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Segmented Asset Markets and Optimal Exchange Rate Regimes[*]

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Abstract

This paper revisits the issue of the optimal exchange rate regime in a flexible price environment. The key innovation is that we analyze this question in the context of environments where only a fraction of agents participate in asset market transactions (i.e., asset markets are segmented). Under this friction alternative exchange rate regimes have different implications for real allocations in the economy. In the context of this environment we show that flexible exchange rates are optimal under monetary shocks and fixed exchange rates are optimal under real shocks. (*JEL: F1, F2*)

KEYWORDS: Optimal exchange rates; Asset market segmentation

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1 Introduction

Fifty years after Milton Friedman's (1953) celebrated case for flexible exchange rates, the debate on the optimal choice of exchange rate regimes rages on as fiercely as ever. Friedman argued that, in the presence of sticky prices, floating rates would provide better insulation from foreign shocks by allowing relative prices to adjust faster. In a world of capital mobility, Mundell's (1963) work implies that the optimal choice of exchange rate regime should depend on the type of shocks hitting an economy: real shocks would call for a floating exchange rate, whereas monetary shocks would call for a fixed exchange rate. Ultimately, however, an explicit cost/benefit comparison of exchange rate regimes requires a utility-maximizing framework, as argued by Helpman (1981) and Helpman and Razin (1979). In such a framework, Engel and Devereux (1998) reexamine this question in a sticky prices model and show how results are sensitive to whether prices are denominated in the producer's or consumer's currency. On the other hand, Cespedes, Chang, and Velasco (2000) incorporate liability dollarization and balance sheets effects and conclude that the standard prescription in favor of flexible exchange rates in response to real shocks is not essentially affected.

An implicit assumption in most, if not all, of the literature is that economic agents have unrestricted and permanent access to asset markets.[1] This, of course, implies that in the absence of nominal rigidities, the choice of fixed versus flexible exchange rates is irrelevant. In practice, however, access to asset markets is limited to some fraction of the population (due to, for example, fixed costs of entry). This is likely to be particularly true in developing countries where asset markets are much smaller in size than in industrial countries. Table 1 shows that even for the United States, the degree of segmentation in asset markets is remarkably high. The table reveals that, as of 1989, 59 percent of U.S. households did not hold any interest bearing assets (defined as money market accounts, certificates of deposit, bonds, mutual funds, and equities). More strikingly, 25

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[1] There are some exceptions when it comes to the related issue of the costs and benefits of a common currency area (see, for example, Neumeyer (1998) and Ching and Devereux (2000), who analyze this issue in the presence of incomplete asset markets).

percent of households did not even have a checking account as late as in 1989. Given these facts for a developed country like the United States, it is easy to anticipate that the degree of asset market segmentation in emerging economies must be considerably higher. Since asset markets are at the heart of the adjustment process to different shocks in an open economy, it would seem natural to analyze how asset market segmentation affects the choice of exchange rate regime.[2]

Table 1 here

This paper abstracts from any nominal rigidity and focuses on a standard monetary model of an economy subject to stochastic real and monetary (i.e., velocity) shocks in which the only friction is that an exogenously-given fraction of the population can access asset markets. The analysis makes clear that asset market segmentation introduces a fundamental asymmetry in the choice of fixed versus flexible exchange rates. To see this, consider first the effects of a positive velocity shock in a standard one-good open economy model in the absence of asset market segmentation. Under flexible exchange rates, the velocity shock gets reflected in an excess demand for goods, which leads to an increase in the price level (i.e., the exchange rate). Under fixed exchange rates, the adjustment must take place through an asset market operation whereby agents exchange their excess money balances for foreign bonds at the central bank. In either case, the adjustment takes place instantaneously with no real effects. How does asset market segmentation affect this adjustment? Under flexible rates, the same adjustment takes place. Under fixed exchange rates, however, only those agents who have access to asset markets (called "traders") may get rid of their excess money balances. Non-traders – who are shut off from assets markets – cannot do this. Non-traders are therefore forced to buy excess goods. The resultant volatility of consumption is costly from a welfare point of view. Hence, under asset market segmentation and in the presence

[2]In closed economy macroeconomics, asset market segmentation has received widespread attention ever since the pioneering work of Grossman and Weiss (1983) and Rotemberg (1984) (see also Chatterjee and Corbae (1992) and Alvarez, Lucas, and Weber (2001)). The key implication of these models is that open market operations reduce the nominal interest rate and thereby generate the so-called "liquidity effect". In an open economy context, Alvarez and Atkeson (1997) and Alvarez, Atkeson, and Kehoe (2002) have argued that asset market segmentation models help in resolving outstanding puzzles in international finance such as volatile and persistent real exchange rate movements as well as excess volatility of nominal exchange rates.

of monetary shocks, flexible exchange rates are superior to fixed exchange rates.

Asset market segmentation also has crucial implications for the optimal exchange rate regime when shocks come from the goods market. We show that when output is stochastic, non-traders in the economy unambiguously prefer fixed exchange rates to flexible exchange rates because pegs provide a form of risk pooling. Under a peg, household consumption is a weighted average of current period and last period's output which implies that the consumption risk of non-trading households is pooled across periods. Under flexible rates, however, the real value of consumption is always current output which implies no intertemporal risk sharing. Trading households, on the other hand, prefer flexible exchange rates to fixed exchange rates since maintaining an exchange rate peg involves injecting or withdrawing money from traders which makes their consumption more volatile under a peg. However, trading households' access to asset markets ensures a much smaller increase in their consumption volatility relative to the reduction in consumption volatility of non-trading households. Using a population share weighted average of the welfare of the two types, we show that under fairly general conditions, the non-traders' preferences dominate the social welfare function. Hence, when output is stochastic, an exchange rate peg welfare dominates a flexible exchange rate regime. In sum, the paper shows that asset market segmentation may be a critical friction in determining the optimal exchange rate regime.

Our paper is related to an older literature on exchange rate regimes. Perhaps the closest paper is Fischer (1977) who showed that in an economy with no capital mobility, fixed exchange rates produced better outcomes than flexible exchange rates when shocks are real while flexible exchange rates are better when shocks originate in the money market. There are two main differences between Fischer (1977) and our paper. First, we solve a micro-founded optimizing model while Fischer obtained his results in the context of a reduced-form model. The reduced-form nature of the model made his analysis unsuitable for a choice-theoretic welfare analysis. Second, we analyze an economy with heterogenous agents whereas Fischer did not. In our model, agent heterogeneity is key to understanding the role of monetary policy in affecting real outcomes. In our framework, monetary policy can react to output disturbances by redistributing resources from one agent to the

other. This channel is critical in achieving the first-best in our model and is missing completely in Fischer's model.

In fact, it is this heterogeneity of agents which also differentiates our work from the work of Helpman and Razin (1982). Helpman-Razin studied an environment with uncertainty and incomplete markets to show that flexible exchange rates can produce higher welfare than fixed exchange rates. However, the key feature of our model is incomplete market participation – some agents are absent from asset markets. In a previous version of this paper we have shown that our results carry over to the complete asset markets case. Hence, what is central for our results is incomplete *market participation*, not incomplete *asset markets*.

The paper proceeds as follows. Section 2 presents the model and the equilibrium conditions while Section 3 describes the allocations under alternative exchange rate regimes and derives the optimal regime under monetary and output shocks. Section 4 studies the optimal, first-best monetary policy rule. Finally, Section 5 concludes. Algebraically tedious proofs are consigned to an appendix.

2 Model

The basic model is an open economy variant of the model outlined in Alvarez, Lucas, and Weber (2001). Consider a small open economy perfectly integrated with world goods markets. There is a unit measure of households who consume an internationally-traded good. The world currency price of the consumption good is fixed at one. The households' intertemporal utility function is

$$W_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \right\}, \tag{1}$$

where β is the households' time discount factor, c_s is consumption in period s, while E_t denotes the expectation conditional on information available at time t.

The households face a cash-in-advance constraint. As is standard in these models, the households are prohibited from consuming their own endowment. We assume that a household consists of a seller-shopper pair. While the seller sells the household's own endowment, the shopper goes

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out with money to purchase consumption goods from other households. We assume that households are heterogenous. In particular, only a fraction λ of the population, called traders, have access to the asset markets, where $0 < \lambda \leq 1$. The rest, $1 - \lambda$, called non-traders, can only hold domestic money as an asset. We restrict λ to be strictly positive. As will become clearer below, the model has a discontinuity at $\lambda = 0$. Since all money injections occur in asset markets, the monetary authority has no way of introducing money into the economy if there are no traders at all.

There are two potential sources of uncertainty in the economy. First, each household receives a random endowment y_t of the consumption good in each period. We assume that y_t is an independently and identically distributed random variable with mean $\bar{y}$ and variance σ_y^2.[3] Second, following Alvarez et al., we assume that the shopper can access a proportion v_t of the household's current period (t) sales receipts, in addition to the cash carried over from the last period (M_t), to purchase consumption. We assume that v_t is an independently and identically distributed random variable with mean $\bar{v} \in [0, 1]$ and variance σ_v^2. In the following we shall refer to these v shocks as velocity shocks.[4]

The timing runs as follows. First, both the endowment and velocity shocks are realized at the beginning of every period. Second, the household splits. Sellers of both households stay at home and sell their endowment for local currency. Shoppers of the non-trading households are excluded from the asset market and, hence, go directly to the goods market with their overnight cash to buy consumption goods. Shoppers of trading households first carry the cash held overnight to the asset market where they trade in bonds and receive any money injections for the period. They then proceed to the goods market with whatever money balances are left after their portfolio

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[3] We could allow for different means and variances for the endowments of traders and non-traders without changing our basic results.

[4] There are alternative ways in which one can think about these velocity shocks. Following Alvarez, Lucas, and Weber (2001) one can 'think of the shopper as visiting the seller's store at some time during the trading day, emptying the cash register, and returning to shop some more'. The uncertainty regarding v can be thought of as the uncertainty regarding the total volume of sales at the time that the shopper accesses the cash register. Alternatively, one can think of this as representing an environment where the shopper can purchase goods either through cash or credit. However, the mix of cash and credit transactions is uncertain and fluctuates across periods.

rebalancing. After acquiring goods in exchange for cash, the non-trading-shopper returns straight home while the trading-shopper can re-enter the asset market to exchange goods for foreign bonds. After all trades for the day are completed and markets close, the shopper and the seller are reunited at home.

2.1 Households' problem

2.1.1 Non-traders

The non-trader's cash-in-advance constraint is given by:

$$M_t^{NT} + v_t S_t y_t = S_t c_t^{NT}, \tag{2}$$

where M_t^{NT} is the beginning of period t nominal money balances while S_t is the period t exchange rate (the domestic currency price of foreign currency). Equation (2) shows that for consumption purposes, the non-traders can augment the beginning of period cash balances by withdrawals from current period sales receipts v_t (the velocity shocks). Notice that while writing (2) we have assumed that the cash-in-advance constraint binds in equilibrium.[5] Appendix 6.1 provides sufficient conditions to ensure that (2) indeed holds for all t.

Money balances at the beginning of period $t+1$ are given by sales receipts net of withdrawals for period t consumption:

$$M_{t+1}^{NT} = S_t y_t (1 - v_t), \tag{3}$$

where S_t denotes the domestic currency price of consumption goods at time t.

The usual flow constraint follows from combining (2) and (3):

$$M_{t+1}^{NT} = M_t^{NT} + S_t y_t - S_t c_t^{NT}. \tag{4}$$

Given the cash-in-advance (2), it follows that:

$$c_t^{NT} = \frac{M_t^{NT} + v_t S_t y_t}{S_t}. \tag{5}$$

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[5] Relaxing this assumption will add complexity to our analysis without qualitatively changing or adding any further insights to the results we obtain. Note further that it is a standard assumption in the literature; see for example Alvarez, Atkeson, and Kehoe (2002), and Alvarez, Lucas, and Weber (2001).

2.1.2 Traders

The traders begin any period with assets in the form of money balances and bond holdings carried over from the previous period. Armed with these assets the shopper of the trader household visits the asset market where she rebalances the household's asset position and also receives the lump sum asset market transfers from the government. Thus, for any period t, the accounting identity for the asset market transactions of a trader household is given by

$$\hat{M}_t^T = M_t^T + (1 + i_{t-1})\frac{B_t}{\lambda} - \frac{B_{t+1}}{\lambda} + S_t(1 + r)f_t - S_t f_{t+1} + \frac{T_t}{\lambda}, \tag{6}$$

where $\hat{M}_t^T$ denotes the money balances with which the trader leaves the asset market and M_t^T denotes the money balances with which the trader entered the asset market. Also, B denotes aggregate one-period nominal government bonds, i is the interest rate on these bonds, f are foreign bonds (denominated in terms of the consumption good), r is the exogenous and constant world real interest rate, and T are aggregate (nominal) lump-sum transfers (i.e., negative taxes) from the government.[6,7] Note that nominal bonds maturing at date t pay an interest rate i_{t-1} since this rate was contracted in $t - 1$.[8]

After asset markets close, the shopper proceeds to the goods market with $\hat{M}^T$ in nominal money balances to purchase consumption goods. Like non-traders, traders can also augment these starting money balances with random withdrawals from current sales receipts to carry out goods purchases.

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[6] We assume that these transfers are made in the asset markets, where only the traders are present. Note that since B and T denote aggregate bonds and aggregate transfers, their corresponding per trader values are B/λ and T/λ since traders comprise a fraction λ of the population.

[7] The assumption of endogenous lump-sum transfers will ensure that any monetary policy may be consistent with the intertemporal fiscal constraint. This becomes particularly important in this stochastic environment where these endogenous transfers will have to adjust to ensure intertemporal solvency for *any* history of shocks. To make our life easier, these transfers are assumed to go only to traders. If these transfers also went to non-traders, then (5) would be affected.

[8] Note that though traders do have access to asset markets, these markets are incomplete. More specifically, traders do not have access to asset markets where they can trade in state contingent assets spanning all states. Hence, as will become clear below, random shocks can induce wealth effects and consumption volatility for traders as well, despite their access to competitive world capital markets. In an appendix to this paper, we analyzed the complete markets case and show how the same key results obtain. Hence, our results on the optimal exchange rate regime under asset market segmentation do not depend on whether asset markets for traders are complete or not. The appendix is available from the authors upon request.

Thus, the cash-in-advance constraint for a trader is given by[9]

$$S_t c_t^T = \hat{M}_t^T + v_t S_t y_t. \tag{7}$$

Combining equations (6) and (7) gives

$$M_t^T + \frac{T_t}{\lambda} + v_t S_t y_t = S_t c_t^T + \frac{B_{t+1}}{\lambda} - (1 + i_t)\frac{B_t}{\lambda} + S_t f_{t+1} - S_t(1 + r)f_t, \tag{8}$$

In this set-up the only reason that traders hold money overnight is the separation between markets. In particular, if the seller could access the asset market at the end of the day, then the trading household would use all their remaining sales receipts from the period to buy interest bearing bonds. Thus, period-t sales receipts net of withdrawals become beginning of next period's money balances

$$M_{t+1}^T = S_t y_t (1 - v_t). \tag{9}$$

Note that since v, S, and y are all exogenous, the traders' money holdings evolve exogenously over time.

A trader chooses c_t, B_{t+1} and f_{t+1} to maximize (1) subject to the flow constraint (8). Combining first-order conditions, we obtain:

$$u'(c_t^T) = \beta(1 + r)E_t \left\{ u'(c_{t+1}^T) \right\}, \tag{10a}$$

$$\frac{u'(c_t^T)}{S_t} = \beta(1 + i_t)E_t \left\{ \frac{u'(c_{t+1}^T)}{S_{t+1}} \right\}. \tag{10b}$$

Equation (10a) is the standard Euler equation for the trader which relates the expected marginal rate of consumption substitution between today and tomorrow to the return on savings (given by $1 + r$) discounted to today. Equation (10b), on the other hand, determines the optimal holdings of nominal bonds. Equations (10a) and (10b) jointly determine the modified interest parity condition for this economy which reflects the standard portfolio choice between safe and risky assets.

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[9]In equilibrium $i_t > 0$, which implies that the cash-in-advance constraint for traders always binds. It can be shown that the sufficient conditions for non-traders' cash-in-advance constraints to bind, derived in appendix 6.1, are sufficient to ensure $i_t > 0$ in equilibrium.

2.2 Government

The government in this economy holds foreign bonds (reserves) which earn the world rate of interest r. The government can sell nominal domestic bonds, issue domestic money, and make lump sum transfers to the traders. Thus, the government's budget constraint is given by

$$S_t h_{t+1} - (1+r)S_t h_t + (1+i_{t-1})B_t - B_{t+1} + T_t = M_{t+1} - M_t, \tag{11}$$

where B denotes the amount of nominal government bonds held by the private sector, h are foreign bonds held by the government, M is the aggregate money supply, and T is government transfers to the traders. Equation (11) makes clear that the money supply can be altered in three ways: through open market operations, through interventions in the foreign exchange market, or through transfers. Importantly, all three methods impact only the traders since they are the only agents present in the asset market.

2.3 Equilibrium conditions

Equilibrium in the money market requires that

$$M_t = \lambda M_t^T + (1-\lambda)M_t^{NT}. \tag{12}$$

The flow constraint for the economy as a whole (i.e., the current account) follows from combining the flow constraint for non-traders (equation (4)), traders (equations (7) and (9)), and the government (equation (11)) and money market equilibrium (equation (12)):

$$\lambda c_t^T + (1-\lambda)c_t^{NT} = y_t + (1+r)k_t - k_{t+1}, \tag{13}$$

where $k \equiv h + \lambda f$ denotes per-capita foreign bonds for the economy as a whole.

To obtain the quantity theory, combine (3), (9), and (12) to get:

$$\frac{M_{t+1}}{1-v_t} = S_t y_t. \tag{14}$$

Notice that the stock of money relevant for the quantity theory is end of period t money balances (i.e., M_{t+1}). This reflects the fact that, unlike standard CIA models (in which the goods market

opens before the asset market and shoppers cannot withdraw current sales receipts for consumption), in this model (i) asset markets open before goods market open (which allows traders to change this period's money balances for consumption purposes); and (ii) both traders and non-traders can access a fraction of current sales receipts.

Combining (3) and (5) gives the consumption of non-traders:

$$c_t^{NT} = \frac{(1 - v_{t-1}) S_{t-1} y_{t-1} + v_t S_t y_t}{S_t}. \tag{15}$$

To derive the consumption of traders, we use equation (9) to substitute for M_t^T in equation (8). Then, subtracting $S_t y_t$ from both sides allows us to rewrite (8) as

$$(1 + r) f_t - f_{t+1} + \frac{(1 + i_t)\frac{B_t}{\lambda} - \frac{B_{t+1}}{\lambda} + \frac{T_t}{\lambda}}{S_t} + y_t - c_t^T = \frac{M_{t+1} - M_t}{S_t},$$

where we have used equation (14) to get $M_{t+1} - M_t = [(S_t y_t - S_{t-1} y_{t-1}) - (v_t S_t y_t - v_{t-1} S_{t-1} y_{t-1})]$. Using equation (11) in the equation above gives

$$\frac{k_{t+1}}{\lambda} - (1 + r)\frac{k_t}{\lambda} = y_t - c_t^T + \left(\frac{1 - \lambda}{\lambda}\right)\left(\frac{M_{t+1} - M_t}{S_t}\right), \tag{16}$$

where k_0 is given exogenously. Equation (16) gives the trader's flow constraint in equilibrium. The left hand side captures the net acquisition of foreign assets (per trader) by the economy while the right hand side gives periodic trader income net of consumption. Given the precise monetary regime, we can iterate forward equation (16) and impose the trader household's first order condition for optimal consumption (equation (10a)) to derive the trader's policy function for consumption along a rational expectations equilibrium path.

It is worth noting that the last term on the right hand side of (16) captures the source of redistribution in this economy. Any changes of money supply occur through central bank operations in the asset market where only traders are present. Hence, the traders receive the entire incremental money injection while their own increase in money balances is only a fraction λ of the total. This leads to redistribution of $\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{M_{t+1} - M_t}{S_t}\right)$ from non-traders to traders. Note that as $\lambda \to 1$ this term goes to zero. It is important to note that this channel exists solely due to asset market segmentation.

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3 Alternative exchange rate regimes

Having described the model and the equilibrium conditions above, we now turn to allocations under specific exchange rate regimes. We will look at two pure cases: flexible exchange rates and fixed exchange rates. The end goal, of course, is to evaluate the welfare implications under the two regimes. In all the policy experiments below, we shall assume that the initial distribution of nominal money balances across the two types of agents is invariant. In particular, we assume that $M_0^T = M_0^{NT} = \bar{M}$.

In order to make the analytics of the welfare comparisons tractable, we shall also assume from hereon that the periodic utility function of both agents is quadratic:

$$u(c) = c - \zeta c^2. \tag{17}$$

To focus our results, we shall proceed by analyzing the effect of each shock in isolation. In particular, we first study an environment where the only shock is the velocity shock and then go to the other case where the only shock is the real shock.

3.1 Velocity shocks only

In this subsection we focus solely on velocity shocks. Hence, we set $\sigma_y^2 = 0$. Thus, there is no uncertainty about the endowment process. Every period all households receive the fixed endowment $\bar{y}$.

3.1.1 Flexible exchange rates under velocity shocks

We assume that under flexible exchange rates, the monetary authority sets a constant path of the money supply:

$$M_t = \bar{M}.$$

Further, the government does not intervene in foreign exchange markets and, for simplicity, we assume that initial foreign reserves are zero. Then, the government's flow constraint reduces to:

$$(1 + i_{t-1})B_t - B_{t+1} + T_t = 0. \tag{18}$$

The quantity theory equation (14) determines the exchange rate:

$$S_t = \frac{\bar{M}}{(1 - v_t)\bar{y}}. \tag{19}$$

The exchange rate will thus follow the velocity shock and be high (low) when the shock v is high (low).

Using (19), consumption of non-traders (given by equation(15)) under flexible exchange rates can be written as:

$$c_t^{NT,flex} = \bar{y} , \qquad t \geq 0. \tag{20}$$

Equation (20) shows that consumption of non-traders remains constant at all times. Intuitively, under floating exchange rates, prices change in proportion to the velocity shocks. Since the velocity shock is common to all agents, there is no redistribution of purchasing power between agents.

To determine consumption of traders under the floating exchange rate regime, we can iterate forward equation (16) under the condition $M_t = \bar{M}$ to get

$$c_t^{T,flex} = r\frac{k_0}{\lambda} + \bar{y} , \qquad t \geq 0, \tag{21}$$

where we have used the fact that under the quadratic utility specification adopted above, equation (10a) – which describes the optimal consumption plans for traders – reduces to $c_0 = E_0(c_t)$ for all $t > 0$. Hence, under flexible exchange rates, consumption of traders is also constant over time. The intuition is the same as before. Since, prices change in proportion to their velocity shock, there are no real balance effects on the traders. Hence, their consumption remains invariant over time.

3.1.2 Fixed exchange rates under velocity shocks

Under fixed exchange rates, the monetary authority sets a constant path of the exchange rate equal to $\bar{S}$. In particular, we assume that the nominal exchange rate is fixed at

$$\bar{S} = \frac{\bar{M}}{(1 - \bar{v})\bar{y}}. \tag{22}$$

In effect, we are assuming that at time $t = 0$ the monetary authority pegs the exchange rate at the deterministic equilibrium level.

Under this specification, it is easy to see from equation (15) that consumption of non-traders under a fixed exchange rate is given by

$$c_t^{NT,peg} = \bar{y}\left[1 + (v_t - v_{t-1})\right], \tag{23a}$$

$$c_0^{NT,peg} = \bar{y}\left[1 + (v_0 - \bar{v})\right]. \tag{23b}$$

Equation (23a) shows that under an exchange rate peg, consumption of non-traders will fluctuate by the full amount of their velocity shock. Intuitively, velocity shocks change the nominal balances that non-traders have available for consumption. Since the price level is now fixed, any change in nominal balances also implies a one-for-one change in real balances and, hence, affects the consumption of non-traders.

To determine the consumption of traders we again iterate forward on equation (16) by using the Euler equation $c_0 = E_0(c_t)$ and after imposing the condition $S_t = \bar{S}$ for all t, we get

$$c_0^{T,peg} = r\frac{k_0}{\lambda} + \bar{y}\left[1 + \left(1 - \frac{1}{\lambda}\right)\frac{r}{1+r}\left[(v_0 - \bar{v}) + E_0\left\{\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^t(v_t - v_{t-1})\right\}\right]\right]. \tag{24}$$

In deriving (24) we have used the fact that under pegged exchange rates, equation (14) implies that $M_{t+1} - M_t = -(v_t - v_{t-1})\bar{S}\bar{y}$.[10]

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[10]More generally, consumption of traders under fixed exchange rates at any point in time $t > 1$ is given by

$$c_t^{T,peg} = r\frac{k_t}{\lambda} + y\left[1 + \left(1 - \frac{1}{\lambda}\right)E_t\left\{\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(v_s - v_{s-1})\right\}\right].$$

To understand the consumption function of traders, note that under fixed exchange rates, the nominal value of GDP remains unchanged, i.e., $S_t \bar{y} = \bar{S} \bar{y}$ for all t. The quantity theory relationship requires that aggregate nominal money balances plus the aggregate withdrawal from current period sales be sufficient to purchase current nominal output. To keep nominal output unchanged over time, any change in cash withdrawals from current receipts, i.e., $v_t \neq v_{t-1}$, must be met by the monetary authority with an offsetting change in aggregate nominal money balances. This intervention must happen through transactions in the asset market where only traders are present. On a per trader basis then, the proportional change in nominal money balances needed for keeping the exchange rate fixed is $-\frac{1}{\lambda}(v_t - v_{t-1})$. Thus, under fixed exchange rates, a velocity shock of Δv not only changes real balances of traders by the full amount but also changes their real balances by $-\frac{1}{\lambda}$ due to central bank intervention. The net effect is $1 - \frac{1}{\lambda}$ which is the term that shows up in the coefficient on the velocity shocks in equation (24).

3.1.3 Optimal exchange rate regime

Having described allocations under the alternative exchange rate arrangements, we now turn to the key focus of the paper: determination of the optimal exchange rate regime. We shall conduct our analysis by comparing the unconditional expectation of lifetime welfare at time $t = 0$ (i.e., before the revelation of any information at time 0). In terms of preliminaries, it is useful to define the following:

$$
W^{i,j} = E\left\{\sum \beta^t \left[c_t^{i,j} - \zeta \left(c_t^{i,j}\right)^2\right]\right\}, \quad i = T, NT, \quad j = flex, \; peg, \tag{25a}
$$

$$
W^j = \lambda W^{T,j} + (1 - \lambda)W^{NT,j}, \quad j = flex, \; peg. \tag{25b}
$$

Equation (25a) gives the welfare for each agent under a specific exchange rate regime where the relevant consumption for each type of agent is given by the consumption functions derived above for each regime. Equation (25b) is the aggregate welfare for the economy under each regime which is the sum of the regime specific individual welfares weighted by their population shares. Note that the quadratic utility specification implies that the expected value of periodic utility can be

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written as

$$E\left(c - \zeta c^2\right) = E(c) - \zeta \left[E(c)\right]^2 - \zeta Var(c). \tag{26}$$

where $var(c)$ denotes the variance of consumption.

Proposition 1 *When velocity shocks are the only source of uncertainty in the economy, the flexible exchange rate regime welfare-dominates the fixed exchange rate regime for both agents and hence, is the optimal exchange rate regime for the economy. When all agents in the economy are traders, i.e., $\lambda = 1$, the fixed and flexible exchange rate regimes are welfare equivalent.*

Proof. It is easy to see that $E(c_t^{NT,flex}) = E(c_t^{NT,peg}) = \bar{y}$ while $E(c_0^{T,flex}) = E(c_0^{T,peg}) = r\frac{k_0}{\lambda} + \bar{y}$. Hence, for both types of agents, expected consumption under the two regimes is identical. However, $Var(c_t^{T,peg}) > Var(c_t^{T,flex}) = 0$ and $Var(c_t^{NT,peg}) > Var(c_t^{NT,flex}) = 0$ for all t. From the expression for expected periodic utility given by (26), it then follows directly that

$$W^{i,flex} > W^{i,peg}, \quad i = T, NT.$$

For $\lambda = 1$ it follows directly from equations (21) and (24) that

$$c_0^{T,flex} = c_0^{T,peg} = rk_0 + \bar{y}.$$

Hence, consumption for traders is identical under both regimes. Moreover, since no stochastic terms enter the consumption function, welfare of traders (and hence aggregate welfare as well) must be identical under both regimes. ■

Intuitively, under flexible exchange rates the adjustment of the price level is proportional to the velocity shock of both agents. Hence, flexible exchange rates completely insulate the real balances of both agents which allows them to smooth consumption completely. Under fixed exchange rate on the other hand, a wealth redistribution occurs across agents due to velocity shocks. Specifically, in order to keep the exchange rate unchanged, the monetary authority intervenes in the asset market to accommodate the average effect of the velocity shock. This affects transfers to traders which induces redistributions. As a result, consumption of non-traders fluctuates over time while consumption of traders is affected by a wealth effect coming from asset market transfers. Hence,

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for $\lambda < 1$, welfare under flexible exchange rates is greater than welfare under fixed exchange rates for both agents. Thus, aggregate welfare under flexible exchange rates is *unambiguously greater* than under fixed rates.

For $\lambda = 1$, our result is similar to the well known result of Helpman and Razin (1979) who showed the welfare equivalence between fixed and flexible exchange rates for representative agent economies with perfect capital mobility where agents are subject to cash-in-advance constraints. Intuitively, under flexible exchange rates the price level adjusts exactly in proportion to the trader's velocity shock which leaves her real balances unchanged and thereby insulates her completely from any wealth effects due to real balance fluctuations. Symmetrically, when exchange rates are fixed, the monetary authority pegs the exchange rate by exactly offsetting the aggregate velocity shock through a corresponding intervention in asset markets. When all agents are in the asset market, the intervention amount in asset markets corresponds exactly to the size of the trader's velocity shock which leaves their real balances unchanged. As in the flexible exchange rate case, this intervention effectively insulates traders from any wealth effects due to their velocity shocks. Hence, the two regimes are identical from a welfare standpoint.

3.2 Output shocks only

We now turn to the issue of real shocks and their effects in this model. To focus on this issue, we assume that $v_t = \bar{v}$ for all t and $\sigma_v^2 = 0$. In other words, there is no uncertainty regarding the velocity realization. However, we now assume that output, y_t, is i.i.d. with mean $\bar{y}$ and variance σ_y^2.

To analyze the welfare trade-offs under fixed and flexible exchange rates we shall continue to assume that under flexible exchange rates $M_t = \bar{M}$ for all t while under fixed exchange rates $S_t = \bar{S} \ (= \bar{M}/(1 - \bar{v})\bar{y})$.

The quantity theory equation in this case is given by

$$\frac{M_{t+1}}{1 - \bar{v}} = S_t y_t.$$

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Note that $M_t = \bar{M}$ implies that under a flexible exchange rate regime, nominal income is constant over time. Hence, nominal money balances of both types are also constant over time.

The above implies that consumption allocations for non-traders under the two regimes, using equation (15), are given by

$$c_t^{NT,flex} = y_t, \tag{27a}$$

$$c_t^{NT,peg} = (1 - \bar{v}) y_{t-1} + \bar{v} y_t. \tag{27b}$$

Note that in deriving equation (27a) we have used the fact $S_{t-1} y_{t-1} = S_t y_t$ under flexible rates. Similarly, iterating forward on the periodic budget constraint for the trading households, equation (16), and imposing the relevant monetary regime on the result gives the consumption allocations for traders under the two regimes:

$$c_t^{T,flex} = r\frac{k_t}{\lambda} + (1 - \beta)y_t + \beta\bar{y}, \tag{28}$$

whereas

$$c_t^{T,peg} = r\frac{k_t}{\lambda} + \underbrace{(1 - \beta)\frac{1 - (1 - \lambda)(\bar{v} + \beta(1 - \bar{v}))}{\lambda}}_{\Psi_1} y_t - \underbrace{(1 - \beta)\frac{(1 - \lambda)(1 - \bar{v})}{\lambda}}_{\Psi_2} y_{t-1}$$
$$+ \underbrace{\beta\frac{1 - (1 - \lambda)(\bar{v} + \beta(1 - \bar{v}))}{\lambda}}_{\Psi_1/r}\bar{y}. \tag{29}$$

It is easy to check that $E(c_t^{NT,flex}) = E(c_t^{NT,peg}) = \bar{y}$ and $E\left(c_t^{T,flex}\right) = E\left(c_t^{T,peg}\right) = r\frac{k_0}{\lambda} + \bar{y}$. Hence, expected consumption of both types is identical under the two regimes.

However the variance of consumption is different. Specifically,

$$Var(c_t^{NT,flex}) = \sigma_y^2, \tag{30a}$$

$$Var(c_t^{NT,peg}) = \sigma_y^2 [1 - 2\bar{v}(1 - \bar{v})] < \sigma_y^2, \tag{30b}$$

$$Var(c_0^{NT,peg}) = \bar{v}^2\sigma_y^2 < \sigma_y^2 \tag{30c}$$

Hence, for non-trading households, consumption volatility is lower under a peg relative to a flexible exchange rate regime. Given that expected consumption is identical under the two regimes while

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volatility is lower under a peg, it follows that non-traders always prefer a fixed exchange rate regime to a flexible rate regime when shocks are real.[11]

To understand the intuition, note that under flexible exchange rates, a constant path of nominal money balances implies that the real value of last period's sales (in terms of current prices) is always equal to current output. Hence, current consumption (which is a weighted average of current and last period's real sales revenues) is just current output. Thus, the entire variance of current output is reflected in the variance of current consumption. Under an exchange rate peg on the other hand, the real value of last period's sales is always last period's output. Hence, current consumption is a weighted average of last period and current period's output. The resulting lower variance of consumption under a peg reflects a form of risk pooling: the consumption risk is pooled across periods.

The variance of consumption of trading households also differs across the two regimes. In the appendix (6.2) we show that

$$VAR\left(c_t^{T,flex}\right) = (t+1)(1-\beta)^2 \sigma_y^2, \tag{31a}$$

$$Var\left(c_t^{T,peg}\right) = (t+1)\ \Psi_1^2\ \sigma_y^2, \tag{31b}$$

where Ψ_1 is as defined in equation (29). Notice that for $\lambda = 1$, $\Psi_1 = 1 - \beta$ and $\Psi_2 = 0$; then equation (29) reduces to (28), and $VAR\left(c_t^{T,flex}\right) = Var\left(c_t^{T,peg}\right)$.

Our welfare metric is given, as before, by equation (25b). In order to compare welfare across regimes we define $\Delta W \equiv W^{flex} - W^{peg}$. Substituting equations (30a-31b) in (25a) and (25b) gives

$$\Delta W = \lambda \Delta W^T + (1-\lambda)\Delta W^N = \frac{1}{\lambda}\left(c_2\lambda^2 + c_1\lambda + c_0\right), \tag{32}$$

where $c_2 = (1-\bar{v})^2\left((1-\beta)^2 - \frac{1+\beta}{1-\beta}\right) + 2(1-\bar{v})\left(\frac{1}{1-\beta} - (1-\beta)\right)$; $c_1 = -(1-\bar{v})^2\left(2(1-\beta)^2 - \frac{1+\beta}{1-\beta}\right) - 2(1-\bar{v})\left(\frac{1}{1-\beta} - (1-\beta)\right)$; $c_0 = (1-\bar{v})^2(1-\beta)^2$.

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[11]We should note that this result is crucially dependent on the household being able to consume some fraction of current sales, i.e., $v > 0$. If $v = 0$ then $c_t^{NT,flex} = y_t$ and $c_t^{NT,peg} = y_{t-1}$. Hence, both expected consumption and the variance of consumption would be identical under the two regimes.

Lemma 1 $c_2\lambda^2 + c_1\lambda + c_0 = 0$ *has two roots:* $\lambda_1 = 1$ *and* $\lambda_2 = \frac{(1-\beta)^2}{\left((1-\beta)^2 - \frac{1+\beta}{1-\beta}\right) + \frac{2}{1-\bar{v}}\left(\frac{1}{1-\beta} - (1-\beta)\right)} \in$ $\left(0, r^2\right)$ *for* $\bar{v} > 0$.

Proof. See Appendix. ∎

Proposition 2 *When endowment shocks are the only source of uncertainty in the economy and* $\bar{v} > 0$, *the optimal exchange rate regime is a fixed exchange rate for all* $\lambda > \lambda_2 \in \left(0, r^2\right)$ *while flexible exchange rates are optimal for all* $\lambda < \lambda_2$. *When all agents in the economy are traders, i.e.,* $\lambda = 1$, *the fixed and flexible exchange rate regimes are welfare equivalent.*

Proof. See Appendix. ∎

Proposition 2 shows that whenever the share of traders is above a critical threshold, fixed exchange rates are the optimal regime under real shocks. To understand the intuition behind this result, it is helpful to note that while both types of agents face the same shock, their ability to cope with them is asymmetric. In particular, trading households have an extra instrument – financial assets – with which to smooth out their consumption flow in response to shocks. Thus, the welfare losses of traders in shifting from a flexible exchange rate regime to a peg is always smaller than the corresponding loss of a non-trading household moving from a peg to a flexible exchange rate regime. Thus, the preferences of non-traders typically dominates the overall welfare criterion.

The only caveat to this intuition occurs for very small values of λ. In particular, when $\lambda < \lambda_2$, a very small number of trading households have to bear the burden of maintaining an exchange rate peg for the entire economy by accepting all the monetary injections or withdrawals. Due to their very small numbers, the resultant consumption volatility of traders under a peg becomes very large. At the limit, consumption volatility of traders goes to infinity as λ tends to zero. However, as $\lambda_2 < r^2$, the range in which flexible rates are optimal is very small. This suggests that exchange rate pegs are, in general, the optimal regime under output shocks.[12]

The welfare equivalence of the two regimes when $\lambda = 1$ can be understood as follows. Under

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[12]Note that we have imposed the restriction $\lambda > 0$ throughout the paper. This reflects the fact that from a monetary policy viewpoint, the model has a discontinuity at $\lambda = 0$ since the monetary authority has no mechanism to introduce money into the economy if there are no traders. Hence, the discussion above applies only to λ strictly greater than zero.

flexible exchange rates there are no monetary injections or withdrawals. On the other hand, to maintain a peg, the government either injects or withdraws money through open market operations. These transfers however are entirely internal to the economy and, hence, do not affect the (exogenous) income stream of the economy. In either regime, the trading households smooth their consumption through asset markets. As the economy's income stream is identical under the two regimes, and as all agents are identical, the Ricardian equivalence implies that the allocations will also be identical.

4   Optimal monetary policy

Having compared fixed and flexible exchange rate regimes, an obvious question is whether there are other welfare superior policy rules. In particular, what is the first-best monetary policy in this environment?

An efficient first-best allocation under the model's environment requires that ex-post the marginal utility of consumption be equalized across all agents. This in turn implies that the consumption of traders and non-traders be equal, i.e., $c_t^T = c_t^{NT}$ for all t. Recall that the trader's optimal consumption follows a random walk, i.e., $c_t^T = E\left\{c_{t+1}^T\right\}$. The first best policy then implies that $c_t^{NT} = E\left\{c_{t+1}^{NT}\right\}$. To derive c_t^T, we first rewrite the economy's resource constraint (13) by iterating forward as

$$\sum_{s=t}^{\infty} \beta^{s-t} \left[\lambda c_s^T + (1-\lambda)c_s^{NT}\right] = (1+r)\,k_t + \sum_{s=t}^{\infty} \beta^{s-t} y_s.$$

The above must hold with probability one; therefore, it will also hold under expectations. Using the first-best allocation and the trader's optimality condition the resource constraint under expectations yields

$$c_t^T = c_t^{NT} = rk_t + (1-\beta)\,y_t + \beta\bar{y} \tag{33}$$

We now need to find the money growth rate μ_t that can implement the first-best. To do so, rewrite

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the non-trader's flow constraint (4) as

$$c_t^{NT} = y_t - \frac{M_{t+1}^{NT} - M_t^{NT}}{S_t}.$$

From the money market equilibrium, $M_{t+1}^{NT} = M_{t+1} = (1 + \mu_t) M_t = (1 + \mu_t) M_t^{NT}$. Then using the quantity theory equation (14) we get

$$c_t^{NT} = \frac{1 + \mu_t v_t}{1 + \mu_t} y_t \tag{34}$$

Combining (33) with (34) yields

$$\mu_t = \frac{\beta (y_t - \bar{y}) - r k_t}{\bar{y} + r y_t - v_t \beta^{-1} y_t + r k_t}$$

For expositional convenience, let us consider the case with $k_t = 0$. Then

$$\mu_t = \frac{y_t - \bar{y}}{\bar{y} + r y_t - v_t \beta^{-1} y_t} \tag{35}$$

A few features of this policy rule are noteworthy. First, since the monetary authority chooses μ after observing the realizations for y and v, this rule is implementable. Second, equation (35) makes clear that when there are no shocks to output, i.e., $y_t = \bar{y}$ for all t, the optimal policy is to choose $\mu_t = 0$ for all t independent of the velocity shock.[13] But this is precisely the flexible exchange rate case. Hence, under velocity shocks a flexible exchange rate regime with a constant money supply implements the first-best allocation.

A third interesting feature of equation (35) is that the optimal monetary policy is procyclical. In particular, it is easy to check that

$$\frac{\partial \mu_t}{\partial y_t} = \frac{\bar{y} (1 + r) (1 - v_t)}{(\bar{y} + r y_t - v_t \beta^{-1} y_t)^2} > 0.$$

Note that the latter inequality above follows from the fact that v is strictly bounded above by one. The intuition for this result is that, ceteris paribus, an increase in output raises consumption through two channels. Current sales revenue rise and hence raise the cash available for consumption.

---

[13]When $k_0 = 0$, and $y_t = \bar{y}$ for all t, (13) and (33) imply that $k_t = 0$ for all t; thus $\mu_t = 0$ is indeed the first-best policy for all t.

Moreover, an increase in output appreciates the currency thereby raising the real value of money balances brought into the period. To counteract these expansionary effects on consumption, the optimal monetary policy calls for an expansion in money growth so as to depreciate the currency and thus inflate away the nominal gains. Fourth, the optimal policy response to velocity shocks depends on the level of output relative to its mean level. In particular,

$$\frac{\partial \mu_t}{\partial v_t} = \frac{y_t(y_t - \bar{y})}{\beta(\bar{y} + ry_t - v_t\beta^{-1}y_t)^2} \gtrless 0.$$

Thus, when output is above the mean level, an increase in v calls for an increase in money growth while if output is below the mean then the opposite is true. Intuitively, an increase in v_t has two opposing effects on real balances available for consumption. First, it raises real balances through appropriating a higher proportion of current sales. Second, a higher v_t depreciates the currency thereby deceasing the real value of money balances brought into the period. When output is equal to the mean level, absent a change in policy, these effects completely neutralize each other. On the other hand, when output is above (below) the mean, the current sales effect is stronger (weaker) than the exchange rate effect. Hence, an increase (decrease) in μ provides the appropriate correction by depreciating (appreciating) the currency.

5 Conclusion

The determination of the optimal exchange rate regime for an open economy is one of the oldest issues in international economics. While there exists a very long and old literature on this topic, most of the work in this area has been conducted in the context of environments with some sort of nominal rigidity – either in wages or in prices. In this paper we have studied an entirely different environment wherein the key friction is in asset markets. In particular, we have analyzed a model in which only a fraction of agents trade in assets. In this environment fixing exchange rates entails central bank interventions in the asset market where only a fraction of agents are present. Hence, monetary shocks (shocks to velocity in our context) under fixed exchange rate regimes cause redistributions across agents thereby generating consumption volatility. On the other hand,

when exchange rates are flexible, monetary shocks cause changes in the price level which insulate agents' real balances. Thus, asset market segmentation causes an inherent welfare bias towards flexible exchange rate regimes when shocks are monetary. On the other hand we have also found that when the economy faces output shocks, under fairly general conditions, fixed exchange rates unambiguously welfare dominate flexible rates.

It is worth nothing that while we have derived the optimal state contingent money growth rule which implements the first-best allocation in this economy, we have not undertaken a detailed qualitative and quantitative comparison of state-contingent rules with optimal non-state contingent rules which would also include the Friedman rule. This is an interesting research topic and is the focus of attention in a related paper (Lahiri, Singh, and Végh (2004)).

We have also ignored the issue of endogeneity of market segmentation. In particular, one would expect that agents endogenously choose to be traders or non-traders with the choice being sensitive to the cost of participating in asset markets as well as the prevailing exchange rate and/or monetary regime. However, we see no reason to believe that this would change our key results. As should be clear from the intuition provided in the paper, what matters for our results is that, at every point in time, some agents have access to assets market while others do not. What particular agents have access to asset markets and whether this group changes over time should not alter the essential arguments. A formal check of this conjecture is left for future work.

6 Appendix

6.1 Sufficient conditions for binding cash-in-advance constraints for non-traders:

At each t, the non-traders maximize (1) subject to their period t budget constraint (4) and

$$M_{t+1}^{NT} \geq (1 - v_t) \ S_t \ y_t. \tag{36}$$

The above equation can be interpreted as the effective cash in advance constraint: if in any period the non-traders do not exhaust the within-period cash available for consumption, the cash remaining at the end of the period then must exceed the RHS. The optimality condition can then be summarized by

$$u'(c_t) \geq \beta E_t \left\{ \frac{S_t}{S_{t+1}} u'(c_{t+1}) \right\}, \qquad \begin{cases} = & \text{if } M_{t+1} > (1 - v_t) S_t y_t, \\ > & \text{only if } M_{t+1} = (1 - v_t) S_t y_t, \end{cases} . \tag{37}$$

Thus for the cash in advance constraint to always bind in equilibrium (37) must hold with inequality for all t. Below, we consider flexible and fixed exchange rates alternately and establish sufficient conditions that ensure that cash in advance constraints bind under both velocity and output shocks.

6.1.1 Flexible exchange rates

Velocity shocks only Here, $c^{NT} = \bar{y}$. Using (19) with (37) implies that the CIA will always bind if

$$\frac{1 - v_{\max}}{1 - \bar{v}} > \beta \tag{38}$$

Output shocks only Here, $c^{NT} = y_t$. Using (14) with (37) implies that the CIA will always bind if

$$\frac{\{y - 2 \zeta y^2\}_{\min}}{\bar{y} - 2 \zeta \left(\bar{y}^2 + \sigma_y^2\right)} > \beta \tag{39}$$

6.1.2 Fixed exchange rates

Velocity shocks only Here $S_t = S_{t+1} = \bar{S}$, and $c_t^{NT} = \bar{y}\,(1 - v_{t-1} + v_t)$. Then (37) implies that the CIA will always bind if

$$\frac{1 - 2\,\zeta\,\bar{y}\,[1 - v_{\min} + v_{\max}\,]}{1 - 2\,\zeta\,\bar{y}\,[\,1 - v_{\max} + \bar{v}]} > \beta \tag{40}$$

Output shocks only Here, $c_t^{NT} = (1 - v_{t-1})\,y_{t-1} + v_t y_t$. Then (37) implies that the CIA will always bind if

$$\left\{ \frac{1 - 2\,\zeta\,[(1 - \bar{v})\,y_{\max} + \bar{v}\,y_t]}{1 - 2\,\zeta\,[(1 - \bar{v})\,y_t + \bar{v}\,\bar{y}]} \right\}_{\min} > \beta \tag{41}$$

6.1.3 An example:

Establishing sufficient conditions requires imposing restrictions on the shock processes. For the utility specification $c - \zeta\,c^2$, let $\zeta = 0.2$. Let the output and velocity processes be uniformly distributed as $y \in [0.96, 1.04]$ and $v \in [0.18, 0.22]$. This approximately implies a % standard deviation of 2.5 for the output process and 6% for the velocity process. Then for $\beta = 0.96$ it is easily checked that (38) - (41) are satisfied

6.2 Expressions for variances under output shocks

First, from equation (29), we obtain

$$Var\left[c_0^{T,peg}\right] = \Psi_1^2 \sigma_y^2. \tag{42a}$$

$$Var\left(c_t^{T,peg}\right) = r^2 Var\left[\frac{k_t}{\lambda}\right] + \left(\Psi_1^2 + \Psi_2^2\right)\sigma_y^2 - 2\,r\,\Psi_2 Cov[k_t, y_{t-1}], \ t \geq 1 \tag{42b}$$

where Ψ_1 and Ψ_2 are as defined in (29). From equations (29) and (16), we get

$$\frac{k_{t+1}}{\lambda} = \frac{k_t}{\lambda} + \beta \underbrace{\frac{1 - (1 - \lambda)\,(\bar{v}\, - (1 - \beta)\,\gamma\,)}{\lambda}}_{(\Psi_1 + \Psi_2)/r}y_t - \beta \underbrace{\frac{(1 - \lambda)\,\gamma}{\lambda}}_{\Psi_2/r}y_{t-1}$$
$$- \beta \underbrace{\frac{1 - (1 - \lambda)\,(\bar{v}\, + \beta\,\gamma\,)}{\lambda}}_{\Psi_1/r}\bar{y}$$

Assuming $k_0 = 0$ in the above implies $E\left(\frac{k_t}{\lambda}\right) = 0$ and

$$
\begin{aligned}
VAR\left(\frac{k_t}{\lambda}\right) &= \left((t-1)\left(\frac{\Psi_1}{r}\right)^2 + \left(\frac{\Psi_1 + \Psi_2}{r}\right)^2\right)\sigma_y^2, \\
Cov\left(k_t, y_{t-1}\right) &= \frac{\Psi_1 + \Psi_2}{r}\,\sigma_y^2,
\end{aligned}
$$

Using (42a) and (42b) with the previous two results yields

$$
Var\left(c_t^{T,peg}\right) = (t+1)\ \Psi_1^2\ \sigma_y^2, \text{ for all } t.
$$

Using the above expression the variance term of traders' life-time utility under a peg is obtained as

$$
-\frac{\Psi_1^2\ \sigma_y^2}{(1-\beta)^2} = -\left(\frac{1 - (1-\lambda)\,(\bar{v}\ + \beta\ (1-\bar{v})\ )}{\lambda}\right)^2 \tag{44}
$$

For the case of flexible exchange rates, from equation (28), we get

$$
Var\left[c_t^{T,flex}\right] = r^2 Var\left[\frac{k_t}{\lambda}\right] + (1-\beta)^2\,\sigma_y^2. \tag{45}
$$

Then, from equations (16) and (28), we get

$$
\frac{k_{t+1}}{\lambda} = \frac{k_t}{\lambda} + \beta y_t - \beta\bar{y},
$$

which directly yields $Var\left[\frac{k_t}{\lambda}\right] = t\beta^2\sigma_y^2$. Then, from (45) we get

$$
Var\left[c_t^{T,flex}\right] = (t+1)\,(1-\beta)^2\,\sigma_y^2
$$

Using the above expression, the variance term of traders' life-time utility under flexible exchange rates is obtained as

$$
-\zeta\sigma_y^2. \tag{46}
$$

6.3   Proof of Lemma 1

Notice that the expected consumption of both traders and non-traders are equal under the two regimes. Hence the difference in welfare solely arises due to the variance component. The welfare

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gain, $W^{flex} - W^{peg}$, as defined in (32), thus can be obtained by using (30a) - (30c) with (44) and (46) to yield

$$
\begin{aligned}
\Delta W &= W^{flex} - W^{peg} = \lambda\,\zeta\left[\left(\frac{(1-(1-\lambda)\,(\bar{v}\ +\beta\ (1-\bar{v})\ ))}{\lambda}\right)^2 - 1\right]\sigma_y^2 \\
&+\frac{1-\lambda}{1-\beta}\,\zeta\left[\left(\bar{v}^2+\beta\ (1-\bar{v})^2\right)-1\right]\sigma_y^2,
\end{aligned}
$$

which after collecting terms yields the second equality in (32) in the main text. Further algebraic factorization yields

$$
\begin{aligned}
\Delta W &= \left(\frac{1}{a^2\left((1-\beta)^2-\frac{1+\beta}{1-\beta}\right)+2a\left(\frac{1}{1-\beta}-(1-\beta)\right)}\right)\frac{1}{\lambda}\,(\lambda-1)\,* \qquad (47)\\
&\left(\lambda-\frac{a^2\,(1-\beta)^2}{a^2\left((1-\beta)^2-\frac{1+\beta}{1-\beta}\right)+2a\left(\frac{1}{1-\beta}-(1-\beta)\right)}\right),
\end{aligned}
$$

where $a = 1-\bar{v}$. Hence the two roots of $\Delta W = 0$ are $\lambda_1 = 1$ and $\lambda_2 = \frac{(1-\beta)^2}{\left((1-\beta)^2-\frac{1+\beta}{1-\beta}\right)+\frac{2}{1-\bar{v}}\left(\frac{1}{1-\beta}-(1-\beta)\right)}$.
Notice that λ_2 is monotonically decreasing in $\bar{v}$ which is bounded above by 1. For $\bar{v} = 0$ some algebra yields

$$
\lambda_2 = \frac{(1-\beta)^2}{\beta^2} = r^2,
$$

while, obviously, $\lambda_2 = 0$ if $\bar{v} = 1$. Hence, for $\bar{v} \in (0,1)$, $\lambda_2 \in \left(0, r^2\right)$.

6.4   Proof of Proposition 2

The proposition follows directly from (47) where it is easy to see that $\Delta W \gtrless 0$ iff for $\lambda \lessgtr \lambda_2$, and $\Delta W = 0$ for $\lambda = 1$.

Table 1: US Household ownership of financial assets, 1989

Checking account	Interest-bearing assets		Total
	No	Yes	
No	19%	6%	25%
Yes	40%	35%	75%
Total	59%	41%	100%

Source: Mulligan and Sala-i-Martin (2000). Data from the Survey of Consumer Finance.

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