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Discrete Double Auctions with Artificial Adaptive Agents: A Case Study of an Electricity Market Using a Double Auction Simulator

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Discrete Double Auctions with Artificial Adaptive Agents: A Case Study of an Electricity Market Using a Double Auction Simulator [1,2]

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Abstract

A key issue raised by previous researchers is the extent to which learning versus market structure is responsible for the high efficiency regularly observed for the double auction in human-subject experiments. In this study, a computational discrete double auction with discriminatory pricing is tested regarding the importance of learning agents for ensuring market efficiency. Agents use a Roth-Erev reinforcement learning algorithm to determine their bid and ask prices. The experimental design focuses on two treatment factors: market capacity; and a key Roth–Erev learning parameter that controls that degree of agent experimentation. For each capacity setting, it is shown that changes in the learning parameter have a substantial systematic effect on market efficiency.

I. Introduction

In the last decade, agent-based computational economics (ACE) modeling has gained attention as another way to understand socioeconomic phenomena. A growing number of research articles, books, and conferences are now applying this method to social science problems. General information about this method can now easily be found on the Internet. Among these, www.iastate.edu/tesfatsi/ace.htm and www.jasss.soc.surrey.ac.uk are good places to start.

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[2] A creative component work, submitted to obtain degree in Master of Science in Economics

One of the advantages of ACE modeling to be stressed in this study is the capability of imitating the behavior of real agents under hypothetical market conditions. This becomes especially important for problems that are too complicated to be analyzed by using an analytical approach. In ACE models, the effects of proposed structural changes for different market types can be evaluated and analyzed before they are applied to a real market condition. For example, given the proposed restructuring of a market, an ACE model could be used to investigate and apply several competing proposed market protocols without disturbing or interrupting the current market operation.

Several researchers have taken advantage of the ACE method to study several different protocols for restructuring in an electricity market. Bower and Bunn [1], for example, evaluate the market efficiency of two different proposed auction pricing mechanisms for the England and Wales electricity market: a uniform-price auction; and a discriminatory-price auction. In their agent modeling, they use naïve reinforcement learning. Other researchers, Nicolaisen et al. [8] [9], report experimental market power and efficiency outcomes under systematically varied concentration and capacity conditions. They test two different learning algorithms in the context of a discriminatory-price double auction. In their earlier paper [8], they test a genetic algorithm (GA) learning algorithm that enabled traders to engage in social mimicry. In their following research [9], they apply individual reinforcement learning by using a modified Roth-Erev (MRE) learning algorithm.

Gode and Sunder [4] have pointed out that the effect of market structure should be given special consideration in economic modeling. Precisely, they stress the importance of separating the effect of learning agents from the effect of market structure. In their research, under a continuous double auction protocol, the learning capability of agents is not a substantial factor for the determination of high efficiency outcomes. They observed that even zero-intelligence agents with a binding budget constraint could consistently result in high market efficiency. In other words, market structure has a more dominant role than agent learning in determining market efficiency for the continuous double auction.

As one of the main objectives of this study, the Gode and Sunder argument will be studied for our market framework. Although Nicolaisen et al. [9] are aware of this issue, they do not try directly to test this issue in their research. The framework used in this study is similar to the framework used in Nicolaisen et al.[8][9], which is a discrete double auction with price discrimination. However, the original Roth-Erev learning algorithm is applied instead of the modified version.

A special feature of the Roth-Erev learning algorithm is that it tries to mimic the learning behavior of real humans. The algorithm incorporates basic principles of human learning behavior well-known in the psychology literature, such as the law of effect and the power of practice. In their studies, Roth and Erev calibrate the learning parameters for their algorithm against results obtained in experiments with human subjects. In the calibration process, they obtain a "best fit" set of learning parameters that best mimics human learning behavior observed in a wide range of games.

The experimental design of this study focuses on two treatment factors: market capacity; and a key Roth-Erev learning parameter e that controls the degree of agent experimentation. Learning parameter e has a nonnegative value. As e greater than zero agent will not quickly become locked in to a particular strategy. Three different specifications for market capacity are tested. For each of these three market structures, eight different specifications are tested for the experimentation parameter e. Two of the learning parameter specifications represent special circumstances: a no-learning case; and a "best fit " learning case. Efficiency and market power outcomes are reported for each treatment in order to separate the effects of learning from the effects of market structure in the tested market framework.

To implement this study, a Java-based object-oriented general double auction simulator (DASim) is developed. DASim constructs adaptive artificial agents to represent real traders. Agents are capable of strategizing their ask and bid prices and quantities as they have the ability to learn via the Roth-Erev algorithm. DASim also incorporates a general matching mechanism that acts as a clearing-house. It accepts traders' asks and bids, matches these asks and bids to determine market-clearing prices and quantities, and then announces to each trader his resulting price and quantity.

The DASim is applied to the study of a restructured electricity market. Agents in the auction are sellers and buyers of electricity. Sellers represent generation companies that generate electricity with a certain capacity. Buyers represent distribution companies that purchase electricity to be redistributed to meet a certain level of retail demand. The sellers and buyers repeatedly submit asks and bids to the clearing-house in an attempt to maximize their profits.

The main contribution of this study is to show that, for a discrete double auction with price discrimination, learning has a substantial role in determining the efficiency of market outcomes. Specifically, as the experimentation parameter e in the Roth-Erev learning algorithm increases, market efficiency decreases monotonically. Moreover, systematic effects are also observed for other market outcomes such as agent market powers and obtained profits. At the same time it is observed that the incidence of coordination failure increases. These findings indicate that changes in the key experimentation parameter e for the well-known Roth-Erev reinforcement-learning algorithm can result in systematic changes in market outcomes for the discrete double auction, in contrast to the Gode and Sunder finding for the continuous double auction.

II. Double Auction Simulator (DASim) Frame Work

DASim is mainly supported by five different classes, which are DASim class, Agent class, Matching class, RandomGen class, and Stats class. Each of these classes has a particular function. The class structure diagram is shown as in Figure 1.

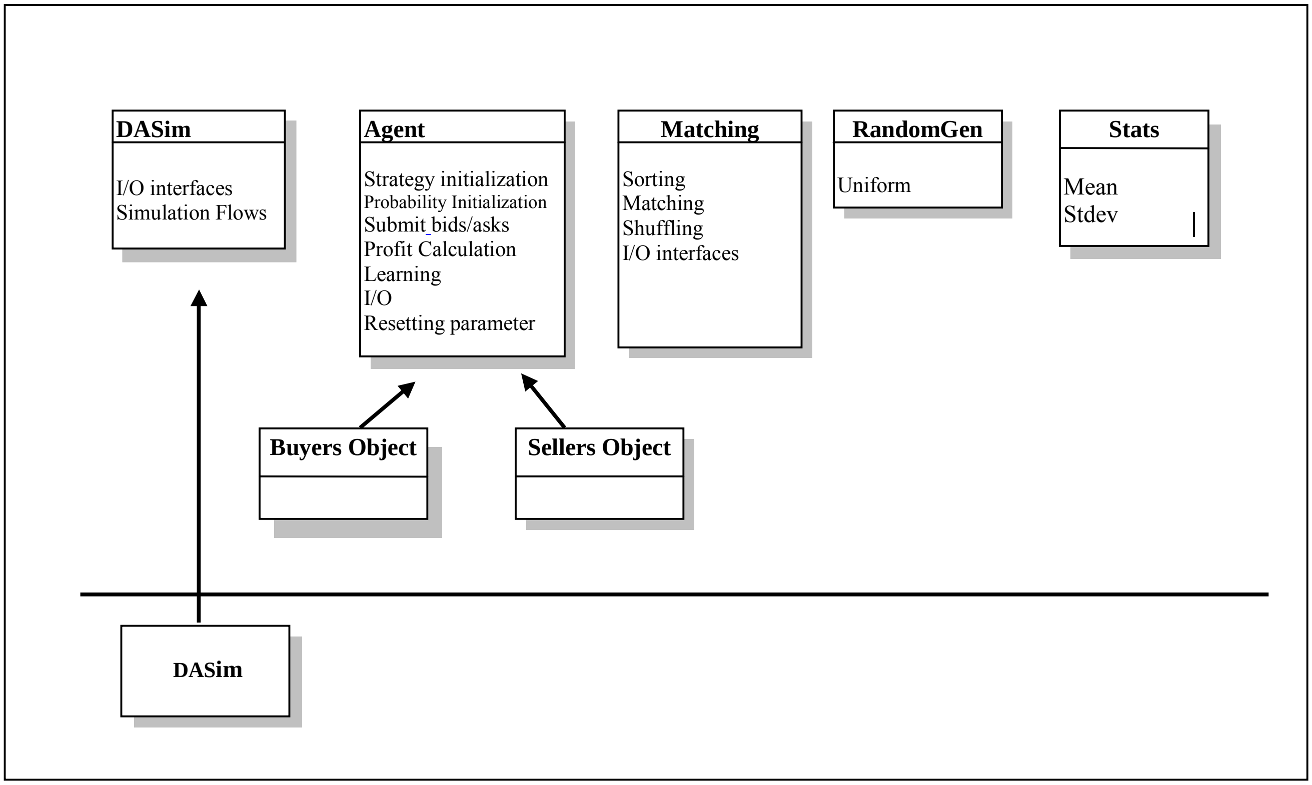

Figure 1 DASim Class Structure

The basic attributes, methods and function of each class are as follows:

1. DASim class.

DASim is the main class of this program; therefore it controls the overall activity of the program. This class decides the sequence of what agents should do, which agent's information should be passed to other classes, and decides which data results will be presented. Basically it controls the traffic of information and the sequence of actions that happens in the double auction.

2. Agent class.

It creates objects that represent sellers and buyers. Each object will symbolize a trader with all of its unique attributes such as marginal cost (MC) or marginal revenue (MR) values and individual learning parameters. These artificial traders are equipped with a learning capability to mimic the ability of agents to strategize ask and bid prices in the auction in order to maximize their own profits. Other routine capabilities of these agents are initializing their strategy sets, submitting ask or bid prices, and calculating profits.

3. Matching class.

Matching class is constructed by several methods that represent the behavior of the clearinghouse. It accepts ask and bid prices from sellers and buyers, and processes them to determine the auction winner. The detailed process for determining the auction involves some steps. This class provides the necessary means for implementing all of those steps which are sorting the ask and bid, matching the highest bid and the lowest ask, determining the auction price, deciding the auction winner and, finally, sending I information back to the relevant traders.

4. RandomGen class.

RandomGen class is a pseudo random number generator. It generates random numbers that are needed in this simulator. In this class, we have the ability to control the seed for generating random numbers. As a result we can control a particular random number sequence that we want to generate. The idea of this seed control is to ensure the repeatability of the experimentation. One of the main benefits of using Java platform is that the random number generated is independent of the platform and machines. In this framework, the only type of pseudo random number available is a uniform random number generator. It could easily be expanded to other types if needed.

5. Stats class.

Basically this class provides statistical calculations for the experimentation outcomes. Specifically, it calculates the average and variance of the experimentation results. Again, it could easily be expanded to another statistical measure if needed.

The general sequence and interaction among the classes are shown in Figure 2. Mainly it can be explain in four steps. **First** is the initialization step. After all input setup parameters are received, the Agent class creates objects which represent sellers and buyers. **Second** is the bidding step. Buyers and sellers submit asks and bids. Asks and bids are selected from pre-defined sets of actions in accordance with choice probabilities. Agents use a reinforcement learning algorithm to update their choice probability distribution in every auction round.

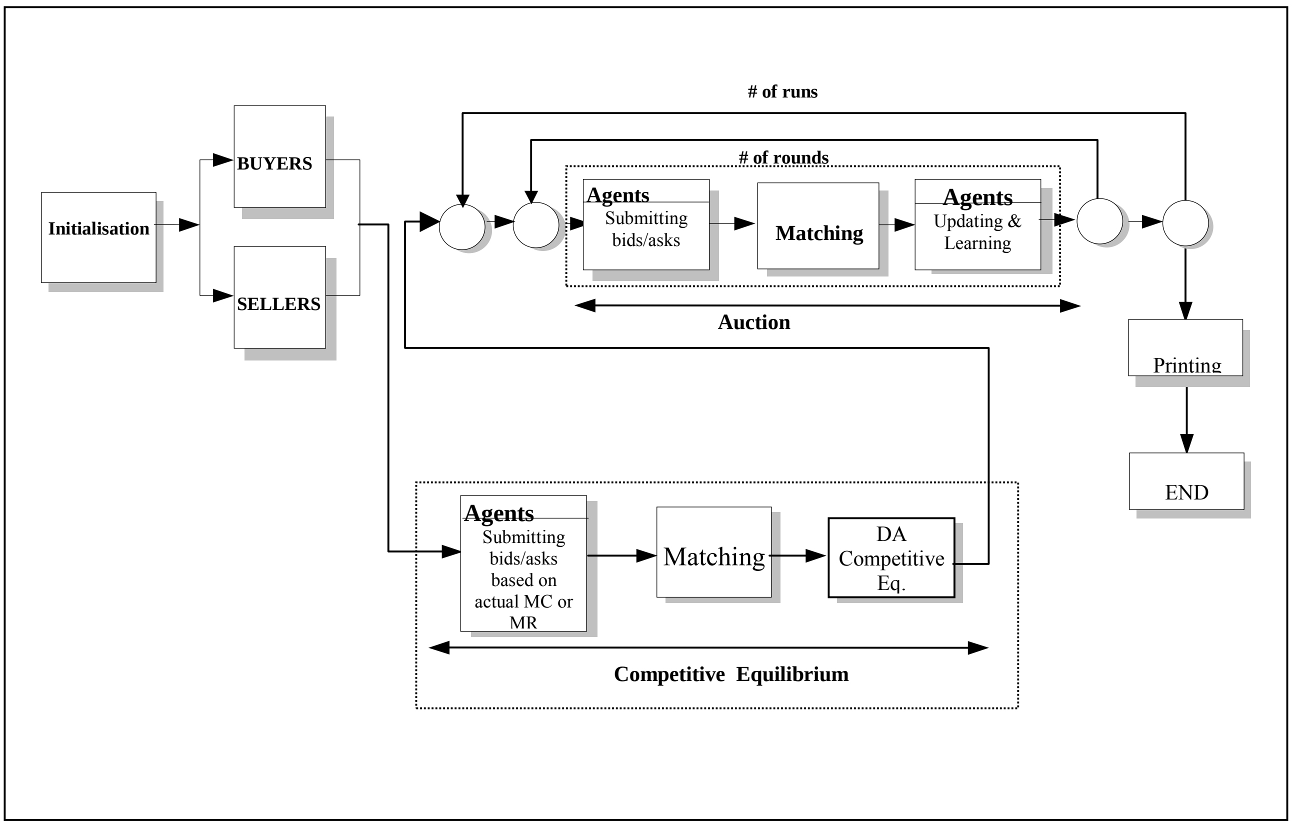

Figure 2. DASim Flow Diagram

Third is the matching step. The clearing house successively matches the lowest ask price and the highest bid price to determine the matched buyer-seller pairs, sets the price for each matched buyer-seller pair at the midpoint of their bid and ask prices, and communicates this price back to each matched buyer-seller pair. **Fourth,** given the auction results, each trader calculates his profits and uses this profit calculation to update his choice probabilities for selecting a bid or ask price in the next auction round.

As shown in figure 2, the second, third and fourth steps are done repeatedly depending on how many runs and how many auction rounds per runs the experimenter decides to conduct for each experimental treatment. Note that, as shown in figure 2, after initialization steps these steps (step 2,3, and 4) are also conducted once under competitive equilibrium conditions. These conditions require each seller to set his ask price at the level of his true marginal cost and each buyer to set his bid price at the level of his true marginal revenue. he outcomes that result under these competitive conditions will later be used as benchmarks against which the auction outcomes will be compared.

III. Electricity Market Framework

The electricity market framework used in this study is adapted from Nicolaisen et al. [8],[9]. There are a small numbers of agents that represent several sellers and buyers that submit asks and bids repeatedly to maximize their profits. Sellers have multi-unit capacities and different marginal costs. Similarly, buyers have multi-unit demands and different marginal revenues. A matching mechanism run by an independent clearinghouse accepts and matches the ask and bid prices. Once ask and bid prices are matched, the clearinghouse determines a price and quantity for each matched pair and sends these outcomes back to the traders. The traders continuously update their ask and bid prices in each period based on the obtained profit in each period.

There are some important technical assumptions used in this auction framework.

1. In each auction round, each seller submits to the clearinghouse an ask price together with its maximum capacity.
2. In each auction round, each buyer submits to the clearinghouse a bid price together with its maximum demand level.
3. Traders' budget constraints are binding. A buyer always submits a bid price lower than or equal to his reservation price, i.e., his true MR.. On the other hand, a seller always submits an ask price higher than or equal to his reservation price, i.e., his true MC.
4. There are no binding transmission constraints. Traders can sell or buy as much capacity as they own or they need.
5. Contracts are binding. All traders are obliged to carry out the trades determined by the auction results.
6. Information is imperfect and private. All information available in this framework is private information. A trader does not have any direct knowledge of the prices and quantities characterizing the trades of other auction participants.
7. No second market is available. A seller always acts only as a seller, and a buyer always acts only as a buyer. Reselling is prohibited.

A detailed illustration of how this market works is shown in Figure 2. First, each trader selects an ask or bid price from a fixed interval of feasible price offers using a choice probability distribution initially specified to be a uniform distribution. The clearinghouse receives these bid and ask prices, sorts the bid prices from the highest to the lowest, and sorts the ask prices from the lowest to the highest. Once sorting is done, the matching process follows. The highest bid price is matched to the lowest ask price, and the price set for this matching is the midpoint between the ask and bid prices. The next highest bid price is then matched to the next lowest ask price and so on until all remaining bid prices are below all remaining ask prices. The clearinghouse then stops the matching process. This price resolving mechanism is called price discrimination. Unlike the uniform price which determining a single price for all the traders, price discrimination determines a separate price for each matched bid and ask price; namely, the mid-point of the two.

To make all these things more clear, we give an example with hypothetical ask and bid prices. Suppose sellers and buyers submit ask and bid prices as shown in Table I.

Sellers	**Ask Price**	**Quantities**
1	36	40
2	18	40
3	14	40
Buyers	**Bid Price**	**Quantities**
1	27	10
2	15	10
3	14	10
4	28	10
5	16	10
6	14	10

Table I. Hypothetical Ask and Bid Prices during an Auction under Market Structure 1

The clearinghouse then will conduct the sorting and matching process as shown in the following table.

SELLERS			Matched	BUYERS		
Sellers	Quantities	Ask Price	Auction Price	Bid Price	Quantities	Buyers
3	10	14	21	28	10	4
3	10	14	20	26	10	1
3	10	14	15	16	10	2
3	10	14	14.5	15	10	5
2	40	18	Not Match	14	10	3
1	40	36	Not Match	14	10	6

Table II. Sorting and Matching Mechanism by Clearinghouse under Hypothetical Ask and Bid Price

Note that, based on private information assumption, the information as shown in Table II will be transferred privately to each trader. Each trader applies its learning capability, and updates its choice probability distribution based on this information.

Sellers	Auction Price	Quantities
1	0	0
2	0	0
3	21	10
3	20	10
3	15	10
3	14.5	10
Buyers	**Auction Price**	**Quantities**
1	20	10
2	15	10
3	0	0
4	21	10
5	14.5	10
6	0	0

Table III. Auction Results from Hypothetical Ask and Bid Prices.

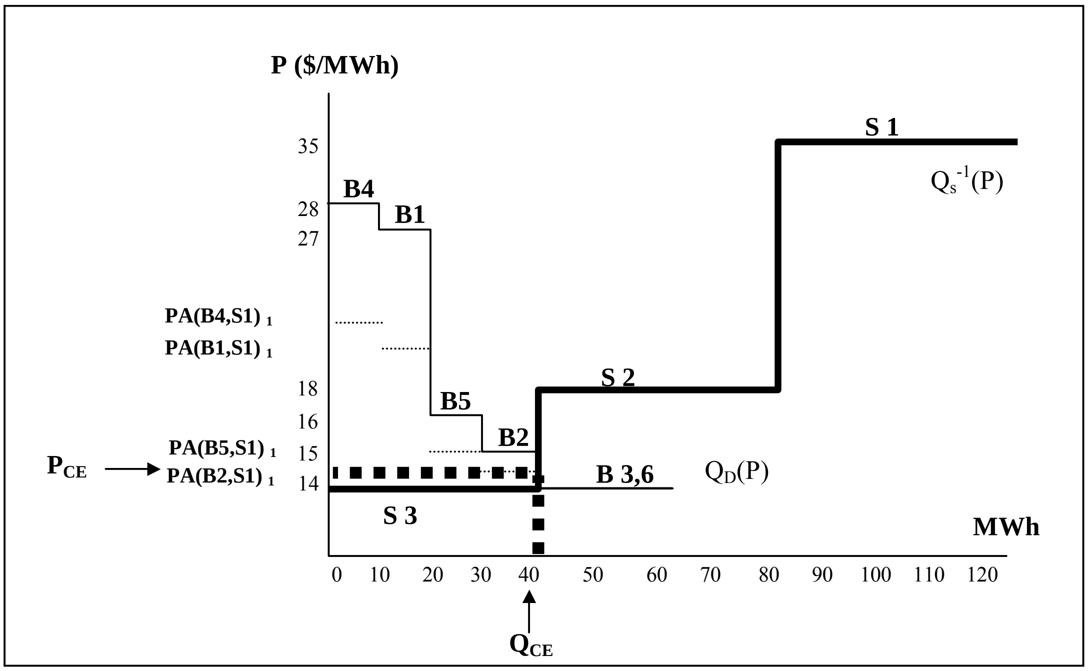

Figure 3. Aggregate Supply and Aggregate Demand under Hypothetical Ask and Bid Prices.

IV. Definitions for Market Power and Market Efficiency

In this section we define several key measures that will be used to evaluate outcomes for the auction market, such as individual market power, aggregate market power, and market efficiency. We also clarify the difference between structural market power levels and experimental market power levels. As we will explain later in this section, these measures are basically calculated from the profits obtained by the traders. Structural market power levels are obtained from the competitive equilibrium outcomes. In contrast, experimental market power levels are obtained from the auction operating under the price-discrimination pricing mechanism. Since it is important to understand how profits are calculated, this chapter begins with the explanation of the profit calculation under competitive equilibrium. It then concludes by explaining the calculation of profits under the auction protocol.

IV. A. Competitive equilibrium profit calculation

By definition in this market structure, a competitive equilibrium (*CE*) is obtained when traders believe that their quantity choices have no effect on the market price *P*. Therefore, the price *P* is an exogenous parameter to their trades. The formal approach for deriving the competitive equilibrium is shown in Nicolaisen et al. [9]. Under the competitive equilibrium calculation, **all sellers and buyers submit their real MC and MR levels as their asks and bids.**

A competitive equilibrium is when $Q^{-1}{}_{s}(p)$, the aggregate inverse supply function, is equal to $Q^{-1}{}_{D}(p)$, the aggregate inverse demand function. As shown in Figure 4a, for example, under market structure 1, the competitive equilibrium price and equilibrium quantity are at the intersection point of $Q^{-1}{}_{D}(p)$ and $Q^{-1}{}_{s}(p)$, which occurs at Q_{CE} and P_{CE}. Note that it is possible that the equilibrium price is not unique. In figure 4a, the equilibrium price could be anything in the interval [11,17] as every point in this interval fulfils the criteria of $Q^{-1}{}_{s}(p) = Q^{-1}{}_{D}(p)$. For concreteness, if an interval of possible equilibrium prices exists, we will take the competitive equilibrium price to be the mid-point of the interval. This convention is the same convention that is used in determining auction prices. As a result, market structure 1 has a competitive equilibrium price of P_{CE} =(17+11)/2=14, and an equilibrium quantity of Q_{CE} = 40. The dashed thick line projects the competitive equilibrium point to the price and quantity axes.

Following the calculation of the competitive equilibrium, the competitive profit for a seller *i* in market structure *j* can be calculated by

$$\Pi SCE_i = (P_{CE} - MC_{ij}) * QSCE_{ij} \tag{1}$$

where

i	= 1,2,3 respectively refers to seller 1, seller 2, and seller 3.
j	= 1,2,3 respectively refers to market structure 1, 2, and 3
ΠSCE_{ij}	= Competitive equilibrium profits for seller i at market structure j
MC_{ij}	= Marginal Cost for seller i at market structure j
$QSCE_{ij}$	= competitive equilibrium quantity sold by seller i at market structure j

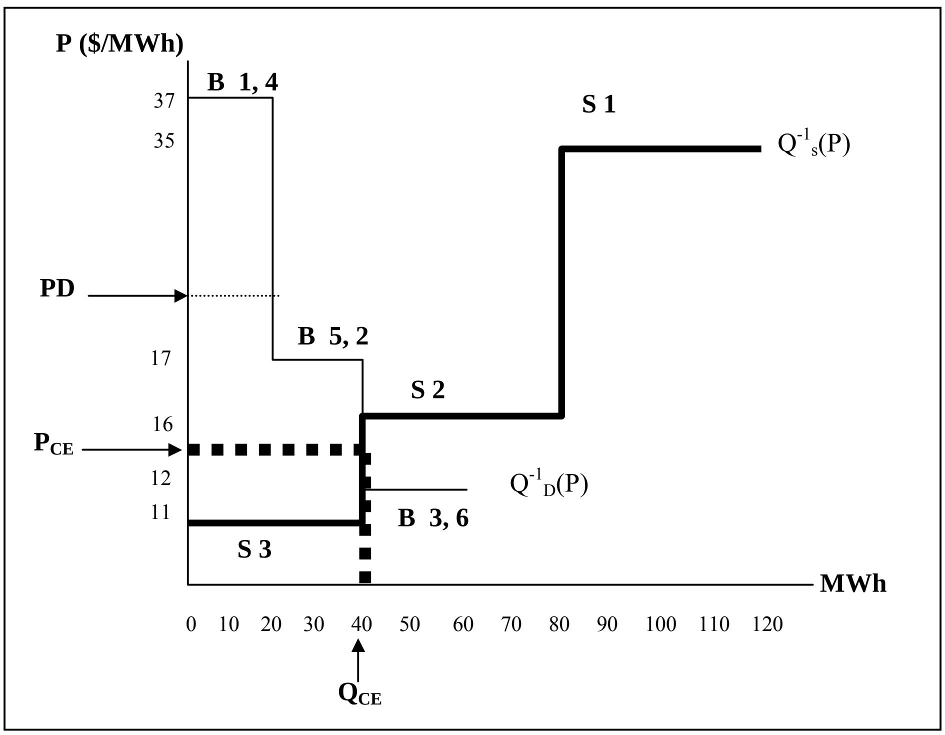

Figure 4a. Aggregate Supply and Aggregate Demand Under Actual MC and Actual MR in **Market Structure 1**

Total seller profit under competitive equilibrium in market structure *j* is denoted by $\Pi\Pi SCE_j$ which is the summation of all profits obtained by all of the sellers.

$$\Pi TSCE_j = \sum_{i=1}^{3} \Pi SCE_{ij}$$

(2)

Meanwhile, buyer *k*'s profit in market structure *j* can be calculated by :

$$\Pi BCE_j = (MR_{kj} - P_{CE}) * QBCE_{kj}$$

(3)

where

k	= 1,2,3,4,5,6 which respectively refers to buyer 1 to buyer 6
j	= 1,2,3 respectively refers to market structure 1, 2, and 3
ΠBCE_{kj}	= Competitive equilibrium profits for buyer *k* at market structure *j*
MR_{kj}	= Marginal Revenue for buyer *k* at market structure *j*
$QBCE_{kj}$	= Competitive equilibrium quantity obtained by buyer *k* at market structure *j*

Total buyer profit under competitive equilibrium in market structure *j* is denoted by $\Pi TBCE$, which is the summation of all profits obtained by all of the buyers.

$$\Pi TBCE_j = \sum_{k=1}^{6} \Pi BCE_{kj}$$

(4)

Any traders that actually trade in the competitive equilibrium are called **infra marginal (IM)** traders. As seen in Figure 4a, in market structure 1 the IM traders are seller 3, buyer 1, buyer 2, buyer 4 and buyer 5. In contrast, any trader that fails to trade in the competitive equilibrium is called an **extra marginal (EM)** trader. As seen in Figure 4a, in market structure 1 the EM traders are seller 2, seller 3, buyer 3, and buyer 6.

To complete the illustration, using relations analogous to equations (1) and (3), the competitive equilibrium profit of each individual trader is calculated as follows (note that P_{CE1} = \$14/MwH)

Trader's ID	Notation	Status	Profit
Seller 1	ΠSCE_{11}	EM	0
Seller 2	ΠSCE_{21}	EM	0
Seller 3	ΠSCE_{31}	IM	(14 - 11)*40 = 120
Buyer 1	ΠBCE_{11}	IM	(37 - 14)*10 = 230
Buyer 2	ΠBCE_{21}	IM	(17 - 14)*10 = 30
Buyer 3	ΠBCE_{31}	EM	0
Buyer 4	ΠBCE_{41}	IM	(37 - 14)*10 = 230
Buyer 5	ΠBCE_{51}	IM	(17 - 14)*10 = 30
Buyer 6	ΠBCE_{61}	EM	0

Table IV a. Individual Competitive Equilibrium Profit Calculation in Market Structure 1, IM = Infra-marginal trader, EM = Extra-marginal trader.

The aggregate total competitive equilibrium profits for buyers and sellers in market structure 1 are therefore as follows :

$$\Pi TSCE_1 = \Pi SCE_{11} + \Pi SCE_{21} + \Pi SCE_{31} = 0 + 0 + 120 = 120$$

$$\Pi TBCE_1 = \Pi BCE_{11} + \Pi BCE_{21} + \Pi BCE_{31} + \Pi BCE_{41} + \Pi BCE_{51} + \Pi BCE_{61}$$

$$= 230 + 30 + 0 + 230 + 30 + 0 = 520$$

Following these same steps and referring to Figures 4b and 4c, the competitive equilibrium prices and quantities for market structures 2 and 3, respectively, are P_{CE2} = \$16.5 /MwH, Q_{CE2}= 40 MwH, and P_{CE3}= \$26.5 /MwH, Q_{CE3}= 20 MwH. The detailed calculation of the individual traders' competitive equilibrium profits for market structures 2 and 3 are shown in the following Table IV B.

Market Structure 2				Market Structure 3			
Trader's ID	Notation	Status	Profit	Trader's ID	Notation	Status	Profit
Seller 1	ΠSCE_{12}	EM	0	Seller 1	ΠSCE_{13}	EM	0
Seller 2	ΠSCE_{22}	IM	(16.5 - 16)*20 = 10	Seller 2	ΠSCE_{23}	IM	(26.5 - 16)*10 = 105
Seller 3	ΠSCE_{32}	IM	(16.5 - 11)*20 = 110	Seller 3	ΠSCE_{33}	IM	(26.5 - 11)*10 = 155
Buyer 1	ΠBCE_{12}	IM	(37 - 16.5)*10 = 205	Buyer 1	ΠBCE_{13}	IM	(37 - 26.5)*10 = 105
Buyer 2	ΠBCE_{22}	IM	(17 - 16.5)*10 = 5	Buyer 2	ΠBCE_{23}	EM	0
Buyer 3	ΠBCE_{32}	EM	0	Buyer 3	ΠBCE_{33}	EM	0
Buyer 4	ΠBCE_{42}	IM	(37 - 16.5)*10 = 205	Buyer 4	ΠBCE_{43}	IM	(37 - 26.5)*10 = 105
Buyer 5	ΠBCE_{52}	IM	(17 - 16.5)*10 = 5	Buyer 5	ΠBCE_{53}	EM	0
Buyer 6	ΠBCE_{62}	EM	0	Buyer 6	ΠBCE_{63}	EM	0

Table IV b. Individual Competitive Equilibrium Profit Calculations
for Market Structures 2 and 3, IM = Infra-marginal trader, EM = Extra-marginal trader.

The aggregate total competitive equilibrium profits for buyers and sellers in market structures 2 and 3 are as follows:

$$\Pi TSCE_2 = \Pi SCE_{12} + \Pi SCE_{22} + \Pi SCE_{32} = 0 + 10 + 110 = 120$$

$$\Pi TBCE_2 = \Pi BCE_{12} + \Pi BCE_{22} + \Pi BCE_{32} + \Pi BCE_{42} + \Pi BCE_{52} + \Pi BCE_{62}$$

$$= 205 + 5 + 0 + 205 + 5 + 0 = 420$$

$$\Pi TSCE_3 = \Pi SCE_{13} + \Pi SCE_{23} + \Pi SCE_{33} = 0 + 105 + 155 = 260$$

$$\Pi TBCE_3 = \Pi BCE_{13} + \Pi BCE_{23} + \Pi BCE_{33} + \Pi BCE_{43} + \Pi BCE_{53} + \Pi BCE_{63}$$

$$= 105 + 0 + 0 + 105 + 0 + 0 = 210$$

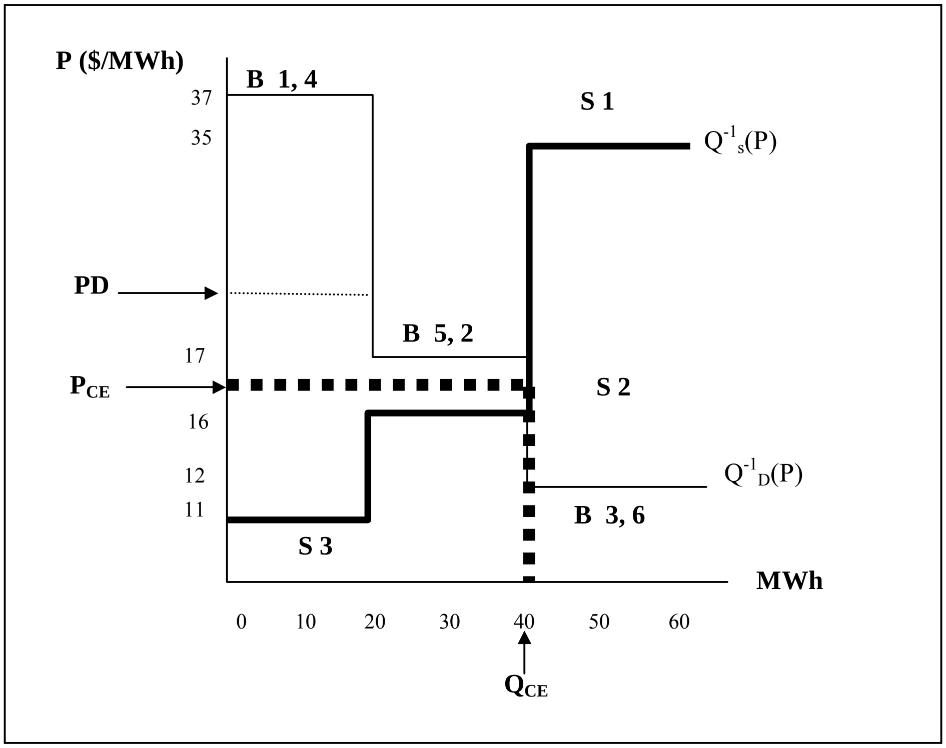

Figure 4b. Aggregate Supply and Aggregate Demand under Actual MC and actual MR in **Market Structure 2**

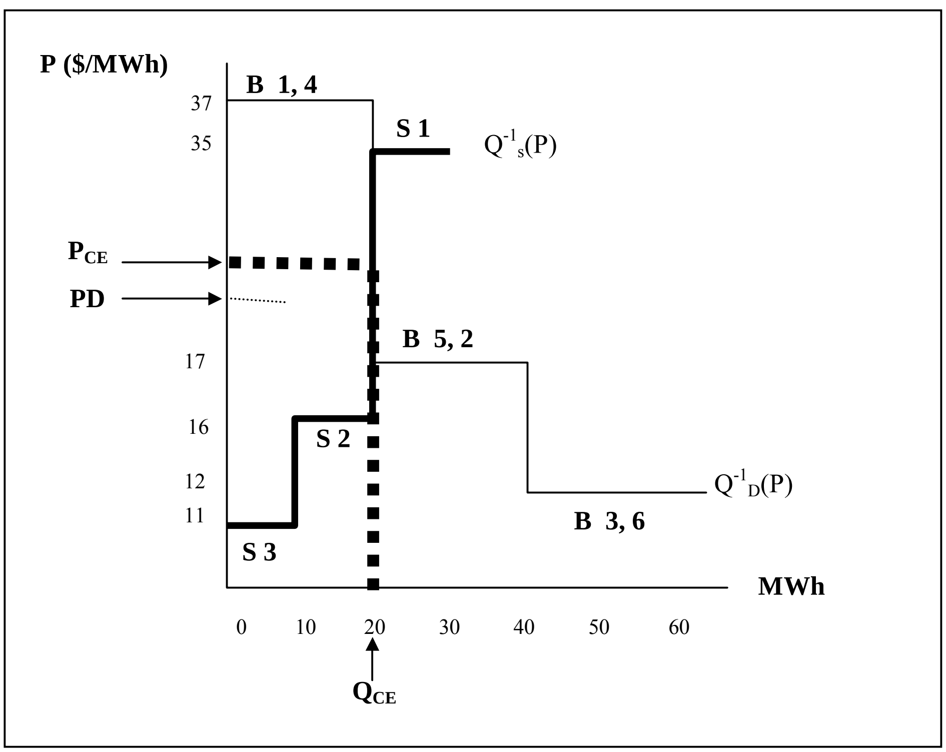

Figure 4c. Aggregate Supply and Aggregate Demand under Actual MC and actual MR in **Market Structure 3**

IV. B. Auction profit calculation

There are two main differences between profit calculations for the competitive equilibrium and profit calculations for the auction. First, unlike in the competitive conditions, in the auction the sellers and buyers will likely submit ask and bid prices that are different from their true MC and MR levels. Intuitively, in order to obtain higher profits, a seller will ask higher than its MC, and a buyer will bid lower than its MR. Second, the auction uses a discriminatory pricing mechanism instead of a uniform pricing mechanism as used for the competitive equilibrium. As a result, the auction might result in a different price for each matched seller-buyer pair.

Recall that the technique to determine the auction's equilibrium prices was explained in section III and illustrated in Figure 3. In that section, it is assumed that under market structure 1 sellers and buyers submit hypothetical ask and bid prices as shown in Table 1. After a matching process carried out by the clearinghouse, as shown in Table 2, the auction results are as reported in Table 3. Referring to Figure 3, the dashed thin lines show the auction prices for the respective matched traders. These lines are at the midpoint of the ask and bid prices submitted by the matched seller-buyer pairs.

Formally, the auction profit for an individual trader *i* in market structure *j* is given by

$$\Pi SA_{ij} = \sum_{k=1}^{6} \left(PA(B_k, S_i)_j - MC_{ij} \right) * QAS_i(B_k)_j \tag{5}$$

for sellers, and by

$$\Pi BA_{kj} = \sum_{i=1}^{3} \left(MR_{kj} - PA(B_k, S_i)_j \right) * QAB_k(S_i)_j \tag{6}$$

for buyers, where

i = 1,2,3 respectively refers to seller 1, seller 2, and seller 3.

j = 1,2,3 respectively refers to market structure 1, 2, and 3

k = 1,2,3,4,5,6 respectively refers to buyer 1 through buyer 6

ΠSA_{ij} = Auction profit for seller i under market structure j

MC_{ij} = Marginal cost for seller i under market structure j

$PA(B_k, S_i)_j$ = Price discrimination price between seller i and buyer k under market structure j

$QAS_i(B_k)_j$ = Quantity matched between seller i and buyer k under market structure j

ΠBA_{kj} = Auction profit for buyer k under market structure j

MR_{kj} = Marginal revenue for buyer k under market structure j

$QAB_k(S_i)_j$ = Quantity matched between buyer k and seller i under market structure j

By construction, $QAS_i(B_k)_j = QAB_k(S_i)_j$.

Note that the values of $PA(B_k, S_i)_j$, $QAS_i(B_k)_j$ and $QAB_k(S_i)_j$ are decided by the clearinghouse matching mechanism.

By using the above notation, Table 3 formally can be reported as follows :

Sellers	**Auction Price**	**Quantities**
1	$PA(., S_1)_1=0$	$QSA_1(.)=0$
2	$PA(., S_{21}=0$	$QSA_2(.)=0$
3	$PA(B_4,S_3)_1=21$	$QSA_3(B_4)=10$
3	$PA(B_1S_3)_1=20$	$QSA_3(B_4)=10$
3	$PA(B_5S_3)_1=15$	$QSA_3(B_4)=10$
3	$PA(B_2S_3)_1=14.5$	$QSA_3(B_4)=10$
Buyers	**Auction Price**	**Quantities**
1	$PA(B_1S_3)_1=20$	$QBA_1(B_4)=10$
2	$PA(B_2S_3)_1=14.5$	$QBA_2(B_4)=10$
3	$PA(B_5, .)_1=0$	$QBA_3(B_4)=0$
4	$PA(B_4S_3)_1=21$	$QBA_4(B_4)=10$
5	$PA(B_5S_3)_1=15$	$QBA_5(B_4)=10$
6	$PA(B_6, .)_1=0$	$QBA_6(B_4)=0$

Table V. Table III Presentation by Using Notation in Equations (5) and (6), the Auction Results from Hypothetical Ask and Bid Prices

By applying equations (5) and (6) , individual profits from the auction are as follows :

Traders ID	Notion	Profit
Seller 1	ΠSA_{11}	0
Seller 2	ΠSA_{21}	0
Seller 3	ΠSA_{31}	(21-11)x10 + (20-11)x10 + (15-11)x10 + (14.5 -10)x10 = 265
Buyer 1	ΠBA_{11}	(37 - 20)*10 = 170
Buyer 2	ΠBA_{21}	(17 - 14.5)*10 = 25
Buyer 3	ΠBA_{31}	0
Buyer 4	ΠBA_{41}	(37 - 21)*10 = 160
Buyer 5	ΠBA_{51}	(17 - 15)*10 = 20
Buyer 6	ΠBA_{61}	0

Table VI. Individual Auction Profit Calculations for the Hypothetical Ask and Bid Prices under Market Structure 1

The total seller and buyer auction profits under market structure j are respectively denoted by $PTSA_j$ and $PTBA_j$ .which have formulas as follows :

$$\Pi TSA_j = \sum_{i=1}^{3} \Pi SA_{ij} \tag{7}$$

$$\Pi TBA_j = \sum_{i=1}^{6} \Pi BA_{ji} \tag{8}$$

Under the hypothetical asks and bids scenario under market structure 1, as shown in table VI, the aggregate profits for seller and buyers are:

$\Pi TSA_1 = \Pi SA_{11} + \Pi SA_{21} + \Pi SA_{31} = 0 + 0 + 265 = \mathbf{265}$

$\Pi TBA_1 = \Pi BA_{11} + \Pi BA_{21} + \Pi BA_{31} + \Pi BA_{41} + \Pi BA_{51} + \Pi BA_{61}$

$= 170 + 25 + 0 + 160 + 20 + 0 = \mathbf{375}$

One special case of the calculation of auction profits is when all sellers and buyers ask and bid their actual MC and MR levels. In this case, we can refer to Figure 4a to 4c, for market structure 1, 2 and 3. The dashed thin lines show the auction discriminatory prices for the variously matched traders. As shown in Table VII, asterisk signs (*) mark the individual auction profit calculations for this special case

Trader's ID	**Market Structure 1**		**Market Structure 2**		**Market Structure 3**	
	Notation	**Profit**	**Notation**	**Profit**	**Notation**	**Profit**
Seller 1	ΠSA^*_{11}	0 - EM	ΠSA^*_{12}	0 - EM	ΠSA^*_{13}	0 - EM
Seller 2	ΠSA^*_{21}	0 - EM	ΠSA^*_{22}	10	ΠSA^*_{23}	105
Seller 3	PSA^*_{31}	320	PSA^*_{32}	260	PSA^*_{33}	130
Total	ΠTSA^*_{1}	320	ΠTSA^*_{2}	270	ΠTSA^*_{3}	235
Buyer 1	ΠBA^*_{11}	130	ΠBA^*_{12}	130	ΠBA^*_{13}	130
Buyer 2	ΠBA^*_{21}	30	ΠBA^*_{22}	5	ΠBA^*_{23}	0 - EM
Buyer 3	ΠBA^*_{31}	0 - EM	ΠBA^*_{32}	0 - EM	ΠBA^*_{33}	0 - EM
Buyer 4	ΠBA^*_{41}	130	ΠBA^*_{42}	130	ΠBA^*_{43}	105
Buyer 5	ΠBA^*_{51}	30	ΠBA^*_{52}	5	ΠBA^*_{53}	0 - EM
Buyer 6	ΠBA^*_{61}	0 - EM	ΠBA^*_{62}	0 - EM	ΠBA^*_{63}	0 - EM
Total	$\boldsymbol{\Pi TBA^*_{1}}$	**320**	$\boldsymbol{\Pi TBA^*_{2}}$	**270**	$\boldsymbol{\Pi TBA^*_{3}}$	**235**

Table VII. Individual and Aggregate Auction Profit Calculations when Sellers and Buyers Submit their Actual MC and MR Levels. EM denotes an extra-marginal trader

IV. C. Market Power

Mas-Collel , Winston, and Green, [8] defines market power as "the ability to alter profitably prices away from competitive levels". In this auction framework, traders could alter their profits away from the competitive levels by submitting asks and bids different than their actual MC or MR levels. Individual, aggregate, and structural market power levels will next be defined.

IV. C.1. Auction Market Power

The market power for buyer k under market structure j is defined as :

$$MPB_{kj} = \frac{\Pi BA_{kj} - \Pi BCE_{kj}}{\Pi BCE_{kj}} \tag{9}$$

The market power for seller *i* under market structure *j* is defined as :

$$MPS_{ij} = \frac{\Pi SA_{kj} - \Pi SCE_{kj}}{\Pi SCE_{kj}} \tag{10}$$

=

Aggregate buyers market power under market structure *j* is defined as :

$$AMPB_j = \frac{\Pi TBA_j - \Pi TBCE_j}{\Pi TBCE_j} \tag{11}$$

Aggregate sellers market power under market structure *j* is defined as :

$$AMPS_j = \frac{\Pi TSA_j - \Pi TSCE_j}{\Pi TSCE_j} \tag{12}$$

IV. C2. Structural Market Power

Structural market power is a special case of auction market power, when all sellers and buyers ask and bid their true MC and MR levels. The expression for structural market power has the same basic form as the expression for as auction market power. Hereafter, an asterisk will be used to distinguish a structural market power calculation from a regular auction market power calculation.

The structural market power for buyer *k* under market structure *j* is defined as :

$$MPB^*_{kj} = \frac{\Pi BA^*_{kj} - \Pi BCE_{kj}}{\Pi BCE_{kj}} \tag{13}$$

The structural market power for seller *i* under market structure *j* is defined as :

$$MPS^*_{ij} = \frac{\Pi SA^*_{kj} - \Pi SCE_{kj}}{\Pi SCE_{kj}} \tag{14}$$

Aggregate buyers structural market power under market structure *j* is defined as:

$$AMPB^*_j = \frac{\Pi TBA^*_j - \Pi TBCE_j}{\Pi TBCE_j} \tag{15}$$

Aggregate sellers structural market power under market structure *j* is defined as :

$$AMPS^*_j = \frac{\Pi TSA^*_j - \Pi TSCE_j}{\Pi TSCE_j} \tag{16}$$

IV. D. Efficiency

The efficiency of market *j* operating under the auction protocol is defined to be total auction profits divided by total profits attained in competitive equilibrium, which is expressed as follows:

$$EA_j = \frac{\Pi TSA_j + \Pi TBA_j}{\Pi TSCE_j + \Pi TBCE_j} \qquad \in \ [0,1] \tag{17}$$

V. Roth- Erev Learning Algorithm

This learning algorithm is characterized by three main parameters: *a scaling parameter s(1);* a *recency parameter r;* and an *experimentation parameter e*. In the initial auction round, each trader assigns an equal "propensity value" to each of his feasible actions (price offers). The trader then updates these propensity values at the end of every subsequent auction round based on the profits he obtains during this auction round.

More precisely, at the beginning of the first auction round, the propensity value that a trader *j* assigns to a feasible action *k* is given by the following equation :

$$q_{jk}(1) = s(1)\, X/K \;; \tag{18}$$

where

$q_{jk}(n)$: trader j's propensity value for action at the beginning of auction round n;

$s(1)$: scaling parameter;

K : total number of feasible actions for each trader;

X : average trader profits in any given auction round (a scaling measurement).

As shown above, at the beginning of the initial auction round, each feasible action for each trader has an equal propensity value. Once propensity values are obtained, the probability with which agent *j* chooses action *k* is calculated as follows:

$$p_{jk}(n+1) = \frac{q_{jk}(n+1)}{\sum_{m=1}^{K} q_{jm}(n+1)} \tag{19}$$

Since the initial propensities are all assigned equal values, equation (19) ensures that the initial choice probability distribution for each trader is a uniform distribution over his set of *K* feasible actions.

The propensity updating mechanism is explained as follows. Suppose that, in the n^{th} auction round, trader j chooses action k' and receives a profit $R(j,k',n)$ For the next auction round, the propensity that trader j associates with each of his feasible actions k is then updated as follows:

$$q_{jk}(n+1) = (1-r)q_{jk}(n) + E(j,k,k',n,K,e) \tag{20}$$

where

$q_{jk}(n)$: the propensity assigned by trader j to action k at the beginning of auction round n

r : recency parameter

e : experimentation parameter

$E(\bullet)$: update function

The *recency* parameter r reduces the importance of past experience, since r has a value between zero and one. The update function $E(.)$ is shown as follow:

$$E(j,k,k',n,K,e) = \begin{cases} R(j,k',n)(1-e), & k = k' \\ R(j,k',n)\dfrac{e}{K-1}, & \mathrm{k} \neq \mathrm{k}' \end{cases} \tag{21}$$

The selected action k' is assigned a reward or penalty consisting of the profit $R(j,k',n)$ multiplied by the factor $(1-e)$. The unselected action also obtains a reward or penalty based on the profit obtained by the chosen action. However, this reward or penalty consists of the profit multiplied by $e/(K-1)$, hence it differs from the reward or penalty for the selected action.

Finally, the updated propensity values will change each trader's choice probability distribution for determining the selection of an action for the next auction round. This learning process takes place in each auction round, for the entire 1000 auction rounds each constituting one run of the auction. Ideally, this updating mechanism should result in the convergence of the choice probability distributions of the traders to some ultimate distributions that are sharply peaked at "best" actions for the traders.

VI. Experimental Design

The experimental design focuses on two treatment factors: market capacity; and the key Roth–Erev experimentation parameter *e* that controls that degree of agent experimentation. The market structures used in this research are adapted from Nicolaisen et al. [8],[9]. The changes in market capacity will cause a shift in structural market power advantages. As shown in the upper-left corners of Tables VIII-X, structural market power advantages shift from the sellers to the buyers as the market structure moves from structure 1 to structure 3.

Market Structure	**1**	**2**	**3**
Seller's Capacity	40	20	10
Buyer's Capacity	10	10	10

Table VIII. Three Different Market Structures

For each market structure, 8 different values for the experimentation parameter *e* are tested, ranging from $e = 0$ to $e = 1$. Specifically, these values are as follows: {1.0, $(K\text{-}1)/K$, 0.8, 0.5, 0.3, 0.2, 0.1, 0.0). K is the number of feasible actions for each trader. In this study, $K = 30$. Two of the *e* values represent special cases of human behavior. The first case (NL) is when the traders have no learning capability. The second case ("Best Fit") is when e represents the “best fit” to human behavior as determined by Roth and Erev [2]. The no-learning situation NL happens when $e = (K\text{-}1)/K$, and the “best fit” situation is when $e = 0.2$. The derivation of no-learning situation will be explained in discussion section.

In this study, the trader's MC and MR values and their strategy domains are also adapted from Nicolaisen et al.[9]. The values of MC or MR for individual traders are shown in Table IX. Note that there are three different values for both MC and MR. These values represent three different types of operating cost: expensive, average, and cheap. Each seller has a feasible action (price offer) set that consists of 30 feasible actions (K = 30) spaced uniformly over the interval [MC, MC + $40/MwH]. Since each action is equally spaced, the difference from one action to the next is $1.33/MwH. Similarly, each buyer has a feasible action (price offer) set that consists of 30 feasible actions spaced uniformly over the interval [MR-40, MR] if MR-40 > 0 , or [0, MR] if MR-40 < 0. The spacing between successive actions will then be $1.33/MwH and MR/30, respectively. With these conditions, sellers will never submit an ask price lower than their MC, and buyers will never submit a bid price lower than zero or higher than their MR. In other words, all traders are restricted to ask and bid prices within their budget intervals.

Sellers	**MC ($/MWh)**
1	35
2	16
3	11
Buyers	**MR ($/MWh)**
1	37
2	17
3	12
4	37
5	17
6	12

Table IX. Linear Revenue and Cost Curve.

The values of the two parameters s(1) and r for the Roth-Erev reinforcement learning algorithm are set the same as in Nicolaisen [9] for their "best fit" case: namely, $s(1)$ = 9.00, r=0.10.

A treatment therefore consists of a particular market structure 1, 2, or 3 together with a particular setting for the experimentation parameter e. For each treatment, 100 runs are generated using 100 different seed values for the pseudo-random number generator. Each run consists of 1000 auction rounds. Finally at the end of each run (i.e., at the end of the 1000th round), data is collected and is averaged across the 100 runs. To have a better sense of outcome variability, the mean and standard deviation of outcomes across the 100 runs are presented.

VII. Experimental Results

Tables V – VII report structural market power outcomes, individual and aggregate market power outcomes, and efficiency auction outcomes for three different market structures. For each market structure, 8 different values are tested for the experimentation parameter value *e*.

All evaluated outcomes are recorded at the end of each auction round for each run. In this computational experiment, we generate 100 runs for each different treatment. Therefore, there will be 100 sets of evaluated outcomes for each treatment. The mean values of outcomes across these 100 runs are taken to represent the evaluated outcomes for the treatment. In addition to average values, standard deviations are also presented to give a clearer picture of the variability in the observations. An asterisk mark is given to a mean market power outcome if it is within one standard deviation of the structural market power outcome.

As shown in Tables X - XII and Figures 3 - 12, as *e* decreases, all the evaluated outcomes such as market power levels and market efficiency increase. At the same time, as shown in Table XI, the sources of inefficiency decrease.

Figures 5 - 7 contrast the individual ask and bid prices for the no-learning case NL with the ask and bid prices under learning with positive e values, for each run for the three different market structures. It shows a significant difference between the observations. In all market structures, under no learning individual traders submit random asks and bids. In contrast, when individual traders have learning capability, they submit ask or bid prices in a much more stable range, with a smaller variance. However, the bid and ask prices of some traders still have a relatively large variance.

The price offers for Seller 3, Buyer 3 and Buyer 6 are not depicted because they are extra-marginal traders that have too high a marginal cost and too low a marginal revenue, respectively. As a result, they could not be matched in any market structure. On the other hand, there is no guarantee that any infra-marginal traders will sell or buy something in the auction. As shown in Table XIII, except in market structure 3 with $e = 0$, the event that an infra-marginal trader fails to trade always happens in all market structures and for every value of e. Another important observation is also shown in Table XIII. In every market structure, matching always happens when $e = 0$, that is, complete coordination failure is never observed.

VIII Discussion

A . Efficiency Market Power Outcomes

From Tables VIII – X and Figure 8, we can observe a systematic relation between the experimentation parameter e and efficiency outcomes. As e increases, the efficiency decreases monotonically. This happens consistently for all tested market structures. Generally, as e increases from 0 to 1, the efficiency outcomes in the three market structures decreases from 94.3 % to 23.1%, 93.2 % to 25%, and 100 % to 23.1 %, respectively.

The reasons behind the decreasing efficiency are explained in Table XI and Figure 9, which show the frequency counts of the sources of inefficiencies for each treatment. As e increases from zero to one, coordination failure (when all the agents fail to trade) increases form zero to about 50 (half the number of runs). Fifty percent of coordination failure alone will cause a 50 % decreases in the market efficiency outcomes. Moreover, even if coordination failure does not happen, inefficiency can still exist when any of the IM traders fails to trade and or any of the EM traders manages to trade.

This observation is confirmed by looking at Figures 10 –12, which shows the profit earnings for each of the market structure2. Infra-marginal traders mostly show a decreasing profit trend as *e* increases. Meanwhile, in some market structures EM traders, could manage to trade. However, no significant trends are observed for the EM traders.

The best efficiency outcomes for this computational experiment (at $e = 0$) are about the same as the ones found in the same market structure under the modified Roth-Erev (MRE) learning algorithm in Nicolaisen et al.[9]. Under MRE the efficiency observed under market structures 1,2,and 3, respectively, in Nicolaisen et al.[9] are 91.8 %, 94.2 % , and 100 %.. Similarly, by using original Roth–Erev learning algorithm with $e = 0$, we obtain efficiency levels of 94.3 %, 93.2 % and 100% respectively for market structures 1,2, and 3.

B. Market power outcomes

Nicolaisen et al.[9] tries to predict the market power outcomes in a similar auction framework by determining analytically derived structural market power level. They found that there is a consistency between **relative *structural market power*** **(SMP)** and ***experimental relative market power (EMP)*** outcomes. Specifically, they found that if sellers attained higher market power in the auction than buyers when calculated in terms of structural market power, they also obtained higher market power in the experimental auction.

Recall that structural market power measures the basic advantages or disadvantages offered to individual traders by the structure of a particular auction market. The artificial adaptive agents then try to exploit their advantages, or offset their disadvantages, by strategically submitting bid and ask prices. The traders with an advantage will try to maintain or even enhance their advantage, while the traders with a disadvantage will try to offset or even eliminate it. A positive SMP for traders of a particular type (sellers or buyers) means that these traders have an advantage under the auction protocol. As observed in the auction outcomes, they lose some of this advantage in absolute terms because their EMP is less than their SMP. However, they maintain their relative advantage, because the sign of EMP is the same as the sign of SMP. On the other hand, a negative SMP for traders of a particular type means that these traders have a disadvantage under the auction protocol. These traders gain in absolute terms in the experimental auction because their EMP is higher than their SMP. However, they are not able to overcome their relative disadvantage, because the sign of EMP remain the same as the sign of SMP. The exact extent of the absolute gain and decline in SMP for traders of each type depends on the particular market structure. In general, then, the learning capabilities of the traders permit them to reduce the absolute differences in their SMP but not their relative differences.

C. Analysis of the Roth-Erev learning Algorithm

Tables X – XII and Figures 8 - 17 show that there is a strong relation between the value of the experimentation parameter *e* and the experimental auction outcomes. As *e* decreases, the "good" outcome measures such as market efficiency, individual and aggregate market power levels, and individual profits, show an increasing pattern. On the other hand, the "bad" outcome e measures such as coordination failure show a decreasing pattern. The immediate question raised by these results is whether the outcomes under the RE learning algorithm are intended to be this highly sensitive to changes in e or whether this is something that Roth and Erev did not observe when they developed and tested this algorithm for different types of games ?

As mentioned earlier, RE learning algorithm is only one type of reinforcement learning algorithm. Under this type of learning algorithm any feasible action that has a good outcome will be rewarded by an increase in its relative "weight" or "strength".

The strength given to a particular action is related to how good the outcomes attained using this action have been. In other words, the strength of an action is proportional to the magnitude of its benefits. On the other hand, a chosen action that results in "bad" outcomes will be penalized by having its strength reduced relative to other actions. In the long run, as the learning process continues, the choice probability will be more tightly massed around the action that has resulted in the best outcomes (the "superior action"). It is expected that, after a sufficient period of learning, the probability distribution over feasible actions will be bell shaped and centred around the superior action. If the learning process is "long enough" the choice probability may even converge to a single peak at the superior action. This is the desirable long run outcome in any application making use of reinforcement learning.

In this auction framework at hand, the convergence to a single action choice is not observed for almost all of the tested e parameter values. The only treatments that show convergence to a single action are those for which $e = 0$. In all cases, **bounded-convergence** happens. . Under bounded-convergence, the probability of choosing the superior action never converges to 1. Rather, even at the very end of an experimental run, there is still a substantial positive probability that traders will choose actions other than their superior actions. In this study, it is observed consistently that the choice probability of the superior action will converge to $(1 - e)$. Meanwhile, the other choices will maintain a positive probability level with no particular probability distribution. In most cases the inferiors action are observed to have flat choice probabilities, with value of $e/K\text{-}1$.

This bounded-convergence conjecture is consistent with what we observe in Table XIII. As e decreases, the choice probability distribution becomes more tightly centred around the superior action, implying there is a smaller chance that traders will choose actions other than their superior actions. At $e = 0$ it is observed that the traders always choose their superior actions by the end of the auction. This explains why coordination failure (all traders fail to trade) is not happened when $e = 0$.

On the other hand, as e increases (note that every trader has the same value of e), the chance of choosing actions in the tails of the choice probability distribution increases. Consequently, the chance that traders will choose actions other than their superior actions increases. Under the worst-case scenario, each trader could simultaneously choose an inferior action that is extremely different than his superior action. Under this condition, complete coordination failure could occur. Table XIII shows that, as e increases, the chance that traders deviate from their superior actions increases. In consequence, coordination failure occurs more often, and the chance of experiencing the worst-case scenario increases.

What causes this bounded convergence to occur, with resulting losses in market efficiency ? This question is analytically difficult to answer since it involves the strategic behavior of all of the traders. However, intuitively, bounded convergence occurs because of the way in which the updating mechanism handles zero-profit events. In this auction, a trader might obtain zero profit when submitting particular ask or bid price k' in some auction round n. In this case, however, equation (24) implies that the trader's choice probabilities will remain unchanged for the next

$$E(j,k,k',n,K,e) = 0 \text{, for all k}$$
$$\text{if } \ R(j,k',n) = 0$$

auction round $n+1$. To see this, note from equation (21) that

(22)

Therefore, the propensity updating equations reduce to

$$q_{jk}(n+1) = (1-r)q_{jk}(n) \tag{23}$$

for all k.

This means that trader *j*'s propensity values for auction round *n+1* are equal to his current propensity values times the common scaling factor *(1-r)*. Thus the choice probability that trader *j* associates with any feasible action *k* at the beginning of the next auction round *n+1* is given by:

$$p_{jk}(n+1) = \frac{q_{jk}(n+1)}{\sum_{m=1}^{k} q_{jm}(n+1)} = \frac{(1-r)q_{jk}(n)}{\sum_{m=1}^{k}(1-r)q_{jm}(n)} = \frac{q_{jk}(n)}{q_{jm}(n)} = p_{jk}(n) \tag{24}$$

This implies that, when any trader *j* obtains a zero profit outcome, his choice probability distribution is not updated in response to this outcome. The only condition under which a trader's choice probability distribution will be updated is when the trader obtains a positive profit outcome.

As mentioned earlier, intuitively, the convergence of the traders' choice probability distributions to a nice bell shape, peaked at "superior" actions, will only occur if good action choices are rewarded and bad action choices are penalized. In the current auction framework, however, since traders use the RE learning algorithm, no penalties are imposed on actions leading to zero profits.

The conclusion that can be obtained from these observations is as follows. In any treatment in which *e* takes on a positive value, the choice probability distributions will tend to assign a persistently positive probability (bounded away from zero) to the collection of actions resulting in zero profits. This will cause the traders to choose actions other than the superior actions, even in the "long run." Therefore, only in treatments with $e=0$ should one expect to observe high market efficiency.

D. The Importance of Learning in Discrete Double Auctions

The objective of this section is to clarify the Gode and Sunder [1] conjecture about the importance of learning for market efficiency under a **continuous double auction** (CDA) market structure. Gode and Sunder claim that structure of the CDA is primarily responsible for the high market efficiency they observe in their CDA experiments. They claim that high market efficiency is attained even with zero-intelligence agents as long as budget constraints are binding on these agents.

To show the importance of learning in the current discrete-auction framework, a direct comparison of outcomes with learning and without learning is made. Under RE learning algorithm, the no-learning case can be obtained by setting the value of e such that the update function will equalize the impact between a selected action k and any unselected action k'. Referring to equation (21), this can be done as follows:

$$R(j,k',n)(1-e) = R(j,k',n)\frac{e}{K-1} \tag{25}$$

$$
\begin{aligned}
&\Leftrightarrow (1-e) = \frac{e}{K-1} \\
&\Leftrightarrow e\left(\frac{1}{K-1}+1\right) = 1 \\
&\Leftrightarrow e\left(\frac{\mathrm{K}}{\mathrm{K\text{ - }1}}\right) = 1 \\
&\Leftrightarrow e = \left(\frac{K-1}{K}\right)
\end{aligned}
\tag{26}
$$

where K = number of feasible actions for any trader ***j.***

Thus, when $e = (K\text{-}1)/K$, the traders lose their learning capability. However, as discussed in Section VI, budget constraints are always binding on the traders in the current auction framework, in the sense that the traders are not permitted to choose actions (price offers) that would definitely result in negative profits. This resembles the binding budget constraint that Gode and Sunder impose on their zero-intelligence agents.

The contrast in outcomes between learning ($e = 0$) and no learning ($e=[K\text{-}1]/K$) can be observed in Tables X - XII. These tables and figures show that, in moving from the no-learning case to the learning case, market efficiency for each of the three tested market structures increases by 396%, 336 %, and 394 %, respectively. Specifically, market efficiency improves from 0.238 to 0.943 in market structure 1, from 0.277 to 0.932 in market structure 2, and from 0.24 to 1 in market structure 3.

The contrast in outcomes between the no-learning case and the learning case are also clearly seen in Figure 5 – 8. When traders have no learning capability, the ask and bid prices of the traders in the final auction round display a random pattern across runs. On the other hand, when the traders have a learning capability, the infra-marginal (IM) traders submit ask and bid prices that cluster around a particular action with a smaller variance.

Therefore, we can conclude that in the discrete double auction (DDA) with discriminatory pricing, learning plays a very significant role. This contrasts sharply with the finding of Gode and Sunder for their CDA market structure: namely, that market structure, not learning, is the primary determinant of market efficiency.

IX. Concluding Remarks

Several important and interesting points are raised by the findings reported in the current study.

1. For a discrete double auction with discriminatory pricing, the learning capability of the traders has substantial effects on market efficiency. This finding is the opposite of what Gode and Sunder [4] found in their continuous double auction experiments. .

2. Although structural market power (SMP) is a good predictor of the relative market power levels attained by sellers and buyers in the experimental auction, as claimed in [9], learning nevertheless has important systematic effects on these experimental market power (EMP) levels in absolute terms. The differences between SMP levels and EMP levels can be used to measure the importance of learning in the market. The more substantial the difference in SMP and EMP, the more important is the learning for obtaining high market efficiency outcomes.

3. Use by traders of the Roth-Erev reinforcement learning algorithm results in systematic errors in the context of the current auction framework unless the Roth-Erev experimentation parameter e is set to 0. More precisely, it is observed that the traders' selected actions result in ever greater efficiency losses as e increases. The reason is that the traders are not responsive to zero-profit outcomes when e takes on a positive value. This lack of responsiveness prevents the convergence of the traders' choice probability

distributions to bell-shaped distributions sharply peaked at a "superior" action. In the present computational experiment, this type of convergence is only observed when $e = 0$.

4. Some potential future research areas that could extend this study are :

    a. Find an analytical proof of the problem with the Roth-Erev learning algorithm that has been found in this study, and improve the algorithm if possible

    b. The current computational experiment assumes that traders use the same learning algorithm with the same parameter values. More interesting results might be found if the traders were instead permitted to use different learning algorithms, or a learning algorithm with parameter values that differed across the traders.

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MARKET STRUCTURE			NO LEARNING			e = 1.0		
	MP			MP	Stdev		MP	Stdev
All Sellers	1.667		All Sellers	-0.365	0.742	All Sellers	-0.373	0.752
All Buyers	-0.385		All Buyers	-0.853	0.197	All Buyers	-0.86	0.192
Seller[1]	N/A		Seller[1]	0	0	Seller[1]	0	0
Seller[2]	N/A		Seller[2]	0	0	Seller[2]	0	0
Seller[3]	1.667		Seller[3]	-0.609	0.681	Seller[3]	-0.608	0.688
Buyer[1]	-0.435		Buyer[1]	-0.816	0.281	Buyer[1]	-0.831	0.272
Buyer[2]	0		Buyer[2]	-0.972	0.200	Buyer[2]	-0.972	0.207
Buyer[3]	N/A		Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	-0.435		Buyer[4]	-0.857	0.249	Buyer[4]	-0.859	0.247
Buyer[5]	0		Buyer[5]	-0.983	0.164	Buyer[5]	-0.982	0.176
Buyer[6]	N/A		Buyer[6]	0	0	Buyer[6]	0	0
			Efficiency	0.238	0.228	Efficiency	0.231	0.270
e= 0.8			**e = 0.5**			**e = 0.3**		
	MP	Stdev		MP	Stdev		MP	Stdev
All Sellers	-0.278	0.746	All Sellers	0.077	0.682	All Sellers	0.317	0.618
All Buyers	-0.798	0.219	All Buyers	-0.616	0.247	All Buyers	-0.43	0.219
Seller[1]	0	0	Seller[1]	0	0	Seller[1]	0	0
Seller[2]	0	0	Seller[2]	0	0	Seller[2]	0	0
Seller[3]	-0.538	0.740	Seller[3]	-0.192	0.823	Seller[3]	0.085	0.846
Buyer[1]	-0.768	0.298	Buyer[1]	-0.584	0.335	Buyer[1]	-0.379	0.288
Buyer[2]	-0.972	0.176	Buyer[2]	-0.879	0.283	Buyer[2]	-0.745	0.355
Buyer[3]	0	0	Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	-0.783	0.305	Buyer[4]	-0.58	0.351	Buyer[4]	-0.403	0.326
Buyer[5]	-0.983	0.164	Buyer[5]	-0.863	0.322	Buyer[5]	-0.722	0.366
Buyer[6]	0	0	Buyer[6]	0	0	Buyer[6]	0	0
Efficiency	0.299	0.292	Efficiency	0.514	0.302	Efficiency	0.71	0.261
e = 0.2			**e = 0.1**			**e = 0.0**		
	MP	Stdev		MP	Stdev		MP	Stdev
All Sellers	0.487	0.645	All Sellers	0.262	0.576	All Sellers	0.569	0.500
All Buyers	-0.355	0.212	All Buyers	-0.243	0.253	All Buyers	-0.202	0.095
Seller[1]	0	0	Seller[1]	0	0	Seller[1]	0	0
Seller[2]	0	0	Seller[2]	0	0	Seller[2]	0	0
Seller[3]	0.326	0.846	Seller[3]	0.209	0.655	Seller[3]	0.46	0.709
Buyer[1]	-0.327	0.265	Buyer[1]	-0.208	0.288	Buyer[1]	-0.185	0.100
Buyer[2]	-0.594	0.349	Buyer[2]	-0.48	0.392	Buyer[2]	-0.288	0.438
Buyer[3]	0	0	Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	-0.319	0.276	Buyer[4]	-0.212	0.281	Buyer[4]	-0.191	0.095
Buyer[5]	-0.602	0.377	Buyer[5]	-0.508	0.395	Buyer[5]	-0.323	0.453
Buyer[6]	0	0	Buyer[6]	0	0	Buyer[6]	0	0
Efficiency	0.803	0.259	Efficiency	0.852	0.274	Efficiency	0.943	0.118

Table X. Structural Market Power, Auction Market Powers and Efficiency Outcomes Under Different Values of Experimental Parameter e in Market Structure 1

MARKET STRUCTURE	MP		NO LEARNING	MP	Stdev	e = 1.0	MP	Stdev
All Sellers	1.25		All Sellers	-0.421	0.780	All Sellers	-0.486	0.770
All Buyers	-0.357		All Buyers	-0.081	0.266	All Buyers	-0.826	0.261
Seller[1]	N/A		Seller[1]	0	0	Seller[1]	0	0
Seller[2]	0		Seller[2]	1.615	6	Seller[2]	1.327	6
Seller[3]	1.364		Seller[3]	-0.606	0.789	Seller[3]	-0.651	0.767
Buyer[1]	-0.366		Buyer[1]	-0.785	0.346	Buyer[1]	-0.804	0.341
Buyer[2]	0		Buyer[2]	-0.872	0.919	Buyer[2]	-0.841	1.048
Buyer[3]	N/A		Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	-0.366		Buyer[4]	-0.829	0.302	Buyer[4]	-0.843	0.292
Buyer[5]	0		Buyer[5]	-1	0.000	Buyer[5]	-1	0.000
Buyer[6]	N/A		Buyer[6]	0	0	Buyer[6]	0	0
			Efficiency	0.277	0.345	Efficiency	0.25	0.341

e= 0.8	MP	Stdev	e = 0.5	MP	Stdev	e = 0.3	MP	Stdev
All Sellers	-0.296	0.810	All Sellers	-0.052	0.642	All Sellers	0.054	0.623
All Buyers	-0.768	0.283	All Buyers	-0.643	0.265	All Buyers	-0.392	0.279
Seller[1]	0	0	Seller[1]	0	0	Seller[1]	0	0
Seller[2]	1.865	6	Seller[2]	3.561	6	Seller[2]	1.554	5
Seller[3]	-0.493	0.864	Seller[3]	-0.381	0.824	Seller[3]	-0.083	0.812
Buyer[1]	-0.752	0.352	Buyer[1]	-0.633	0.362	Buyer[1]	-0.393	0.367
Buyer[2]	-0.747	1.374	Buyer[2]	-0.704	1.175	Buyer[2]	-0.516	1.060
Buyer[3]	0	0	Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	-0.779	0.316	Buyer[4]	-0.63	0.351	Buyer[4]	-0.38	0.381
Buyer[5]	-0.95	0.501	Buyer[5]	-0.793	0.778	Buyer[5]	-0.769	0.562
Buyer[6]	0	0	Buyer[6]	0	0	Buyer[6]	0	0
Efficiency	0.337	0.365	Efficiency	0.495	0.322	Efficiency	0.707	0.295

e = 0.2	MP	Stdev	e = 0.1	MP	Stdev	e = 0.0	MP	Stdev
All Sellers	0.27	0.404	All Sellers	0.092	0.359	All Sellers	0.222	0.362
All Buyers	-0.306	0.224	All Buyers	-0.128	0.182	All Buyers	-0.151	0.084
Seller[1]	0	0	Seller[1]	0	0	Seller[1]	0	0
Seller[2]	0.548	4	Seller[2]	1.119	2	Seller[2]	2.872	4
Seller[3]	0.244	0.632	Seller[3]	-0.001	0.417	Seller[3]	-0.019	0.636
Buyer[1]	-0.29	0.305	Buyer[1]	-0.105	0.226	Buyer[1]	-0.159	0.122
Buyer[2]	-0.693	0.612	Buyer[2]	-0.27	0.453	Buyer[2]	-0.57	0.500
Buyer[3]	0	0	Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	-0.301	0.319	Buyer[4]	-0.142	0.257	Buyer[4]	-0.122	0.089
Buyer[5]	-0.778	0.432	Buyer[5]	-0.3	0.482	Buyer[5]	-0.58	0.467
Buyer[6]	0	0	Buyer[6]	0	0	Buyer[6]	0	0
Efficiency	0.822	0.232	Efficiency	0.921	0.170	Efficiency	0.932	0.095

Table XI. Structural Market Power, Auction Market Powers and Efficiency Outcomes Under Different Values of Experimental Parameter e in Market Structure 2

MARKET STRUCTURE			NO LEARNING			e = 1.0		
	MP			MP	Stdev		MP	Stdev
All Sellers	-0.096		All Sellers	-0.78	0.274	All Sellers	-0.787	0.279
All Buyers	0.119		All Buyers	-0.735	0.335	All Buyers	-0.747	0.336
Seller[1]	0		Seller[1]	0	0	Seller[1]	0	0
Seller[2]	0		Seller[2]	-0.773	0	Seller[2]	-0.794	0
Seller[3]	-0.161		Seller[3]	-0.784	0.476	Seller[3]	-0.782	0.399
Buyer[1]	0.095		Buyer[1]	-0.78	0.476	Buyer[1]	-0.776	0.494
Buyer[2]	0		Buyer[2]	0	0.000	Buyer[2]	0	0.000
Buyer[3]	0		Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	0.143		Buyer[4]	-0.695	0.537	Buyer[4]	-0.723	0.528
Buyer[5]	0		Buyer[5]	0	0.000	Buyer[5]	0	0.000
Buyer[6]	0		Buyer[6]	0	0	Buyer[6]	0	0
			Efficiency	0.24	0.272	Efficiency	0.231	0.272
e= 0.8			**e = 0.5**			**e = 0.3**		
	MP	Stdev		MP	Stdev		MP	Stdev
All Sellers	-0.707	0.277	All Sellers	-0.506	0.321	All Sellers	-0.355	0.290
All Buyers	-0.654	0.330	All Buyers	-0.434	0.379	All Buyers	-0.269	0.355
Seller[1]	0	0	Seller[1]	0	0	Seller[1]	0	0
Seller[2]	-0.742	0	Seller[2]	-0.539	1	Seller[2]	-0.375	1
Seller[3]	-0.684	0.453	Seller[3]	-0.484	0.483	Seller[3]	-0.342	0.430
Buyer[1]	-0.656	0.567	Buyer[1]	-0.435	0.581	Buyer[1]	-0.268	0.558
Buyer[2]	0	0.000	Buyer[2]	0	0.000	Buyer[2]	0	0.000
Buyer[3]	0	0	Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	-0.654	0.550	Buyer[4]	-0.437	0.581	Buyer[4]	-0.272	0.573
Buyer[5]	0	0.000	Buyer[5]	0	0.000	Buyer[5]	0	0.000
Buyer[6]	0	0	Buyer[6]	0	0	Buyer[6]	0	0
Efficiency	0.317	0.268	Efficiency	0.526	0.316	Efficiency	0.683	0.247
e = 0.2			**e = 0.1**			**e = 0.0**		
	MP	Stdev		MP	Stdev		MP	Stdev
All Sellers	-0.318	0.277	All Sellers	-0.2	0.263	All Sellers	-0.062	0.182
All Buyers	-0.178	0.369	All Buyers	-0.135	0.272	All Buyers	0.077	0.224
Seller[1]	0	0	Seller[1]	0	0	Seller[1]	0	0
Seller[2]	-0.325	0	Seller[2]	-0.189	0	Seller[2]	-0.05	0
Seller[3]	-0.314	0.402	Seller[3]	-0.208	0.341	Seller[3]	-0.07	0.184
Buyer[1]	-0.147	0.510	Buyer[1]	-0.075	0.390	Buyer[1]	0.074	0.253
Buyer[2]	0	0.000	Buyer[2]	0	0.000	Buyer[2]	0	0.000
Buyer[3]	0	0	Buyer[3]	0	0	Buyer[3]	0	0
Buyer[4]	-0.209	0.538	Buyer[4]	-0.196	0.136	Buyer[4]	0.08	0.230
Buyer[5]	0	0.000	Buyer[5]	0	0.000	Buyer[5]	0	0.000
Buyer[6]	0	0	Buyer[6]	0	0	Buyer[6]	0	0
Efficiency	0.744	0.265	Efficiency	0.829	0.245	Efficiency	1	0.000

Table XII. Structural Market Power, Auction Market Powers and Efficiency Outcomes Under Different Values of Experimental Parameter e in Market Structure 3

Market Structure 1

	NL	0.8	0.5	0.3	0.2	0.1	0
IM fail to trade							
Seller3	71	64	42	27	22	14	11
Buyer1	16	18	22	11	7	4	0
Buyer4	22	23	24	17	8	5	0
Buyer2	48	55	69	60	35	19	6
Buyer5	52	61	71	58	40	23	5
EM manage to trade							
Seller2	21	26	30	28	20	12	16
Seller1	0	0	0	0	0	0	0
Buyer3	0	1	0	0	0	1	0
Buyer6	0	0	0	0	0	1	0
All agents	51	41	14	3	4	6	0

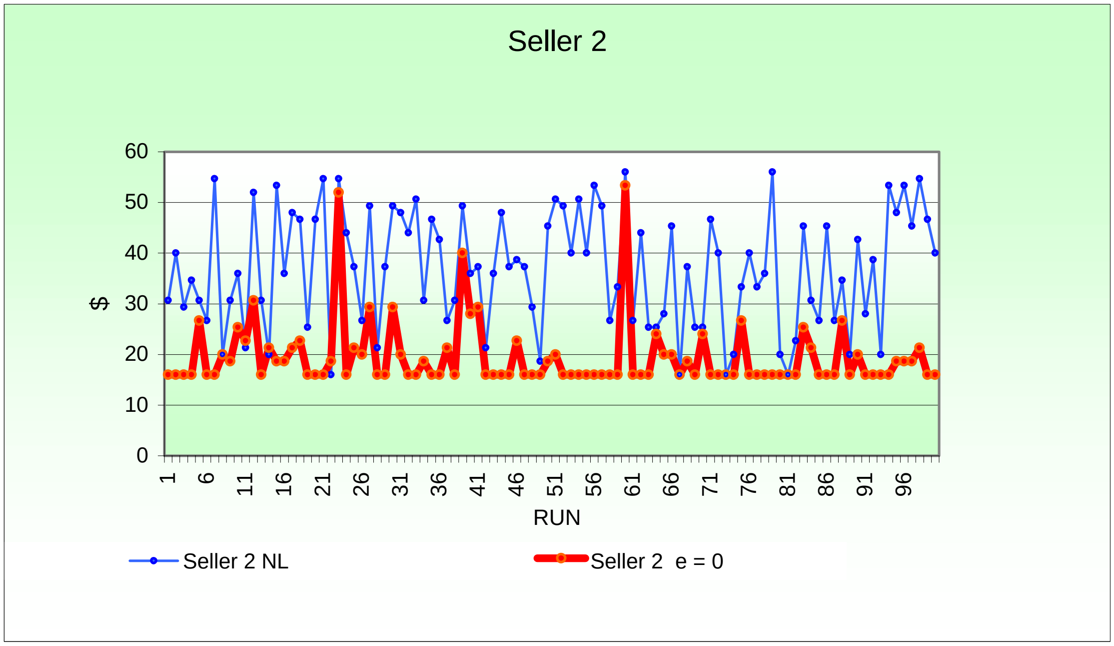
Market Structure 2

	NL	0.8	0.5	0.3	0.2	0.1	0
IM fail to trade							
Seller3	73	63	50	35	16	6	8
Seller2	77	72	55	43	66	10	26
Buyer1	16	19	27	15	12	4	0
Buyer4	19	19	25	15	13	6	0
Buyer2	46	52	73	60	71	15	53
Buyer5	47	54	72	66	72	17	53
EM manage to trade							
Seller1	0	0	0	0	0	0	0
Buyer3	0	0	0	0	0	0	0
Buyer6	0	0	0	0	0	0	0
All agents	54	46	20	8	2	1	0

Market Structure 2

	NL	0.8	0.5	0.3	0.2	0.1	0
IM fail to trade							
Seller3	70	59	37	22	16	9	0
Seller2	77	68	44	34	20	17	0
Buyer1	26	30	22	24	16	9	0
Buyer4	21	25	28	25	24	15	0
EM manage to trade							
Seller1	0	1	1	0	0	0	0
Buyer2	1	1	2	1	1	0	0
Buyer5	1	0	0	1	0	0	0
Buyer3	0	0	0	0	0	0	0
Buyer6	0	0	0	0	0	0	0
All agents	53	38	16	2	1	1	0

Table XIII. Sources of Inefficency Frequency Counts



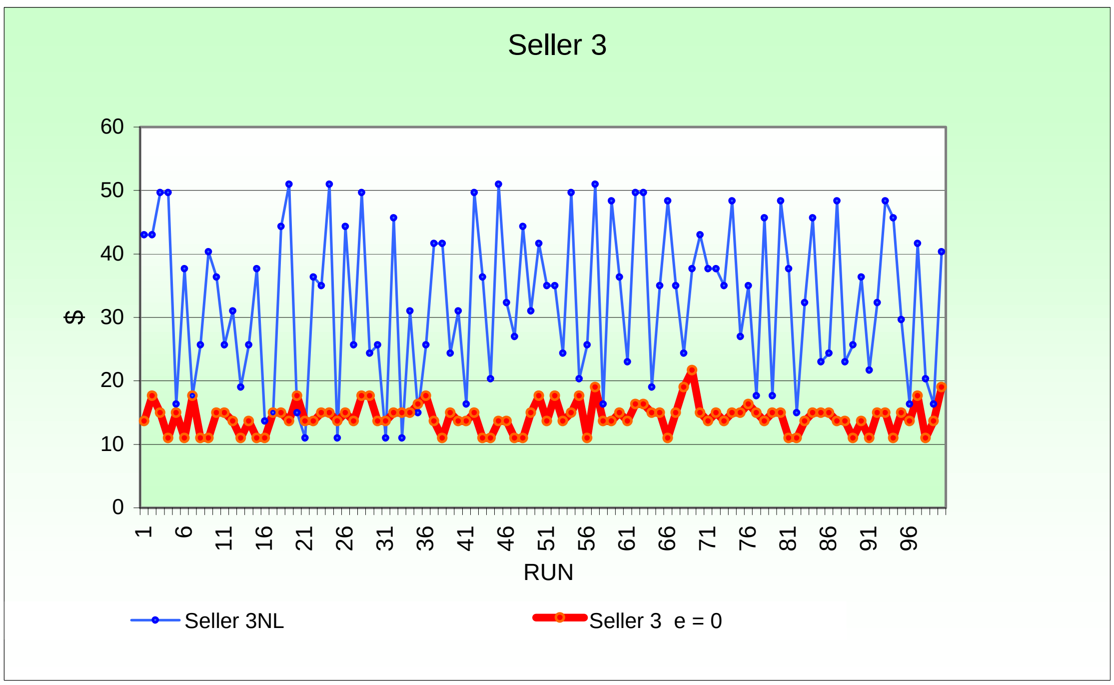
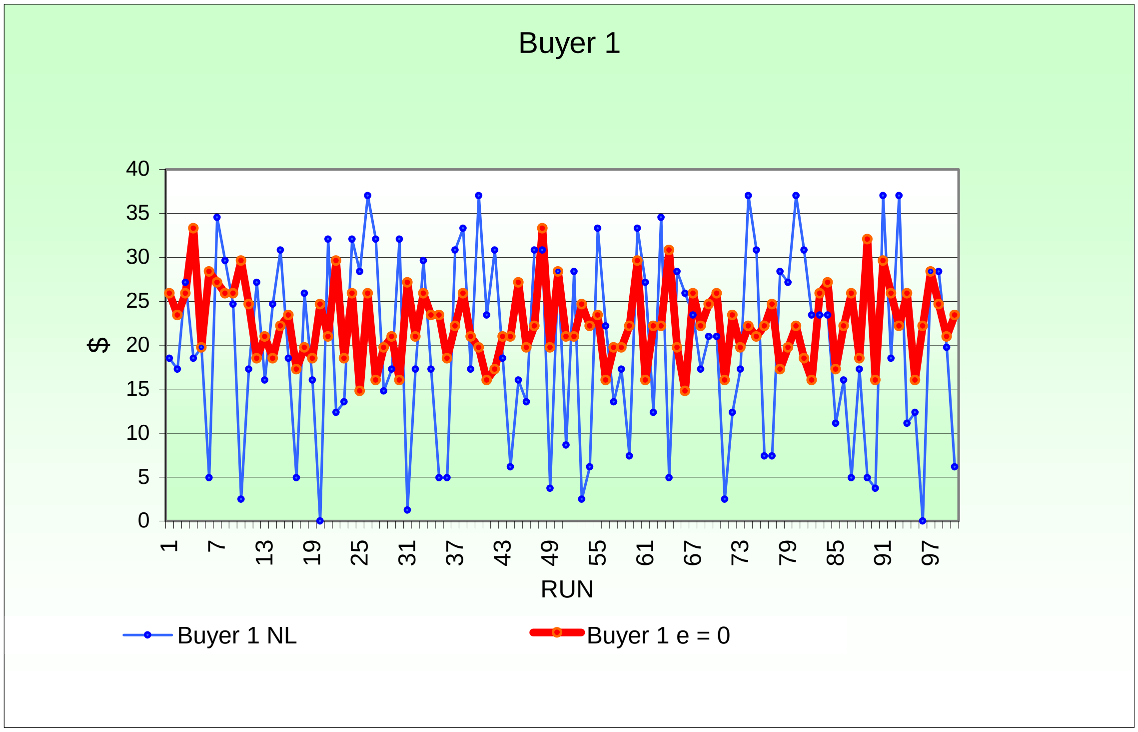


Figure 5A. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round No Learning vs Learning (e=0), for **Market Structure 1**, Seller2 and Seller 3



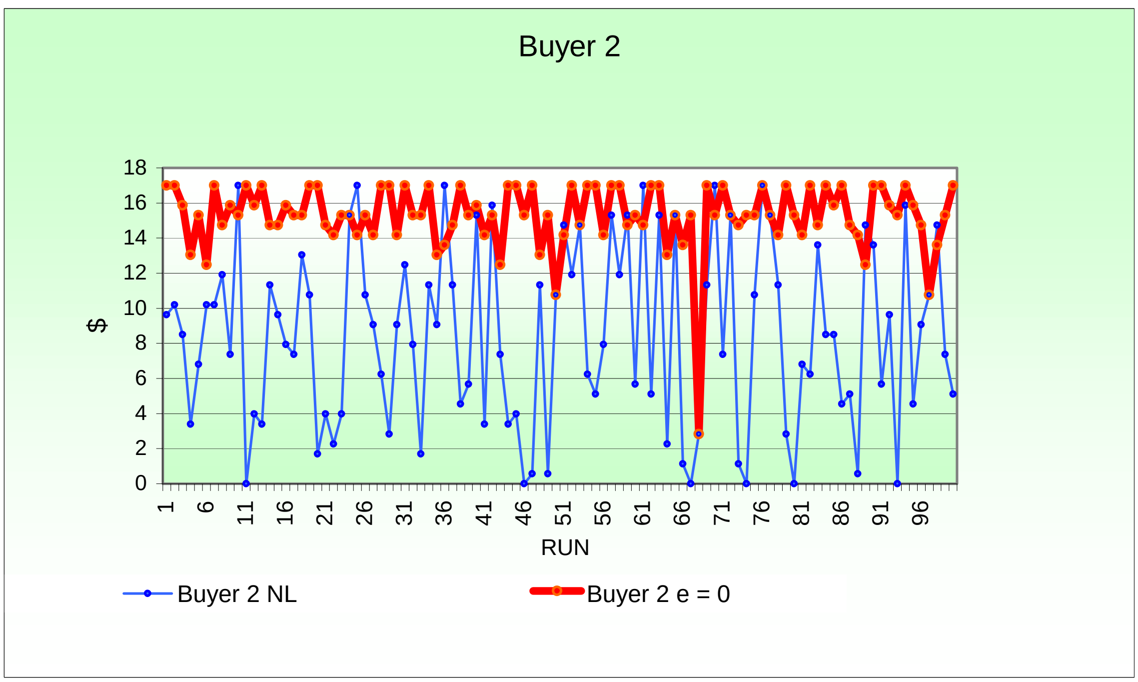
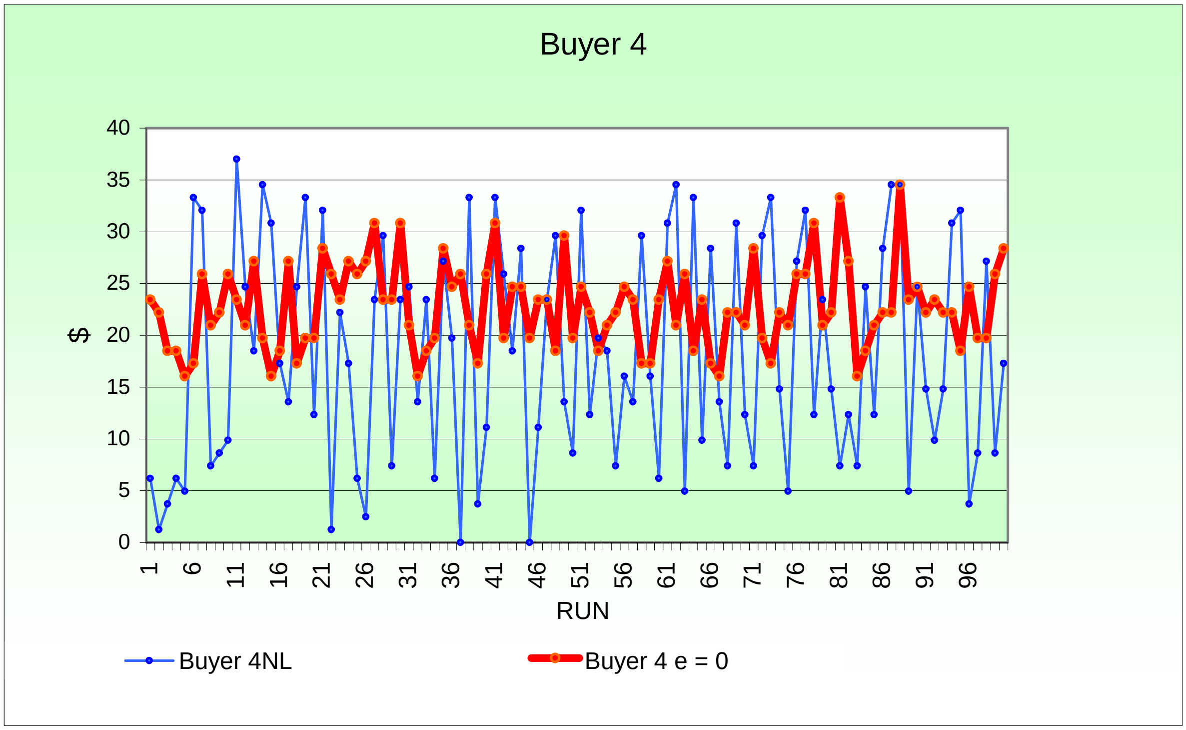


Figure 5B. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round No Learning vs Learning (e=0), for **Market Structure 1**, Buyer 1 and Buyer 2



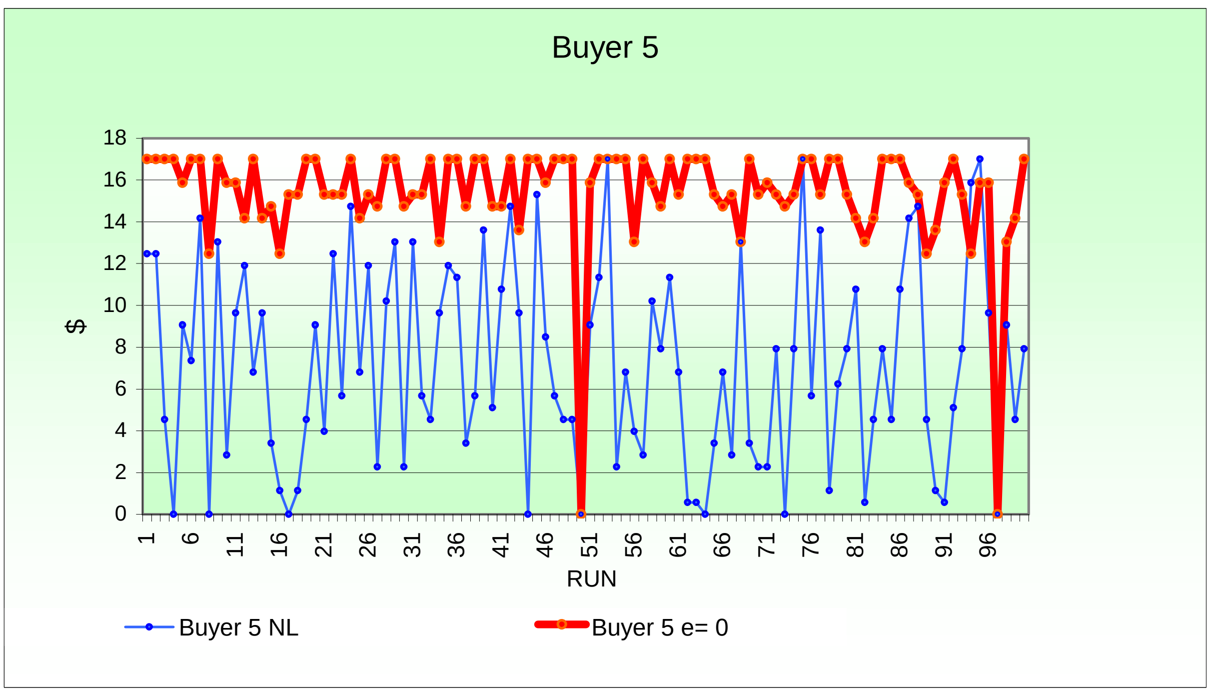
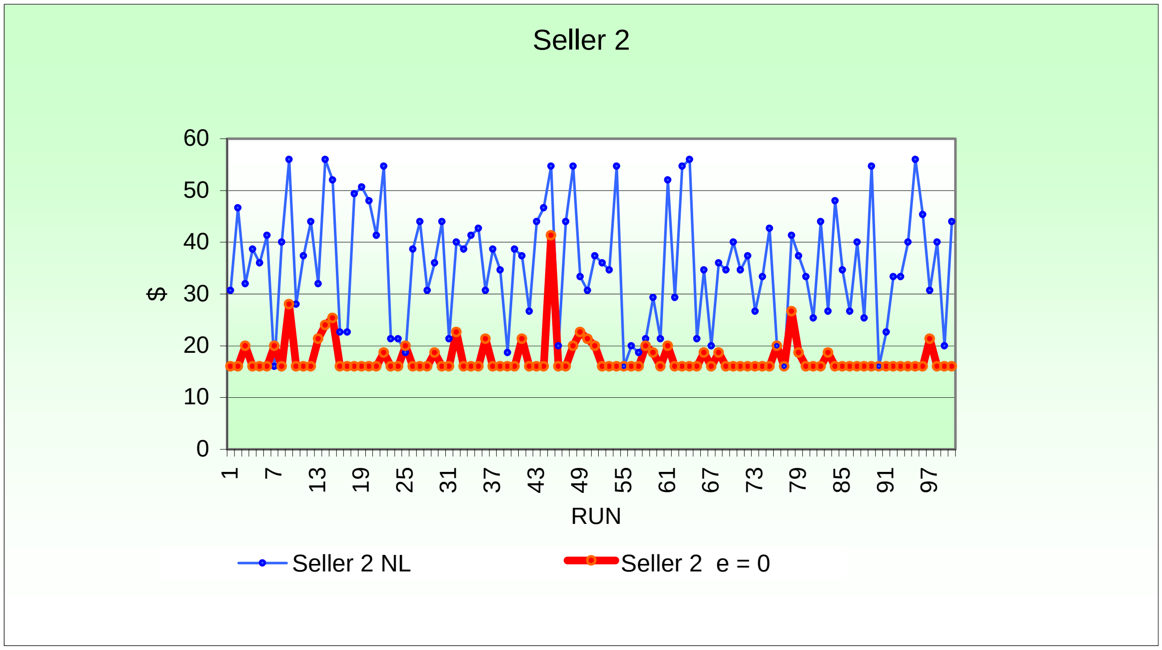


Figure 5C. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round
No Learning vs Learning (e=0), for **Market Structure 1**, Buyer 3 and Buyer 4



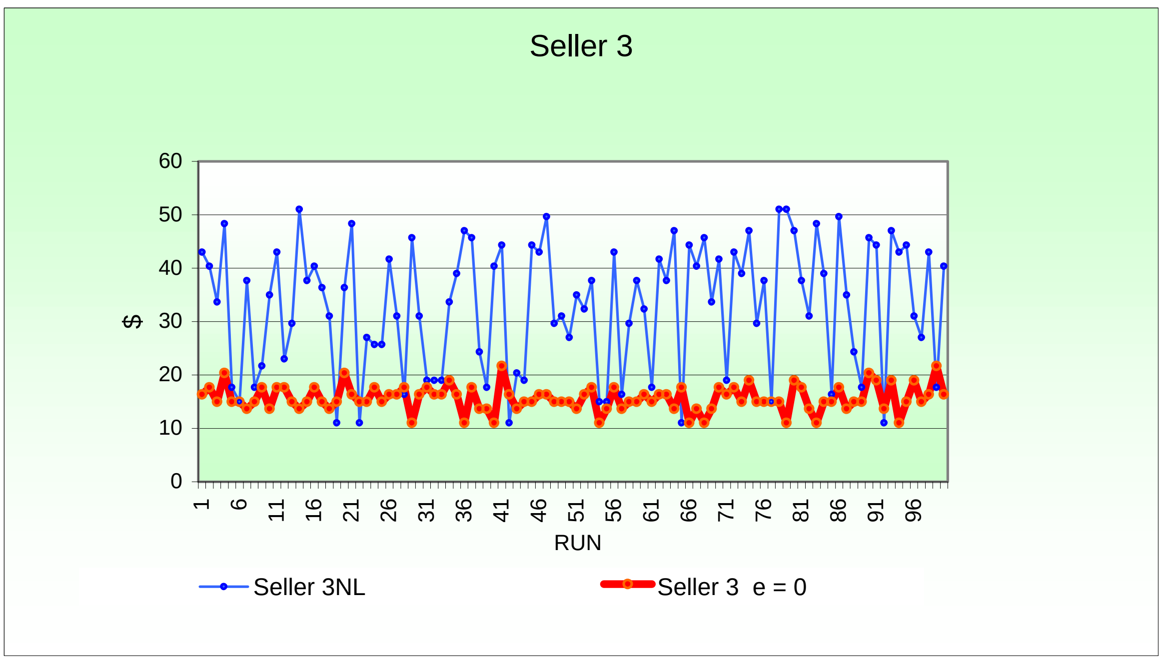
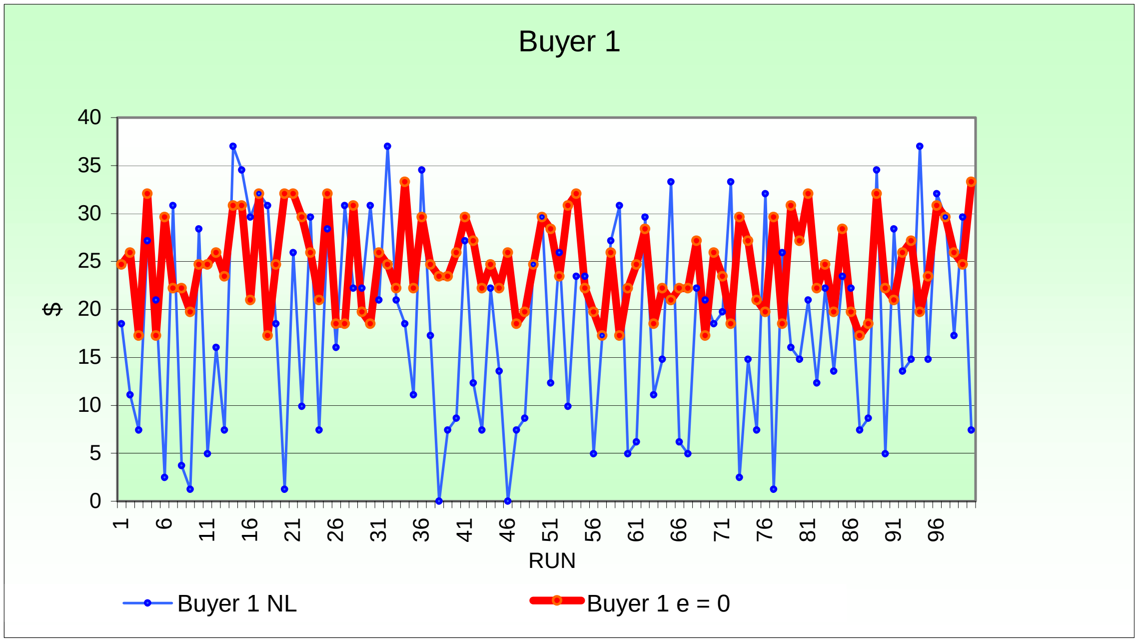


Figure 6A. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round No Learning vs Learning (e=0), for **Market Structure 2**, Seller 2 and Seller 3



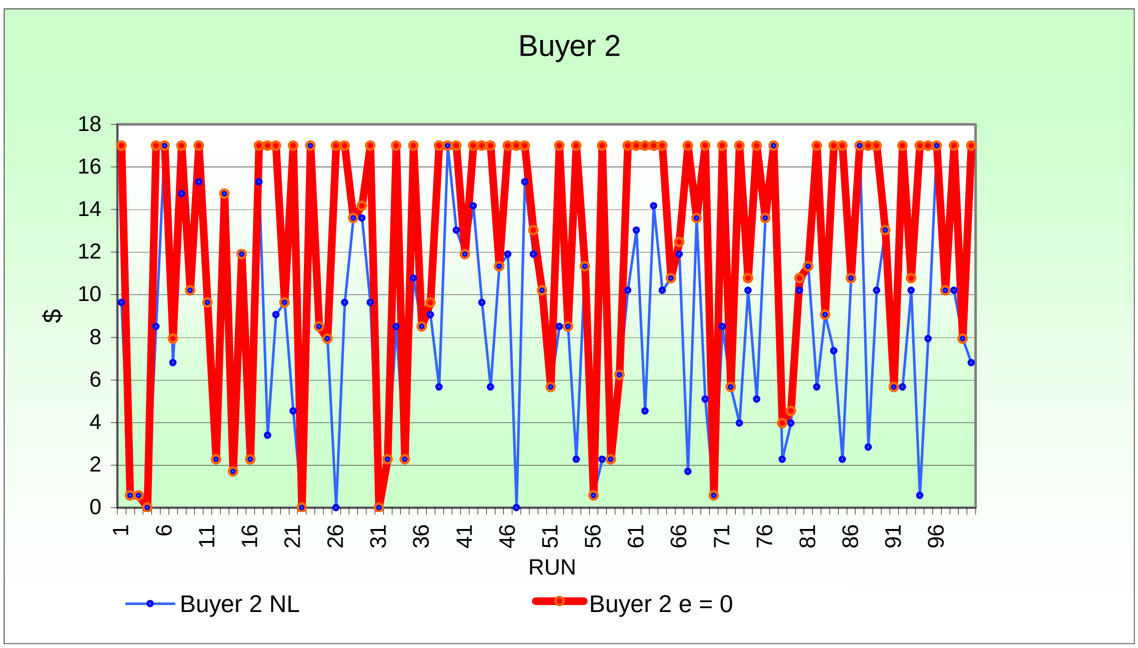
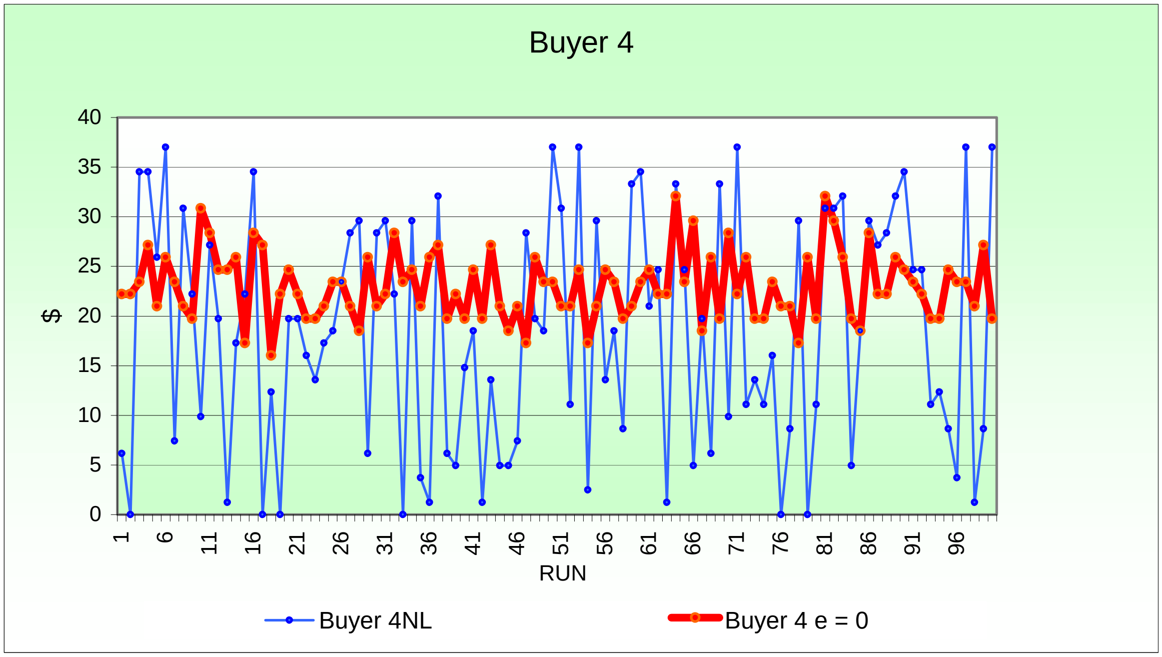


Figure 6B. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round No Learning vs Learning (e=0), for **Market Structure 2**, Buyer 2 and Buyer 3



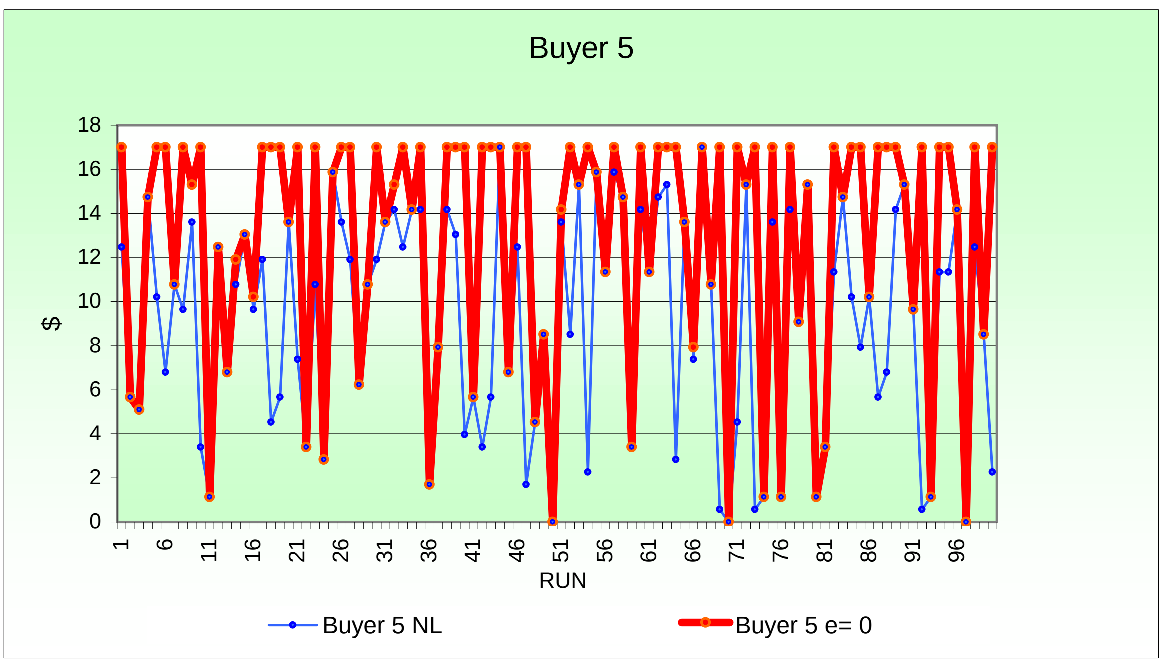
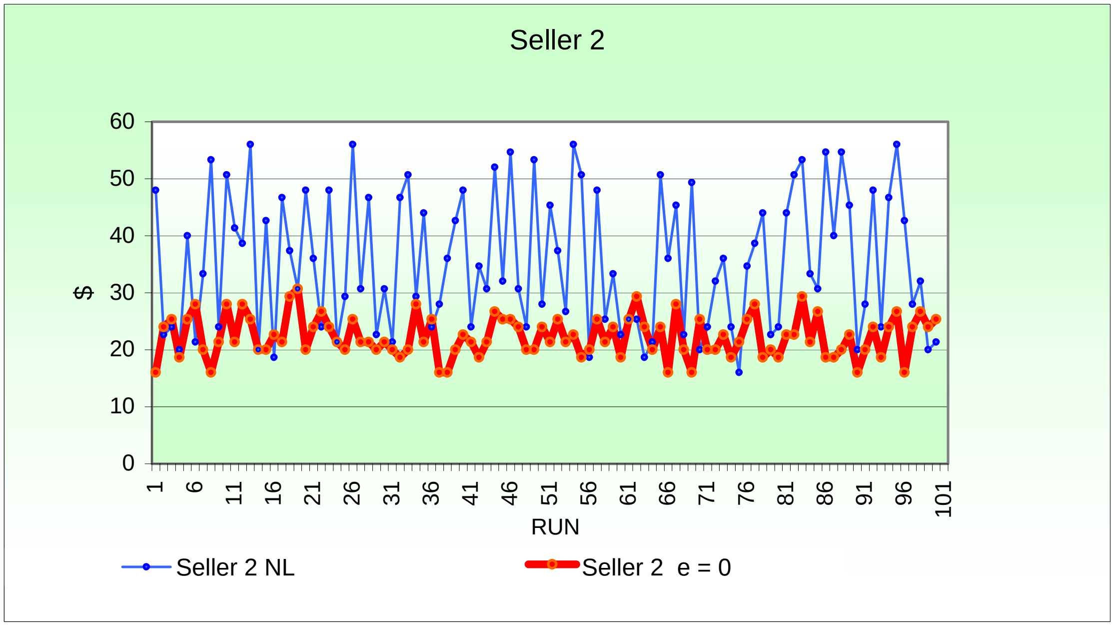


Figure 6C. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round No Learning vs Learning (e=0), for **Market Structure 2**, Buyer 4 and Buyer 5



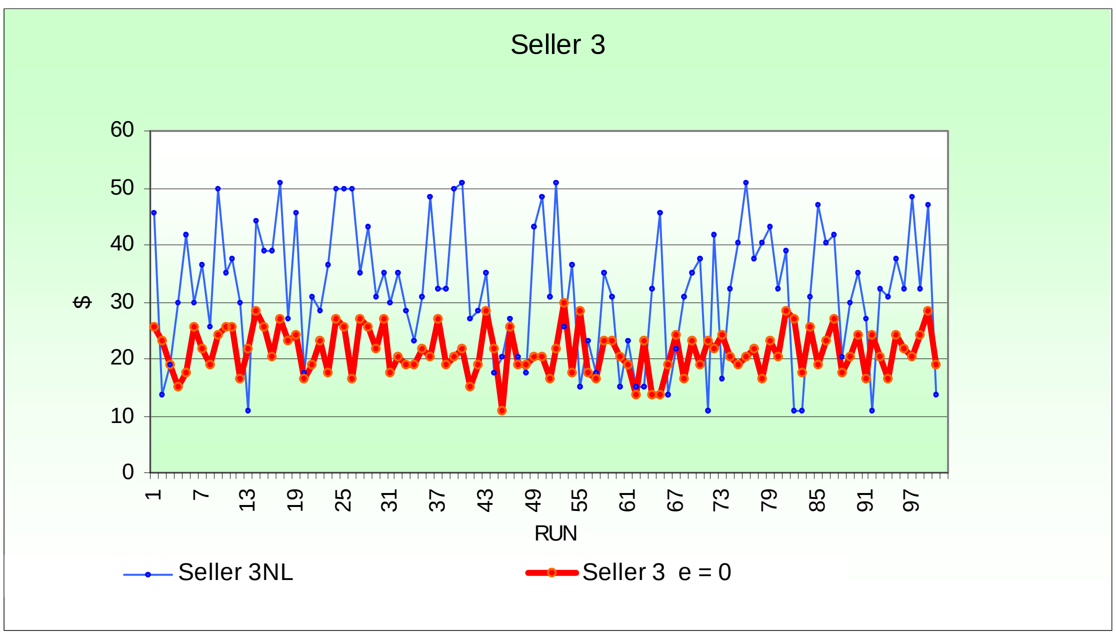
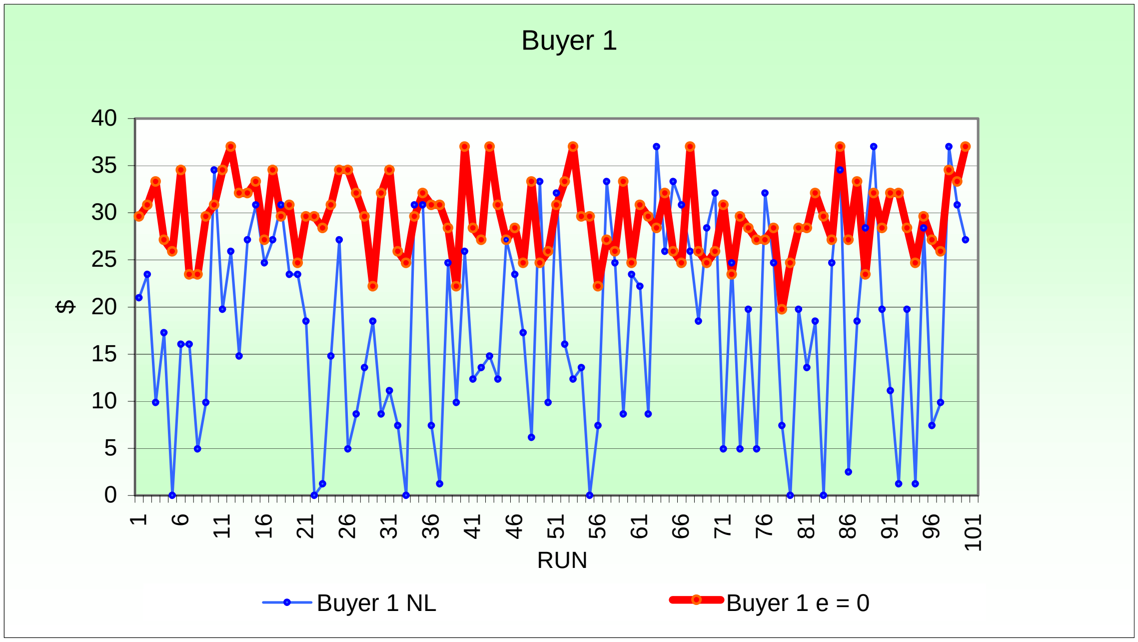


Figure 7A. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round No Learning vs Learning (e=0), for **Market Structure 3**, Seller 2 and Seller 3



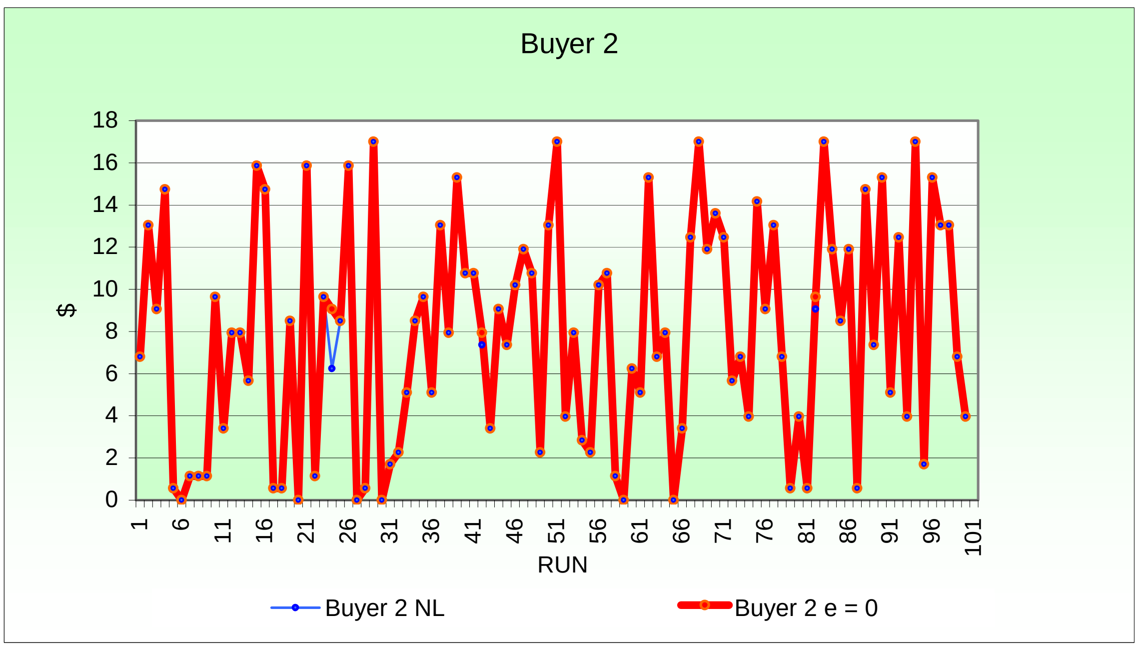
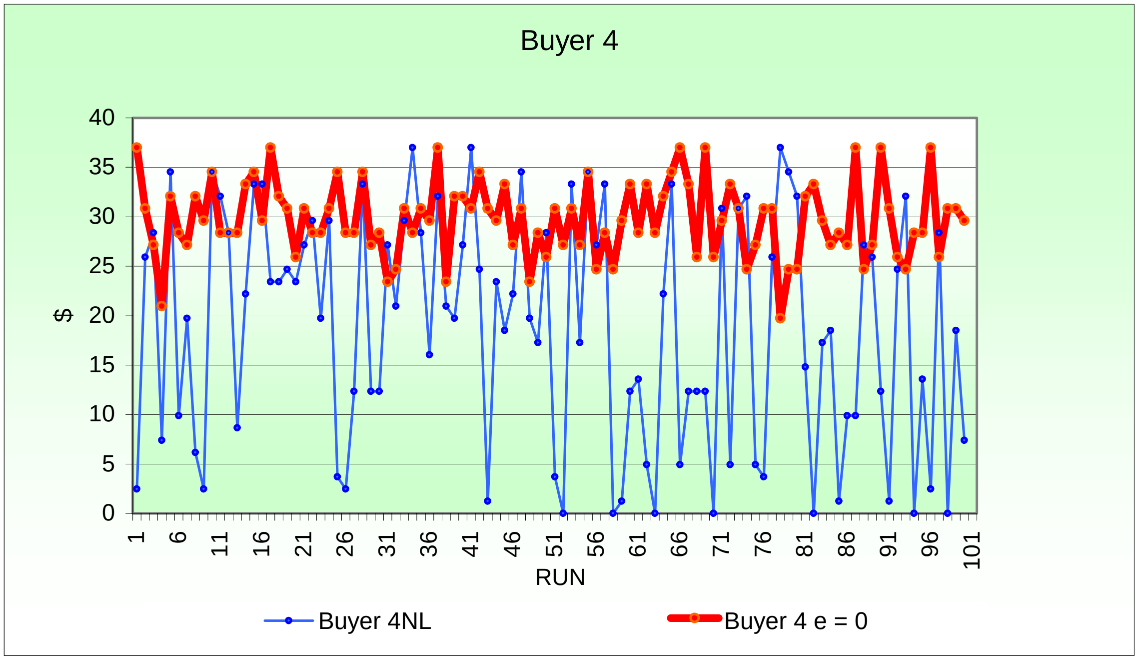


Figure 7B. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round
No Learning vs Learning (e=0), for **Market Structure 3**, Buyer 1and Buyer 2



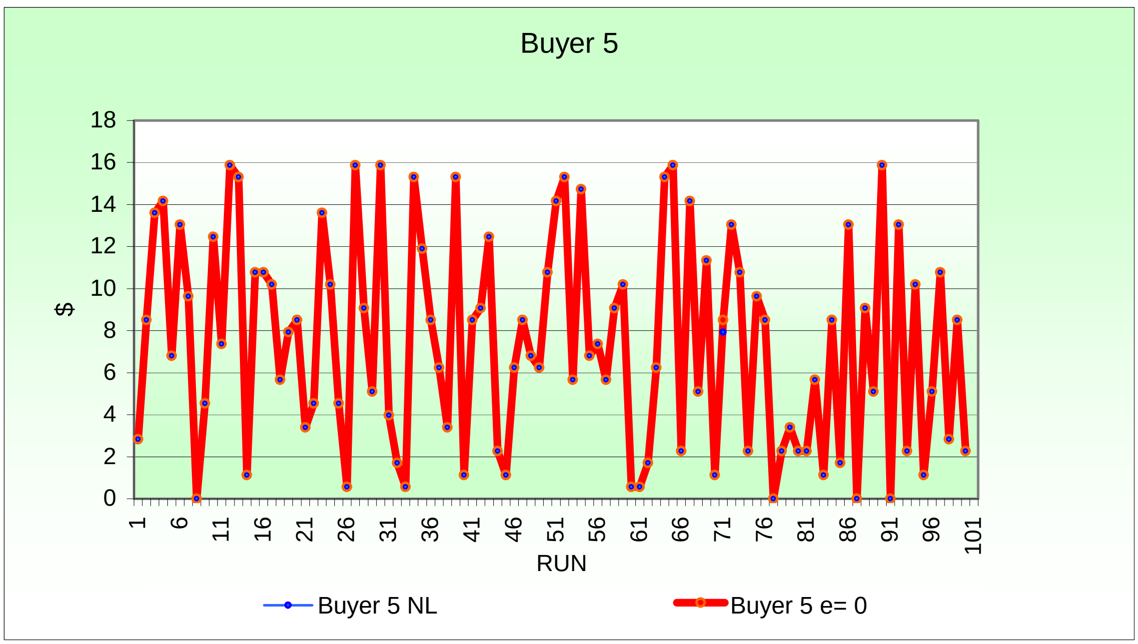


Figure 7C. Comparison of Ask and Bid Prices Recorded at the 1000th Auction Round
No Learning vs Learning (e=0), for **Market Structure 3**, Buyer 4 and Buyer 5

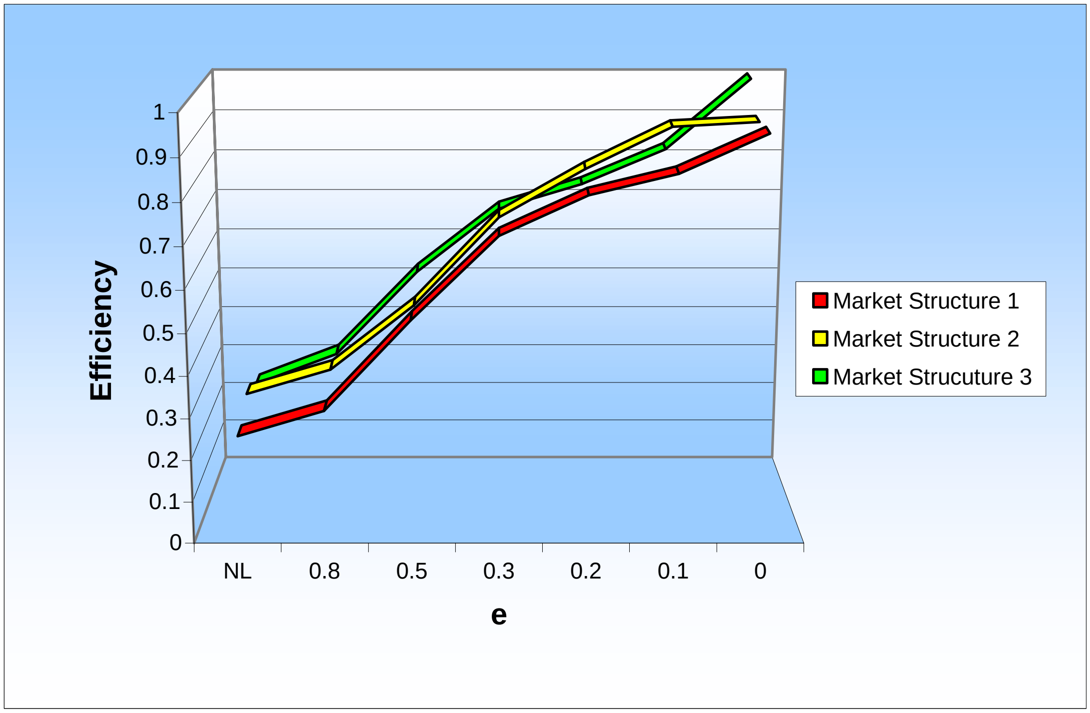


Figure 8. Efficiency Vs Experimentation Parameter *e* for Different Market Structure

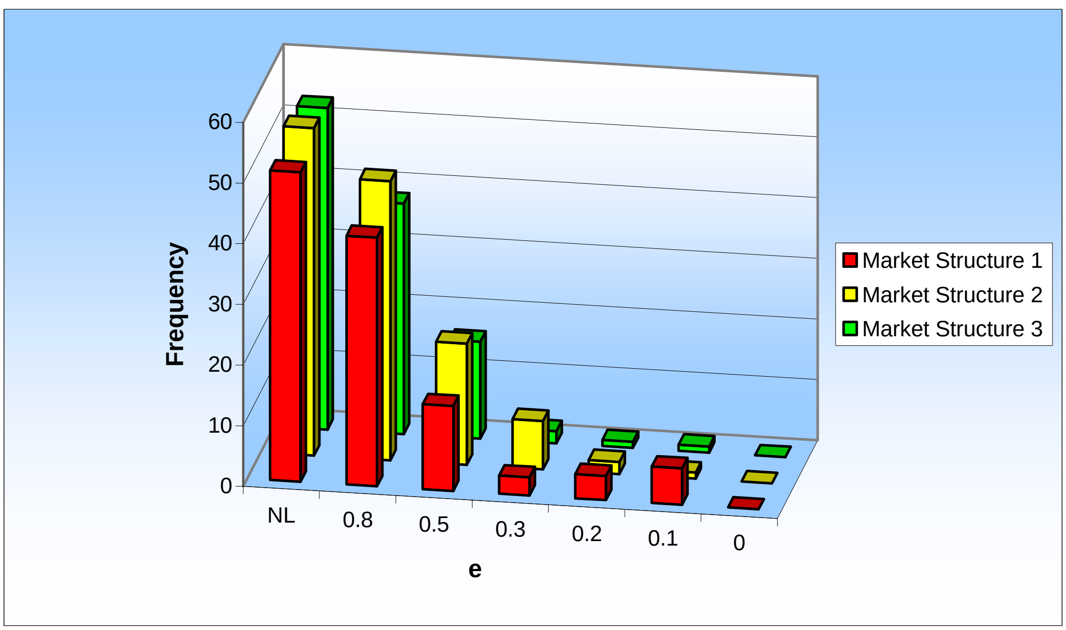
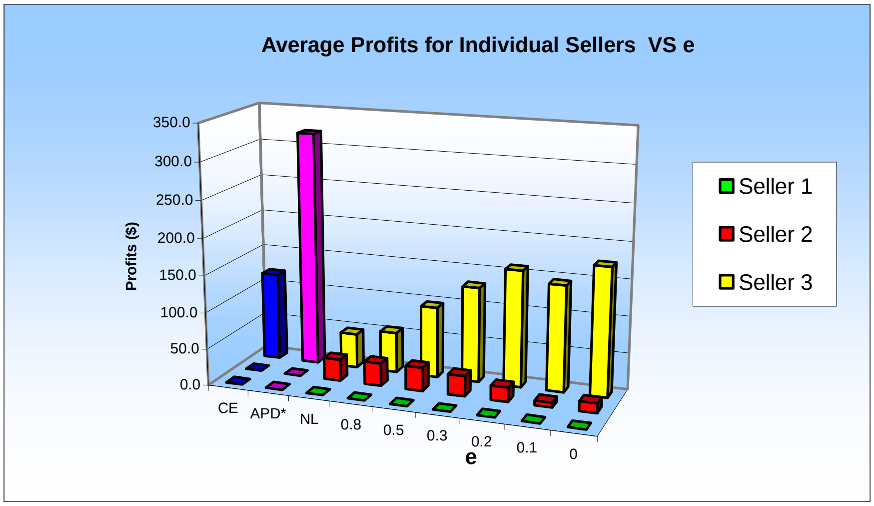
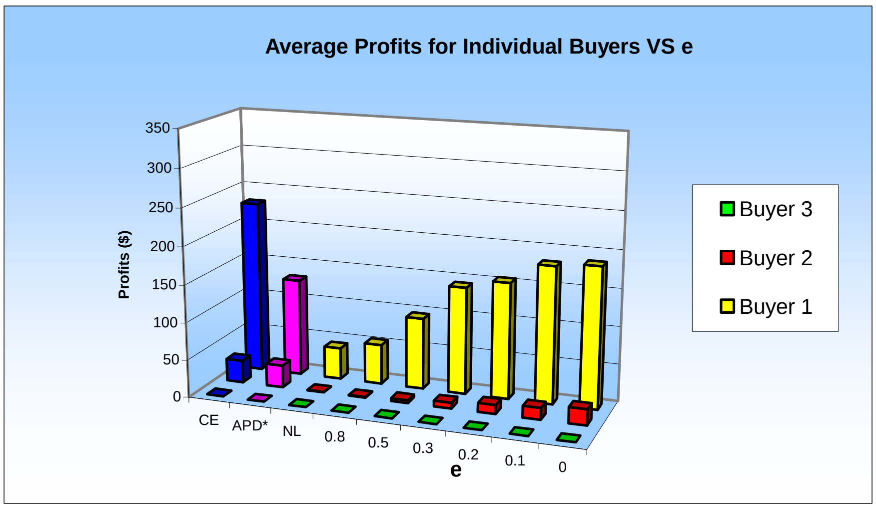


Figure 9. Fail to Trade Frequency Vs Experimentation Parameter *e*





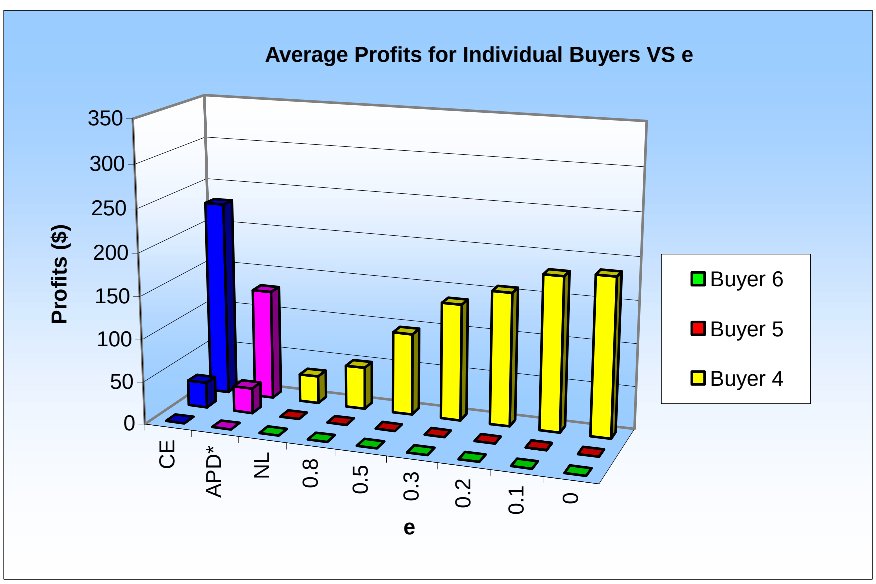
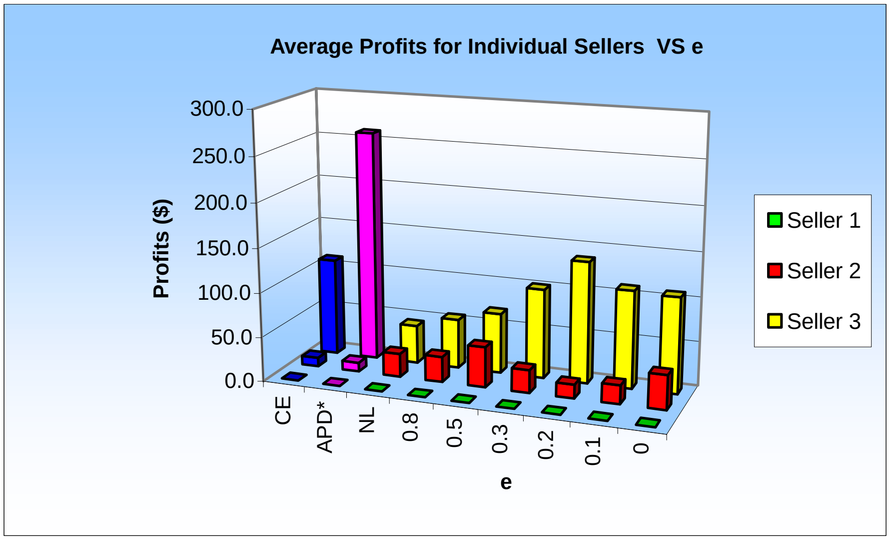
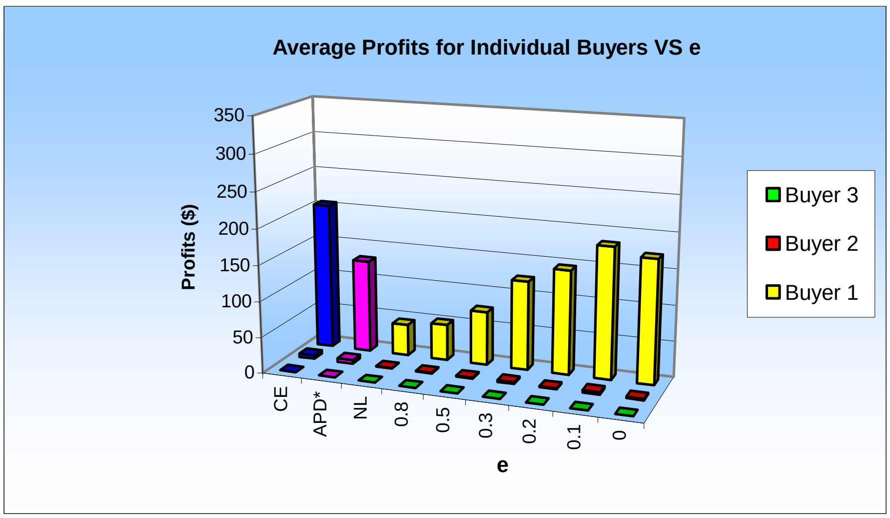


Figure 10. Average Total Profits Earn in Final Auction Run for Buyer and Seller in **Market Structure 1** Vs Experimentation Parameter e
CE = Calculated Profits Under Competitive equilibrium
APD* : Calculated Profits Under Auction Protocol when Seller and Buyer Bids their Actual MR and MC





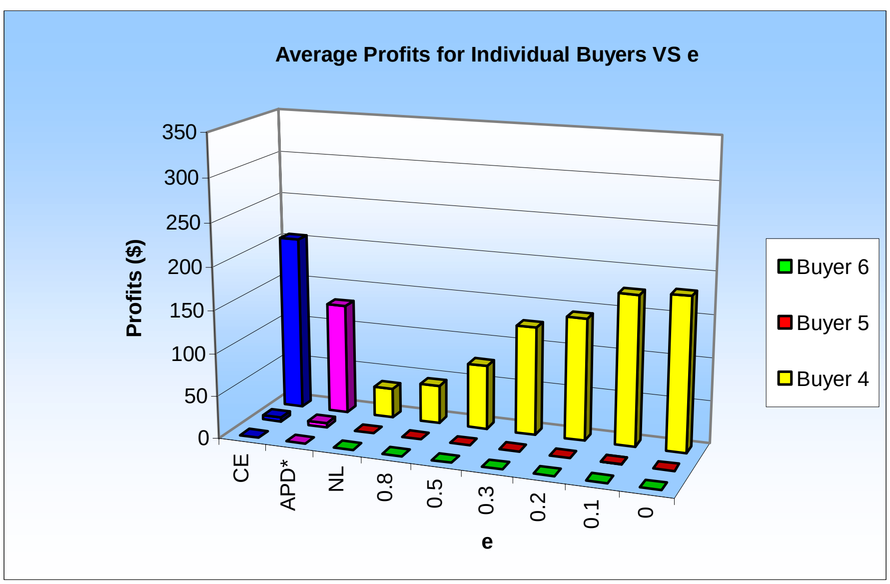
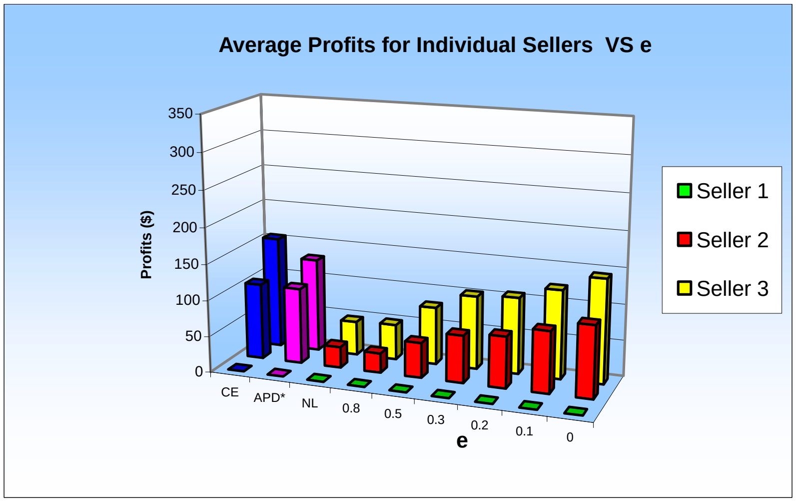
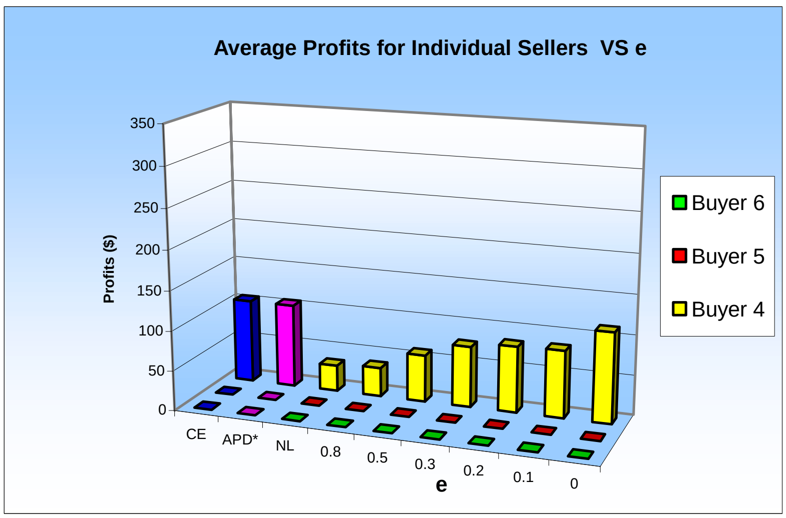


Figure 11. Average Total Profits Earn in Final Auction Run for Buyer and Seller in **Market Structure 2** Vs Experimentation Parameter e
CE = Calculated Profits Under Competitive equilibrium

APD* : Calculated Profits Under Auction Protocol when Seller and Buyer Bids their Actual MR and MC





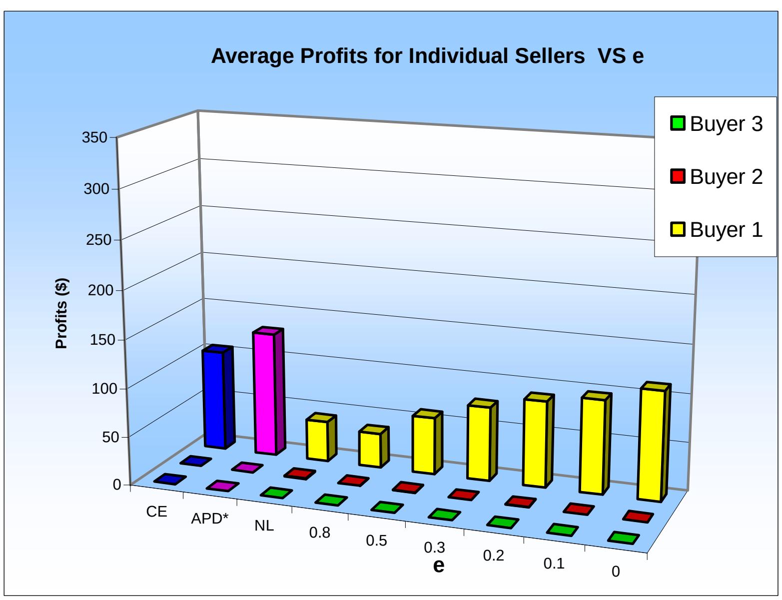
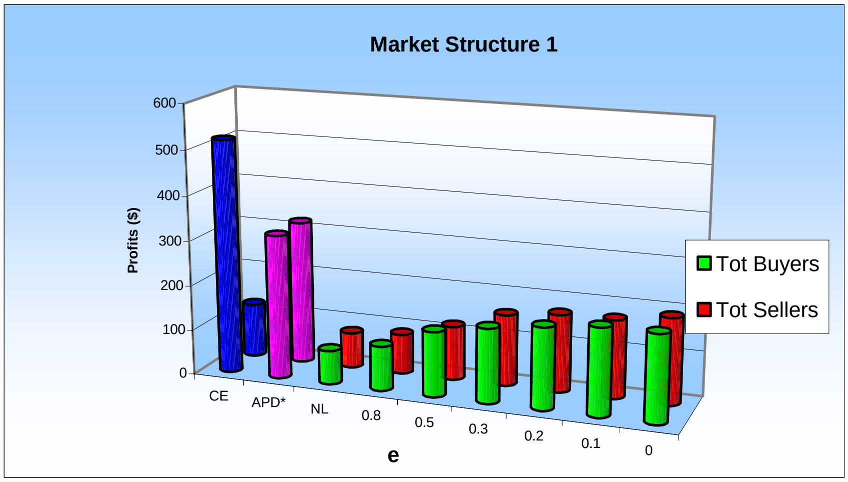
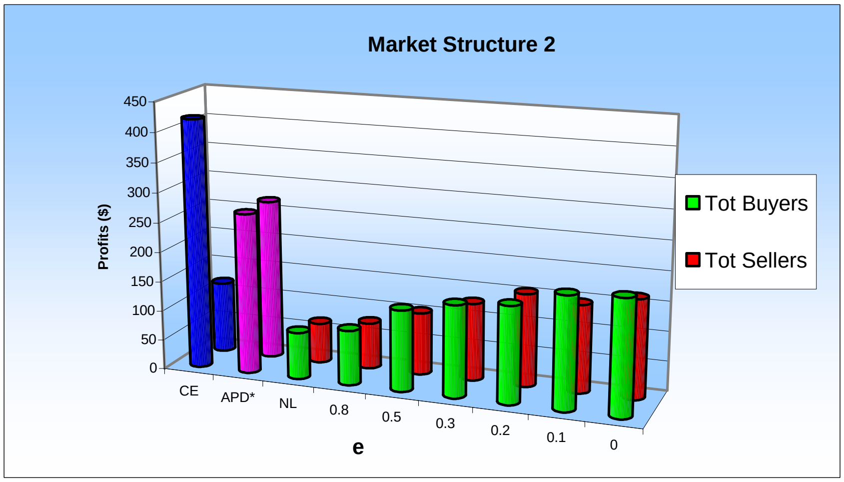


Figure 12. Average Total Profits Earn in Final Auction Run for Buyer and Seller in **Market Structure 2** Vs Experimentation Parameter e

CE = Calculated Profits Under Competitive equilibrium

APD* : Calculated Profits Under Auction Protocol when Seller and Buyer Bids their Actual MR and MC





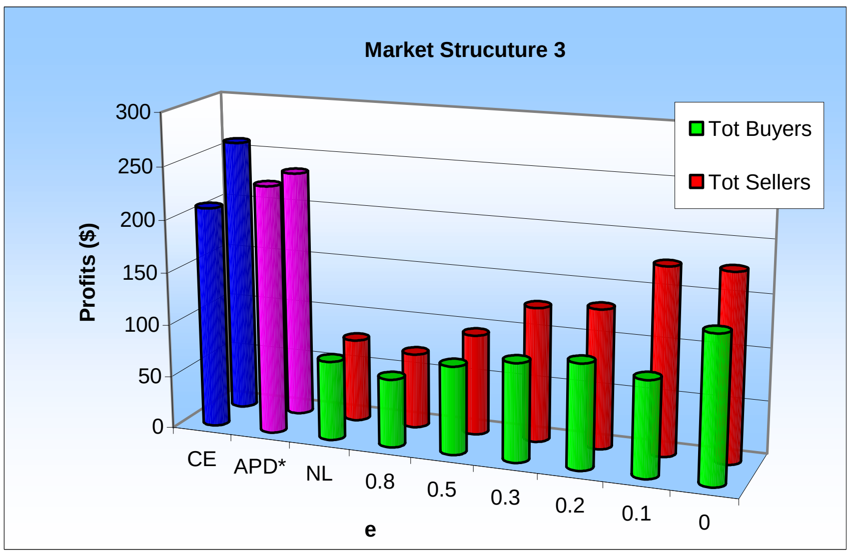


Figure 13. Average Aggregate Profits Earned in Final Auction Run for Buyers and Sellers Vs Experimentation Parameter *e*

CE = Calculated Profits Under Competitive equilibrium

APD* : Calculated Profits Under Auction Protocol when Seller and Buyer Bids their Actual MR and MC

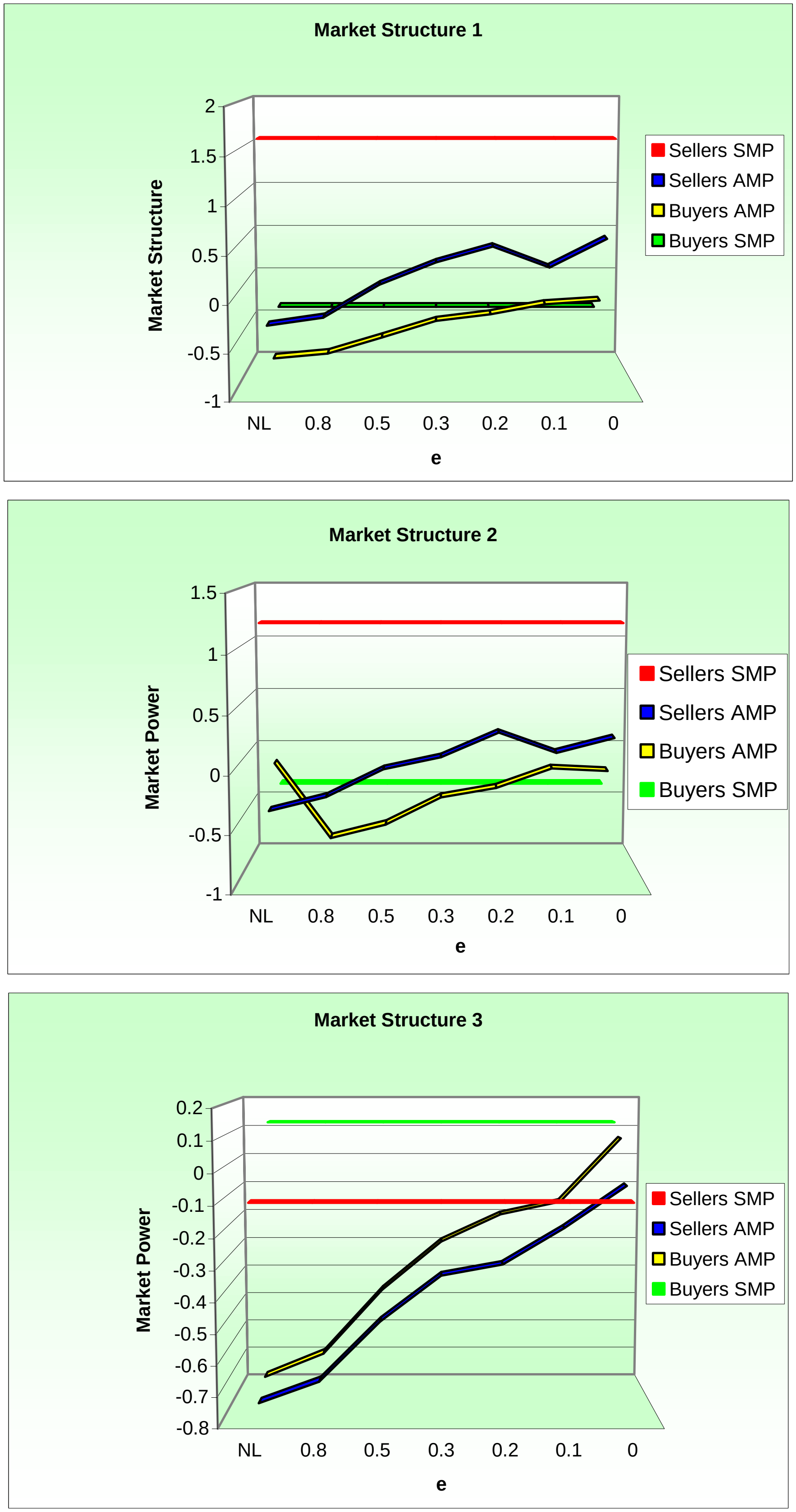


Figure 14. Aggregate Structural market Power (SMP) and Aggregate Auction Market Power (AMP) For Buyers and Sellers Using Different Tested Values of The Experimentation Parameter *e*

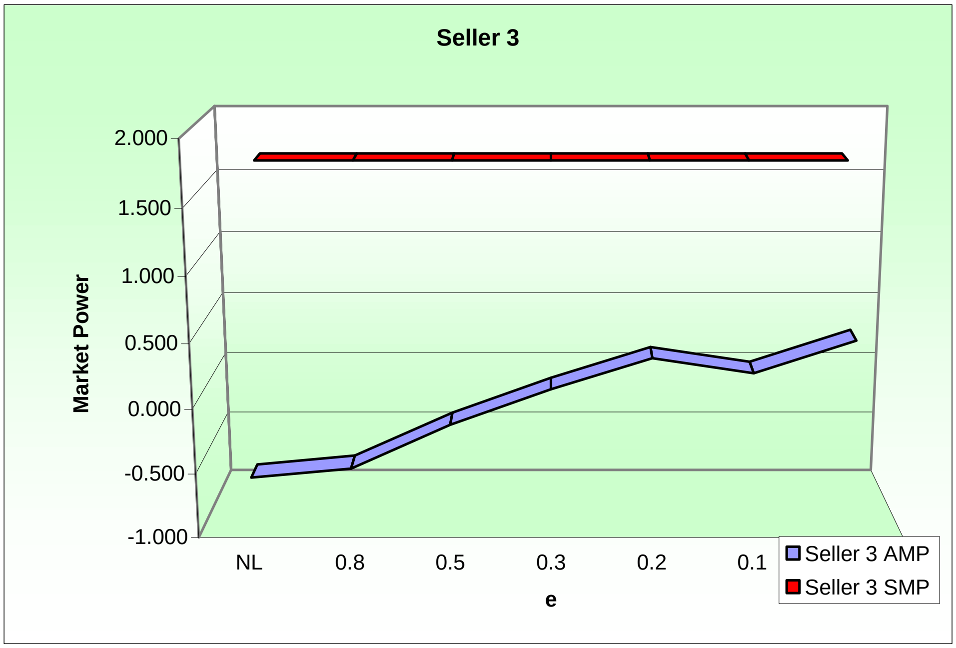
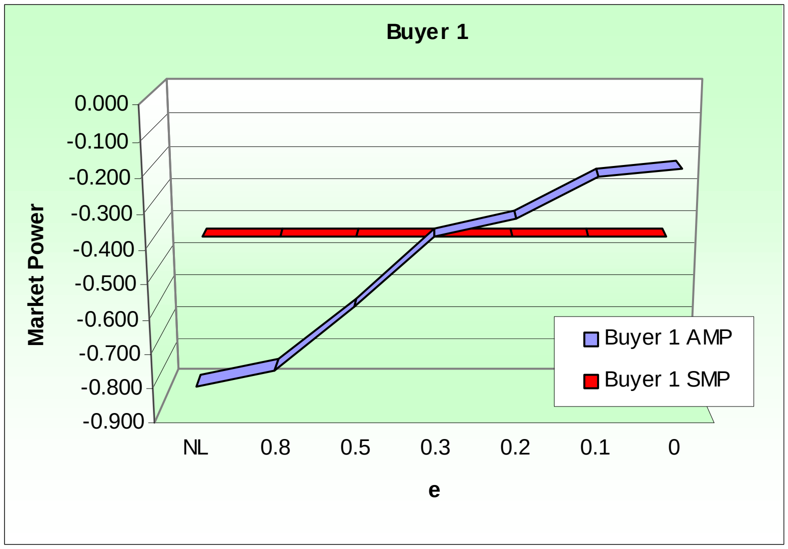


Figure 15 A. Individual Structural Market Power (SMP*) and Individual Auction Market Power (AMP*) under **Market Structure 1**
Using Different Tested Values of The Experimentation Parameter *e* for Seller 3
Note : Seller 1, and Seller 2 are Extra Marginal Agents



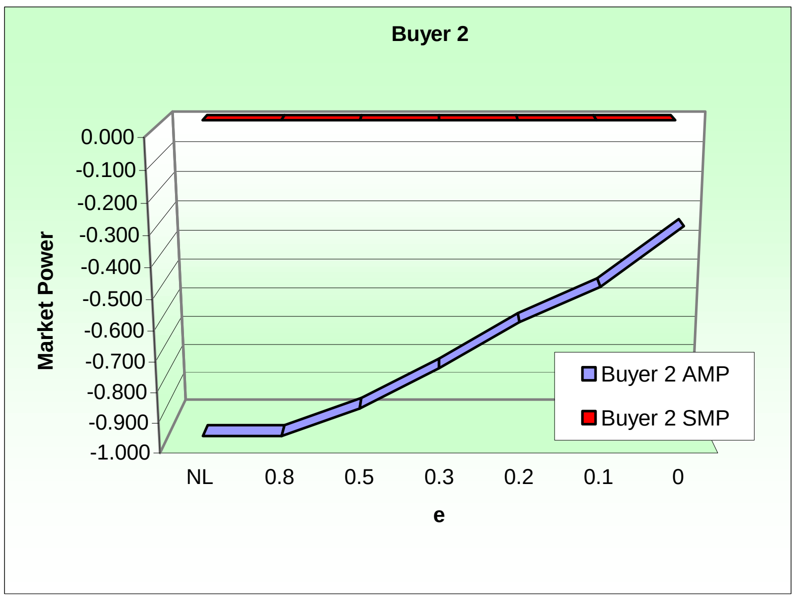
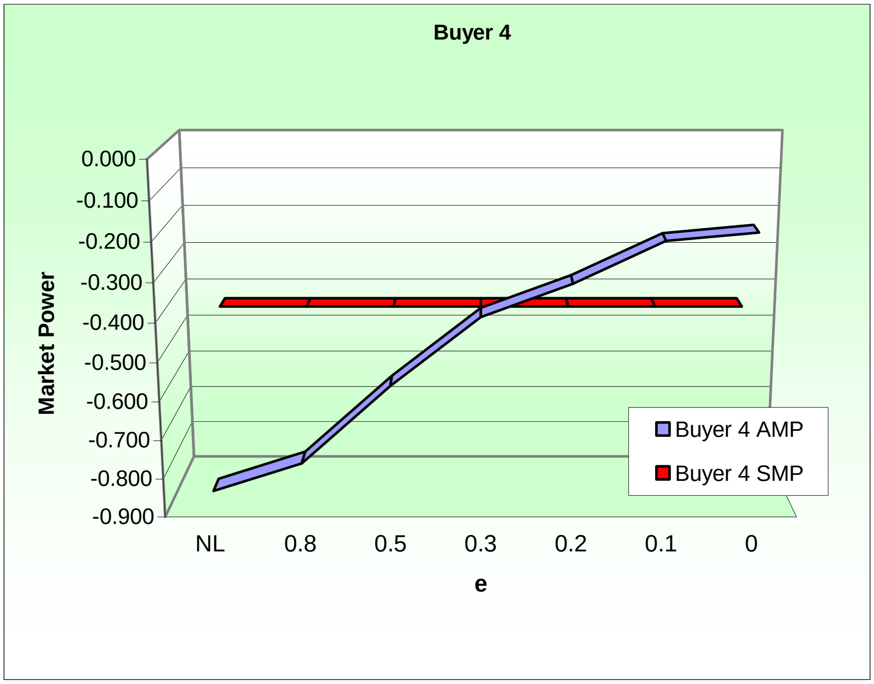


Figure 15 B. Individual Structural Market Power (SMP*) and Individual Auction Market Power (AMP*) under **Market Structure 1**
Using Different Tested Values of The Experimentation Parameter *e* for Buyer 1 and Buyer 2



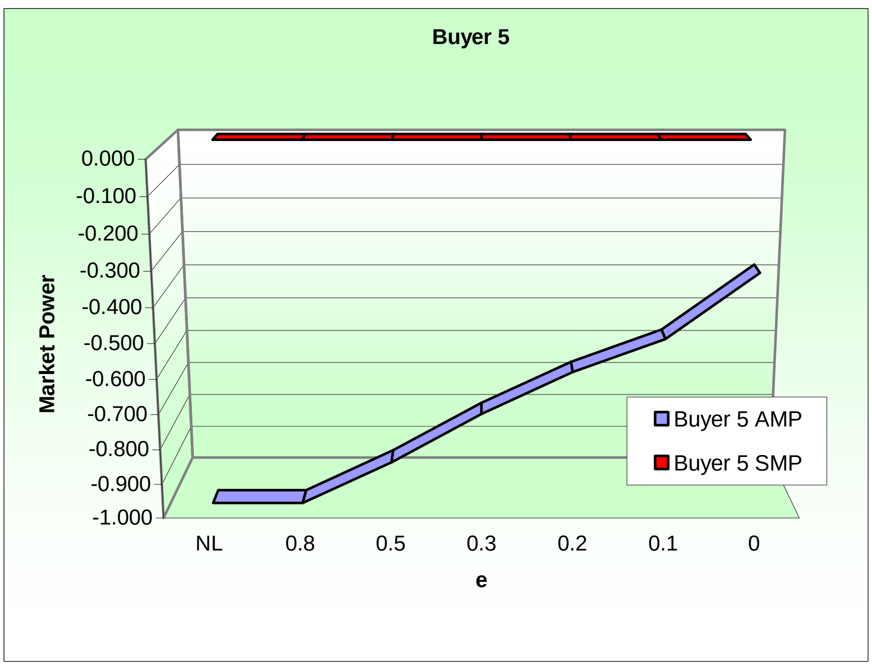


Figure 15 C. Individual Structural Market Power (SMP*) and Individual Auction Market Power (AMP*) under **Market Structure 1**
Using Different Tested Values of The Experimentation Parameter *e* for Buyer 4 and Buyer 5

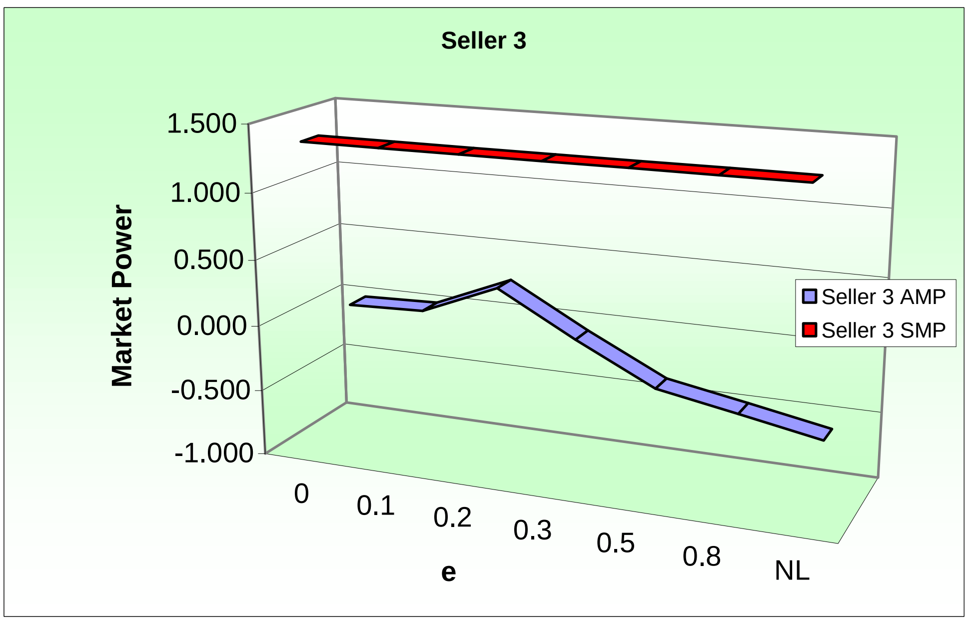
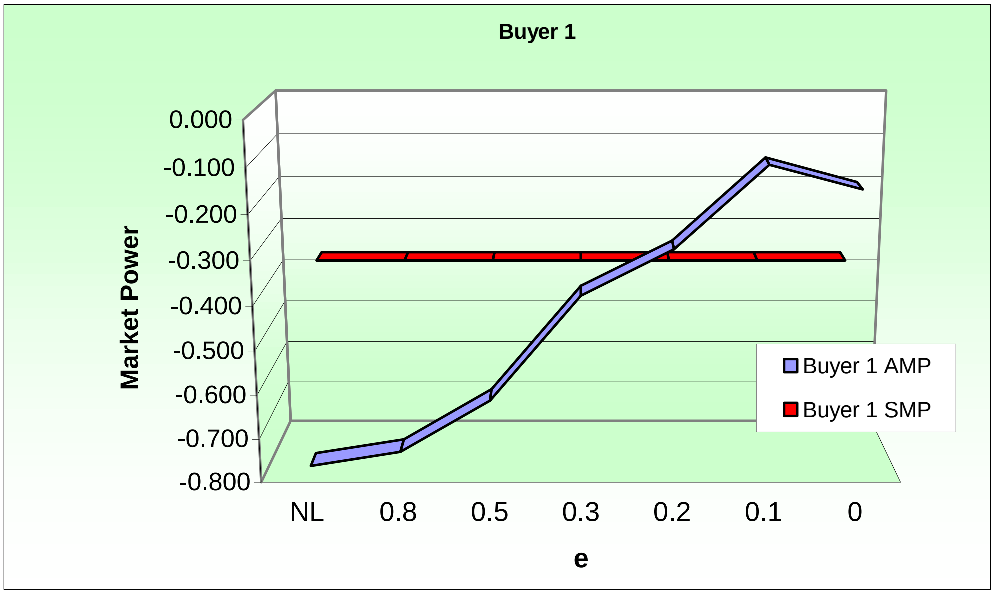


Figure 16 A. Individual Structural Market Power (SMP*) and Individual Auction Market Power (AMP*) under **Market Structure 2**
Using Different Tested Values of The Experimentation Parameter *e* for Seller2 and Seller 3



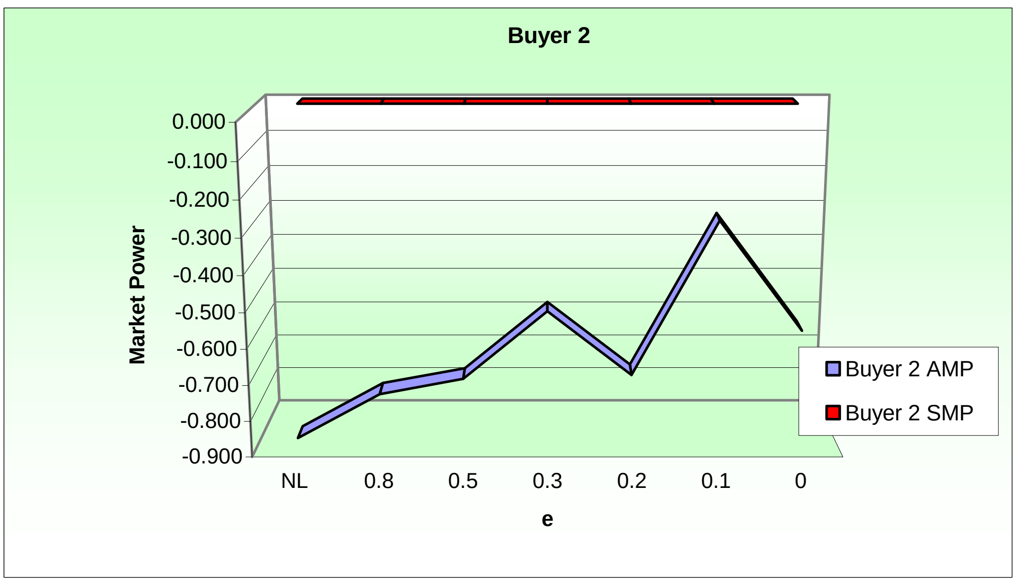
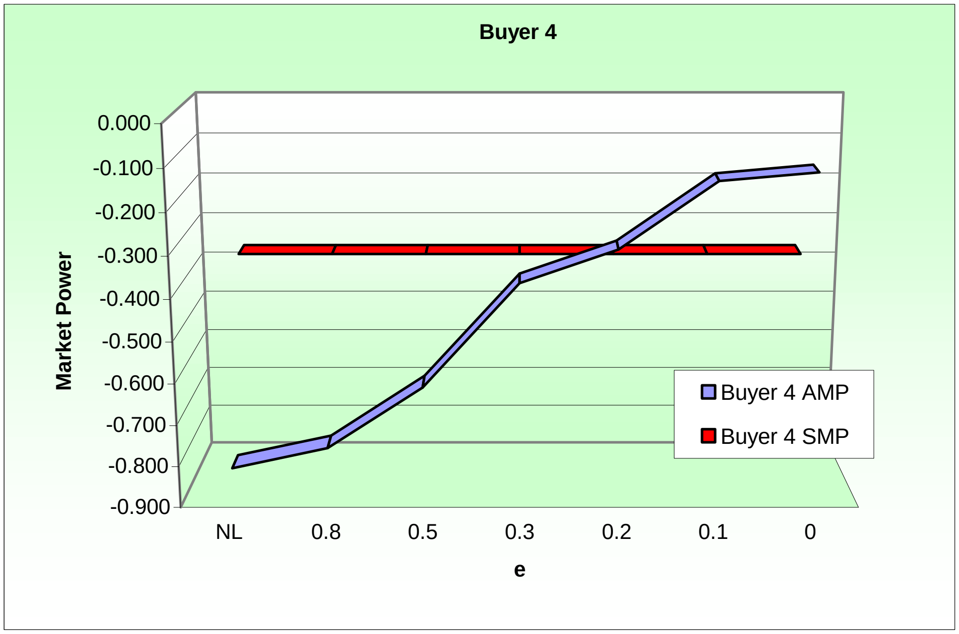


Figure 16 B. Individual Structural Market Power (SMP*) and Individual Auction Market Power (AMP*) under **Market Structure 2**
Using Different Tested Values of The Experimentation Parameter *e* for Buyer 1 and Buyer 2



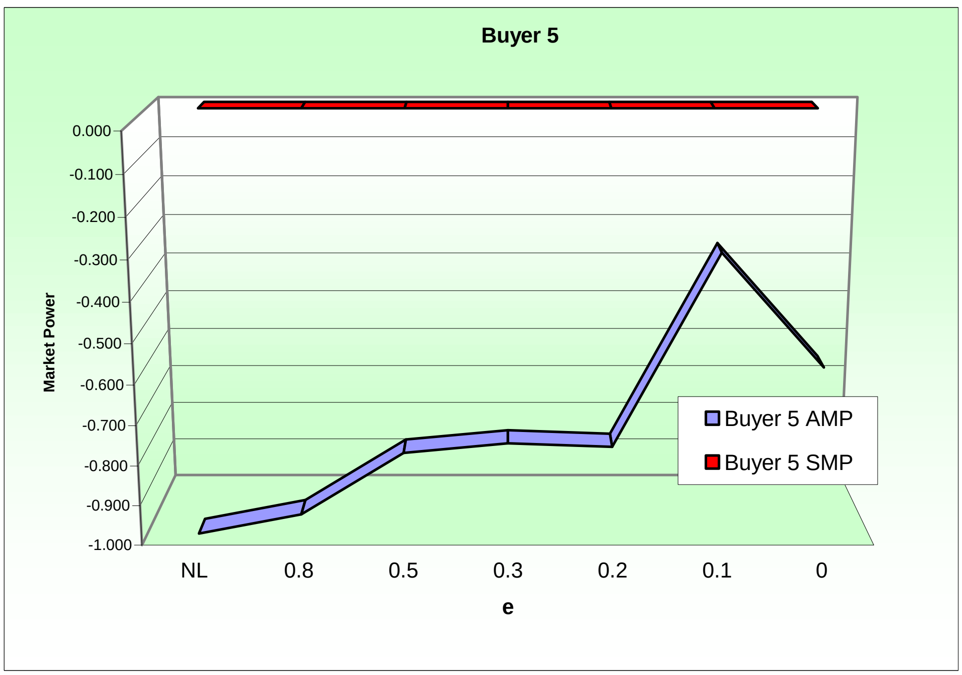
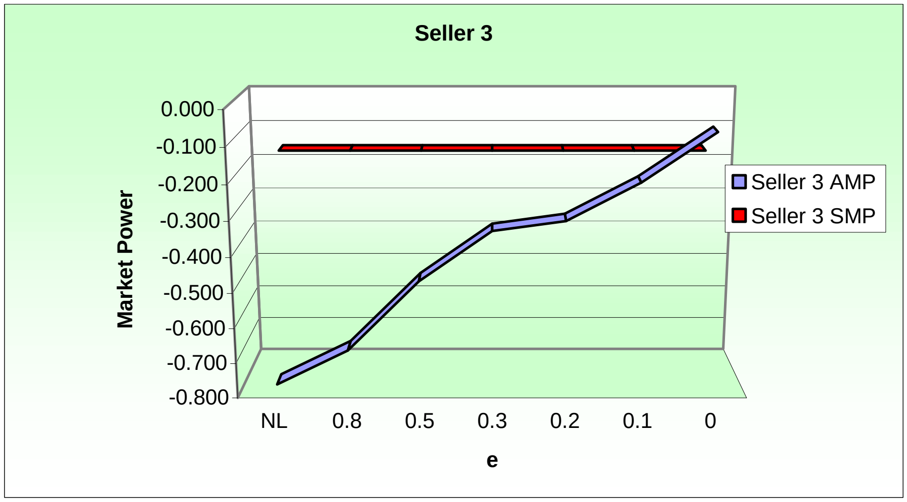
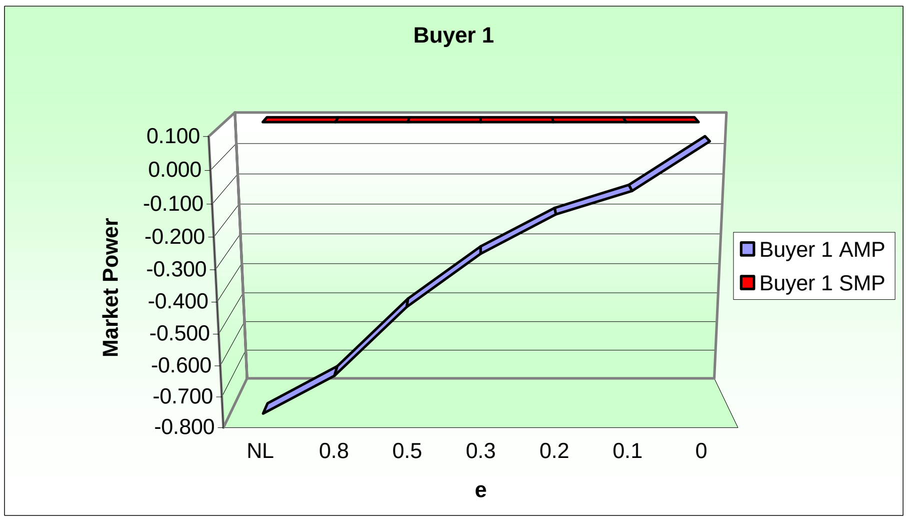


Figure 16 C. Individual Structural Market Power (SMP*) and Individual Auction Market Power (AMP*) under **Market Structure 2**
Using Different Tested Values of The Experimentation Parameter *e* for Buyer 4 and Buyer 5





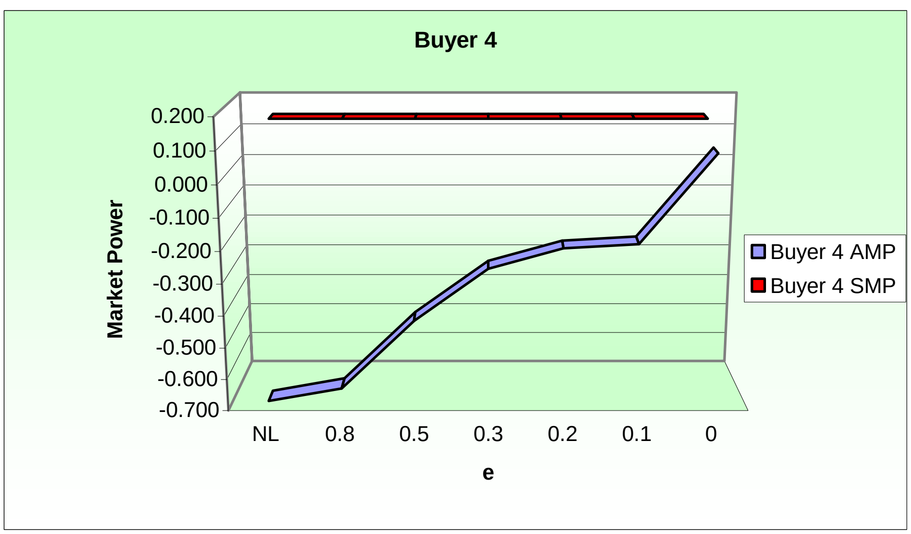


Figure 17. Individual Structural Market Power (SMP*) and Individual Auction Market Power (AMP) Under **Market Structure 3** Using Different Tested Values of The Experimentation Parameter *e*
Note : Other Sellers and Buyers are Extra Marginal Agents