Banco Central de Chile
Documentos de Trabajo

Central Bank of Chile
Working Papers

N° 463

Abril 2008

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NONLINEAR DYNAMIC IN THE CHILEAN STOCK MARKET: EVIDENCE FROM RETURNS AND TRADING VOLUME

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Resumen

En este trabajo se investiga la existencia de un posible comportamiento no lineal en las series de retornos y volumen transado para el caso del mercado accionario chileno. Para capturar las posibles no linealidades de las series se estiman modelos autoregresivos de transición suave (modelos STAR), los cuales son contrastados con alternativas lineales. Para investigar la relación empírica entre ambas variables, se complementa el análisis univariado con la estimación de vectores autoregresivos con cambios de régimen (modelos MS-VAR). La evidencia econométrica apoya la idea de que el mercado accionario chileno se encuentra caracterizado por la presencia de patrones no lineales en ambas series, así como en su relación conjunta. En conjunto, estos resultados sugieren que una adecuada evaluación de la hipótesis de eficiencia de mercado para el caso de la Bolsa de Santiago debe considerar un enfoque no lineal, a diferencia de estudios de previos que utilizan un marco lineal para testear esta hipótesis.

Abstract

In this paper we investigate the possible presence of nonlinear dynamics for stock index returns and trading volume at the Chilean Stock Market. To capture any nonlinear behavior in the series we estimate Smooth Transition Autoregressive (STAR) models and test them against the linear alternatives. As a complement to this univariate approach, we use Markov-Switching Vector Autoregressive (MS-VAR) models to investigate the empirical relationship between both variables. The results clearly show that the Chilean Stock Market is characterized by the presence of nonlinear patters in both series (trading volume and stock returns) as well as in their joint relationship. The presence of nonlinearities is a key issue in testing the Efficient Market Hypothesis (EMH), according to which stock returns and trading volume should be not related. Previous researches on the efficiency using data from the Chilean stock market, using linear models, support the hypothesis. However, the nonlinear patters we found in the data are a clear signal of misspecification problems in a testing procedure based on a linear approach.

We would like to thank Fabián Gredig, Pete McMenamin, Alvaro García and an anonymous referee for their helfpful comments on an early draft. Víctor Sanhueza provided valuable research assistant. Any views expressed are those of the authors and are not necessarily shared by the Central Bank of Chile or the University of Santiago-Chile. E-mail: raranda@usach.cl; pjaramillo@bcentral.cl.

1 Introduction

An old Wall Street adage states that *"it takes trading volume to make prices move"*. This can help us to understand the long standing interest among financial economists for studying the relation between stock returns and trading volumes. Given that the stock of a company reflects investors' expectations about the future prospect of the firm, new information about fundamentals (the future course of dividends or discount rates) causes investors to change their expectations and is the main reason for stock price changes. This is a general statement of the Efficient Market Hypothesis (Fama 1970, 1991), which in turns implies that one cannot earn abnormal profits by buying and selling stocks. However, the release of new information does not necessarily induce stock price to move since investors may evaluate the news heterogeneously (good or bad, for example) and, on average, despite of its importance to individual investors, such information does not noticeably affect prices (Gurgul et. al, 2005).

Empirical evidence on the EMH has been contradictory (see Karpoff, 1987). Some evidence show that price increases are positively correlated with trading volume, even though the relationship between trading volume and price falls is more ambiguous. Typically, the price-volume relationship depends on the rate of information flow and dissemination to the market, the extent to which market prices convey information, the size of the market and the existence of short-selling constraints. Price changes can be interpreted as the market re-evaluation of new information, while the corresponding volume is an indicator of investors disagreement about the meaning of this information. In this sense, trading volume is a critical complement in the process that generates stock returns and volatilities. In addition, Karpoff points out that several empirical tests about the price-volume relationship are based on the wrong assumption about the functional relationship between these variables, as well as this relation being monotonic.

Empirical studies of modern stock markets tend to confirm the existence of a positive relationship between volumes and prices in developed markets, even though the models describing the data were mostly assumed to be linear, although GARCH models were also used. However, it is generally recognized that asset markets, in general, and equities markets, in particular, are characterized by the occurrence of low frequency, high amplitude shocks. Because of that, a linear dynamic model of stock market returns may provide a misleading specification of market movements (Bradley and Jansen, 2004). The introduction of nonlinear dynamics leads to conclude that zero serial correlation in stock returns implies a statistical independence if and only if the joint probability distribution is normal. The importance of this condition was made clear with the discovery of nonlinear dependence in stock market returns, first reported by Hinich and Patterson (1985). Today it is know that the lack of linear dependence (serial or autocorrelation) does not rule out nonlinear dependence in stock returns which may even become predictable. This implies that in order to make the EMH test operational a few additional features of the regressions are needed. More precisely, any support either for the mixture of distributions model or the sequential arrival of information hypothesis is taken as evidence against the EMH.

It is known that in small emerging capital markets, the acquisition of information is more costly than in developed markets. This can explain the relatively limited amount of research on emerging financial markets, considering that the unique characteristics of emerging stock markets provide excellent opportunities to study the effects of market microstructure on stock returns and the efficiency of these markets. A natural question then is to figure out the extent in which the empirical findings for highly liquid stock markets of industrial countries should hold also for emerging stock markets. The peculiar characteristics of risk and returns in emerging stock markets represents the focus of a recent literature, which may provide interesting insights of historical cases of emerging markets.

In summary, in an efficient market prices should adjust instantaneously toward their fundamental values and trading volume contains no information about future price developments. Are stock returns and trading volume related at the Santiago Stock Exchange? If they are, is the relationship linear or nonlinear? The purpose of this paper is to reexamine the evidence on the stock return and volume relationship in an emerging economy. By using a sample from the stock index return and volume for the Chilean stock market, our contribution to the literature on emerging stock markets are the following. First, we implement a high frequency analysis with daily information for the variables of interests, instead of monthly data as in the previous literature concerning emerging markets. Second, we use linear and nonlinear Granger causality tests to clear up any positive correlation between stock index return and volume because this issue is relevant for the EMH testing. Third, we look for any nonlinear pattern in the dynamic by formulating and estimating univariate Smooth Transition Autoregressive (STAR) models, and test this model against the linear alternative. Fourth, the analysis of nonlinearities is fully complemented by estimating and testing a nonlinear bivariate Markov Swithching Vector Autoregressive (MS-VAR) models for the relationship between trading volume and stock returns. To our knowledge this is the first time that all these characteristics are combined in a work for the Chilean stock market. Our preliminary evidence and econometric results clearly show that the Chilean Stock Market is characterized by the presence of nonlinear patters in both series (trading volume and stock returns) as well as in their joint relationship.

The structure of the paper is as follows. The next section briefly summarizes some of the literature concerning the relation between stock price and volume. Section 3 describes our data and the econometric methodology we use in this study. Section 4 reports and discuss our main results. Finally, section 5 provides some conclusions and discusses limitations.

2 Literature Review

Market folklore and empirical evidence suggest that trading volume is positive related to stock returns, and two stylized facts are frequently mentioned. First, the correlation between trading volume and the absolute value of price changes is positive; that is, a large increase in volume is usually accompanied by either a large rise or a large fall in prices. Second, the correlation between volume and returns is also positive. However, there is not sufficient scientific evidence supporting those findings. A major limitation has been the lack of substantial theory linking trading volume directly to stock returns, although researchers have examined indirect links through models of information arrivals and stock returns. Examples include Admati and Ffeiderer (1988), Barclay, Litzenberger and Warner (1990), Barclay and Warner (1993), Brock and Kleidon (1992), Easley and O'Hara (1992), among others. These articles are generally based on the economics of information and tend to be focused toward microstructure issues.

From this literature we found several reasons for a possible relation between returns and trading volume (Hiemnstra and Jones, 1994). For example, models of sequential arrival of information (Copeland, 1976; Jennings, Stark and Fellighan, 1981) postulates that new information that reaches the market is not disseminated to all participants simultaneously, but to one investor at a time; final information equilibrium is reached only after a sequence of transitional equilibria. Hence, due to sequential information flow, lagged trading volume may have predicted power for current absolute stock returns and lagged absolute stock returns could have predictive power for current trading volume. A second explanation for causal relationship between returns and trading volume is based on the mixture of distributions model. In this model, if trading is used to measure the disagreement as traders revise their reservation prices based on the arrival of new information the greater the disagreement; that is, the larger the level of trading volume, the large the absolute price change. Thus, there is a positive causal relation running from trading volume to absolute stock returns. This is, of course, implies that knowledge of the behavior of volume can marginally improve conditional price change forecasts based on past price change forecasts alone.

Noise trader models provide a third explanation for the causal relation; these type of models can reconcile the difference between short- and long-run autocorrelation properties of aggregate stock returns. Aggregate stock returns are positively autocorrelated in the short run, but negatively autocorrelated in the long run. Since noise traders do not trade on the basis of economic fundamentals, they impart transitory mispricing components to stock prices in the short run. The temporary component disappears in the long run, producing mean reversion in stock returns. A positive causal relation from stock returns to volume is consistent with the positive feedback trading strategies of noise traders, for which the decision to trade is conditioned on past stock price movements.

From a practical point of view, market participants carefully watch the volume of trade, which presumably conveys valuable information about future price movements. What we can learn from volume depends on why investors trade and how trades with different motives relate to prices. Two reasons are often mentioned for why investors trade stocks: to rebalance their portfolios for risk sharing and to speculate on their private information. These two types of trades, called hedging and speculative trades, results in different return dynamics. When a subset of investors sells a stock for hedging reasons, the stock's price must decrease to attract other investors to buy. Since the expectation of future stock payoff remains the same, the decrease in the price causes a low return in the current period and a high expected return for the next period (e.g., negative return autocorrelation). However, when a subsets of investors sell a stock for speculative reasons, its price decrease reflecting the negative private information about its future payoff. Since this information is usually only partially impounded into the price, the low return in the current period will be followed by a low return in the next period when the negative private information is further reflected in the price (e.g., positive return autocorrelation).

On a daily base, movements in stock market prices and expected returns may occur for two reasons. Informational trades, due to public information that causes all investors to change their valuation of the stock market because of new information about fundamental shocks affecting it. Non-informational trades, due to non-informational factors such as interactions among different groups of investors with heterogeneous information, exogenously shift misperceptions of future stock payoffs, irrational noise trading, or by over-confident investors who over-estimate the precision of their private signal about security values; heterogeneous information and investment opportunities, and shifts in the risk aversion of some traders. It is very difficult to distinguish between these two different views of stock market movements using data on stocks alone. If public information that affects all investors arrives, then stock market trading volume may not be significantly affected; however, selling pressure by non-informational traders must have a substantial effect on trading volume. Therefore, the two types of trades can be distinguished by looking at trading volume.

From the empirical side of the literature, while earlier research on the topic mainly focuses on the contemporaneous relationship between returns and volume, more recent studies examine causal dynamics (Karpoff, 1987). For example, Smirlok and Starks (1992), Gallant, Rossi and Tauchen (1992) and Hiemstra and Jones (1994) point out significant linear and nonlinear dynamics between trading volume and returns and conclude that more can be learned by studying prices jointly with volume. On the other hand, Blume, Easley and O'Hara (1994) examine the information content of volume in a theoretical context. These authors show that lagged volume could be useful for predict price movements when prices are noisy and market participants cannot obtain the full information signal from price alone; their model is consistent with the widespread use of technical analysis in financial markets.

Campbell, Grossman and Wang (1993) find that trades due to heterogeneous investors (i.e., non-informational traders) that are accompanied by high trading volume are expected to be associated with a low serial correlation in stock returns because market makers buying stocks would require higher expected returns to compensate for their bearing additional risk. They suggest the use of data on stock market trading volume as a means of distinguish between these two types of trade. They provide a model whose implications are consistent with this distinction.

Lee and Rui (2000, 2001) follow the approach of Campbell et al. (1993). They empirically identifying the components of stock returns and trading volume due to non-informational and informational traders, examining whether the components due to non-informational traders can account for the empirical relationship between trading volume and serial correlation of stock returns. Also, they report that for the trading volume-serial correlation in the stock returns relationship, the evidence is consistent with theoretical predictions that non-informational components can account for high trading volume accompanied by a low serial correlation in stock returns.

Empirical evidence has also shown that the return and trading volume time-series properties are best described using nonlinear models. For example, the returns data oftend reveals a volatility clustering phenomenon associated with GARCH of large (small) shocks of either sign tending to follow large (small) shocks.

The evidence of nonlinearity in returns and trading volume is not limited to the case in which these series are individually described. Hiemstra and Jones (1994) report uni-directional linear Granger causality from returns to volume in contrast to bidirectional nonlinear causality between these variables; they also alter stock returns with Exponential GARCH (EGARCH) to control for volatility persistence, and still find nonlinear causality running from volume to stock returns. Silvapulle and Choi (1999) get similar results focusing on the emerging Korean stock market. Campbell, Grossman and Wang (1993) find a negative relation between daily stock index return autocorrelations and trading volume; they assume that two types of investors exist in the market: noninformational investors who want to sell stocks for exogenous reasons, and market makers who are willing to buy stocks to accomodate the market selling pressure but who require compensation for taking the risk in the form of a lower stock price or a higher expected stock return. For such traders, stock return reversals tend to cause an abnormally large increase in volume, as prices tend to fall, increasing the trading volume as long as the reallocation of risk between heterogeneous traders is completed. Therefore, large trading volume will be associated with relatively large negative serial correlation of returns.

Saatcioglu and Starks (1998) examines the stock price-volume relation in a set of Latin American markets (Argentina, Brazil, Chile, Colombia, Mexico, and Venezuela). They document a positive relation between volume and both the magnitude of price change and price change itself, but they do not find strong evidence on stock price changes leading volume, in contrast to the evidence reported by studies on developed markets. They conclude that the set of emerging markets with different institutions and information flows than developed markets, do not present similar stock price-volume lead-lag relation to the preponderance of studies employing data from developed countries. Sarantis (2001) finds that STAR models are useful in describing asymetric cycles in stock price growth rates in most industrial countries.

3 Data and Econometric Approach

3.1 The Data and Some Preliminar Evidence

In financial markets, the price of a stock depends not only on the asset exchanged and the timing of the trade, but also on the trade volume (number of stock shares) and on the investor's characteristics. Then quoted prices could differ from the true prices involved in transactions. We must also take into account the spread between bid and ask prices that arise from the need to cover any cost involved in financial intermediation; this spread can be very informative about the liquidity and efficiency of the stock market. However, all these problems can be overcome if we consider a price index. In this paper we consider the closure price of stocks given by an index of selective stocks.[1] Figure 1 below shows the evolution of this index from January 2, 1989 to October 25, 2007. As we can see, preliminary there is some evidence of nonlinearity.

The behavior of returns is particularly interesting. As we said before, in an efficient market the path of prices and return per period are unpredictable. The EMH hypothesis implies that the expected value of tomorrow's price p_{t+1}, given all relevant information up to and including today (Ω_t) should equal today's price p_t, possible up to a deterministic growth component (a drift). In testing the EMH the model commonly used is $p_t = \mu + p_{t-1} + \varepsilon_t$, where $\varepsilon_t \overset{iid}{\sim} D(0, \sigma^2)$ and D is some distribution, or returns follow a random walk with drift $\Delta p_t = \mu + \varepsilon_t$. As a first approximation we can ask if the random walk model is a good characterization of the actual behavior of stock returns in the Santiago Stock Exchange. Figure 1 depicts the actual behavior of stock prices for the whole sample, together with alternative simulated paths (100 draws) for prices from the random walk model[2]. As we can see, in the long run the random walk model is far from being a good approximation for the actual behavior of stock prices in Chile.

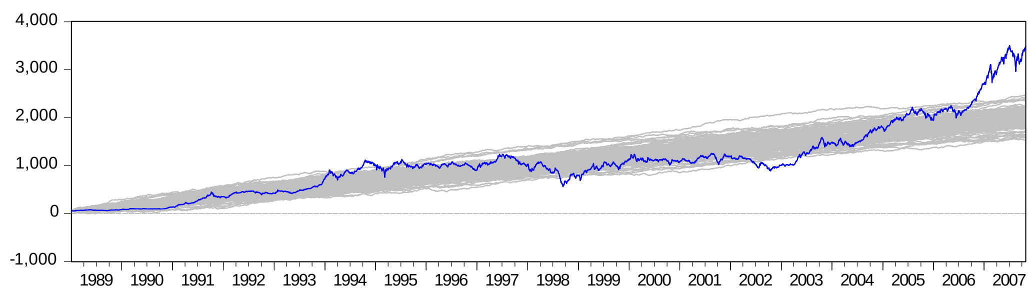

Figure 1. Stock Price Index and Random Walk Simulations

As it is frequently found in many economic and financial time series, figure 1 also shows the presence of a long-run positive trend. Besides this, some changes in the level of the series are observed, both in the short and the medium run. This implies that the data generating process for stock prices would be better characterized by changing means, which in turns implies different regimes in the time series. This is important because changing regimes are one of the source of nonlinearities in time series processess. A useful transformation is to consider

[1] The index used is the IPSA, "Índice de Precios Selectivo de Acciones" (Index of Selective Stock Prices). This index comprises the forty most traded stocks in the Santiago Stock Exchange, selected annualy.

[2] The data generation process of the random walk model is $y_t = 0.4 + y_{t-1} + \varepsilon_t$ with initial value 48.69 (index value in January 2, 1989). The innovations are normally distributed with standard deviation 3. This specification show a better fit in the sample.

returns instead of prices, defined by $r_{t+1} = \frac{p_{t+1} - p_t}{p_t}$, which can be approximated by[3],

$$r_{t+1} = \ln(p_{t+1}) - \ln(p_t)$$

The time series for the level and the first difference of prices (returns) and trading volume over the entire sample used in our empirical analysis - from July 18, 1995 to October 25, 2007 - are depicted in Figure 2. This figure also show the time series of trading volume and its first difference.

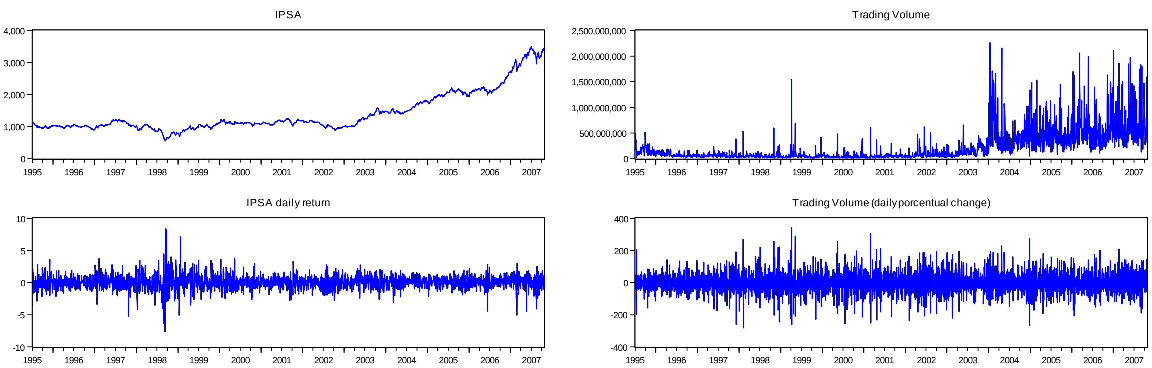

Figure 2. Returns and trading volume (level and first differences)

We see that both variables appear to be nonlinear and heteroskedasticity is a possible source of this characteristic in our data. The Table 1 presents some descriptive statistics for both series.

	Level		First Differences	
Sample: July 1995 - October 2007	IPSA Index	Trading Volume	Return	Change of Trading Volume
Mean	1408.7	2.20E+08	0.036	0.055
Median	1125.3	86536764	0.00	0.00
Maximum	3499.5	2.26E+09	8.4	342.1
Minimum	554.7	3224875	-7.7	-282.7
Std. Dev.	627.5	2.95E+08	1.05	64.86
Skewness	1.58	2.58	0.10	0.07
Kurtosis	4.79	11.27	8.86	4.89
Jarque-Bera	1767.748	12718.66	4602.153	481.6268
Probability	0.00	0.00	0.00	0.00
Observations	3214	3214	3213	3213

Source: Own Elaboration

Table 1: Descriptive Statistics

The statistics reports clear evidence of excess of skewness and kurtosis in both series. The Jarque-Bera test rejects the null of normality for both variables at conventional significance levels. To get further insights on the statistical distributions of the series, Figure 3 shows the empirical distribution compared to the normal distribution.

[3] This transformation tends to subestimate the true value for returns, $\widetilde{r}_{t+1}$. In fact, it can be shown that $\widetilde{r}_{t+1} = \ln(1 + \frac{p_{t+1} - p_t}{p_t}) = r_{t+1} - \frac{r_{t+1}^2}{2}$.

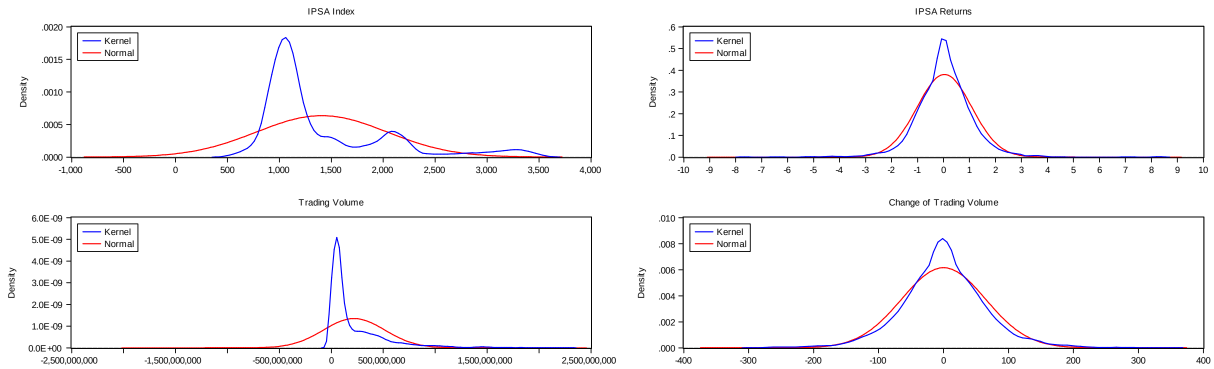

Figure 3: Empirical Distributions (IPSA Index and Trading Volume)

The figure is very illustrative of the leptokurtic nature of returns, and the existence of at least two modes perfectly identifiable. Again, this is evidence of possible nonlinearities in the data.

A natural approach to modeling economic time series with nonlinear models seems to be to define different states of the world or regimes, and to allow for the possibility that the dynamic behavior of economic variables depends on the regime that occur at any given point in time. However, problems immediately arise: there is a vast and growing number of possible models. Roughly speaking, there are two main classes of regime dependent statistical models: the so called Smooth Transition Regression (STR) family and the popular Markov-Switching models proposed by Hamilton (1989).

The STR models, on one hand, are a general class of state-dependent reduced form, nonlinear time series models in which the transition between states is generally endogenously generated; they encompass as particular cases the Exponential Autoregressive (EAR), the Threshold Autoregressive (TAR) and the SETAR models. In particular, Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993) and Teräsvirta (1994) promote a family of univariate business cycle models called Smooth Transition Autoregressive (STAR) models; these models can be viewed as a combination of the SETAR and the EAR models. In Markov-Switching models, on the other hand, the transition between regimes is assumed to be exogenously generated by a Markov Chain process. This implies that one can never be certain that a particular regime has occurred at a particular point in time, but can only assign probabilities to the occurrence of the different regimes.

One difficulty with the Markov-Switching models is that they imply a sharp regime switch and therefore a small number (usually two) of regimes. This assumption is too restrictive compared to STAR models which can be considered as a regime-switching model that allow for two regimes where the transition between one regime to other is smooth. Moreover, the main advantage in favour of STAR models is that changes in economic aggregates are influenced by changes in the behavior of many different agents and it is highly unlikely that all agents react simultaneously to a given economic signal. In financial markets, for example, with a large number of investors, each switching at different times (probably due to heterogeneous objectives and beliefs), a smooth transition or a continuum of states between the extremes appears more realistic. Thus, when considering aggregate economic series, the time path of any structural change is liable to be better captured by a model whose dynamic undergo gradual, rather than instantaneous adjustment between regimes. The STAR models allow exactly this kind of gradual change whilst being flexible enough that the conventional change arises as a special case. In the following we give some details on the structure of these two nonlinear models.

3.2 The Smooth Transition Autoregressive (STAR) Model

The Smooth Transition Autoregressive (STAR) model is a generalization of a two-regime system in which the transition between the two extreme regimes is smooth. The STAR models are estimated when the linearity hypothesis is strongly rejected for at least one transition variable. This model links two linear autoregressive models by a bounded transition function. Different transition functions characterize different dynamic properties of data, resulting in different specication for the STAR models (see van Dijk, 1999; van Dijk et. al., 2000, Krolzig, 2002, Potter, 1999; and Teräsvirta, 1994). The general structure of this type of model is:

$$y_t = \left[\alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}\right] + \Phi(y_{t-d}, \delta)\left[\beta_0 + \sum_{i=1}^{p} \beta_i y_{t-i}\right] + u_t \tag{1}$$

where u_t is an independent and identically distributed random variable with mean zero and variance σ^2 (or alternately a martingale difference sequence[4]). $\Phi(y_{t-d}, \delta)$ is the transition function which it's a continuous function that is bounded between 0 and 1. Throughout this paper we assume that the transition variable is the lagged endogenous variable (y_{t-d}), wher d is the *"delay"* parameter whose value is a positive integer. Additionally, in this specification two linear AR component are connected using a bounded nonlinear transition function.

Therefore, to empirically implement the STAR model, we must first select the autoregressive order of autoregression p and then choose d by varying it and selecting the value of d that minimizes the $p - value$ in a linearity test. Different choices for the transition function give rise to two different types of regime-switching models with a smooth transition: the Logistic STAR (LSTAR) model, in which the transition function is the logistic function:

$$\Phi_L(y_{t-d}, \delta) = \{1 + \exp[-\gamma(y_{t-d} - c)]\}^{-1}; \quad \text{with } \gamma > 0, \tag{2}$$

where $\delta = (y, c)'$, and the Exponential STAR (ESTAR) model, in which the transition function is modeled as an exponential function:

$$\Phi_E(y_{t-d}, \delta) = 1 - \exp[-\gamma(y_{t-d} - c)^2]. \tag{3}$$

It is straightforward to extend the model to allow for exogenous variables as additional regressors. The transition variable can also be an exogenous variable, or a (possibly nonlinear) function of lagged endogenous

[4] The normality assumption is needed if the specification test are derived as Lagrange Multiplier (LM)- type test; if they are interpreted as tests based on artificial regressions, then a martingale difference assumption is sufficient (Teräsvirta, 1994).

variables. It is also the possible to include a linear time trend as a transition variable (Lin and Teräsvirta, 1994).

If a logistic STAR model of order p is chosen, high and low trading volume/stock returns may have rather different dynamics, and the change in dynamic from one regime to the other is smooth. Parameters change monotonically and the transition variable deviates from a fixed point c, the threshold between the two regimes. In an exponential STAR of order p, volume/returns may move rapidly between very small and very large values for which local dynamics are stable. The parameter γ determines the smoothness of the change in the value of the transition function, and thus the smoothness of the transition from one regime to the other. In this study we assume that the conditional variance of u_t is constant.

There are several useful extensions of the basic STAR model are proposed in the literature; models for vector time series, models for multiple regimes, or time varying nonlinear propierties (see, for example, van Dijk et. al., 2000).

3.3 Markov-Switching Vector Autoregressive (MS-VAR) Model

Since previos discussions suggest that both stock returns and trading volume should be related, in this part we formulate and estimate a Markov Switching-Vector Autoregressive (MS-VAR) model, an extension of a regime switching model proposed by Hamilton (1989). The MS-VAR models provide a flexible framework allowing for heteroskedasticity, occasional shifts, reversing trends and forecast performed in a nonlinear way (for details see Krolzing, 1998). In the general representation of Markov-switching vector autoregressions of order p and M regimes, all parameters of the autoregression are conditioned on the state s_t of the Markov chain. Let M denote the number of feasible regimes, so that $s_t \in \{1, ..., M\}$. It is assumed that each regime has a $VAR(p)$ representation with parameters $\upsilon(m)$, $\sum_m$, A_{1m},...., A_{jm}, $m = 1, ..., M$, such that

$$y_t = \begin{cases} \upsilon_1 + A_{11}y_{t-1} + ... + A_{p1}y_{t-p} + \sum_1^{1/2} u_t, & \text{if } s_t = 1 \\ \\ \upsilon_M + A_{1M}y_{t-1} + ... + A_{pM}y_{t-p} + \sum_M^{1/2} u_t, & \text{if } s_t = M \end{cases} \tag{4}$$

where $u_t \sim NID(0, I_K)$.[5]

The conditional probability density of an observed vector of time series y_t is given by:

$$p(y_t \mid Y_{t-1}, s_t) = \begin{cases} f(y_t \mid Y_{t-t}, \theta_1) & \text{if } s_t = 1 \\ \quad ... \\ f(y_t \mid Y_{t-t}, \theta_M) & \text{if } s_t = M \end{cases}$$

where θ_M is the VAR parameter vector in regime $m = 1, ..., M$ and Y_{t-1} are the observations.

[5] Even at this early stage a complication arises if the mean adjusted form is considered. The conditional density for y_t depends not only on s_t but also on $s_{t-1}, ..., s_{t-p}$, i.e., M^{p+1} different conditional means of y_t can be distinguished (see, Krolzig, 1997).

4 Empirical Results

In this section we first address the linear or nonlinear dependence in the data. Then we use alternative models to capture any nonlinear patterns eventually detected in the data, following the advice of Hiemstra and Jones (1994), whom provide empirical evidence for argumenting that more can be learned about the stock market dynamic by studying the joint dynamics of stock prices and trading volume rather than by focusing only on the univariate dynamics of stocks prices. Before that we examine the issue of nonstationarity, given that there is some visual evidence on nonstationarity in both variables.

As we said before, the time series for stock returns and change of trading volume show some nonlinearities and possibly heteroskedasticity too. Because causality tests can be sensitive to nonstationarities associated with structural breaks, it is important to analize periods where the univariate and bivariate stochastic processes generating stock prices and trading volume can be considered as stationary. In testing for stationarity we apply a battery of unit roots tests, including the standard Augmented Dickey-Fuller (ADF) tests, the Phillips-Perron tests, the Dickey-Fuller tests with GLS Detrending (DFGLS), the Kwiatowski, Phillips, Schmidt, and Shin (KPSS) test, and the Elliot, Rothemberg and Stock Point Optimal (ERS) test. The reason for using this set of tests is that standard unit root tests suffer from poor size and power, and also because there is some problems with the assumption of nonstationarity rather than stationarity as the null hypothesis for the test (Maddala and Kim, 1998). The results are reported in Table 2.

Test		ADF	Phillips-Perron	ADF-GLS	ERS	KPSS
Null Hypothesis		unit root	unit root	unit root	unit root	stationary
Return		-13.49	-46.93	-2.22	0.62	0.02
Change of Trading Volume		-82.97	-498.73	-82.98	0.07	0.06
Critical Values						
	1%	-3.96	3.96	-3.48	0.22	0.22
	5%	-3.41	5.62	-2.89	0.15	0.15
	10%	-3.13	6.89	-2.57	0.12	0.12

Source: Own Elaboration

As we can see from the table, returns and percentage changes in Chilean stock market trading volume are stationary at conventional signicance levels. On the other hand, looking at the first difference of returns and volume in Figure 2, larger variances than in the surrounding periods suggest that the data may not be generated by the same data-generating process during the whole sample period. However, what is in appariance a structural break may also be due to nonlinearity, which can be modeled with a constant parameter model. As the sample includes a great number of observations (around 3,100 obs.), it is reasonable to assume that we would observe regime shifts in the data. To motivate the possibility of modeling different regimes, lets consider the Figure 4 which show the residuals from a linear model, in which we have regressed the logarithm of returns (and trading volume) on a constant and a trend.

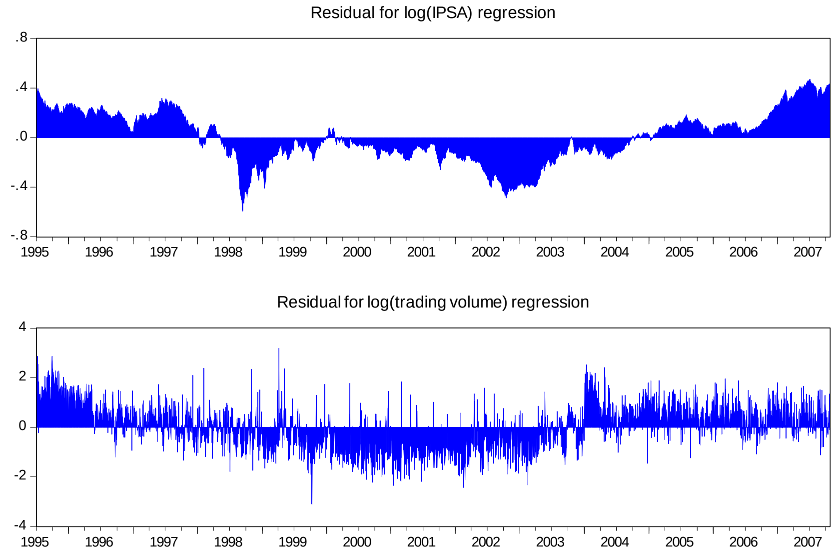

Figure 4: Residuals from a linear regression model (on a constant and a trend)

It is observed that returns and trading volume tend to stay either above or below a trend, and the changes around the trend have been quite abrupt. However, if we expect that the change in model parameters have been smooth, this can be modeled by a nonlinear STAR model. In order to find out whether the data support the fact that periods with large price movements are also periods with larger than average trading volume, and viceversa (Karpoff, 1987), the next figure shows the moving correlation coefficients for both stock returns and trading volume in four different moving windows (7, 30, 90 and 360 days).

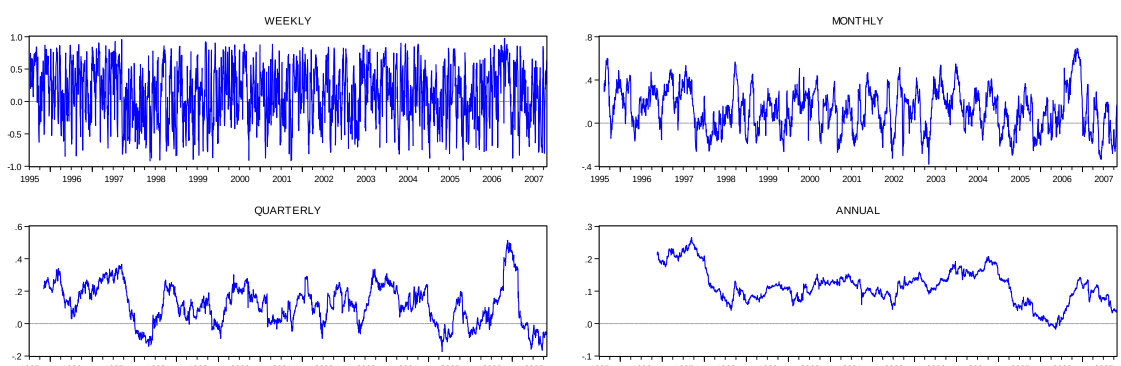

Figure 5: Rolling correlations between returns and trading volume

As we can see, the evidence suggests a positive contemporaneous correlation between returns and trading volume with coefficients close to 0.11 in full sample. The standard deviations change according the frequency data (0.42 at weekly frequency and 0.05 annual), as in Gallant et. al. (1993) .

4.1 Linear and Nonlinear Granger Causality Tests

To get further insights on nonlinearities, we test for Granger causality between trading volume and stock returns, where Δy_t is the logarithmic difference of returns and trading volume, Δy_t^2 is the squared logarithmic difference of returns and volume, and $|\Delta y_t|$ is the volatility (absolute value of logarithmic difference) of returns and volume. Causality tests can provide useful information on whether knowledge of past stock returns (trading volume) movements improves short-run forecasts of current and future movements in trading volume (stock returns) (Rashid, 2007).

As mentioned earlier, the results of stationarity tests provided evidence that returns in the stock index and percentage volume changes on the Santiago Stock Exchange are stationary at their level for the entire sample period. Table 3A reports the results of the Granger causality tests. Lag lengths on the dependent and independent variables and ranges of p-values are reported.

Linear Granger Causality Test			
Null Hypothesis (Lags 1 to 4)	Δy_t	Δy_t^2	$\|\Delta y_t\|$
Return does not Granger Cause Volume	(0.00 - 0.48)	(0.06 - 0.22)	(0.00 - 0.03)
Volume does not Granger Cause Return	(0.75 - 0.98)	(0.15 - 0.55)	(0.00 - 0.26)
Null Hypothesis (Lags 5 to 12)			
Return does not Granger Cause Volume	(0.02 - 0.13)	(0.29 - 0.77)	(0.02 - 0.11)
Volume does not Granger Cause Return	(0.25 - 0.98)	(0.52 - 0.96)	(0.29 - 0.61)

Table 3A: Linear Granger Causality Test (p-values ranges)

The results of Granger causality are very sensitive to lag order. Therefore we use a lag intervals. Focusing on the rejection of the null hypothesis of Granger noncausality at the 5% significance level, the Granger test is able to reject that stock returns and absolute returns do not cause volume changes for the entire sample, but not from trading volume to returns and absolute returns. We interpret this evidence as suggesting that there is no evidence of bidireccional causality between stock returns and trading volume.

The traditional Granger Causality Tests is only useful to examine any linear relation in the variables; however, it is unable to explore the nonlinear relationship between two varibles. To address this issue, and following Hiemstra and Jones (1994), we use a modified version of Baek and Brock's (1992) nonlinear Granger causality test to expose the nonlinear interactions between stock returns and percentage volume changes.[6] Table 3B reports the results on nonlinear causality.

[6] Codes for implementing the modified Baek and Brock test for nonlinear causality were developed in R, a freeware software, and are available upon request.

Nonlinear Granger Causality Test			
Null Hypothesis (Lags 1 to 4)	Δy_t	Δy_t^2	$\|\Delta y_t\|$
Return does not Granger Cause Volume	(2.78 - 6.11)	(1.60 - 2.71)	(-4.71 - 0.29)
Volume does not Granger Cause Return	(-9.05-1.47)	(-9.60 - 1.06)	(2.16 - 8.84)
Null Hypothesis (Lags 5 to 12)			
Return does not Granger Cause Volume	(-5-54 - 4.93)	(-5.40 - 4.82)	(-4.41 - 9.44)
Volume does not Granger Cause Return	(-5.32 - 5.03)	(-2.81 - 2.99)	(-4.12 - 0.67)

Table 3B: NonLinear Granger Causality Test (F–statistics ranges)

As the reported results suggests, there is evidence of bidirectional nonlinear Granger causality between stock returns and trading volume. This result holds for all the common lag lengths used in conducting the test. None of the standardized test statistics is lower than 4.60, which is strong statistical evidence in favor of nonlinear causality in both directions. Again the results suggests that a nonlinear modeling approach may be useful in describing the behavior of stock returns and volume. This is very important because the evidence of nonlinearity has strong implications on the EMH, given that it implies that stock returns are potentially predictable. For example, if investors could have profitably operated a trading rule, net of all transaction costs, that exploits some sort of nonlinear patterns in the data, it would be at odds with the weak-form of the EMH, which postulates that even nonlinear combinations of previous prices are not useful predictors of future prices (Brooks, 1996; Brooks and Hinich, 1999; McMillan and Speight, 2001).

4.2 Testing Linearity against TAR and STAR Models

Having reported evidence on nonlinear causality running in both directions, the next step is to model the behavior of both series using nonlinear models. Since these models are based on autoregressive structures, the first problem we faces in searching for the appropriate econometric specification is to select the right lag structure; this is a non trivial exercise when using nonliner models. A common approach is to start estimating an $AR(p)$ model assuming that the selected lag order p is the same in both regimes of the nonlinear model. Consequently, we fit an $AR(p)$ model to both variables (returns and trading volume). Table 4 shows the best $AR(p)$ specifications for different lags order. The lag order were selected by the Hannan-Quinn information criteria.

Sample: *July 1995 - October 2007*	IPSA Index Return	First Difference of Trading Volume
Constant	0.00 (1.48)	0.00 (0.20)
Lag 1	0.19 (7.78)	-0.60 (28.45)
Lag 2	0.01 (0.50)	-0.50 (22.18)
Lag 3	-0.01 (0.41)	-0.44 (17.81)
Lag 4	0.02 (0.66)	-0.37 (14.16)
Lag 5	0.06 (2.17)	-0.27 (10.80)
Lag 6	0.02 (0.62)	-0.24 (9.95)
Lag 7	-0.06 (1.90)	-0.19 (8.74)
Lag 8	-	-0.17 (8.66)
Lag 9	-	-0.11 (6.29)
Adjusted R-squared	0.04	0.28
Hannan-Quinn IC	-6.32	1.64
Obs.	3206	3204

Source: Authors Elaboration

Note: Test - t in parenthesis

Table 4: Best Linear Model Specifications

Table 5 reports son diagnosis statistics for both models estimated. As we can see, there is statistical evidence that residuals are not white noise, and that there is evidence skewness and excess kurtosis in the residuals. Both null hypothesis are rejected even at a 1% level. Substantial excess of kurtosis as well as moderate negative (positive) skewness in residuals suggest the presence of mainly negative (positive) outliers in the trading volume series.

	Residuals from AR(p) Model for Return	Residuals from AR(p) Model for Volume
Mean	4.60E-06	-1.80E-18
Median	-0.000161	-0.031598
Maximum	0.083156	3.375921
Minimum	-0.069504	-2.583491
Std. Dev.	0.010215	0.547035
Skewness	0.203976	0.49802
Kurtosis	8.220312	5.040744
Jarque-Bera	3660.314	688.4236
Probability	0	0
Sum	0.014743	-2.89E-15
Sum Sq. Dev.	0.334192	958.4892
Observations	3204	3204

Source: Own Elaboration

Table 5: Diagnosis Models

To capture nonlinear dynamics, Threshold Autoregressive (TAR) models allow the model parameters to change according to the value of a weakly exogenous threshold variable. Following Tsay (1989) and Hansen (1997), we now introduce two approaches for testing threshold nonlinearity and estimating the unknown parameters in the associated models . The Tsay's nonlinearity test centers on the use of an arranged autoregression with recursive least squares estimation, while the Hansen's Sup-LR tests has the advantage that the threshold can be simultaneously estimated with the other parameters in the model, so we can construct valid confidence intervals for the estimated threshold. The results for return and trading volume are presented in the Table 6 and 7.

	Return		Trading Volume	
	Tsay F-stat	P-val	Tsay F-stat	P-val
d=1	5.713	0.000	9.979	0.000
d=2	3.288	0.001	4.647	0.000
d=3	1.796	0.073	2.633	0.003
d=4	7.032	0.000	1.871	0.045
d=5	4.955	0.000	1.691	0.077
d=6	4.721	0.000	1.052	0.396
d=7	4.366	0.000	2.311	0.011
d=8	-	-	0.946	0.489
d=9	-	-	0.198	0.997

Source: Own elaboration

Table 6: Tsay Nonlinearity Test

Using Tsays's tests, the null hypothesis of the no threshold nonlinearity is actually rejected for both linear AR models from delay 1 to 5. As an practica approach, Tsay suggested to choose the delay parameter such as to maximize the F-stat ($d = \text{argmax} F(\cdot)$). For both variables, the results indicate that $d = 1$ is appropriate. On the other hand, using Hansen tests (Table 7) the null hyphotesis of no threshold nonlinearity is reject for both returns and trading volume, with threshold value of 0.83% and -47.09%, respectively.

	Return	Trading Volume
Threshold Estimate	0.83	-47.09
F-test for no threshold	48.71	101.0
Bootstrap P-Value	0.00	0.00
Trimming percentage	0.10	0.10
Bootstrap Replications	1000	1000

Table 7. Hansen Sup-LR Nonlinearity

The most important questions that needs to be answered when considering regime-switching models is whether the additional regime relative to the single regime in a linear AR model add significantly to explaining the dynamic behavior fo the time series (Franses and van Dijk, 2000). A natural approach is to take he linear model as the null hypothesis and the regime-switching model as the alternative. However, any statistical test that takes a regime switching-model as the alternative suffer from the problem of so-called unidentified nuisance parameters under the null, which is the case with the STAR model. This implies that the test statistic has nonstandard asymptotic distributions. Notwithstanding, Luukonnen, Saikkonen and Teräsvirta (1988) demonstrate that conventional distribution theory is still applicable in the case of a nonlinear model. Luukonnen, Saikkonen and Teräsvirta uggest approximate the transition function $\Phi(y_{t-d}, \delta)$ with a Taylor approximation around $\gamma = 0$ to obtain an auxiliary regression which is then used to testing the null.[7] Additionaly we also

[7] Luukkonen et al (1988) test is a conventional Lagrange multiplier (LM) test with an asymptotic χ^2 distribution. See Franses and van Dijk (2000), and Zivot and Wang (2006), chapter 18.

implement the Granger and Teräsvirta (1993) tests for nonlinearity, which is robust to heteroskedastic errors.

	Return		Trading Volume	
Test	LL-S-T test	G-T Test	LL-S-T test	G-T Test
Lag 1	0.00	0.13	0.00	0.00
Lag 2	0.00	0.64	0.00	0.03
Lag 3	0.00	0.71	0.00	0.04
Lag 4	0.00	0.46	0.00	0.12
Lag 5	0.00	0.04	0.02	0.07
Lag 6	0.00	0.49	0.02	0.59
Lag 7	0.00	0.48	0.02	0.09
Lag 8	-	-	0.07	0.12
Lag 9	-	-	0.94	0.70

Source: Own Elaboration

Note:

LL-S-T Test: Luukkonen, Saikkonen and Teräsvirta (1998).

G-T Test: Granger and Teräsvirta (1993).

Null Hypothesis for LL-S-T and G-T tests: no smooth threshold nonlinearity

Table 8: STAR Nonlinearity Test

Assuming that the errors are homoskedastic, the results reported indicate that the null of no smooth threshold nonlinearity is rejected at conventional significance levels for both series. However, when the possibility of heteroskedastic errors is allowed, as it is common in many financial time series, the Granger and Teräsvirta procedure shows that we can reject de null only for the trading volume series, which is not clear in the case of the returns series. We interpret this as evidence in favour of a STAR type of model. Additionally, we developed a ARCH test for autoregressive conditional heteroskedasticity (ARCH) in the residuals (Engle 1982)[8]. As the no-ARCH hypothesis is also rejected at the 1% level, this leads us to assume a nonconstant conditional variance in the error processess; moreover, this may be also a signal of a nonlinear conditional mean (Teräsvirta, 1994; van Dijk, 1999).

4.3 LSTAR and ESTAR Estimations

After testing for nonlinear behavior in both time series and having rejected a linear model against a nonlinear STAR model, we proceed now with the specification and estimation of univariate STAR models for both series.[9]. In selecting the models, we had follow a sequential approach based on the transition variable considered and different specifications for the transition function (conditional to the transition variable) and the variables included in the linear and nonlinear parts of the STAR model. That is, we first specified a linear AR model of order p for the time series under analysis. Then we tested the null hypothesis of linearity against the alternative

[8] This particular heteroskedasticity specification was motivated by the observation that in many financial time series, the magnitude of residuals appeared to be related to the magnitude of recent residuals. ARCH in itself does not invalidate standard LS inference. However, ignoring ARCH effects may result in loss of efficiency. In both cases, p-values equal to 0.

[9] The STAR models estimations in OX package (see www.doornik.com) and Finmetrics a S-Plus module.

of STAR nonlinearity. If linearity was rejected, we selected the appropriate transition variable. Then we estimated the parameters in the selected STAR model and evaluated the model. Finally, we modify the model if necessary. Since the parameter γ determines the smoothness of the transition between regimes, a higher value for this parameter is a clear indication for abrupt changes between regimes, and should also be an important source of information about the properties of the models. The results of the estimation of STAR models are reported in table 9 for different values of the transition variable.

			LSTAR Transition		**ESTAR Transition**	
Return	*Lags*	*Transition Variable*	γ	$c-threshold$	$\widetilde{\gamma}$	$c-threshold$
Model A	7	y_{t-1}	0.152	2.455	3.264	-1.063
Model B	26	y_{t-10}	0.267	-1.694	-3.197	0.119
Model C	26	y_{t-17}	0.256	-2.406	-18.757	0.425
Trading Volume						
Model A	9	y_{t-1}	1.211	-104.75	-3.176	83.313
Model B	9	y_{t-2}	0.711	-91.498	-3.386	53.745
Model C	9	y_{t-4}	5.606	117.803	-4.504	-35.194

Table 9. Coefficients from LSTAR and ESTAR Models

As we see from the table, the Logistic STAR estimation for returns shows that in model A, with 7 lags for the dependent variable and y_{t-1}, as transition variable, the transition is smoother around a threshold of 2.45%. In model B, with 26 lags for the dependent variable and y_{t-10} as transition variable, the result are suggesting a faster transition between the two states, with a threshold of -1.6%, while in model C the transition between regimes is similar to model B but with values closer to -2.4%. When the transition function is exponential, the values for the threshold are less fluctuating, ranging from -1.06% to 0.42%, and with a strong transition in the model A case ($\widetilde{\gamma} = 3.26$).[10]

For trading volume the results show a strong aggreement about the optimal lags for the dependent variable (9 lags) in the three models with different transition variables (y_{t-1}, y_{t-2} and y_{t-4}, respectively). In this case, the threshold are highly variable, fluctuating from -104% to 117% for the LSTAR model and from -35% to 83% for the ESTAR model. All coefficients are statistical significants at conventional levels. Figure 6A and 6B show the transition functions versus transition variables for return and trading volume for the three alternative transition variables and transtin functions (Logistic and Exponential STAR) used.

[10] The new parameter $\widetilde{\gamma}$ can be transformed to the original parameter γ as follows: $\gamma = \exp(\widetilde{\gamma})/\sigma_z^2$, where σ_z^2 is the sample variance of the transition variable z_t. The transformation has numerical propierties: its scale free, the new parameter lies in $(-\infty, \infty)$ and is unconstrained; and its a linear function of the logarithm of γ, wich is more dampened than γ.

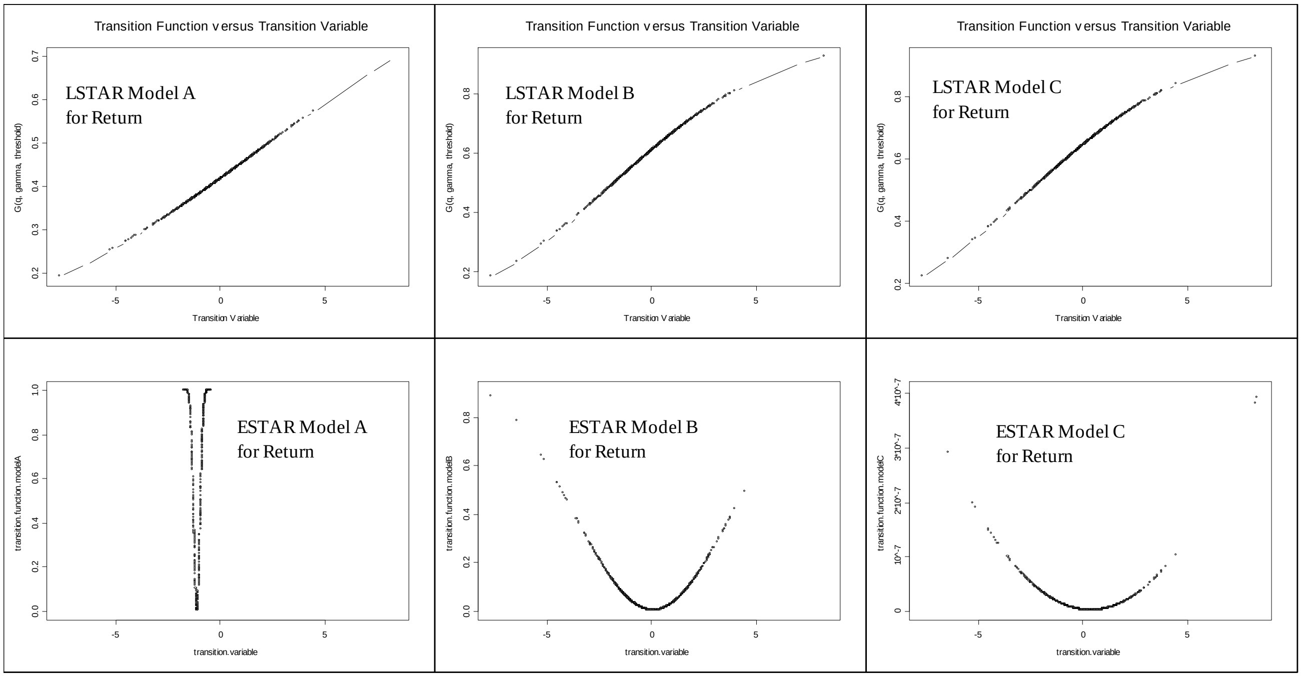

Figure 6A. LSTAR and ESTAR Estimations for Return

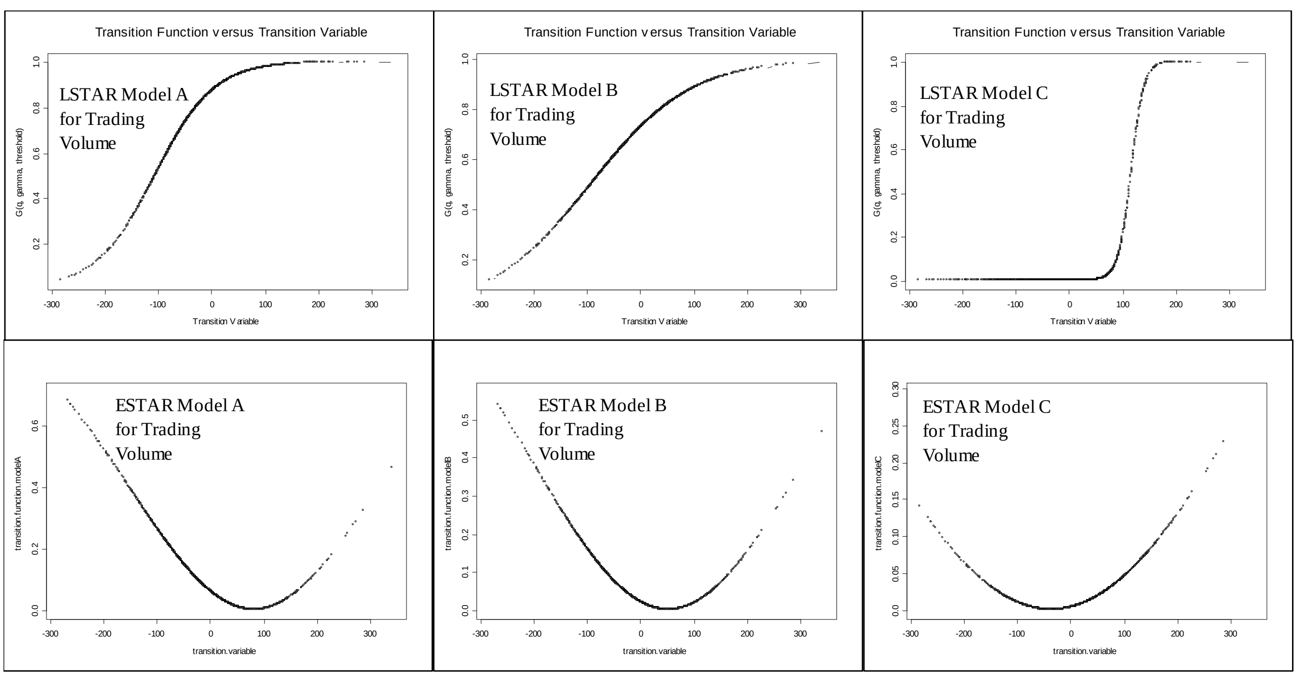

Figure 6B. LSTAR and ESTAR Estimations for Trading Volume

The information provided by the estimation of STAR models is important for our purposes, since they give strong support to the existence of significant nonlinearities both in returns and trading volume. Certainly there is a lot of ways to model nonlinearities (for example, TAR (SETAR) models, STAR, G(ARCH) y Markov chain, besides the whole family of models derived from them), but here we consider the STAR model for practical reasons. First, as an univariate models they can be useful for short run projections with high frequency data without requiring any further information or additional assumptions which would increase the complexity of the analysis. Second, recognized the advantages of a bivariate of a multivariate approaches in modeling, the source of nonlinearity in the data, this task is out of the objectives of this paper. To accomplish this, we would need to look not only at idiosyncratic factors underlying the behavior of the variables, but also at global factors given the strong evidence of co movement in financial markets (see Brooks and Del Negro, 2003; Pindyck and Rotemberg, 1990; among others). Besides, in order to adequately capture the dynamic of returns and trading volume with a model incorporating the additional factors, we need to use data of lower frequency. Third, a model with high frequency data it is of interest for traders and market analyst that periodically follow the Chilean Stock Market because we provide information on threshold values and particular specifications for both variables.

4.4 MS-VAR Estimations[11]

In order to capture the bidirectional causality detected with nonlinear causality tests, we estimate a first order markov switching autorregresions models for both stock index and trading volume. The MS-VAR models allow for a great variety of specifications (see Krolzig, 1998). We estimated three different models: (1) MS-Mean Variance Model: $y_t = \mu(s_t)+u_t$; (2) MS-VAR(p) General Model: $y_t = \beta(s_t)x_t+u_t$; and (3) MS-Switch Intercept Model: $y_t = \mu(s_t) + \beta(s_t)x_t + u_t$. Table 10 present the results for nine models with different characteristics on switch (or not) in variance and/or variance assumption (heteroscedastic). As we can see from the table, model M3 is the best according to the BIC and HQ information criteria. The null of residual normality is strongly rejected in models M2, M4, M6, M7, M8 y M9 but not in model M3. The Figure 7 shows the evolution of returns and the filtered and smoothed probabilities, together with the residuals of the equations for both variables (index returns and trading volume) in the VAR for model M3.

For all nine MS_VAR models estimated the results show that for the year 2007 the process can be characterized by the presence of four clearly identifiable stages in both variables. These models consider two regimes (high and low) which are consistent with the positive cycle displayed by a wide range of common stocks that year in the Chilean stock market. Our results imply that, by jointly modeling the dynamic of trading volume and stock index, we are able to capture the feedback runnig from both variables, as suggested by nonlinear causality tests. Again, this is further evidence favouring a nonlinear modeling approach for the Chilean stock market.

[11] The models were estimated using MSVARlib of Gauss, developed by Benoît Bellone, and are available in http://bellone.ensae.net/download.html.

	Information Criteria		Normality Test
Models	BIC	HQ	[p-values]
M1 = MS-Mean Variance Model	-0.361	-0.398	0.98
M2 = MS-Mean Variance Model (switch variance)	0.200	-0.017	0.00
M3 = MS-VAR(p) General Model	-0.805	-0.843	0.51
M4 = MS-VAR(p) General Model (switch variance)	-0.162	-0.199	0.00
M5 = MS-Switch Intercept Model	-0.794	-0.831	0.16
M6 = MS-Switch Intercept Model (switch variance)	-0.140	-0.178	0.00
M7 = MS-Mean Variance Model (switch variance and heterosc.)	0.020	-0.017	0.00
M8 = MS-VAR(p) General Model (switch variance and heterosc.)	-0.157	-0.194	0.00
M9 = MS-Switch Intercept Model (switch variance and heterosc.)	-0.136	-0.174	0.00

Table 10: Results of MS–VAR Estimations

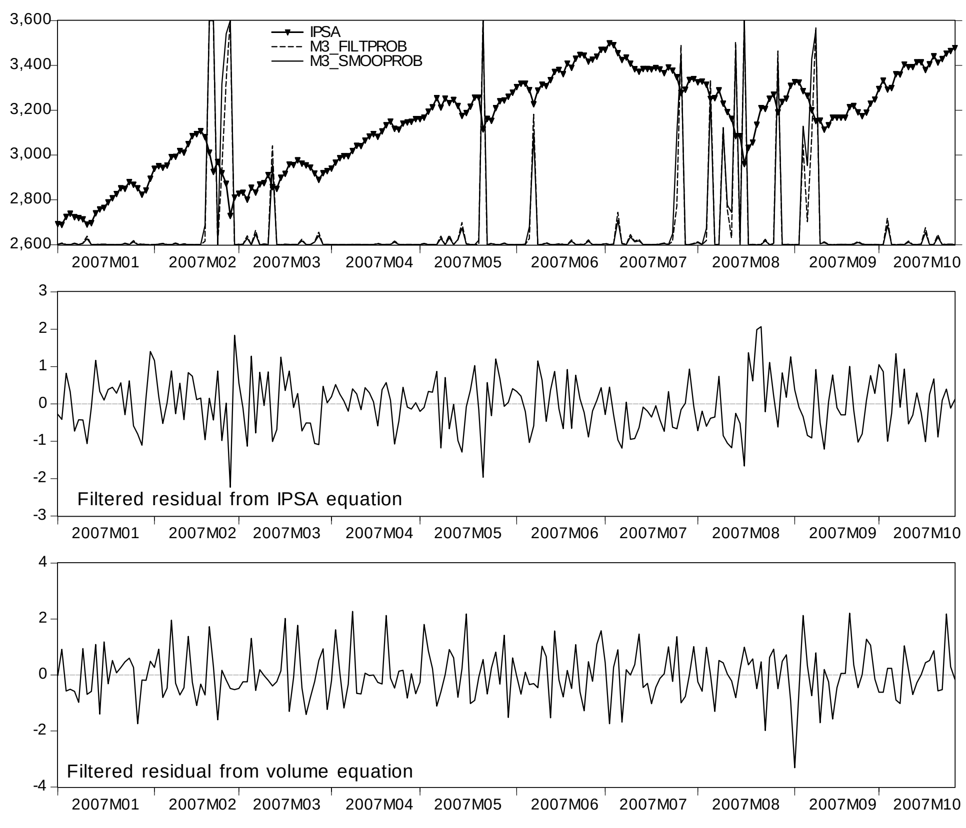

Figure 7. IPSA, Filtered and Smoothed probabilities, and Residuals from Model 3

As we said before, the presence of nonlinearities is a key issue in testing the Efficient Market Hypothesis, according to which stock returns and trading volume should be not related. Previous research on the efficiency using data from the Chilean stock market (see, for example, Solarzando, 1998; Parisi and Acevedo, 2001; Marshall and Walker, 2002; and Zuñiga, 1993), using linear models, support the hypothesis. However, the nonlinear patters we found in the data are a clear signal of misspecification problems in a testing procedure based on a linear approach.

5 Concluding Remarks

In this article we examine the relation between stock index returns and trading volume in an emerging financial market. Using daily data from the Santiago Stock Exchange, the Chilean Stock Market, we first tests for stationarity and linear causality between the stock index returns and trading volume. The results of this linear causality test show that stock returns and volume have no predictive power for one another. However, because of the distribution of the returns and volume series provides some evidence of nonlinear dependence, we formally tests for and finds evidence of significant nonlinearities in the returns and volume series. Besides, our results show evidence of bidirectional nonlinear causality between both variables. Given that, we further estimate both univariate and multivariate nonlinear models in order to capture these nonlinearities. To our knowledge, this is the first time that these issues are addressed for the Chilean Stock Market.

The results of nonlinear causality tests reported in this article are consistent with te predictions of more than one of the competing explanatios for the presence of a causal relationship between stock returns and trading volume. For example, causality from trading volume to stock returns is consistent with the sequential information arrival models and the mixture of distribution model. Also, a significant causal relationship from stock returns to trading volume is implied by the noise trading model.

The finding of a significant nonlinear causal relationship between price variability and trading volume can be of interest to market regulators, as they decide on the effectiveness or the appropriateness of market restrictions. The results also have some practical implications for traders and market analysts, because the strong nonlinear causal relationships between stock index returns and trading volume implies that knowledge of current trading volume improves the ability to forecats future returns. This improvement of short-term return predictability should lead to the construction of more accurate hedge ratios and improvements in investment strategies.

In terms of the implications for the Efficient Market Hypothesis, the fact that lagged trading volume contains information useful for predicting stock market returns may imply a degree of inefficiency in the Chilean Stock Market. Such inefficiency may be caused by some sort of consensus between traders in condition their prices on the trading patterns of other traders or in previous day's trading volume as a measure of the market consensus.

The next step is to model the joint distribution of stock returns and trading volume with nonlinear multivariate models using alternative variables as a threshold in the nonlinear models. By including these variables we would be able to analyze, for example, any potential influence of macroeconomic and financial factors in explaining this nonlinear joint dynamic of stock returns and trading volume and to test for the efficiency of the stock market. Given that macroeconomic variables are unavailable at higher frequency; the analysis should be carry out using low frequency (monthly) data. This line of research is explored in an upcoming paper.

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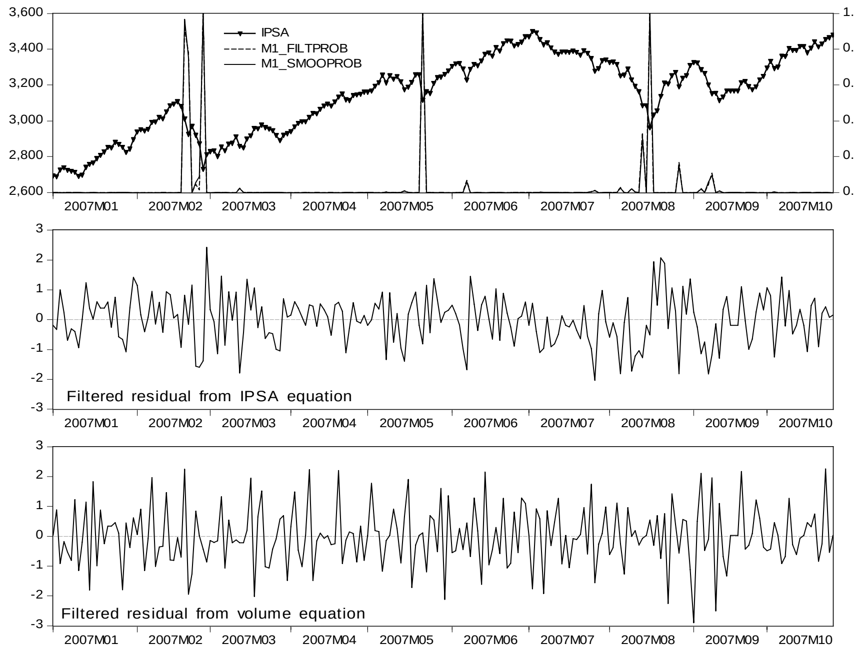
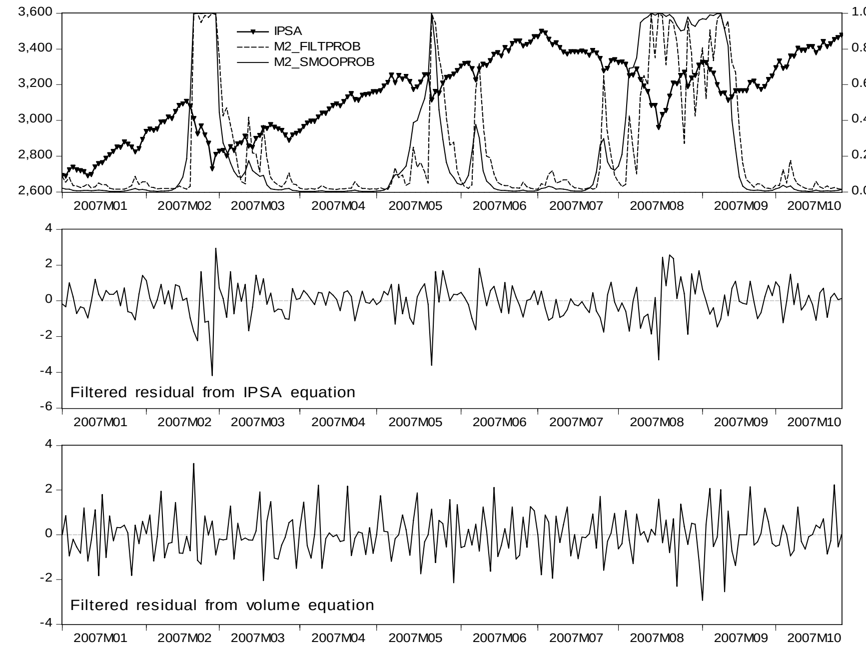
Appendix

Figure A1. Filtered and Smoothed probabilities

Panel A. Model 1 (*)



Panel B. Model 2 (**)



Source: Own Elaboration

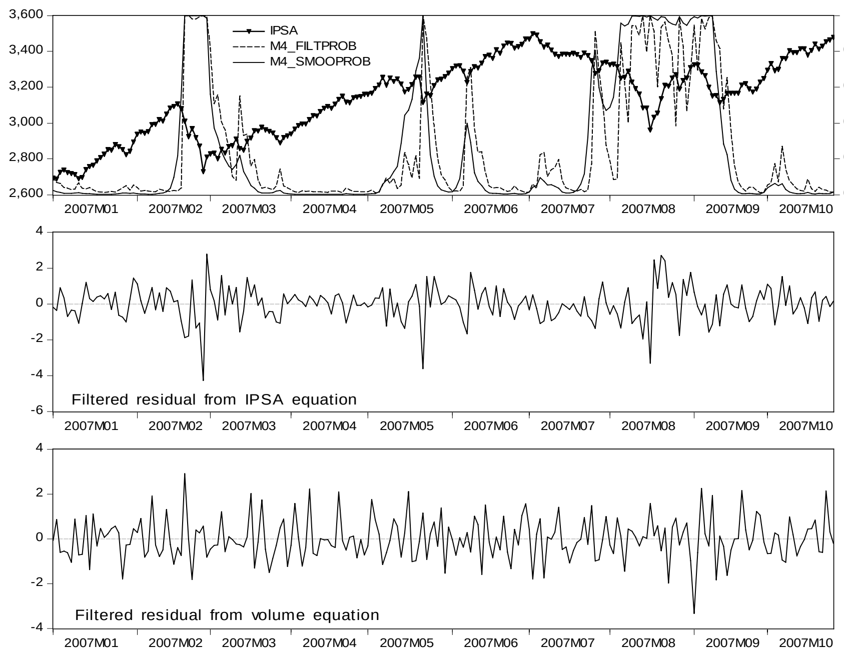
<u>Notes</u>:

* Mean Variance MS-VAR(1) model with two feasible regimes (high and low) and full variance

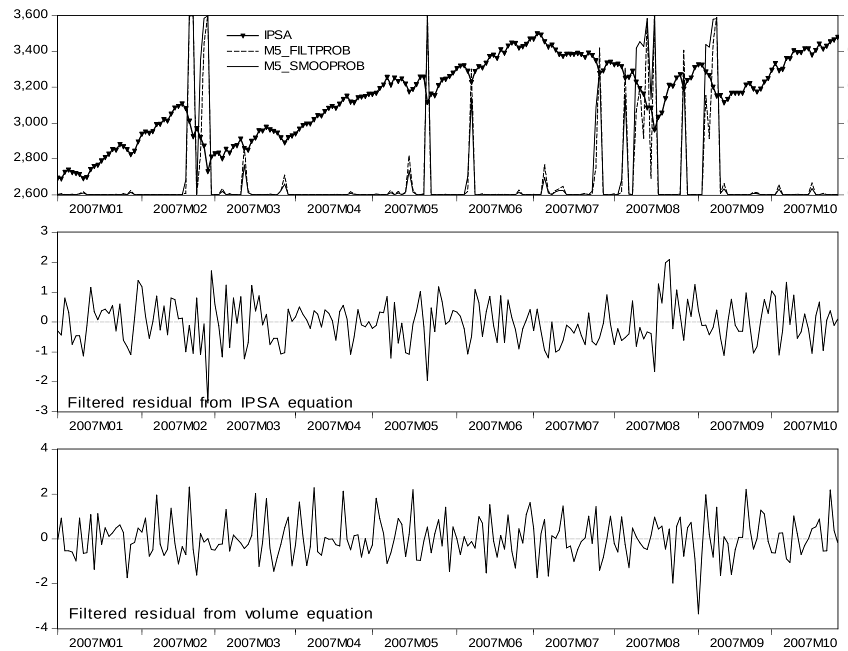
** Mean Variance MS-VAR(1) model with two feasible regimes (high and low), switch in variance and full variance

Figure A2. Filtered and Smoothed probabilities

Panel A. Model 4 (*)



Panel B. Model 5 (**)



Source: Own Elaboration

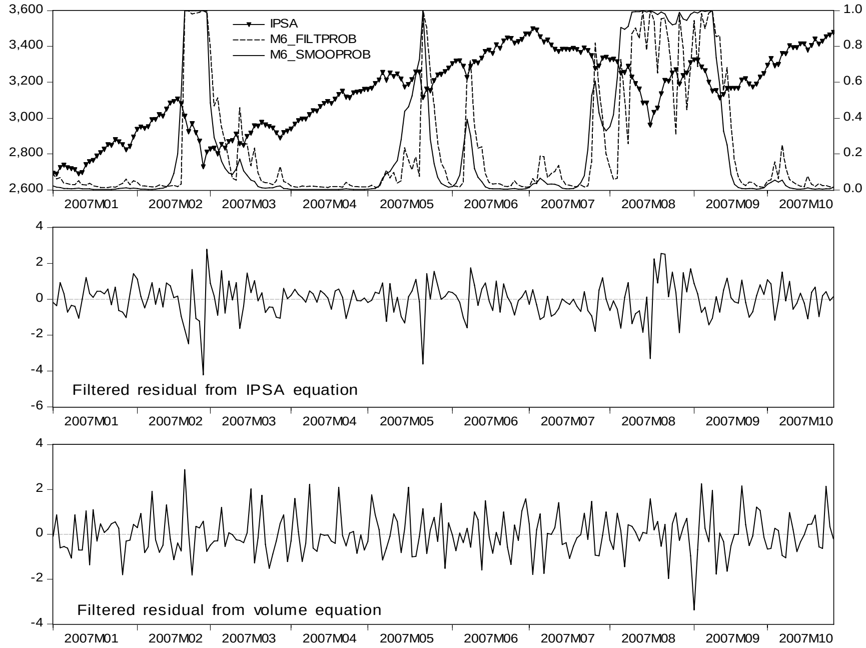
Notes:

* General MS-VAR(1) model with two feasible regimes (high and low), switch in variance and full variance

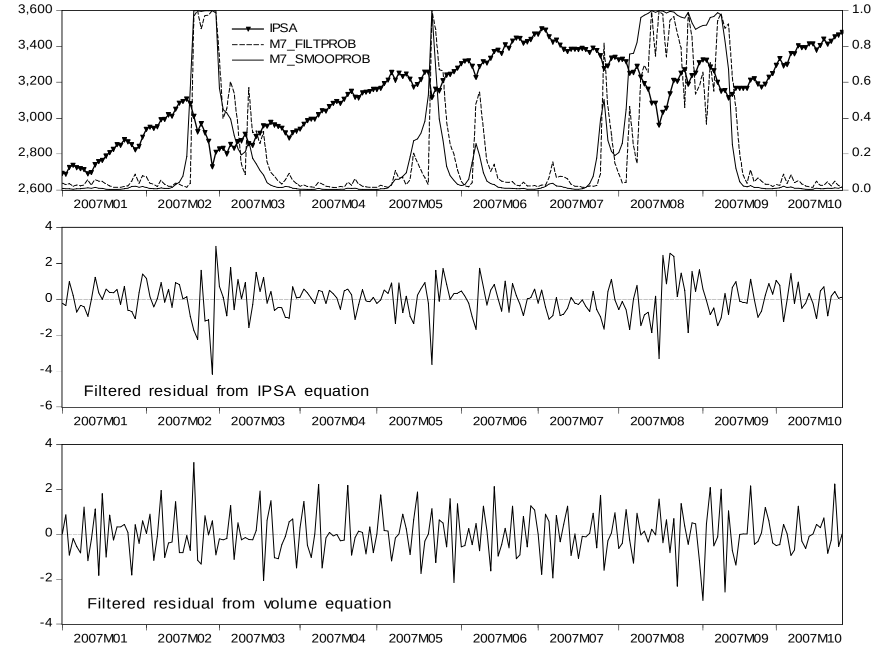
** Switching Intercept MS-VAR(1) model with two feasible regimes (high and low) and full variance

Figure A3. Filtered and Smoothed probabilities

Panel A. Model 6 (*)



Panel B. Model 7 (**)



Source: Own Elaboration

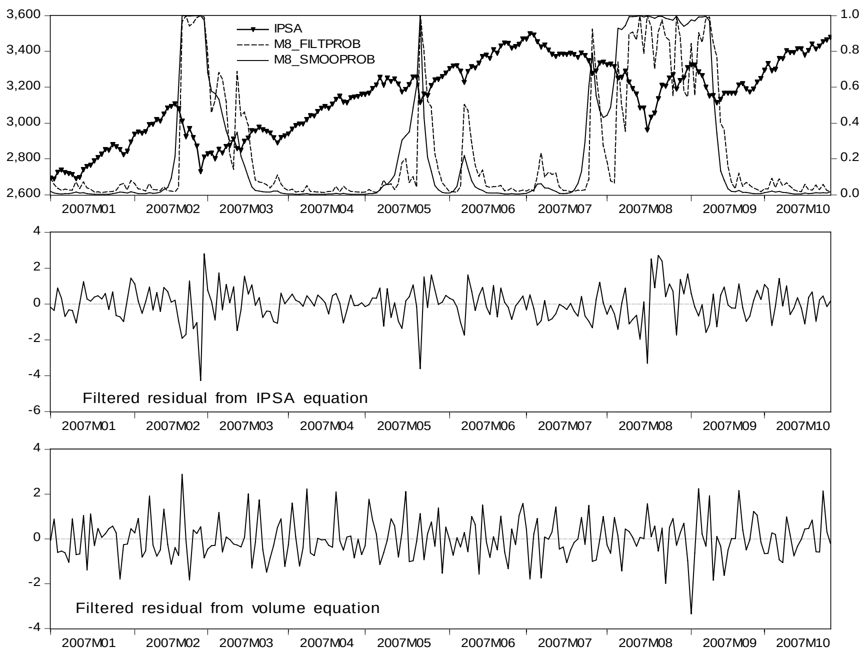
<u>Notes</u>:

* Switching Intercept MS-VAR(1) model with two feasible regimes (high and low), switching in variance and full variance

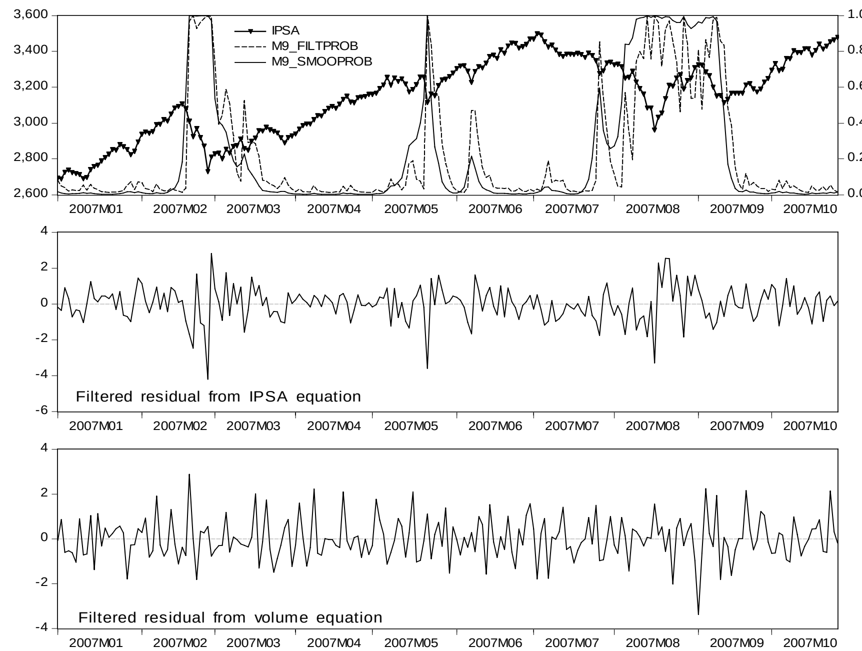
** Mean Variance MS-VAR(1) model with two feasible regimes (high and low), switching in variance and heteroscedastic

Figure A4. Filtered and Smoothed probabilities

Panel A. Model 8 (*)



Panel B. Model 9 (**)



Source: Own Elaboration

<u>Notes</u>:

* General MS-VAR(1) model with two feasible regimes (high and low), switching in variance and heteroscedastic

** Switching Intercept MS-VAR(1) model with two feasible regimes (high and low), switching in variance and heteroscedastic

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