"



5 These are two recurrent concepts in patent race models (Tirole, 1988, chapter 10). The persistence of monopoly was studied by, among others, Gilbert and Newbery (1982) and Reinganum (1983). The notion of leapfrogging was introduced by Fudenberg et al. (1983). Whereas our model does not focus on these two issues, it does emphasize that they may be directly affected by the specific features of the relevant IPR system.

3. Equilibria in the Improvement Games

We characterize the equilibrium solution of the improvement games first and, by standard backward induction principles, analyze the initial games next, under both IPR regimes that we have described. As explained in more detail in what follows, we will focus on "Markov strategies," whereby the history of the game is allowed to affect strategies only through state variables that summarize the payoff-relevant attributes of the strategic environment (Fudenberg and Tirole, 1991, chapter 13). Thus, our equilibrium concept will be that of *Markov Perfect Equilibrium* (MPE), that is, a profile of Markov strategies that yields a subgame perfect Nash equilibrium (Maskin and Tirole, 2001).

3.1. Improvement Game under the Full Patent Regime

As noted, here we do not really have a game, just an optimization problem where, at each stage, the firm that is allowed to invest has to choose an action from $\{I, N\}$. Such a firm is effectively a monopolist in the improvement game. If it chooses action I at any one stage, success will occur with probability p and hence the expected payoff to choosing action I in that stage is $-c + p\delta\Delta/(1-\delta)$ (because success yields a stage payoff Δ forever starting with the next period). Hence action I is optimal in any one stage i.f.f. $c/\Delta \le \delta p/(1-\delta) \equiv x_0$. Naturally, if it is optimal for such a monopolist to choose action I at any one stage, then it is optimal to do so in every stage and hence the investment rule does not depend on k. If the condition $c/\Delta \le x_0$ for the optimality of action I holds, the expected payoff of the patent holder at the start of the improvement game when the state is k, labeled $V_M(k)$, therefore is

$$V_M(k) = \frac{k\Delta - c}{1-\delta} + \frac{\delta p \Delta}{(1-\delta)^2} \quad . \tag{1}$$

If, on the other hand, $c/\Delta > x_0$, then the patent holder's optimal action would be N and the payoff would be $V_M(k) = k\Delta/(1-\delta)$.

3.2. Improvement Game(s) under the Research Exemption Regime

In the improvement game under the RE regime, firms are asymmetric. The firm with the last success is the Leader who can earn returns from the market (in proportion to the number of extra innovation steps that it has relative to the competitor, which we have denoted as k). The other firm, labeled as the Follower, does not earn current returns but has the same opportunities to engage in R&D as the other firm. As discussed earlier, $k = 1, 2, 3, \ldots$ represents one of the "state" variables of the game. The other state variable of the game is the identity of the Leader, $\ell \in G \equiv \{1,2\}$. Together, (k,ℓ) summarize all the payoff-relevant information of the history of the game leading up to any particular subgame.

We consider only *Markov strategies*, so that the strategy of a firm depends only on the state of the game. The state space of the game is $S \equiv G \times \mathbb{N}$, where G is the set of players defined earlier, and $\mathbb{N} \equiv \{1,2,\ldots\}$ is the set of natural numbers. A Markov strategy here is defined as a function $\sigma_i : S \to [0,1]$, $i \in G$. Specifically, the strategy $\sigma_i(\ell,k)$ tells us the probability that player i will attach to action I when the state is (ℓ,k). Thus, at any stage of the game with the same state, the Markov strategy σ_i specifies the same probability distribution over available actions. Although the use of Markov strategies is somewhat restrictive, it is standard in the dynamic oligopoly models in general and in the models of innovation races in particular (e.g., Bar, 2006; Hörner, 2004).

Alternatively, we can characterize the strategy of the two "types" of firms. Conditional on being a Leader, the only payoff-relevant state is the number of innovation steps k that the Leader has over the Follower. Similarly, conditional on being the Follower, the only relevant state is again the number of innovations steps k that the Leader is enjoying. [Note: the stage and continuation payoffs to the Follower actually do not depend on k. But because k affects the Leader's payoffs, a Markov strategy for the Follower must also condition on k.] Thus, with some abuse of notation, we can write the strategy of the Leader as $\sigma_L(k)$ and the strategy of the Follower as $\sigma_F(k)$.[6]

At any stage of the game, the expected payoff of a firm for the subgame starting at that point, for given strategies of the two firms, depends on the firm being a Leader or a Follower. For given strategies of the two firms, the payoff to the Follower does not depend on how many steps behind the Follower is lagging the Leader. The payoff to the Leader, on the other hand, does depend on the number of leads it has. Thus, for a given strategy profile $\sigma \equiv (\sigma_L, \sigma_F)$, for the game Γ_k we can write the payoff to the Follower as $V_F(\sigma_L, \sigma_F)$ and the payoff to the Leader as $V_L(\sigma_L, \sigma_F, k)$. These value functions must satisfy the following recursive equations:

$$\begin{aligned} V_L(\sigma,k) = \Delta k &+ \sigma_L \sigma_F \left[-c + q\delta V_L(\sigma,k+1) + q\delta V_F(\sigma) + (1-2q)\delta V_L(\sigma,k)\right] \\ &+ \sigma_L(1-\sigma_F)\left[-c + p\delta V_L(\sigma,k+1) + (1-p)\delta V_L(\sigma,k)\right] \\ &+ (1-\sigma_L)\left[\sigma_F\left(p\delta V_F(\sigma) + (1-p)\delta V_L(\sigma,k)\right) + (1-\sigma_F)\delta V_L(\sigma,k)\right] \end{aligned} \tag{2}$$

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6 Hörner (2004) similarly uses Markov strategies where the state space is the set of integers. But note that the stage payoff in Hörner depends only on whether the firm is a Leader or a Follower, whereas in our model stage payoffs ($k\Delta$) are state-dependent.

$$V_F(\sigma) = \sigma_F\sigma_L\left[-c + q\delta V_L(\sigma,1) + (1-q)\delta V_F(\sigma)\right] \\ + \sigma_F(1-\sigma_L)\left[-c + p\delta V_L(\sigma,1) + (1-p)\delta V_F(\sigma)\right] + (1-\sigma_F)\delta V_F(\sigma) \quad . \tag{3}$$

As discussed earlier, we have a family of improvement games Γ_k, each of which differs only in the number of improvement steps that the Leader has over the Follower—the number k that identifies the state variable of the game. Under our Bertrand pricing assumption, only the highest quality of the product is sold in the market and the per-period (gross) return to the firm selling it is $k\Delta$. To find the MPE we start with the simplest case in which $\sigma_L(k) = \sigma_F(k) = 1$ for all $k = 1,2,\ldots$.

Lemma 1. Suppose that, in the improvement game with a research exemption, $\sigma_L(k) = 1$ and $\sigma_F(k) = \phi \in [0,1]$, for all $k = 1,2,\ldots$. Then,

(i) $$V_F(\sigma) = \frac{\phi\Delta q\delta\left[1-\delta(1-2\phi q-(1-\phi)p)\right]}{(1-\delta)\left(1-\delta(1-2\phi q)\right)\left(1-\delta(1-\phi q)\right)} - \frac{\phi c\left(1-\delta(1-(1+\phi)q)\right)}{(1-\delta)\left(1-\delta(1-2\phi q)\right)} \equiv V_F \tag{4}$$

(ii) $$V_L(\sigma,k) = \frac{-c+\phi q\delta V_F}{1-\delta(1-\phi q)} + \frac{\Delta k}{1-\delta(1-\phi q)} + \frac{\Delta\delta\left(\phi q+(1-\phi)p\right)}{\left(1-\delta(1-\phi q)\right)^2} \quad . \tag{5}$$

The proof of this result is confined to the Appendix. Thus, when the Leader invests in every period with probability one while the Follower invests with the same probability $\phi \in [0,1]$ in every period, Lemma 1 provides close-form expression for the value of being the Leader or the Follower (conditional on the constant, but arbitrary, mixing probability ϕ). These expressions will prove useful in establishing the MPE for the improvement game claimed in Proposition 1. Note that the value to being the Follower does not depend on the number of leads possessed by the Leader. This is because, if successful in the stage R&D race, the new Leader obtains a one-step lead over the other firm (under our Bertrand pricing assumption). The value to being a Leader, on the other hand, increases with k, the number of improvement steps of the Leader not matched by the Follower, as well as being increasing in the stage payoff Δ and decreasing in R&D cost c.

Next we establish a complete characterization of the conditions under which the Follower and/or the Leader actually invest in the equilibrium of the improvement games. For that purpose, we define the threshold levels:

$$x_0 \equiv \frac{\delta p}{1-\delta} \tag{6}$$

$$x_1 \equiv \frac{q\delta\left(1-\delta(1-p)\right)}{(1-\delta)\left(1-\delta(1-q)\right)} \tag{7}$$

$$x_2 \equiv \frac{q\delta}{\left(1-\delta(1-q)\right)} \quad . \tag{8}$$

Note that, under the assumed structure of the model, $x_0 > x_1 > x_2$. Given that, the firms' equilibrium investment decisions in the improvement game are as follows.

Proposition 1. Then MPE of the improvement game satisfies the following conditions:

(i) If $c/\Delta \le x_2$ then $\sigma_L(k)=1$ and $\sigma_F(k)=1$ for all $k=1,2,\ldots$.

(ii) If $x_2 \le c/\Delta \le x_1$ then $\sigma_L(k)=1$ and $\sigma_F(k)=\phi\in[0,1]$ for all $k=1,2,\ldots$.

(iii) If $x_1 \le c/\Delta \le x_0$, then $\sigma_L(k)=1$ and $\sigma_F(k)=0$ for all $k=1,2,\ldots$.

(iv) If $x_0 \le c/\Delta$, then $\sigma_L(k)=\sigma_F(k)=0$ for all $k=1,2,\ldots$.

The proof, confined to the Appendix, relies on establishing that neither Leader nor Follower has a one-stage deviation from the proposed strategy that would increase his payoff. Because this game is continuous at infinity—that is, the difference between payoffs from any two strategy profiles will be arbitrary close to zero provided that these strategy profiles coincide for a sufficiently large number of periods starting from the beginning of the game—Theorem 4.2 in Fudenberg and Tirole (1991) implies that the proposed strategy profile is the MPE.

Thus, when the R&D cost c is low enough, relative to the stage reward Δ, both firms invest with probability one in every stage. In this case the value functions of the Leader and of the Follower reduce to

$$V_L(\sigma,k)=\frac{\Delta-c}{(1-\delta)}+\frac{(k-1)\Delta}{\left(1-\delta(1-q)\right)} \tag{9}$$

$$V_F=\frac{q\delta\Delta-\left(1-\delta(1-q)\right)c}{(1-\delta)\left[1-\delta(1-q)\right]} \quad . \tag{10}$$

Note that the value of being a Leader when $k>1$ is decreasing in the R&D success probability. Intuitively, when both firms engage in R&D in every period, the Leader with more than one step lead has more to lose than to gain from the R&D context. As for the Follower, $V_F \to 0$ as $c/\Delta \to x_2$. But were the Follower to choose action N for all $c/\Delta \ge x_2$, the value to being a Leader would jump from $V_L(\sigma,k)$ as in equation (9) to V_M as given in equation (1). But then, if the firm that is a Follower in any one stage believes that future Followers always choose action

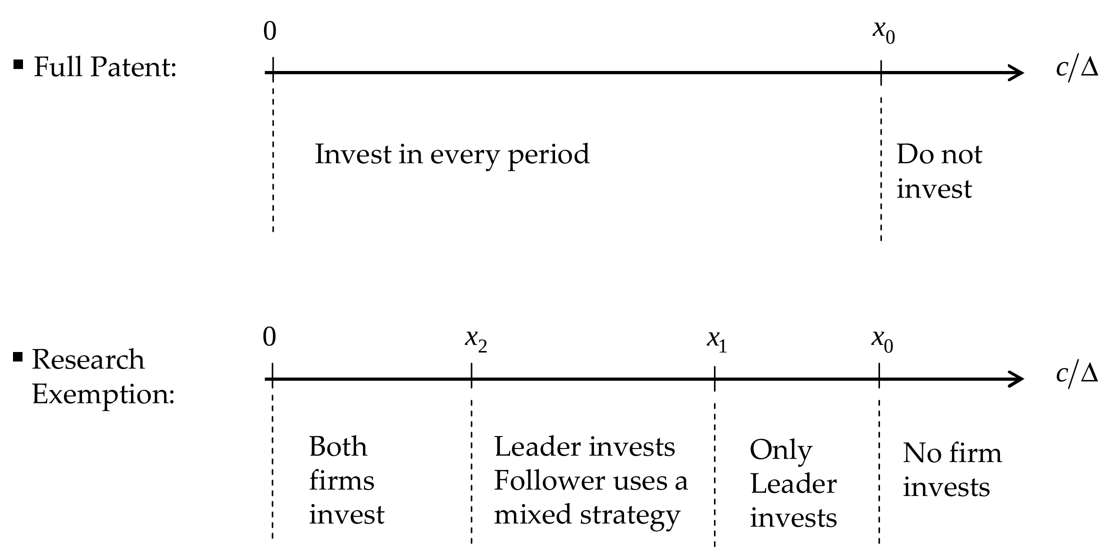
N, then by deviating to I in that stage, the firm would obtain a positive probability of becoming an uncontested Leader, with an associated strictly positive payoff. Thus, $\sigma_F(k)=0$ for all k cannot be part of an equilibrium when $x_2 < c/\Delta$ but c/Δ is close to x_2. The MPE in the domain $x_2 \le c/\Delta \le x_1$, therefore, entails the Follower's use of a mixed strategy, whereby the Follower invests with probability $\phi \in [0,1]$ in all stages. Specifically, as derived in the Appendix, the mixing probability ϕ in this domain is the positive root that solves the quadratic equation

$$-c\left[1-\delta\left(1-q(1+\phi)\right)\right]\left(1-\delta(1-\phi q)\right)+\Delta q\delta\left(1-\delta(1-2\phi q)+\delta(1-\phi)p\right)=0 \quad . \tag{11}$$

At $c/\Delta = x_1$, equation (11) yields $\phi = 0$. At this point the Follower drops out of the improvement game and only the Leader finds it profitable to invest. In fact, it can be verified that, when evaluated at $c/\Delta = x_1$ and $\phi = 0$, the Leader's payoff is equal to the monopolist's payoff. For $x_1 \le c/\Delta \le x_0$ only the Leader invests (with probability one) in the improvement stage, whereas for $x_0 < c/\Delta$ no firm invests. Thus, for $x_1 \le c/\Delta$ the FP regime and the RE regimes are equivalent as far as the improvement game is concerned.

The conclusions of Proposition 1 are illustrated in Figure 2, which represents the type of equilibrium strategies that apply for various ranges of the parameter ratio c/Δ. When R&D is

Figure 2. Types of Markov perfect equilibria in the improvement games



too costly, relative to the expected payoff, no innovation takes place; the range of parameters that supports this outcome is the same under either regime (i.e., $c/\Delta > x_0$). With a more favorable cost/benefit ratio, the incumbent in the FP regime will find it worthwhile to engage in improvements. In this parameter space the RE regime supports only one firm if $x_1 < c/\Delta \le x_0$, and two firms if $0 \le c/\Delta \le x_1$.

The payoff to the two firms in this type of equilibrium is of some importance. By using the expression in equation (4) of Lemma 1, and evaluating it at the ϕ, which solves the equilibrium condition in (11), we find that $V_F = 0$ in the domain $x_2 \le c/\Delta \le x_1$. The payoff to the Leader, on the other hand, at the ϕ that solves (11), is:

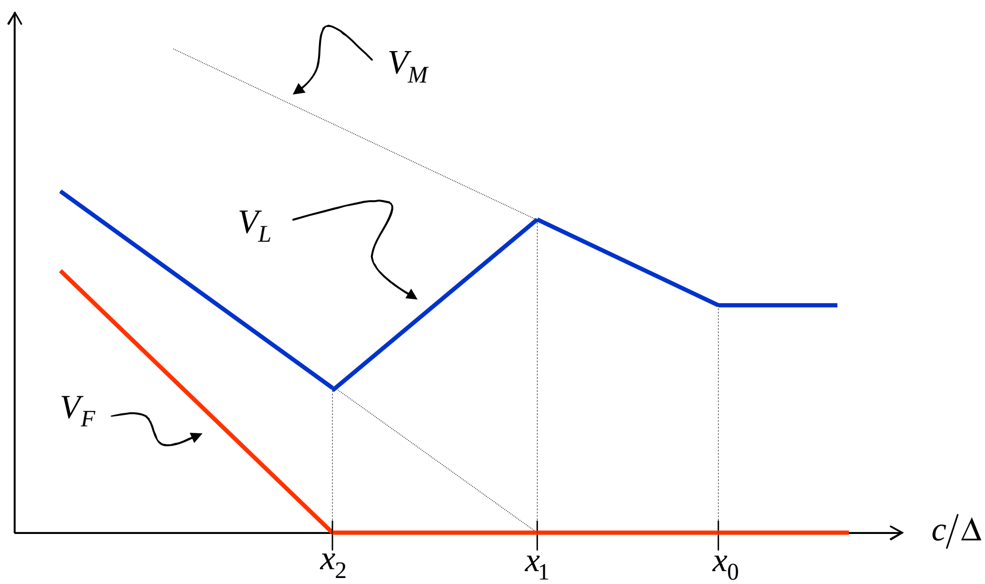
$$V_L(\sigma,k) = \frac{c}{q\delta} + \frac{(k-1)\Delta}{1-\delta(1-\phi q)} \quad . \tag{12}$$

Thus, in the domain $x_2 \le c/\Delta \le x_1$, the payoff to the Leader is increasing in the R&D cost c. That is, the gain from the weakening R&D competition (the Follower invests with a decreasing probability as c increases) more than outweighs the direct negative impact of R&D cost. That the Leader's payoff must be increasing on some part of the domain when $x_2 \le c/\Delta$ is clear when one notes that the monopolist's payoff at $c/\Delta = x_0$ and the Leader's payoff at $c/\Delta = x_2$ satisfy

$$V_M(k)\Big|_{c/\Delta = x_0} = \frac{k\Delta}{(1-\delta)} \quad > \quad \frac{k\Delta}{\big(1-\delta(1-q)\big)} = V_L(\sigma,k)\Big|_{c/\Delta = x_2} \quad . \tag{13}$$

The equilibrium payoff to the Leader and the Follower are illustrated in Figure 3. The threshold levels x_0, x_1, and x_2 that we have identified satisfy intuitive comparative statics properties, such as $\partial x_0/\partial p > \partial x_1/\partial p > \partial x_2/\partial p > 0$ and $\partial x_0/\partial \delta > \partial x_1/\partial \delta > \partial x_2/\partial \delta > 0$. More interestingly, the foregoing analysis shows that, in a well-defined sense, under the RE regime the Leader has a stronger incentive to invest in improvements than does the Follower. This property of the MPE reflects the carrot-and-stick nature of the incentives at work here, what Beath, Katsoulacos, and Ulph (1989) call the "profit incentive" and the "competitive threat." The carrot is the same for both contenders—a successful innovation brings an additional per-period reward of Δ. But the stick differs. For the Follower, failure to innovate when the opponent is successful does not change its situation (recall that the value function of the Follower is invariant to the state of the game). But for the Leader, failure to innovate when the opponent is successful implies the loss of the current gross returns $k\Delta$.

Figure 3. Equilibrium payoffs in the improvement games



4. Equilibrium in the Initial Game

The initial investment game has a structure similar to that of the improvement game. The major differences are the following: (i) the cost of investment in R&D is equal to $c_0 \geq c$; (ii) both firms are in exactly the same position and the per-period profit flow in the investment game is equal to zero; and (iii) the game ends as soon as one of the firms obtains the first successful innovation. We will consider the FP regime first.

4.1. Full Patent Regime

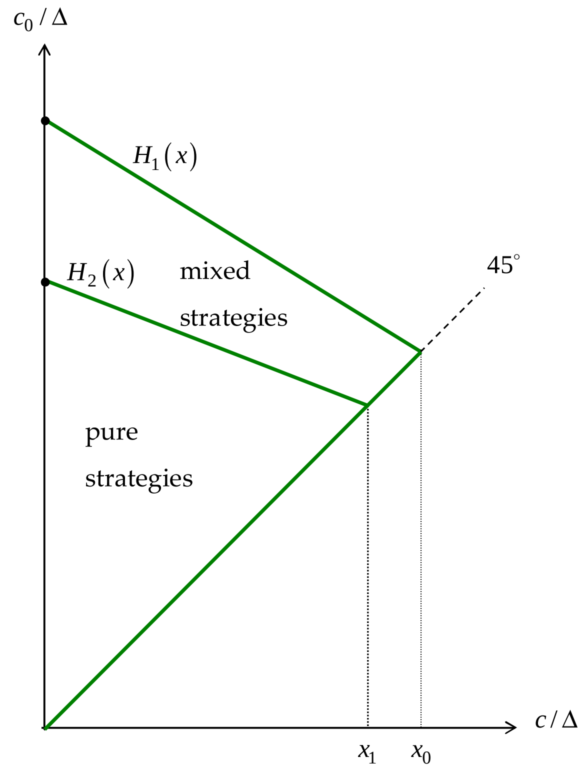
We find that the equilibrium depends critically on the postulated asymmetry between initial innovation and follow-on improvements. To facilitate exposition, it is useful to refer to Figure 4, which illustrates the parametric regions of the types of equilibria that arise. The regions of interest are defined by the following functions:

$$H_1(x) \equiv \frac{p\delta}{1-\delta}\left(\frac{1-\delta+p\delta}{1-\delta}-x\right) \tag{14}$$

$$H_2(x) \equiv \frac{q\delta}{1-\delta}\left(\frac{1-\delta+p\delta}{1-\delta}-x\right) \quad . \tag{15}$$

For notational simplicity, let σ_0 denote the strategy $\sigma(k)$ when $k=0$, that is, the probability of investment of a given firm in the initial investment game. We can then state the following results (details of the proof are in the Appendix).

Figure 4.
Equilibrium with
"full patent" protection



Proposition 2. The symmetric equilibrium of the investment game under the FP regime is given by the strategy profile (σ_0, σ_0), where σ_0 satisfies the following conditions:

(i) If $c/\Delta > x_0$, then $\sigma_0 = 0$.

(ii) If $c/\Delta \le x_0$ and $c_0/\Delta > H_1(c/\Delta)$, then $\sigma_0 = 0$.

(iii) If $c/\Delta \le x_0$ and $c_0/\Delta < H_2(c/\Delta)$, then $\sigma_0 = 1$.

(iv) If $c/\Delta \le x_0$, and $H_2(c/\Delta) \le c_0/\Delta \le H_1(c/\Delta)$, then $\sigma_0 = \dfrac{p\delta V_M - c_0}{(p-q)\delta V_M}$,

where V_M is the value function, at the start of the first improvement game, for the patent holder who will be investing in every period (as derived in equation (1), with $k = 1$).

As one would expect, for a given value of c, relatively low values of initial R&D cost c_0 will induce both firms to invest with probability one, as in part (iii) of Proposition 2. If the R&D cost parameters c and/or c_0 are large enough (as in parts (i) and (ii) of Proposition 2), on the other hand, neither firm invests. For intermediate values of the R&D cost parameters, as exactly identified in part (iv) of Proposition 2, each firm would want to invest if the other does not. Thus, in addition to such pure-strategy equilibria, here we have a (symmetric) mixed-strategy equilibrium. Note that the mixed-strategy equilibrium converges to a pure-strategy equilibrium in the appropriate limit: $\sigma_0 \to 0$ as $c_0/\Delta \to H_1(c/\Delta)$ and $\sigma_0 \to 1$ as $c_0/\Delta \to H_2(c/\Delta)$. Thus, with respect to Figure 4, in equilibrium both firms randomize between investing and not when the parameter vector $(c/\Delta, c_0/\Delta)$ lies in the area labeled "mixed strategies," and both firms invest with probability one when the parameter vector lies in the area labeled "pure strategies."

4.2. Research Exemption Regime

The equilibrium of the investment game under the RE regime similarly depends on the relative magnitude of the R&D costs that characterize the initial innovation as opposed to the follow-on improvements. As derived earlier, under RE regime one can distinguish three intervals of values of c/Δ in which the strategy of the Follower and the resulting equilibrium in the improvement stage is qualitatively different: $[0, x_2]$, $[x_2, x_1]$ and $[x_1, x_0]$. In what follows we will analyze the equilibrium of the initial stage in these cases. The various possibilities that arise are illustrated in Figure 5, where the parametric regions of interest are defined by the functions $H_1(x)$ and $H_2(x)$ defined earlier, and by the following functions:

$$H_3(x) = \frac{p\delta}{1-\delta}(1-x) \tag{16}$$

$$H_4(x) = \frac{q\delta(1-\delta+2\delta q)}{(1-\delta+p\delta)(1-\delta+\delta q)} - \frac{\delta(2q-p)}{(1-\delta+p\delta)}x \tag{17}$$

$$H_5(x) = \frac{p}{q}x \tag{18}$$

Functions $H_3(x)$ and $H_4(x)$ determine the threshold levels of c_0 and the resulting strategy profiles for a given value of $(c/\Delta) \in [0, x_2]$, and the function $H_5(x)$ does the same for the parametric region $(c/\Delta) \in [x_2, x_1]$. The following proposition characterizes the equilibrium of the investment game under the RE regime for all values of $c/\Delta \geq x_1$.

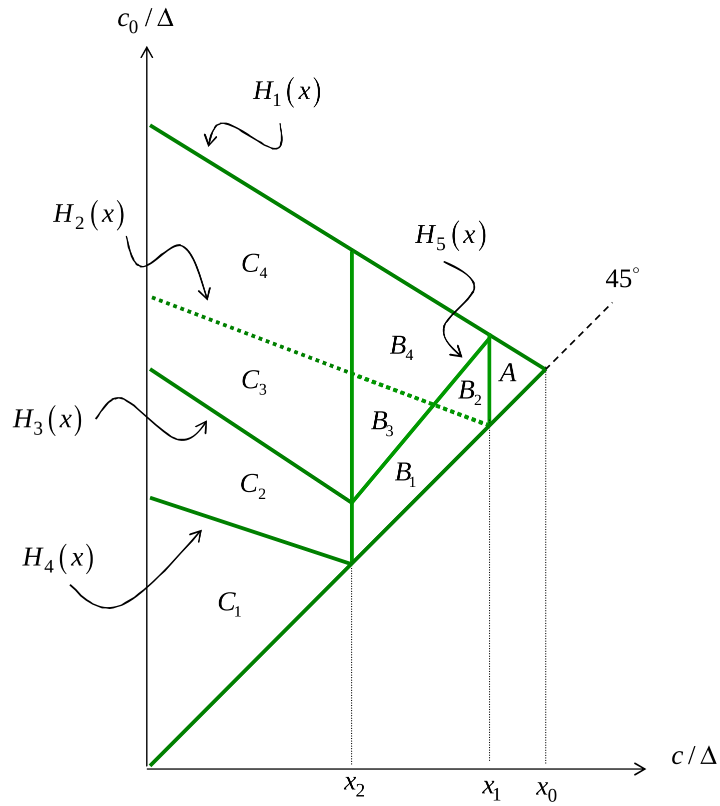
Proposition 3. Suppose that $c/\Delta \geq x_1$. Then the strategy profile (σ_0, σ_0) constitutes the symmetric equilibrium of the investment game under the research exemption regime i.f.f. it satisfies the following conditions:

(i) If $c/\Delta > x_0$, then $\sigma_0 = 0$.

(ii) If $x_1 \leq c/\Delta \leq x_0$ and $c_0/\Delta > H_1(c/\Delta)$, then $\sigma_0 = 0$.

(iii) If $x_1 \leq c/\Delta \leq x_0$ and $c_0/\Delta \leq H_1(c/\Delta)$, then $\sigma_0 = \dfrac{p\delta V_M - c_0}{(p-q)\delta V_M}$.

Figure 5.
Equilibrium with the "research exemption"



The results of this proposition follow directly from observing that, as was shown in Proposition 1, when $c/\Delta \geq x_1$ the Follower does not invest at the improvement stage. This implies that payoffs of the Leader and the Follower are identical to the payoffs of the patent holder and of the firm that did not innovate under the FP regime, respectively. Therefore the resulting equilibrium must also be identical to the one obtained under the FP regime (see Proposition 2). It is also readily verified that $H_2(x_1) = x_1$. This implies that there is no pure-strategy equilibrium in the investment game in this case.

Next we consider the interval $[x_2, x_1]$. Recall that in this case both the Leader and the Follower take part in the improvement game, but the payoff of the Follower is equal to zero. The resulting equilibrium at the investment stage is characterized as follows.

Proposition 4. Suppose that $x_2 \leq c/\Delta \leq x_1$ and let $V_1 \equiv V_L(\sigma, 1)$ denote the payoff of the winner of the investment game (i.e., the first Leader), as given by equation (5). Then the strategy profile (σ_0, σ_0) constitutes the symmetric equilibrium of the investment game under the research exemption regime i.f.f. it satisfies the following conditions:

(i) If $c_0 = c$, then $\sigma_0 = 1$.

(ii) If $c/\Delta \le c_0/\Delta \le H_5\left(c/\Delta\right)$, then $\sigma_0 = \dfrac{p\delta V_1 - c_0}{(p-q)\delta V_1}$.

(iii) If $c_0/\Delta > H_5\left(c/\Delta\right)$, then $\sigma_0 = 0$.

The proof of this result is given in the Appendix. Thus, in the initial investment game we can have an equilibrium in which both firms invest with probability one even if $x_2 \le c/\Delta$ (that is, even though, at the improvement stage, under these conditions the Follower will only play a mixed strategy).

Finally, consider the case $(c/\Delta) \in [0, x_2]$, that is, when both the Leader and the Follower invest with probability one in the improvement stage.

Proposition 5. Suppose that $c/\Delta \le x_2$. Then the strategy profile (σ_0, σ_0) constitutes the symmetric equilibrium of the investment game under the research exemption regime i.f.f. it satisfies the following conditions:

(i) If $c_0/\Delta \le H_4\left(c/\Delta\right)$, then $\sigma_0 = 1$.

(ii) If $c_0/\Delta > H_3\left(c/\Delta\right)$, then $\sigma_0 = 0$.

(iii) If $c/\Delta < x_1$ and $H_4\left(c/\Delta\right) \le c_0/\Delta \le H_3\left(c/\Delta\right)$, then $0 \le \sigma_0 \le 1$.

The proof of the proposition is given in the Appendix, where the quadratic equation defining σ_0 for part (iii) is also explicitly derived. With respect to Figure 5, therefore, pure strategies are used in the parameter regions labeled C_1, and symmetric mixed strategies are used in regions A, B_1 and C_2. As one might expect, the equilibrium strategies in the initial game reflect the nature of equilibrium at the improvement stage. Recall that, in the improvement game, the Follower will not take part whenever $c/\Delta > x_1$. If this condition is satisfied, once one of the firms succeeds in completing the first innovation step, its rival will immediately drop out of the race. This type of equilibrium is similar to the one obtained by Fudenberg et al. (1983) in the context of a race with a known finish line, and by Hörner (2004) in an infinite-horizon setting. Specifically, the incentives to invest in R&D is highest when the firms compete for the *entire market*, i.e., when the winner of the initial game faces no competition afterwards. In particular, note that whenever $c/\Delta < x_1$, no investment takes place if $c_0/\Delta \ge H_3(0) = x_0$. But, when $x_1 < c/\Delta < x_0$ we can find a range of c_0/Δ such that $c_0/\Delta \ge x_0$

and still both firms invest with positive probability in equilibrium, as can be seen with the aid of Figure 5. The same conclusion applies to the case $x_2 < c/\Delta < x_1$, when the Leader faces a Follower that randomizes and does not invest with probability one in each period.

Comparing the equilibrium outcomes under the FT and RE regimes, we note that in the parameter regions C_4 and B_4 of Figure 5 we have no initial R&D investment under the RE regime, whereas the FP regime leads to some initial investment (given by the mixed-strategy equilibrium). Similarly, in regions C_3 and B_3 of Figure 5 we again have no initial R&D investment under the RE regime, whereas under the FP regime both firms invest with probability one in the initial game. Thus, it is apparent that the presence of an RE clause unambiguously weakens the initial incentive of firms to invest in R&D. The welfare consequences of these weakened investment incentives are analyzed next.

5. Welfare Analysis

Having characterized the MPE of the model, we can now turn to the normative implications of the analysis. We consider first the returns, from an *ex ante* perspective, to the two firms, and next derive the aggregate welfare of the economy.

5.1. Firms' Expected Profit

The expected profit of the two firms at time zero, before the initial research investment c_0 is made, depends on the particular equilibrium solution that applies to the region of the parameter space. The regions of interest (labeled A, B_1 to B_4, and C_1 to C_4) are illustrated in Figure 4. Our findings are as follows.

Proposition 6. The firms' expected profits under the FP regime are never lower, and can be strictly higher, than those under the RE regime. Specifically:

(i) Firms' expected profit under RE and FP regimes are the same if $c_0/\Delta \geq H_2(c/\Delta)$.

(ii) Firms' expected profit under the FP regime is higher than under the RE regime

(a) if $x_2 < c/\Delta < x_1$ and $c_0/\Delta \leq H_2(c/\Delta)$,

(b) if $c/\Delta < x_2$ and $c_0/\Delta \leq H_2(c/\Delta)$.

The domain of part (i) of this proposition encompasses the parameter space labeled as A, B_2, B_4, and C_4 in Figure 5. In area *A* the firms have exactly the same equilibrium strategies under either regime (see Propositions 1, 2, and 3): in the improvement games only the Leader invests whenever $c/\Delta > x_1$. Consequently, the firms have the same behavior in the initial game as well. The firms' equilibrium strategy is to invest with probability one in the parameter space of area A (earning a positive expected payoff). In the area C_4 there is no investment in the initial game under the RE regime, whereas firms invest with a mixed strategy under the FP regime (but earn a zero expected payoff). In area B_2 firms randomize in the investment game under both regimes. Finally, in area B_4 there is a mixed-strategy equilibrium under the FP regime and none of the firms invests under the RE regime. For the domain of part (ii)(a), *ex ante* expected profits are positive under the FP regime and zero under the RE regime (because none of the firms invests in the investment game in area B_3, and because firms randomize in area B_1). The domain of part (ii)(b) encompasses areas C_1, C_2, and C_3 in Figure 5. Consider area C_1 first. Under either regime, both firms invest with probability one in both the investment game and the improvement games. Because firms have the same probability of success, it follows that both firms prefer the FP regime, *ex ante*, i.f.f. $V_M \geq V_L + V_F$. By using the expressions derived in Lemma 1 (for the case $\phi = 1$), this inequality is equivalent to

$$\frac{p\delta\Delta}{1-\delta} \geq \frac{q\delta\Delta}{1-\delta+q\delta} - c \tag{19}$$

which is clearly satisfied. Turning to the parameter space comprising area C_2, we note that here firms invest with probability one in the FP regime, whereas they randomize in the mixed-strategy equilibria under the RE regime. The expression for the expected profit of each firm under the FP regime solves the recursive equation $V_0^{FP} = -c_0 + q\delta V_M + (1-2q)\delta V_0^{FP}$, and thus:

$$V_0^{FP} = \frac{q\delta V_M - c_0}{1-(1-2q)\delta} \ . \tag{20}$$

whereas under the RE regime expected profit is given by

$$V_0^{RE} = \frac{\sigma_0 p\delta V_F}{1-\delta(1-\sigma_0 p)} \tag{21}$$

where σ_0 is the investment probability in the equilibrium mixed strategy. As shown in the Appendix, a sufficient condition for $V_0^{FP} > V_0^{RE}$ holds. Finally, for the parameter space of area C_3, firms invest with probability one in the initial game and enjoy a positive profit, whereas there is no investment (and zero profit) under the RE regime.

Thus, Proposition 6 establishes that firms, *ex ante*, would never prefer the RE regime over the FP regime. This result differs from that of Bessen and Maskin (2002), where a (suitably defined) weaker patent system, in a similar sequential innovation setting, can produce higher *ex* *ante* returns to the innovating firms than a full patent system. The root of that result is a complementarity assumption that is appealing in a sequential setting: the presence of a competitor increases the probability that future profitable innovations (improvements) may be undertaken (although it erodes the firm's expected profit in a given stage innovation race). The former effects counter the latter (standard) effect and can lead to a firm benefiting from its innovation being used by others for future innovations. A flavor of Bessen and Maskin's complementarity assumption is present in our model as well: prior to knowing the identity of the winner of the initial innovation stage, an RE may be appealing because it guarantees the possibility of taking part in future (profitable) innovation stages. But the specific structure of the IPR regimes that we have modeled, and the explicit requirement of an MPE solution, in our setting ensure that the FP protection is preferred *ex ante* by the firms.

5.2. Welfare

Because under the Bertrand pricing condition that we have used the sum of firms' profits does not coincide with social welfare, we have to take into account consumer surplus when evaluating efficiency of patents and research exemptions. First we compute the expected social welfare starting at stage one of the improvement game. Let W_i denote this welfare measure when there are i firms ($i=1,2$) investing (in equilibrium) in every period of the game, and let W_ϕ denote the corresponding welfare measure when the Leader invests with probability one and the Follower invests with probability ϕ, evaluated at the beginning of the improvement game. Clearly, W_1 coincides with monopoly profits V_M because the monopolist captures the entire surplus when it is the only one to invest in every period. Hence,

$$W_1 \equiv V_M = \frac{\Delta - c}{1-\delta} + \frac{p\delta\Delta}{(1-\delta)^2}\,. \tag{22}$$

On the other hand, the situation in which two firms invest in every period from the social point of view is the same as the situation in which there is a monopolist with cost $2c$ and success probability $2q > p$ that invests in every period. Hence the sum of firms' profits and consumer surplus is equal to the profits of such a monopolist. Therefore,

$$W_2 = \frac{\Delta - 2c}{1-\delta} + \frac{2q\delta\Delta}{(1-\delta)^2}\,. \tag{23}$$

The measure of social welfare when the Follower randomizes between investing and not can be shown to be given by the following expression (see the Appendix for an explicit derivation):

$$W_\phi = \frac{\Delta - c(1+\phi)}{1-\delta} + \frac{\Delta(\phi 2q\delta + (1-\phi)p\delta)}{(1-\delta)^2}\,. \tag{24}$$

Note that, as one would expect, when $\phi = 0$ we have $W_\phi = W_1$, and when $\phi = 1$ we have $W_\phi = W_2$. Similarly to the analysis of the equilibrium of the investment game, we will compare welfare under the two IPR regimes in the three possible cases: $(c/\Delta) \in [0, x_2]$, $(c/\Delta) \in [x_2, x_1]$ and $(c/\Delta) \in [x_1, x_0]$. Note that for the case $(c/\Delta) \in [x_1, x_0]$, we have shown that the equilibrium strategies of firms are exactly the same in both regimes. This implies that the social payoffs are equal. It turns out that in the two remaining cases it is possible to characterize social welfare ranking only for a subset of the domain of possible values of $(c/\Delta, c_0/\Delta)$. We present these analytic results in the following two propositions and then perform numerical analysis of the remaining cases.

Proposition 7. Suppose that $c/\Delta \in [0, x_2]$. The social payoffs under the RE and FP regimes are related as follows:

(i) If $H_3(c/\Delta) < c_0/\Delta < H_2(c/\Delta)$, then the FP regime yields higher welfare.

(ii) If $c_0/\Delta \le H_4(c/\Delta)$, then

(a) if $(1-p)(2-p) \ge (1-\delta)/\delta$, the RE regime yields a higher welfare;

(b) if $(1-p)(2-p) < (1-\delta)/\delta$, the FP regime gives higher social welfare if $(1-p)x_0 < c/\Delta \le x_1$ but the RE regime yields higher welfare if $0 \le c/\Delta \le (1-p)x_0$.

For the case of part (i), with FP protection both firms invest with probability one; hence, the social payoff is positive and greater than the social payoff with the RE (which is zero because none of the firms invests in equilibrium). For part (ii), here both firms invest with probability one in both investment and improvement games. The question of whether the RE is better than the FP regime is essentially the same as the question of whether it is better to have two firms (as under the RE regime) or one firm (as under the FP regime) in the improvement game. Thus, the RE regime yields higher welfare i.f.f. $W_2 \ge W_1$, that is, whenever

$$\frac{c}{\Delta(1-p)} \le \frac{\delta p}{1-\delta} \equiv x_0 \quad . \tag{25}$$

We know that in this region $c/\Delta < x_1$. We conclude that in this region the RE regime will yield a higher welfare as long as parameter values satisfy the following inequality:

$$(1-p)x_0 \equiv \frac{\delta p(1-p)}{1-\delta} \ge \frac{q\delta}{1-\delta+q\delta} \equiv x_1 \quad \Leftrightarrow \quad (1-p)(2-p) \ge (1-\delta)/\delta \, . \tag{26}$$

Proposition 8. Suppose that $c/\Delta \in [x_2, x_1]$. The social payoffs under the RE and FP regimes are related as follows:

(i) For all values of $(c/\Delta, c_0/\Delta)$ that satisfy the condition $H_5(c/\Delta) < c_0/\Delta < H_2(c/\Delta)$ (region B_3) the FP regime yields higher welfare.

(ii) For all values of $(c/\Delta, c_0/\Delta)$ that satisfy the condition $H_2(c/\Delta) < c_0/\Delta < H_5(c/\Delta)$ (region B_2) the RE regime yields higher welfare.

(iii) For all values of $(c/\Delta, c_0/\Delta)$ that satisfy $\max\{H_2(c/\Delta), H_5(c/\Delta)\} < c_0/\Delta < H_1(c/\Delta)$, that is, region B_4, there is no difference in welfare between the two IP regimes.

For the parameter region of part (i), with FP protection both firms invest with probability one; hence, the social payoff is positive and greater than the social payoff with the RE, which is equal to zero because none of the firms invests in equilibrium. For part (ii), firms randomize in the investment game under both IP regimes. Even though expected profits are zero under both IP regimes, the RE regime yields a higher welfare because firms do not appropriate the whole consumer surplus (under our Bertrand pricing assumption). Finally, for part (iii), firms randomize under the FP regime (earning zero expected profit), and there is no investment under the RE regime. We conclude that welfare is equal to zero in both cases.

5.3. An Illustration

Propositions 8 and 9 do not say anything conclusive about the welfare ranking of the two IPR regimes when the parameters of interest fall in areas C_2 and B_1 of Figure 5. It turns out that either welfare ranking is possible in these areas, depending on parameter values. That much can easily be established by deriving explicit expressions for the welfare functions of interest that are then numerically evaluated for alternative parameter values. Suppose that both firms invest with probability σ_0 in the investment game. Then the expected social payoff of the whole game, labeled $W_0(\sigma_0)$, is defined by the following recursive equation:

$$\begin{aligned} W_0(\sigma_0) = \sigma_0{}^2\left[-2c_0 + 2q\delta W_i + (1-2q)\delta W_0(\sigma_0)\right] \\ + 2\sigma_0(1-\sigma_0)\left[-c_0 + p\delta W_i + (1-p)\delta W_0(\sigma_0)\right] + (1-\sigma_0)^2\,\delta W_0(\sigma_0) \end{aligned} \tag{27}$$

yielding (upon some simplification)

$$W_0(\sigma_0) = \frac{\sigma_0 p(2-\sigma_0 p)\delta W_i - 2\sigma_0 c_0}{1-\delta\left[1-2\sigma_0\left((1-\sigma_0)p + \sigma_0 q\right)\right]} \tag{28}$$

where W_i is equal to either W_1, W_2, or W_ϕ, depending on the equilibrium of the improvement game. The social welfare measures under the FP and RE regimes are given by, respectively,

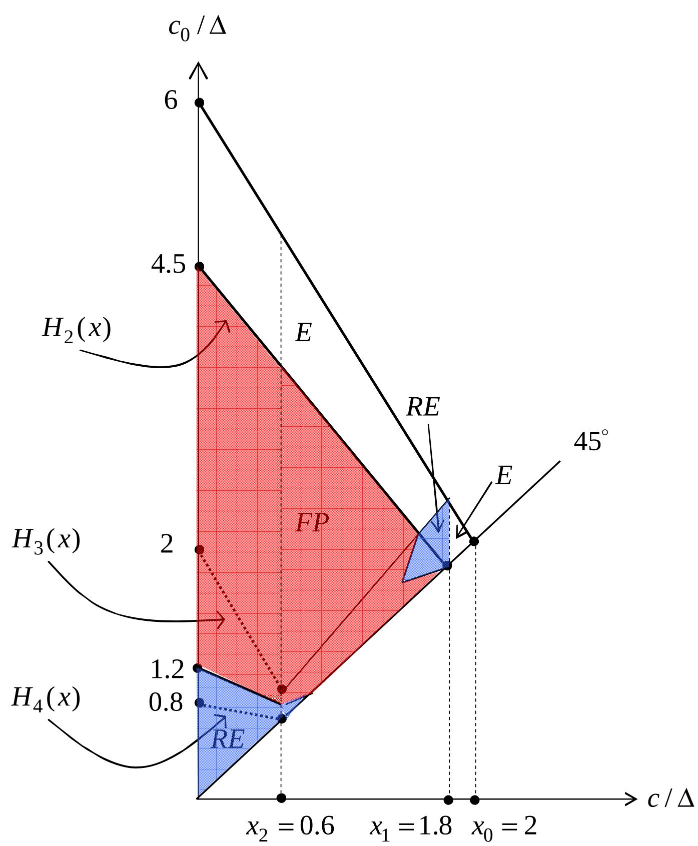
$$W^{FP} = W_0(1) = \frac{p(2-p)\delta W_1 - 2c_0}{1-\delta+2q\delta} \tag{29}$$

$$W^{RE} = W_0(\sigma_0) = \frac{\sigma_0 p\left(2-\sigma_0 p\right)\delta W_i - 2\sigma_0 c_0}{1-\delta\left[1-2\sigma_0\left((1-\sigma_0)p+\sigma_0 q\right)\right]} \tag{30}$$

where in (30) W_i is equal to either W_2 or W_ϕ.

These welfare functions can now be compared for any given set of parameter values (upon calculation of the equilibrium mixed-strategy parameter σ_0). Consider, for example, $\Delta = 1$ (without loss of generality), and suppose that $p = 0.5$ and $\delta = 0.8$.[7] The welfare comparison of the two IPR regimes that we obtain in this case is summarized in Figure 6 where, for concreteness, the various regions are drawn to scale (i.e., given $p = 0.5$ and $\delta = 0.8$).

Figure 6.
Welfare comparisons
(case of $\delta = 0.8$ and $p = 0.5$)



7 These parameter values broadly reflect the nature of plant breeding, where the probability of success of a research program may be good, but where it usually takes several years to bring a new variety to the market. For example, $\delta = 0.8$ corresponds to a research period of five years if the annual discount rate is approximately equal to 4.5 percent.

The un-shaded regions in Figure 6 (labeled E) represent the parameter space where the FP and RE regimes are equivalent in terms of social welfare. In the rightmost portion of this parameter space (region A in Figure 5) there is no difference in welfare because the equilibrium is the same under the two intellectual property regimes. In the other portion of this parameter space, welfare equivalence results because no investment takes place under the RE regime, whereas under the FP regime all the surplus is competed away by the two firms (who engage in a mixed-strategy equilibrium in the initial investment game). In the red-colored regions of Figure 6, labeled FP, the FP regime is better from the social point of view; these regions correspond to parts (i) and (ii) of Proposition 7, part (i) of Proposition 8, and the conclusions of the analysis of regions C_2 and B_1 discussed in the foregoing. Finally, in the blue-colored regions of Figure 6, labeled RE, the RE regime dominates patents from the social point of view. These regions were described in part (ii) of Proposition 7 and part (ii) of Proposition 8 and in the context of the analysis of regions C_2 and B_1.

The fact that the parameter space in which the RE regime dominates is disjoint exhibits one of the simplifying features of our model. Specifically, the assumption that the entire surplus created by the innovation can be extracted by a monopolist patent holder means that there is no residual consumer surplus in region B_2; and in this region there is no expected profit either under the FP regime, although some investment takes place, because the mixed-strategy equilibria competes away all the expected profit. Under the RE regime, firms earn zero initial expected profits (they also play a mixed strategy in both the initial and the improvement games). But given the Bertrand pricing assumption, consumers can capture some of the benefits of innovation here, and thus the RE regime dominates the FP regime in this region. In other words, the limited avenue for R&D benefit spillover to consumers that we allow in our model somewhat slants the comparison in favor of the RE regime. Whereas this result underlies a limiting feature of the model (which could, of course, be relaxed, at the cost of making the characterization of the results even more cumbersome), it does reinforce the significance of the parameter space where we have shown that the FP regime dominates.

5.4. On Licensing

In this paper we have assumed that, under both intellectual property regimes, no licensing takes place between competing firms. The type of licensing that we might consider here is for the right to carry out R&D (there is clearly no incentive for the Leader and patent holder to license the right to produce). Because licensing is a central theme in studies of cumulative innovation (e.g., Green and Scotchmer, 1995), it might be useful to articulate how licensing would affect our results. First note that, unlike some other quality ladder models in this area, here we have assumed that ideas are not scarce in that both the initial innovator and the other firm can pursue the follow-on innovation. But we have also implicitly assumed that firms can operate only one project at a time (i.e., each firm has a given stock of R&D capabilities), so that, in principle, licensing the ability to perform product-improving R&D might be useful.

Under the RE regime, it is clear that there is no scope for licensing because the lagging firm has free access to the latest innovation for R&D purposes (or, to put it differently, follow-on innovations are patentable and non-infringing). Under the FP regime, on the other hand, the winner of the initial game would find it profitable to license the right to innovate if the monopoly profit from investing in the two separate projects is higher than the profit from a single project. In fact, because in our setting the monopolist captures the entire surplus from innovation, this condition is equivalent to whether it is better, from the social point of view, to have one or two firms engaged in R&D.[8] In part (ii) of Proposition 7 we have shown that two firms are better than one i.f.f. $(1-p)x_0 \geq c/\Delta$. Therefore, in this domain, licensing could occur. Because in our setting the monopolist fully internalizes the social benefit of innovation, allowing for licensing arrangements would improve the welfare properties of the FP regime without affecting the nature of the equilibrium under the RE regime. We should conclude, therefore, that if licensing were allowed in this model the FP regime would weakly dominate the RE in every case. But we caution against this overly strong conclusion. In our model it is not particularly meaningful to consider licensing because we do not explicitly model an asymmetric information structure, a feature that has been shown to be critical in the licensing of technology, especially in a cumulative innovation setting (Gallini and Wright, 1990; Bessen, 2004).

6. Conclusion

Recent court decisions have renewed interest, both in the United States and abroad, in the question of whether patent law reform should include a statutory research exemption (Merrill, Levin, and Myers, 2004; Thomas, 2004; Rimmer, 2005). Conversely, for the case of plant breeder's rights (an intellectual property right system that already possesses a well-defined research exemption), there has been considerable debate on whether the access provided by the research exemption should be curtailed (Le Buanec, 2004). Little economic research on this feature of intellectual property rights exists, however. In this paper we attempt to fill this gap in the policy analysis of intellectual property rights by studying the welfare properties of the

8 The presumption that firms can carry out only one project at a time rules out the "invariance" effect of Sah and Stiglitz (1987).

research exemption and its ability to provide incentives for R&D investment when the innovation process is sequential and cumulative. We develop a dynamic model of production and R&D competition in which the cost of the initial innovation effort differs from the cost of subsequent improvements. In this framework we derive explicit solutions for the Markov perfect equilibria of the investment and improvement games and analyze the social welfare properties of full patent and research exemption regimes.

Among the findings of the paper, it turns out that the firms themselves always prefer (*ex ante*) the full patent protection regime. The social ranking of the two intellectual property regimes, on the other hand, depends on the relative magnitudes of costs of initial innovation and improvements. In particular, there exists a range of improvement cost parameters in which the social ordering of the two regimes depends on the magnitude of the initial innovation cost: for low values of this initial cost the research exemption regime yields a higher welfare, whereas when the initial cost is large the full patent regime is optimal from the social point of view. This implies that the research exemption is most likely to provide inadequate incentives when there is a large cost of establishing a research program, as is arguably the case for the plant breeding industry (where developing a new variety typically takes several years). On the other hand, when both initial and improvement costs are small relative to the expected profits (perhaps the case of the software industry noted by Bessen and Maskin, 2002), the weaker incentive to innovate is immaterial (firms engage in R&D anyway), and the research exemption regime results in a higher social payoff.

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Appendix.

Proof of Lemma 1.

Consider the situation where the Leader invests with probability one in every period, whereas the Follower invests with the same probability $\phi \in [0,1]$ in every period (i.e., $\sigma_L(k)=1$ and $\sigma_F(k)=\phi$, $\forall k$). As in the text, the value to the Follower is written as V_F (this value is independent of the state), whereas for the Leader we simplify the notation and write $V_k \equiv V_L(\sigma_L(k),\sigma_F(k),k)$. From the recursive equations in (2) and (3) we have

$$V_k = \Delta k - c + \phi[q\delta V_{k+1} + q\delta V_F + (1-2q)\delta V_k] + (1-\phi)[p\delta V_{k+1} + (1-p)\delta V_k] \tag{31}$$

$$V_F = \phi\left[-c + q\delta V_1^L + (1-q)\delta V_F\right] + (1-\phi)\delta V_F \quad . \tag{32}$$

Hence, for the Leader we have

$$V_k = \alpha + \beta k + \gamma V_{k+1} \qquad k = 1,2,...$$

where the parameters α, β and γ are defined as follows:

$$\alpha \equiv \frac{-c + \phi q \delta V_F}{(1-\phi\delta(1-2q)-(1-\phi)(1-p)\delta)}$$

$$\beta \equiv \frac{\Delta}{(1-\phi\delta(1-2q)-(1-\phi)(1-p)\delta)}$$

$$\gamma \equiv \frac{\phi q\delta + (1-\phi)p\delta}{(1-\phi\delta(1-2q)-(1-\phi)(1-p)\delta)} < 1 \ .$$

Assuming that the following convergence condition holds,

$$\lim_{n\to\infty} \quad \gamma^n V_{k+n} \to 0 \tag{33}$$

the general solution to the value of the Leader can be written as

$$V_k = \frac{\alpha}{1-\gamma} + \frac{\beta k}{1-\gamma} + \frac{\beta\gamma}{(1-\gamma)^2} \quad . \tag{34}$$

Note that, by using (34), the term in the convergence condition (33) reduces to

$$\gamma^n V_{k+n} = \gamma^n \left(\frac{\alpha}{1-\gamma} + \frac{\beta\gamma}{(1-\gamma)^2} + \frac{\beta k}{1-\gamma} \right) + \frac{\beta}{1-\gamma}\gamma^n n \quad .$$

Given that $\gamma < 1$, it follows that $\gamma^n \to 0$ as $n \to \infty$, and also that $n\gamma^n \to 0$ as $n \to \infty$, so that (33) holds. From the previous definitions of the parameters α, β and γ, it then follows that

$$V_k = \frac{-c + \phi q \delta V_F}{1-\delta(1-\phi q)} + \frac{\Delta k}{1-\delta(1-\phi q)} + \frac{\Delta\delta(\phi q + (1-\phi)p)}{(1-\delta(1-\phi q))^2} \quad . \tag{35}$$

This expression is conditional on V_F, which satisfies (32). But for $k = 1$ we have

$$V_1 = \frac{-c + \phi q \delta V_F}{1-\delta(1-\phi q)} + \frac{\Delta}{1-\delta(1-\phi q)}\left[\frac{1-\delta\left(1-2\phi q-(1-\phi)p\right)}{1-\delta(1-\phi q)}\right] . \tag{36}$$

Upon solving the system of equations given by (32) and (36) we obtain:

$$V_1 = \frac{-c\left(1-\delta\left[1-\phi q(1+\phi)\right]\right)}{(1-\delta)\left(1-\delta(1-2\phi q)\right)} + \frac{\Delta}{(1-\delta)} + \frac{\Delta\left(\delta(1-\phi)p\right)}{(1-\delta)\left(1-\delta(1-2\phi q)\right)} \tag{37}$$

$$V_F = \phi\left(\frac{\Delta q \delta\left[1-\delta(1-2\phi q-(1-\phi)p)\right]}{(1-\delta)\left(1-\delta(1-2\phi q)\right)\left(1-\delta(1-\phi q)\right)} - \frac{c\left(1-\delta(1-(1+\phi)q)\right)}{(1-\delta)\left(1-\delta(1-2\phi q)\right)}\right) . \tag{38}$$

Equations (35) and (38) contain the results claimed in Lemma 1.

Proof of Proposition 1.

Part (i). If both firms invest in every period, then, from the recursive equations (2) and (3), their value functions are given by

$$V_L(\sigma,k) = k\Delta - c + q\delta V_F + q\delta V_L(\sigma,k+1) + (1-2q)\delta V_L(\sigma,k) \tag{39}$$

$$V_F = -c + q\delta V_L(\sigma,1) + (1-q)\delta V_F \ . \tag{40}$$

Consider the Leader first, and suppose that now the Leader deviates by not investing in state s. Then its expected payoff is

$$V_L(\hat{\sigma},s) = s\Delta + p\delta V_F + (1-p)\delta V_L(\hat{\sigma},s) \quad \Rightarrow \quad V_L(\hat{\sigma},s) = \frac{s\Delta + p\delta V_F}{1-\delta(1-p)} \ . \tag{41}$$

Here, $\hat{\sigma} = (\hat{\sigma}_L, \sigma_F)$ and $\hat{\sigma}_L(k) = \sigma_L(k)$ for all $k \neq s$. Assume that $s = 1$. Then the one-stage deviation under consideration would be profitable i.f.f. $V_L(\hat{\sigma},1) \geq V_L(\sigma,1)$. From Lemma 1, when $k = 1$ and both firms invest in every period, the value functions of Leader and Follower reduce to

$$V_L(\sigma,1) = \frac{\Delta - c}{(1-\delta)} \tag{42}$$

$$V_F(\sigma) = \frac{\Delta q \delta}{(1-\delta)\left(1-\delta(1-q)\right)} - \frac{c}{(1-\delta)} \ . \tag{43}$$

By using these expressions and (41), we find that $V_L(\sigma,1) \geq V_L(\hat{\sigma},1)$ i.f.f.

$$\frac{c}{\Delta} \leq \frac{p\delta}{1-\delta(1-q)} \ . \tag{44}$$

For the purposes of part (i) of Proposition 1 we observe that

$$x_2 \equiv \frac{q\delta}{1-\delta(1-q)} < \frac{p\delta}{1-\delta(1-q)} \quad . \tag{45}$$

Therefore we conclude that deviating by not investing in state $s=1$ cannot be profitable for the Leader when $c/\Delta \le x_2$. Since not investing in state $s=1$ yields a strictly lower payoff to the Leader, the Leader will not choose this action with positive probability in any arbitrary deviation when the lead is equal to one. Hence we conclude that the Leader has no profitable deviation in this state. Next, by using equation (5) of Lemma 1 we can write

$$V_L(\sigma,s+1) - V_L(\hat{\sigma},s+1) = V_L(\sigma,s) - V_L(\hat{\sigma},s) + \frac{\Delta\delta(p-q)}{\left(1-\delta(1-q)\right)\left(1-\delta(1-p)\right)} \quad . \tag{46}$$

If $V_L(\sigma,1) \ge V_L(\hat{\sigma},1)$, then, because $p > q$, it follows by induction that $V_L(\sigma,k) \ge V_L(\hat{\sigma},k), \forall k$. Thus, the Leader does not have a profitable deviation at any stage. Next consider the Follower. Conditional on the Leader investing in every period, the value to the Follower of investing in every period is V_F as given by (43), whereas the value of deviating to not investing in the first stage is $V_F\left(\hat{\sigma}\right) = \delta V_F\left(\sigma\right)$. Hence, $V_F(\sigma) \ge V_F(\hat{\sigma})$ whenever $V_F \ge 0$ which, from (43), is equivalent to $c/\Delta \le q\delta/\left(1-\delta(1-q)\right) \equiv x_2$.

Part (ii). Because the value of being a Follower does not depend on the state k of the game, the Follower can follow the same stationary strategy at all states. Thus, consider the candidate equilibrium profile $\sigma \equiv \left(\sigma_L, \sigma_F\right)$ where $\sigma_F(k) = \phi \in [0,1], \forall k$ and $\sigma_L(k) = 1, \forall k$. From part (i), $\phi = 1$ i.f.f. $c/\Delta \le q\delta/\left(1-\delta(1-q)\right) \equiv x_2$. For $c/\Delta < x_2$ and close enough to x_2, suppose that $\phi \in (0,1)$. Then, in any one stage, the Follower must be indifferent between actions I and N (given that the rest of the game accords with the strategy profile σ), that is, $V_F^I = V_F^N$ where

$$V_F^I = -c + q\delta V_1 + (1-q)\delta V_F$$

$$V_F^N = \delta V_F \quad .$$

By using the expressions derived in Lemma 1, we find that $V_F^I = V_F^N$ requires ϕ to solve the quadratic equation (11). Note that $\phi \to 1$ as $c/\Delta \to x_2$ and $\phi \to 0$ as $c/\Delta \to x_1$. By construction, the Follower does not have a one-stage profitable deviation from $\sigma_F(k) = \phi, \forall k$. As for the Leader, the value of $\sigma_L(k) = 1, \forall k$ when the Follower plays $\sigma_F(k) = \phi, \forall k$ is given by $V_L(\sigma,k)$ in Lemma 1. Deviating at stage s only by choosing action N at that stage yields payoff

$$V_L(\hat{\sigma},k) = \Delta k + \phi p\delta V_F + (1-\phi p)\delta V_L(\sigma,k) \quad .$$

Because, as shown, in the postulated mixed-strategy equilibrium the Follower's payoff $V_F = 0$, $V_L(\sigma,k) \geq V_L(\hat{\sigma},k)$ holds as long as $V_L(\sigma,k) \geq \Delta k / (1-\delta(1-\phi q))$, which, by using the result of Lemma 1, is equivalent to

$$\frac{c}{\Delta} \leq \frac{\delta(\phi q + (1-\phi)p)}{1-\delta(1-\phi q)} \quad . \tag{47}$$

This inequality can be shown to hold for all $\phi \in [0,1]$ that solve equation (11). Because equation (11) applies to $x_2 \leq c/\Delta \leq x_1$, then in this domain the Leader does not have a profitable deviation from $\sigma_L(k) = 1, \forall k$.

Part (iii). If $\sigma_F(k) = 0, \forall k$, then the situation is isomorphic to that of the FP protection environment and, as established earlier, it is indeed optimal for the Leader to invest whenever $c/\Delta \leq x_0$. Given $\sigma_L(k) = 1, \forall k$, it follows from the proof of part (ii) that the Follower does not have a profitable one-stage deviation when $x_1 < c/\Delta$.

Part (iv). If the firms play according to the strategy profile $\sigma_L(k) = \sigma_F(k) = 0, \forall k$, then the payoffs are given by $V_L(\sigma,1) = \frac{\Delta}{1-\delta}$ and $V_F(\sigma) = 0$. Suppose that the Leader considers the strategy $\hat{\sigma}_L(k)$ such that $\hat{\sigma}_L(1) = 1$ and $\hat{\sigma}_L(k) = \sigma_L(k) = 0, \forall k > 1$ (i.e., the Leader deviates by investing in state $k = 1$ only). Then the Leader's expected payoff can be written as

$$V_L(\hat{\sigma}_L, \sigma_F, 1) = V_L(\sigma_L, \sigma_F, 1) - c + p\delta \frac{\Delta}{1-\delta} \quad . \tag{48}$$

Thus, $V_L(\hat{\sigma}_L, \sigma_F, 1) - V_L(\sigma_L, \sigma_F, 1) > 0$ holds i.f.f. $\Delta p \delta - c(1-\delta) > 0$, that is i.f.f. $c/\Delta < x_0$. We conclude that the Leader has no profitable one-state deviation in this case. Now, for the Follower, consider the strategy $\hat{\sigma}_F(k)$ such that $\hat{\sigma}_F(1) = 1$ and $\hat{\sigma}_F(k) = \sigma_F(k) = 0, \forall k > 1$ (i.e., the Follower deviates by investing in state $k = 1$ only). Then its expected payoff is given by

$$V_F(\sigma_L, \hat{\sigma}_F) = V_F(\sigma_L, \sigma_F) - c + p\delta \frac{\Delta}{1-\delta} \tag{49}$$

and again we find that $V_F(\sigma_L, \hat{\sigma}_F) - V_F(\sigma_L, \sigma_F) > 0 \iff c/\Delta < x_0$.

Proof of Proposition 2

Part (i). We will show that for each firm it is optimal not to invest, given that its rival does not invest. The winner of the investment game would obtain a payoff equal to $\Delta/(1-\delta)$, so that the payoff from investing in the initial game while the other firm does not invest is

$$V_0 = -c_0 + p\delta \frac{\Delta}{1-\delta} + (1-p)\delta V_0 \quad \Rightarrow \quad V_0 = \frac{1}{(1-(1-p)\delta)} \left(\frac{p\delta\Delta}{1-\delta} - c_0 \right). \tag{50}$$

Therefore such a firm will invest i.f.f.

$$\frac{p\delta\Delta}{1-\delta} - c_0 \geq 0 \quad \Leftrightarrow \quad \frac{p\delta}{1-\delta} \geq \frac{c_0}{\Delta} \geq \frac{c}{\Delta}.$$

Because here $x_0 < c/\Delta$, the best response of such firm is not to invest.

Part (ii). We will show that no firm can deviate profitably by switching to $\sigma_0 = 1$. Because $c/\Delta \leq x_0$ by assumption, the payoff of the winner of the investment game is given by the V_M of equation (1) (with $k=1$). Consider the payoff to the firm from playing $\sigma_0 = 1$ given that its rival plays $\sigma_0 = 0$. This satisfies

$$V_0 = -c_0 + p\delta V_M + (1-p)\delta V_0 \quad . \tag{51}$$

Therefore such a firm will find it profitable i.f.f.

$$V_0 = \frac{p\delta V_M - c_0}{1-(1-p)\delta} \geq 0 \quad \Leftrightarrow \quad \frac{p\delta}{1-\delta} \left(\frac{1-\delta+p\delta}{1-\delta} - \frac{c}{\Delta} \right) \geq \frac{c_0}{\Delta} \quad . \tag{52}$$

Part (iii). Consider the situation in which both firms invest with probability one. Then each firm's value function is given by

$$V_0 = -c_0 + q\delta V_M + (1-2q)\delta V_0 \quad . \tag{53}$$

Because the firm that does not innovate obtains a zero payoff, both firms invest in equilibrium if

$$V_0 = \frac{q\delta V_M - c_0}{1-(1-2q)\delta} \geq 0 \quad \Leftrightarrow \quad \frac{q\delta}{1-\delta} \left(\frac{1-\delta+p\delta}{1-\delta} - \frac{c}{\Delta} \right) \geq \frac{c_0}{\Delta} \quad . \tag{54}$$

Part (iv). Because here we have $c_0/\Delta \leq H_1(c/\Delta)$, we know from (ii) that in the absence of competition each firm will find it profitable to invest. On the other hand, since $c_0/\Delta \geq H_2(c/\Delta)$, we know from (iii) that if its rival is investing a firm will find it profitable not to invest. This implies that there exist two pure-strategy Nash equilibria in this domain, which require the two firms to behave asymmetrically. But there also exist a pair of mixed strategies which, because of their symmetry, may be more appealing. To compute the symmetric mixed-strategy equilibrium, suppose that firm 2 randomizes between investing and not with probability σ_0. Then the payoff of firm 1, conditional on investing or not investing, respectively, satisfies the following recursive equations:

$$V_0^1 = \sigma_0 \left(q\delta V_M + (1-2q)\delta V_0^1 \right) + (1-\sigma_0) \left(p\delta V_M + (1-p)\delta V_0^1 \right) - c_0 \tag{55}$$

$$V_0^1 = \sigma_0 (1-p)\delta V_0^1 + (1-\sigma_0)\delta V_0^1 \quad . \tag{56}$$

From (56) it follows that $V_0^1 = 0$. In a non-degenerate mixed-strategy equilibrium each firm is indifferent between its two (pure) strategies. Hence, the second firm's equilibrium mixing probability must satisfy

$$\sigma_0 q\delta V_M + (1-\sigma_0)p\delta V_M - c_0 = 0 \qquad \Leftrightarrow \qquad \sigma_0 = \frac{p\delta V_M - c_0}{(p-q)\delta V_M}.$$

Proof of Proposition 4

If both firms invest with probability one in the investment game, then the value function of each firm is given by

$$V_0 = -c_0 + q\delta V_1 + (1-2q)\delta V_0 \qquad \Leftrightarrow \qquad V_0 = \frac{q\delta V_1 - c_0}{1-(1-2q)\delta}.$$

On the other hand, if only one firm invests, then its value function is given by

$$V_0^I = -c_0 + p\delta V_1 + (1-p)\delta V_0^I \qquad \Leftrightarrow \qquad V_0^I = \frac{p\delta V_1 - c_0}{1-(1-p)\delta}$$

and the value function of the firm that does not invest (V_0^N) is equal to zero. Therefore, both firms invest in equilibrium i.f.f. $V_0 \geq V_0^N$, that is, $\frac{q\delta V_1 - c_0}{1-(1-2q)\delta} \geq 0$. The last expression can be written as $q\delta V_1 \geq c_0$. By using equation (9), which implies that $V_1 = V_L(\sigma,1) = c/(q\delta)$, we can write this last condition simply as $c \geq c_0$. By assumption we are limiting consideration to the case $c \leq c_0$; therefore, firms will invest with probability one only when $c = c_0$. On the other hand, none of the firms invests in equilibrium if $V_0^I < 0$, that is, $\frac{p\delta V_1 - c_0}{1-(1-p)\delta} < 0$. The last expression is equivalent to $p\delta V_1 < c_0$, or

$$H_5\left(\frac{c}{\Delta}\right) \equiv \frac{p}{q}\frac{c}{\Delta} < \frac{c_0}{\Delta}.$$

Note that this implies that $H_5(x)$ is a linear function with $H_5(x_1) = H_1(x_1)$ and $H_5(x_2) = H_3(x_2)$ (see Figure 5). Finally, if $c/\Delta < c_0/\Delta \leq H_5(c/\Delta)$ then both firms must randomize in a symmetric equilibrium. In such a symmetric mixed-strategy equilibrium, if the second firm invests with probability σ_0, then for the first firm to be indifferent between investing and not we must have

$$\sigma_0 q\delta V_1 + (1-\sigma_0) p\delta V_1 - c_0 = 0 \qquad \Leftrightarrow \qquad \sigma_0 = \frac{p\delta V_1 - c_0}{(p-q)\delta V_1}.$$

Proof of Proposition 5

Part (i). If both firms invest with probability one in the investment game, then the value function of each firm is given by

$$V_0 = -c_0 + q\delta V_L + q\delta V_F + (1-2q)\delta V_0 \qquad \Leftrightarrow \qquad V_0 = \frac{q\delta(V_L + V_F) - c_0}{1-(1-2q)\delta}. \tag{57}$$

On the other hand, if only one firm invests, then its value function is given by

$$V_0^I = -c_0 + p\delta V_L + (1-p)\delta V_0^I \qquad \Leftrightarrow \qquad V_0^I = \frac{p\delta V_L - c_0}{1-(1-p)\delta} \tag{58}$$

and the value function of the firm that does not invest is given by

$$V_0^N = p\delta V_F + (1-p)\delta V_0^N \qquad \Leftrightarrow \qquad V_0^N = \frac{p\delta V_F}{1-(1-p)\delta}. \tag{59}$$

Therefore, both firms invest in equilibrium i.f.f.

$$V_0 \geq V_0^N \qquad \Leftrightarrow \qquad \frac{q\delta(V_L + V_F) - c_0}{1-(1-2q)\delta} \geq \frac{p\delta V_F}{1-(1-p)\delta}.$$

By using the expressions for V_L and V_F derived earlier, the last expression can be rearranged to yield the claimed parametric domain.

Part (ii). Suppose that a firm faces a rival that does not invest. From (v) we know that such a firm will find it profitable to invest i.f.f.

$$V_0^I = \frac{p\delta V_L - c_0}{1-(1-p)\delta} \geq 0 \qquad \Leftrightarrow \qquad \frac{p\delta(\Delta - c)}{1-\delta} \geq c_0 \qquad \Leftrightarrow \qquad c_0/\Delta \leq H_3\left(c/\Delta\right). \tag{60}$$

Part (iii). The results in (i) and (ii) imply that in this case a firm that faces no rival will find it optimal to invest. On the other hand, if the rival is investing, then it is optimal not to invest. It is clear that the symmetric equilibrium must involve mixed strategies. To compute them, suppose that one of the firms invests in each period of the investment stage with probability $\sigma_0 \in [0,1]$. In equilibrium its rival must be indifferent between investing and not. In particular, we have

$$V_0^I = \sigma_0\left(-c_0 + q\delta V_L + q\delta V_F + (1-2q)\delta V_0^I\right) + (1-\sigma_0)\left(-c_0 + p\delta V_L + (1-p)\delta V_0^I\right)$$

$$\Leftrightarrow \quad V_0^I = \frac{(\sigma_0 q + (1-\sigma_0)p)\delta V_L + \sigma_0 q \delta V_F - c_0}{1-\delta(1-p)(1-\sigma_0 p)} \tag{61}$$

and

$$V_0^N = \sigma_0 \left(p\delta V_F + (1-p)\delta V_0^N \right) + (1-\sigma_0)\delta V_0^N \quad \Leftrightarrow \quad V_0^N = \frac{\sigma_0 p \delta V_F}{1-\delta(1-\sigma_0 p)} . \tag{62}$$

The equilibrium mixing probability must satisfy $V_0^I = V_0^N$, implying

$$\frac{(\sigma_0 q + (1-\sigma_0)p)\delta V_L + \sigma_0 q \delta V_F - c_0}{1-\delta(1-p)(1-\sigma_0 p)} = \frac{\sigma_0 p \delta V_F}{1-\delta(1-\sigma_0 p)} . \tag{63}$$

This defines a quadratic equation in σ_0 of the form $a \cdot \sigma_0{}^2 + b \cdot \sigma_0 + e = 0$, where

$$a \equiv -p\delta^2 (p-q)(V_L - V_F) < 0 \tag{64}$$

$$b \equiv \delta\left[(p-q)\left(V_L(3\delta - 1) - V_F(1+\delta)\right) - c_0 p \right] \tag{65}$$

$$e \equiv (1-\delta)(p\delta V_L - c_0) \geq 0 \tag{66}$$

and where V_F and $V_L \equiv V_L(\sigma, 1)$ are given by equations (4) and (9), respectively. The equilibrium mixing probability is the root of this equation that belongs to the unit interval.

Completion of the Proof of Proposition 6

Note that (21) is monotonically increasing in σ_0, achieving its maximum on $[0,1]$ at $\sigma_0 = 1$. Thus, a sufficient condition for $\Pi^{FP} > \Pi^{RE}$ in this case is

$$\frac{q\delta V_M - c_0}{1-(1-2q)\delta} > \frac{p\delta V_F}{1-\delta(1-p)} \quad \Rightarrow \quad (q\delta V_M - c_0)(1-\delta(1-p)) > (p\delta V_F)(1-(1-2q)\delta) \tag{67}$$

provided that c_0 is such that we still are in region *D*, that is, $\frac{c_0}{\Delta} \leq \frac{p\delta}{1-\delta}\left(1 - \frac{c}{\Delta}\right) \equiv H_3(c/\Delta)$.

The LHS of the inequality in (67) is decreasing in c_0, so take the upper value $\overline{c}_0 = \frac{p\delta\Delta}{1-\delta}\left(1 - \frac{c}{\Delta}\right)$.

Recalling the expressions for V_M and V_F (equation (1) and Lemma 1), and evaluating them at $\overline{c}_0$, the inequality of interest reduces to

$$\frac{\delta}{(1-\delta)^2}\left[(q-p)(\Delta - c)(1-\delta) + qp\Delta\right](1-\delta(1-p)) > p\delta\left[\frac{-c(1-\delta(1-q)) + q\delta\Delta}{(1-\delta)[1-\delta(1-q)]}\right](1-(1-2q)\delta).$$

Note that the LHS is increasing in c and the RHS is decreasing in c. Hence, evaluate both at the lower bound $c = 0$, so that the resulting sufficient condition simplifies to

$$(1-\delta(1-p))(1-\delta(1-q)) > \frac{(2-p)\delta}{(1+\delta-p)}(1-\delta)(1-\delta(1-2q)) .$$

We can now verify that the inequality is always satisfied because, given that $p \in (0,1)$ and $\delta \in (0,1)$, we have $(2-p)\delta < (1+\delta-p)$ and

$$\big(1-\delta(1-p)\big)\big(1-\delta(1-q)\big) > \big(1-\delta\big)\big(1-\delta(1-2q)\big) \quad \Leftrightarrow \quad (1-\delta)(p-q)+\delta pq > 0 .$$

Derivation of the Function W_ϕ

Suppose that the Leader invests in all periods, and the Follower invests with probability ϕ in each period. Let $W_\phi(k)$ denote the expected total surplus at stage k. Then we have

$$W_\phi(k) = \Delta k - c(1+\phi) + \phi\Big[2q\delta W_\phi(k+1) + (1-2q)\delta W_\phi(k)\Big] + (1-\phi)\Big[p\delta W_\phi(k+1) + (1-p)\delta W_\phi(k)\Big].$$

This can be written as

$$W_\phi(k) = \alpha + \beta k + \gamma W_\phi(k+1) \qquad k = 1,2,...$$

where the parameters α, β, and γ are defined as

$$\alpha \equiv \frac{-c(1+\phi)}{\big(1-\phi\delta(1-2q)-(1-\phi)(1-p)\delta\big)}$$

$$\beta \equiv \frac{\Delta}{\big(1-\phi\delta(1-2q)-(1-\phi)(1-p)\delta\big)}$$

$$\gamma \equiv \frac{\phi 2q\delta + (1-\phi)p\delta}{\big(1-\phi\delta(1-2q)-(1-\phi)(1-p)\delta\big)} < 1 .$$

The general solution to the value of the Leader can be written as

$$W_\phi(k) = \frac{\alpha}{1-\gamma} + \frac{\beta k}{1-\gamma} + \frac{\beta\gamma}{(1-\gamma)^2} .$$

Using the definitions of the parameters α, β, and γ given above, and simplifying for the case $k = 1$, yields

$$W_\phi(1) = \frac{\Delta - c(1+\phi)}{1-\delta} + \frac{\Delta(\phi 2q\delta + (1-\phi)p\delta)}{(1-\delta)^2} \equiv W_\phi .$$