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# TESTING FOR UNIT ROOTS USING ECONOMICS

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## Resumen

Este trabajo considera las implicaciones económicas de tener una raíz unitaria en los procesos estocásticos de variables tales como el consumo o el PIB. Usando una variedad de modelos, se desarrollan tests indirectos de raíz unitaria basados en claras distinciones que deberían aparecer cuando la variable de escala es estacionaria en diferencia o estacionaria en tendencia. Se muestra que estos tests no presentan el indeseable trade-off entre tamaño y poder que caracteriza a los tests tradicionales de raíz unitaria, y se aplican a una variedad de países.

## Abstract

This paper considers the economic implications of having a unit root (UR) on the stochastic process of variables such as consumption or GDP. Using a variety of models, we develop indirect tests for unit roots based on sharp distinctions that should arise when the scale variable is difference stationary (DS) or trend stationary (TS). We show that these tests do not feature the undesirable size-power trade-off that characterizes traditional UR tests and apply them to a range of countries.

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# 1 Introduction

Starting with the influential work of Nelson and Plosser (1982), unit root tests have literally exploded in the literature of econometrics. This is justifiable given the striking statistical differences between series that have a unit root (UR) and those that do not. For instance, while transitory shocks have permanent effects on the levels of difference-stationary series, this is not the case for trend-stationary time series. Furthermore, while difference-stationary time series have unbounded mean square forecast errors, these are bounded for series that are not integrated.

Despite all these differences, as Section 2 will show, statistical tests for the presence of unit roots provide an interesting example of the struggle between size and power, which, as noted by Christiano and Eichenbaum (1990), cannot be settled with finite samples. This is so because in most cases these tests are performed without the aid of economic theory and consist entirely of automated statistical procedures. With economic theory virtually silent, mechanized test results don't appear to conform to established economic principles, yielding results, such as the investment, unemployment, inflation and even interest rates (real and nominal) being characterized as I(1) processes (Rose, 1988).

This paper develops tests for unit roots that combine the empirical implications of general equilibrium models with established econometric results and applies them to several time series processes. Section 2 presents a brief review of the tests for unit roots that are routinely used in the profession and stresses their main weaknesses. Section 3 develops indirect tests for unit roots with the aid of a variety of general equilibrium models. Section 4 provides Monte Carlo evidence comparing traditional unit root tests with those proposed here. Section 5 applies the tests developed for several time series of different countries. Finally, Section 6 concludes.

# 2 Unit Root Tests: A Brief Review

Unit root tests are usually used to evaluate whether a series is better characterized as difference or trend-stationary.[1] Given the important statistical differences between series with a unit root and those that do not, it is often assumed that visual inspection alone can provide a guide with respect to their order. Example 1 shows that this is definitely not the case, given that even when processes share the same realizations of innovations, series that have unit roots and series that do not can behave similarly.

**Example 1 (Visual inspection)** *Consider the following stochastic processes for series y and z:* $y_t = \begin{cases} y_{t-1} + \varepsilon_t \\ 0.9y_{t-1} + \varepsilon_t \end{cases}$ *and* $z_t = \begin{cases} 0.05 + z_{t-1} + \varepsilon_t \\ 0.05t + \varepsilon_t \end{cases}$ *with* $\varepsilon_t \sim N\left(0, 0.1^2\right)$*. In each case, the first processes correspond to a pure random walk and a random walk with drift; while the last correspond to a stationary AR(1) and a trend-stationary*

[1] Hamilton (1994) provides a textbook introduction to this topic.

*process. Figure 1 presents realizations of size 100 for each of these processes, with* $\{\varepsilon_t\}_{t=1}^{100}$ *fixed. We labeled the resulting series as y1, y2, z1 and z2 randomly. Can you tell which have unit roots?*

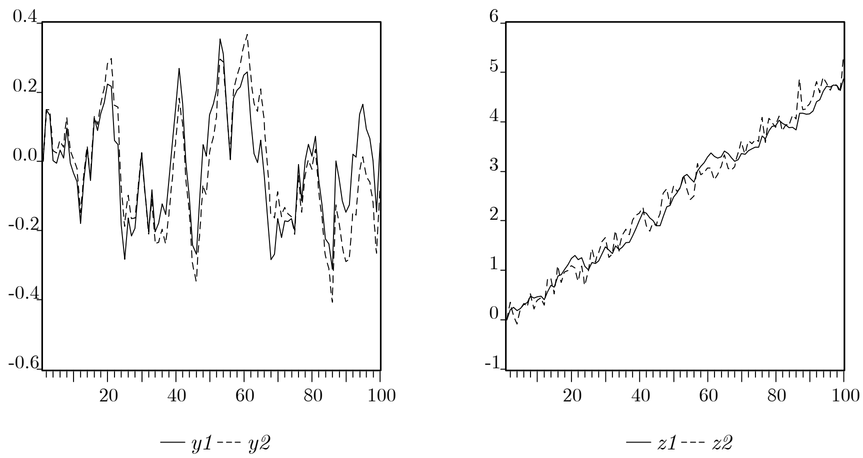


Figure 1: Where are the Unit Roots?

Given that visual inspection can not serve as a guide for distinguishing between series with or without unit roots, a battery of statistical tests has been devised to address this issue.[2] The most popular is the Augmented Dickey-Fuller (ADF) test which is generally used as follows:

$$y_t = \alpha + \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t \tag{1}$$

where $p$ denotes the number of lags necessary in order to make $\varepsilon_t$ white noise.[3] In order to test the hypothesis: $\mathrm{H}_0 : \rho = 1$ (i.e., $y_t$ contains a unit root) one obtains the OLS estimator for $\rho$ $(\widehat{\rho})$ and calculates "$t$-like" statistics for the null. As is well documented (Dickey and Fuller, 1979), the asymptotic distribution of this test is non-standard and if conventional $t$ tests were used, Type I errors would occur.

**Example 2 (Why $t$ tests do not work)** *Consider the following stochastic process for y:* $y_t = 0.1 + y_{t-1} + \varepsilon_t$; $\varepsilon_t \sim N(0,1)$. *Figure 2 presents the results of a Monte Carlo experiment in which 1,000 samples, each of size 500 were generated for y. The*

[2] For those interested, *y2* and *z1* were the series with unit roots in Figure 1.

[3] Oftentimes, (1) includes a linear deterministic trend, but may also consider a non-linear deterministic trend (Bierens, 1997).

*solid line denotes the empirical distribution of the "t-like" test for unit roots, while the dashed line denotes the pdf of a standard normal distribution. The area denoted by A corresponds to the Type I error that would occur if the test were asymptotically normal. The area denoted by C corresponds to the Type I error that would occur if the "correct" critical value were used. Thus the area A+B denotes the size distortion of standard asymptotic tests. The vertical lines denote 5% critical values for a standard normal and the ADF test.*

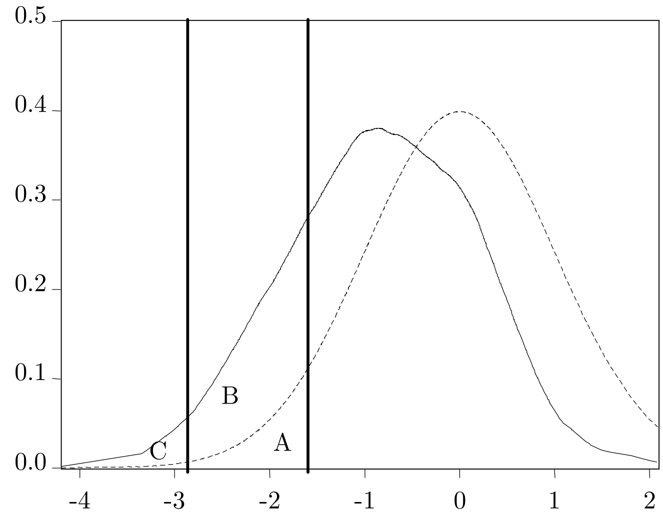


Figure 2: Type I Errors and the ADF Test.

While the above example shows that sizable Type I errors can occur when the "incorrect" critical values are used to test the null of a unit root, ADF tests present important problems in several respects: First, the test is sensitive to the choice of $p$ and variants of the test that allow $\varepsilon_t$ in (1) to be serially correlated are often used (Phillips and Perron, 1988). Second, ADF tests incur in important Type II errors, when the true data-generating-process (DGP) is stationary but close to a unit root (Cochrane, 1991). Still a differenet type of critique revealing the problems of the power of ADF tests arises when the true DGP is that of a series having occasional changes in level or trend (Perron, 1989).

**Example 3 (Break in level and Type II errors)** *Consider the following stochastic process for y:* $y_t = \begin{cases} 0.1 + 0.95 y_{t-1} + \varepsilon_t & \text{for } t \leq T/2 \\ 0.5 + 0.95 y_{t-1} + \varepsilon_t & \text{for } t > T/2 \end{cases}$ ; $\varepsilon_t \sim N(0,1)$. *Figure 3 presents the results of a Monte Carlo experiment in which 1,000 samples, each of size 500 were generated for y. The solid line corresponds to the empirical distribution of the "t-like" test for unit roots. The area denoted by A corresponds to the Type II error* *that would occur if the ADF critical values were used. In this case approximately 90% of the times a unit root would not be rejected. The vertical line represents the 5% critical value for the ADF test.*

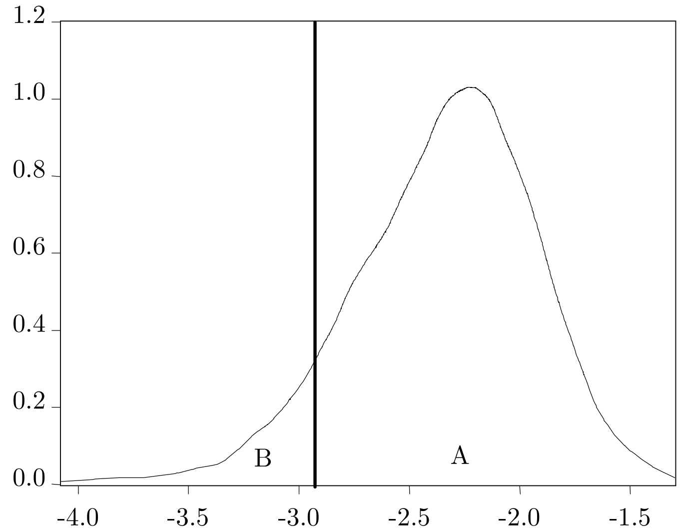


Figure 3: Type II Errors and the ADF Test.

If we use an alternative process more general than (1), like the one just described, ADF tests would tend to conclude (incorrectly) that a unit root was present. On the other hand, Perron's (1989) test for unit roots relies on the assumption that the econometrician knows the precise date of the break. Zivot and Andrews (1992) develop tests that consider the possibility of a break in level and/or trend occurring on an unknown date. Other variants of this test also allow for departures of normality in the innovations.

Murray and Nelson (2000) recently challenged what appeared to be a broad consensus with respect to modelling U.S. GDP as a trend-stationary time series process (with or without breaks) by showing size distortions of tests for unit root, when the true DGP was that of a difference-stationary time series, with additive outliers.

What this brief review indicates is that statistical procedures for testing the presence of unit roots by considering their univariate time series properties reveal a constant struggle between size and power. Although the same could be said about any null hypothesis test, the statistical and economic implications of a unit root make this null particularly important. The following section attempts to show that if economics and not merely statistics is brought into consideration, unit roots have sharp and distinctive implications that can be used to test for their presence indirectly.

# 3 Unit Roots and their Economic Implications

This section describes unit root tests that can be derived from a variety of established general equilibrium models. We begin by deriving tests for real endowment economies and show that most of their implications can be generalized for a variety of dynamic models. The main reason for beginning with simple endowment economies is that they are easy to solve analytically and offer a starting point for evaluating the observable implications of different driving processes.

Starting with Lucas (1978), several papers have considered the properties of asset prices in fully specified endowment economies. To name two, Campbell (1986) and Burnside (1998) describe analytical methods that can be used to solve for asset prices in endowment economies with Gaussian innovations. What is puzzling is that even though these results are well established, to my knowledge no one has used the empirical implications they provide for testing for unit roots. As I will show below, when the endowment is characterized by a difference-stationary process, it presents sharp restrictions on the joint behavior of endowment growth and asset prices, which other stochastic processes (such as a trend-stationary endowment) do not.

## 3.1 A Simple Endowment Economy

Consider a closed endowment economy with a representative agent that is interested in maximizing:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

subject to the budget constraint

$$y_t + \left(1 + r_{t-1}\right) b_{t-1} \geq c_t + b_t$$

where $y_t$ is the level of endowment, $c_t$ is the level of consumption, $b_t$ is the demand for a risk-free private bond paying a net return of $r_t$ in the following period (this return is known at date $t$), $u\left(\cdot\right)$ is strictly increasing and strictly concave, $\beta > 0$ is the subjective discount factor, and $E_t$ denotes the expectation operator conditional on the information available at time $t$.

Under the above conditions, the gross return on this asset is given by:

$$\left(1 + r_t\right) = \frac{1}{\beta E_t \left(\frac{u'(y_{t+1})}{u'(y_t)}\right)} \tag{2}$$

which simply states that the asset's gross return is a function of the intertemporal marginal rate of substitution (stochastic discount factor).

Consider now the special case of (2), in which we impose a Constant Relative Risk Aversion (CRRA) utility function with the Arrow-Pratt relative risk aversion coefficient denoted by $\gamma$ (inverse of the intertemporal elasticity of substitution). Then (2) can be expressed as:

$$(1+r_t) = \frac{1}{\beta E_t \left(\frac{y_{t+1}}{y_t}\right)^{-\gamma}} \tag{3}$$

Note that to determine the return on this asset we need to solve (3) by explicitly introducing a law of motion for the endowment process. Take two such cases: the first assumes that the (log of the) endowment process is difference-stationary (DS), and the second that it is trend-stationary (TS):

$$\begin{aligned} &\text{DS}: \Delta \ln y_t = \alpha + \sum_{i=1}^{k} \delta_i \Delta \ln y_{t-i} + \varepsilon_t, \text{ where } \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right) \\ &\text{TS}: \ln y_t = \eta + \alpha t + \sum_{i=1}^{l} \delta_i \ln y_{t-i} + v_t, \text{ where } v_t \sim N\left(0, \sigma_v^2\right) \end{aligned} \tag{4}$$

where $\varepsilon$ and $v$ are innovations and $k$ and $l$ denote the number of lags necessary to produce them. Under these assumptions, the return of the asset is:

$$\ln(1+r_t) = \begin{cases} a_\varepsilon + \gamma \sum_{i=1}^{k} \delta_i \Delta \ln y_{t+1-i} & \text{(DS)} \\ a_v + \gamma \sum_{i=1}^{l} \delta_i \Delta \ln y_{t+1-i} - \gamma v_t & \text{(TS)} \end{cases} \tag{5}$$

where $a_j = \alpha\gamma - \ln\beta - 0.5\gamma^2\sigma_j^2$ for $j = \varepsilon, v$.

Several important implications of (5) merit attention: first, independently of the source of non-stationarity of the endowment process, the real interest rate is stationary. Second, when the endowment process is DS, the real interest rate is a deterministic function of contempory and past values for the growth rate of the endowment, while for the TS process a white noise innovation has to be considered. Third, in either case, the real interest rate Granger causes the endowment growth.[4] Finally, the univariate representation of $\ln(1+r_t)$ can be characterized as an ARMA$(k, k-1)$, when the endowment is DS, and as an ARMA$(l, l-1)$ process, when the endowment is TS.[5]

To conclude, this example shows that the relationship between real interest rates and endowment growth rates can be used to construct an indirect test for unit roots in the endowment. This is true despite the fact that the univariate representations for the real interest rate with DS or TS endowment processes may be observationally equivalent. The theory predicts that when the endowment is DS, the real interest rate is a deterministic function of present and past growth rates.

[4] The reader can verify that if (5) is used in (4), the endowment process can be characterized as $\Delta \ln y_{t+1} = \alpha - a_j/\gamma + \ln(1+r_t)/\gamma + j_{t+1}$ for $j = \varepsilon, v$.

[5] This is readily verifiable once we substitute the result obtained on footnote 4 back to (5).

### 3.2 Breaks and Additive Outliers

As discussed in Section 2, conventional unit root tests lead to serious Type I errors when the true data-generating process (DGP) corresponds to a (trend-) stationary time series with breaks in levels (or trend). On the other hand, as recently noted, unit root tests that consider the alternative of trend-stationarity with breaks in levels lack of power when the true DGP is that of a DS time series with additive outliers (Murray and Nelson, 2000).[6]

Next, we show that the difficulties in distinguishing processes with traditional unit root tests disappear once we consider their impact on variables like the real interest rate. We continue to focus on endowment economies and allow for the possibility of a break in level in both processes.[7]

If we modify (4) to allow for a break in level in period $t = T_0$, the laws of motion for the endowment are:

$$\begin{aligned} &\text{AO}: \Delta \ln y_t = \alpha + \sum_{i=1}^{k} \delta_i \Delta \ln y_{t-i} + \zeta O_t + \varepsilon_t, \text{ where } \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right) \\ &\text{BL}: \ln y_t = \eta + \alpha t + \sum_{i=1}^{l} \delta_i \ln y_{t-i} + \vartheta L_t + v_t, \text{ where } v_t \sim N\left(0, \sigma_v^2\right) \end{aligned} \tag{6}$$

where $O$ is one in period $T_0$ and zero otherwise, and $L$ is zero for $t < T_0$ and one thereafter.

It is important at this juncture to mention that the data available to the econometrician and the agents of our theoretical economy, may differ. As Zivot and Andrews (1992) discuss, the date of an eventual break in level is generally not known to the econometrician and tests must be constructed accordingly, while agents may make their decisions based on different information. At this point, I will assume that the agents know the precise date of the break in level and I will discuss its effects in section 3.3.

Given the laws of motion described in (6), once again we use (3) to solve for the one period real interest rate and obtain:

$$\ln\left(1 + r_t\right) = \begin{cases} a_\varepsilon + \gamma \sum_{i=1}^{k} \delta_i \Delta \ln y_{t+1-i} + \gamma \zeta O_{t+1} & \text{(AO)} \\ a_v + \gamma \sum_{i=1}^{l} \delta_i \Delta \ln y_{t+1-i} + \gamma \vartheta \Delta L_{t+1} - \gamma v_t & \text{(BL)} \end{cases} \tag{7}$$

As in the simple case described in (5), difference- and trend-stationary endowment processes differ significantly in how their corresponding real interest rates are

[6] AO denotes a unit root process with an additive outlier and BL a trend-stationary process with a break in level.

[7] A trivial extension that does not change the fundamental conclusions corresponds to the case in which we allow for a break in trend.

determined. The only difference in cases in which additive outliers or breaks in levels are allowed is when the interest rate shifts proportionately one period before the break occurs, but the properties of the relationship between the interest rate and the growth rate would otherwise remain unaltered.[8] That is, in the case of a DS endowment process, we would still find a deterministic relationship between the interest rate and present and past values for the growth rate of endowment once we include a one-period dummy. If the econometrician does not know the date of the potential break, sequential tests can be performed for this purpose.

### 3.3 Markov Switching Regimes

The previous setup assumed the endowment process was subject to a break in level both in the DS and the TS processes. An increasingly popular specification considers processes like GDP better characterized as subjected to Markov Switching regimes (Hamilton, 1994).

To evaluate the effects of such specifications, we consider the case in which the current state of the economy is observed by the agent but not by the econometrician. In this case, trivial extensions of (7) can be performed to accommodate Markov Switching Regimes. For that purpose, consider that $O_t$ and $L_t$ from (6) are now indicator functions that take the value of one when the state is, say, $s_t = 1$ and zero otherwise (when $s_t = 0$). Given a $2 \times 2$ matrix of transition probabilities $P$, with $p_{i,j}$ denoting the probability that state $j$ will follow state $i$ ($\Pr[s_{t+1} = j | s_t = i]$), the real interest rates in these economies would correspond to:[9]

$$\ln(1 + r_t) = \begin{cases} a_\varepsilon + \gamma \sum_{i=1}^{k} \delta_i \Delta \ln y_{t+1-i} - \ln\left(p_{i,0} + p_{i,1} e^{-\gamma\zeta}\right) & \text{(MSD)} \\ a_v + \gamma \sum_{i=1}^{l} \delta_i \Delta \ln y_{t+1-i} - \gamma v_t - \ln n_t & \text{(MST)} \end{cases} \quad (8)$$

where:

$$n_t = \begin{cases} p_{0,0} + p_{0,1} e^{-\gamma\vartheta} & \text{if } s_t = 0 \\ p_{1,1} + p_{1,0} e^{\gamma\vartheta} & \text{if } s_t = 1 \end{cases}$$

Once again, the main difference between processes with and without unit roots is that in the case of the former the real interest rate is a deterministic function of present and past growth rates of the endowment process. Thus, even when the econometrician may not directly observe $s_t$, when a unit root is present it can easily be recovered by noticing that there are only two possible values for the last term

[8] In the case of series with breaks in trends, the difference from (7) would be that the real interest rate would have a permanent change in level beginning in period $T_0 - 1$.

[9] MSD (MST) denotes the difference- (trend-) stationary Markov switching process, with the state being observable for the agent. This example can be easily generalized for an $n$-state, $k$th-order Markov Switching Process.

in the first expression in (8). Furthermore, it would be expected that the breaks in level in the DS process should be infrequent, in which case $p_{1,1}$ would be small. At any rate, just as in the previous cases, indirect tests for unit roots in the endowment process are based solely on their time series implications for the interest rate, thus the econometrician would not even need to estimate the law of motion of the endowment process. In fact, this indirect test would need to include all possible combinations of dummy variables for every period, to take into account the changes in the constant term in case a unit root is present.[10]

One case not considered here would be if neither the agent nor the econometrician directly observes the current state $s$; in that case, both the agent and the econometrician could presumably use a filter such as the one described in Kim and Nelson (1999) to infer it. In that case, the last expressions in (8) should be modified to include the probability of ending in different states using the information available for each period. We will return to this subject in section 4.

### 3.4 Monetary Economies

Up to this point, our indirect tests were based on the time series implications of a DS or TS endowment process for the real interest rate. Nevertheless, a risk-free perfectly indexed instrument does not exist. Thus, while instructive, the previous tests would be difficult to conduct with actual data. Given this, we next consider how nominal and *ex-ante* real interest rates for a risk-free one period nominal bond would be set in the presence of alternative stochastic processes for the endowment.

As customary with monetary economies and time separable utility functions, the nominal interest rate ($i_t$) for a one period risk-free bond that is redeemed in period $t{+}1$ satisfies:

$$(1+i_t)^{-1} = \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \frac{P_t}{P_{t+1}} \right] \tag{9}$$

where $P_t$ denotes the price level on period $t$.

If nominal interest rates are positive, in order for money to be valued in equilibrium, it must play a role that nominal bonds do not. If we assume (as in Lucas, 1980) that it is needed to buy goods, and impose a cash-in-advance (CIA) constraint of the form $M_t \geq P_t c_t$, it will hold with equality when $i_t > 0$. If, as in the previous case, we consider a CRRA utility function, (9) can be expressed as:

$$(1+i_t)^{-1} = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{1-\gamma} \frac{M_t}{M_{t+1}} \right] \tag{10}$$

[10] This is easier said than done, given that for a sample size $T$, there are $\sum_{i=0}^{T} \binom{T}{i} = 2^T$ possible combinations of such dummy variables. Given that half of them are redundant, we would "only" need to consider $2^{T-1}$ of such combinations. For a sample size of say $T$=25, this amounts to almost 17 million combinations.

In order to solve for (10) we need to describe the laws of motion for $c$ and $M$. If we consider an endowment economy with either no government expenditures (where the revenue that arises from seigniorage is returned via lump sum transfers) or where government expenditures are a fixed fraction of the endowment, the law of motion of the (log of the) endowment process and of the (log of the) consumption process will coincide up to a proportionality constant. Here we also consider two possible cases for these laws of motion; one in which both $c$ and $M$ are DS and another in which they are TS and denote these cases as CDS and CTS respectively.[11]

$$\text{CDS}: \begin{bmatrix} \Delta \ln c_t \\ \Delta \ln M_t \end{bmatrix} = \begin{bmatrix} \alpha_c + A(L)\Delta \ln c_t + B(L)\Delta \ln M_t \\ \alpha_M + C(L)\Delta \ln c_t + D(L)\Delta \ln M_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{M,t} \end{bmatrix}$$
$$\text{CTS}: \begin{bmatrix} \ln c_t \\ \ln M_t \end{bmatrix} = \begin{bmatrix} \alpha_c t + A(L)\ln c_t + B(L)\ln M_t \\ \alpha_M t + C(L)\ln c_t + D(L)\ln M_t \end{bmatrix} + \begin{bmatrix} v_{c,t} \\ v_{M,t} \end{bmatrix}$$

where $A(L), B(L), C(L), D(L)$ are lag polynomials and

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{M,t} \end{bmatrix} \sim N(0, \Sigma_\varepsilon) \text{ and } v_t = \begin{bmatrix} v_{c,t} \\ v_{M,t} \end{bmatrix} \sim N(0, \Sigma_v)$$

Defining $\mu_t$ as $E_t \begin{pmatrix} \Delta \ln c_{t+1} & \Delta \ln M_{t+1} \end{pmatrix}'$ and $a'$ as $\begin{pmatrix} 1-\gamma & -1 \end{pmatrix}$, we can solve (10) explicitly:

$$\ln(1+i_t) = \begin{cases} -\ln\beta - a'\mu_t - \frac{1}{2}a'\Sigma_\varepsilon a & \text{(CDS)} \\ -\ln\beta + a'\mu_t + \frac{1}{2}a'\Sigma_v a & \text{(CTS)} \end{cases} \tag{11}$$

where

$$\mu_t = \begin{cases} \begin{bmatrix} \alpha_c + A(L)\Delta \ln c_{t+1} + B(L)\Delta \ln M_{t+1} \\ \alpha_M + C(L)\Delta \ln c_{t+1} + D(L)\Delta \ln M_{t+1} \end{bmatrix} & \text{(CDS)} \\ \begin{bmatrix} \alpha_c + A(L)\Delta \ln c_{t+1} + B(L)\Delta \ln M_{t+1} - v_{c,t} \\ \alpha_M + C(L)\Delta \ln c_{t+1} + D(L)\Delta \ln M_{t+1} - v_{M,t} \end{bmatrix} & \text{(CTS)} \end{cases}$$

Once again, an important feature that emerges is that when $c$ is DS, the nominal interest rate becomes a deterministic function of present and past values of the growth rates of consumption and money. When $c$ is TS, the relationship is stochastic.

Furthermore, notice that the inflation rate in period $t+1$ ($\pi_{t+1}$) can be derived from the CIA constraint:

$$(1+\pi_{t+1}) = \frac{M_{t+1}}{M_t}\frac{c_t}{c_{t+1}}$$

[11] Trivial extensions that will not affect the essence of our results may consider the situation where one is TS and the other DS.

in which case, the expected inflation $\left(\pi^e_{t+1}\right)$ can be derived as in (11) by defining $b'$ as $\begin{pmatrix} -1 & 1 \end{pmatrix}$ and obtaining:

$$\ln\left(1+\pi^e_{t+1}\right) = \ln E_t\left(1+\pi_{t+1}\right) = b'\mu_t + \frac{1}{2}b'\Sigma_i b \quad \text{for } i=\varepsilon, v \tag{12}$$

Finally, combining (11) and (12) we derive the *ex-ante* real interest rate $(r^e_t)$ for each economy as $\ln\left(1+r^e_t\right) = \ln\left(1+i_t\right) - \ln\left(1+\pi^e_{t+1}\right)$.

$$\ln\left(1+r^e_t\right) = \begin{cases} -\ln\beta + \gamma g_t - d'\text{vech}\left(\Sigma_\varepsilon\right) & \text{(CDS)} \\ -\ln\beta + \gamma g_t - d'\text{vech}\left(\Sigma_v\right) - \gamma v_{c,t} & \text{(CTS)} \end{cases} \tag{13}$$

with $g_t = \left(\alpha_c + A(L)\Delta\ln c_{t+1} + B(L)\Delta\ln M_{t+1}\right)$, $d' = \begin{pmatrix} 1-\gamma+\frac{1}{2}\gamma^2 & \gamma-2 & 1 \end{pmatrix}$, and vech($\Sigma_i$) denoting the 3×1 vector that is obtained from vertically stacking those elements on or below the principal diagonal of $\Sigma_i$ $(i=\varepsilon, v)$.

Just as in the previous cases, there is a significant difference between DS and TS processes for how the *ex-ante* real interest rate is determined. In the first case, it is still a deterministic function of present and past values of the growth rate of consumption and money, while in the later, a white noise term must be added.

How would our results change if we considered a more general and realistic setup, in which money is not demanded solely based on CIA constraints or when consumption and output are endogenously determined? Even if we cease to consider endowment economies, equation (9) would still be satisfied and in equilibrium will determine the nominal interest rate. We can modify our analysis by considering how agents form their expectations about the growth rate of consumption and inflation, without explicitly solving the policy function of consumption or considering the law of motion of the inflation rate (which is endogenously determined). Let us assume that agents form their expectations according to a VAR process of the form:[12]

$$\text{GDS}: \begin{bmatrix} \Delta\ln c_t \\ \Delta\ln P_t \end{bmatrix} = \begin{bmatrix} \alpha_c + A(L)\Delta\ln c_t + B(L)\Delta\ln P_t \\ \alpha_P + C(L)\Delta\ln c_t + D(L)\Delta\ln P_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{P,t} \end{bmatrix}$$

$$\text{GTS}: \begin{bmatrix} \ln c_t \\ \ln P_t \end{bmatrix} = \begin{bmatrix} \alpha_c t + A(L)\ln c_t + B(L)\ln M_t \\ \alpha_M t + C(L)\ln c_t + D(L)\ln P_t \end{bmatrix} + \begin{bmatrix} v_{c,t} \\ v_{P,t} \end{bmatrix}$$

If we consider a CRRA utility function, and define $\mu_t$ as $E_t\begin{pmatrix} \Delta\ln c_{t+1} & \Delta\ln P_{t+1} \end{pmatrix}'$ and $a'$ as $\begin{pmatrix} -\gamma & -1 \end{pmatrix}$, we obtain a expression that is identical to (11) except that in this case $\mu'_t = E_t\begin{pmatrix} \Delta\ln c_{t+1} & \Delta\ln P_{t+1} \end{pmatrix}$ and $\Sigma_i$ is the variance-covariance matrix of the innovations of the VAR just described. Thus, even in this setup, the difference between DS and TS processes for consumption remains the fact that in the former case a deterministic relationship exists between nominal interest rates, growth rates of consumption and the price level. A similar conclusion can be reached for the *ex-ante* real interest rate following the steps taken to obtain (13).[13]

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[12] We denote by GDS (GTS) to the VAR process that is consistent with a DS (TS) process for consumption. Given that the error terms must be innovations, we could also include other "exogenous" variables to the VAR without modifying the spirit of our conclusions.

[13] In this section we assumed that when $\ln c$ was DS (TS), $\ln M$ and/or $\ln P$ shared this charac-

# 4 Finite Sample Properties of UR Tests

The previous section presented a wide variety of models for which the time series implications for variables such as the real interest rate, nominal interest rate, and *ex-ante* real interest rate differ significantly depending on whether the scale variable (GDP or consumption) is DS or TS. Here, we present the finite sample properties of traditional unit root tests and compare them with those of the indirect tests just described. For doing so, we conducted several Monte Carlo experiments with different specifications for the forcing variables.

The Monte Carlo experiments consisted of generating 10,000 samples, each of size $T$=172, which corresponds to a sample size comparable to that of quarterly observations of U.S. GDP from 1957:1 to 1999:4. Each Monte Carlo experiment used a data generating process broadly consistent with the specifications discussed in the previous section.

To make these results comparable with other Monte Carlo experiments, each specification was estimated using quarterly data from the U.S. and treated as the true DGP for simulating artificial samples. Table 1 reports some of these specifications.[14]

| | Specification |
|---|---|
| DS | $\alpha = \underset{(0.001)}{0.005};\ \delta_1 = \underset{(0.073)}{0.320}$ |
| TS | $\alpha = \underset{(0.0001)}{0.00026};\ \delta_1 = \underset{(0.078)}{1.297}; \delta_2 = \underset{(0.079)}{-0.331}$ |
| AO | $\alpha = \underset{(0.001)}{0.005};\ \delta_1 = \underset{(0.070)}{0.331};\ \zeta = \underset{(0.009)}{0.032}O_{78:2}$ |
| BL | $\alpha = \underset{(0.0001)}{0.00033};\ \delta_1 = \underset{(0.071)}{1.250};\ \delta_2 = \underset{(0.071)}{-0.295};\ \vartheta = \underset{(0.001)}{0.004}L_{58:2}$ |
| MSD | $\alpha = \underset{(0.005)}{-0.012};\ \delta_1 = \underset{(0.081)}{0.247};\ \zeta = \underset{(0.001)}{0.007};\ p_{0,0} = \underset{(0.270)}{0.349};\ p_{1,0} = \underset{(0.030)}{0.047}$ |
| MST | $\alpha = \underset{(0.0001)}{0.0003};\ \delta_1 = \underset{(0.070)}{1.297};\ \delta_2 = \underset{(0.069)}{-0.331};\ \vartheta = \underset{(0.001)}{0.0016};\ p_{0,0} = \underset{(0.387)}{0.893};\ p_{1,0} = \underset{(0.032)}{0.049}$ |

Table 1: Selected Specifications Used in Monte Carlo Experiments

Once DGPs consistent with the different models described on the previous sections were defined, 10,000 artificial samples were generated for each, yielding simulated values for GDP, GDP growth, and interest rates.[15] Using these values, we first

teristic; the results of the tests described in the next section would not be affected if one were TS and the other DS. Additive outliers, breaks in trends, and Markov switching processes can also be included and use tests that are similar to the ones described above.

[14]The specifications used to obtain DGPs for the models consistent with CDS, CTS, GDS, and GTS are not reported due to space limitations. The VAR models were either VAR(2) or VAR(4).

[15]The specifications reported in Table 1 were consistent with real economies, thus only real rates were simulated from them. The models that used VAR processes for GDP, money and/or prices simulated nominal and *ex-ante* real rates.

conducted traditional unit root tests by using only the simulated values of GDP. Four of such tests were conducted: ADF tests (Dickey and Fuller, 1979), PP tests (Phillips and Perron, 1988), KPSS tests (Kwiatowski et al., 1992), and ZA tests (Zivot and Andrews, 1992).

As discussed in previously, these tests use univariate statistical representations of the scale variable and conduct hypothesis testing based solely on them. As we showed, when a UR is present, the interest rate (either real or nominal) must be deterministically associated with the growth rate of the scale variable, and in monetary economies with either the growth rate of money or inflation. Thus, our indirect tests are rather simple; depending on the particular model considered, if a UR is present, a regression of the interest rate and present and past values of growth rates of the scale, money and/or inflation must fit perfectly; that is the $R^2$ of such a regression must be equal to 1. Thus, all indirect tests search for the maximum value of $R^2$ using the other variables as regressors. Where the true DGP is that of a DS process with an additive outlier, we introduce a dummy variable, $O$, that takes the value of 1 on each possible date, and recover the maximum $R^2$ after considering all possible dates. Table 2 describes each test.

| | Specification | $H_0$ |
|---|---|---|
| ADF | $y_t = \eta + \alpha t + \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \varepsilon_t$ | DS |
| PP | $y_t = \eta + \alpha t + \rho y_{t-1} + \varepsilon_t$ | DS |
| KPSS | $y_t = \eta + \alpha t + \varepsilon_t$ | TS |
| ZA | $y_t = \eta + \alpha t + \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \zeta O_t + \varepsilon_t$ | DS |
| Indirect Tests | $\ln(1 + r_t) = \eta + \sum_{i=1}^{p} \delta_i \Delta y_{t+1-i} + \varepsilon_t$ | DS |

Table 2: Unit Root Tests. The dependent variable in the case of indirect tests is either the real interest rate, the nominal interest rate or the ex-ante real interest rate and the regressors are the growth rate of the endowment and money or prices, depending on the model considered. The test considers the specification with the best fit.

The only specification where a simple algorithm for conducting the test is not available is when the true DGP corresponds to the case of MSD. As already discussed, in that case there are literally millions of combinations that should be considered in order to perform the UR test. In this case, we opted for a compromise in which genetic algorithms were used to search for the highest $R^2$. For each sample, we randomly selected 500 possible combinations of sequences of dummy variables and used traditional cross-over, mutation and selection procedures to look for the best possible sequence of dummies with which to run the regressions.[16] Given that in this case, even when a UR is present it is almost impossible to obtain the perfect fit that the theory predicts, our Monte Carlo experiments allow us to obtain critical values for the test. The critical value for the $R^2$ at a 5% significance level was 0.85. Thus,

[16] Bauer (1994) presents a textbook introduction to Genetic Algorithms.

only when the $R^2$ exceeds this value, we would not reject the null hypothesis of a UR.

| | Direct Tests | | | | Indirect Tests | |
|---|---|---|---|---|---|---|
| | ADF | PP | KPSS | ZA | R | |
| DS | 0.056 | 0.061 | **0.288** | 0.057 | 0.000 | |
| TS | **0.157** | **0.158** | 0.091 | **0.115** | **1.000** | |
| AO | 0.051 | 0.055 | **0.269** | 0.064 | 0.000 | |
| BL | **0.092** | **0.110** | 0.201 | **0.112** | **1.000** | |
| MSD | 0.055 | 0.055 | **0.299** | 0.103 | 0.050 | |
| MST | **0.144** | **0.142** | 0.094 | **0.111** | **0.930** | |
| | ADF | PP | KPSS | ZA | I | Ex |
| CDS | 0.051 | 0.055 | **0.288** | 0.058 | 0.000 | 0.000 |
| CTS | **0.230** | **0.234** | 0.055 | **0.150** | **1.000** | **1.000** |
| GDS | 0.042 | 0.048 | **0.291** | 0.047 | 0.000 | 0.000 |
| GTS | **0.113** | **0.112** | 0.120 | **0.090** | **1.000** | **1.000** |

Table 3: Size and Power of Tests. For specifications AO and BL, sequential dummy variables were included and the best model considered. For MSD and MST the best model was obtained using genetic algorithms as optimization routines. For the models CDS(GDS) and CTS(GDS) the I test corresponds to regressions of the nominal interest rate and the growth rates of the scale and money (prices). The tests denoted by Ex use identical specifications as the I tests, but in these cases the dependent variable is the ex-ante real interest rate.

Table 3 reports the size and power of each of the tests performed. Numbers reported in bold characters denote the power of a test, while numbers in normal characters report the size. As can be observed, traditional UR tests indicate the problems outlined in section 2; in particular, even though these tests show no important size distortions, their lack of power is astonishing. On the other hand, the only test that considered that the process was TS under the null (KPSS) has not only a considerable lack of power (although not as notorious as the tests based on the null of DS), but also presents important size distortions. This last feature is not new and remains present even when the critical values reported in Rothman (1997) are used.

On the other hand, the indirect tests developed in this paper, show no size or power distortions. Given that in all cases the theoretical model predicts that when a UR is present, interest rates and growth rates of the scale and money or prices should present a perfect fit, the indirect tests would only recognize a UR when such a relation is present.

# 5 UR Tests in Practice

The usefulness of the tests can be assessed once we confront them with actual data. Given that some of the indirect tests require for the econometrician to observe real interest rates, I proxied them by using actual ex-post real rates of nominal risk free bonds for seven countries, and used two measures for such a rate: Treasury Bills and 90-day deposit rates. As some of the tests use *ex-ante* real interest rates, we obtained them by estimating ARMA models for the inflation rate, and used one-step-ahead forecasts to obtain our estimates of the *ex-ante* real rate.

| | ADF | PP | KPSS | ZA | $T$ |
|---|---|---|---|---|---|
| United States | -2.34 | -1.99 | **0.31** | -3.95 | 172 |
| United Kingdom | -2.14 | -2.22 | **0.21** | -3.60 | 172 |
| Switzerland | -2.41 | -2.08 | **0.13** | -3.75 | 140 |
| Sweden | -2.50 | **-3.46** | **0.15** | -3.75 | 124 |
| Japan | -1.22 | **-3.71** | 0.47 | -2.68 | 172 |
| Canada | -0.97 | -0.69 | 0.47 | -2.72 | 172 |
| Australia | -2.20 | -2.01 | 0.33 | **-4.81** | 162 |

Table 4: Traditional Unit Root Tests for GDP

Table 4 reports the results of applying traditional UR tests to the GDP of each country included. Where values are reported in bold characters, a UR is rejected. Once again, if we relied merely on these tests it would be difficult to conclude anything from them for most of the countries considered. Furthermore, TS alternatives are usually preferred even in countries where a UR is not rejected in all four tests, when using less a conservative level (for example, if the ZA test considered a 10% level, only Japan and Canada would not reject the null).

On the other hand, Table 5 shows that in all cases, the theoretical relationship predicted by different models is very different from actual data. In particular, in all countries with relatively long time series, the corresponding values for $R^2$ are rather small, providing strong evidence against the UR hypothesis for GDP.

It may be rightly claimed that these results depend crucially on the underlying economic structure that produced them. If actual economies differed significantly from the stylized economies considered, these tests would not be valid if taken at face value. Two considerations merit attention when evaluating this claim: first, several recent papers have taken the opposite route from the one taken here. For example, Lau (1997) and Lau (1999) show that endogenous growth models are consistent with

| | 90-day Deposit Rate | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | R | RAO | RM | IM | IP | ExM | ExP | $T$ |
| United States | **0.09** | **0.15** | **0.20** | **0.13** | **0.57** | **0.01** | **0.23** | 148 |
| United Kingdom | **0.19** | **0.30** | **0.33** | **0.32** | **0.26** | **0.10** | **0.54** | 168 |
| Switzerland | **0.23** | **0.37** | **0.38** | **0.40** | **0.64** | **0.35** | **0.35** | 76 |
| Sweden | **0.18** | **0.24** | **0.34** | **0.25** | **0.55** | **0.32** | **0.26** | 124 |
| Japan | **0.22** | **0.47** | **0.49** | **0.18** | **0.58** | **0.40** | **0.61** | 172 |
| Canada | **0.16** | **0.23** | **0.30** | **0.16** | **0.53** | **0.18** | **0.32** | 116 |
| Australia | **0.11** | **0.20** | **0.24** | **0.06** | **0.26** | **0.02** | **0.38** | 109 |
| | Treasury Bill Rate | | | | | | | |
| United States | **0.04** | **0.09** | **0.17** | **0.08** | **0.60** | **0.05** | **0.33** | 172 |
| United Kingdom | **0.21** | **0.34** | **0.35** | **0.39** | **0.42** | **0.11** | **0.61** | 172 |
| Switzerland | **0.22** | **0.36** | **0.37** | **0.51** | **0.60** | **0.27** | **0.18** | 80 |
| Sweden | **0.16** | **0.22** | **0.25** | **0.24** | **0.25** | **0.29** | **0.24** | 124 |
| Japan | **0.25** | **0.53** | **0.54** | **0.16** | **0.59** | **0.47** | **0.81** | 133 |
| Canada | **0.12** | **0.17** | **0.19** | **0.12** | **0.52** | **0.07** | **0.17** | 172 |
| Australia | **0.08** | **0.16** | **0.19** | **0.08** | **0.19** | **0.02** | **0.28** | 122 |

Table 5: Indirect Unit Root Tests ($R^2$ reported). R, RAO and RM are the tests based on the null hyphotesis that GDP follows a process that is consistent with DS, AO, and MSD and uses ex-post real interest rates. IM(P) correspond to the results of the I test using money(prices). ExM(P) correspond to the Ex test, but now using the ex-ante real interest rate as dependent variable.

unit roots, and thus use statistical results of traditional UR tests to see if a theory has any support; but, given the problems discussed above with this statistical approach, the inconclusiveness of the tests makes their implications dubious. On the other hand, our approach considers a wide range of models, which are implicitly taken into account in the statistical tests, and presents their economic implications. If the tests do not capture reality in a one-to-one correspondence, at least they provide first or second order approximations; thus given the huge distance between what these theories predict and what is actually observed in the data, it would be very difficult to make the case for a UR in any real economy.

Finally, all these tests imply that when the scale variable has a unit root, interest rates should satisfy a deterministic relationship with growth rates of the scale (and possibly other variables). Even if the theoretical model were correct, these conditions may be never be satisfied with actual data given that the variables used in practice may contain measurement errors. In such a case, even if “real” variables satisfied these relationships, they would not be replicated with observed data. If measurement errors were to be blamed for the results of Table 5, these should be considerable.

# 6 Concluding Remarks

This paper develops several unit root tests based on economic theory. The advantage of pursuing this methodological path is that the unreliable nature of traditional UR testing is due, at least in part, to the fact that it relies exclusively on statistical representations of univariate processes. Furthermore, as extensive evidence shows, these tests have unusually low power.

Here we consider a variety of general equilibrium models that take into consideration several of the most popular specifications for the traditional UR tests, as the laws of motion of scale variables, and show that when a UR is present, a deterministic relationship between interest rates and growth rates of the scale, money or prices should arise. Given that this differs sharply from TS processes, simple and powerful tests can be constructed, building on the theory.

We show that when these tests are applied to actual data, it is very difficult to make the case for a UR in GDP.

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