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## Public and Private Expenditures on Health in a Growth Model

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**ABSTRACT.** This paper introduces endogenous longevity risk in an otherwise standard overlapping generations model with capital. In the model, an agent may increase the length of her old age by incurring investments in her own health funded from her wage income. Such private health investments are more “productive” if accompanied by complementary tax-financed public health programs. The public input in private longevity is socially desirable; yet its presence may expose the economy to aggregate chaotic fluctuations.

**Keywords:** longevity, public health, chaos

**JEL Classification:** E10, J10, O10, O40

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## 1. INTRODUCTION

According to the World Health Organization, the life expectancy at birth in 2003 for Swaziland was 35 years; the same for Japan was 82 years.<sup>1</sup> In 2002, the life expectancy at age *sixty* in Sierra Leone was 8 years; that in Japan was nearly 20 years.<sup>2</sup> These stupendous gaps in longevity between countries present a challenge to epidemiologists and economists. In 2000, the World Health Organization (WHO, 2000) released a major report in which it argued that “the differing degrees of efficiency with which health systems organize and finance themselves, and react to the needs of their populations explain much of the widening gap in death rates [...] between countries...” In effect, the report argued that longevity differences between countries had a lot to do with the economics of health systems and much less about medicine (see Navarro, 2000).<sup>3</sup>

Health systems have two components, a public and a (often ignored) private one. As classified in the World Development Report (1993), public health programs “work in three ways: they deliver specific *health services to populations* (for example, immunizations), they promote healthy *behavior*, and they promote healthy *environments*”.<sup>4</sup> That same report also eloquently argues that “what people do with their lives [...] affects their health far more than anything governments can do”.<sup>5,6</sup> This paper presents a simple framework to

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<sup>1</sup>The data are taken from <http://www.who.int/entity/whr/2005/annex/annex1.xls>. The gap between the highest ten life expectancy countries and the lowest ten is about 40 years.

<sup>2</sup>In 2003, only 4% of the population in Swaziland were of age 60 or higher compared to 25% in Japan.

<sup>3</sup>A quick glance at Figure 1 suggests support for this line of thinking. If one plots cross country life expectancy at birth against per capita total expenditures on health (in US \$), the plot has a clear upward slope. Similarly, if plots life expectancy at birth against per capita expenditures on health (in US \$), a similar picture emerges. The same WHO report presents the message as follows: “If Sweden enjoys better health than Uganda — life expectancy is almost exactly twice as long — it is in large part because it spends exactly 35 times as much per capita in its health systems”.

<sup>4</sup>The World Development Report (1993) documents that nearly 60% of world spending on health (roughly 8% of the world’s income) comes from governments. In developing countries, government spending from general tax revenues amounts to roughly half of the 2-7% of GNP allocated to public health.

<sup>5</sup>Private efforts to improve health and longevity may include annual diagnostic health screening, opportunity cost of regular exercise, taking vitamins, nutrients, and other supplements, eating organically grown food, health benefits from quitting unhealthy habits such as smoking, etc. In developing countries, these may also include out-of-pocket expenses for essential medication or clinical services provided by an (often) unregulated private health sector. As Fabricant, Kamara, and Mills (1999) document, such expenses range from 2%-5% of household income among the upper income groups in India, Vietnam, Bangladesh and Nepal to as high as 10-25% for the poorer quintiles in Azerbaijan and other countries.

<sup>6</sup>Similar sentiments (“individual vs. social responsibility for health”) are echoed in the United States

study this dynamic complementarity between public health programs and private efforts to improve health and longevity.

To that end, the paper presents an otherwise standard overlapping generations model with production (a la Diamond, 1965) modified to include longevity risk. In the model economy, young agents work for a competitive wage and all survive to old age (the second period). However, any agent is alive only for a fraction of the second period of her life. Agents may “self-protect” by incurring personal expenses that reduce this longevity risk; this makes longevity a choice variable of the agent. All agents care only about their old age consumption while alive. Hence the allocation problem for young agents is to decide how much to invest in one’s longevity and how much to save for the future. Our assumptions imply that longevity and youthful savings are both normal goods. We assume that tax-financed public health programs exist whose main purpose is to complement private investments to improve health and longevity. Specifically, we assume that the marginal impact on longevity of a marginal increase in private health investment rises with more public expenditure on health.

At the margin, the principal tension is as follows: longevity is desirable and necessary to enjoy the returns from past physical investments and yet that longer life is costly to acquire (in terms of lower physical investment and hence lost future utility). In general equilibrium, lower investment in capital translates into lower future wage income (and hence smaller tax revenues). This in turn leads to reduced public health expenditures which diminishes the public health complementarity with private health investments making longevity harder to achieve.

Our main results are as follows. Under some mild assumptions on the utility and the longevity function, we can prove the existence of a unique non-trivial steady state which is locally stable. We are able to demonstrate under some parametric specifications that, in a stationary lifetime welfare sense, the tax-financed public input into private longevity may be socially desirable. In other words, from within the model itself, we can justify the presence of the public input into the private longevity production function by appealing to

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starting with the 1979 Surgeon General’s Report on Health Promotion and Disease Prevention.

a welfare criterion.<sup>7</sup>

We go on to study the dynamics of the law of motion for the capital-labor ratio. Under the assumption of convexity of the longevity function, we can show that the time-map of the capital-labor ratio is unimodal and non-monotonic. By means of a reasonable numerical example, we are able to verify that our time map satisfies a set of sufficient conditions for topological chaos outlined in Mitra (2001).

It is important to point out that the nonmonotonicity of the time map is entirely a consequence of the *interaction* of private and public investments in health. This fact has an important implication: tax-financed public expenditures aimed at raising people's longevity may introduce endogenous volatility in an economy where such fluctuations are *impossible* in their absence. In conjunction with the welfare result alluded to earlier, the previous message becomes somewhat stronger: for now one can argue (at least in a stationary welfare sense) that the public input in private longevity is socially desirable even though its presence may expose the agents to a "bewildering" (Azariadis, 1993) set of dynamic chaotic equilibria.

Our paper is in line with a new yet burgeoning literature that incorporates mortality concerns in growth models. The seminal paper in this literature is Chakraborty (2004), an exploration of a new connection between pervasive ill-health and economic growth. Chakraborty introduces endogenous mortality in an otherwise standard overlapping generations model with production; in particular, the probability with which a young agent survives on to old age depends on public health expenditures which are in turn funded by income taxes on labor income. Chakraborty shows that if the starting capital stock is sufficiently low, the country is more likely to be permanently stuck in a poverty trap. Finlay (2005) and Haaparanta and Puhakka (2004) generalize the environment and broaden the scope of the results in Chakraborty (2004).<sup>8</sup> Aisa and Pueyo (2004) produce a continuous time overlapping generations model where tax-financed public revenues can provide a pro-

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<sup>7</sup>This sets our work apart from much of the literature where the public input is never rationalized internally.

<sup>8</sup>Our analysis and its focus differs from that in Chakraborty (2004) in several ways. First, we allow longevity to be chosen optimally by private agents themselves. Second, we focus on old-age longevity and not on young-age mortality.

duction externality or improve social health (average longevity). In their setup, a longer life expectancy results in an increase in savings (as in Chakraborty, 2004); it also increases the available workforce and may increase growth and even average longevity.

The novelty of our work relative to these papers lies in our exclusive focus on the interaction between the public and private component of health systems and its consequences for aggregate volatility. Indeed, our result on the existence of complex dynamics (chaotic equilibria) is of some independent interest. There is a vast literature studying the possibility of complex dynamics in general equilibrium growth models, especially in overlapping generations models.<sup>9</sup> As is well-known in this literature, complex dynamics can emerge under assumptions of limited market participation, imperfect competition, multiple sectors etc.; additionally, as discussed in Azariadis (1993), a sufficiently strong income effect which, in turn, produces “backward-bending” savings functions can also produce complex dynamics in overlapping generations models. In our model, chaotic dynamics emerge in a relatively standard economy; indeed, in our setup, optimal savings is independent of its return and sans the public input in private longevity, our model economy would not produce any endogenous fluctuations of any kind. Thus, ours is not yet another paper demonstrating the presence of complex dynamics in neoclassical growth models. Our novelty lies in its emphasis of the role of endogenous longevity and the interaction between the two components of health systems in generating these complex dynamics.

The rest of the paper is organized as follows. In Section 2, we present the details of the specification of the model economy and derive the solution to the agents’ problem. Section 3 presents our main results concerning the general equilibrium law of motion for the capital-labor ratio while Section 4 concludes. Proofs of major results are in the appendix.

## 2. THE MODEL

**2.1. Environment.** We consider an economy consisting of an infinite sequence of (potentially) two-period lived overlapping generations, and an infinitely-lived government. Let  $t = 1, 2, \dots$  index time. At each date  $t > 1$ , a new generation comprised of  $N$  identical members appears. Each agent is endowed with one unit of labor when young and is retired when

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<sup>9</sup>See Lorenz (1993) and Mitra, Majumdar, and Nishimura (2000) for surveys of this literature.

old. In addition, the initial old agents are endowed with  $k_1 > 0$  units of capital.

There is a single final good produced using a standard neoclassical production function  $F(K_t, L_t)$  where  $K_t$  denotes the capital input and  $L_t$  denotes the labor input at  $t$ . The final good can either be consumed in the period it is produced, or it can be stored to yield capital at the very beginning of the following period. Capital is assumed to depreciate 100% between periods. Let  $k_t \equiv K_t/L_t$  denote the capital-labor ratio (capital per young agent). Then, output per young agent at time  $t$  may be expressed as  $f(k_t)$  where  $f(k_t) \equiv F(K_t/L_t, 1)$  is the intensive production function. We assume that  $f(0) = 0$ ,  $f' > 0 > f''$ , and that the usual Inada conditions hold. For reasons of analytical tractability, we assume the Cobb-Douglas form:  $f(k) = Ak^\alpha$ ,  $\alpha \in (0, 1)$ ,  $A > 0$ .

Let  $c_{2,t+1}$  denote the consumption of the final good by a representative old agent born at  $t$ . All such agents have preferences representable by the utility function  $U(c_{2,t+1}) = E_t u(c_{2,t+1})$  where  $u : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$  is twice-continuously differentiable, strictly increasing, and strictly concave in its arguments.<sup>10</sup> We will specialize to the commonly used form  $u(c) = c^{1-\sigma}/(1-\sigma)$ , with  $0 < \sigma < 1$ .

All young agents survive to old age (the second period). However, any agent is alive only for a fraction  $\theta \in (0, 1]$  of the second period of her life. In this sense,  $1+\theta$  captures the notion of longevity in our model.<sup>11</sup> Agents may influence their longevity (“self protect”, Shavell, 1979) by undertaking private investments ( $x$ ) in their own health.<sup>12</sup> These investments have to be privately funded from wage income (see below). Following Dranove (1998), one could interpret these expenses as “preventive” medicine to prevent “premature death”.

We assume that  $\theta$  is strictly increasing and strictly concave in  $x$ . For much of what we present below, we assume a simple constant-elasticity functional form for  $\theta$  :

$$\theta(x; \eta) = b\eta x^{b\eta}; \quad b, \eta > 0, \quad \theta \in (0, 1) \tag{1}$$

where  $b$  is a parameter. Strict concavity of  $\theta(x)$  would then imply the restriction  $b\eta \in (0, 1)$ .

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<sup>10</sup>See Hall and Jones (2004) for a discussion on why  $u > 0$  has to hold and for anomalies with utility functions “multiplied with life expectancy”.

<sup>11</sup>It is in this sense that ours is a model of longevity in old age as opposed to mortality during youth. Our formulation has the added advantage in that it allows us to abstract away from issues relating to unintended bequests of the dead. These concerns are addressed in Aisa and Pueyo (2004).

<sup>12</sup>For a description of these investments, see Footnote 5.

Following common practice in the literature, one can also interpret  $\theta(x)$  as a “longevity production function”. Some empirical justification for the constant-elasticity functional form for  $\theta$  is indirectly provided in Hall and Jones (2004).

The government imposes a distortionary tax  $\tau$  on agents’ incomes. If  $w_t$  is the wage rate, then the government raises revenue of amount  $\mathcal{V}(w_t; \tau) = \tau w_t$ . We assume that  $\eta$  is a function of  $\mathcal{V}(w_t; \tau)$  :

$$\eta(\mathcal{V}(w_t; \tau)) = 1 + \mathcal{V}(w_t; \tau); \quad (2)$$

henceforth, following Chakraborty (2004), we will label  $\mathcal{V}(w_t; \tau)$  the public investment in health/longevity. This formulation nicely captures the essence of the interaction between the public and the private components to health systems. If private efforts to improve health are not forthcoming, then a superior public health system does not help much, and vice versa.

Notice from (1) that

$$\frac{x\theta'(x)}{\theta(x)} = b\eta = b\eta(\mathcal{V}(w_t; \tau)). \quad (3)$$

In other words, the elasticity of longevity with respect to private investment in longevity is itself influenced by public expenditures in the health sector. Additionally, the marginal impact on longevity of a marginal increase in private health investment is increased with more public expenditure on health. The better and larger the public health system, the larger is the impact of additional private investment on agents’ longevity. As will also be clear below, the formulation in (1)-(3) provides a lot of analytical tractability. In passing, note from (2) if  $\tau = 0$ ,  $\mathcal{V}(w_t; \tau) = 0$  and  $\eta(\cdot) = 1$ , and then  $\theta(x) = bx^b$  captures the *pure* effect of private investments in longevity.

This interactive formulation may be motivated roughly as follows. Imagine the public health care system in a country is in a total state of dysfunction and disarray. In that environment, regular health check-ups by inept physicians or good doctors using poor out-dated medical technology (e.g., in the case of faulty mammograms) may not reveal serious yet preventable maladies. Regular exercise may improve general health but may not be



enough to ward off post-operative infections in unsanitary hospital conditions. It may also be that the public health care system imposes no quality control on dispensed medicines and supplements and as such their consumption provides little longevity benefits. Finally, as argued in Chakraborty (2004), “public health expenditure in new medical facilities, sanitation improvements, disease control and inoculation programs augments private health capital by reducing the economy-wide risk of contacting fatal diseases.” Such complementarity is also stressed by Dow, Philipson, and Sala-i-Martin (1999) and Liu and Neilson (2005).

**2.2. Trade and markets.** Young agents supply their labor endowment inelastically in competitive labor markets, earning a wage income of  $w_t$  at time  $t$ , where

$$w_t \equiv w(k_t) = f(k_t) - k_t f'(k_t). \quad (4)$$

Capital is traded in competitive capital markets, and earns a gross real return of  $R_{t+1}$  between  $t$  and  $t + 1$ , where

$$R_{t+1} = f'(k_{t+1}). \quad (5)$$

For  $f(k) = Ak^\alpha$ , it follows that  $w(k) = (1 - \alpha)Ak^\alpha$  and  $R(k) = A\alpha k^{\alpha-1}$ . For future reference, note that  $w'(k) > 0$  and  $w''(k) < 0$ . Each young agent born at date  $t \geq 1$  maximizes

$$U(c_{2,t+1}) = \theta(x_t) u(c_{2,t+1})$$

subject to

$$x_t + s_t = (1 - \tau) w_t \quad (6)$$

and

$$c_{2,t+1} = R_{t+1} s_t, \quad (7)$$

where  $s_t$  denote savings of a young agent. Return to savings are made available to old agents right at the very beginning of the second period of their life.<sup>13</sup> The agent takes

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<sup>13</sup>This allows us to abstract away from issues relating to the eventual fate of the savings of those who die before receiving any return on them; see Chakraborty and Das (forthcoming) for details.

$w_t, R_{t+1}, \eta(\cdot)$ , and  $\tau$  as given. Then the agent's problem is to choose  $x_t$  by solving

$$\max_{x_t} \theta(x_t) \cdot \frac{[R_{t+1} \{(1-\tau)w_t - x_t\}]^{1-\sigma}}{1-\sigma}.$$

Below we will focus on interior solutions to this problem.

**Lemma 1.** *If*

$$0 < b\eta < \sigma, \tag{A.0}$$

*then  $\theta(x)u(c(x))$  is strictly concave in  $x$  and the agent's problem has a unique solution in the interior.*<sup>14</sup>

The first order condition to the agent's problem is given by<sup>15</sup>

$$\theta'(x_t)u(c_{t+1}) = \theta(x_t)u'(c_{t+1}) \tag{8}$$

which easily reduces to

$$\frac{\theta'(x_t)}{\theta(x_t)} \{(1-\tau)w_t - x_t\} = (1-\sigma).$$

The intuition for (8) is as follows. In our environment, an agent can influence future utility by one of two possible means: she may invest an additional unit in her own longevity which raises her longevity by  $\theta'(x_t)$  but reduces her total utility from consumption; alternatively, she can invest an additional unit in physical capital which changes her future utility by  $u'(c_{t+1})$  while reducing her longevity. These two options should appear equivalent to an agent. Using (3), the optimal private health investment can be computed to be

$$x_t = \frac{(1-\tau)b\eta(\cdot)}{1-\sigma + b\eta(\cdot)}w_t, \tag{9}$$

and the optimal savings function is given by

$$s_t = \frac{(1-\tau)(1-\sigma)}{1-\sigma + b\eta(\cdot)}w_t. \tag{10}$$

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<sup>14</sup>To see that the condition in Lemma 1 may not be that restrictive, consider an example with  $\eta = c \frac{\tau w(k)}{1+\tau w(k)}$ . Here  $\eta \in (0, c) \forall k > 0$  and hence the condition would reduce to  $0 < bc < \sigma$ , a simple condition on parameters.

<sup>15</sup>For a general utility function  $u$ , the first order condition is given by  $-\theta'(x^*)u(c^*) + \theta(x^*)u'(c^*)R = 0$ . When  $u$  has the CRRA form, this reduces to (8).

Recall that  $\eta$  is a function of  $\mathcal{V}(w_t; \tau)$ .

A nice implication of our assumptions on preferences etc. is that agents' optimal saving is realistically independent of the rate of return. Also from (9)-(10) it follows that *ceteris paribus*, longevity and youthful savings are both normal goods. As agents' incomes rise, they increase their saving in the form of capital and their own longevity. Furthermore, an investigation of (9)-(10) reveals that *ceteris paribus*, private investments in health rises and optimal saving in capital falls as public expenditure on health rises. An intuition for this is as follows. Recall private investments in health are more "productive" (yield more longevity) when public expenditure on health rises. When  $\sigma < 1$ , agents at the margin value longevity over consumption; as such they reduce savings in the physical asset and invest in their own health.

### 3. GENERAL EQUILIBRIUM

**3.1. Steady states.** In general equilibrium, using  $s_t = k_{t+1}$  and (4), we can rewrite (10) as

$$k_{t+1} = \frac{(1-\tau)(1-\sigma)}{1-\sigma+b\eta(\cdot)} w(k_t) \equiv g(k_t). \quad (11)$$

Eq. (11) is the equilibrium law of motion for the capital-labor ratio. Given a  $k_1$  and  $\tau$ , sequences  $\{k_t\}_{t=2}^{\infty}$  that satisfy (11) and (A.0) constitute valid dynamic competitive equilibria. Once this solution sequence is known, it is possible to compute  $\{x_t\}_{t=1}^{\infty}$ ,  $\{\theta_t\}_{t=1}^{\infty}$ ,  $\{w_t\}_{t=1}^{\infty}$ , etc.

For future reference, note that

$$\frac{dk_{t+1}}{dk_t} = \frac{(1-\tau)(1-\sigma)w'(k_t)}{1-\sigma+b\eta(\tau w(k_t))} \left[ 1 - \frac{b\tau\eta'(\tau w(k_t))w(k_t)}{1-\sigma+b\eta(\tau w(k_t))} \right]. \quad (12)$$

Note if  $\eta'(\cdot) < 0$  obtains, then as is evident from (12), the law of motion in (11) represents an increasing relationship between the capital-labor ratios at two successive dates. In this case, there is no possibility for the time map  $g(\cdot)$  to exhibit complex dynamics of any kind.

We begin by studying steady state equilibria, time-invariant solutions to (11). At a steady state,  $k_{t+1} = k_t = k^*$ . Then using  $w(k) = (1-\alpha)Ak^\alpha$ , equation (11) reduces to

$$k^* = A(1 - \tau)(1 - \sigma)(1 - \alpha) \frac{(k^*)^\alpha}{1 - \sigma + b\eta(A\tau(1 - \alpha)(k^*)^\alpha)}. \quad (13)$$

It is immediately clear that  $k^* = 0$  is a solution to (13). Define  $H(k) \equiv A(1 - \tau)(1 - \sigma)(1 - \alpha)(k^*)^{\alpha-1} - b\eta(A\tau(1 - \alpha)(k^*)^\alpha)$ . To find the non-trivial steady states, we rewrite (13) as  $H(k) = 1 - \sigma$ .

**Proposition 1.** (i) Suppose the function  $\eta(\mathcal{V}(w_t; \tau))$  satisfies

$$\eta(\cdot) \in [0, \infty) \quad (A.1)$$

$$\eta'(0) < \infty \text{ and } \eta'(\infty) < \infty \quad (A.2)$$

then a non-trivial steady state solution to (11) exists; (ii) An additional sufficient condition for the existence of a unique non-trivial steady state ( $k^*$ ) is  $\eta'(\cdot) > 0$ .

Existence of steady states follows from the mild regularity assumptions made on the public input function; uniqueness follows from assuming that the elasticity of longevity with respect to private investment in health is *increasing* in the size of the public health program. Henceforth, we will assume that

$$\eta'(\cdot) > 0 \quad (A.3)$$

holds. Under (A.3), it follows there is a unique non-trivial steady state  $k^*$  [no closed form expression for  $k^*$  is however possible to obtain]. For future reference, note that if  $\tau = 0$ , then  $\eta(0) = 1$  and (13) has a unique closed form solution given by

$$\tilde{k} = \left[ \frac{A(1 - \sigma)(1 - \alpha)}{(1 - \sigma) + b} \right]^{1/(1-\alpha)}. \quad (14)$$

**Lemma 2.** *The trivial steady state is locally unstable.*

Unlike Chakraborty (2004), there is no possibility of poverty traps here. We go on to study the equilibrium comparative static properties of  $k^*$ .

**Proposition 2.** a)

$$\frac{\partial k^*}{\partial \tau} < 0$$

b)

$$\left. \begin{array}{l} \frac{\partial \mathcal{V}(w_t; \tau)}{\partial \tau} \geq 0 \text{ if } \tau \leq 1 - \alpha \\ \frac{\partial \mathcal{V}(w_t; \tau)}{\partial \tau} < 0 \text{ if } \tau > 1 - \alpha \end{array} \right\}$$

As the tax rate rises, ceteris paribus private investment in health and capital investment both fall. This reduces future wages which further reduces capital investment. When it comes to tax revenue (public expenditures on health), there is a standard Laffer curve: when the tax rate is low (high) enough, a further rise in the tax rate raises (reduces) public expenditures on health.

Thus far we have assumed  $\tau$  to be exogenously given. It is possible to “endogenize”  $\tau$  in several ways: as discussed in Chakraborty (2004) or Finlay (2005),  $\tau$  may be computed either by appealing to a median voter theorem or by computing the socially optimal tax rate  $\tau^*$ . We follow the latter route. For our purposes, define  $\tau^*$  to be the tax rate that maximizes the stationary lifetime welfare of agents alive in the steady state. Notice that  $x^*(k^*(\tau)) = (1 - \tau)w(k^*(\tau)) - k^*(\tau)$ , and  $c^*(k^*(\tau)) = R(k^*(\tau))s(k^*(\tau)) = \alpha f(k^*(\tau))$  where  $k^*(\tau)$  is the steady state capital-labor ratio computed using (11). Then lifetime stationary welfare (indirect utility as a function of the tax rate) can be written as

$$W(\tau) \equiv [\theta(x^*(k^*(\tau))) \cdot u(c^*(k^*(\tau)))] ; \quad (15)$$

hence

$$\tau^* = \arg \max_{\tau} [\theta(x^*(k^*(\tau))) \cdot u(c^*(k^*(\tau)))]$$

We would like to be able to show that  $\tau^* \in (0, 1)$  but given the analytical messiness, we are only able to prove the following local result.

**Proposition 3.** *Define*

$$C_1 \equiv \alpha(1 - \sigma) + b, \quad C_2 \equiv \frac{(2\alpha - 1)(1 - \sigma) + \alpha b}{1 - \sigma} b\eta'(0) \left( \frac{(1 - \sigma)(1 - \alpha)}{1 - \sigma + b} \right)^{\frac{1}{1 - \alpha}},$$

$$\bar{A} \equiv \left( -\frac{C_1}{C_2} \right)^{1 - \alpha}, \quad \text{and } \bar{\alpha} \equiv \frac{1 - \sigma}{2(1 - \sigma) + b\eta'(0)}.$$

*Then, (1) if  $\alpha \geq \bar{\alpha}$ ,  $W'(0) < 0$ ; (2) if  $\alpha < \bar{\alpha}$ , and  $A > \bar{A}$ ,  $W'(0) < 0$ ; (3) if  $\alpha < \bar{\alpha}$ , and  $A < \bar{A}$ ,  $W'(0) > 0$ .*

Recall that  $\tau = 0 \Rightarrow \eta(k) = 1$  and hence  $\theta(x) = bx^b$  [no public input into the private longevity production function]. When  $W'(0) > 0$  obtains, it is socially desirable (welfare improving) to raise the tax rate locally around  $\tau = 0$ . Given our specification of the  $\theta(\cdot)$  function, the choice of a positive tax rate implies that the public input in the private longevity production function is desirable. That is, at least in a stationary welfare sense, under some of the parametric conditions specified in Proposition 3, one can partially justify government support of private efforts to increase longevity. Below we will show that such support may expose the economy to endogenous fluctuations.

We close with a numerical example where  $W$  is strictly concave in  $\tau$  and an interior socially optimal tax rate obtains.

**Example 1.** *Suppose  $\eta(\cdot) = 1 + 0.5 [\mathcal{V}(w; \tau)]^8$ ,  $f(k) = 20k^{0.6}$ ,  $\sigma = 0.9$ , and  $b = 0.1$ . Then  $\tau^* \approx 0.21$  and  $W(\tau^* \approx 0.21) > W(\tau = 0)$  as described in Figure 2.*

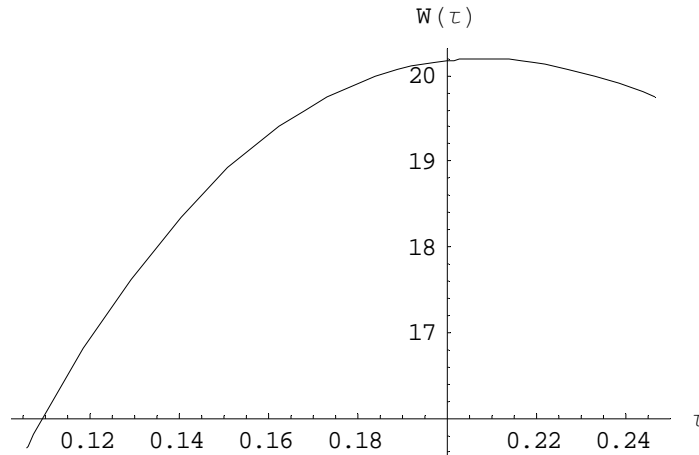


Figure 2: Optimal tax rate for Example 1

**3.2. Complex dynamics.** We are now in a position to piece together more information about the law of motion (11). From (12), it is clear that if  $\eta'(\cdot) < 0$  holds, then the law of motion is monotonic and no complex dynamics are possible. The next result outlines a further restriction on  $\eta(\cdot)$  which in conjunction with (A.0) and (A.3) would constitute a set of necessary conditions for the existence of equilibrium complex dynamics.

**Proposition 4.** *If along with (A.0) and (A.3),*

$$\eta''(\cdot) > 0 \tag{A.5}$$

*holds, then the law of motion in (11) is non-monotonic. Indeed, the time map  $g(\cdot)$  in (11) is unimodal.*

As is well-known, a necessary condition for the time map  $g(\cdot)$  to exhibit any kind of complex dynamics is that  $g$  be non-monotonic. We explore the possibility of periodic and chaotic behavior below.

Mitra (2001) offers a set of sufficient conditions for chaos in unimodal maps (like  $g$ ). Mitra (2001) focuses solely on dynamical systems  $(X, g)$ , where the state space  $X$  is an interval on the non-negative part of the real line. The map,  $g$ , is required to be a continuous

function from  $X$  to  $X$ , unimodal with a maximum at  $\hat{k}$  with  $g(\hat{k}) > \hat{k}$ , and the unique steady state ( $k^*$ ) must satisfy  $k^* > \hat{k}$ . For such maps, Mitra (2001) states the following theorem (his Proposition 2.3, pg. 142) which we restate for the sake of completeness.

**Theorem** (Mitra, 2001) Let  $(X, g)$  be a dynamical system. If  $g$  satisfies  $g^2(\hat{k}) < \hat{k}$  and  $g^3(\hat{k}) < k^*$ , then  $(X, g)$  exhibits topological chaos.

Below we verify by means of a numerical example that the time map  $g(\cdot)$  in (11) satisfies these conditions.

**Example 2.** Suppose the parametric specification is identical to that in Example 1. Set  $\tau = \tau^* = 0.21$  computed in Example 1. Then, it is possible to verify that assumptions (A.0) and (A.3) are verified, and the law of motion  $g(\cdot)$  has the configuration in Figure 3. Also,  $\hat{k} = 0.37$ ,  $k^* = 0.68$ ,  $g^2(\hat{k}) = 0.03$ , and  $g^3(\hat{k}) = 0.4$  implying the conditions in the theorem above are satisfied. Hence, for this parametric specification, the time map  $g(\cdot)$  in (11) exhibits topological chaos.

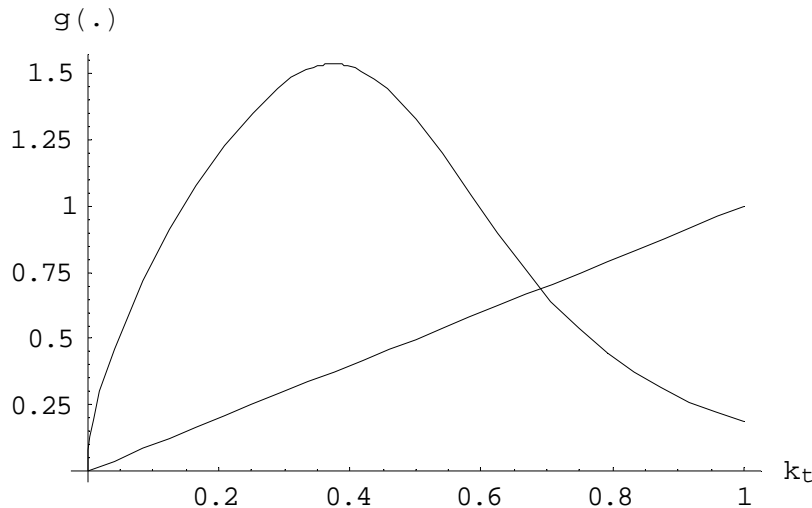


Figure 3: The time map for  $g(\cdot)$  in Example 2

Many similar examples are easy to generate.<sup>16</sup>

<sup>16</sup>A few words about the realism of the above example are in order. As has been argued (see Chakraborty, 2004)  $\alpha$  may easily be rationalized to be above 0.5 simply by broadening the concept of capital to include



It is important to point out that the nonmonotonicity of  $g(\cdot)$  [a necessary condition for existence of periodic and chaotic equilibria] is entirely a consequence of the *interaction* of private and public investments in health. To see this, note that in the absence of the public input, we can write  $\theta(x) = bx^b$  [with  $\tau = 0$ ] and hence (11) would be replaced by  $k_{t+1} = \frac{(1-\sigma)}{1-\sigma+b}w(k_t)$ . Since  $w'(\cdot) > 0$ , it is easy to see that this would not produce a non-monotonic time map. The message is clear: tax-financed public expenditures aimed towards raising people's longevity may introduce endogenous volatility in an economy where such fluctuations are impossible in their absence. In conjunction with the welfare result in Example 1 (which uses the same parametric specification as in Example 2) the previous message becomes somewhat stronger; for now one can argue at least in a stationary welfare sense, the public input in private longevity is desirable even though its presence may expose the agents to a “bewildering” (Azariadis, 1993) set of dynamic chaotic equilibria.

#### 4. CONCLUSION

Health systems comprise of two wings, a public and a private one. A major goal of the public wing (public health programs) is to promote healthy behavior. Of course, such public efforts to improve health, longevity, and well-being need private backing too. This paper presents a simple framework to study this dynamic complementarity between public health programs and private efforts to improve health and longevity. More specifically, it introduces endogenous longevity risk in an otherwise standard overlapping generations model with capital. In the model, an agent may privately invest in her own longevity by incurring expenses funded from her wage income. Such private health investments are more “productive” if accompanied by complementary tax-financed public health programs. We find that the public input in private longevity may be socially desirable; yet its presence may expose the economy to aggregate chaotic fluctuations otherwise not possible in the absence of the public input.

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human capital. The above parametric specification produces a range of  $\theta \in (0.4, 0.8)$ . This is in line with cross country WHO data suggesting that the range of ages people survive after age 60 is 7-18 years. The choice of  $\sigma$  follows standard practice in the mortality literature, e.g., Shepard and Zeckhauser (1984).

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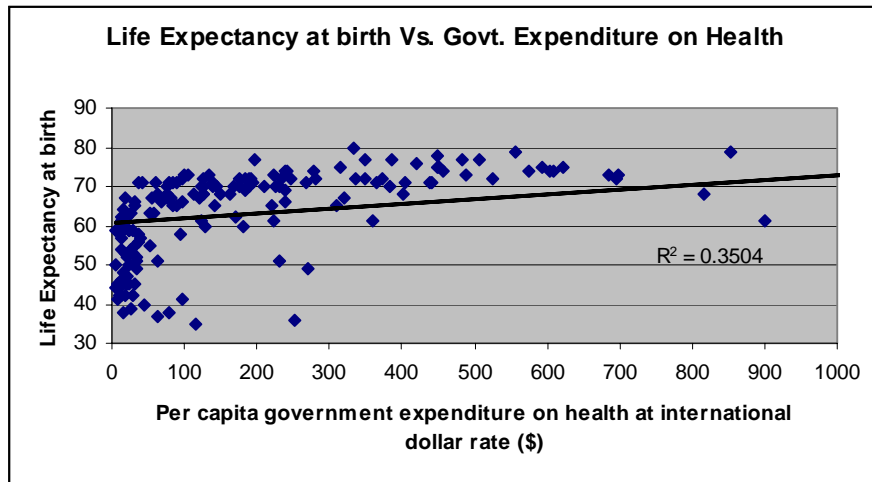
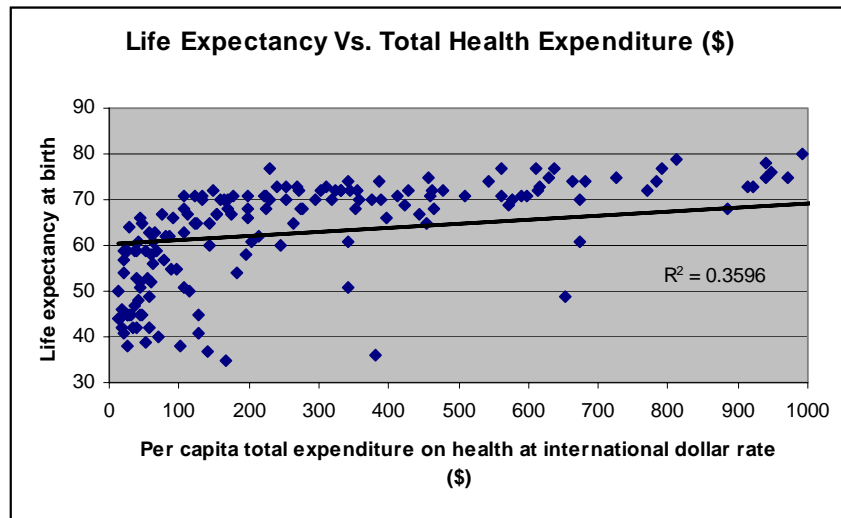


Figure 1

<sup>16</sup>The data is taken from <http://www.who.int/whr/annexes/en/>.

## A. PROOF OF LEMMA 1

A sufficient condition for  $\theta(x)u(c)$  to be concave is that its Hessian

$$\begin{vmatrix} \theta''(x)u(c) & \theta'(x)u'(c) \\ \theta'(x)u'(c) & \theta(x)u''(c) \end{vmatrix}$$

be negative definite, i.e., a)  $\theta''(x)u(c) < 0$ , and b)  $\theta''(x)u(c)\theta(x)u''(c) - (\theta'(x)u'(c))^2 > 0$ . The assumed concavity of  $\theta$  ensures (a). To ensure (b), we require

$$\theta''(x)u(c)\theta(x)u''(c) > (\theta'(x)u'(c))^2. \quad (16)$$

Using the specified utility function and longevity function, (16) reduces to

$$b^2\eta^2(b\eta - 1)x^{b\eta-2}b\eta x^{b\eta} \frac{c^{1-\sigma}}{1-\sigma} (-\sigma)c^{-\sigma-1} > b^4\eta^4 x^{2b\eta-2} c^{-2\sigma},$$

which further reduces to  $b\eta < \sigma$ .

## B. PROOF OF PROPOSITION 1

Solutions to  $H(k) = 1 - \sigma$  determine non-trivial steady state capital-labor ratios. It is easy to check that under (A.1)

$$\begin{aligned} H(0) &= \lim_{k \rightarrow 0} H(k) = \infty - b\eta(0) = \infty \\ H(\infty) &= \lim_{k \rightarrow \infty} H(k) = 0 - b\eta(\infty) < 0 \end{aligned}$$

holds. Since  $H(\cdot)$  is a continuous function, there exists for sure a  $k^*$  such that  $H(k^*) = 1 - \sigma$ . If  $H(\cdot)$  is monotonically decreasing, then there must exist one and only one non-trivial steady solution. Since

$$H'(k) = -A(1 - \tau)(1 - \sigma)(1 - \alpha)^2(k^*)^{\alpha-2} - A\alpha b\tau(1 - \alpha)(k^*)^{\alpha-1}\eta'(A\tau(1 - \alpha)(k^*)^\alpha),$$

it follows that if  $\eta'(\cdot) > 0$ , then  $H'(\cdot) < 0$ .

## C. PROOF OF LEMMA 2

Since  $w'(k) = \infty$  and both  $\eta(0)$  and  $\eta'(0)$  are finite (via assumption (A.1)-(A.2)), it follows from (12) that the trivial steady state is unstable.

## D. PROOF OF PROPOSITION 2

a) Straightforward differentiation of  $H(k) = 1 - \sigma$  reveals

$$\frac{\partial k^*}{\partial \tau} = -\frac{A(1-\sigma)(1-\alpha)(k^*)^{\alpha-1} + b\eta'(\cdot)A(1-\alpha)(k^*)^\alpha}{A(1-\tau)(1-\sigma)(1-\alpha)^2(k^*)^{\alpha-2} + A\tau\alpha(1-\alpha)b\eta'(\cdot)(k^*)^{\alpha-1}} \quad (17)$$

Using (A.4), the result follows.

b) It follows from the definition of  $\mathcal{V}(\cdot)$  that the sign of  $\frac{\partial \mathcal{V}(\cdot)}{\partial \tau}$  is the same as the sign of  $(k^* + \tau\alpha\frac{\partial k^*}{\partial \tau})$ . Using (17), one can verify that

$$\text{sign}\left(k^* + \tau\alpha\frac{\partial k^*}{\partial \tau}\right) = \text{sign}\frac{(1-\sigma)(1-\tau-\alpha)}{(1-\tau)(1-\alpha)(1-\sigma) + \tau\alpha b\eta'(\cdot)k^*}k^*$$

It follows that if  $\tau \leq 1 - \alpha$ , then  $\frac{\partial \mathcal{V}(\cdot)}{\partial \tau} \geq 0$ .

## E. PROOF OF PROPOSITION 3

We start by rewriting the expression for stationary welfare in (15) as  $W(\tau) = \theta((1-\tau)w(k^*) - k^*)u(R(k^*)k^*)$ . It follows that

$$\begin{aligned} W'(\tau) &= \left[-w(k^*) + (1-\tau)w'(k^*)\frac{\partial k^*}{\partial \tau} - \frac{\partial k^*}{\partial \tau}\right]\theta'(x^*)u(c^*) \\ &+ \left[R'(k^*)\frac{\partial k^*}{\partial \tau}k^* + R(k^*)\frac{\partial k^*}{\partial \tau}\right]\theta(x^*)u'(c^*) \end{aligned} \quad (18)$$

Using the first order conditions to the agent's problem (see footnote 15), the expression for  $W'(\tau)$  in (18) reduces to

$$\begin{aligned} W'(\tau) &= [-w(k^*) + (1-\tau)\frac{\partial w}{\partial k}\frac{\partial k^*}{\partial \tau}]\theta'(x^*)u(c^*) + k^*\frac{\partial R}{\partial k}\frac{\partial k^*}{\partial \tau}\theta(x^*)u'(c^*) \\ &= -w(k^*)\theta'u + A\alpha(1-\alpha)(k^*)^{\alpha-1}[(1-\tau)\theta'(\cdot)u(\cdot) - \theta(\cdot)u'(\cdot)]\frac{\partial k^*}{\partial \tau} \end{aligned} \quad (19)$$

From the first order condition, we have  $\theta(\cdot)u'(\cdot) = \frac{1}{R}\theta'(\cdot)u(\cdot)$ . Replacing  $\theta(\cdot)u'(\cdot)$  by  $\theta'(\cdot)u(\cdot)$ , eq. (19) now becomes

$$W'(\tau) = \frac{w(k^*)}{k^*}\theta'(\cdot)u(\cdot)\left[-k^* + \alpha\left(1-\tau - \frac{1}{R}\right)\frac{\partial k^*}{\partial \tau}\right]$$

which evaluated at  $\tau = 0$  yields

$$W'(0) = \frac{w(\tilde{k})}{\tilde{k}}\theta'(\cdot)u(\cdot)\left[-\tilde{k} + \alpha\left(1 - \frac{1}{R}\right)\frac{\partial k^*}{\partial \tau}\Big|_{\tau=0}\right], \quad (20)$$

where  $\tilde{k}$  is defined in (14).

From (17), we know that  $\frac{\partial k^*}{\partial \tau}|_{\tau=0} = -\frac{(1-\sigma)\tilde{k} + b\eta'(0)\tilde{k}^2}{(1-\sigma)(1-\alpha)} < 0$ . Then (20) reduces to

$$W'(0) = A(1-\alpha)\tilde{k}^{\alpha-1}\theta'(\cdot)u(\cdot) \left[ -\tilde{k} + \alpha\left(1 - \frac{1}{A\alpha}\tilde{k}^{1-\alpha}\right)\left(-\frac{((1-\sigma)\tilde{k} + b\eta'(0)\tilde{k}^2)}{(1-\sigma)(1-\alpha)}\right) \right]$$

which further simplifies to

$$W'(0) = -A(1-\alpha)\tilde{k}^{\alpha-1}\theta'(\cdot)u(\cdot) \left[ \left\{ \frac{\alpha(1-\sigma) + b + \frac{(2\alpha-1)(1-\sigma) + \alpha b}{(1-\sigma)}b\eta'(0)\tilde{k}}{(1-\alpha)(1-\sigma+b)} \right\} \tilde{k} \right].$$

Since  $A(1-\alpha)\tilde{k}^{\alpha-1}\theta'(\cdot)u(\cdot) > 0$  and  $\tilde{k} > 0$ , we have

$$W'(0) \geq 0 \Rightarrow \alpha(1-\sigma) + b + \frac{(2\alpha-1)(1-\sigma) + \alpha b}{(1-\sigma)}b\eta'(0)\tilde{k} \leq 0$$

or

$$W'(0) \geq 0 \Rightarrow C_1 + C_2A^{\frac{1}{1-\alpha}} \leq 0 \tag{21}$$

where  $C_1, C_2$  are defined in the statement of the proposition.

Case I: Since  $C_1 > 0$ , it is obvious that if  $\alpha > 1/2$ , the left hand side of the equation (21) is positive, and hence  $W'(0) < 0$ .

Case II: If  $\alpha < 1/2$ , eq(21) can be rewritten as

$$W'(0) \geq 0 \Rightarrow A \leq \left(-\frac{C_1}{C_2}\right)^{1-\alpha} \equiv \bar{A}$$

if  $\bar{A} > 0$ . That is  $W'(0) > 0$  if  $A < \bar{A}$  and  $W'(0) < 0$  if  $A > \bar{A}$ . The condition  $\bar{A} > 0$  can then be reduced to  $\alpha < \frac{1-\sigma}{2(1-\sigma)+b\eta(0)} \equiv \bar{\alpha}$ , where  $\bar{\alpha} < 1/2$ . Hence, if  $\alpha \geq \bar{\alpha}$ , then  $C_2 \geq 0$ , which also implies the left hand side of the equation (21) is positive, i.e.,  $W'(0) < 0$ .

#### F. PROOF OF PROPOSITION 4

From (12), it follows that

$$\begin{aligned} \text{sign}\left(\frac{dk_{t+1}}{dk_t}\right) &= \text{sign}\left[1 - \frac{b\tau\eta'(\tau w(k_t))w(k_t)}{1-\sigma + b\eta(\tau w(k_t))}\right] \\ &= \text{sign}\left[1 - \sigma + b\eta(\tau w(k_t)) - b\tau\eta'(\tau w(k_t))w(k_t)\right] \end{aligned}$$

Let  $J(k) \equiv b\tau\eta'(\tau w(k_t))w(k_t) - b\eta(\tau w(k_t))$ . Then using (A.1)-(A.5), it follows that  $J(0) < 0$ ,  $J(\infty) = \infty$ , and

$$\begin{aligned} J'(k) &= b\tau\eta''(\tau w(k))\tau w'(k)w(k) + b\tau\eta'(\tau w(k))w'(k) - b\tau\eta'(\tau w(k))w'(k) \\ &= b\tau\eta''(\tau w(k))\tau w'(k)w(k) > 0 \end{aligned}$$

Therefore, there exists a single  $\hat{k}$  such that  $J(\hat{k}) = 1 - \sigma$ . Hence, when  $k < \hat{k}$ ,  $J(\hat{k}) < 1 - \sigma$  and when  $k > \hat{k}$ ,  $J(\hat{k}) > 1 - \sigma$ .