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March 2009

Working Paper # 09005

Department of Economics
Working Papers Series

Ames, Iowa 50011

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April 28, 2009

Abstract

Applying auction theory to the toxic-asset rescue plan currently released by the United States Treasury Department, this paper demonstrates an equilibrium where moderately poor bidders outbid rich bidders in such auctions. After defeating their rich rivals and acquiring the toxic assets, such bidders will default on government-provided loans whenever the toxic assets turn out to be unsalvageable. An alternative mechanism is discussed.

*I am grateful to an anonymous referee for insightful comments.

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1 Introduction

The United States Treasury Department has recently published two plans ([3]) to rescue the financial sector by auctioning off its "toxic assets." One is the Legacy Loan Program (LLP) for risky home loans. The other is the Legacy Securities Program (LSP) for risky mortgage-backed securities. The main feature of the plans is to subsidize the buyers of the toxic assets with government-provided loans and equities.

Paul Krugman [2] has speculated that the rescue plan is an offer of a gamble "to play heads I win, tails the taxpayers lose." At first glance, this gamble analogy might sound merely rhetorical, because even in the case of "tails" (that a toxic asset turns out to be unsalvageable), a private investor who has bought the toxic asset does not necessarily walk away from the loss, as to walk away he needs to default thereby forfeiting as least part of his own assets including the good ones.

However, applying some straightforward logic of auction theory, I find that the "heads I win and tails the taxpayers lose" scenario is an outcome of high probability if the toxic assets are auctioned off according to the rescue plans.

The main problem of these plans is due to the loan subsidies. When a toxic asset turns out to be unsalvageable, the investor who has acquired it may be tempted to default on the loan. The equilibrium analysis presented below shows that, unless bidders initially endowed with moderately poor assets can be excluded from the auction, such bidders will outbid their richer rivals and, upon winning, will default when the acquired toxic assets turn out to be unsalvageable, leaving the loss to taxpayers.

2 The Model

A toxic asset is to be auctioned off to a set of bidders. The value of the asset is common to all of them, and is equal to either v ("salvageable") with probability π or zero ("unsalvageable") with probability $1 - \pi$. The parameters $v > 0$ and $\pi \in (0, 1)$ are commonly known.

Each bidder is initially endowed with a quantity w of asset. Bidders may differ from one another in the quantities w that they have. Call w the *type* of a bidder.

The toxic asset is sold to one of the bidders via a Vickrey (second-price) auction with zero reserve price. (Our result can be extended trivially to the case where the auction format is an English auction. The extension to Dutch and first-price auctions may be done in the spirit of Section 3.4.3 of Zheng [4].)

Suppose a bidder wins the auction at a price p. Then the government provides a fraction $\gamma \in [0, 1)$ of p as a loan at an interest rate $r \geq 0$ and pays half of the the rest as equity, so the winner pays $\frac{1}{2}(1-\gamma)p$ up front. Given the bidder's initial asset w, we require the budget constraint

$$\frac{1}{2}(1-\gamma)p \leq w. \tag{1}$$

Then the value of the toxic asset is realized. Denote the value by V. Then the winner decides whether to default on its obligation of the loan.

If the winner does not default, it pays back its share of the loan, i.e., $\frac{1}{2}\gamma p(1+r)$, and gets its share of the value of the toxic asset; hence it gets

$$\frac{1}{2}\left(V - \gamma p(1+r)\right) \tag{2}$$

currently. With the opportunity cost for its upfront payment $\frac{1}{2}(1-\gamma)p$ accounted according to the market interest rate R, which is greater than the rate r provided by the government, the winner's payoff in the entire game is equal to

$$\frac{1}{2}\left(V - \gamma p(1+r)\right) - \frac{1}{2}(1-\gamma)p(1+R) = \frac{1}{2}V - \frac{1}{2}p(1+R-\gamma(R-r)). \tag{3}$$

If the winner defaults, it forfeits the toxic asset and the current value of its own asset, which is w less the upfront payment $\frac{1}{2}(1-\gamma)p$; so the winner gets

$$-(1+R)\left(w - \frac{1}{2}(1-\gamma)p\right). \tag{4}$$

Hence the defaulting winner's total payoff from the whole game is equal to

$$-(1+R)\left(w - \frac{1}{2}(1-\gamma)p\right) - \frac{1}{2}(1-\gamma)p(1+R) = -(1+R)w. \tag{5}$$

This model corresponds to the LLP, with $\gamma \approx 0.86$. The LSP differs from the LLP in that the government provides loans and equities before the investor buys any toxic asset. However, LSP has the same feature of combining government-privided loans and equities, with $\gamma = 1/2$. Hence the incentive for an investor to default is similar to that in the model.

The published provisions of the LLP and LSP seem to indicate that a private investor does not have to borrow from the government up to γ of the price for the toxic asset. However, as long as the borrowing rate r offered by the government is below the market rate R (otherwise there is no point of offering the loan subsidy), the investor would opt for taking full advantage of the government loan up to γ.

3 The Equilibrium

Let us start by analyzing a winner's decision of default given the realization of the value V of the toxic asset.

Lemma 1 *Suppose a bidder with initially owned asset $w > 0$ has won the toxic asset at a price p. When $V = v$, the winner does not default unless*

$$v = (1+r)\gamma p \quad \textit{and} \quad (1-\gamma)p = 2w \tag{6}$$

and is indifferent about default if (6) holds. When $V = 0$, the winner defaults if

$$p\left(1 + R - \gamma(R-r)\right) > 2w(1+R) \tag{7}$$

and does not default if the opposite of (7) holds.

Proof Given the realized value V of the toxic asset and the already paid the upfront payment $\frac{1}{2}(1-\gamma)p$, a winner defaults if his payoff (2) from keeping the asset is less than the payoff (4) from default, and does not default if the inequality is reversed. Since the government would not provide a loan that it knows can never be returned, $\frac{1}{2}(v - \gamma p(1+r)) \geq 0$; by the budget constraint (1), $-(w - \frac{1}{2}(1-\gamma)p) \leq 0$. Thus, when $V = v$, (2) is greater than or equal to (4), so the winner weakly prefers not to default, and the indifference holds only when the two weak inequalities hold as equalities, i.e., Eq. (6). When $V = 0$, $\frac{1}{2}(0 - \gamma p(1+r)) < -(1+R)\left(w - \frac{1}{2}(1-\gamma)p\right)$ is equivalent to (7). ■

Anticipating their future decision characterized by Lemma 1, bidders calculate their expected payoffs from winning the toxic asset when its value V is still uncertain. Let $u(p,w)$ denote this expected payoff for a bidder whose initial asset is w and who wins the toxic asset at price p such that p satisfies the budget constraint (1).

Lemma 2 *For any $w \geq 0$ and any $p \geq 0$ that satisfies the budget constraint (1),*

$$u(p,w) = \begin{cases} \frac{\pi v}{2} - \frac{p}{2}(1 + R - \gamma(R-r)) & \text{if } p \leq \frac{2w(1+R)}{1+R-\gamma(R-r)} \\ \left(\frac{\pi v}{2} - (1-\pi)(1+R)w\right) - \frac{p}{2}\pi(1 + R - \gamma(R-r)) & \text{if } p \geq \frac{2w(1+R)}{1+R-\gamma(R-r)}. \end{cases} \tag{8}$$

Proof In calculating a winner's payoff, we may assume without loss that a winner does not default when $V = v$, as the only possible exception is when he is indifferent about default

(Lemma 1). Thus, when (7) holds, Lemma 1 implies that the winner defaults if and only if $V = 0$, hence from Eqs. (3) and (5) we have

$$u(p, w) = \pi\left(\frac{1}{2}v - \frac{1}{2}p(1 + R - \gamma(R - r))\right) + (1 - \pi)\left(-(1 + R)w\right),$$

which is the second branch of (8). When the opposite of (7) holds, Lemma 1 implies that the winner does not default, so Eq. (3) gives

$$u(p, w) = \pi \cdot \frac{1}{2}v - \frac{1}{2}p(1 + R - \gamma(R - r)),$$

which is the first branch of (8). Finally, when $p = \frac{2w(1+R)}{1+R-\gamma(R-r)}$, one can easily show that the two branches are equal to each other. ■

Lemma 2 implies that the bidders' preferences in our setup is a special case of the private-value model, with the private value for a type-w bidder being $\frac{\pi v}{2}$ when the going price p is below the cutoff $\frac{2w(1+R)}{1+R-\gamma(R-r)}$, and otherwise $\frac{\pi v}{2} - (1 - \pi)(1 + R)w$.

Assume that the penalty for violating the budget constraint (1) is larger than any possible payoff a bidder may obtain. Then our model is a standard Vickrey auction with private values and budget constraints. As a fact in auction theory (e.g., Che and Gale [1]), the dominant strategy equilibrium in this game is that every bidder submits a bid equal to either a threshold price $\tilde{p}(w)$ such that $u(\tilde{p}(w), w) = 0$ or the highest price subject to the budget constraint (1), whichever is lower. Let us calculate the threshold $\tilde{p}(w)$.

Lemma 3 *Extend the function u to the entire $\mathbb{R}^2_+$ according to Eq. (8), regardless of the budget constraint. For any $w \geq 0$, there exists a unique $\tilde{p}(w) > 0$ with $u(\tilde{p}(w), w) = 0$, and*

$$\tilde{p}(w) = \begin{cases} \frac{\pi v - 2(1-\pi)(1+R)w}{\pi(1+R-\gamma(R-r))} & \text{if } (1 + R)w \leq \frac{\pi}{2}v \\ \frac{\pi v}{1+R-\gamma(R-r)} & \text{if } (1 + R)w \geq \frac{\pi}{2}v. \end{cases} \tag{9}$$

Proof By Eq. (8), the function $u(\cdot, w)$ is continuous and strictly decreasing, with $u(0, w) = \pi v/2 > 0$ and $u(p, w) < 0$ for sufficiently large p. Thus, there exists a unique $\tilde{p}(w) > 0$ such that $u(\tilde{p}(w), w) = 0$. In Eq. (8), $u(p, w)$ turns from the first branch to the second branch when $p = \frac{2w(1+R)}{1+R-\gamma(R-r)}$, where $u(p, w) = \frac{\pi}{2}v - (1 + R)w$. Thus, there are only two cases:

a. $u\left(\frac{2w(1+R)}{1+R-\gamma(R-r)}, w\right) \geq 0$, i.e., $(1 + R)w \leq \frac{\pi}{2}v$, then with $u(\cdot, w)$ strictly decreasing, $\tilde{p}(w)$ must be the root of the second branch of (8), i.e.,

$$\left(\frac{\pi v}{2} - (1 - \pi)(1 + R)w\right) - \frac{\tilde{p}(w)}{2}\pi(1 + R - \gamma(R - r)) = 0,$$

which gives the first branch of (9).

b. $u\left(\frac{2w(1+R)}{1+R-\gamma(R-r)}, w\right) \leq 0$, i.e., $(1+R)w \geq \frac{\pi}{2}v$, then $\tilde{p}(w)$ is the root of the first branch of (8), i.e.,

$$\frac{\pi v}{2} - \frac{\tilde{p}(w)}{2}(1+R-\gamma(R-r)) = 0,$$

which gives the second branch of (9).

Note that the two branches of Eq. (9) are equal to each other when $(1+R)w = \frac{\pi}{2}v$. ■

Now we are ready to characterize the equilibrium.

Proposition 1 *There is an equilibrium where any bidder with any quantity $w \geq 0$ of initially owned asset submits a bid $p^*(w)$ such that*

$$p^*(w) = \begin{cases} \frac{2w}{1-\gamma} & \text{if } w \leq \frac{(1-\gamma)\pi v}{2((1-\gamma)(1+R)+\pi\gamma(1-r))} \\ \frac{\pi v - 2(1-\pi)(1+R)w}{\pi(1+R-\gamma(R-r))} & \text{if } \frac{(1-\gamma)\pi v}{2((1-\gamma)(1+R)+\pi\gamma(1-r))} \leq w \leq \frac{\pi v}{2(1+R)} \\ \frac{\pi v}{1+R-\gamma(R-r)} & \text{if } w \geq \frac{\pi v}{2(1+R)}, \end{cases} \tag{10}$$

and, if the bidder is the winner and the highest losing bid is p, defaults if and only if the realized value of the toxic asset is 0 and (7) holds.

Proof By the private-value payoff function characterized by Lemma 2 and the budget constraint (1), a fact in auction theory (e.g., Che and Gale [1]) implies that the dominant strategy equilibrium in the Vickrey auction is that every bidder submits a bid equal to

$$p^*(w) = \min\left\{\tilde{p}(w), \frac{2w}{1-\gamma}\right\}. \tag{11}$$

By Eq. (9), as w increases from 0 to $\frac{\pi v}{2(1+R)}$, $\tilde{p}(w)$ decreases from $\frac{v}{1+R-\gamma(R-r)}$ to $\frac{\pi v}{1+R-\gamma(R-r)}$ and then stays at that level constantly as w increases further. Thus, there exists a unique $\hat{w} > 0$ such that

$$\tilde{p}(\hat{w}) = \frac{2\hat{w}}{1-\gamma}.$$

Since

$$\left.\frac{2w}{1-\gamma}\right|_{w=\frac{\pi v}{2(1+R)}} = \frac{\pi v}{(1-\gamma)(1+R)} > \frac{\pi v}{1+R-\gamma(R-r)},$$

we have $\hat{w} < \frac{\pi v}{2(1+R)}$. Thus, $\hat{w}$ must be the solution such that the first branch of the right-hand side of (9) is equal to $\frac{2w}{1-\gamma}$, i.e.,

$$\frac{\pi v - 2(1-\pi)(1+R)\hat{w}}{\pi(1+R-\gamma(R-r))} = \frac{2\hat{w}}{1-\gamma},$$

which yields

$$\hat{w} = \frac{(1-\gamma)\pi v}{2((1-\gamma)(1+R)+\pi\gamma(1-r))}. \tag{12}$$

Then (11) implies (10). The rest of the proposition directly follows from Lemma 1. ■

4 The Default-Exacerbating Consequence

The policy-maker's goal of auctioning off the toxic assets, I presume, is to encourage private investors to share with taxpayers the risk of toxic assets and to mitigate the default crisis in the financial sector. The auction plan, however, fares poorly in fulfilling the goal.

First, let us inspect the equilibrium bid function characterized by Proposition 1.

Corollary 1 *In the equilibrium characterized by Proposition 1,*

a. a bidder's bid $p^*(w)$ is strictly increasing in w when $w \leq \hat{w}$, strictly decreasing in w when $\hat{w} \leq w \leq \frac{\pi v}{2(1+R)}$, and is equal to the constant $\frac{\pi v}{1+R-\gamma(R-r)}$ when $w \geq \frac{\pi v}{2(1+R)}$,

b. $p^*(w) \geq \frac{\pi v}{1+R-\gamma(R-r)}$ when $w \geq w_*$, where

$$w_* = \frac{(1-\gamma)\pi v}{2(1+R-\gamma(R-r))}. \tag{13}$$

Proof Claim (a) follows from the equilibrium bid function, Eq. (9). To prove claim (b), note that there exists uniquely a $w_* \in \left(0, \frac{\pi v}{2(1+R)}\right)$ at which the first branch of the right-hand side of (10) is equal to the third branch in (10). Solving that equation yields (13). ■

Thus, bidders with initially owned assets between w_* and $\frac{\pi v}{2(1+R)}$ bid higher than bidders with larger initially owned assets. Why do such "poor" bidders bid more aggressively than their richer rivals? That is because with little initially owned asset a winner would have little to lose if he defaults when the toxic asset turns out to be unsalvageable. (The bidders with types less than w_* cannot bid aggressively, due to their severe budget constraints.)

Hence the auction may result in an adverse outcome: financially capable bidders lose the auction to some of their financially constrained rivals. As formalized in the next corollary, such financially constrained bidders are more prone to bankruptcies, thereby exacerbating the default problem already troubling the policy-makers.

Corollary 2 *Suppose there are at least two bidders with types above w_* and at least one of them has types less than $\frac{\pi v}{2(1+R)}$, then any winner of the auction defaults when ex post $V = 0$.*

Proof Denote w for the type of the winner and p for the price for the toxic asset. By hypothesis, there is at least one bidder whose type is in the interval $\left(w_*, \frac{\pi v}{2(1+R)}\right)$. By Corollary 1(a), the bid from such a bidder is higher than the bid from any bidder with types in $\left[\frac{\pi v}{2(1+R)}, \infty\right)$. Thus, $w < \frac{\pi v}{2(1+R)}$. By hypothesis, there are at least two bidders whose types are above w_*, hence Corollary 1(b) implies that the highest losing bid, i.e., p, is at least as high as $\pi v/(1 + R - \gamma(R - r)$. Thus, $p\,(1 + R - \gamma(R - r)) \geq \pi v > 2w(1 + R)$, so (7) holds. Then Lemma 1 implies that the winner defaults if $V = 0$. ■

By Corollary 2, unless only one bidder participates in the auction, which would generate a depressing zero price for the toxic asset, the probability with which default occurs conditional on the toxic asset being unsalvageable is greater than or equal to

$$\text{Prob}\left\{w^{(2)} \geq w_* \text{ and some bidder's type is in } \left(w_*, \frac{\pi v}{2(1+R)}\right)\right\}, \tag{14}$$

where $w^{(2)}$ denotes the second highest type among the bidders. Although the outcome of "heads I win, tails the taxpayers lose" speculated by Krugman does not occur for sure, it is more probable when there are more bidders, because the probability (14) goes to one as the number of bidders goes to infinity, as long as $\frac{\pi v}{2(1+R)} > w_*$, which is guaranteed by $\gamma > 0$.

One might hope to avoid this default-exacerbating outcome by excluding the private investors of types below $\frac{\pi v}{2(1+R)}$ from the auction. But a truthful diagnosis of the financial health of various firms may be costly, if not impossible, for the government to obtain. After all, how many people knew a year ago the financial troubles of AIG and the like?

5 Alternative Mechanisms

The driving force for the buyers of toxic assets to default is the loans provided to them by the government. Thus, a trivial mechanism that eliminates the default problem is to offer no loan to these buyers. That amounts to setting the parameter γ to be zero, so that $w_* = \frac{\pi v}{2(1+R)}$ by Eq. (13). Then the interval in (14) is degenerate, and the budget constraint (1) becomes $p \leq 2w$, implying that the condition (7) for default never holds.

This no-loan mechanism, however, might offer little help to stimulate the demand for the toxic assets, given that many private investors are currently financially constrained.

An alternative mechanism, which provides loans and may mitigate the default danger, has been analyzed by an early article of mine, Zheng [4]. There, the winning bidder is allowed to borrow from the government only up to the amount by which the price of the object exceeds the winner's initially endowed wealth. In that article, it is proved that if the interest rate for such a loan is above a threshold, $\frac{1-\pi}{\pi}$, then rich bidders win and the probability of default is low. That is because, to finance the same payment in that mechanism, a rich bidder would have less debt liability than a poor bidder. Thus, the cost to finance a payment is low for rich bidders and high for poor bidders, while the cost to default is high for rich bidders and low for poor bidders. When the borrowing rate is above the threshold, the financing cost outweighs the default cost, so rich bidders outbid poor bidders and default only with a small probability. As long as the borrowing rate is still below the market rate, the bidders are still financially subsidized. With the bidding competition intensified by the subsidy, the expected revenue and sometimes even the expected profit for the government are higher than those without the subsidy ([4, Proposition 4.2]).

To implement the alternative mechanism, the government needs to have a truthful assessment on a winning bidder's asset after the auction. That is no easy task, but still more doable than getting a truthful assessment on every bidder's asset before the auction, which is needed to rule out the type of adverse outcomes demonstrated in the previous section.

A cautionary note, however, is that the alternative mechanism is also prone to the default problem if the financial subsidy is overly generous. It is proved in Zheng [4] that there is a "high bids and broke winners" equilibrium similar to the one in this paper if the borrowing rate is below the threshold $\frac{1-\pi}{\pi}$. With such a low borrowing rate, the financing cost is outweighed by the default cost, so rich bidders cannot outbid poor bidders.

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