Theory Appendix to "Computer Adoption and Returns in Transition"

1 The model

The model consists of N small open economies indexed by $j \in \{1, 2.., N\}$. In each economy, there is an overlapping generations of individuals who live for two periods; in each period a continuum of individuals is born. In addition, there is a continuum of two types of firms. Both types of firms produce a single homogenous good. While type I firms use labor only, type II firms combine each worker with a computer to produce output. The primitives and the problems of each entity is described below.

The small open economy assumption implies that agents can borrow and lend freely at a gross interest rate R determined in the world markets.[1]

To ease notation, we avoid time subscripts below since the choice problem of all generations is identical.

1.1 Individuals

An individual indexed by i is identified by a pair $\{\xi_i, \tau_i\}$, where ξ_i denotes her innate ability to learn, and τ_i is a stand-in for her desire to be up-to-date with information/computer technology and/or her preference for technological services such as the information that is accessible through the Internet (which is distinct from consumption of homogenous goods). We assume that ξ_i and τ_i are jointly distributed over support $\left[\underline{\xi}, \overline{\xi}\right]$ and $[1, \overline{\tau}]$ with CDF (PDF) given by $G(\xi, \tau)$ $(g(\xi, \tau))$. An individual knows her ξ and τ. Her lifetime utility is given by

$$U = \ln c_{iy} + \rho \ln c_{io} + C_i \;\; (1+\rho) \;\; \ln \tau_i \tag{1}$$

where c_{iy} and c_{io} denote the consumption of homogenous goods in her young and old age, respectively. The variable C_i is a computer adoption variable takes a value of either 1 or 0; it is 1 if the individual adopts computer, 0 otherwise. Thus, the individual can enjoy computers and its services only if her adoption choice is unity. On the other hand, c_{iy} and c_{io} are continuous variables.

[1] The structure of financial markets in a closed economy can be easily endogenized by having agents live for three periods, and by modifying their human capital profile to ensure that agents borrow (save) in their first (second) period of life. This will unnecessarily complicate the analysis without yielding any further insights.

An individual is endowed with a unit of time in both periods. In the first period she has "raw" human capital that equals her unit time endowment, and she can only work in the type I firm at a wage rate $\bar{w}$. In the first period, however, she can educate herself by investing H_i in education that increases her human capital (effective labor) to $h(\xi_i, H_i) > 1$. It is assumed that an individual's human capital is perfectly observable to the firms.

With enhanced human capital, the wage she can earn in a type I firm equals $\bar{w}\ h(\xi_i, H_i)$. Alternatively, she can train herself in computer/information technology that enables her to not only enjoy its services, but also qualifies her to work in a type II firm that offers a wage proportional to $h(\xi_i, H_i)$ (more of this below). However, learning computer/information technology costs $k(\xi_i, H_i)$.

Assumptions It is assumed that

$$h_1 > 0, h_{22} < 0; h_2 > 0;\ h_{12} \geq 0. \tag{2}$$

The first set implies that human capital is increasing in education but with diminishing returns. Next, the higher the innate ability, the higher is the human capital for a given amount of education. Finally, the returns to education are non-decreasing in an agent's innate ability.

We also assume

$$k_1 < 0, k_{22} > 0; k_2 < 0;\ k_{12} = 0. \tag{3}$$

The first set implies that the cost of adopting computer/information technology is decreasing and convex in the amount of education. Next, it is decreasing in the individual's innate ability. The last assumption on the separability of the cost function in ξ_i and H_i is not necessary and is merely done for the sake of analytical simplicity.

It bears emphasis that the cost function k can be potentially location-specific. In particular, a location with poorer infrastructure is likely to have a higher cost. For simplicity, however, we relegate the location-specific cost differences to the firms' problem below.

1.2 Firms

All firms are perfectly competitive. A type I firm uses a unit of human capital to produce a unit of output. Perfect competition then implies that $\bar{w} = 1$.

Type II firms combine each worker with a computer/IT terminal to produce output. These terminals enhance labor productivity: each unit of human capital produces $\eta > 1$ unit of output. However, setting up each terminal costs f_j per period, where the subscript j captures the notion that the costs are location-specific: a location with a poorer infrastructure entails a higher cost. Under perfect competition, an individual with human capital $h(\xi_i, H_i)$ is offered a wage that equals $\eta\ h(\xi_i, H_i) - f_j$.

1.3 The household's problem

Given the wage structure, we now solve the problem for an individual located in location 1. To ease notation, we drop subscript j below; the solution can be easily applied to any location.

The problem of an agent i is

$$\max_{H_i, c_{iy}, c_{io}, C_i \in \{0,1\}} \{\ln c_{iy} + \rho \ln c_{io} + C_i\ \ (1+\rho)\ \ \ln \tau_i\} \tag{4}$$

subject to

$$c_{iy} + H_i + C_i\ k(\xi_i, H_i) = 1 + x_i \tag{5a}$$

$$c_{io} + R\ x_i = C_i\left(\eta\ h(\xi_i, H_i) - f_j\right) + (1 - C_i)\, h(\xi_i, H_i) \tag{5b}$$

where x_i denotes an individual's borrowing when young. The LHS (RHS) in both (5a) and (5b) represent expenditures (resources available) to an agent in periods 1 and 2 respectively. As is standard, to avoid trends in variables, below we assume that $R = \rho^{-1}$.

As C_i is a discrete choice, we first solve the household's choice problem separately for both $C_i = 0$ and $C_i = 1$. Then, we compare the resulting indirect utilities to identify households who choose to adopt or not to adopt computer/IT technology.

1.3.1 $C_i = 0$

Here, by substituting (5a) and (5b) in (4), the household's problem can be reduced to

$$\max_{H_i, x_i} \left\{\ln(1 + x_i - H_i) + \rho\ \ln\left(h(\xi_i, H_i) - \rho^{-1}\ x_i\right)\right\} \tag{6}$$

The first order conditions are

$$\frac{1}{1 + x_i - H_i} = \frac{\rho}{h(\xi_i, H_i) - \rho^{-1}\ x_i} h_2(\xi_i, H_i), \tag{7a}$$

$$\frac{1}{1 + x_i - H_i} = \frac{1}{h(\xi_i, H_i) - \rho^{-1}\ x_i}. \tag{7b}$$

Substituting (7b) in (7a) yields

$$h_2(\xi_i, H_i^*) = \rho^{-1} \tag{8}$$

where H_i^* is the optimal level of education when i does not train herself in computers. Finally solving for x_i^* from (7b) and substituting it in (6) yields the indirect utility:

$$W^*(\xi_i) = (1+\rho)\left[\ln\left(1 - H_i^* + \rho\, h(\xi_i, H_i^*)\right) - \ln(1+\rho)\right]. \tag{9}$$

1.3.2 $C_i = 1$

Here, by substituting (5a) and (5b) in (4), the household's problem can be reduced to

$$\max_{H_i, x_i} \left\{\ln\left(1 + x_i - H_i - k(\xi_i, H_i)\right) + \rho\ \ln\left(\eta\ h(\xi_i, H_i) - f - \rho^{-1}\ x_i\right)\right\} \tag{10}$$

The first order conditions are

$$\frac{1 + k_2(\xi_i, H_i)}{1 + x_i - H_i - k(\xi_i, H_i)} = \frac{\rho}{\eta\ h(\xi_i, H_i) - f - R\ x_i}\eta\ h_2(\xi_i, H_i) \tag{11a}$$

$$\frac{1}{1 + x_i - H_i - k(\xi_i, H_i)} = \frac{1}{\eta\ h(\xi_i, H_i) - f - R\ x_i} \tag{11b}$$

Substituting (11b) in (11a) yields

$$\frac{\eta\ h_2(\xi_i, H_i^{**})}{1 + k_2(\xi_i, H_i^{**})} = \rho^{-1} \tag{12}$$

where H_i^{**} is the optimal level of education when i does train herself in computers. Finally solving for x_i^{**} from (7b) and substituting it in (6) yields the indirect utility:

$$W^{**}(\xi_i) = (1+\rho)\left[\begin{array}{c} \ln\left(1 - H_i^{**} - k(\xi_i, H_i^{**}) + \rho\ \left(\eta\ h(\xi_i, H_i^{**}) - f\right)\right) \\ -\ln(1+\rho) + \ln\tau_i \end{array}\right] \tag{13}$$

The following Lemma characterizes the choice of education as a function of innate ability. It also establishes

that for an individual the amount of education is higher if she also chooses to train in computers.

Lemma 1 *Both H_i^* and $H_i^{**}$are weakly increasing in ξ_i. Furthermore, $H^{**} > H^*$.*

Proof. See Section 2.1. ■

Intuitively, a higher innate ability increases the marginal product of education, and therefore the individual chooses a higher amount.[2] The second result $H^{**} > H^*$ is due to the feature that education not only increases one's human capital, but also reduces the cost of learning computers, as assumed in (3), in case the individual decides to train in computer/IT.

[2]Notice in particular that if $h_{12} > 0$, then both H_i^* and H_i^{**} are strictly increasing in ξ_i.

1.3.3 The choice of computer adoption

Comparing indirect utilities given by (9) and (13) implies that $C_i = 1$ if and only if

$$\begin{aligned}&\left(1 - \underbrace{H_i^{**} - k\left(\xi_i, H_i^{**}\right)}_{\text{cost of education and computer adoption}} + \underbrace{\eta\, h\left(\xi_i, H_i^{**}\right) - f}_{\text{earning in type II firm}}\right)\tau_i \\ \geq\ & 1 - \underbrace{H_i^{*}}_{\text{cost of education}} + \underbrace{h\left(\xi_i, H_i^{*}\right)}_{\text{earning in type I firm}}\end{aligned} \tag{14}$$

where H_i^* and H_i^{**} solve (8) and (12) respectively. Equation (14) leads to the following proposition.

Proposition 1 *Given η and f, the computer adoption rule for an individual i is*

$$C_i = \begin{cases} 1, & \text{if } \xi_i \geq \hat{\xi}\left(\tau_i, \eta, f\right) \\ 0, & \text{if } \xi_i < \hat{\xi}\left(\tau_i, \eta, f\right) \end{cases}$$

Moreover, $\hat{\xi}_1 < 0, \hat{\xi}_2 < 0, \hat{\xi}_3 > 0.$

Proof. See Section 2.2. ■

The intuition behind the results stated in Proposition 1 is simple. Given the individual's preference for technology and the market wages on a job with computers, the higher her innate ability the more likely she is to adopt computers. The innate ability has a direct as well as indirect effect on computer learning. A higher ability facilitates a lower computer/IT learning cost. Second, a higher ability makes her achieve a higher level of education that also reduces her cost of technology adoption: an *indirect* effect.

The higher the individual's (hedonic) preference for technology, the lower is the ability at which she decides to adopt computers. A higher productivity in type II firms (i.e., jobs with computers) leads to higher equilibrium wages, thus inducing agents of lower abilities to also adopt computers and opt for working in type II firms. Similarly, a higher cost of IT investment by firms lowers equilibrium wages. Then, only relatively higher ability individuals will adopt and work on computer jobs. This is more likely to be observed in countries with relatively poorer infrastructure.

An interesting question is: can an individual with $\xi_i = \hat{\xi}$ be indifferent between the two choices even if her earnings in type II firms fall below that in type I firm? To answer this, first note from (14) that if she chooses to work in type II firms her education H^{**} will be higher than had she chosen to work in a type I firm. In addition, she has to bear a cost of adopting computer/IT. However, if her preference for computer/IT services were strong enough, i.e., τ_i is large enough such that it compensates for her pecuniary loss, she would still choose to work in the type II firm. A similar argument holds for individuals with $\xi_i > \hat{\xi}$.

Thus, the model allows for instances where an individual chooses to work in type II firms even when wages are lower, or more specifically, even when wages net of cost of education and computer adoption are lower relative to the other alternative of working in type I firms.

2 Proofs

2.1 Proof of Lemma 1

Proof. Applying Implicit function theorem to (8) and (12) directly yields

$$\begin{aligned} \frac{dH_i^*}{d\xi_i} &= -\frac{h_{21}\left(\xi_i, H_i^*\right)}{h_{22}\left(\xi_i, H_i^*\right)} \geq 0 \\ \frac{dH_i^{**}}{d\xi_i} &= -\frac{\rho\ \eta\ h_{21}\left(\xi_i, H_i^{**}\right)}{\rho\ \eta\ h_{22}\left(\xi_i, H_i^{**}\right) - k_{22}\left(\xi_i, H_i^{**}\right)} \geq 0 \end{aligned}$$

For the second part, assume $H^{**} \leq H^*$. Then, since $\eta > 1$ and $h_{22} < 0$, $\eta\ h_2\left(\xi_i, H_i^{**}\right) > h_2\left(\xi_i, H_i^*\right)$. Since the denominator in (12) is less than unity, $\frac{\eta\ h_2\left(\xi_i, H_i^{**}\right)}{1+k_2\left(\xi_i, H_i^{**}\right)} > h_2\left(\xi_i, H_i^*\right) = \rho^{-1}$, which contradicts (12). Hence, $H^{**} > H^*$. ■

2.2 Proof of Proposition 1

Proof. Fix τ_i, η, and f. By Envelope Theorem the derivative of the LHS in (14) $-k_1\left(\xi_i, H_i^{**}\right) + \eta\ h_1\left(\xi_i, H_i^{**}\right) > 0$ and that of the RHS is $h_1\left(\xi_i, H_i^*\right) > 0$. Thus, both sides of (14) are increasing in ξ_i.

To prove the first part we need to show that $-k_1\left(\xi_i, H_i^{**}\right) + \eta\ h_1\left(\xi_i, H_i^{**}\right) > h_1\left(\xi_i, H_i^*\right)$. This is done by noting that $-k_1\left(\xi_i, H_i^{**}\right) > 0$; $h_{12} > 0$, and $H_i^{**} > H_i^*$ from Lemma 1.

The second set of results is obvious from (14). ■