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Working Papers

N° 474

Junio 2008

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Documento de Trabajo **N° 474**

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DEPOSIT INSURANCE, MORAL HAZARD AND THE RISK OF RUNS

Nancy Silva
Gerencia Estabilidad Financiera
Banco Central de Chile y
London School of Economics

Resumen

La efectividad de los sistemas de garantías de depósito para eliminar corridas bancarias varía con el tamaño del seguro, y con el nivel de responsabilidad en la supervisión de la banca de la agencia a cargo de administrar esta garantía. Cuando la agencia no tiene responsabilidades supervisoras, una garantía parcial preserva el rol de monitoreo de los depositantes y reduce la ocurrencia de corridas bancarias, pero no las elimina por completo. Cuando la agencia se involucra en labores de supervisión, las corridas van desapareciendo a medida que la información del regulador se vuelve más precisa. Sin embargo, mientras menor es la protección ofrecida a los depositantes, menos bancos insolventes son intervenidos. Las garantías de depósito inducen riesgo moral, al hacer aumentar el retorno de los depósitos a la vista, aunque este efecto parece ser menor mientras más amplio es el mandato de la agencia de seguros. Por lo tanto, las economías deberían preferir esquemas en que la agencia de seguro de depósitos tiene un mayor grado de responsabilidad en la supervisión de la banca.

Abstract

The effectiveness of deposit insurance in eliminating panic runs varies with the size of coverage and the degree of supervisory involvement of the agency in charge of insurance. When the agency is not involved in the supervision of banks, partial insurance preserves the monitoring role of depositors and reduces the region for which runs occur, but it is unable of completely eliminating them. When the agency has a high degree of supervisory involvement, even with partial insurance panic runs disappear as the regulator's signal becomes more precise. However, the smaller the protection offered to depositors, the higher is forbearance. Deposit insurance induces moral hazard by increasing the equilibrium value of the demand deposit contract in the interim period, though this effect seems to be smaller under a broad mandate. Therefore, a scheme where the insurance agency has more supervisory involvement should be preferred.

The author is thankful to Viral Acharya, Charles Goodhart and Margaret Bray for their insightful comments and suggestions; and to Steven Seelig, for his empirical examples motivated this research.
E-mail: nsilvas@bcentral.cl.

1. INTRODUCTION

Bank runs cause real economic problems. General activity typically declines substantially during panic runs, as the payment system is suspended and productive investment is terminated due to the unwillingness to make new loans.

Panics runs were frequent in the United States during the National Banking Era (1864-1913), with an average seasonal pattern of 5 years (Miron, 1986). The introduction of deposit insurance and regulation during the 1930's has been regarded by many authors and policymakers as one of the main causes of the decrease in the rate of bank failures in that country (Friedman and Schwartz, 1963; Diamond and Dybvig, 1983; Williamson, 1995). Although this formula was not imitated outside the USA until the 1960's (initially in India and later in Europe); since the 1980's, with the incidence of escalating banking crises, financial stability and consumer protection concerns has led to the widespread establishment of explicit limited deposit guarantees.[1]

Nevertheless, experience has demonstrated that limited insurance is not sufficient to protect banks from runs in a weakened financial system. In the last two decades, at least twelve countries[2] have temporarily extended explicit full coverage during times of serious financial distress. Some of them did not even have an explicit system of deposit insurance before their crises.

The success of blanket guarantees in stopping bank runs has been mixed. Funding constraints and macroeconomic stability appear to be key limitations to their effectiveness. In Norway, after the failure of the three largest banks nearly depleted the capital of the private insurance fund, runs were out of control and the government was forced to establish a public insurance fund, which ended by controlling about 85 percent of commercial bank assets by the end of 1991 (Ongena et al, 2000). A blanket guarantee was also introduced in Ecuador in December 1999, after runs in the largest bank of the country started to spread to other institutions. After a long and painful process of deposit freezing , the blanket guarantee worked for a time, until the authorities decided to default on the external debt and unilaterally reschedule the domestic debt. This prompted a three-pronged debt/currency/banking crisis, which no deposit guarantee could have stopped.

Although deposit insurance is a popular tool among policymakers, even partial protection is a controversial issue among economists. Many authors agree that deposit insurance is a source of moral hazard, that by reducing the incentives of depositors to monitor their banks it damages financial stability by encouraging risk-taking. Deposit insurance can indeed be very costly, that cost being typically born by taxpayers. For example, the USA Savings and Loan Crises (1986-1995) involved a loss of US$153 billion, of which US$124 billion were born by US taxpayers. In the 1980's the insurance limit was raised from US$40,000 to US$100,000; encouraging depositors to continue funding an already risky industry (which was reflected on raising interest rates). According to Jameson (2003), however, deposit insurance was not the only responsible for this collapse, as loose regulation on Savings and Loans activity meant that institutions were able to use these new funds to gamble their way into profit.

A successful guarantee must then be accompanied by efficient regulation, in order to prevent the negative effect of moral hazard on financial stability. In an empirical work Demirguç-Kunt and Detragiache (1999) find that explicit deposit insurance in-

[1] In a survey conducted by the IMF and the World Bank among 85 different systems of deposit insurance, 67 countries were offering an explicit and limited deposit guarantee in normal times, with varying types of funding (ex-ante, ex-post), membership (compulsory in all but seven countries) and mandate across economies (see Demirguç-Kunt and Sobaci, 2000).

[2] Ecuador, Finland, Honduras, Indonesia, Japan, Korea, Malaysia, Mexico, Nicaragua, Norway, Thailand and Turkey.

creases the vulnerability of the banking system, particularly when the coverage is more extensive, as its presence tends to make economies more vulnerable to raises in real interest rates, exchange rate depreciation, and to runs triggered by currency crises. However, good institutions (used as an estimate of a good regulatory environment) perform an important role in curbing this negative effect.

Diamond and Dybvig (1983) were the first to propose deposit insurance as a mechanism to stop inefficient runs on solvent banks, in a model with perfect information and deterministic returns. Runs in this multiple equilibrium model are a consequence of coordination failure among depositors, and turn out to be a self-fulfilling prophecy. With the introduction of insurance the "bad" sunspot equilibrium (runs) is always eliminated, and the policy becomes costless as it is never used in equilibrium. Since then, and despite the limited predictive capability of this model, authors have taken for granted the power of deposit insurance as an instrument for eliminating inefficient runs, and have concentrated instead on studying the problem of pricing of insurance and its effect on banks' moral hazard.

This paper will come back to the study of the effects that deposit insurance has on the equilibrium behaviour of depositors and banks, while abstracting from the problem of insurance pricing. In particular, I want to consider whether the empirical findings described above can be supported by this model. I will consider Goldstein and Pauzner's (2000) model of information based bank runs, where private information allows for a unique equilibrium. I will show that while consumers achieve better risk sharing in a competitive banking system than in autarky, more solvent projects are liquidated as uninsured depositors fail to coordinate in a subset of fundamentals, and run on banks they know to be solvent. When introducing deposit insurance, I show that its effectiveness in eliminating panic runs varies with the size of coverage and the degree of supervisory involvement of the agency in charge of insurance. Under a narrow mandate (when the agency is not involved in the supervision of banks), a deposit insurance contract preserving the monitoring role of depositors involves offering less than full protection. The trade off is that panic runs cannot be completely eliminated with a partial guarantee, although it does reduce the region of fundamentals for which that occurs. Under a broad mandate (with a high degree of supervisory involvement), I show that panic runs tend to disappear for any level of insurance as the regulator's signal becomes more precise, given that liquidity assistance is committed to solvent but illiquid banks. Moreover, it is cost efficient never to provide liquidity to insolvent banks. However, only extremely insolvent banks are closed, and those with enough funds to cover the payment of the final period guarantee are allowed to continue in operation. Therefore, the smaller the protection offered to depositors, the higher is forbearance. All these results hold irrespective of the specific values of the guarantee, which in particular might imply the social cost of deposit insurance to be lower under a broad mandate. Finally, I show that deposit insurance induces moral hazard by increasing the equilibrium value of the demand deposit contract in the interim period, but this effect seems also to be smaller under a broad mandate. Limited insurance can contain moral hazard up to some extent, justifying the observed conduct of governments across the world in normal times.

Given the combination of these results, and the empirical evidence provided by other authors, a scheme where the DIC has more supervisory involvement (broad mandate) or a high degree of coordination with the supervisory authority should be preferred.

The paper is organised as follows. Section 2 introduces the benchmark model of information based deposit runs, as developed by Goldstein and Pauzner (2000). Deposit insurance is justified because of the inefficient liquidation of solvent banks in equilibrium. Section 3 introduces deposit insurance under two possible mandates for the insurer. The equilibrium under a narrow mandate is discussed in section 4, and that under a broad mandate, in section 5. Section 6 solves for the optimal demand deposit contract offered by banks, and compares moral hazard under the two mandates. Policy implication and possible extensions are discussed in section 7. Finally, conclusions are given in section 8.

2. A MODEL OF INFORMATION-BASED BANK RUNS

One of the simpler and better known models explaining the inherent fragility associated to the banking system belongs to Diamond and Dybvig (1983). The maturity mismatch between long-term loans financed with short-term deposits exposes banks to the risk of runs. Crucially, public information on the quality of a bank's investment portfolio leads to multiple equilibria, one of which involves coordination failure among depositors, who run on a solvent bank solely because they fear other depositors will do the same. As a result, the probability of runs is undetermined in this model, seriously limiting its usefulness as an instrument to evaluate policies designed to reduce banking fragility.

Goldstein and Pauzner (2000) modify this model, using global games' techniques.[3] By replacing common knowledge on the bank's fundamentals by noisy private signals received by depositors, a unique equilibrium emerges in which fundamentals act as a mechanism to coordinate agent's beliefs towards a more efficient outcome.

Consider an economy with three periods $t \in \{0, 1, 2\}$ and a perfectly competitive banking industry, where all banks have access to the same two investment technologies at the planning period ($t = 0$). Banks are risk neutral and decide whether to invest in a liquid technology, returning 1 unit of consumption at $t + 1$ per unit invested at t; or in a stochastic, long-term, partially illiquid technology, returning 1 if liquidated in the interim period ($t = 1$), and R if liquidated at $t = 2$. The long-term return function, $R = R(\theta)$, is a continuous and increasing function of a random variable θ, uniformly distributed in the interval $[0, 1]$, that represents underlying fundamentals of the projects financed by a bank. After receiving deposits, banks decide which project to invest in. Assuming that $E[R(\theta)] > 1$, investment in the risky project is superior to storage and, therefore, all resources are pulled on it. Because the zero profit condition implies all banks will offer exactly the same contract, it is possible to restrict the analysis to one representative bank.[4]

A continuum of mass one consumers receive 1 unit of endowment –let say money– at $t = 0$, that they invest in the representative bank, which offers a demand deposit contract (c_1, c_2).[5] Depositors are risk averse, with preferences represented by a concave and increasing utility function, $u(c_1, c_2)$, with coefficient of relative risk aversion higher than 1. Depositors are uncertain about their time of consumption. With probability $1 - \pi$ a depositor is patient, meaning she enjoys consumption only at $t = 2$. With complementary probability, π, she is impatient and consumes only in the interim

[3] Global games, first studied by Carlsson and van Damme (1993), are games of incomplete information where players observe noisy signals of an uncertain underlying economic state or fundamental, which determines the payoffs of the game. For a review of the theory see Morris and Shin (2002).

[4] Assume depositors cannot invest in more than one bank, so the contract that one bank offers does not affect the payoffs of depositors on a different bank.

[5] Every bank in this economy is ex-ante identical, therefore it is possible to normalise the size of the representative bank to 1 (there is no equity at $t = 0$). Limiting the analysis to demand deposit contracts is a standard assumption in the literature, not restrictive, because these contracts are effectively observed in banks (Alonso, 1996; Gale, 2000). This assumption, however, implies that this model does not solve for the optimal contractual form.

period.[6] At $t = 1$, types are privately realised and all impatient depositors withdraw to consume. Patient depositors evaluate the expected payoff at the final period, conditional on their beliefs on the response of their counterparts, and decide whether to withdraw or to remain.

Let n be the total number of withdrawals at $t = 1$, so $n - \pi$ is the number of patient depositors withdrawing in the interim period ($n \in [\pi, 1]$). If $n > 1/c_1$, the bank does not have enough resources to pay the promised value of deposits, c_1, and it is liquidated. Thus, each consumer demanding early withdrawal receives $1/n$ and those waiting until the second period receive 0. On the other hand, if $n \leq 1/c_1$, the bank survives to the final period and all depositors demanding early withdrawal receive c_1. For simplicity, assume that the bank is always liquidated at $t = 2$, and because there was no equity in the initial period, remaining customers equally share the value of final assets, i.e. $c_2(\theta, n) = \dfrac{(1 - nc_1)R(\theta)}{(1 - n)}$.[7]

Naturally, $1 \leq c_1 \leq 1/\pi$, otherwise depositors would prefer not to invest in the bank (first inequality), or runs would be triggered by the demand of impatient depositors alone (second inequality).

Patient depositors' payoffs are summarised on the following table:

	$n \leq 1/c_1$	$n > 1/c_1$
$t = 1$	c_1	$1/n$
$t = 2$	$c_2(\theta, n)$	0

TABLE 1: Patient depositors' payoffs in the game without insurance.

At the beginning of period 1, each depositor receives a private, non-verifiable signal on the true value of the fundamental, $\theta_i = \theta + \varepsilon_i$; where ε_i are i.i.d. random variables, uniformly distributed in the interval $[-\varepsilon, \varepsilon]$. This distributional assumption implies that signals are equally informative among depositors.[8]

If a consumer were sure that $\theta < \theta_L$ –where θ_L is defined as the solution to $c_2(\theta_L, \pi) = c_1$–[9] withdrawal would be a strictly dominant strategy, no matter the value of n (see table 1). In the present model, a consumer knows that $\theta < \theta_L$ if her signal satisfies $\theta_i < \theta_L - \varepsilon$, and every consumer receive signals below this level if $\theta < \theta_L - 2\varepsilon$. Hence, the interval $[0, \theta_L - 2\varepsilon]$ is the *lower dominance region,* where all patient depositors withdraw independent of the actions of other players. On the other hand, if a consumer were sure that $\theta = 1$, she would know the bank's return to be at its highest possible level and, therefore, she should prefer to remain. By continuity of the payoff function, there exists θ_U such that if $\theta_i > \theta_U + \varepsilon$ a patient depositor remains, and every consumer receive signals above this level if $\theta > \theta_U + 2\varepsilon$. The interval $[\theta_U + 2\varepsilon, 1]$ is the *upper dominance region*, where patient depositors always remain.[10]

[6] With no discounting, the utility of impatient agents is $u(c_1, c_2) = u(c_1)$, while that of patient depositors is simply $u(c_1, c_2) = u(c_2)$.

[7] This simplification becomes natural under the assumption of perfect competition in an economy with a finite planning horizon. In practical terms, it means that in the end the bank actually behaves as a mutual fund.

[8] The uniform distributional assumption is consistent with the Laplacian "*principle of insufficient reason*" –that one should apply a uniform prior to unknown events–, because it implies that around the switching point the number of agents remaining or withdrawing are uniformly distributed. When the payoff of a dominant action is increasing in the true value of the fundamentals (action monotonicity), Morris and Shin (2002) show that this action coincides with the equilibrium action. Action monotonicity is satisfied by the payoffs of the present model.

[9] Patient depositors wait if $E[u(c_2(\theta, \pi))] \geq u(c_1)$ which, given that the utility function is concave and increasing, implies $E[c_2(\theta, \pi)] \geq c_1$. As for very low realizations of θ, $c_2(\theta, \pi) < c_1$, there must exist $\theta = \theta_L$ such that $c_2(\theta_L, \pi) = c_1$.

[10] The existence of the upper dominance region is not directly implied by the payoffs of this game,

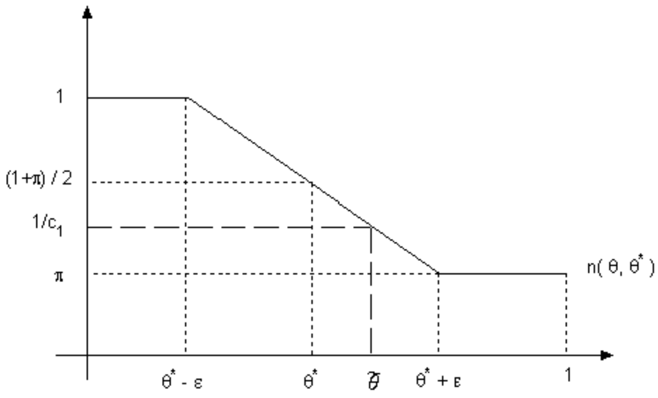

FIG. 1 Number of total early withdrawals.

Goldstein and Pauzner (2000) concentrate on "*equilibrium on switching strategies*", this is, an equilibrium in monotone strategies with threshold θ^*, such that if $\theta_i < \theta^*$ patient depositors withdraw, and remain if $\theta_i > \theta^*$. Indeed, this type of solution turns out to be the only equilibrium of the game.[11]

After receiving a signal θ_i, depositor i knows that the true value of θ lies in the interval $[\max\{\theta_i - \varepsilon, 0\}, \min\{\theta_i + \varepsilon, 1\}]$. With incomplete information, depositors must condition their beliefs upon their private signals, which are positively correlated with the private signals of others. With every patient depositor following the same equilibrium strategy, a consumer rationally anticipates that if $\forall i\ \theta_i < \theta^*$, everybody will withdraw. This will be the case for values of $\theta < \theta^* - \varepsilon$. In the same way, a consumer knows that all patient depositors will wait if $\forall i\ \theta_i > \theta^*$, which is always the case if $\theta > \theta^* + \varepsilon$. Finally, and because of the uniform distributional assumption on ε_i, in the intermediate region a rational player will assign a uniform distribution to her beliefs on the number of patient depositors withdrawing early. Hence, when the equilibrium threshold is θ^*, the number of early withdrawals, n, will be given by the following non-increasing function of θ (figure 1):

$$n(\theta, \theta^*) = \begin{cases} 1 & if \quad \theta < \theta^* - \varepsilon \\ \dfrac{1+\pi}{2} - \dfrac{(1-\pi)(\theta - \theta^*)}{2\varepsilon} & if \quad \theta^* - \varepsilon \leq \theta \leq \theta^* + \varepsilon \\ \pi & if \quad \theta > \theta^* + \varepsilon \end{cases} \tag{1}$$

as it was for the case of the lower dominance region. If the signal is very high, patient depositors remain provided that other depositors wait as well, and enough of them for the long-term technology not to be completely liquidated in the interim period. Alternative explanations could justify this behaviour. For example, Dasgupta (2002) proposes that when very high returns are guaranteed, a bank becomes an attractive target for potential purchase by a larger, more liquid bank, which would make it optimal for patient depositors to wait. Alternatively, for very high signal banks the supervisory authority could be willing to act as a LoLR (an explanation that will become natural later on, when studying the case with insurance under a broad mandate), rescuing them when facing a liquidity shock. Anticipating that, patient depositors should remain.

[11] Making strong use of the uniform distributional assumption on the noise, Goldstein and Pauzner (2000) show that for any feasible belief $n(\theta)$, the regions where $\Delta(\theta_i, n(\theta)) \leq 0$ and $\Delta(\theta_i, n(\theta)) > 0$ are complementary connected intervals, and therefore any equilibrium of the game must be monotone. Dasgupta (2002) extends this result to general distributional assumptions on $n(\theta)$, and show that, for this game, there are no non-monotone equilibria in the set of all feasible beliefs over the actions of other agents.

In deciding whether to withdraw or to remain, a depositor must evaluate the (conditional) expected utility of these two actions. Denoting by $\delta(\theta,\theta^*)$ the difference of utilities between waiting until $t=2$ and withdrawing at $t=1$:

$$\delta(\theta,\theta^*) \doteq \delta(\theta,n(\theta,\theta^*)) = \begin{cases} u\left(c_2(\theta,n)\right) - u(c_1) & \text{if } n(\theta,\theta^*) \leq 1/c_1 \\ u(0) - u(1/n) & \text{if } n(\theta,\theta^*) > 1/c_1 \end{cases},$$

each consumer evaluates:

$$\Delta(\theta_i,\theta^*) \doteq E_{\theta_i}[\delta(\theta,\theta^*)] = \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \tfrac{1}{2\varepsilon}\delta(\theta,\theta^*)d\theta.$$

This means that upon receiving a signal θ_i, if a patient depositor's conditional expected utility of remaining is higher than the utility of withdrawing, this is if $\Delta(\theta_i,\theta^*) > 0$, she will wait to withdraw in the final period. Otherwise, if $\Delta(\theta_i,\theta^*) < 0$, she will quit the bank at $t=1$. Finally, if $\Delta(\theta_i,\theta^*) = 0$ a depositor will be indifferent between the two actions.

Theorem 1 (Goldstein and Pauzner, 2000). *There exists a unique equilibrium threshold θ^* in the interval $]\theta_L-\varepsilon,\theta_U+\varepsilon[$ satisfying $\Delta(\theta^*,\theta^*)=0$, $\Delta(\theta_i,\theta^*)<0$ for all $\theta_i<\theta^*$, and $\Delta(\theta_i,\theta^*)>0$ for all $\theta_i>\theta^*$. Moreover, $\theta^*(c_1)$ is increasing in c_1.*

The equilibrium threshold can be computed as the solution to[12]

$$\int_{\pi}^{1/c_1} u\left(\frac{1-nc_1}{1-n}R\left(\theta^*+\frac{\varepsilon}{1-\pi}(1+\pi-2n)\right)\right)dn = \int_{\pi}^{1/c_1} u(c_1)dn + \int_{1/c_1}^{1}\{u(1/n)-u(0)\}\,dn \quad (2)$$

The existence of the equilibrium comes from the continuity of $\Delta(\theta_i,\theta^*)$ in both arguments, the dominance regions, and the monotonicity of $\Delta(\theta_i,\theta_i)$ as a function of one variable. That $\theta^*(c_1)$ is increasing in c_1 can be intuitively justified. If the payment in the interim period increases, more of the risky project has to be liquidated to pay early withdrawals. Therefore, the incentive for patient depositors to wait should decrease, both because the expected payoff at $t=2$ is lower, and because each agent assigns a higher probability to the event of a run.[13]

Proposition 1. *The probability of a bank run is equal to $\theta^*+\dfrac{\varepsilon}{1-\pi}\left(1+\pi-\dfrac{2}{c_1}\right)$, and it is increasing in the level of risk sharing offered by the demand deposit contract (c_1).*

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[12] Using $\theta(n) = \theta^* + \dfrac{\varepsilon}{1-\pi}(1+\pi-2n)$, the inverse function of $n(\theta,\theta^*)$ in the region $[\theta^*-\varepsilon,\theta^*+\varepsilon]$ to change variables, and rearranging terms:

$$\Delta(\theta^*,\theta^*) = \int_{\theta^*-\varepsilon}^{\widetilde{\theta}} \{u(0)-u(1/n)\}\,d\theta + \int_{\widetilde{\theta}}^{\theta^*+\varepsilon} \{u(c_2(\theta,n))-u(c_1)\}\,d\theta = 0.$$

$$\Leftrightarrow \int_{1/c_1}^{1} \{u(1/n)-u(0)\}\,dn = \int_{\pi}^{1/c_1} \{u(c_2(\theta(n)))-u(c_1)\}\,dn.$$

[13] A formal proof can be found in Goldstein and Pauzner (2000), and a simpler version in Dasgupta (2002).

See proof in the appendix.

A higher c_1 represents a gain in risk sharing, as more resources from the final period are passed to early consumers. However, it also implies an increase in the probability of runs. Goldstein and Pauzner (2000) prove that $c_1 > 1$ provided that the probability of $R(\theta) < 1$ is small enough.

2.1. Inefficient Liquidation

Private information allows for the coordination of depositors' actions, in such a way that bank runs are avoided for sufficiently high values of the fundamentals (when $\theta > \theta^* + \varepsilon$). A natural question is then whether the equilibrium behaviour of depositors is desirable in terms of financial stability. Is it possible that panic runs persist for a region of the fundamental, such that solvent banks can be liquidated? I will show in this section that depositors will still fail to coordinate in a subset of the fundamental, and run on banks they know to be solvent.

Assume for a moment that both the safe and risky technologies are available to depositors for direct investment at $t = 0$. Also assume that $E[u(R(\theta))] > u(1)$, so that depositors invest all their resources in the long-term risky project. At $t = 1$ all impatient depositors withdraw. Suppose that patient depositors still receive private signals. Risk aversion implies that if $E_{\theta_i}[u(R(\theta))] \leq u(1)$ they should liquidate the project at $t = 1$, while if $E_{\theta_i}[u(R(\theta))] > u(1)$ they should hold it to the final period. Thus, in *autarky*, if a consumer evaluates the project to be solvent she should wait and liquidate it in the final period.

In an intermediated system, both the definition of solvency and the behaviour of depositors will clearly depend on the promised value of deposits at $t = 1$.

Definition 1. When a bank offers a demand deposit contract paying $c_1 \geq 1$ in the interim period, the bank is said to be **fundamentally solvent** if $c_2(\theta, \pi) \geq c_1$, this is, if $\theta \geq \theta_L$.

A bank is fundamentally solvent if when only impatient consumers withdraw, the payoff at $t = 2$ is at least as good as the maximum certain payoff at $t = 1$. According to this definition, solvency is a property that cannot be verified in the interim period. No player in this game (not even the bank itself) is able to observe the true value of θ until $t = 2$. However, depositors observing signals $\theta_i > \theta_L + \varepsilon$ can be sure that the bank is solvent. Hence, if $\theta > \theta_L + 2\varepsilon$, everybody receive signals above $\theta_L + \varepsilon$ and all patient depositors know the bank is solvent. Is it then possible for solvent banks to go bankrupt in this model? Or put differently, is it possible that in equilibrium $\theta^* - \varepsilon > \theta_L + 2\varepsilon$, so that for certain values of θ all depositors run on a solvent bank?

Notice that if $c_1 = 1$, the solvency criteria would be the same as in autarky, and if $\varepsilon \to 0$ there would be no pure panic runs in equilibrium. Taking limit when $\varepsilon \to 0$ in equation 2, we obtain $u(R(\theta^*)) = u(1)$, which implies that $\theta^*(1) = \theta_L(1)$. That is, when the noise is negligible and the contract offers the value of liquidation of the project at $t = 1$, pure panic runs are eliminated (indeed, even if the noise were not negligible, partial runs do not occur because $\forall \varepsilon > 0$, $\widetilde{\theta}(1) = \theta^*(1) - \varepsilon$). However, as no risk sharing is offered, this contract does not improve on the autarkic solution.

For the case $c_1 > 1$, take limit as ε goes to zero in equation 2:

$$\int_{\pi}^{1/c_1} u\left(\frac{1 - nc_1}{1 - n} R(\theta^*)\right) dn = \int_{\pi}^{1/c_1} u(c_1) dn + \int_{1/c_1}^{1} \underbrace{\{u(1/n) - u(0)\}}_{>0} dn$$

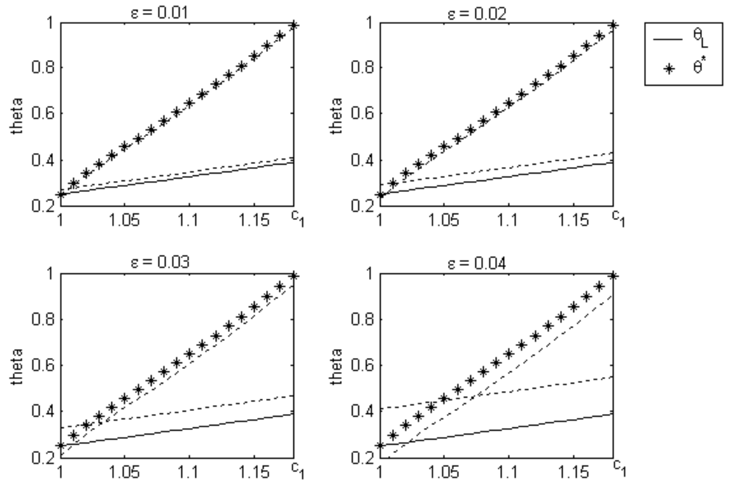

FIG. 2 Equilibrium and solvency thresholds without insurance.

If θ^* were equal to θ_L then $c_2(\theta^*, \pi) = c_1$, but as $c_2(\theta, n)$ is decreasing in $n \geq \pi$, $\theta^* > \theta_L$ would be needed only to compensate the first term in the RHS. As the second term is strictly positive, θ^* needs to further increase, which implies that for $c_1 > 1$ and $\varepsilon = 0$, $\theta^* > \theta_L$ and pure panic runs occur in this region.

For the case of a strictly positive amount of noise, consider the following numerical example: $u(c) = \ln(\frac{1}{2} + c) + 1$, which is increasing, concave, and has an index of relative risk aversion higher than 1; and $R(\theta) = (\theta + \rho)^2$, which is continuous and increasing in θ, and satisfies $Eu(R(\theta)) \geq u(1)$. Also consider the following values for the parameters: $\pi = 1/3$, $\rho = 0.75$. Figure 2 shows the solvency and equilibrium threshold levels (θ_L and θ^*, respectively) for different values of c_1 and ε. The dotted lines represent the levels $\theta_L + 2\varepsilon$ and $\theta^* - \varepsilon$, between which pure panic based runs occur. For example, if $c_1 = 1.1$ and $\varepsilon = 0.01$, $\theta_L = 0.3261$ and $\theta^* = 0.5948 \gg \theta_L$, and for all values of the fundamental in the interval $[\theta_L + 2\varepsilon, \theta^* - \varepsilon[= [0.3461, 0.5848[\neq \varnothing$, all patient depositor run on the bank even though they know it is solvent. Observe that as c_1 increases, the region of pure panic runs becomes larger. Moreover, the smaller the noise the smaller the value of c_1 for which panic runs occur.

In conclusion, while a competitive banking system offers better risk sharing for consumers; more solvent projects are liquidated than in autarky. This is costly to society, as output is reduced and jobs are destroyed.

3. DEPOSIT INSURANCE IN A MODEL OF INFORMATION BASED BANK RUNS

For the case with deterministic returns ($R(\theta) = R > 1$ constant), Diamond and Dybvig (1983) show that the introduction of a guarantee on deposits paying the outside option or autarkic solution, $(1, R)$, acts as a mechanism that ex-ante eliminates the inefficient equilibrium (runs on solvent banks). Such a guarantee could be credibly financed by a tax on early withdrawals, in an amount depending on n,[14] it is always effective in deterring runs (even when offering only partial coverage), and so it is not used in equilibrium. In a model with stochastic returns, however, the effectiveness of the insurance policy in eliminating panic runs will vary with the size of the guarantee and the degree of supervisory involvement of the agency in charge of insurance.

Consider a Deposit Insurance Corporation (DIC), offering an insurance contract (g_1, g_2), in case the bank fails. The DIC can operate under a *narrow mandate* –common in Europe–, acting basically as a pay box to compensate insured depositors of failed banks when instructed by the appropriate authority; or under a *broad mandate* –common in Asia and the Americas– where it also monitors the condition of the banking industry and take responsibility for the resolution of failed insured institutions.

In order to stress the differences between the two systems, I will abstract from the presence of a regulatory authority in the case of a narrow mandate DIC. In this case, the agency will simply pay the guarantee every time the bank does not have enough resources to repay depositors claims, regardless of the bank being insolvent or just illiquid. Under a broad mandate, however, the DIC will also have the ability to monitor the bank's activities –which is modelled by a private signal on θ–, providing lender of last resort (LoLR) assistance to solvent but illiquid banks with positive probability, and resolving inefficient banks according to a *least-cost* criteria.

Under a narrow mandate, the timing of the game with deposit insurance is as follows:

- At $t = 0$ the DIC announces the level of insurance it is going to offer (g_1, g_2). Depositors receive 1 unit of endowment (money) that they invest in the representative bank, which offers a demand deposit contract $(c_1, c_2(\theta, n))$. After receiving deposits, the bank invests in the risky asset.
- At $t = 1$ all impatient depositors withdraw. Patient depositors observe private signals on θ ($\theta_i = \theta + \varepsilon_i$) and decide whether to withdraw or to remain. The DIC observes the realisation of n. If $n > 1/c_1$, the bank's assets are liquidated and transferred to the DIC for the payment of the guarantee. If $n \leq 1/c_1$, the bank continues in operation until the final period.
- At $t = 2$, if the bank is open, remaining patient depositors are paid $\max\{c_2(\theta, n), g_2\}$. If $c_2(\theta, n) < g_2$ remaining assets are transferred to the DIC for the payment of the guarantee. If the bank went bankrupt at $t = 1$, remaining depositors receive g_2.

On the other hand, when the DIC operates under a broad mandate, the timing of the game is as follows:

- At $t = 0$ the DIC announces the level of insurance it is going to offer (g_1, g_2). Depositors receive 1 unit of endowment (money) that they invest in the re presentative bank, which offers a demand deposit contract $(c_1, c_2(\theta, n))$. After receiving deposits, the bank invests in the risky asset.

[14] Considering a sequential servicing constraint, each consumer withdrawing early pays $\tau = c_1 - 1$ in taxes, that are immediately deposited back in the bank by the government, to make these resources available to pay other depositors. If $n = \pi$ the money is returned to depositors for consumption in the interim period. However, if $n > \pi$ each depositor withdrawing early consumes only 1, and patient depositors waiting to the second period receive R. Chari (1989) criticises this solution, arguing that depositors could consume their money before paying the tax, therefore making the scheme impossible to implement. However, if the government arranged for the tax to be directly paid by banks (as it usually happens in economies with well developed tax collection systems), this problem would be solved.

- At $t = 1$ all impatient depositors withdraw. Patient depositors observe private signals on θ ($\theta_i = \theta + \varepsilon_i$) and decide whether to withdraw or to remain. The DIC observes the realisation of n and its own private signal ($s = \theta + \xi_s$, $\xi \leq \varepsilon$), upon which decides whether to leave the bank open –sometimes providing liquidity assistance– or to close it and pay the guarantee, in which case all of the bank's assets are passed onto the DIC and all depositors claiming early withdrawal are paid out g_1.
- At $t = 2$, if the bank is open, remaining patient depositors are paid $\max\{c_2(\theta, n), g_2\}$; if $c_2(\theta, n) < g_2$ remaining assets are transferred to the DIC for the payment of the guarantee in the second period. If the bank went bankrupt at $t = 1$, remaining depositors are paid g_2.

Deposit guarantees are usually expressed as a percentage of the principal or nominal value of deposits at the time of a bank failure, or as a limit up to which deposits can be recovered. Thus, a natural constraint for the value of insurance is $g_1 = g_2 = g \leq c_1$. Indeed, following the definition of solvency, if g were strictly higher than c_1 the DIC would have to pay the guarantee in the second period to depositors in a solvent bank, even if this were not subject to runs in the interim period.

The funding of a deposit insurance system varies from country to country. I consider here an ex-post funded system, getting resources through a government tax on withdrawals, as in Diamond and Dybvig (1983). However, as in Goldstein and Pauzner (2000), I will drop the "*sequential servicing constraint*" assumption and allow the bank to observe the length of the queue (n) before paying out depositors, so that all customers withdrawing at a given period receive exactly the same payoff. I assume that deposits are senior to other claims, so that when a bank fails at date t, its assets –or their liquidation value– are transferred to the DIC for the payment of the deposit guarantee. Therefore, the government can directly tax all early withdrawals at a constant rate equal to $\tau = c_1 - g$, transferring the revenue to the DIC for the payment of the guarantee en in the final period.

The following analysis is divided in two phases. First, I study the equilibrium behaviour of depositors and the DIC under the two mandates. Second, I study the optimal decision problem for the bank. A complete formulation of the game should include a payoff function for the DIC, in order to compute the optimal level of insurance offered in the planning period. This paper will not solve for that problem, but I offer a discussion of the ideas that should be considered in the concluding section.

In order to be able to make comparisons later, I will denote by θ^* the equilibrium threshold in the benchmark model without deposit insurance, and by θ_g^* the one obtained in the model with insurance.

4. EQUILIBRIUM UNDER A NARROW MANDATE

(Interim Period Sub-Game)

Under a narrow mandate and once the DIC and the bank have announced their respective contracts (g and c_1), all impatient depositors withdraw in the interim period and patient depositors, observing private signals on θ, decide whether to withdraw or to remain. At this stage this is the only relevant decision, because the action of the DIC is directly determined by the strategies played by patient depositors: the guarantee is paid out if and only if $n > 1/c_1$.[15]

[15] Notice that the final period payoff is a result of the actions taken in the interim period, and that no relevant decision is made in the last stage of the game.

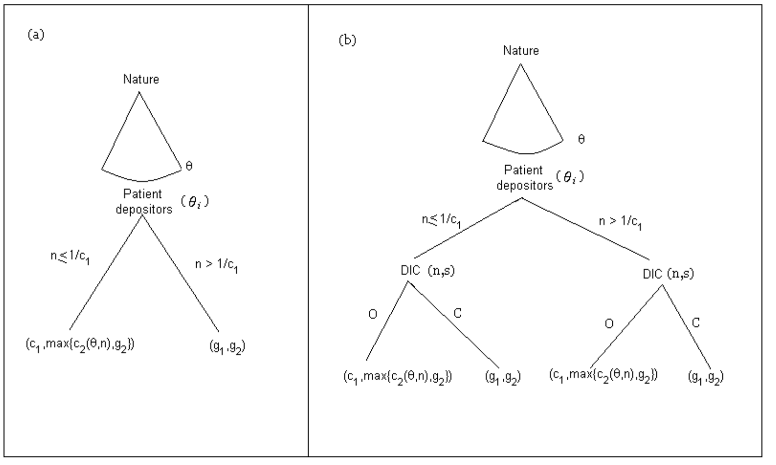

FIG. 3 Extended form of the game in the interim period, when the DIC operates under (a) a narrow mandate and (b) a broad mandate.

Figure 3.a gives a representation of the extended form of this sub-game. According to this, patient depositors' payoffs are given by table 2.

	$n \leq 1/c_1$	$n > 1/c_1$
$t = 1$	c_1	g
$t = 2$	$\underbrace{\max\{c_2(\theta, n), g\}}_{\geq g}$	g

TABLE 2: Patient depositors' payoffs in the game with insurance under a narrow mandate.

Denote by $n(\theta)$ a feasible belief ($\pi \leq n(\theta) \leq 1$) about the aggregate behaviour of patient depositors consistent with the information received, and by $\delta_g(\theta, n(\theta))$ the difference of payoffs between waiting until $t = 2$ and withdrawing at $t = 1$, once the deposit guarantee g is in place:

$$\delta_g(\theta, n(\theta)) = \begin{cases} u\left(\max\{c_2(\theta, n), g\}\right) - u(c_1) & \text{if } n(\theta) \leq 1/c_1 \\ 0 & \text{if } n(\theta) > 1/c_1 \end{cases}.$$

After receiving a private signal, θ_i, each consumer evaluates :

$$\Delta_g(\theta_i, n(\theta)) = E_{\theta_i}[\delta_g(\theta, n(\theta))] = \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} \tfrac{1}{2\varepsilon} \delta_g(\theta, n(\theta)) d\theta,$$

the conditional expected premium to wait when deposits are insured.

Given that $c_1 \geq g$, the lower and upper dominance regions can be defined as in the model without insurance. For $\theta < \theta_L$, the bottom left of table 2 is always less than or equal to c_1, hence to withdraw is weakly dominant. For very high levels of the fundamentals ($\theta > \theta_U$), depositors should remain, independently of the actions of other players.[16]

[16] When $\theta > \theta_L$ it is weakly dominant to remain. Nevertheless, as in this region the payoff of a single depositor will depend on the strategy chosen by other patient consumers, assume it is given again by the interval $[\theta_U + 2\varepsilon, 1]$.

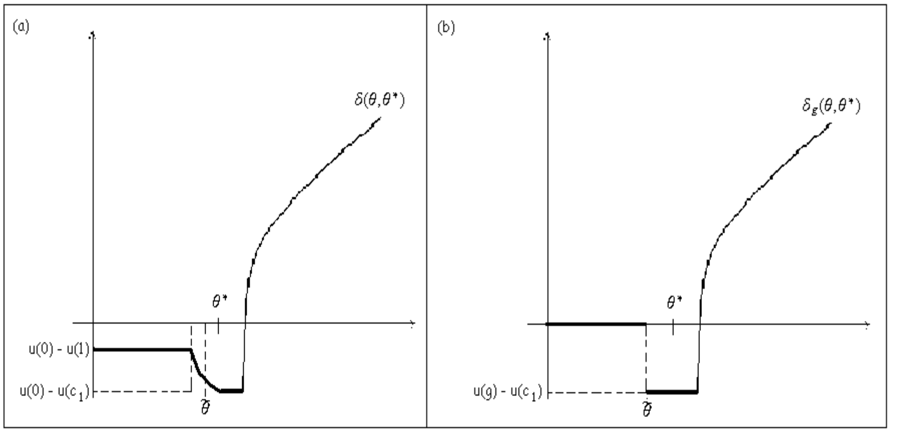

FIG. 4 Premium to wait to withdraw in the final period (a) without deposit insurance and (b) with deposit insurance.

If a switching point θ^* exists, a belief consistent with the existence of the dominance regions and the uniform distribution of the noise is again given by $n(\theta,\theta^*)$, in equation 1. Denote by $\delta_g(\theta,\theta^*) \doteq \delta_g(\theta, n(\theta,\theta^*))$ and $\Delta_g(\theta_i,\theta^*) \doteq \Delta_g(\theta_i, n(\theta,\theta^*))$.

Remark 1. Notice that if $g < c_1$ the function $\delta_g(\theta,\theta^*)$ has a discontinuity at $\theta = \widetilde{\theta}$. Nevertheless, the function $\Delta_g(\theta_i,\theta^*)$ is still continuous in both arguments (see figures 4 and 5).

PROPOSITION 2. *For any given value of the guarantee and as a function of one variable, $\Delta_g(\theta^*,\theta^*)$ is strictly increasing.*

See proof in the appendix.

If $c_1 = g$, $\delta_g(\theta,\theta^*) \geq 0$ for all θ, which implies that $\Delta_g(\theta_i,\theta^*) \geq 0$ for all θ_i. Hence, patient depositors do not monitor their banks ($n(\theta) = \pi,\ \forall\theta$). Such a solution means that insolvent banks are never liquidated, making the guarantee very expensive for low states of the fundamentals. Therefore, I will concentrate in the case $c_1 > g$.

PROPOSITION 3. *If $c_1 > g$, there exists a unique equilibrium in switching strategies, θ_g^*, such that a patient consumer withdraws if $\theta_i < \theta_g^*$ and remains if $\theta_i > \theta_g^*$.*

Proof. Call $\Delta_g(\theta^*) = \Delta_g(\theta^*,\theta^*)$.

$\forall\theta \leq \widetilde{\theta} - \varepsilon,\ \Delta_g(\theta) = 0$ (see figure 5).

$$\forall\theta \in]\widetilde{\theta} - \varepsilon, \theta_L - \varepsilon],\ \Delta_g(\theta) = \int_{\theta-\varepsilon}^{\theta+\varepsilon<\theta_L} \tfrac{1}{2\varepsilon}\left\{u\left(\max\left\{c_2(\theta,n),g\right\}\right) - u(c_1)\right\} d\theta < 0,$$

as for any $n \geq \pi$ and $\theta < \theta_L$, $c_2(\theta,n) < c_1$.

$$\Delta_g(\theta_U + \varepsilon) = \int_{\theta_U}^{\theta_U+2\varepsilon} \tfrac{1}{2\varepsilon}\left\{u\left(\max\left\{c_2(\theta,n),g\right\}\right) - u(c_1)\right\} d\theta > 0,$$

as for $\theta > \theta_U$, $n = \pi$ and $c_2(\theta,n) > c_1$.

By continuity, there exists $\theta_U + \varepsilon > \theta_g^* > \theta_L - \varepsilon$ such that $\Delta_g(\theta_g^*) = 0$, and proposition 2 implies this solution is unique. ∎

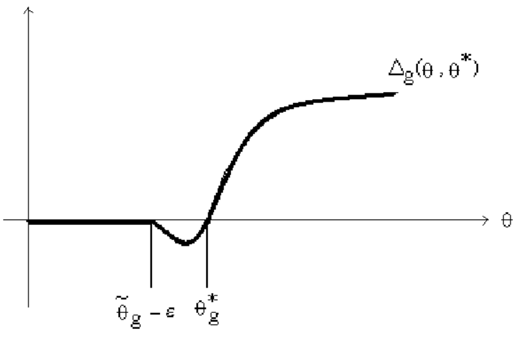

FIG. 5 Expected premium to wait until in the final period with deposit insurance.

Proposition 4. *The monotone equilibrium threshold, θ_g^*, defines the unique strategy surviving iterated deletion of strictly dominated strategies over the set of all feasible beliefs on the actions of players.*

Proof. Whatever the value of g, strategies in this game are complementary. In the region where $n \leq 1/c_1$, the payoff to remain is non-decreasing in the number of players waiting to withdraw in the second period. In the region $n > 1/c_1$ the payoff is constant, therefore non-decreasing in the number of players withdrawing. Hence, the result follows as a direct application of the results in Morris and Shin (2002) for binary actions games, with a continuum players, strategic complementarity, and a unique monotone equilibrium threshold. ∎

Summing up, a unique equilibrium preserving the monitoring role of depositors exists in the model with deposit insurance if $c_1 > g$, and it is such that if a patient depositor receives a signal $\theta_i \leq \theta_g^*$, $\Delta_g(\theta_i, \theta_g^*) \leq 0$, so she withdraws; while if $\theta_i > \theta_g^*$, $\Delta_g(\theta_i, \theta_g^*) > 0$, and the depositor remains.

Using that $n(\theta, \theta_g^*)$ is linear in the interval $[\theta_g^* - \varepsilon, \theta_g^* + \varepsilon]$ to change variables and rearrange terms, it is possible to see that θ_g^* solves:

$$\int_{\pi}^{1/c_1} u\left(\max\left\{\frac{1-nc_1}{1-n}R\left(\theta_g^*+\frac{\varepsilon}{1-\pi}(1+\pi-2n)\right), g\right\}\right) dn = \int_{\pi}^{1/c_1} u(c_1) dn \quad (2)$$

Proposition 5. *The monotone equilibrium threshold for the game of information based bank runs with insured deposits ($c_1 > g$), satisfies the following properties:*

1. *θ_g^* increases in c_1 iff*
$$u(g) > u(c_1) - c_1^2 \int_{\pi}^{1/c_1} \left\{u'(c_1) - u'(c_2(n))\frac{\partial c_2}{\partial c_1}\mathbb{1}_{\{n \leq \hat{n}\}}\right\} dn.$$
2. *θ_g^* is decreasing in g. This is, a higher value of insurance increases the incentives to remain.*

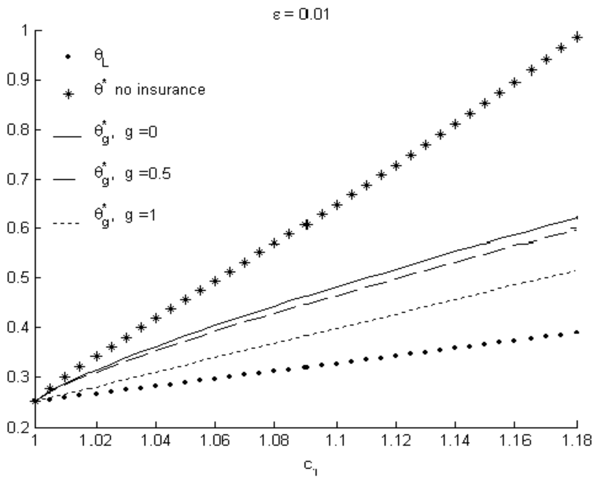

FIG. 6 Equilibrium and solvency thresholds with and without insurance.

See proof in the appendix.

THEOREM 2. *Under a narrow mandate, a deposit insurance contract preserving the monitoring role of depositors involves $g < c_1$. Nonetheless, inefficient liquidation of solvent banks (panic runs) will persist for this type of insurance, the less so the higher the guarantee.*

Proof. Reconsider equation 3 when the signal's noise vanishes. Taking limit when $\varepsilon \to 0$:

$$\int_{\pi}^{1/c_1} u\left(\max\left\{c_2(\theta_g^*, n), g\right\}\right) dn = \int_{\pi}^{\widehat{n}} u\left(c_2(\theta_g^*, n)\right) dn + \int_{\widehat{n}}^{1/c_1} u\left(g\right) dn = \int_{\pi}^{1/c_1} u(c_1) dn,$$

with $\widehat{n}$ defined by $c_2(\theta_g^*, \widehat{n}) = g$.

Because $g < c_1$, $c_2(\theta_L, \pi) = c_1$, and $c_2(\theta_g^*, n)$ decreasing in n; θ_g^* needs to be higher than θ_L for this equality to hold, which means that in equilibrium depositors will still run on some solvent banks. Finally, it is clear from this equation that the higher g the smaller the region of panic runs. ∎

For a strictly positive amount of noise (e.g. $\varepsilon = 0.01$) consider the numerical example in figure 6. For every value of $c_1 > 1$ the equilibrium threshold is above θ_L, although the gap is smaller for higher levels of insurance (fewer banks are li quidated). Consider the case $g = 1$ and $c_1 = 1.1$. For these parameters, $\theta_L = 0.3261$ and the equilibrium threshold equals $\theta_g^* = 0.3928$. Hence, for every θ in the region $[\theta_L + 2\varepsilon, \theta_g^* - \varepsilon[= [0.346\,1, 0.3828[\neq \varnothing$, all patient depositor continue running on a bank they know to be solvent. This region, however, is substantially smaller than the one without insurance, in page 8.

Blanket guarantees in times of crisis are usually designed to protect the principal value of deposits, in order to enhance market confidence and secure the purchasing power of consumers. In this model, a blanket guarantee would translate into $g = 1$. However, despite the high level of protection depositors still run on some solvent banks. Notice that this result does not obey to a lack of confidence in the deposit insurance system, or to a macroeconomic shock affecting the economy. It emerges naturally as an equilibrium in a model with asymmetric information, where depositors rationally anticipate the reaction functions of their counterparts.

5. EQUILIBRIUM UNDER A BROAD MANDATE

Under a broad mandate, and once the DIC and the bank have announced their respective contracts (g and c_1), all impatient depositors withdraw in the interim period and patient depositors, observing private signals on θ, decide whether to withdraw or to remain. The DIC moves after depositors have played, observing the realisation of n and its own private signal s, which comes from the monitoring of the bank's activities. Based on the information revealed by these two variables, the DIC must decide whether to leave the bank open –in which case liquidity assistance may sometimes be required– or to close it and pay the guarantee.

Figure 3.b gives a representation of the extended form of the sub-game faced by depositors and the DIC in the interim period.

5.1. DIC's Sub-game

In this version of the game, bankruptcy is not determined solely by the actions of depositors, but it can also be the efficient outcome of supervision and prudential regulation. On a theoretical level, Repullo (2000) justifies the allocation of supervisory activities to the DIC every time withdrawals are large enough to pose a systemic threat: *"deposit insurance...institutions have become responsible for dealing with solvency problems, leaving Central Banks with the exclusive role of handling liquidity problems"*. As the present model does not study the problem of separation of activities between the central bank and the banking supervisor, for simplicity, the DIC will be allowed to deal with both solvency and liquidity issues.

Hence, as opposed to the case with a narrow mandate, this time the DIC has access to private information which can be used to decide a closure rule and a LoLR policy for banks. At $t = 1$ the DIC receives a private, non verifiable signal $s = \theta + \xi_s$, where ξ_s is uniformly distributed on $[-\xi, \xi]$. Given the supervisory role assigned to this agency, preferential access to information will naturally imply that $\xi \leq \varepsilon$; meaning that, on average, the DIC's signal is more informative than the signals of depositors. Knowing the value of s, the DIC corrects the conditional probability distribution of θ and estimates that the true value of θ follows a uniform distribution in the interval $[s - \xi, s + \xi]$.

Once patient depositors have played, the realisation of n becomes observable to all players, in particular to the DIC. As a function of θ (which is indeed a 1-1 relationship in the region of partial runs), n also carries information about the true state of the bank. For a given value of n, the DIC has the option to close the bank based on its estimated solvency, or leave it open, in which case liquidity assistance could be provided under exceptional circumstances.

The IMF's code of best practice requires the resolution of a failing bank to be decided according to a "*least cost*" criteria (Hoelscher and Quintyn, 2003). Using this idea, I define the objective function of the DIC at this stage of the game as to minimise the cost of resolution of a bank.[17]

I will first study the case of perfect information ($\xi = 0$), so that s perfectly reveals the true value of θ in the interim period. Later, I will extend these results to the case of a noisy signal ($\xi > 0$).

[17] Although this assumption still allows for very interesting results, a complete welfare analysis would require a more general definition of the DIC's objectives (see section 7.1).

5.1.1. *Closure rule*

When the DIC decides to close a bank, the guarantee on deposits must be paid. I am assuming deposits are senior to other claims, therefore, when a bank fails all of its assets are passed onto the DIC, and the agency has to decide how to manage available resources. Suppose the DIC has the option to issue debt against the future value of the bank's assets, with an expected return of $E_s[R(\theta)]$.[18] Of course, the DIC can also liquidate the assets in the interim period, obtaining a certain return of 1. Hence, the net expected cost of the decision of closing a bank in the interim period equals

$$g - \max\{E_s[R(\theta)], 1\}.$$

As I am assuming the DIC to have perfect information, the term $E_s[R(\theta)]$ simplifies to $R(\theta)$.

RESULT 1. *When the DIC has perfect information ($\xi = 0$), it is cost efficient for it to close the bank if remaining assets are insufficient to cover the value of the guarantee in the final period ($c_2(\theta, n) < g$), and leave it open otherwise.*

See proof in the appendix.

5.1.2. *LoLR policy*

According to Bagehot's doctrine, a LoLR should lend only to solvent banks experiencing liquidity problems (Bagehot, 1873) . In the present framework,

DEFINITION 2. A bank faces a liquidity shock if $n > \pi$ at $t = 1$.

i. For a given realisation of n, a bank is **liquid** in both periods if $c_2(\theta, n) \geq c_1$. Notice that any liquid bank must also be solvent.

ii. A bank is **fundamentally solvent but illiquid** if $\theta \geq \theta_L$, $n > \pi$ and $c_2(\theta, n) < c_1$.

Solvency is a property of the bank which cannot be verified until the final period. However, and as for the moment I am assuming $\xi = 0$, the DIC can perfectly observe the true value of θ in the interim period.

Clearly, if for a given value of n a bank were liquid, no assistance would be required. If it were fundamentally solvent but illiquid, however, there would be room for a LoLR.

PROPOSITION 6. *When the DIC has perfect information ($\xi = 0$), a cost efficient LoLR policy is to rescue fundamentally solvent but illiquid banks.*

See proof in the appendix.

Therefore, the DIC should commit liquidity assistance to solvent but illiquid banks, and such commitment should be public information in order to deter panic runs. Should the DIC, under some circumstances, also commit liquidity to insolvent banks? The answer is no, and it will be proved in what follows.

Define by $\underline{\theta}$ the value the fundamental satisfying $c_2(\underline{\theta}, \pi) = g$. Clearly, as $g < c_1$, $\underline{\theta} < \theta_L$. Consider the case where the bank is fundamentally insolvent and it is also facing a liquidity shock:

[18] Debt issuance will require the DIC to provide funds to pay the guarantee in the first period. Nonetheless, as the bank's assets have been transferred to it, the counterpart risk of this loan should be minimal.

$$n > \pi,\ c_2(\theta, n) < g \text{ and } \max\{\underline{\theta}, 0\} \leq \theta < \theta_L.$$

As $c_2(\theta, n) < g$, the closure rule determines that it should be closed. However, as $\theta \geq \underline{\theta}$, $c_2(\theta, \pi) \geq g$ and by lending the excess withdrawal at $t = 1$, the DIC could secure a higher return for remaining patient depositors in the final period.

PROPOSITION 7. *The DIC should never commit liquidity assistance to fundamentally insolvent banks.*

See proof in the appendix.

When combining the two policies (closure rule and LoLR) it is possible to conclude that, with perfect information, solvent banks are never allowed to fail, and it is only when an insolvent bank experiences large withdrawals in the first period –and large enough for remaining assets to be insufficient to cover the payment of the guarantee in the final period– that the bank is closed by the DIC. This result indicates that, despite the supervisory role assigned to this agency, depositors retain some monitoring power, but also that the DIC lacks the commitment to close all insolvent banks in the interim period (if $g < c_1$). Indeed, the smaller the guarantee, the higher the share of insolvent banks that might be allowed to survive.

Summarising the previous results:

THEOREM 3. *When the DIC has perfect information and operates under a broad mandate, a cost efficient policy is to leave a bank open if it is fundamentally solvent or if its remaining assets are enough to pay the guarantee on deposits in the final period. Otherwise, the bank should be closed. The LoLR should lend only to fundamentally solvent banks facing liquidity shocks (table 3).*

Proof. The only part of this proposition that has not been proved yet is that a bank should be closed independent of the actions of consumers if $\theta < \underline{\theta}$ (provided this value is non negative). This result is immediate, because $c_2(\theta, \pi) < g$ clearly implies that $c_2(\theta, n) < g$ for all $n \geq \pi$. Following result 1 and proposition 7, the bank should be closed. ∎

$\boldsymbol{\theta}$	**Second period return**	**Closure rule**	**LoLR policy**	**Observations**
$\theta < \underline{\theta}$	$c_2(\theta, \pi) < g$	Close	No	Bank fundamentally insolvent
$\underline{\theta} \leq \theta < \theta_L$	$c_2(\theta, n) < g$	Close	No	
	$c_2(\theta, n) \geq g$	Open	No	
$\theta \geq \theta_L$	$c_2(\theta, \pi) \geq c_1 \geq g$	Open	Yes	Bank fundamentally solvent, lend $(n - \pi)c_1$ iff $n > \pi$ and $c_2(\theta, n) < c_1$

TABLE 3: Closure and LoLR policies with a broad mandate and perfect information.

The assumption that the DIC lends at a discounted rate normalised to zero does not contradict other assumptions in the model –the safe technology return was also normalised to zero–, neither it is uncommon in the literature. Allen and Gale (1998), for example, study the problem of a central bank providing emergency liquidity assistance in a model where early liquidation of assets is costly, and they also normalise the lending interest rate to zero. Other authors have argued that a LoLR should lend at a high penalty rate, in order to stop public funds from being used to finance regular investment (see Bagehot (1873) and Repullo (2000)). However, as the present model considers only one representative bank, it cannot take into account interbank lending as an alternative source of liquidity for solvent banks, as they do.

5.1.3. Case of imperfect information ($\xi > 0$)

Having established these results, moving to the case of imperfect information is simple. Considering a positive but small amount of noise, all the previous equations in θ can be rewritten in terms of their conditional expected value, which will allow for computing cost efficient closure and LoLR policies.

Define by s_L the value of the signal satisfying $E_{s_L}[c_2(\theta,\pi)] = c_1$, $s^* = s^*(n)$ such that $E_{(s^*,n)}[c_2(\theta,n)] = g$, and $\underline{s}$ such that $E_{\underline{s}}[c_2(\theta,\pi)] = g$ (if the solution is non-negative, and zero otherwise). Notice that for any given value of n, these parameters are uniquely determined because $c_2(\theta,n)$ is increasing in θ.

Proposition 8. *For small non-negative values of the DIC's signal noise, a cost efficient policy is to leave the bank open if $s \geq s_L$ or if $s \geq s^*(n)$, and close it otherwise.*

Proof. Same as above, replacing all expressions by their conditional expected values. ∎

The DIC can anticipate that a bank is solvent if $s > \theta_L + \xi$, and insolvent if $s < \theta_L - \xi$. Because $c_2(\theta,\pi) < c_1$ if $\theta < \theta_L$, and $c_2(\theta,\pi) > c_1$ if $\theta > \theta_L$, it follows from the definition of s_L that $\theta_L - \xi \leq s_L \leq \theta_L + \xi$. Indeed, when $s > s_L$ the expected value of $c_2(\theta,\pi)$ is computed for larger values of θ, and because $c_2(\theta,\pi)$ is increasing in θ this implies that $E_s[c_2(\theta,\pi)] > c_1$. The opposite is true when $s < s_L$. Therefore, conditional on its private information, the DIC estimates a bank to be solvent if $s \geq s_L$, and insolvent if $s < s_L$.

Proposition 9. *For small non-negative values of ξ, the DIC should commit liquidity assistance to a bank facing runs if and only if $s \geq s_L$ and $E_s[c_2(\theta,n)] < c_1$.*

Corollary 1. $\lim_{\xi \to 0} s_L(c_1) = \theta_L(c_1)$. *This is, when the information gathered by the DIC becomes extremely precise, only insolvent banks are allowed to fail.*

These results justify the principle of *"creative ambiguity"*: depositors cannot anticipate if the LoLR will provide liquidity assistance for a subset of the fundamentals ($\theta \in]\theta_L - \xi, \theta_L + \xi[$). Nevertheless, this is not a consequence of the LoLR randomising over its set of actions (playing an equilibrium in mixed strategies, as in Freixas et al. (1999)). The DIC's strategy is perfectly determined and rational, but it is not observed by consumers due to asymmetric information.

Indeed, two kind of errors are possible when $\xi > 0$. With positive probability the DIC can mistakenly allow a solvent bank to fail (by refusing liquidity assistance), or else bail out an insolvent bank. However, the information contained in $n(\theta)$ has not yet been taken into account. If a monotone equilibrium threshold for depositors θ_g^* exists, n can be expressed as an invertible function of θ in the region of partial runs (this is, where $\pi < n < 1$).

Proposition 10. *If $\theta_g^* \leq \theta_L + \varepsilon$, $n(\theta,\theta_g^*) > \pi$ reveals the true value of θ at $t = 1$. In this case, no matter what the value of ξ is, the DIC determines its closure and LoLR policies as in the case of perfect information (theorem 3). If $\theta_g^* > \theta_L - \varepsilon$, the DIC also gets perfect information in the region of no runs ($n(\theta,\theta_g^*) = \pi$).*

Proof. Assume a monotone equilibrium threshold θ_g^* existed, such that $\theta_L - \varepsilon \leq \theta_g^* \leq \theta_L + \varepsilon$. A value of $n(\theta)$ consistent with this equilibrium is given by equation 1. Hence,

$$n = \begin{cases} \pi & \text{if } \theta_i > \theta_g^* \text{ for all } i, & \text{if } \theta > \theta_g^* + \varepsilon \geq \theta_L, \text{ the bank is solvent} \\ \in]\pi, 1[& \text{if } \theta_g^* - \varepsilon < \theta < \theta_g^* + \varepsilon, & n \text{ fully reveals } \theta \\ 1 & \text{if } \theta_i < \theta_g^* \text{ for all } i, & \text{if } \theta < \theta_g^* - \varepsilon \leq \theta_L, \text{ the bank is insolvent} \end{cases}$$

The next step is to determine that a monotone equilibrium does indeed exists for depositors.

5.2. Patient Depositors' Sub-game

Once a patient depositor has received a private signal on θ, she will construct beliefs about the behaviour of other players, the value of the last period return, and the action chosen by the DIC, upon which she will derive her optimal strategy.

Dominance regions: Whatever the relationship between the first and second period guarantee, an upper dominance region does exist for this game. Formally, a depositor can be sure that the bank is solvent if $\theta_i > \theta_L + \varepsilon \geq \theta_L + \xi$. Because $s_L \leq \theta_L + \xi$, she also knows that the bank will be bailed out if facing runs. All patient depositors receive signals above this value if $\theta > \theta_L + 2\varepsilon$, and anticipating that $c_2^L = c_2(\theta, \pi) \geq c_1$ whatever the value of n, they wait until the final period.[19] Hence, the *upper dominance region* for this game is given by $[\theta_L + 2\varepsilon, 1]$.[20]

If $\underline{\theta}$ exists ($c_2(\underline{\theta}, \pi) = g$), $R(.)$ increasing implies that $\underline{\theta} - \xi \leq \underline{s} \leq \underline{\theta} + \xi$. If the DIC's signal is below $\underline{s}$ the bank will be closed. Therefore, if $\forall i\ \theta_i < \underline{\theta} - \varepsilon$, a depositor knows the bank will be closed no matter what strategies are played by other players. All depositors will receive signals below this level if $\theta < \underline{\theta} - 2\varepsilon$. Hence, if $\underline{\theta} > 2\varepsilon$ the optimal reaction of depositors in this region will be to run, in which case a *lower dominance region* exists.[21]

In the intermediate region ($[\underline{\theta} - 2\varepsilon, \theta_L + 2\varepsilon]$), only sufficiently illiquid insolvent bank will be closed, this is, those for which $s < s^*(n)$. In this region, depositors' payoff will depend upon their actions in the following way:[22]

	$n \geq 1/c_1$	$n < 1/c_1$	
		$s < s^*(n)$	$s \geq s^*(n)$
$t = 1$	g	g	c_1
$t = 2$	g	g	$\max\{c_2(n, \theta), g\}$

TABLE 4: Patient depositors' payoffs in the game with insurance and a broad mandate.

Consistent with the dominance regions, $n = \pi$ if $\theta > \theta_L + 2\varepsilon$ and $n = 1$ if $\theta < \underline{\theta} - 2\varepsilon$. In the intermediate region, the action taken by the DIC will be directly determined by the behaviour of patient depositors, which payoffs are described in table 4. The entry in the bottom right corner satisfies $E_s\left[c_2(n, \theta)\right] \geq g$.

Once again, $n(\theta, \theta^*)$ in equation 1 defines a belief consistent with these dominance regions and the uniform distribution of the noise.

As in the case with a narrow mandate, it is possible to prove that:

[19] The upper index *"L"* in c_2^L stands for the "liquidity" assistance by the LoLR.

[20] In the game without insurance, additional assumptions were required for depositors to unconditionally remain for higher realisations of θ. With the DIC operating under a broad mandate, these assumptions are endogenised by means of the commitment of liquidity to solvent but illiquid banks.

[21] For low values of the guarantee, $\underline{\theta}$ might well not exist. However, if coordination induces $\theta_L - \varepsilon \leq \theta_g^*$, this should not be a problem (because $\theta_L > 2\varepsilon$) and indeed, as previously established, it will provide the DIC with high quality information to assess the solvency of banks, through the information contained in $n(\theta, \theta_g^*)$.

[22] $n \geq 1/c_1 \Longrightarrow s < s^*(n)$. If $s \geq s^*(n)$ the bank will remain open and the depositors' final payoff will be higher than g. Anticipating that, depositors should wait, so $n < 1/c_1$, which is a contradiction.

Proposition 11. *When the DIC operates under a broad mandate and offers insurance $g < c_1$ in both periods, there exists a unique equilibrium threshold θ_g^*, such that depositors remain if $\theta_i > \theta_g^*$ and withdraw otherwise. This threshold satisfies $\underline{\theta} - \varepsilon \leq \theta_g^* \leq \theta_L + \varepsilon$.*

Proof. Similar to proposition 3. ∎

Corollary 2. $\lim_{\varepsilon \to 0} \theta_g^*(c_1) \leq \theta_L(c_1)$, *which says that as the noise of depositor's signals vanishes, they run only on insolvent banks.*

As $\xi \leq \varepsilon$, when ε tends to zero ξ must go to zero as well, and the outcome of the game with a broad mandate insurance agency achieves a social optimum, in the sense that liquidity assistance is targeted exclusively to solvent banks ($\lim_{\xi \to 0} s_L = \theta_L$), but is not used in equilibrium as they do not experience runs ($\lim_{\varepsilon \to 0} \theta_g^*(c_1) \leq \theta_L(c_1)$). Extremely insolvent banks are closed ($\lim_{\xi \to 0} \underline{s} = \underline{\theta}$), and those with enough funds to cover the payment of the final period guarantee are allowed to continue operating. All these results hold irrespective of the specific values of insurance provided, which in particular might imply the insurance policy to be less expensive under a broad mandate.

6. MORAL HAZARD: DEMAND DEPOSIT CONTRACT WITH INSURANCE

(Bank's Planning Period Sub-Game)

The previous sections solved for the equilibrium strategy of depositors and the DIC taking as given the value of c_1, and without considering the effect that deposit insurance may have on the value of the demand deposit contract. Too gene rous a protection could generate moral hazard, if because of limited liability banks choose excessively high values of c_1, making a bank fundamentally insolvent for most realisation of θ. If insurance were sufficiently generous for depositors to be willing to accept this contract, with a high probability the DIC would end up paying the guarantee in the interim period.

Many authors agree that deposit insurance induces moral hazard. In fact, a guarantee on deposits can be seen as a callable put option on the agency offering insurance (Merton, 1977; Acharya and Dreyfus, 1989), which value increases monotonically in the volatility of the investment portfolio, and then is maximised at the highest possible level of risk.

This section will derive a condition for the optimal value of c_1 under both mandates. Because the equations become intractable for positive values of the noise, I will solve for the limit case when ε tends to zero. An analytical solution is possible when abstracting from the effect that changes in c_1 have on the equilibrium threshold. In that case, it is possible to show that the effect of deposit insurance is to reduce the level of risk sharing, this effect being stronger under a broad mandate. However, when considering the impact of changes in c_1 on the equilibrium threshold, comparative statics show that the net effect of insurance is to increase the value of c_1, which I interpret as a raise in moral hazard because the probability of runs is increasing in c_1 (at least for high levels of the guarantee). Limited insurance can contain moral hazard up to some level, justifying the observed conduct of governments across the world of offering only partial insurance on deposits in normal times, in order to encourage depositors to keep monitoring their banks.

6.1. Narrow Mandate DIC

At the planning period, the bank calculates the value of c_1 that maximises the ex-ante expected utility of consumers:

$$\max_{c_1} Eu(c_1) = \int_{\theta_g^*+\varepsilon}^{1} \left\{\pi u(c_1) + (1-\pi)u\left(\max\left\{\frac{1-\pi c_1}{1-\pi}R(\theta), g\right\}\right)\right\} d\theta$$

$$+ \int_{\widetilde{\theta}_g}^{\theta_g^*+\varepsilon} \left\{n(\theta,\theta_g^*)u(c_1) + (1-n(\theta,\theta_g^*))u\left(\max\left\{\frac{1-n(\theta,\theta_g^*)c_1}{1-n(\theta,\theta_g^*)}R(\theta), g\right\}\right)\right\} d\theta$$

$$+ \int_{0}^{\widetilde{\theta}_g} u\left(g\right) d\theta$$

For $\theta > \theta_g^* + \varepsilon$ all depositors remain and receive $\max\{c_2(\theta,\pi), g\}$. When $\theta < \theta_g^* - \varepsilon$, all withdraw, the bank goes bankrupt and depositors are paid the guarantee. In the intermediate region, depositors' withdrawals are decreasing in the value of the fundamental according to the formula of $n(\theta,\theta_g^*)$, and the guarantee is paid each time the bank's resources are insufficient to cover demanded deposits (i.e. when $\theta < \widetilde{\theta}_g$).

When $\varepsilon \to 0$ the region $[\theta_g^* - \varepsilon, \theta_g^* + \varepsilon]$ converges to $\{\theta_g^*\}$, and as payoffs in this region are discontinuous at $\widetilde{\theta}_g \longrightarrow \theta_g^*$, the problem reduces to:

$$\max_{c_1} Eu(c_1) = \int_{\theta_g^*(c_1)}^{1} \{\pi u(c_1) + (1-\pi)u\left(\max\left\{c_2\left(\theta,\pi\right), g\right\}\right)\} d\theta$$
$$+ \lim_{\varepsilon\to 0^+} \left\{n(\theta,\theta_g^*)u(c_1) + (1-n(\theta,\theta_g^*))u(\max\left\{c_2\left(\theta_g^*, n(\theta,\theta_g^*)\right), g\right\})\right\}$$
$$- \lim_{\varepsilon\to 0^-} \left\{n(\theta,\theta_g^*)u(g) + (1-n(\theta,\theta_g^*))u(g)\right\} + \int_{0}^{\theta_g^*(c_1)} u(g) d\theta$$

It was established before that when $\varepsilon \to 0$, $\theta_g^*(c_1) > \theta_L(c_1)$, hence the term under the first integral $\max\{c_2(\theta,\pi), g\} = c_2(\theta,\pi) \geq c_1 \geq g$. Also, $\lim\limits_{\varepsilon\to 0^+} n(\theta,\theta_g^*) = \pi$ and $\lim\limits_{\varepsilon\to 0^-} n(\theta,\theta_g^*) = 1$, and the previous expression becomes:

$$\max_{c_1} Eu(c_1) = \int_{\theta_g^*(c_1)}^{1} \{\pi u(c_1) + (1-\pi)u\left(c_2\left(\theta,\pi\right)\right)\} d\theta$$
$$+\pi u(c_1) + (1-\pi)u(c_2\left(\theta_g^*,\pi\right)) - u(g) + \int_{0}^{\theta_g^*(c_1)} u(g) d\theta$$

A sufficient condition for the value of c_1 maximising this function is given by equation

$$\pi \int_{\theta_g^*(c_1)}^{1} \left\{u'(c_1) - R(\theta)u'\left(c_2(\theta,\pi)\right)\right\} d\theta + \pi\left\{u'(c_1) - R(\theta_g^*(c_1))u'\left(c_2(\theta_g^*(c_1),\pi)\right)\right\} =$$
$$\frac{\partial\theta_g^*(c_1)}{\partial c_1}\left[\pi u(c_1) + (1-\pi)u\left(c_2(\theta_g^*(c_1),\pi)\right) - (1-\pi c_1)R'(\theta_g^*(c_1))u'\left(c_2(\theta_g^*(c_1),\pi)\right) - u(g)\right] \quad (4)$$

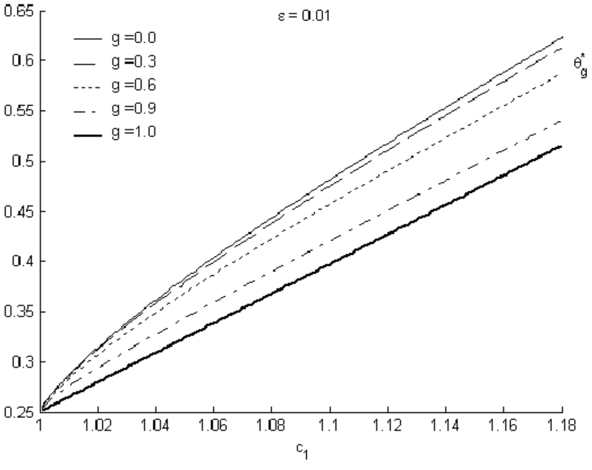

FIG. 7 Equilibrium threshold with insurance, as a function of c_1 and g.

Hence, the marginal gain from risk sharing due to the transfer of consumption from patient to impatient depositors (LHS) equals the positive marginal cost associated to the increase in the probability of runs (RHS).

PROPOSITION 12. *When abstracting from the effect that changes in the demand deposit contract have on the equilibrium threshold of depositors* $\left(\frac{\partial \theta_g^*(c_1)}{\partial c_1}=0\right.$ *and* $\left.\frac{\partial \theta^*(c_1)}{\partial c_1}=0\right)$*, the introduction of deposit insurance reduces the level of risk sharing offered by banks.*

See proof in the appendix.

Given that the bank's portfolio has not changed, the reduction in the interest paid on deposits could be explained as the bank "*free riding*" on the reduction of risk on deposits, resulting from the DIC's guarantee.

The effect of c_1 on the equilibrium threshold, however, is not nil. For the case without insurance, theorem 1 established that $\frac{\partial \theta^*(c_1)}{\partial c_1}> 0$. For the case with insurance under a narrow mandate, proposition 5 showed that $\theta_g^*(c_1)$ is increasing in c_1 if and only if

$$u(g) > u(c_1) - c_1^2 \int_{\pi}^{1/c_1} \underbrace{\left\{ u'(c_1) - u'(c_2(n)) \frac{\partial c_2}{\partial c_1} \mathbb{1}_{\{n \leq \hat{n}\}} \right\}}_{>0} dn.$$

This inequality is clearly satisfied when $g = c_1$, then by continuity the result extends to a small neighbourhood of values of $g < c_1$. For the parameters of my numerical example, simulations for different values of g show that indeed $\frac{\partial \theta_g^*(c_1)}{\partial c_1}> 0$ (see figure 7).

Define by $\Psi(c_1, g) = 0$ the equation implicitly defined by the optimal condition of this problem, equation 4. Notice that $Eu(c_1)$ is quasi-concave on c_1, as it is the

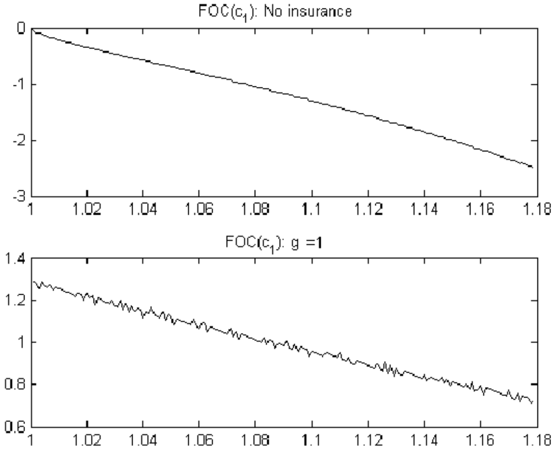

FIG. 8 Optimal value of c_1 without insurance and with $g = 1$.

composition of monotonic and concave functions. Therefore, given that c_1^g – the equilibrium value of the demand deposit contract in the interim period in the model with insurance– maximises $Eu(c_1)$, $\frac{\partial \Psi}{\partial c_1}(c_1^g) = SOC(c_1^g) \leq 0$.

Partial differentiation of equation 4 with respect to g gives $\frac{\partial \Psi}{\partial g} = \frac{\partial \theta_g^*(c_1)}{\partial c_1} u'(g) > 0$. Hence, by the implicit function theorem $\frac{\partial c_1^g}{\partial g} = -\frac{\frac{\partial \Psi}{\partial c_1}(c_1^g)}{\frac{\partial \Psi}{\partial g}} \geq 0$, which justifies the intuitive idea that as $g \leq c_1$, the higher the protection to depositors the less liable are banks for losses, and the higher the interest rate they offer (higher c_1).

Under a narrow mandate, as in the benchmark model, the probability of failure in the interim period is given by

$$p \doteq prob\{n > 1/c_1\} = \theta_g^* + \frac{\varepsilon}{1-\pi}\left(1 + \pi - \frac{2}{c_1}\right).$$

Hence, if g is sufficiently high for $\frac{\partial \theta_g^*}{\partial c_1} > 0$, the probability of failure in the interim period is increasing in c_1 :

$$\frac{\partial p}{\partial c_1} = \frac{\partial \theta_g^*}{\partial c_1} + \frac{2\varepsilon}{(1-\pi)\, c_1^2} > 0$$

On the other hand, for small values of g (limited protection) proposition 5 could be violated, so that $\frac{\partial \theta_g^*}{\partial c_1} < 0$ in which case $\frac{\partial c_1^g}{\partial g} < 0$ and the probability of runs could decrease with the level of insurance:

$$\frac{\partial p}{\partial g} = \frac{\partial \theta_g^*}{\partial g} + \frac{2\varepsilon}{(1-\pi)\, c_1^2}\frac{\partial c_1^g}{\partial g} < 0$$

These results can be summarised in the following theorem.

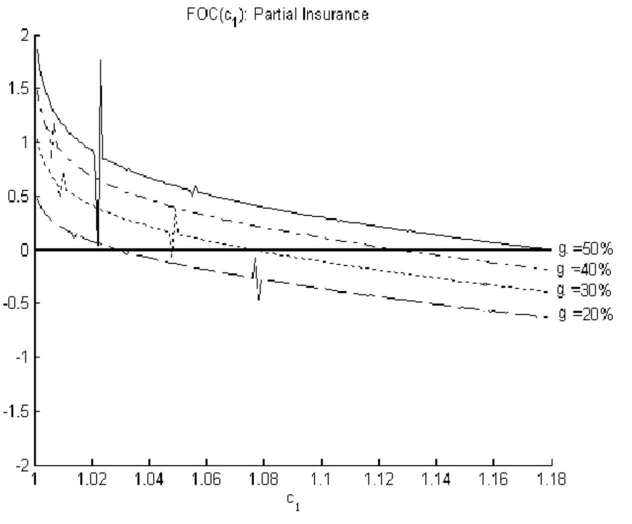

FIG. 9 Limiting moral hazard – optimal value of c_1 with partial insurance.

THEOREM 4. *High levels of deposit insurance increases the equilibrium level of risk sharing offered by banks ($c_1^g > c_1^{ng}$), and therefore the probability of runs. Partial insurance can limit moral hazard.*

Figure 8 plot the FOC for the determination of the optimal demand deposit contract, as described by equations 4 and 5 (in the appendix). Without insurance, the level of risk sharing offered is very small and actually very close to 1. When offering full principal insurance ($g = 1$), the FOC is positive and decreasing for the values of c_1 considered in this simulation, implying the optimal level of risk sharing will be higher. Limited liability and a high level of insurance increase the incentives for the bank to take on more risk.

Partial insurance can limit moral hazard, as seen in figure 9, while improving on intertemporal risk sharing (compared to the non-insurance case).

6.2. Broad Mandate DIC

Consider again $\varepsilon \to 0$, which implies that ξ also vanishes. From section 5, $\lim_{\xi\to 0} s_L(c_1) = \theta_L(c_1)$, $\lim_{\xi\to 0} \underline{s}(c_1) = \underline{\theta}(c_1)$ and $\underline{\theta} \leq \lim_{\varepsilon\to 0} \theta_g^*(c_1) \leq \theta_L(c_1)$. Thus, in the region $\underline{\theta}(c_1) \leq \theta \leq \theta_L(c_1)$, as $\lim_{\varepsilon\to 0^+} n(\theta, \theta_g^*) = \pi$ and $\lim_{\varepsilon\to 0^-} n(\theta, \theta_g^*) = 1$, the closure of insolvent banks is strictly determined by the actions of depositors.

The problem faced by the bank in the planning period is the same as before:

$$\max_{c_1} Eu(c_1) = \int_{\theta_g^*(c_1)}^{1} \{\pi u(c_1) + (1-\pi) u\left(c_2(\theta,\pi)\right)\} d\theta$$

$$+ \pi u(c_1) + (1-\pi) u(c_2\left(\theta_g^*, \pi\right)) - u(g) + \int_0^{\theta_g^*(c_1)} u(g) d\theta,$$

and a sufficient condition for the optimal value of c_1 is given by equation 4.

As mentioned in section 2.1, when ε equals zero inefficient liquidation (panic runs) occurs in the model without insurance for any value of $c_1 > 1$. Therefore, $\theta^*(c_1) > \theta_L(c_1) \geq \theta_g^*(c_1)$.

Knowing this relationship, when abstracting from the effect that changes in the demand deposit contract have on the equilibrium threshold of depositors $\left(\frac{\partial \theta_g^*(c_1)}{\partial c_1} = \frac{\partial \theta^*(c_1)}{\partial c_1} = 0\right)$, the introduction of a deposit guarantee reduces the level of risk sharing offered by banks: $c_1^{ng} > c_1^g$. However, the shifting is stronger in this case.[23]

Hence, the effect of deposit insurance on the optimal demand deposit contract can be decomposed in two factors. When $\frac{\partial \theta_g^*}{\partial c_1} = 0$ its effect is to reduce the level of risk sharing offered by banks with respect to the case without an explicit guarantee. When the impact of changes in c_1 on the probability of runs is included, the opposite effect is observed, and c_1^g increases in an amount related to the level of the promised protection. In the case of a broad mandate the net effect of insurance is expected to be a raise in c_1^g, although smaller than in the case of a narrow mandate, as the first factor (a shift to the left of c_1^{ng}) is stronger.[24]

7. ROBUSTNESS

In what follows, I discuss some policy implication arising from the model, and how the results might change under alternative specifications and assumptions.

7.1. Deposit Insurance Contract (DIC's Planning Period Sub-game)

The choice of the optimal amount of coverage is a relevant issue not discussed in this model, as the value of g was considered an exogenous parameter influencing the outcome of the game. The IMF typically suggests the world average of per capita GDP as a rough rule of thumb for adequate coverage. In practice, however, coverage limits vary widely from country to country. For example, the percentage of the value of deposits covered is almost negligible in Sri Lanka and Estonia, and only about 10 percent in Brazil and Tanzania; but above 65 percent in the USA, and more than 70 percent in Norway, India and Japan (see statistical appendix in Garcia (2000)).

A complete definition of the game (and therefore a full welfare analysis of deposit insurance) would need to specify a payoff function for the DIC, in order to determine the optimal level of insurance offered at the planning period. Such a function should include the effect that a deposit guarantee has on both the depositors' equilibrium threshold and the demand deposit contract offered by banks.

The objectives of deposit insurance usually include the protection of depositors and financial stability concerns. Garcia (2000) argues that because many deposit insurance schemes include all deposit taking institutions, consumer protection is a number one concern. Financial stability, in his opinion, would be the main concern if membership were confined only to systemically important banks. Taking this author's point of view, the DIC should determine g in order to maximise the ex-ante expected utility of

[23] Denote by $\theta_g^N(c_1)$ the equilibrium threshold under a narrow mandate and by $\theta_g^B(c_1)$, the one under a broad mandate. Because $\theta_g^N(c_1) \geq \theta_L(c_1) \geq \theta_g^B(c_1)$, from the proof of proposition 12 in the appendix it can be seen that the integration region in $LHS_{(4)}(c_1)$ is larger in the latter case and includes smaller values of θ, where the function $\varpi(\theta, c_1)$ is negative. Hence, an even smaller value of c_1 is needed in order to make $LHS_{(4)}(c_1^g) = 0$.

[24] c_1^{ng} stands for the equilibrium value of the demand deposit contract in the interim period in the model without insurance.

consumers, while considering a funding constraint (money raised through an ex-post tax, in the case of this model), and the effect that deposit insurance has on the risk of runs and on moral hazard. On the other hand, Rochet (1999) notices that prudential authorities themselves tend to insist more on the prevention of systemic risks as the main issue. Under this view, g should be chosen in order to minimise the probability of joint failure of systemic institutions.

Coverage limits are in practice determined by very complex political processes. In the USA, for example, it has been discussed whether deposit insurance should be indexed to living costs.[25] Indeed, the choice of the limit coverage could become time-inconsistent when facing a systemic crises (the ex-ante chosen value of the guarantee could become ex-post inefficient), particularly so if the insurer has a narrow mandate or if its signal is too noisy under a broad mandate. In either case, some solvent banks would go bankrupt, weakening even more the financial system. The authorities could then decide to temporarily increase the guarantee in order to contain runs, however hard reducing it later to contain moral hazard (Garcia, 2000).

7.2. Credibility and Blanket Guarantees

Blanket guarantees are usually designed in times of crisis to protect the principal value of deposits, in order to enhance market confidence and secure the purchasing power of consumers. When $g = c_1$ runs are completely eliminated, but so is market discipline. This is why deposit guarantees usually cover the principal value of deposits and not the interest accrued on them. The results in this paper show, however, that if $g < c_1$ and the agency in charge of insurance does not get involved in the supervision of banks, depositors will still run on some solvent banks, the more so the lower the guarantee. This result does not obey to a lack of confidence in the insurance scheme, nor to a macroeconomic shock affecting the economy, but it emerges naturally as an equilibrium in a model with asymmetric information, where depositors rationally anticipate the reactions of their counterparts.

For a deposit guarantee scheme to be credible and operational, initial funding is required. The IMF code of best practice establishes that the funding of a deposit insurance scheme should be adequate and perceived as sufficient to maintain public confidence, and that upon failure legal priority over assets should be given to the DIC on behalf of depositors, as this model assumes.

Some countries run schemes with an ex-ante funding, charging participants institutions an insurance premium. Others, as in this model, run ex-post funded systems. Whatever the scheme applied, and given the scope of the losses involved when a bank fails, for insurance to be credible government backing may be needed. Indeed, in the majority of countries with explicit systems, while deposit insurance is privately funded by their member institutions some implicit or explicit government backing always exists.

7.3. Macroeconomic Shocks

If in the interim period depositors were uncertain about the available funds for the payment of the guarantee in the subsequent period, they could precipitate a run. Consider $g < c_1$, the value of the guarantee in the interim period, and $g_2 < g$ its expected value in the final period. It is not difficult to prove that a monotonic equilibrium exists for this game, and that the equilibrium threshold is decreasing in g_2. Therefore, the

[25] See *Alan Greenspan: Deposit Insurance.* Testimony before the Committee on Banking, Housing, and Urban Affairs, U.S. Senate, Washington, D.C., 26 February 2003. BIS Review 10/2003.

smaller the expected insurance in the final period, the more banks would be suffering from runs. Such a situation could arise because of fears of asset depletion (corruption), macroeconomic shocks, or an attack on the currency.

If the economy experienced a macroeconomic shock in the interim period, reducing future returns from $R(\theta)$ to, let say, $R(\theta) - k$, the effect would be twofold. First, the solvency threshold would move upwards, increasing the probability of runs. With more banks suddenly becoming insolvent, the DIC might not have enough resources to cover the payment of the guarantee and, if this were anticipated, more patient depositors would withdraw. Second, the promise of liquidity assistance by a lender of last resort would probably not be enough to overcome this effect, as fewer bank would be bailed out in a model with a broad mandate, because expected returns are lower.

7.4. Twin crisis

The present model considers only one representative bank, therefore, a run on the bank could be identified with a run in the currency. If not enough funds can be readily available for the payment of deposit insurance, its expected value in the last period (g_2, as in section 7.3) might be reduced, raising the equilibrium threshold and therefore runs. Increasing pressure to cover guaranteed deposits could force the government into devaluation, in which case the value of the guarantee in the final period would be further reduced, generating a spiral reaction.

Devaluation could be introduced in this model as in Repullo (2000), where the government provides liquidity assistance (broad mandate) in the form of bonds that can be traded for consumption goods in the interim period, which determines their equilibrium price level. Doubts that the guarantee could be paid in the final period would precipitate a collapse in the price of these bonds, which could be interpreted as a currency crisis. Devaluation could also be modelled as in Chang and Velasco (1999), by linking the equilibrium exchange rate of the economy to the same fundamental of the bank.

7.5. Moral Hazard

The net effect of deposit insurance on the equilibrium demand deposit contract was to raise its value and, for sufficiently high levels of the guarantee, also the risk of runs. This result is consistent with empirical evidence showing that an increase in the volume of insured funds is usually accompanied by a sharp rise in interest rates (e.g. in the Savings and Loan crisis).

Comparative statics showed that moral hazard can be limited by offering partial insurance. Indeed, the IMF code of best practice establishes that limits on coverage should be "*low enough to encourage depositors and sophisticated creditors to monitor and discipline their banks*".

A different source of moral hazard, not covered in this model, is proposed by Bond and Crocker (1993), linking deposit insurance to the level of capitalisation of banks. Their model, though, concentrates on the effect that insurance pricing has on the level of optimal reserves. In the present model banks do not hold reserves, because of the assumptions about the risky technology (that its return in the interim period equals that of the safe asset). Indeed, banks cannot shift to riskier investment projects as a result of the introduction of insurance, simply because these projects are not available. One possibility for studying this phenomenon would be to consider higher costs of early liquidation, in the sense that only a fraction $\mu < 1$ of the original investment could be recovered at $t = 1$. As in that case banks would need to keep reserves to pay impatient depositors withdrawals in the interim period, it would be possible to study how the introduction of a guarantee on deposits would shift the composition of the investment portfolio.

7.6. Closure Rule

Strictly speaking, closure is only one of the possible options for dealing with failed banks. Others involve the intervention or takeover of the institution (transferring the control of the bank to the DIC), the merger or sale of the bank to a stronger institution, purchase an assumption, or a bridge bank for the administration of good assets. In the present model with a single representative bank, many of these options are not viable.

Acharya and Yorulmazer (2007a,b) show that the acquisition of failed banks by stronger institutions can be welfare improving, and even make systemic crisis less likely by reducing the correlation between banks' portfolios. The present model could be extended to study this type of policy in the following way. Consider two ex-ante identical banks that ex-post differ only on the realization of θ. If one bank became insolvent (Bad) while the other is solvent (Good), the DIC (broad mandate) could consider merging both if their combined return were enough to pay patient depositors in both banks at least c_1 in the final period:

$$E_{s_1,s_2}[(1-\pi c_1)R(\theta_G)+R(\theta_B)] \geq \pi c_1 + 2(1-\pi)c_1,$$

where s_j is the DIC's private signal on bank j.

This would require the DIC to provide liquidity assistance to the stronger institution, in order to satisfy patient depositors' redemptions in the failed bank. This would be a different form of LoLR assistance, as the one proposed in Acharya and Yorulmazer (2007a,b). Given the banks have no equity; the "purchase" would effectively be a transfer of assets from the failed bank to the solvent one (if viable).

The equilibrium for the depositors' sub-game will depend on the structure of the signals they receive. If they obtained two equally informative signals (for example, if they held deposits in both banks), anticipating the merger patient depositors in the insolvent institution would prefer to wait. On the other hand, if depositors could monitor only one bank the equilibrium would depend on the order of the game, this is, which bank nature chooses to play first, as in Dasgupta (2002). Otherwise, the DIC would need to act pre-emptively, based solely on its signals and therefore missing the information contained in the number of runs on each bank.

The effect of these policies on moral hazard is not clear.

7.7. Other Extensions

Deposit guarantees are designed to protect small and usually uninformed depositors. This gives a trade-off, because more sophisticated depositors tend not to be covered, and could exercise monitoring power independent of the DIC. Including this type of players in the game could provide an additional explanation for the failure of blanket guarantees sometimes.

Finally, it would be interesting to compare ex-ante versus ex-post funded systems. I have chosen to discuss an ex-post tax funded system, as in Diamond and Dybvig (1983). In practice, some countries do run ex-post funded systems, charging a fee to surviving institutions participating in the scheme (e.g. in the U.K.). If an ex-ante premium were charged to the bank, the equations of the model would be modified. I expect, however, that the main results would not change.

8. CONCLUDING REMARKS

In this paper I have introduced deposit insurance in a model of information based bank runs. The model has a unique equilibrium, which allows for a proper evaluation of the effects of insurance on the behaviour of depositors, banks, and the insurer. I have shown that while consumers achieve better risk sharing in a competitive banking system than in autarky, more solvent projects are liquidated as uninsured depositors fail to coordinate in a subset of fundamentals, and run on banks they know to be solvent. While deposit insurance may prevent panic runs up to some extent, its effectiveness varies with the size of coverage and the degree of supervisory involvement of the agency in charge of insurance. I have considered two possible mandates. Under a narrow mandate, and abstracting from the presence of any other regulatory authority, its main responsibility is to pay the guarantee every time a bank has insufficient resources to cope with withdrawals. Under a broad mandate, the insurer also has responsibility for the resolution of insolvent and/or illiquid banks, and the ability to provide emergency liquidity assistance as a lender of last resort.

Under a narrow mandate, a deposit insurance contract preserving the monitoring role of depositors involves offering less than full protection. The trade off is that panic runs cannot be completely eliminated with a partial guarantee, although it does reduce the region of fundamentals for which that occurs. Under a broad mandate, I showed that panic runs tend to disappear for any level of insurance as the regulator's signal becomes more precise. Given that liquidity assistance is committed to solvent but illiquid institutions, depositors do not run on solvent banks. Moreover, it is cost efficient for the authority never to provide liquidity to insolvent banks. However, only extremely insolvent banks are closed, and those with enough funds to cover the payment of the final period guarantee are allowed to continue in operation. Therefore, the smaller the protection offered to depositors, the higher is forbearance. All these results hold irrespective of the specific values of the guarantee, which in particular might imply the social cost of deposit insurance to be lower under a broad mandate.

Finally, I showed that deposit insurance induces moral hazard by increasing the equilibrium value of c_1, but this effect seems also to be smaller under a broad mandate. Limited insurance could contain moral hazard up to some level, justifying the observed conduct of governments across the world in normal times.

Under a narrow mandate, pure panic runs persist even when depositors' signals become very precise, the more so the lower the guarantee. Under a broad mandate, on the other hand, panic runs are eliminated even with partial insurance, which reduces moral hazard but increases forbearance. Which one should be preferred?

Both mandates are equally popular among economies. In their survey, Demirguç-Kunt and Detragiache (1999) report that 34 out of 67 deposit insurance systems have a narrow constitution, but they also show that the negative externalities imposed by deposit insurance on financial stability can be curbed by effective regulation, a result in line with the main conclusions of this paper. Indeed, during recent years some countries (e.g. France) have stated to move from narrow to broad mandate schemes (Garcia, 2000).

Therefore, a scheme where the DIC has more supervisory involvement (broad mandate), or else a high degree of coordination with the authority in charge of supervision, should be preferred.

9. APPENDIX

Proof of proposition 1

Proof. For any feasible value of c_1, there exists a unique equilibrium threshold $\theta^*(c_1)$ and, therefore, the function determining the number of early withdrawals, $n(\theta, \theta^*(c_1))$, is also uniquely defined. A bank goes bankrupt if and only if depositors run on the bank in the interim period, that is, if and only if $n > 1/c_1$. Define by $\widetilde{\theta}(c_1)$ the value of θ such that $n(\widetilde{\theta}(c_1), \theta^*(c_1)) = 1/c_1$.
As n is strictly decreasing in θ in the region $[\theta^*(c_1) - \varepsilon, \theta^*(c_1) + \varepsilon]$ (or equivalently, for values of n in between π and 1), $n(\theta, \theta^*(c_1)) > 1/c_1$ if and only if $\theta < \widetilde{\theta}(c_1)$ (see figure 1). Therefore, $prob\{n > 1/c_1\} = prob\left\{\theta < \widetilde{\theta}(c_1)\right\} = \widetilde{\theta}(c_1)$, given that θ is uniformly distributed in $[0, 1]$. Using the inverse function of $n(\theta, \theta^*(c_1))$,
$\widetilde{\theta}(c_1) = \theta^* + \dfrac{\varepsilon}{1-\pi}\left(1 + \pi - \dfrac{2}{c_1}\right)$.
Notice that $\left|1 + \pi - \dfrac{2}{c_1}\right| \leq 1 - \pi$ (see figure 1), therefore $0 < \theta^* - \varepsilon \leq \widetilde{\theta} \leq \theta^* + \varepsilon < 1$, and the probability is well defined and non-degenerated. Finally, it increases in c_1 because $\dfrac{\partial\widetilde{\theta}}{\partial c_1} = \dfrac{\partial\theta^*}{\partial c_1} + \dfrac{2\varepsilon}{(1-\pi)(c_1)^2} > 0$. ∎

Proof of proposition 2

Proof. Consider $\theta^1, \ \theta^2 \in [0, 1]$ such that $\theta^1 < \theta^2$. I want to prove that $\Delta_g(\theta^1, \theta^1) < \Delta_g(\theta^2, \theta^2)$.

$$\Delta_g(\theta^j, \theta^j) = \int_{\widetilde{\theta}^j}^{\widehat{\theta}^j} \{u(g) - u(c_1)\}\, d\theta + \int_{\widehat{\theta}^j}^{\theta^j+\varepsilon} \left\{u\left(\frac{1 - n(\theta, \theta^j)c_1}{1 - n(\theta, \theta^j)} R(\theta)\right) - u(c_1)\right\} d\theta,$$
$$j = 1, 2,$$

where $\widetilde{\theta}^j$ is defined by $n(\widetilde{\theta}^j, \theta^j) = 1/c_1$ and satisfies $\widetilde{\theta}^j = \theta^j + \dfrac{\varepsilon}{1-\pi}\left(1 + \pi - \dfrac{2}{c_1}\right)$; and $\widehat{\theta}^j$ is such that $c_2(\widehat{\theta}^j, n(\widehat{\theta}^j, \theta^j)) = g$, $\widetilde{\theta}^j \leq \widehat{\theta}^j \leq \theta^j + \varepsilon$.

$\widehat{\theta}^j$ is uniquely defined, as in the interval $[\theta^j - \varepsilon, \theta^j + \varepsilon]$, $c_2(\theta, n(\theta, \theta^j))$ is strictly increasing in θ. For the same reason it is true that $\widehat{\theta}^1 - \widetilde{\theta}^1 > \widehat{\theta}^2 - \widetilde{\theta}^2$, as when θ^j is higher a relatively smaller value of $\widehat{\theta}^j$ is required for $c_2(\widehat{\theta}^j, n(\widehat{\theta}^j, \theta^j)) = g$.

Finally, notice that from the definition of $n(\theta, \theta^j)$, if $\theta \in [\theta^1 - \varepsilon, \theta^1 + \varepsilon]$, $\theta' \in [\theta^2 - \varepsilon, \theta^2 + \varepsilon]$, and $\theta - \theta^1 = \theta' - \theta^2$ then $n(\theta, \theta^1) = n(\theta', \theta^2)$. This basically establishes that over the intervals $[\widetilde{\theta}^j, \theta^j + \varepsilon]$ $\ j = 1, 2$, the functions $n(\theta, \theta^j)$ take exactly the same values.

This information is sufficient to conclude that the function $\Delta_g(\theta^*, \theta^*)$ is increasing. Although the first integral is higher for $j = 1$ (because the function is constant and the integration region is larger), the argument under the second integral is increasing in θ –remember that n takes the same values in both regions– and the integration region is larger and ranges for higher values of the fundamentals for $j = 2$. Hence, the loss in the former is compensated by the gains in the latter when

$j = 2$, $(c_2(\theta, n(\theta,\theta^2)) > c_2(\theta, n(\theta,\theta^1)) > g)$, implying that $\Delta_g(\theta^2,\theta^2) > \Delta_g(\theta^1,\theta^1)$. ∎

Proof of proposition 5

Proof. Define

$$\Phi(\theta_g^*, c_1, g) \doteq \int_{\pi}^{\widehat{n}} \{u(c_2(n)) - u(c_1)\} dn + \int_{\widehat{n}}^{1/c_1} \{u(g) - u(c_1)\} dn = 0,$$

where $c_2(n) = \dfrac{1-nc_1}{1-n} R(\theta(n))$ and $c_2(\widehat{n}) = g$.

1. $$\frac{\partial \Phi}{\partial \theta_g^*} = \int_{\pi}^{\widehat{n}} u'(c_2(n)) \left(\frac{1-nc_1}{1-n}\right) R'(\theta(n)) dn + \frac{\partial \widehat{n}}{\partial \theta_g^*} \{u(g) - u(c_1)\} - \frac{\partial \widehat{n}}{\partial \theta_g^*} \{u(g) - u(c_1)\} > 0,$$

because by assumption $u(.)$ and $R(.)$ are increasing.

$$\frac{\partial \Phi}{\partial c_1} = \int_{\pi}^{\widehat{n}} \left\{u'(c_2(n))\frac{\partial c_2}{\partial c_1} - u'(c_1)\right\} dn + \frac{\partial \widehat{n}}{\partial c_1} \{u(g) - u(c_1)\}$$
$$- \int_{\widehat{n}}^{1/c_1} u'(c_1) dn + \frac{\partial (1/c_1)}{\partial c_1} \{u(g) - u(c_1)\} - \frac{\partial \widehat{n}}{\partial c_1} \{u(g) - u(c_1)\},$$

where $\dfrac{\partial c_2}{\partial c_1} = \dfrac{-n}{1-n} R(\theta(n)) < 0$.

Simplifying,

$$\frac{\partial \Phi}{\partial c_1} = \int_{\pi}^{\widehat{n}} u'(c_2(n))\frac{\partial c_2}{\partial c_1} dn - \int_{\pi}^{1/c_1} u'(c_1) dn + \left(\frac{1}{c_1}\right)^2 \{u(c_1) - u(g)\}.$$

By the Implicit Function theorem,

$$\frac{\partial \theta_g^*}{\partial c_1} = -\frac{\frac{\partial \Phi}{\partial c_1}}{\frac{\partial \Phi}{\partial \theta_g^*}} > 0 \Leftrightarrow \frac{\partial \Phi}{\partial c_1} < 0$$

$$\Leftrightarrow u(g) > u(c_1) - c_1^2 \int_{\pi}^{1/c_1} \underbrace{\left\{u'(c_1) - u'(c_2(n))\frac{\partial c_2}{\partial c_1} \mathbb{1}_{\{n \le \widehat{n}\}}\right\}}_{>0} dn,$$

where $\mathbb{1}_{\{n \le \widehat{n}\}} = \begin{cases} 1 & if\ n \le \widehat{n} \\ 0 & \sim \end{cases}$.

2. $$\frac{\partial \Phi}{\partial g} = \frac{\partial \widehat{n}}{\partial g} \{u(g) - u(c_1)\} + \int_{\widehat{n}}^{1/c_1} u'(g) dn - \frac{\partial \widehat{n}}{\partial g} \{u(g) - u(c_1)\}$$

$$\Rightarrow \frac{\partial \Phi}{\partial g} = \int_{\widehat{n}}^{1/c_1} u'(g) dn > 0 \Rightarrow \frac{\partial \theta_g^*}{\partial g} = -\frac{\frac{\partial \Phi}{\partial g}}{\frac{\partial \Phi}{\partial \theta_g^*}} < 0$$ ∎

Proof of result 1

Proof. Assume $n \leq 1/c_1$ and compare the costs of the two actions in the interim period:[26]

Close	*Open*	
$g - \max\{R(\theta), 1\}$	0	, *if* $c_2(\theta, n) \geq g$
	$(1-n)g - (1-nc_1)R(\theta)$	, *if not*

Closing the bank means the guarantee has to be paid at a cost equal to $g - \max\{R(\theta), 1\}$. If the bank is left open, and remaining assets are enough to pay depositors at least the value of the guarantee in the final period, this action has no cost to the DIC. However, if funds in the bank are insufficient, the guarantee must be honoured at a cost equal to $(1-n)g - (1-nc_1)R(\theta)$.

A least cost criteria implies that if $c_2(\theta, n) \geq g$ it is better to leave the bank open. Indeed, this rule is Pareto optimal, even if $g - \max\{R(\theta), 1\} \leq 0$ (in which case the DIC should be indifferent between the two actions, as its objective is to minimise the cost of bank resolution, not to make a profit from this operation). In order to see that, look at the welfare of depositors:

Close	*Open*
$u(g)$	$nu(c_1) + (1-n)u(c_2(\theta, n))$

Because $c_1 \geq g$ and $c_2(\theta, n) \geq g$, depositors are better off if the bank is allowed to survive to the final period.

On the other hand, if $c_2(\theta, n) < g$ closure is the least cost solution. Comparing the costs of the two actions:

$$g - \max\{R(\theta), 1\} \leq (1-n)g - (1-nc_1)R(\theta)$$
$$\Leftrightarrow ng \leq \max\{R(\theta), 1\} - (1-nc_1)R(\theta).$$

By contradiction, assume $ng > \max\{R(\theta), 1\} - (1-nc_1)R(\theta)$.

i. If $R(\theta) \geq 1 : ng > R(\theta) - (1-nc_1)R(\theta) = nc_1 R(\theta) \Leftrightarrow 1 \geq \frac{g}{c_1} > R(\theta)$ which is a contradiction.

ii. If $R(\theta) < 1 : ng > 1 - (1-nc_1)R(\theta) \Leftrightarrow R(\theta) > \frac{1-ng}{1-nc_1} \geq 1$, which is again a contradiction.

∎

Proof of proposition 6

Proof. Suppose the bank is fundamentally solvent but illiquid.

$$c_2(\theta, \pi) = \frac{1-\pi c_1}{1-\pi} R(\theta) \geq c_1 \Leftrightarrow (1-\pi c_1)\, R(\theta) - (n-\pi)\, c_1 \geq (1-n)\, c_1.$$

If the DIC provides liquidity assistance for a maximum of $(n-\pi)\, c_1$ in the interim period, the inequality above establishes that the residual return when liquidating only πc_1 units in the interim period minus the repayment of the loan –at zero interest rate–, is enough to secure remaining patient depositors to receive a least c_1 at $t = 1$. In other words, lending money to a fundamentally solvent bank has zero cost for the DIC.

Committing liquidity assistance to fundamentally solvent but illiquid banks is indeed Pareto optimal, in terms of consumers' welfare:

[26] If $n > 1/c_1$ all assets would be liquidated in the interim period, in which case to allow the bank to operate until the second period would not be an option, unless it receives a loan from the central bank. However, a bail out would not be efficient in this case, because the bank is insolvent (see propositions 7, 10 and 11).

i. If $c_2(\theta, n) < g$ it was argued before that the bank should be closed. However, comparing the welfare of depositors it is possible to see that the bank should be bailed out (as this policy has zero cost):

Close		*LoLR + Open*
$u(g)$	$<$	$nu(c_1) + (1-n)u(c_2^L)$ where $c_2^L = c_2(\theta, \pi) \geq c_1 \geq g$

ii. If $g \leq c_2(\theta, n) < c_1$ it was established before that the bank should be allowed to survive until the final period. Comparing the welfare of depositors when just leaving the bank open against the situation where the DIC also provides liquidity assistance:

Open		*LoLR + Open*
$nu(c_1) + (1-n)u(c_2(\theta, n))$	$<$	$nu(c_1) + (1-n)u(c_2^L)$ where $c_2^L \geq c_1 > c_2(\theta, n)$

∎

Proof of proposition 7

Proof. Compare the costs of the two policies for the DIC:

Close	*LoLR + Open*
$g - \max\{R(\theta), 1\}$	$(n-\pi)\, c_1 - \{(1-\pi c_1)\, R(\theta) - (1-n)g\}$

The net cost of bailing out the bank equals the cost of the loan minus whatever asset can be recovered from the bank in the final period.

Closure is the least cost solution if and only if

$$g - \max\{R(\theta), 1\} \leq (n-\pi)\, c_1 - \{(1-\pi c_1)\, R(\theta) - (1-n)g\}$$
$$\Leftrightarrow ng \leq (n-\pi)\, c_1 + \max\{R(\theta), 1\} - (1-\pi c_1)\, R(\theta).$$

By contradiction, assume $ng > (n-\pi)\, c_1 + \max\{R(\theta), 1\} - (1-\pi c_1)\, R(\theta)$.

i. If $R(\theta) \geq 1 : ng > (n-\pi)\, c_1 + R(\theta) - (1-\pi c_1)R(\theta)$
$\Leftrightarrow 0 > n\underbrace{(c_1 - g)}_{\geq 0} + \pi c_1 \underbrace{(R(\theta) - 1)}_{\geq 0}$ which is a contradiction.

ii. If $R(\theta) < 1 : ng > (n-\pi)\, c_1 + 1 - (1-\pi c_1)R(\theta)$
$\Leftrightarrow \underbrace{(1-\pi c_1)}_{>0}\underbrace{(R(\theta) - 1)}_{<0} > n\underbrace{(c_1 - g)}_{\geq 0}$, which is again a contradiction.

∎

Proof of proposition 12

Proof. I want to compare the equilibrium condition for c_1 under a narrow mandate against the one obtained by Goldstein and Pauzner (2000) in the model without insurance:

$$\pi \int_{\theta^*(c_1)}^{1} \left\{ u'(c_1) - R(\theta) u'\left(\frac{1-\pi c_1}{1-\pi} R(\theta)\right)\right\} d\theta$$
$$= \frac{\partial \theta^*(c_1)}{\partial c_1} \left[\pi u(c_1) + (1-\pi) u \left(\frac{1-\pi c_1}{1-\pi} R(\theta^*(c_1))\right) - u(1)\right] \quad (5)$$

Take $\dfrac{\partial \theta_g^*(c_1)}{\partial c_1} = \dfrac{\partial \theta^*(c_1)}{\partial c_1} = 0$. I will prove that $c_1^g < c_1^{ng}$ (where the c_1^{ng} stands for the equilibrium without insurance, and c_1^g for the one with insurance).

Define $\varpi(\theta, c_1) \doteq u'(c_1) - R(\theta)u'\left(\frac{1-\pi c_1}{1-\pi}R(\theta)\right)$. The following result is required.

RESULT 2. $\varpi(\theta, c_1)$ is increasing in θ and decreasing in c_1.

Proof. In order to see that $\varpi(\theta, .)$ is increasing in θ, notice that $\varpi(\theta, .)$ is differentiable and

$$\begin{aligned}\frac{\partial \varpi}{\partial \theta} &= -R'(\theta)u'\left(c_2(\theta,\pi)\right) - R(\theta)u''\left(c_2(\theta,\pi)\right)\left(\frac{1-\pi c_1}{1-\pi}\right)R'(\theta) \\ &= -R'(\theta)\{u'\left(c_2(\theta,\pi)\right) + u''\left(c_2(\theta,\pi)\right)c_2(\theta,\pi)\} \\ &= \underbrace{-\frac{R'(\theta)}{u'(c_2)}}\left\{\underbrace{1 + \underbrace{\frac{c_2 u''(c_2)}{u'(c_2)}}_{<-1}}_{<0}\right\} > 0,\end{aligned}$$

because by assumption $u(.)$ index of relative risk aversion is higher than 1.

In the same way, $\varpi(., c_1)$ is differentiable and

$$\frac{\partial \varpi}{\partial c_1} = u''(c_1) + \frac{\pi}{1-\pi}\left[R(\theta)\right]^2 u''\left(c_2(\theta,\pi)\right) < 0,$$

because $u(.)$ is concave.■

c_1^{ng} is the solution to

$$\pi \int_{\theta^*(c_1^{ng})}^{1} \left\{u'(c_1^{ng}) - R(\theta)u'\left(\frac{1-\pi c_1^{ng}}{1-\pi}R(\theta)\right)\right\} d\theta = 0,$$

the LHS of equation 5 (FOC without insurance).

Evaluating the LHS of equation 4 (FOC with insurance) in c_1^{ng}:

$$\begin{aligned}LHS_{(4)}(c_1^{ng}) = &\underbrace{\pi \int_{\theta^*(c_1^{ng})}^{1} \left\{u'(c_1^{ng}) - R(\theta)u'\left(\frac{1-\pi c_1^{ng}}{1-\pi}R(\theta)\right)\right\} d\theta}_{=0} \\ &+ \int_{\theta_g^*(c_1^{ng})}^{\theta^*(c_1^{ng})} \left\{u'(c_1^{ng}) - R(\theta)u'\left(\frac{1-\pi c_1^{ng}}{1-\pi}R(\theta)\right)\right\} d\theta \\ &+ \pi\left\{u'(c_1^{ng}) - R(\theta_g^*(c_1^{ng}))u'\left(\frac{1-\pi c_1^{ng}}{1-\pi}R(\theta_g^*(c_1^{ng}))\right)\right\}\end{aligned}$$

RESULT 3. For c_1 given, $\theta_g^*(c_1) < \theta^*(c_1)$.

Proof. Proposition 5 can be generalised to different values of the insurance offered in each period. It is easy to prove (by implicit differentiation) that θ_g^* is decreasing in the gap $g_1 - g_2$. As the case without insurance can be described as a particular case where $g_1 = 1/n$ and $g_2 = 0$, $\theta_g^*(c_1) < \theta^*(c_1)$ $\forall g_1 = g_2$. ■

RESULT 4. $\varpi(\theta, c_1)$ increasing in θ implies $\varpi(\theta_g^*(c_1^{ng}), c_1^{ng}) < \varpi(\theta^*(c_1^{ng}), c_1^{ng}) < 0$.

Proof. $\pi \int_{\theta^*(c_1^{ng})}^{1} \varpi(\theta, c_1^{ng})d\theta = 0$ and $\frac{\partial \varpi}{\partial \theta} > 0$, implies that the function $\varpi(., c_1^{ng})$ is not constant and must change of sign in the interval $[\theta^*(c_1^{ng}), 1]$. Being increasing, this means it has to be positive for high values of θ, and negative for small values of θ. In particular, $\varpi(\theta^*(c_1^{ng}), c_1^{ng}) < 0$. ■

From result 4, it follows that $LHS_{(4)}(c_1^{ng}) < 0$. Hence, as $\varpi(\theta, c_1)$ is decreasing in c_1, and $\varpi(\theta, c_1^{ng})$ is negative over the region $[\theta_g^*(c_1^{ng}), \theta^*(c_1^{ng})]$, $LHS_{(4)}(c_1^g) = 0$ if and only if $c_1^g < c_1^{ng}$.

Remember I have assumed $\dfrac{\partial \theta_g^*(c_1)}{\partial c_1} = \dfrac{\partial \theta^*(c_1)}{\partial c_1} = 0$, and then a change in c_1 does not change the limits of integration in the equations. ▮

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