IOWA STATE UNIVERSITY

Trade and Welfare Under Alternative Exchange Rate Regimes

Rajesh Singh

February 2004

Working Paper # 04008

Department of Economics
Working Papers Series

Ames, Iowa 50011

Trade and Welfare Under Alternative Exchange Rate Regimes[1]

RAJESH SINGH

Iowa State University

rsingh@iastate.edu

This draft: June 2002

Abstract

This paper compares the welfare under two standard alternative exchange rate regimes, fixed and flexible, in a stochastic dynamic general equilibrium two-country setting. Conventional wisdom holds that countries often prefer low exchange-rate variability to stabilize trade. This may explain the observed 'fear of floating' in emerging markets – although most of them claim to adopt a flexible system, in reality they often intervene to peg. We show that under incomplete capital markets a fixed exchange rate regime unambiguously increases trade and improves welfare. This provides a potential explanation for the observed exchange rate policies in emerging markets.

1   Introduction

In the debate over the merits of fixed versus floating exchange rates, the professional opinion has never remained fixed. At the time of Bretton Woods (1944), the architects of the postwar system favored fixed rates, as it was considered that the flexible rates were partly to be blamed for the economic instability during the interwar period. However by 1960s, most of the economists advocated a return to flexible rates, due to the widening U.S. balance-of-payments disequilibrium that eventually led to the breakdown of the Bretton Woods system. In 1980s, fixed exchange-rates were back in vogue again, as a credible way to achieving monetary stability, especially in high-inflation developing countries. Finally, the financial crises in Mexico (1994-95), East Asia (1997-98), Russia (1998), and Brazil (1999) swung the opinion back to favoring flexible exchange-rates, as fixed exchange rates were held to be the prime culprits in creating moral hazards in these insufficiently regulated financial markets.

There clearly seems to be a divergence of views between the international financial institutions (such as the IMF) and the emerging markets policy makers. The evidence points out that a majority of emerging market countries opt for fixed exchange-rate systems, even though they claim to follow a floating regime[1], perhaps in order to remain in the good books of the international aid agencies. For developing countries, Calvo and Reinhart (2000) find that the exchange rate variation in comparison with their money supply, interest rates, and even terms of trade is too small to justify their official claims. They conclude that there seems to be a widespread 'fear of floating' among these countries due to a lack of monetary policy credibility associated with a floating system, and also due to their large foreign currency denominated liabilities.

Yet, recently the debate has taken a new turn. Some researchers have claimed that the countries are (or should be) opting for corner solutions[2]. A key proponent of the bipolar view, Stanley Fischer[3] further qualifies this argument:

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[1]See Levy-Yeyati and Sturzenegger (2000)

[2]See for instance Eichengreen (1994,98) and Frankel et. al. (2000)

[3]See Fischer (2001)

To put the point graphically, if exchange rate arrangements lie along a line connecting hard pegs like currency unions, currency boards, and dollarization on the left, with free floating on the right, the intent of the bipolar view is not to rule out everything but the two corners, but rather pronounce as unsustainable segment of that line representing a variety of soft pegging exchange rate arrangements.

While a consensus is generally emerging that 'no single currency regime is right for all countries at all times'[4], the basic underlying arguments to evaluate the alternatives remain the same. In all recent financial crises episodes, a common feature has been a large currency mismatch between the assets and liabilities of the private sector. It is well known that emerging market countries often encourage foreign currency borrowings through insuring currency risk by a commitment to fixed exchange rates. Calvo and Reinhart (2000), argue that the mismatch between domestic currency assets and dollar denominated liabilities is the main rationale behind the prevalent 'fear-of-floating' in emerging markets. Both in the traditional and the current on-going debate over flexible versus fixed exchange-rate systems, the latter is argued, *inter alia*, to stabilize and improve international trade and investment. On the other hand, it is also argued that with free capital mobility the agents should be able to hedge their currency risks which weakens the case for fixed exchange rates. Surprisingly, however, the open economy literature has yet to provide theoretical underpinnings to these widely accepted arguments in an optimizing and dynamic general-equilibrium framework.

In this paper, we attempt to fill this gap by evaluating both trade and welfare under alternative exchange-rate systems in a standard neoclassical framework under various capital-market structures. In particular, we focus on the issue of currency mismatch between assets and liabilities of the private sector. We find that when the agents have hedging opportunities through trade in nominal bonds, a flexible exchange-rate system is welfare-superior to a fixed exchange-rate system. Under the former, both trade and consumption variabilities are smaller. On the other hand, with incomplete markets, a fixed exchange-rate system unambiguously increases trade in intermediate goods, and it is strictly welfare improving under fairly general conditions. More interestingly, we also find that the countries may find it optimal to 'manage-float' under certain circumstances.

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[4]See Frankel (1999)

Specifically, we show that there is a threshold level of uncertainty, in terms of the variance of the monetary innovations, beyond which both countries may find the fixed exchange-rate system to be welfare improving. Finally, with some restrictions on capital markets such as trade in only single currency nominal bonds, the initial conditions may be the key to an evaluation of the alternative systems. In particular, each may be superior to the other, depending on the inter-country initial level (and composition) of nominal claims. Again, the level of uncertainty turns out to be crucial to the choice of a system.

To address these issues, we present a two-country stochastic general equilibrium model with optimizing representative households. The countries freely trade both in intermediate and final goods. While the trade in final (consumption) good takes place in spot markets, trade in intermediate goods is through bilateral credit and is contracted one period in advance of the payment. Money is needed for transactions; both for purchasing consumption goods in spot markets as well as for making payments for input trade. While spot consumption purchases are done with domestic currency, trade payments are made in the seller currency. This, essentially, is the source of currency-risk in our model, since agents earn cash revenues in domestic currency, whereas the payments for the input purchase need to be made in foreign currency. Further, these decisions are made before monetary uncertainty is realized. We model uncertainty by assuming jointly symmetric stochastic processes for money growth innovations, where a fixed exchange-rate system amounts to an assumption that they are perfectly correlated[5].

In our benchmark model with complete markets, we show that the evaluation of an optimal regime is irrelevant, as in the resulting Arrow-Debreu equilibrium the allocations are Pareto optimal. Then we introduce market incompleteness in stages by first allowing trade in nominal bonds denominated in each country's currency, then in only one currency, and finally by completely restricting international trade in bonds. In all these cases, the real effects are obtained primarily through two channels. First, through the supply side as exchange rate risk creates home-bias in input use, which distorts the efficient allocation of resources. Second, through consumption variabil-

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[5]More generally, one can think of these innovations representing both money supply and money demand shocks.

ity that arises from the nominal denomination of international claims. With incomplete markets, fixed exchange rate system hedges currency risk, which obtains an efficient input allocation. On the other hand, fixed exchange rates reduce diversification opportunities available to households by perfectly correlating the returns on nominal bonds. Henceforth, results essentially depend on the existing level of financial development in terms of market completeness.

The traditional models that employ welfare metric for policy evaluation, such as Helpman and Razin (1979) and Helpman (1981), find that flexible exchange rate system is preferred to a fixed exchange rate system. However, these models introduce uncertainty through a one-time level shock, and therefore, abstract from stochastic environments. In Helpman and Razin (1982) where a stochastic environment is explicitly modeled in a two period optimizing framework, uncertain money shocks create a precautionary motive for holding money[6]. Using dynamic general equilibrium models under stochastic environments Stockman (1980) and Svensson (1984) analyze exchange rate determination with incomplete markets and find that the monetary policy has real effects, but they do not delve into the welfare evaluation of different exchange-rate systems. Enders and Lapan (1982) show that the expected utility is higher in the fixed exchange-rate system. However, there is a built-in asymmetry in their model; international capital is allowed to flow only in fixed exchange-rate system.

Similarly, sticky-price 'new open economy macroeconomics' models such as in Obstfeld and Rogoff (1995), Corsetti and Pesenti (1997), Obstfeld and Rogoff (1998), and Obstfeld and Rogoff (2000) mainly address the international transmission of monetary shocks and its welfare impact, but do not evaluate alternative exchange-rate systems. Finally, pricing-to-market models by Devereux and Engel (1999) and Betts and Devereux (2000), who explicitly evaluate welfare under alternative exchange-rate systems broadly find floating rates to be preferable. In the same framework, Bacchetta and van Wincoop (2000) find that trade is unaffected by the exchange-rate system and, in general, both trade and welfare can be higher under either exchange-rate system.

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[6]This motive is essentially created by an asymmetric treatment of currency and bond markets. Our approach clearly differs from theirs, as in our model money is only used for transactions.

In section 2, we present our model and analyze the equilibrium allocations under complete markets. In section 3, the same analysis is carried out under two cases of market incompleteness: (a) with nominal bonds in both currencies and (b) with nominal bonds only in Home currency. Section 4 completely shuts off international trade in bonds. In section 5, we offer some conclusions.

2 The Model

Consider a world consisting of two countries: Home and Foreign. Each country has an infinitely lived representative household with identical preferences over a homogeneous consumption good that is produced in both countries. In particular, at any instant t the Home household maximizes the expected value of the sum of its discounted one period utilities given by

$$U_t = E_t \sum_t^{\infty} \beta \ln c_t \tag{1}$$

where β is the discount factor, and E_t is an expectation operator conditional on the information available at t. Foreign's objective is similarly described. In what follows, we will usually work with variables and equations for Home that will be analogous for Foreign. To economize on notation, we will differentiate Foreign quantities and Foreign currency prices by using an asterisk.

Both Home and Foreign households have identical technologies to produce the consumption good. For Home, it is expressed as

$$Y_t = \theta \left(y_t^H \right)^{\alpha} \left(y_t^F \right)^{\gamma} ; \qquad \alpha + \gamma \leq 1 \tag{2}$$

where θ is a productivity constant, and y^H and y^F are two production inputs. Home and Foreign are endowed uniquely with input types H and F in the amounts of y and y^* respectively. From households' utility functions and production technologies it is implied that the inputs will be completely used. Hence, $y^H + y^{H*} = y$, and $y^F + y^{F*} = y^*$.

Both Home and Foreign households face a cash-in-advance constraint to purchase consumption good. In addition, they also need to pay in cash for the purchase of inputs. There is complete mobility in the goods market; each household can procure inputs as well as consumption good in

both markets at seller-currency prices. As is standard in these type of models, we assume that each household consists of a worker-shopper pair.

The event sequence runs as follows. In the beginning of each period, the worker and the shopper meet and pool their resources together. The resources consist of their country specific input endowments, and the nominal money balances carried overnight from the previous day's sales proceeds. Each household first chooses the allocations of the Home and Foreign inputs. The key assumption of our model is that these trades are made on credit and each household pays in cash during the next period at the pre-contracted seller-currency prices. Further, cash transactions are made between shoppers after markets open. Next, the worker and the shopper separate and the markets for consumption good, currency and loans open. The shopper pays (receives) last period's import (export) cash dues, decides the quantity of nominal assets to carry for the next period, and visits sale centers to purchase consumption good. The worker produces and sells the final good for cash at the home-currency market price. At the end of the day, the household gets together, consumes its purchases and keeps money balances overnight.

Before the day begins, each country's government provides a random lump-sum monetary transfer Γ (Γ^*) to the Home (Foreign) household. In the aggregate, these transfers are made as random fractions x and x^* of their existing money supplies. We denote Home's aggregate money supply carried from period $t - 1$ to t as M_t. Thus, in period t it evolves as $M_{t+1} = (1 + x_t)M_t$. We assume that the distribution of x and x^* is i.i.d. and jointly symmetric[7], with a correlation less than one. Under a flexible exchange-rate system, money supplies are generally different. A fixed exchange-rate system will require that the correlation be one, that is $x_t = x_t^*$ for all t.

Home's combined budget and cash-in-advance constraint in any period is given by

$$N_t + \Gamma_t + Z_t - S_t Z_t^* + B_t - q_t B_{t+1} \geq P_t c_t^H + S_t P_t^* c_t^F \tag{3}$$

[7]By assuming symmetric money growth rates we rule out any asymmetric inflationary effects on the supply side such as in Stockman (1981).

where the seller-currency values of H-input export (Z), and F-input import (Z^*), evolve as

$$Z_t = P_t^H \, y_{t-1}^{H*} \quad \text{and} \quad Z_t^* = P_t^{F*} \, y_{t-1}^F \tag{4}$$

and the household accumulates money balances as sales proceeds

$$N_{t+1} = P_t Y_t \tag{5}$$

P^H and P^{F*} are pre-determined seller-currency prices of H and F type inputs respectively. Although these equilibrium input prices are contracted at the time of delivery, the payments are made in subsequent period. Similarly, P and P^* represent the final good prices in Home and Foreign currency. Further, for Home (Foreign) y^{H*} and y^F denote quantities of H and F input export (import) and import (export) respectively. The households can freely trade in one period nominal bonds denominated in Home currency[8]. Thus, B represents the household's nominal asset holding. Notice that Home consumption has two components: (i) purchased from Home (c^H) and (ii) from Foreign (c^F). Finally, S denotes the nominal exchange rate as number of Home currency units per unit of Foreign currency.

An equilibrium with complete goods market mobility implies that $P_t = S_t P_t^*$. Hence, (3) can be rewritten as

$$N_t + \Gamma_t + Z_t - S_t \, Z_t^* + B_t - q_t B_{t+1} \geq P_t c_t \tag{6}$$

where $c \, (= c^H + c^F)$ denotes total Home consumption.

Since the money supplies are assumed to have a trend, it is convenient to convert all time t nominal variables as a fraction of the aggregate money supply carried forward from time $t-1$ to t (M_t). We define

$$
\begin{aligned}
m_t &= \frac{N}{M_t} & p_t &= \frac{P_t}{M_t} & p_t^H &= \frac{P_t^H}{M_t} \\
b_t &= \frac{B_t}{M_t} & \tau_t &= \frac{\Gamma_t}{M_t} & e_t &= S_t \frac{M_t^*}{M_t}
\end{aligned}
\tag{7}
$$

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[8] In subsequent analysis we expand the set of nominal assets available for international trade. The restriction, at this stage, is only for expositional convenience.

Since individual and aggregate money balances are equal ($N_t = M_t$), in equilibrium $m_t = 1$ $\forall\ t$. Similarly, $m^*, p^*, p^{F*}, z^*, b^*$, and τ^* denote Foreign counterparts. With these variables, the consolidated budget and cash-in-advance constraint (6) becomes

$$m_t + \tau_t + z_t - e_t\, z_t^* + b_t - q_t(1 + x_t)b_{t+1} \geq p_t c_t \tag{8}$$

Using (7), the normalized export and import payments (4), and the nominal money balances carried overnight (5) can be rewritten as

$$z_t = p_t^H\, y_{t-1}^{H*} \quad \text{and} \quad z_t^* = p_t^{F*}\, y_{t-1}^F \tag{9}$$

and

$$m_{t+1} = \frac{p_t Y_t}{(1 + x_t)} \tag{10}$$

Each period the household's choice involves allocating inputs for production and choosing a level of consumption and nominal assets, given its current nominal balances and asset holdings. Home's optimal choices are made by maximizing its discounted lifetime utility (1) subject to its budget and cash-in-advance constraints (8) and (10). The optimal choices are characterized by the first order conditions

$$b_{t+1}: \qquad \frac{q_t(1 + x_t)}{p_t c_t} = \beta E_t\left[\frac{1}{p_{t+1}c_{t+1}}\right] \tag{11}$$

$$y_t^{H*}: \qquad p_{t+1}^H\, y_t^H = \alpha \tag{12}$$

$$y_t^F: \qquad p_{t+1}^{F*}\, y_t^F = \gamma\frac{E_t\left[\frac{1}{p_{t+1}c_{t+1}}\right]}{E_t\left[\frac{e_{t+1}}{p_{t+1}c_{t+1}}\right]} \tag{13}$$

where the operator E_t obtains conditional statistical expectation based on the information available at t.

The first order condition (11) has the standard interpretation. The marginal utility of consumption sacrificed by saving a unit of nominal balances in the current period is equal to its expected

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next-period marginal value discounted for the household's impatience. Given the Cobb-Douglas production function, (12) states that the share of home input income (or its opportunity cost) in nominal GDP is equal to its share parameter α. Finally, (13) states that, in terms of next-period expected marginal utility, the share of Foreign currency import expenditure in Home's nominal GDP is equal to its share parameter γ. Similarly, Foreign first order conditions are

$$b_{t+1}^* : \qquad \frac{q_t(1+x_t)}{e_t\, p_t^* c_t^*} = \beta E_t \left[\frac{1}{e_{t+1} p_{t+1}^* c_{t+1}^*} \right] \tag{14}$$

$$y_t^{H*} : \qquad p_{t+1}^H y_t^{H*} = \alpha\, \frac{E_t \left[\frac{1}{p_{t+1}^* c_{t+1}^*} \right]}{E_t \left[\frac{1}{e_{t+1} p_{t+1}^* c_{t+1}^*} \right]} \tag{15}$$

$$y_t^F : \qquad p_{t+1}^{F*}\, y_t^{F*} = \gamma \tag{16}$$

Therefore, the equilibrium allocations will be determined by the budget constraints and the above first order conditions. In equilibrium, as $S_t = \frac{P_t}{P_t^*}$, it is clear that $e_t = \frac{p_t}{p_t^*}$. Further, as in equilibrium $m = m^* = 1$

$$p_t = \frac{1+x_t}{Y} \quad \text{and} \quad p_t^* = \frac{1+x_t^*}{Y^*} \tag{17}$$

and

$$e_t = \frac{1+x_t}{1+x_t^*} \frac{Y_t}{Y_t^*} \tag{18}$$

Equations (12), (13), (15), and (16) are easily manipulated to obtain the optimal equilibrium input allocations as

$$y_t^H = y\, \frac{E_t \left[\frac{1}{p_{t+1} c_{t+1}^*} \right]}{E_t \left[\frac{1}{p_{t+1}^* c_{t+1}^*} \right] + E_t \left[\frac{1}{p_{t+1} c_{t+1}^*} \right]}; \quad y_t^F = y^*\, \frac{E_t \left[\frac{1}{p_{t+1} c_{t+1}} \right]}{E_t \left[\frac{1}{p_{t+1}^* c_{t+1}} \right] + E_t \left[\frac{1}{p_{t+1} c_{t+1}} \right]} \tag{19}$$

and

$$y_t^{H*} = y\, \frac{E_t \left[\frac{1}{p_{t+1}^* c_{t+1}^*} \right]}{E_t \left[\frac{1}{p_{t+1}^* c_{t+1}^*} \right] + E_t \left[\frac{1}{p_{t+1} c_{t+1}^*} \right]}; \quad y_t^{F*} = y^*\, \frac{E_t \left[\frac{1}{p_{t+1}^* c_{t+1}} \right]}{E_t \left[\frac{1}{p_{t+1}^* c_{t+1}} \right] + E_t \left[\frac{1}{p_{t+1} c_{t+1}} \right]} \tag{20}$$

With the given technology, if $\alpha + \gamma < 1$, it is easy to show that the combined Home and Foreign output is maximized when

$$y_t^H = y_t^{H*} = \frac{y}{2} \quad \text{and} \quad y_t^F = y_t^{F*} = \frac{y^*}{2} \tag{21}$$

But, in general, with incomplete markets the input allocation rules (19) and (20) will not lead to an efficient allocation as in (21). This inefficiency can create opportunities for welfare improvement and, hence, a role for policy intervention. This will be the focus of our analysis in subsequent sections.

2.1   Equilibrium allocations with complete markets

As a benchmark, we will begin our comparative analysis of the two exchange-rate systems under complete markets. Specifically, let us assume that the households can write state-contingent debt contracts. Let $X_t = \{x_t, x_t^*\}$ and let $X^t = \{X_1, ...X_t\}$. Formally, Home budget constraint (inclusive of the cash-in-advance constraint) then becomes

$$m(X^{t-1}) + \tau(x_t) + z(X^{t-1}) - e(X^t)\, z^*(X^{t-1}) + b(X^{t-1}, X_t) -$$
$$\sum_{X_{t+1}} q(X^t, X_{t+1})\,(1 + x_t)\, b(X^t, X_{t+1}) \geq p(X^t)\, c(X^t) \tag{22}$$

Notice that the debt payoffs are contingent only on the realization of monetary shocks. We assume that $\frac{\beta}{x_{\min}} < 1$. In equilibrium, this assumption implies that the nominal interest rate is always positive, i.e. $q\left(X^t\right) < 1$ for any realization of the monetary shock x. Hence, the cash-in-advance constraint always binds. The households' first order conditions with respect to their input choices (12), (13), (15), and (16) remain the same. With state-contingent debt markets the first order conditions related to the choice of nominal asset holdings, (11) and (14) are transformed into

$$
\begin{aligned}
b_{t+1}: && \frac{q(X^t, X_{t+1})\,(1 + x_t)}{p(X^t)\, c(X^t)} &= \frac{\beta}{p(X^{t+1})\, c(X^{t+1})} \\
b_{t+1}^*: && \frac{q(X^t, X_{t+1})\,(1 + x_t)}{p(X^t)\, c^*(X^t)} &= \frac{\beta}{p(X^{t+1})\, c^*(X^{t+1})}
\end{aligned} \tag{23}
$$

Here we have again made use of the fact that $p_t = e_t\, p_t^*$. Equation (23) directly yields

$$\frac{c^*(X^t)}{c(X^t)} = \frac{c^*(X^{t+1})}{c(X^{t+1})} \tag{24}$$

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This is an expected result: with complete markets, Home and Foreign households are fully able to share monetary risk and smoothen their consumption. Let $\frac{c^*}{c} = \kappa$ (a constant). Thus, in every possible state of nature Home and Foreign share of consumption in the total output is constant.

$$c_t = \frac{Y_t + Y_t^*}{1 + \kappa} \quad \text{and} \quad c_t^* = \kappa \frac{Y_t + Y_t^*}{1 + \kappa} \tag{25}$$

Equation (24) implies that (20) will lead to

$$\frac{y^H}{y^F} = \frac{y^{H*}}{y^{F*}} = \frac{y}{y^*} \tag{26}$$

However, the above result does not ensure an efficient resource allocation as in (21) if $\alpha + \gamma < 1$ (although (26) is a sufficient condition for efficient production if $\alpha + \gamma = 1$), which is possible if and only if $y^H = y^{H*} = \frac{y}{2}$ and $y^F = y^{F*} = \frac{y^*}{2}$. In appendix (6.1) we show that complete risk sharing implies efficient production allocations as in(21). Hence, with complete markets

$$Y = Y^* = \theta \left(\frac{y}{2}\right)^\alpha \left(\frac{y^*}{2}\right)^\gamma \tag{27}$$

Thus, Home and Foreign (constant level of) consumption is given as in (25), where κ would be determined by the initial level of wealth and the input ratio parameters of production function. Hence, with complete markets the first-best (Pareto optimal) allocation is achieved.

Clearly, in this set-up the choice of an exchange-rate regime is irrelevant as both systems would lead to the same first-best outcome.

3 Alternative Exchange-Rate Systems under Incomplete Markets

For an exchange-rate regime to be non-neutral, we need situations where households face currency risk on their cross-currency assets and liabilities. In this section, we first consider a set-up where nominal bonds in both Home and Foreign currency are freely traded. We analyze this case with both flexible and fixed exchange rates. Next, we evaluate welfare under the two alternative systems, when only Home currency bonds are internationally traded.

3.1 Case I: Home and Foreign Currency Bonds

With nominal bonds in both currencies, the Home household's budget constraint (8) is

$$m_t + \tau_t + z_t - e_t\, z_t^* + b_t^H + e_t\, b_t^F - q_t(1 + x_t)b_{t+1}^H - e_t\, q_t^*(1 + x_t^*)b_{t+1}^F \geq p_t c_t \tag{28}$$

where b^H and b^F now denote the Home holdings of Home and Foreign bonds, with prices q and q^* respectively. Again, Home and Foreign households' first order conditions with respect to input choices (12), (13), (15), and (16) still hold. In addition the two nominal bond markets give rise to four additional conditions. For Home they are

$$
b_{t+1}^H: \qquad \frac{q_t(1 + x_t)}{p_t c_t} = \beta E_t\left[\frac{1}{p_{t+1}c_{t+1}}\right]
$$

$$
b_{t+1}^F: \qquad \frac{e_t\, q_t^*(1 + x_t^*)}{p_t c_t} = \beta E_t\left[\frac{e_{t+1}}{p_{t+1}c_{t+1}}\right] \tag{29}
$$

and for Foreign

$$
b_{t+1}^{H*}: \qquad \frac{q_t(1 + x_t)}{e_t\, p_t^* c_t^*} = \beta E_t\left[\frac{1}{e_{t+1}p_{t+1}^* c_{t+1}^*}\right]
$$

$$
b_{t+1}^{F*}: \qquad \frac{q_t^*(1 + x_t^*)}{p_t^* c_t^*} = \beta E_t\left[\frac{1}{p_{t+1}^* c_{t+1}^*}\right] \tag{30}
$$

Using the exchange determination equation (18), (29), and (30) yield

$$
\frac{q_t^* Y_t^*}{q_t Y_t} = \frac{E_t\left[\frac{1}{p_{t+1}^* c_{t+1}^*}\right]}{E_t\left[\frac{1}{p_{t+1}c_{t+1}^*}\right]} = \frac{E_t\left[\frac{1}{p_{t+1}^* c_{t+1}}\right]}{E_t\left[\frac{1}{p_{t+1}c_{t+1}}\right]} \tag{31}
$$

From (31), (19), and (20), it is then clear that

$$
\frac{q_t^*}{q_t} = \frac{\frac{y_t^{H*}}{Y_t^*}}{\frac{y_t^H}{Y_t}} = \frac{\frac{y_t^{F*}}{Y_t^*}}{\frac{y_t^F}{Y_t}} \tag{32}
$$

Equation (32) states that the ratios of marginal productivities of inputs (both H and F) in Home and Foreign output are inversely related with their respective gross interest rates (given that the marginal productivity of H and F input in Home (Foreign) production are $\frac{\alpha Y}{y^H}$ and $\frac{\alpha Y}{y^F}$ ($\frac{\alpha Y^*}{y^{H*}}$ and $\frac{\alpha Y^*}{y^{F*}}$) respectively). Intuitively, this relationship can be interpreted as follows: for the Home household selling a unit of Home input obtains P^H units of Home currency at the beginning of

next period, whereas using it in Home production earns $P\frac{\alpha Y}{y^H}$ units of Home currency (that will be available only during next period). In equilibrium, therefore, $P^H = P\frac{\alpha Y}{y^H}$. On the other hand, the Foreign household has to pay P^H units of Home currency in the next period to obtain a unit of H input, which is equivalent to qP^H in the current period. However, it can earn $P^*\frac{\alpha Y^*}{y^{H*}}$ in Foreign currency (again to be available for use during next period). It's equivalent to $q^*P^*\frac{\alpha Y^*}{y^{H*}}$ units of Foreign currency in the current period, which in Home currency is equal to $Sq^*P^*\frac{\alpha Y^*}{y^{H*}}$. Again, in equilibrium $qP^H = q^*P\frac{\alpha Y^*}{y^{H*}}$ (as $P = SP^*$). This implies the first equality in (32) while a similar argument for the use of F input yields the latter.

Further, in appendix (6.2) we show that in equilibrium both nominal bonds offer the same nominal interest rates

$$q_t = q_t^* \quad \forall\, t \tag{33}$$

and that the households completely diversify their next period assets and liabilities

$$b^H + z = b^F - z^* \quad \forall\, t \tag{34}$$

Intuitively, the equality of bond prices (33) can be interpreted as follows. We found that the ratio of bond prices determines the allocation of inputs into Home and Foreign production. Suppose that in any period t, $q_t > q_t^*$ (then $Y_t > Y_t^*$). In turn, this implies that the value of a unit of Home currency during the next period is relatively more valuable than a unit of Foreign currency. Since money supply shocks are symmetrically distributed, this also implies that the Foreign output is expected to be smaller ($Y_{t+1}^* < Y_t^*$ and $Y_{t+1} > Y_t$), which can only happen if the relative bond prices are further expected to diverge i.e. $\frac{q_{t+1}}{q_{t+1}^*} > \frac{q_t}{q_t^*}$. Continuing in this manner, this is only possible if agents expect that $\lim_{t->\infty} q_t^* = 0$. Then $\lim_{t->\infty} Y_t^* = 0$. But given the technology ((2) and if $\alpha + \gamma < 1$), this means that the marginal product of a unit each of the two inputs (together) in Foreign production will be infinitely large relative to that of Home. Hence, this can not be an equilibrium. Therefore, a rational expectations equilibrium requires that the $q_t = q_t^* \; \forall\, t$. The equality of bond prices directly implies that Home and Foreign output are equal

$$Y = Y^* = \theta \left(\frac{y}{2}\right)^\alpha \left(\frac{y^*}{2}\right)^\gamma \tag{35}$$

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Hence, we find that an efficient input allocation is obtained as in (21), which maximizes combined Home and Foreign output.

The second result in (34) has a standard portfolio diversification interpretation. Given a choice of nominal assets with return uncertainty in terms of their real value, the household holds an optimally diversified portfolio. Let us assume that at any moment t, Home household is a net lender. It is found that Home is long in both nominal assets. While it already has a long position in Home currency due to its export credit on H input, it has a short position on the import credit on F input. Therefore it acquires a higher long position in Foreign bonds relative to Home bonds to optimally diversify its asset portfolio. Further given that $q_t = q_t^* \ \forall \ t$, this portfolio consists of equal fractions (let this portfolio fraction be $l_t = b_t^H + z_t = b_t^F - z_t^*$) of claims over each country's total outstanding nominal money supply. This allows Home and Foreign households to consume

$$\frac{c_t}{Y} = 1 + l_t \left(\frac{1}{1 + x_t} + \frac{1}{1 + x_t^*} \right) - q_t(2l_{t+1} - \alpha + \gamma) \ \text{ and}$$

$$\frac{c_t^*}{Y} = 1 - l_t \left(\frac{1}{1 + x_t} + \frac{1}{1 + x_t^*} \right) + q_t(2l_{t+1} - \alpha + \gamma) \tag{36}$$

Finally, using the first order conditions (29) and (30), the equilibrium bond prices and bond holdings are described by the following stochastic difference equations

$$\frac{q_t}{1 + l_t \left(\frac{1}{1+x_t} + \frac{1}{1+x_t^*} \right) - q_t(2l_{t+1} - \alpha + \gamma)}$$
$$= E_t \left[\frac{1}{(1 + x_{t+1}) + l_{t+1} \left(1 + \frac{1+x_{t+1}}{1+x_{t+1}^*} \right) - q_{t+1}(2l_{t+2} - \alpha + \gamma)} \right] \tag{37}$$

and

$$\frac{q_t}{1 - l_t \left(\frac{1}{1+x_t} + \frac{1}{1+x_t^*} \right) + q_t(2l_{t+1} - \alpha + \gamma)}$$
$$= E_t \left[\frac{1}{(1 + x_{t+1}) - l_{t+1} \left(1 + \frac{1+x_{t+1}}{1+x_{t+1}^*} \right) + q_{t+1}(2l_{t+2} - \alpha + \gamma)} \right] \tag{38}$$

with l_0 as given.

Thus, we have shown that a free trade in cross-currency nominal bonds leads to an efficient resource allocation for production. Further, although households still face the risk related with

monetary uncertainty which makes the returns on their bond holdings uncertain, they are able to completely hedge their cross-currency risk by trading in and holding equal amounts of the two nominal bonds.

Having analyzed the equilibrium with flexible exchange rates, let us now look at the equilibrium outcome under the alternative system.

3.1.1 Equilibrium under Fixed Exchange Rates

The exchange rate is fixed by perfectly correlating the monetary shocks of Home and Foreign so that $x = x^*$. To see this, first notice that with two nominal bonds, the bond prices are equal as we discussed above[9]. As before, it directly implies that the input allocations will be efficient and that Home and Foreign outputs will be equal ($Y = Y^*$). Hence, if the cash-in-advance constraint binds (which we assume), clearly Home and Foreign prices grow at the same rate. Thus the exchange rate, $S = \frac{P}{P^*}$ is constant. In the other direction, let us assume that the nominal exchange rate is fixed at $S = \frac{P}{P^*} = \bar{S}$. Then it follows that if $x = x^*$, the peg can be indefinitely maintained. In this case the normalized prices are $p = p^*$. From input allocation rules (19) and (20), then $y^H = y^{H*} = \frac{y}{2}$ and $y^F = y^{F*} = \frac{y^*}{2}$, which implies that $Y = Y^* \; \forall \; t$.

To evaluate households' welfare, let us look at the consumption obtained under fixed exchange rates. For Home

$$\frac{c_t}{Y} = 1 + l_{ft}\frac{1}{1 + x_t} - q_t(l_{ft+1} - \alpha + \gamma) \tag{39}$$

and for Foreign

$$\frac{c_t^*}{Y} = 1 - l_{ft}\frac{1}{1 + x_t} + q_t(l_{ft+1} - \alpha + \gamma) \tag{40}$$

where l_f denotes the total nominal assets held by Home[10]. It is easy to see that while the input-output allocations remain the same, the households, on the other hand, loose the benefit of portfolio diversification as obtained under a flexible exchange rate by holding two imperfectly correlated nominal bonds. The increase in the variance of the asset portfolio (for any given initial asset level)

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[9]However, an arbitrage argument also now leads to the same conculsion. To rule out infinite profit opportunities, in an equilibrium the nominal bond prices will be equal, so that $q = q^*$.

[10]The currency denomination is irrelevant, as has been discussed above.

under a fixed exchange rate is equivalent to an ex-ante decrease in welfare for both Home and Foreign. Thus, a peg is strictly welfare reducing for both countries, when both Home and Foreign currency nominal bonds are available.

The equilibrium bond prices and bond holdings will be described by the difference equations

$$\frac{q_t}{1 + 2l_{ft}\frac{1}{1+x_t} - q_t(2l_{ft+1} - \alpha + \gamma)}$$
$$= E_t\left[\frac{1}{(1 + x_{t+1}) + 2l_{ft+1} - q_{t+1}(2l_{ft+2} - \alpha + \gamma)}\right] \tag{41}$$

and

$$\frac{q_t}{1 - 2l_{ft}\frac{1}{1+x_t} + q_t(2l_{ft+1} - \alpha + \gamma)}$$
$$= E_t\left[\frac{1}{(1 + x_{t+1}) - 2l_{ft+1} + q_{t+1}(2l_{ft+2} - \alpha + \gamma)}\right] \tag{42}$$

with l_0 as given[11].

3.2 Case II: Only Home Bonds

When only Home nominal bonds are internationally traded, the households' combined budget and cash-in-advance constraints, and Home and Foreign first order conditions are given as in (8), (10), (11), (12), (13), (14), (15), and (16). The resulting input allocations between Home and Foreign production are as described in (19) and (20). Under a flexible exchange rate regime, the input allocation ratios are described by

$$\frac{y_t^F}{y_t^{F*}} = \frac{E\left[\frac{1}{p_{t+1}c_{t+1}}\right]}{E\left[\frac{1}{p_{t+1}^*c_{t+1}}\right]} \quad ; \quad \frac{y_t^{H*}}{y_t^H} = \frac{E\left[\frac{1}{p_{t+1}^*c_{t+1}^*}\right]}{E\left[\frac{1}{p_{t+1}c_{t+1}^*}\right]} \tag{43}$$

Using the identity, $E(XY) = E(X)E(Y) + Cov(X, Y)$, we can expand (43)

$$\frac{y_t^F}{y_t^{F*}} = \frac{E\left[\frac{1}{p_{t+1}}\right]E\left[\frac{1}{c_{t+1}}\right] + Cov\left[\frac{1}{p_{t+1}}, \frac{1}{c_{t+1}}\right]}{E\left[\frac{1}{p_{t+1}^*}\right]E\left[\frac{1}{c_{t+1}}\right] + Cov\left[\frac{1}{p_{t+1}^*}, \frac{1}{c_{t+1}}\right]};$$

$$\frac{y_t^{H*}}{y_t^H} = \frac{E\left[\frac{1}{p_{t+1}^*}\right]E\left[\frac{1}{c_{t+1}^*}\right] + Cov\left[\frac{1}{p_{t+1}^*}, \frac{1}{c_{t+1}^*}\right]}{E\left[\frac{1}{p_{t+1}}\right]E\left[\frac{1}{c_{t+1}^*}\right] + Cov\left[\frac{1}{p_{t+1}}, \frac{1}{c_{t+1}^*}\right]} \tag{44}$$

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[11] One can see that the equilibrium bond prices will be lower under fixed exchange rates since the return variability will be higher.

Therefore, to analyze input allocations, it is necessary to look at the consumption equations and determine how they covary with the Home and Foreign consumption good prices. For Home

$$c_t = Y_t + \frac{(z_t + b_t^H)}{1 + x_t}Y_t - \frac{z_t^*}{1 + x_t^*}Y_t^* - q_t b_{t+1}^H Y_t \tag{45}$$

and For Foreign

$$c_t^* = Y_t^* - \frac{(z_t + b_t^H)}{1 + x_t}Y_t + \frac{z_t^*}{1 + x_t^*}Y_t^* + q_t b_{t+1}^H Y_t \tag{46}$$

We will consider three special cases here. (a) Home holdings of its own bonds (b_t^H) is positive. (b) b_t^H is negative and equal to $-z_t$, i.e. it hedges the currency risk acquired by the export credit. (c) b_t^H is negative and equal to $-(z_t + z_t^*)$, i.e. Home is a net borrower. However, it has diversified liabilities, both in Home and Foreign currency as a fraction z_t^* of their respective money supplies.

We reiterate that with cash-in-advacne constraint binding, the commodity prices p_t and p_t^* are given as $\frac{1+x}{Y_t}$ and $\frac{1+x^*}{Y_t^*}$ respectively. Let us, for the moment, assume that $Y \simeq Y^{*12}$. This assumption allows us to further assume that $E\left[\frac{1}{p}\right] \simeq E\left[\frac{1}{p^*}\right]$. Then, the input allocations are mainly determined by the covariance terms in (44).

Case (a): $b_t^H > 0$ When Home has positive bond holdings (b_t^H), it is obvious from (45) that a high realization of x_t (and thus p_t) relative to x_t^* will make Home poorer and the Home would cut both on its current consumption as well as the next-period nominal asset holdings. Thus, p_t and c_t are negatively correlated and hence the covariance term in the numerator of the first equation in (44) is negative. On the other hand, from (46) it is implied that the impact on c_t^* is just the opposite. Hence, the covariance term in the denominator of the second equation in (44) is positive. Similarly, we can argue that a relatively higher realization of x_t^*(and thus p_t^*) will yield a positive covariance term in the denominator of the first equation and a negative covariance term in the numerator of the second equation in (44). If the effects are approximately symmetric, we expect a

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[12] Of course, Y and Y^* are equibrium outcomes. Although Home and Foreign outputs change because of changes in input allocations, if the home-bias is approximately symmetric (as we show), then the effect on the ratio $\frac{Y}{Y^*}$ is of a second order.

symmetric home-bias of use of inputs in Home and Foreign production[13].

Case (b): $b_t^H = -z_t$ When Home holds a negative level of its own currency assets such as to hedge against its export credit claims over Foreign, it is clear from (45) that realizations of x_t are relatively less important than that of x_t^*. In fact, the covariance terms in the numerator of the first equation and in the denominator of the second equation in (44) may turn out to be very small. In this case, however, the realizations of x_t^* would still play its role and would result in a similar bias as discussed in case (a). Clearly, the bias will be smaller.

Case (c): $b_t^H + z_t = -z_t^*$ When $b_t^H + z_t = -z_t^*$, home has a diversified debt portfolio. Changes in x_t and x_t^* affect Home and Foreign current wealth symmetrically. Hence, we can argue that both covariance terms in the first equation of (44) would be positive whereas both in the second will be negative. It is easy to see that this equilibrium is equivalent to the case with two nominal bonds under flexible exchange rates, where the initial conditions (by assumption) are such that, in equilibrium, Home buys zero level of Foreign bonds. As discussed before, there is no home bias in input use in this case.

Case (d): $b_t^H + z_t < -z_t^*$ Continuing this argument, we can say that when $b_t^H + z_t < -z_t^*$, there can be a situation where instead of home-bias, *we may actually have a foreign-bias in input use.*

3.2.1   Equilibrium under Fixed Exchange Rates

We argued above that the home-bias of input use will depend upon initial nominal bond holdings. It can range from a very high home-bias to even an anti-home bias. Let us turn our attention to the situation under fixed exchange rates.

Suppose the money supplies of the two countries are perfectly correlated $(x = x^*)$ and the agents expect the exchange rate to remain pegged at $S = \bar{S}$ (say $= 1$). Notice that with fixed

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[13]However, a Home decrease (increase) in the next period real return on assets due to a bad/good relization of (x, x^*) will also affect future input allocations and thus decrease (increase) home bias, and change next period's Home and Foreign output. But, as assumed, the effect on the ratio $\frac{Y}{Y^*}$ is of a second order and the conclusion will still be valid.

exchange rates two-bond economy is *equivalent* to a one-bond economy and hence the outcome will be similar to the one obtained in the previous section. The rational expectations equilibrium output of the two countries will be equal, i.e. $Y = Y^*$ and the equilibrium can again be described by (39), (40), (41), and (42).

We can now see that a relative welfare evaluation of the two exchange rate systems will depend on the initial nominal debt holdings of Home bonds. If the economy is in case (a) under flexible exchange rates, then a fixed exchange rate system may be welfare improving with a higher combined Home and Foreign output. On the other hand, if Home is already diversified in its debt holdings as in case (c), a peg may be welfare reducing for both countries[14].

4   Incomplete Markets: No International Trade in Bonds

In our analysis under incomplete markets with only one nominal bond, we have implicitly assume that an efficient allocation of inputs will improve the welfare of both countries, which will generally hold if the two countries are symmetric in their inputs, i.e. $\alpha = \gamma$. There, the welfare comparison of the two exchange rate systems basically boils down to evaluating the trade off between the costs and the benefits of efficient input allocation and portfolio diversification[15]. However, even if a fixed exchange rate system allocates resources efficiently, it may not be able to achieve a Pareto optimal outcome, which requires that the welfare of both countries strictly increase[16].

In this section, we assume that while the households can freely borrow and lend domestically, cross-border lending is not permitted. Hence, while the Home household carries cross-currency assets and liabilities through exports (Home currency assets/investments) and imports (Foreign currency negative assets/Foreign investment), it is not able to hedge it through nominal asset holdings. In equilibrium, it is obvious that $b_t = b_{t+1} = 0$. In all that follows, we assume that the

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[14] An explicit welfare evaluation will also have to take other parameters of the model such as the input shares α and γ into account. However, we conjecture that these results will hold under fairly general conditions, such as $\alpha = \gamma$.

[15] An explicit evaluation under general conditions will be very complicated and beyond the scope of this paper

[16] Although we do not deal with this issue here, it is obviously important from the point of view of the implementation of an exchange rate system.

cash-in-advance constraint (8) holds with equality. This is ensured iff $q < 1$, which holds if

$$\beta E_t \left[\frac{c_t}{(1 + x_{t+1})c_{t+1}} \right] < 1 \tag{47}$$

This implies certain restrictions on the distribution of the money growth rate x and the discount factor β. For $\beta < 1$, it can be shown that a sufficiently high value of the mean relative to the standard deviation of x will ensure that (47) holds.

With (8) holding with equality, using equilibrium prices (17) and the fact that net supply of bonds, $b_t = b_{t+1} = 0$, Home and Foreign consumption are

$$c_t = Y_t + \frac{z_t}{1 + x_t} Y_t - \frac{z_t^*}{1 + x_t^*} Y_t^* \tag{48}$$

and

$$c_t^* = Y_t^* - \frac{z_t}{1 + x_t} Y_t + \frac{z_t^*}{1 + x_t^*} Y_t^* \tag{49}$$

Notice that Home and Foreign consumption depend on the current period Home and Foreign outputs, and export payments and import receipts. $\frac{z_t}{1+x_t} Y_t$ denotes the real value of Home (Foreign) export (import) receipts (payments) while $\frac{z_t^*}{1+x_t^*} Y_t^*$ denotes the real value of Home (Foreign) import (export) payments(receipts). As z_t and z_t^* are pre-determined, their current real values depend on the currency prices in terms of consumption goods.

In appendix (6.3) we show that the equilibrium input allocation is stationary[17]. Henceforth, we suppress time subscripts, and use unconditional expectation operator E instead of E_t.

Using equilibrium prices (17), and (12), (13), (15), and (16), it is straightforward to show that

$$\frac{y^H/Y}{y^{H*}/Y^*} = \frac{E\left[\frac{1}{(1+x)c^*}\right]}{E\left[\frac{1}{(1+x^*)c^*}\right]} \quad \text{and} \quad \frac{y^F/Y}{y^{F*}/Y^*} = \frac{E\left[\frac{1}{(1+x)c}\right]}{E\left[\frac{1}{(1+x^*)c}\right]} \tag{50}$$

From (48) and (49) we notice that Home consumption (c) is inversely related to its money growth rate while directly related with that of Foreign. Since $E(XZ) = E(X)E(Z) + Cov(X, Z)$, it is

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[17]The restriction on international borrowings essentially makes the households' choice problem equivalent to a single period problem.

implied that $\frac{y^H/Y}{y^{H*}/Y^*} > 1$ and $\frac{y^F/Y}{y^{F*}/Y^*} < 1$. Hence, the efficiency condition for production (21) does not hold as $\frac{y^H/Y}{y^{H*}/Y^*} \neq \frac{y^F/Y}{y^{F*}/Y^*} \neq 1$. It is clear that with flexible exchange rates there is a home-bias in production as the countries choose a higher proportion of their own inputs. Since input trade gives rise to consumption variability through uncertain currency liabilities in real terms, each country prefers to use more of the input that it is endowed with.

Having characterized optimal input allocations and trade under flexible exchange rates, we now proceed to analyze household welfare under alternative exchange-rate systems. We find that when the countries are symmetric in relation to their input share in production technology, a fixed exchange-rate system is strictly welfare improving. However, when the share parameters α and γ are not equal, we have to resort to numerical computations to show our qualitative results. Therefore to build intuition, at first, we will work with an extremely asymmetric case where only Home endowment is used in production.

4.1 Only Home Input Used in Production: $\gamma = 0$

Here, the technology is described by

$$Y = \theta(y^H)^\alpha \tag{51}$$

The budget constraints of Home and Foreign reduce to

$$m + \tau + p^H y^{H*} \geq pc \; ;$$
$$m^* + \tau^* - \frac{p^H y^{H*}}{e} \geq p^* c^* \tag{52}$$

When the cash-in-advance constraint binds, consumption of Home and Foreign are

$$c = Y + \frac{p^H y^{H*}}{1+x}Y \; ;$$
$$c^* = Y^* - \frac{p^H y^{H*}}{1+x}Y \tag{53}$$

Again, notice that Home and Foreign consumption depend on the current period Home and Foreign outputs, and the real value of import credit due in the current period, $\frac{p^H y^{H*}}{1+x}Y$. Home and Foreign

output are given by

$$Y = \theta \left(\frac{1}{1+k} \right)^{\alpha} y^{\alpha} \; ;$$

$$Y^* = \theta \left(\frac{k}{1+k} \right)^{\alpha} y^{\alpha} \tag{54}$$

where y is the Home endowment of productive input and where $k = \frac{z}{\alpha} = \frac{p^H y^{H*}}{\alpha} = \frac{E\left[\frac{1}{p^* c^*} \right]}{E\left[\frac{1}{ep^* c^*} \right]}$. It is

easy to see that for efficient resource utilization $k = 1$. Thus, k is a measure of home-bias; smaller

k implies that Foreign imports less input from Home than the efficient level. From (12), (15), and

(53), k is obtained as a solution to

$$k^{1-\alpha} = \frac{E\left[\frac{1}{1+x^*} \right] E\left[\frac{1+x}{1+x-\alpha k^{1-\alpha}} \right]}{E\left[\frac{1}{1+x-\alpha k^{1-\alpha}} \right]} \tag{55}$$

For comparing the allocations under the two systems, it is useful to write (53) as

$$c = Y + \frac{\alpha k}{1+x} Y \; ;$$

$$c^* = Y^* - \frac{\alpha k}{1+x} Y \tag{56}$$

Using (56), (55), and (54) we can evaluate (1) as the welfare in a flexible exchange-rate system.

4.1.1   Trade and Welfare under Fixed Exchange Rates

Next we examine this set-up with fixed exchange rates. It is easy to see from (55) that when money

growth shocks are perfectly correlated, i.e. $x = x^*$, home-bias is eliminated as $k = 1$. Therefore,

$$Y = Y^* = \theta \left(\frac{1}{2} y \right)^{\alpha} . \tag{57}$$

Intuitively, there are two underlying factors that obtain efficient allocation of inputs between

Home and Foreign production in switching from a flexible to a fixed exchange-rate system. Consider

Foreign's choice of import each period. By importing a unit of input, Foreign household earns its

marginal revenue in its own currency whereas the marginal cost it incurs is in Home currency.

Hence, there is clearly a currency risk involved in this transaction, that reflects in the equilibrium

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price of the imported input. Essentially, the input price is lower than it would be with no currency risk

$$p^H = \alpha \frac{1+k}{y} \tag{58}$$

while under fixed exchange rates $p^H = \frac{2\alpha}{y}$. A lower price in terms of Home money implies a higher than efficient $(y^H = \frac{y}{2})$ level of input use by Home, so as to equalize marginal revenue from its own production versus selling it to Foreign. Hence, there is a home-bias in input use[18]. With fixed exchange rates, as both Home and Foreign revenues and liabilities have same currency denomination and hence, once the friction related with currency risk is removed, efficient allocation is obtained.

From (56) Home and Foreign consumption are

$$c = Y \left(1 + \frac{\alpha}{1+x} \right) \; ;$$
$$c^* = Y \left(1 - \frac{\alpha}{1+x} \right) \tag{59}$$

Notice that although the combined Home and Foreign output is maximized, both still face consumption risk through monetary uncertainty. Its real value in terms of consumption good is $Y\frac{\alpha}{1+x}$, which is dependent on the current money growth shock. Hence, although fixed exchange rates obtain production efficiency, the consumption uncertainty still remains.

Using (56) and (59), we can evaluate the welfare gain obtained by switching from fixed to flexible exchange-rate system. As we can not obtain closed form solution for the flexible exchange-rate case, we resort to numerical evaluation.

4.1.2 Welfare Gains from Float to Peg: A Numerical Evaluation

Since it is not a calibration exercise, we only look at the qualitative behavior of our model using plausible parameter values. We fix the Home endowment $y = 1$ and the technology parameter $\theta = 1$. For computational convenience, we assume a simple two-state stochastic process where the money growth rates can be either 'high' or 'low' with equal probability. It can be easily ensured
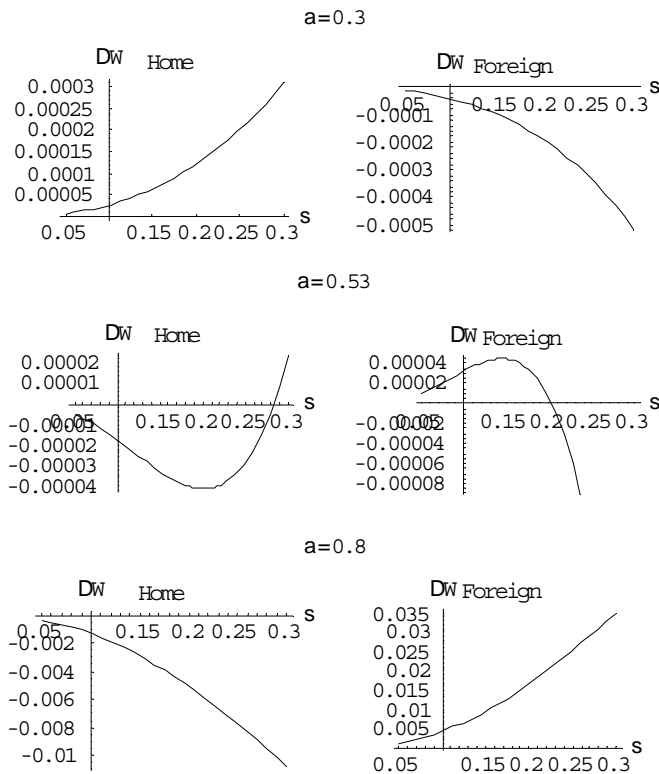
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[18]This is essentially the underlying rationale behind home-bias of input use in all set-ups so far discussed. However, with no nominal bond markets, this effect is more clearly discernible.

that the cash-in-advance constraint binds over a range of plausible parameter values for discount rate β, technology parameter α, average growth rate $\bar{x}$, and its standard deviation σ. Since we conduct our exercise over a wide range of technology parameter $\alpha \in [0.05, 0.95]$, we fix the average value of the growth rate $\bar{x} = 0.5$[19] to ensure that the cash-in-advance always binds over its variance $\sigma \in (0, 0.3)$. However, our main results hold through, even for smaller values of mean and variance of monetary growth rates after restricting the range of plausible values for other parameters.

Below we present a panel of graphs that compare welfare gains obtained by Home and Foreign by switching from flexible to fixed exchange-rate system. These gains are plotted against the level of uncertainty in terms of the standard deviation of money growth distribution. We obtain the plots for three different values of $\alpha = 0.3, 0.52,$ and 0.8.

Figure 1. Welfare Gains from Float to Peg

a=0.3

DW Home

0.0003
0.00025
0.0002
0.00015
0.0001
0.00005

0.05   0.15 0.2 0.25 0.3   s

DW Foreign

-0.0001   0.05   0.15 0.2 0.25 0.3   s
-0.0002
-0.0003
-0.0004
-0.0005

a=0.53

DW Home

0.00002
0.00001

0.15 0.2 0.25 0.3   s

-0.00001
-0.00002
-0.00003
-0.00004

DW Foreign

0.00004
0.00002

0.15 0.2 0.25 0.3   s

-0.00002
-0.00004
-0.00006
-0.00008

a=0.8

DW Home

0.05   0.15 0.2 0.25 0.3   s
-0.002
-0.004
-0.006
-0.008
-0.01

DW Foreign

0.035
0.03
0.025
0.02
0.015
0.01
0.005

0.05   0.15 0.2 0.25 0.3   s

_____

[19]This may seem implausibly high but our purpose is to look at the qualitative results over a wide range of parameter values.

Clearly, these pictures indicate that a change in the exchange-rate system affects Home and Foreign asymmetrically. For low values of α, while it is optimal for the exporting country to switch from a floating system to a peg; it is welfare reducing for Foreign. On the other extreme, for high values of α the situation is reversed. Further, welfare gains depend on the variance of monetary growth distribution. For both low and high values of α, an increase in variance enhances the welfare gain effect. However, for intermediate values the response becomes non-monotonic.

To interpret these results, notice that Home consumes, in addition to its own output, the final consumption good that it imports in lieu of the credit it extends to Foreign. The real value of the credit depends both on the distribution (which reflects in the pre-contracted equilibrium price, i.e. the terms of trade) and the current realization of money growth shocks. Similarly, Foreign's consumption is its own output net of its import credit liabilities towards Home.

In switching from a flexible to a fixed exchange-rate system, first, there is an output effect that equilibrates input trade at the efficient level such that aggregate output is maximized[20]. It increases Foreign output while decreasing Home output. Second, there is a terms of trade effect which improves for Home in switching from a floating exchange-rate system to a peg. Clearly, both of these effects are monotonic with respect to the variance of money growth shocks. Finally, there is an additional third factor: the consumption variability related with the import credit. Compare consumption allocations under flexible (56) and fixed (59) rates. Although under fixed exchange rates the terms of trade improve, and Home gets a larger transfer of final goods from Foreign, it is also implied that a larger part of both Home and Foreign consumption now becomes uncertain.

At lower values of input share parameter α, the allocation of productive input remains near the efficient level even under flexible exchange rates. When the countries switch from a flexible to a fixed exchange-rate system, the terms of trade (and also the export receipts of Home) effect dominates that of the individual output changes. Since Foreign is a net importer, its welfare is lower than that under a fixed system while Home's welfare improves. For higher values of α input

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[20]Under a flexible exchange-rate system, Foreign imports a lower input amount implying a lower level of production, while Home produces at a higher level.

allocations fare relatively far from their efficient levels under a flexible system. Hence, country output changes dominate that of the terms of trade effect. For some intermediate values of α, the two opposing effects give rise to a non-monotonic response of welfare-gains the standard deviation of the money growth rates as observed in the panel II of Figure 1.

One puzzling result is that the switch is never Pareto optimal; one country gains at the cost of the other[21]. This is despite the fact that a fixed exchange-rate system increases the combined Home and Foreign output and, therefore, consumption. This is explained by the third factor: the larger consumption variability under fixed rates. A switch increases consumption variability of both countries as a ratio of their mean consumption. This effect remains dominant enough to rule out any case (a pair of α and σ), where welfare of both countries could improve simultaneously.

4.2 Input Trade and Welfare with Two Factors

To analyze a situation where a switch from flexible to fixed exchange-rate system can be Pareto optimal, we now turn our attention to the two-input general case. We first look at the model variables under a fixed exchange rate and then compare them with allocations under flexible rate.

4.2.1 Fixed Exchange Rates

As before, in a fixed exchange-rate system the monetary growth rates of Home and Foreign are perfectly correlated, i.e. $x = x^*$. Again, home-bias is completely eliminated and it is easily seen that $y^H = y^{H*} = \frac{y}{2}$, $y_F = y_F^* = \frac{y^*}{2}$. Hence, $Y = Y^*$. Also, from (48) and (49)

$$
\begin{aligned}
c &= Y + \frac{(\alpha - \gamma)}{1 + x} Y \; ; \\
c^* &= Y^* - \frac{(\alpha - \gamma)}{1 + x} Y
\end{aligned}
\tag{60}
$$

As before, even though the output is endogenously predetermined, the consumption is still

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[21] It turns out that the combined welfare of the two countries, as a sum of their individual ex-ante utilities, improves when the welfare improves for Foreign. This is simply due to the fact that Foreign's consumption is lower. Any transfer from Home to Foreign subject to the concavity of the welfare function drives this result.

variable and depends on the realization of monetary shock in the current period.

4.2.2 Symmetric Inputs: $\alpha = \gamma$

Let us look at a symmetric case i.e. where the two countries are similar in their endowments, i.e. $\alpha = \gamma$. With fixed exchange rates we have perfect risk sharing

$$c = c^* = Y_{Fixed} = Y^*_{fixed} = (yy^*)^{\frac{1}{2}} \tag{61}$$

However, with flexible exchange rates consumption is variable

$$c = c^* = Y_{flex} + \frac{1}{2}\omega \left[\frac{1}{1+x} - \frac{1}{1+x^*} \right] \tag{62}$$

where ω, the measure of home-bias is obtained from (12) and (15)[22]

$$\omega = \frac{E\left[\frac{1}{pc}\right]}{E\left[\frac{e}{pc}\right]} = \frac{E\left[\frac{1}{p^*c^*}\right]}{E\left[\frac{1}{ep^*c^*}\right]} \tag{63}$$

It is obvious from equilibrium prices (17) and consumption (61) that $\omega < 1$. Further, Home and Foreign outputs are equal

$$Y_{flex} = Y^*_{flex} = \frac{\omega^{\frac{1}{2}}}{1+\omega}(yy^*)^{\frac{1}{2}}$$

Moreover, it is obvious that $Y_{flex} < Y_{fixed}$. Comparing consumption under the two systems we can see that From (61) both countries enjoy a *certain* first-best level of consumption under a fixed exchange rate, whereas under flexible system expected consumption, $E(c) = E(c^*) = Y_{flex}$ $\left(\text{ since } E\left(\frac{1}{1+x}\right) = E\left(\frac{1}{1+x^*}\right)\right)$, is lower. Further, ex-ante utility will even lower as $U\left(E\left(c\right)\right) > E(U(c))$. Hence, a fixed exchange-rate system is strictly welfare-improving for both countries.

4.2.3 Asymmetric Input Shares: $\alpha \neq \gamma$

Using (48), (49), (13) and (15), it can be shown that

$$c = Y + \frac{\alpha\mu}{1+x}Y - \frac{\gamma\omega}{1+x^*}\frac{\mu^\alpha}{\omega^\gamma}Y \ ;$$
$$c^* = Y^* + \frac{\gamma\omega}{1+x^*}\frac{\mu^\alpha}{\omega^\gamma}Y + \frac{\alpha\mu}{1+x}Y \tag{64}$$

---

[22] As one would expect, with symmetric input shares, home-bias is also symmetric.

where the Home and Foreign output Y and Y^* are given by

$$Y = \left(\frac{1}{1+\mu}\right)^\alpha \left(\frac{\omega}{1+\omega}\right)^\gamma \ ;$$
$$Y^* = \left(\frac{\mu}{1+\mu}\right)^\alpha \left(\frac{1}{1+\omega}\right)^\gamma \tag{65}$$

where $\omega \left(= \dfrac{E\left(\frac{1}{pc}\right)}{E\left(\frac{1}{p^*c}\right)}\right)$ and $\mu \left(= \dfrac{E\left(\frac{1}{p^*c^*}\right)}{E\left(\frac{1}{pc^*}\right)}\right)$ are found as solution to

$$\omega = \frac{E_0\left[\dfrac{1}{(1+x)+\alpha\mu-\omega\gamma\left(\frac{1+x}{1+x^*}\right)\frac{\mu^\alpha}{\omega^\gamma}}\right]}{E_0\left[\dfrac{1}{(1+x^*)\frac{\omega^\gamma}{\mu^\alpha}-\omega\gamma+\alpha\mu\left(\frac{1+x^*}{1+x}\right)\frac{\omega^\gamma}{\mu^\alpha}}\right]} \ ;$$

$$\mu = \frac{E_0\left[\dfrac{1}{(1+x^*)+\omega\gamma-\alpha\mu\left(\frac{1+x^*}{1+x}\right)\frac{\omega^\gamma}{\mu^\alpha}}\right]}{E_0\left[\dfrac{1}{(1+x)\frac{\mu^\alpha}{\omega^\gamma}-\alpha\mu+\omega\gamma\left(\frac{1+x}{1+x^*}\right)\frac{\mu^\alpha}{\omega^\gamma}}\right]} \tag{66}$$

Again, we resort to numerical computations to analyze our qualitative results.

4.2.4 Numerical Evaluation of Welfare Gains

We simulate our model for the parameter values of the previous exercise, and conduct our analysis for various combinations of α and γ. To cover whole possible ranges of plausible variations between the two parameters, we mainly consider two cases: $\alpha + \gamma = 0.3$ and 0.6. Within both cases, we try different combinations of the two so as to contrast our results.

Case I: $\alpha + \gamma = 0.3$ The following graphs presents our results

Figure 2. Welfare   Gains   from   Float   to   Peg : a + g = 0.3

a=0.2 , g=0.1

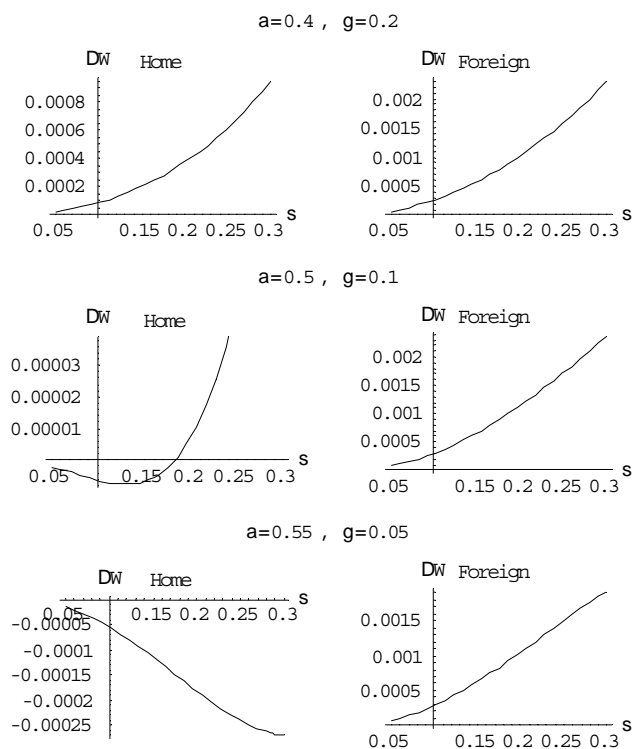a=0.255 , g=0.045

a=0.27 , g=0.03
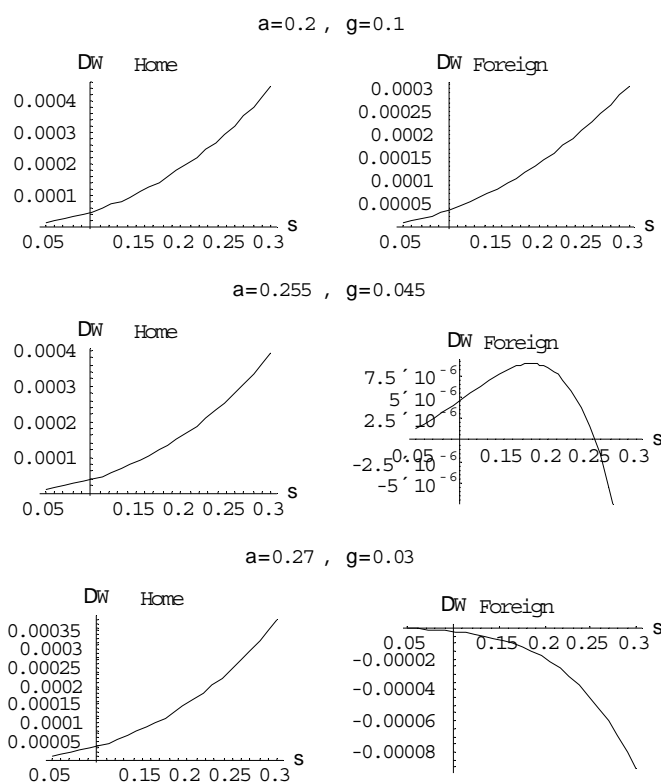
In section 4.1, where only one country is endowed with the productive input, we found that for low values of α Home gains while Foreign suffers in a switch from a flexible to fixed exchange rates. However, as we already know from section 4.2.2 that when both countries' input share parameters are equal, a switch is always optimal. By continuity, the same would hold for slightly dissimilar values of α and γ. In Figure 2, panel I, we observe that both countries improve their welfare, even when the difference is large enough. However, as the difference grows and Foreign input share gets smaller, the terms of trade effect increasingly becomes important. Hence, in panel III we observe the same pattern as that with only one input. For some intermediate differential range as in panel II, for Home the terms of trade effect dominates at all variances, while Foreign's welfare has a non-monotonic response as the one observed for Foreign in panel II of Figure 1. As explained before, this comes as a trade-off between worsened terms of trade and increased Foreign output.

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Next, we use higher values of input share parameters.

Case II: $\alpha + \gamma = 0.6$ Figure 3 presents our results

Figure 3. Welfare Gains from Float to Peg : a + g = 0.6

Again , as in section 4.2.4, we observe above in panel I that both countries' welfare improve even when the difference between α and γ is large enough. However, as the difference grows and Foreign input share gets smaller, the reallocatoin of resources work asymmetrically and we observe a similar pattern as in section 4.1, where only one country is endowed with the productive input. In section 4.1, for large values of α,we found that Home suffers, while Foreign gains, in a switch from a floating to fixed exchange-rate system. Hence, in the last panel we observe the same pattern as that with only one input. For some intermediate differential range as in panel II, Home's welfare gain is non-monotonically related to the variance of monetary shocks, which comes as a trade-off between the improved terms of trade and a lower output as obtained in panel II of Figure 1 in section 4.1. This is an interesting result: Home may find it optimal to have a flexible exchange-rate

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system, if the variance of monetary disturbances is small. For larger levels of uncertainty, it would prefer fixed rates. Thus we have a result where it is optimal for home to '*dirty-float*'.

5 Conclusion

In this paper we attempt to provide a rationale for the 'fear of floating'. By focussing mainly on the issue of currency mismatch between private sector's revenues and liabilities, it is shown that when cross-currency transactions are made in uncertain monetary environments, trade is strictly higher under a fixed exchange-rate system. Furthermore, when markets are incomplete, it may be welfare improving for all countries to opt for a fixed exchange-rate system. However, when the countries are asymmetric in terms of their input endowments, it may be possible that a change from a flexible to a fixed exchange-rate system benefits one country at the cost of the other. More interestingly, the countries may find it optimal to 'manage-float' under certain circumstances. Specifically, we show that there is a threshold level of uncertainty, in terms of the variance of the monetary innovations, beyond which both countries may find the fixed exchange-rate system to be welfare improving.

On the other hand, when agents have hedging opportunities through trade in nominal bonds, a flexible exchange-rate system is welfare-superior to a fixed exchange-rate system. Under the former, both trade and consumption variabilities are smaller. Finally, with some restrictions on capital markets such as trade in only single currency nominal bonds, the initial conditions may be the key to an evaluation of the alternative systems. In particular, each may be superior to the other, depending on the inter-country initial level (and composition) of nominal claims. Again, the level of uncertainty turns out to be crucial to the choice of an exchange-rate system.

Often, developing countries hold debt issued by developed countries, while mutually extending import credits. While developed countries generally leave their exchange rates to the market, developing countries try to peg their exchange rates with that of the former. One conclusion of our model is that the welfare impact of a peg is much higher for a developing country, especially if the input shares in technology are large, and the developed country happens to be relatively richer in capital inputs, which is an empirically plausible case. The empirical validity of this conclusion can

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be easily tested.

In our model, the main source of exchange-rate uncertainty is generated through import credits. Alternatively, this can be modeled as cross-country investments whose returns are prone to exchange-rate uncertainty. Although we have assumed that the transactions take place in seller-currency, the analysis can be easily extended to cases where transactions take place in buyer-currency, or all credit transactions take place in only one currency. Further, instead of exchange-rate risk only distorting the supply side allocations, one can think of the uncertainty also affecting consumption allocations in a two-good consumption model. Last, but not the least, our framework can also be employed for evaluating the alternative systems under real shocks. All these issues are part of the agenda for our future research.

6 Appendix

6.1

It's obvious from (26) that $\frac{y^H}{y^{H*}} = \frac{y^F}{y^{F*}}$ for all t. Using the equilibrium condition $p_t Y_t = 1 + x_t$, equation (24) along with (12) and (15) yields

$$
\frac{y_t^F}{y_t^{F*}} = \frac{y_t^H}{y_t^{H*}} = \frac{E_t\left[\frac{Y_{t+1}}{(1+x_{t+1})(Y_{t+1}+Y_{t+1}^*)}\right]}{E_t\left[\frac{Y_{t+1}^*}{(1+x_{t+1}^*)(Y_{t+1}+Y_{t+1}^*)}\right]} \qquad \text{or}
$$

$$
\frac{y_t^F}{y_t^{F*}} = \frac{E_t\left[\dfrac{1}{(1+x_{t+1})\left(1+\left(\frac{y_{t+1}^F}{y_{t+1}^{F*}}\right)^{-(\alpha+\gamma)}\right)}\right]}{E_t\left[\dfrac{1}{(1+x_{t+1}^*)\left(1+\left(\frac{y_{t+1}^F}{y_{t+1}^{F*}}\right)^{(\alpha+\gamma)}\right)}\right]} \tag{67}
$$

Since X is i.i.d., a unique solution to (50) is obtained as $\frac{y_t^F}{y_t^{F*}} = \frac{y_{t+1}^F}{y_{t+1}^{F*}} = 1$.

6.2

Assuming that the cash-in-advance constraint always binds, and using the equilibrium conditions for prices (17), exchange rates (18), and that the net supply Home and Foreign nominal bonds $b_t^H + b_t^{H*} = 0$, and $b_t^F + b_t^{F*} = 0$, we derive

$$p_{t+1}^* c_{t+1} = (1 + x_{t+1}^*)\frac{Y_{t+1}}{Y_{t+1}^*} + \left(z_{t+1} + b_{t+1}^H\right)\frac{1 + x_{t+1}^*}{1 + x_{t+1}}\frac{Y_{t+1}}{Y_{t+1}^*}$$

$$+ (b_{t+1}^F - z_{t+1}^*) - q_{t+1}b_{t+2}^H(1 + x_{t+1}^*)\frac{Y_{t+1}}{Y_{t+1}^*} - q_{t+1}^* b_{t+2}^F(1 + x_{t+1}^*)$$

$$p_{t+1}c_{t+1}^* = (1 + x_{t+1})\frac{Y_{t+1}^*}{Y_{t+1}} - (z_{t+1} + b_{t+1}^H) - (b_{t+1}^F - z_{t+1}^*)\frac{1 + x_{t+1}}{1 + x_{t+1}^*}\frac{Y_{t+1}^*}{Y_{t+1}}$$

$$+ q_{t+1}b_{t+2}^H(1 + x_{t+1}) + q_{t+1}^* b_{t+2}^F(1 + x_{t+1})\frac{Y_{t+1}^*}{Y_{t+1}}$$

$$p_{t+1}c_{t+1} = (1 + x_{t+1}) + (z_{t+1} + b_{t+1}^H) + (b_{t+1}^F - z_{t+1}^*)\frac{1 + x_{t+1}}{1 + x_{t+1}^*}\frac{Y_{t+1}^*}{Y_{t+1}}$$

$$- q_{t+1}b_{t+2}^H(1 + x_{t+1}) - q_{t+1}^* b_{t+2}^F(1 + x_{t+1})\frac{Y_{t+1}^*}{Y_{t+1}}$$

$$p_{t+1}^* c_{t+1}^* = (1 + x_{t+1}^*) - (z_{t+1} + b_{t+1}^H)\frac{1 + x_{t+1}^*}{1 + x_{t+1}}\frac{Y_{t+1}}{Y_{t+1}^*}$$

$$- (b_{t+1}^F - z_{t+1}^*) + q_{t+1}b_{t+2}^H(1 + x_{t+1}^*)\frac{Y_{t+1}}{Y_{t+1}^*} + q_{t+1}^* b_{t+2}^F(1 + x_{t+1}^*) \tag{68}$$

From the first order conditions (29) and (30), the Home and Foreign consumption ratio is

$$\frac{c_t}{c_t^*} = \frac{E_t\left[\frac{1}{p_{t+1}c_{t+1}^*}\right]}{E_t\left[\frac{1}{p_{t+1}c_{t+1}}\right]} = \frac{E_t\left[\frac{1}{p_{t+1}^* c_{t+1}^*}\right]}{E_t\left[\frac{1}{p_{t+1}^* c_{t+1}}\right]} \tag{69}$$

Using (69) with the bond price ratio equation (31) yields

$$\frac{q_t^* Y_t^*}{q_t Y_t} = \frac{E_t\left[\frac{1}{p_{t+1}^* c_{t+1}}\right]}{E_t\left[\frac{1}{p_{t+1}c_{t+1}}\right]} = \frac{E_t\left[\frac{1}{p_{t+1}^* c_{t+1}}\frac{E_{t+1}\left[\frac{1}{p_{t+2}c_{t+2}^*}\right]}{E_{t+1}\left[\frac{1}{p_{t+2}c_{t+2}}\right]}\right]}{E_t\left[\frac{1}{p_{t+1}c_{t+1}}\frac{E_{t+1}\left[\frac{1}{p_{t+2}c_{t+2}^*}\right]}{E_{t+1}\left[\frac{1}{p_{t+2}c_{t+2}}\right]}\right]}$$

$$= \frac{E_t\left[\frac{1}{p_{t+1}^* c_{t+1}^*}\right]}{E_t\left[\frac{1}{p_{t+1}c_{t+1}^*}\right]} = \frac{E_t\left[\frac{1}{p_{t+1}^* c_{t+1}^*}\frac{E_{t+1}\left[\frac{1}{p_{t+2}c_{t+2}}\right]}{E_{t+1}\left[\frac{1}{p_{t+2}c_{t+2}^*}\right]}\right]}{E_t\left[\frac{1}{p_{t+1}c_{t+1}^*}\frac{E_{t+1}\left[\frac{1}{p_{t+2}c_{t+2}}\right]}{E_{t+1}\left[\frac{1}{p_{t+2}c_{t+2}^*}\right]}\right]} \quad \forall\, t \tag{70}$$

Denoting e, c, c^*, p, and p^* as functions in the state space, the second equality in (70) can be

written as

$$\frac{E_t\left[\frac{1}{p^*(X_{t+1},X^t)c(X_{t+1},X^t)}\right]}{E_t\left[\frac{1}{p(X_{t+1},X^t)c(X_{t+1},X^t)}\right]} = \frac{E_t\left[\frac{1}{p^*(X_{t+1},X^t)c(X_{t+1},X^t)}G(X^{t+1})\right]}{E_t\left[\frac{1}{p(X_{t+1},X^t)c(X_{t+1},X^t)}G(X^{t+1})\right]}$$

$$= \frac{E_t\left[\frac{1}{p^*(X_{t+1},X^t)c^*(X_{t+1},X^t)}\right]}{E_t\left[\frac{1}{p(X_{t+1},X^t)c^*(X_{t+1},X^t)}\right]} = \frac{E_t\left[\frac{1}{p^*(X_{t+1},X^t)c^*(X_{t+1},X^t)}\frac{1}{G(X^{t+1})}\right]}{E_t\left[\frac{1}{p(X_{t+1},X^t)c^*(X_{t+1},X^t)}\frac{1}{G(X^{t+1})}\right]} \tag{71}$$

where

$$G(X^{t+1}) = \frac{E_{t+1}\left[\frac{1}{p(X_{t+2},X^{t+1})c^*(X_{t+2},X^{t+1})}\right]}{E_{t+1}\left[\frac{1}{p(X_{t+2},X^{t+1})c(X_{t+2},X^{t+1})}\right]}$$

We call two functions $f(x_{t+1}, x^*_{t+1}; X^t)$ and $g(x_{t+1}, x^*_{t+1}; X^t)$ as distributionally equivalent (under our assumed i.i.d. and jointly symmetric distribution for $X_{t+1} = \{x_{t+1}, x^*_{t+1}\}$), if $f(x^*_{t+1}, x_{t+1}; X^t) = g(x_{t+1}, x^*_{t+1}; X^t)$ and vice versa. An observation of the expressions in (68) imply that all equalities in (71) will hold if and only if, the functions $p(X^{t+1})c(X^{t+1})$ and $p(X^{t+1})c^*(X^{t+1})$ are distributionally equivalent to $p^*(X^{t+1})c(X^{t+1})$ and $p^*(X^{t+1})c^*(X^{t+1})$ respectively, for all t and X^t. This, in turn leads to the conclusion that all conditional expectations taken at time t are symmetric with respect to the realizations of X_{t+1}, i.e. $G(x_{t+1}, x^*_{t+1}; X^t) \equiv G(x^*_{t+1}, x_{t+1}; X^t)$ for all t and X^t. This further implies that two distributionally equivalent functions (such as $p(X^{t+1})c(X^{t+1})$ and $p^*(X^{t+1})c(X^{t+1})$) when multiplied with conditional expectations functions (such as $G(x_{t+1}, x^*_{t+1}; X^t)$) form resulting functions that are also distributionally equivalent. Further, if functions $p(X^{t+1})c(X^{t+1})$ and $p(X^{t+1})c^*(X^{t+1})$ are distributionally equivalent to $p^*(X^{t+1})c(X^{t+1})$ and $p^*(X^{t+1})c^*(X^{t+1})$ respectively, then it can be shown that all expectation functions are symmetric in the current realizations of x and x^*. By observing the detailed expressions for these functions as in (68), it is implied that the condition for distributions equivalence is met if and only if

$$Y(X_{t+1}, X^t) = Y^*(X_{t+1}, X^t) \tag{72}$$

and

$$b^H(X^t) + z(X^t) = b^F(X^t) - z^*(X^t) \quad \forall \quad X^t \text{ and } t \tag{73}$$

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It directly follows that

$$E_t \left[\frac{1}{p^* (X_{t+1}, X^t) \, c^* (X_{t+1}, X^t)} \right] = E_t \left[\frac{1}{p (X_{t+1}, X^t) \, c^* (X_{t+1}, X^t)} \right] \tag{74}$$

and

$$E_t \left[\frac{1}{p^* (X_{t+1}, X^t) \, c (X_{t+1}, X^t)} \right] = E_t \left[\frac{1}{p (X_{t+1}, X^t) \, c (X_{t+1}, X^t)} \right] \tag{75}$$

In the other direction, if Home and Foreign output are equal as in (72) and that the equality of nominal claims as in (73) holds, then from (31) the nominal bond prices are equal, i.e. $q = q^*$ $\forall\, t$. From (32) and (2) it is then implied that $Y = Y^*$ $\forall\, t$. Moreover, if $q = q^*$, then from the previous discussion it is obvious that equality of nominal claims (73) also holds.

6.3

Using (17), the input allocation rules (12), (13), (15), and (16) yield

$$\frac{y_t^{H*}}{y_t^H} = \frac{E_t \left[\frac{Y_{t+1}^*}{(1+x_{t+1}^*) c_{t+1}^*} \right]}{E_t \left[\frac{Y_{t+1}}{(1+x_{t+1}) c_{t+1}^*} \right]} \; ; \; \frac{y_t^F}{y_t^{F*}} = \frac{E_t \left[\frac{Y_{t+1}}{(1+x_{t+1}) c_{t+1}} \right]}{E_t \left[\frac{Y_{t+1}^*}{(1+x_{t+1}^*) c_{t+1}} \right]} \tag{76}$$

Again, using these conditions (12), (13), (15), and (16), input import credits (normalized) can be quantified

$$z_{t+1} = \alpha \frac{y_t^{H*}}{y_t^H} \quad ; \quad z_{t+1}^* = \gamma \frac{y_t^F}{y_t^{F*}} \tag{77}$$

It is clear from (77), (76), (48) and (49) that $\frac{y_t^{H*}}{y_t^H} = \frac{y_{t+1}^{H*}}{y_{t+1}^H}$ and $\frac{y_t^F}{y_t^{F*}} = \frac{y_{t+1}^F}{y_{t+1}^{F*}}$. Hence, it is implied that $y_t^H = y_{t+1}^H = y^H$, $y_t^{H*} = y_{t+1}^{H*} = y^{H*}$, $y_t^F = y_{t+1}^F = y^F$, and $y_t^{F*} = y_{t+1}^{F*} = y^{F*}$. Hence, $Y_t = Y_{t+1} = Y$, and $Y_t^* = Y_{t+1}^* = Y^*$.

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