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Testing for Strict Stationarity

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Working Paper No. 602 June 2007 ISSN 1473-0278

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May 29, 2007

Abstract

The investigation of the presence of structural change in economic and financial series is a major preoccupation in econometrics. A number of tests have been developed and used to explore the stationarity properties of various processes. Most of the focus has rested on the first two moments of a process thereby implying that these tests are tests of covariance stationarity. We propose a new test for strict stationarity, that considers the whole distribution of the process rather than just its first two moments, and examine its asymptotic properties. We provide two alternative bootstrap approximations for the exact distribution of the test statistic. A Monte Carlo study illustrates the properties of the new test and an empirical application to the constituents of the S&P 500 illustrates its usefulness.
JEL Codes: C32, C33, G12
Keywords: Covariance Stationarity, Strict Stationarity, Bootstrap, S&P500

1 Introduction

The question of whether characteristics of economic and financial series change structure over time has been and continues to be a major preoccupation in econometrics. The search for an answer has taken many guises. This focus is not surprising. Assuming wrongly that the structure of a process remains fixed over time, has very significant and adverse implications. The first obvious implication relates to parametric, or even nonparametric, modelling and concerns the inconsistency of the estimated structure. A distinct, yet related, implication is the fact that structural change is likely to be responsible for most major forecast failures of time series models.

As a result a huge literature on modelling and testing structural change has emerged. Most of the work assumes that structural changes occur rarely and are

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abrupt. Many tests for the presence of structural change of that form exist in the literature starting with the pathbreaking work of Chow (1960) who assumed knowledge of the point in time at which the change occurred. Other tests that relax this assumption have been developed by Brown, Durbin, and Evans (1974), Ploberger and Kramer (1992) and many others. In this context it is worth noting that little is being said about the cause of structural breaks in either statistical or economic terms. Recent work by Kapetanios and Tzavalis (2004) provides a possible avenue for modelling structural breaks and, thus, addresses partially this issue.

Another more recent strand of the literature takes a different approach. In this approach the coefficients of parametric models are assumed to evolve over time. To achieve this the parameters are assumed to be stochastic processes leading to stochastic time-varying coefficient (STVC) models. Such models bear resemblance to simple nonlinear econometric models such as bilinear models (see Tong (1990)). An important question arising out of the use of such models goes to the heart of what structural change is. A relatively uncontroversial definition would be a change in the unconditional moments of the process under investigation. If one were to adopt this definition, use of STVC models may be problematic. To see this we note that, as mentioned above, these models can be viewed as nonlinear time series models. But processes following nonlinear models of that form can be strictly stationary under certain assumptions (see, e.g., Pourahmadi (1988) and Liu and Brockwell (1988)). Another alternative is to assume that coefficients change but in a smooth deterministic way. Such modelling attempts have a long pedigree in statistics starting with the work of Priestley (1965). More recent examples of such work include Dahlhaus (1996), Robinson (1989), Robinson (1991), Orbe, Ferreira, and Rodriguez-Poo (2005), Kapetanios (2006) and Kapetanios (2007).

A vast and related literature concerns whether economic processes behave like random walks or are stationary around a deterministic trend component. Unit root processes change over time but in a very specific way. As a result the questions addressed in this literature form part of the debate on structural change. In this context, a widely used procedure is the KPSS test, proposed by Kwiatkowski, Phillips, Schmidt, , and Shin (1992), for testing stationarity against the unit root alternative. Other related tests of the same hypothesis have been studied in Leybourne and McCabe (1994), Xiao (2001), Giraitis, Kokoszka, and Teyssiere (2003), Hobijn, Franses, and Ooms (2004) and Xiao and Lima (2007). Although most of these tests have been developed with the unit root alternative in mind, they have been used more generally to explore structural change.

Within the aforementioned literatures on testing for structural change focus has almost exclusively been placed on changes in the first two moments of a process. As a result all tests are essentially tests of covariance stationarity. However, while this specific hypothesis may be of great interest, there are cases where it does not address fully the issue of structural change. One leading example is financial time series where changes in higher moments such the skewness and kyrtosis are of interest. Such changes cannot be captured by the aforementioned tests. A further example where these tests may be found lacking concerns series which do not possess higher moments. A majority of existing tests assume the existence of fourth moments. If these moments do not exist then the asymptotic theory on which these tests are based changes drastically (see, e.g., Loretan and Phillips (1996)).

The present paper focuses on the related hypothesis of strict stationarity thereby considering the whole distribution of the process. Appropriate tests for strict stationarity can address the above issues, by focusing on the whole distribution rather than the first two moments. We provide an approach for testing this hypothesis which we operationalise using nonparametric kernel methods. Since the use of kernel methods usually implies that asymptotic approximations are not very accurate we also discuss a bootstrap approximation. Our analysis is based on independent processes. This assumption is relevant for our bootstrap implementation. However, we also discuss extensions to dependent processes and an alternative less accurate bootstrap approximation that takes better account of dependence. A Monte Carlo study illustrates the performance of our methods in small samples. An extensive empirical application to S&P 500 constituents illustrates further the potential of our approach. It is worth noting some further recently available work by Busetti and Harvey (2007), that is related to ours in the sense that focus is placed on changes in the quantiles of the process rather than its moments, thereby providing an alternative to considering the density of the process. Busetti and Harvey (2007) discuss methods based on state space representations for testing the hypothesis of no change in quantiles. However, unlike our work, a particular alternative based on a random walk representation is considered.

This paper is organised as follows: Section 2 discusses the problem and provides the main test approach. Section 3 operationalises the test and provides its asymptotic properties. Section 4 discusses the bootstrap approximations. Sections 5 and 6 provide the Monte Carlo and empirical results of the paper respectively. Finally, Section 7 concludes. The proofs are relegated to an Appendix.

2 Preliminaries

Consider a stochastic process y_i, $i = 1, ..., n$ which can be decomposed into a deterministic component and a stochastic component as in

$$y_i = d_i + x_i \tag{1}$$

The deterministic trend d_i depends on unknown parameters and is specified as $d_i = \gamma' z_i$, where $\gamma = (\gamma_0, ..., \gamma_p)'$ is a vector of trend coefficients and z_i is a deterministic trend of known form, e.g., $z_i = (1, i, ..., i^p)'$. Leading cases for the deterministic component are a constant term, $z_i = 1$, or a linear time trend, $z_i = (1, i)'$. x_i is the stochastic component of y_i. We assume that there exists an estimator, $\hat{\gamma}$, for γ such that $\hat{x}_t - x_t$ is asymptotically negligible for our theoretical results, where $\hat{x}_t = y_t - \hat{\gamma}' z_i$. In what follows we use $\hat{x}_t$ as our data and for notational simplicity refer to it as x_t. We are interested in the null hypothesis that x_i is strictly stationary. More formally, letting $f_i(.)$ denote the probability density function of x_i, the null hypothesis we test is

$$H_0 : f_i = f, \quad \forall i$$

Our entertained alternative is the complement of H_0 and as a result it is extremely general. It encompasses a number of alternatives that have been analysed extensively in the literature such as unit root processes and processes with deterministic breaks in the unconditional mean. Our alternative includes other cases of interest. These include the less analysed case of changes in the unconditional variance and cases which have not been analysed at all such as changes in higher unconditional moments. It is worth noting the findings of Xiao and Lima (2007) who observe that many widely used stationarity tests cannot capture changes in the unconditional variance.

Out test approach has common elements with a widely used approach for constructing covariance stationarity tests. In particular, a principle on which covariance stationarity tests, such as that of Xiao and Lima (2007), are based is that excessive fluctuation in the first two moments of a process indicate departures from covariance stationarity. Excessive fluctuation is gauged by looking at the recursively estimated sample first and second moment of the process. Our focus is not on moments but the density function of the process.

Then, it seems reasonable to consider estimates of the density of the process recursively estimated. Denoting an estimate of the density, at x, based on the first i observations of the sample by $\hat{f}_i(x)$ we consider the fluctuations of the process $\hat{f}_i(x) - \hat{f}_n(x)$ as a function of i and x. Related work on covariance stationarity test considers the maximum over i. We choose to follow the same strategy for both i and x and therefore consider as a test statistic the following

$$T_n = \max_{x \in \mathcal{X}} \max_{i=n_0,\ldots,n-1} \left[\hat{v}_i^{1/2}(x) \left(\hat{f}_i(x) - \hat{f}_n(x) \right) \right] \tag{2}$$

where $\mathcal{X}$ is some a priori chosen grid for x and $\hat{v}_i(x)$ denotes the variance of $\hat{f}_i(x) - \hat{f}_n(x)$. An obvious choice for $\mathcal{X}$ is an evenly spaced grid of sample quantiles from the whole sample.

Of course, we can use a variety of different estimators for the density function. In the next section, we focus on kernel based estimators to operationalise the test based on the test statistic given in (2).

3 The Test

As discussed in the previous Section, a test based on the fluctuations of the recursively estimated density of the process of the form (2), has the potential of capturing deviations from the null hypothesis of strict stationarity. In this section we focus on kernel based estimators of the density of the process. Let

$$\hat{f}_i(x) = \frac{1}{ih} \sum_{j=1}^{i} K\left(\frac{x_j - x}{h} \right)$$

for some kernel function $K(.)$ satisfying $\int K(\psi)\, d\psi = 1$ and some bandwidth parameter h. We make the following assumptions.

Assumption 1 *Under the null hypothesis, H_0, $\{x_i\}_{i=1}^{\infty}$ is a strictly stationary i.i.d. sequence*

Assumption 2 *The kernel function $K(.)$ is a Borel-measurable real valued function that satisfies the following:*

1. $\int K(\psi)d\psi = 1$

2. $\int |K(\psi)|d\psi < \infty$

3. $|\psi||K(\psi)| \to 0$ *as* $|\psi| \to \infty$

4. $\sup |K(\psi)| < \infty$

5. $\int K^{2+\delta}(\psi)d\psi < \infty$ *for some* $\delta > 0$.

Assumption 3 *Under the null hypothesis, H_0, $f(x)$ is continuous at any point x_0 and $\int f(x)dx < \infty$.*

Assumption 4 *$h \equiv h_n$ is such that (i) $h \to 0$, (ii) $nh \to \infty$ and (iii) $nh^5 \to 0$.*

Assumption 5 *There exists an estimator for γ, denoted $\hat{\gamma}$, such that $\hat{\gamma} - \gamma = o_p(\sqrt{nh})$*

Some comments on the assumptions are in order. Assumption 1 is strict but made at this point partly for convenience in deriving the covariance kernel of the limiting process of $\hat{f}_i(x) - \hat{f}_n(x)$ as a function of i, in Theorem 1. It is relaxed below Theorem 1 where we discuss how our results are affected by dependence. It is of relevance in our main bootstrap implementation but again it is relaxed in the context of an alternative bootstrap implementation in Section 4. Assumptions 2-4 are standard in the kernel based density estimation literature and need no further discussion. Assumption 5 ensures that the estimation error arising from the need to estimate γ is negligible. The presence of a second moment for u_t is enough to ensure a parametric rate of convergence for standard estimators for γ which satisfies the assumption. However, since we do not assume the existence of any moments we need to assume Assumption 5.

We then have the following theorem

Theorem 1 *Under assumptions 1-4 $\sqrt{nh}\left(\hat{f}_{[nr]}(x) - \hat{f}_n(x)\right)$, $r \in (0,1)$ converges weakly to a Gaussian process with covariance kernel given by (25) and (41)*

This theorem provides the means for constructing a test since using the above theorem, it follows that the covariance kernel of the limit of the process $\sqrt{nh}\hat{f}_n^{-1/2}(x)\left(\hat{f}_{[nr]}(x)-\hat{f}_n(x)\right)$ contains no nuisance parameters and therefore the distribution of its maximum over r and $\mathcal{X}$, under the null hypothesis, can be tabulated and used for testing the null hypothesis. These critical values will only depend on $\mathcal{X}$.

A major assumption that has been made in the derivation of the covariance kernel of the limit of the process of $\sqrt{nh}\left(\hat{f}_{[nr]}(x)-\hat{f}_n(x)\right)$ is that the data are independent. This assumption can be relaxed straightforwardly using results from Robinson (1983) and Doukhan, Massart, and Rio (1995). The alternative dependence assumption needed is given below.

Assumption 6 *The process $\{y_i\}_{i=1}^{\infty}$ is strictly stationary and β-mixing with mixing coefficients β_m, $m=1,...$ such that $h^{-1}\sum_{m=1}^{n}\beta_m<\infty$.*

Then, the following Theorem holds

Theorem 2 *Under assumptions 2-6 $\sqrt{nh}\left(\hat{f}_{[nr]}(x)-\hat{f}_n(x)\right)$, $r\in(0,1)$ converges weakly to a Gaussian process with the same covariance kernel as in the i.i.d. case.*

These results provide a comprehensive description of the behaviour of the new test under the null hypothesis. However, preliminary investigations suggest that the asymptotic approximation is badly behaved. As a result the next section discusses a bootstrap approximation to the exact distribution of the test statistic.

We conclude this section with a discussion of the behaviour of the test when the null hypothesis does not hold. As the deviations from the null hypothesis can take many forms we cannot provide a general discussion but it is worth exploring the effect of a break in the density function as some point in time denoted n_1 such that $n_1/n=c\in(0,1)$. Let $n_2=n-n_1$. Let the true density prior to and following the break be denoted by $f^{(1)}(x)$ and $f^{(2)}(x)$ respectively. Finally, let $f_1(x)$ and $f_2(x)$ be different on an interval of non-zero Borel measure. Therefore, the alternative hypothesis is

$$H_{1,b}: f_i(x)=\begin{cases} f^{(1)}(x) & \text{if } i\le n_1 \\ f^{(2)}(x) & \text{if } i>n_1 \end{cases}$$

Then, for $r<c$, $\hat{f}_{[nr]}(x)\xrightarrow{p}f^{(1)}(x)$ whereas

$$\hat{f}_n(x)=\frac{1}{nh(n)}\sum_{j=1}^{n}K\left(\frac{x_j-x}{h}\right)=\frac{n_1h(n_1)}{nh}\left[\frac{1}{n_1h(n_1)}\sum_{j=1}^{n_1}K\left(\frac{x_j-x}{h}\right)\right]+$$

$$\frac{n_2 h(n_2)}{nh}\left[\frac{1}{n_2 h(n_2)}\sum_{j=n_1+1}^{n} K\left(\frac{x_j - x}{h}\right)\right] \xrightarrow{p} c^{1+\alpha} f^{(1)} + (1 - c^{1+\alpha}) f^{(2)}$$

where we have assumed that $h = n^{\alpha}$ for some $\alpha < 0$. As a result the test statistic will be $O_p(n^{(1+\alpha)/2})$ providing consistency for the test in the case of abrupt breaks. The above is summarised in the following Theorem.

Theorem 3 *Under Assumptions 2-4 but letting $H_{1,b}$ hold instead of H_0 and assuming that both $f^{(1)}(.)$ and $f^{(2)}(.)$ satisfy the relevant parts of Assumptions 2-4, the test based on the test statistic given by (2), and using the relevant asymptotic critical values discussed below Theorem 1, is consistent.*

Of course it is straightforward to extend Theorem 3 to a local power framework, whereby we can see that the proposed test has power exceeding some assumed significance level if the following local alternative holds

$$H_{1,b,l} : f_i(x) = \begin{cases} f^{(1)}(x) & \text{if } i \leq n_1 \\ f^{(2,n)}(x) & \text{if } i > n_1 \end{cases}$$

where $f^{(2,n)}(x) = f^{(1)}(x) + n^{-(1+\alpha)/2} g(x)$, $g(x)$ is different from zero on an interval of non-zero Borel measure and $g(x)$ is such that for all n, $f^{(2,n)}(x)$ is a proper density function and satisfies the relevant parts of Assumptions 2-4. Similar analyses are, of course, possible for other alternative hypotheses where the structural change in the density function is smooth rather than abrupt, at the additional cost of a more complicated setup for describing such structural changes. Nevertheless, it is worth noting that $H_{1,b}$ (and, therefore, $H_{1,b,l}$) is quite general encompassing all possible abrupt changes in the process, such as, e.g., changes in moments of all orders.

4 A Bootstrap Approximation

In this section we investigate bootstrap approximations to the exact distribution of our test statistic. Our first suggestion is an intuitive bootstrap sample generator based on the density estimate using the whole sample. In particular, using the density estimate obtained over the whole sample we generate bootstrap data as follows: Let $\hat{F}_n(x) = \int_{-\infty}^{x} \hat{f}_n(y)dy$. Then, each observation of the bootstrap sample $x_1^*, \ldots, x_n^*$ is generated by $x_i^* = \hat{F}_n^{-1}(u_i^*)$ where $u_1^*, \ldots, u_n^*$ are i.i.d. random variables distributed uniformly over $(0, 1)$. The integration of $\hat{f}_n(.)$ and the inversion of $\hat{F}_n(.)$ may be done easily numerically. The above scheme is easy to carry out and imposes the null hypothesis onto the bootstrap samples since given some estimate of the density function the bootstrap sample is strictly stationary even if the original sample is not.

We will provide a formal justification of this scheme, referred to as Resampling Scheme IB (for independent boostrap) (RSIB) in what follows. However, a shortcoming of the above scheme is the fact that it assumes independence as the bootstrap sample will be i.i.d. even if the original sample is not. On this we note two things: firstly, extensions are possible along the lines of using estimates of the joint density of $x_t, \ldots x_{t-p}$ for some $p > 0$ to construct the bootstrap samples, but these extensions are both dependent on a choice of p and numerically problematic. Secondly, given the results of Theorems 1 and 2 even if data are dependent, but satisfy the mixing assumption 6, use of RSIB is asymptotically justified.

The issue with the above scheme is the fact that the null hypothesis needs to be imposed on the bootstrap samples, otherwise simply resampling the data using some block resampling scheme would be adequate and would fully accommodate dependence. An alternative approach that provides valid inference without the need to impose the null hypothesis on the bootstrap samples is provided by subsampling. Subsampling was introduced informally by Mahalanobis (1946). Its properties were first discussed formally in Politis and Romano (1994). The method entails resampling without replacement from the original data and constructing samples of smaller size than the original sample. In the case of dependent process subsampling occurs by sequentially resampling $n-b+1$ overlapping blocks of size b from the original sample. Each such block is a resample. By virtue of the fact that, as Politis, Romano, and Wolf (1999, p. 40) put it, 'each subset of size b (taken without replacement from the original data) is indeed a sample of size b from the true model', a more robust approximation to the properties of statistics based on the original sample is feasible. Further, given that the original test statistic will be consistent, for abrupt changes in the density, and $O_p(n^{(1+\alpha)/2})$ whereas the subsample test statistic, being based on a sample of size b, will therefore be $O_p(b^{(1+\alpha)/2}) = o_p(n^{(1+\alpha)/2})$. implies that a test based on subsampling without imposing the null hypothesis on the subsample samples will still be consistent albeit with lower power. We will refer to this subsampling scheme as Resampling Scheme DS (for dependent subsampling) (RSDS).

For the theoretical justification of the two resampling schemes we propose, we introduce the following assumptions and provide the following theorems.

Assumption 7 *The characteristic function of K is absolutely integrable.*

Assumption 8 *Under the null hypothesis, f is uniformly continuous.*

Assumption 9 $h \equiv h_n$ *is such that* $nh^2 \to \infty$.

Let the exact distribution of T in (2) be denoted by $G_n(x, T_n)$. Let the estimates of $G_n(x, T_n)$ using the RSIB and RSDS be denoted $\hat{G}_n^{RSIB}(x, T_n)$ and $\hat{G}_{n,b}^{RSDS}(x, T_n)$ respectively. Finally, let P_n denote the joint probability distribution of the sample $y_1, \ldots, y_n$. Then, the following Theorems hold

Theorem 4 *Under the null hypothesis H_0 and assumptions 1-4 and 7-9*

$$\lim_{n\to\infty} P_n\left[\sup_x \left|\hat{G}_n^{RSIB}(x, T_n) - G_n(x, T_n)\right| > \varepsilon\right] = 0$$

for all $\varepsilon > 0$.

Theorem 5 *Under the null hypothesis H_0 and assumptions 2-9*

$$\lim_{n\to\infty} P_n\left[\sup_x \left|\hat{G}_{n,b}^{RSDS}(x, T_n) - G_n(x, T_n)\right| > \varepsilon\right] = 0$$

5 Monte Carlo Study

In this Section we provide a Monte Carlo study of the new test. We look at the performance of the test under both the null hypothesis and a particular class of alternative hypotheses. Given that we have focused on a discussion of abrupt changes in the discussion on test power in Section 3 we focus on this class of alternative hypotheses. As we have discussed in the Introduction, the new test can be useful in two major additional ways to existing tests. Firstly, as a test of changes that are not captured by changes in the first two moments of the process and secondly as a test that is appropriate when it is suspected that the process does not have higher moments, since most existing tests assume the existence of fourth moments. We investigate both cases in our Monte Carlo study.

We now describe in detail the structure of our study. We focus on the RSIB to construct our test as we wish to focus on a variety of structural change scenarios and do not wish to complicate matters by introducing dependence. We first discuss the setup for the null hypothesis. In this case we generate data using four different distributions. The first is simply the standard normal which provides a benchmark for the test. The rest of the experiments consider t-distributed random variables with v degrees of freedom where we set v to 3, 4 and 5. In the case of $v = 3, 4$ we violate the usual assumption of finite fourth moments on which many stationarity tests are based.

For the alternative hypothesis we consider four different sets of experiments, denoted experiment sets 1, 2, 3 and 4. For experiment set j, $j = 1, ..., 4$, we impose a break at the middle of the sample in the j-th moment of the process. For each experiment set we vary the magnitude of the break to evaluate power performance. For each experiment set we consider breaks of four different magnitudes which, of course, depend in nature on the experiment set j. We refer to these break magnitudes as Breaks A-D. The data generation process for the first half of each sample is simply the standard normal distribution. The data generation process for the second half of each sample for each experiment set is detailed below:

- Experiment Set 1: $x_{[n/2]+1}, ..., x_n \sim$ *i.i.d.* $N(a, 1)$ where $a = 0.25, 0.5, 1, 1.5$
- Experiment Set 2: $x_{[n/2]+1}, ..., x_n \sim$ *i.i.d.* $N(0, a)$ where $a = 1.5, 1.75, 2, 2.5$
- Experiment Set 3: $x_i = (\tilde{x}_i - \hat{\mu})/\hat{\sigma}$ where $\tilde{x}_{[n/2]+1}, ..., \tilde{x}_n \sim$ *i.i.d.* $TPN(1, a^2, 0)$ where $a = 2, 3, 4, 5$ and $\hat{\mu}$ and $\hat{\sigma}^2$ are the sample mean and variance of $\tilde{x}_{[n/2]+1}, ..., \tilde{x}_n$.
- Experiment Set 4: $x_i = (\tilde{x}_i - \hat{\mu})/\hat{\sigma}$ where $\tilde{x}_{[n/2]+1}, ..., \sim$ *i.i.d.* t_a where $a = 5, 4, 3, 2$ and $\hat{\mu}$ and $\hat{\sigma}^2$ are the sample mean and variance of $\tilde{x}_{[n/2]+1}, ..., \tilde{x}_n$.

Some comment is in order for Experiment Set 3. This introduces a break in the third moment. To do this we use the two-part normal distribution (TPN) which allows for skews. A good references for TPN is John (1982). The TPN is closely related to the normal hence its intuitive appeal. It has been used in the Inflation Report produced quarterly by the Bank of England to provide information on the central bank's view of skews in inflation and GDP growth forecasts. Its density is simply made up of splicing together two normalised halves of normal densities. The algebraic form of the $TPN(\sigma_1^2, \sigma_1^2, \mu)$ density is given by

$$\begin{array}{cc} (\pi(\sigma_1 + \sigma_2)^2/2)^{-1/2} \; e^{-1/2(x-\mu)^2/\sigma_1^2} & x < \mu \\ (\pi(\sigma_1 + \sigma_2)^2/2)^{-1/2} & x = \mu \\ (\pi(\sigma_1 + \sigma_2)^2/2)^{-1/2} \; e^{-1/2(x-\mu)^2/\sigma_2^2} & x > \mu \end{array}$$

where μ is the common mean (and mode) of the two normal densities (and the mode of the resulting TPN) and σ_i^2, $i = 1, 2$ are the respective variances. Figure 1 provides the graph of the four densities we consider for Experiment Set 3. For comparability, we also provide graphs of the densities for Experiment Sets 1, 2 and 4, in that Figure. Note that the normalisation that takes $\tilde{x}_t$ to y_t, for Experiment Sets 3 and 4, is one that ensures that the first two moments of the distribution do not change. It worth noting that for Experiment Set 3 the break is not confined to the third moment but affects the fourth moment of the distribution as well. We consider samples sizes of 200 and 400. $\mathcal{X}$ is set to 20 evenly spaced quantiles starting with the lowest decile and finishing with the highest. We set r (the proportion of observations from which we recursively estimate the density) to 0.2 and α where $h = n^{\alpha}$ to -1/5. We carry out 99 bootstrap replications and 500 Monte Carlo replications. Both of these numbers are relatively small but increasing the number of bootstrap replications can only improve the results. Further, the computational cost of running the bootstrap procedure coupled with the numerical cost of integrating $\hat{f}$ and especially inverting $\hat{F}$ is considerable. We have not explored computationally efficient ways of inverting $\hat{F}$.

Results are reported in Table 1 for the size experiments and Table 2 for the power experiments. They make interesting reading. The RSIB resampling scheme performs very well under the null hypothesis for all distributions considered. This is especially encouraging in the case of distributions t_4 and t_3 since in these cases the fourth moment, usually assumed finite in the stationarity testing literature, does not exist.

Moving on to the power experiments we again obtain encouraging results. The power of the test increases with sample size reinforcing the findings of Theorem 4 concerning the consistency of the test. The power is also increasing with respect to the break magnitude, again in accordance with intuition. It is interesting to see how power behaves with respect to the moment that actually suffers the break. The test is most powerful for breaks in the mean as expected. Breaks in the unconditional variance are the second most detectable followed by breaks in the fourth moment. Breaks in the third moment are the least easy to detect. With the exception of third moment breaks, the above ranking is intuitive. Overall, results suggest that even for relatively small sample sizes such as 200 observations, which is a small sample for most financial time series, the test can provide reasonable power as long as the break is relatively pronounced.

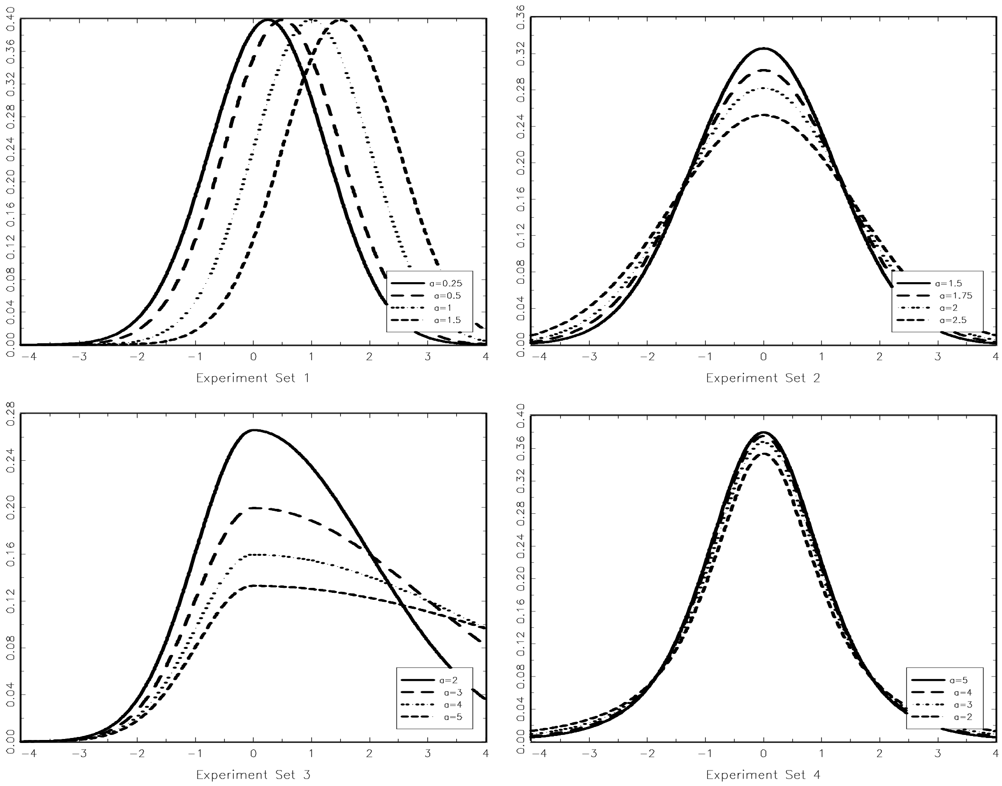
Table 1: Monte Carlo Size Results

	T	
Distribution	200	400
Normal	0.036	0.056
t_5	0.062	0.050
t_4	0.052	0.064
t_3	0.064	0.062

Table 2: Monte Carlo Power Results

		Experiment Set			
Break	T	1	2	3	4
A	200	0.076	0.084	0.050	0.072
	400	0.166	0.202	0.086	0.110
B	200	0.234	0.188	0.094	0.120
	400	0.474	0.342	0.162	0.212
C	200	0.818	0.256	0.108	0.204
	400	0.984	0.550	0.210	0.378
D	200	0.998	0.448	0.106	0.520
	400	1.000	0.828	0.250	0.780

Figure 1: Monte Carlo study Densities



6 Empirical Application to Stock Returns

In this section, we provide an empirical application that illustrates the potential of the new test to detect the presence of structural change. As it is sometimes difficult to draw meaningful conclusions from the empirical analysis of a single series for the performance of a new statistical test, we consider a large dataset such as the S&P 500. Data, obtained from Datastream, are weekly returns and span the period 01/01/1993-20/01/2004 comprising 575 weekly observations. We choose to consider only companies for which data are available throughout the period leading us to have 412 series on which to use our test. We normalise the returns series to have mean equal to zero and variance equal to one prior to applying our test. We apply our test setting $\mathcal{X}$ to 20 evenly spaced quantiles starting with the lowest decile and finishing with the highest, following our Monte Carlo study. We set r (the proportion of observations from which we recursively estimate the density) to 0.2 and α where $h = n^{\alpha}$ to -1/5. The bootstrap replications are set to 149, significantly increasing the value from that used in the Monte Carlo study.

We report the probability values for the test of the strict stationarity null hypothesis, carried out on the 412 company return series in Tables 3-6. Probability values below 0.05, and the company names to which they correspond, are reported in bold typescript for easy identification. As we can see for these Tables a large minority of the series are in fact found to reject the null hypothesis of strict stationarity at the 95% significance level.

These results prompt the obvious question as to what causes the tests to reject. In order to explore further this issue we carry out the following supplementary analysis. We estimate recursively the mean, variance, skew and kyrtosis of the series that reject the null hypothesis of strict stationarity and plot these in Figures 2-5. We set the proportion of observations from which we recursively estimate the above statistics to 0.2 which is equal to the value for r used for the stationarity tests. The plots are in the same order as the company names in the Tables. So, for example, the ninth stock for which there is evidence to reject the hypothesis of strict stationarity is AT&T in Table 3 (as noted next to the company name in the Tables). The relevant plots for this stock are those found last on the first row of plots for each of the Figures 2 to 5, since each row of plots contains nine plots. Likewise, the relevant plots for the 10th stock that rejects (Company Name: Automatic Data Processing) are those found first in the second row of plots, and so on.

Examination of these plots provides some interesting insights. Starting with the recursively estimated mean we see that a number of stock returns exhibit an inverted U shape with the peak around the year 2000 corresponding with the bull market of the late 90's. Despite this, the evidence for a structural change in the mean is not that convincing. Given the pretty strong evidence that stock returns are not unit root process it is clear that, as noted by Xiao and Lima (2007), standard covariance stationarity tests will not have power to detect any other form of structural change.

Next we move to Figure 3 which provides plots of recursively estimated variances.

Here the picture is pretty clear. Most stock returns exhibit an upward trend in unconditional variance. It is worth stressing the distinction between conditional variance which is the focus of many volatility models such as GARCH and stochastic volatility models, and may vary for stationary processes, and unconditional variance which has to be fixed if a process is stationary. Unconditional variance has received less attention than its conditional counterpart. Figure 3 clearly shows that an upward trend in unconditional variance is likely to be the cause of the rejections given by the strict stationarity test. Note further that this upward trend, although the main feature of the recursively estimated variances, is not the only feature. It is apparent mainly for the period 1998-2002. Since 2002 there has been a stabilisation of the variance and in many cases a reduction. Moreover, given that the recursively estimate variance is by necessity backward looking this stabilisation is likely to have started shortly after 2000.

Moving on to the plots of the recursively estimated skews and kyrtoses we note that there is little evidence of a systematic pattern of change across stock returns. It is worth focusing some attention on three stock returns which do not exhibit such strong trends in their recursively estimated unconditional variances. These are Electronic Arts, International Game Technology and Navistar International (the 27th, 44th and 63rd series that reject). Of those, International Game Technology exhibits a clear trend for the recursively estimated mean providing some explanation for the test rejection. For the other two stocks it appears that significant shifts in the skew (Navistar International) and kyrtosis (Electronic Arts, Navistar International) can provide a reason for the test rejections. It is worth noting that the recursive estimates of higher moments are quite sensitive to outliers (which provide, of course, potential evidence of fat tails and other deviations from normality) and as a result one should interpret these plots cautiously.

Overall, we can conclude that there has been a gradual upward shift in unconditional variances for the stocks examined during the late 90's and early 2000's and that this is the main cause of the widespread rejection of the strict stationarity null hypothesis. This is a rather powerful result. It suggests that the consideration of conditional mean and, especially, conditional variance models which assume stationarity is problematic for the period under examination. All estimation results for such models, if they assume stationarity, are therefore suspect. A further important question relates to modelling conditional volatility. Are the shifts we observe changes in conditional or unconditional volatility? Out analysis suggest the unconditional volatility has shifted. If so, modeling via GARCH or stochastic volatility model comes into question. There is another possibility: That swings in conditional volatility are long term ones that involve periods of 3-5 years, or even longer periods. Given our sample period, such swings would appear as unconditional volatility swings. But this possibility simply recasts the problem. These swings cannot be captured by standard volatility models and may require models where structural change is regular but rare. The work of Kapetanios and Tzavalis (2006) provides one modelling avenue in that direction.

7 Conclusion

The presence of structural change in economic and financial series is a major preoccupation in econometrics. A number of tests that have been developed for testing whether a process is unit root or not, such those developed in Leybourne and McCabe (1994), Xiao (2001), Giraitis, Kokoszka, and Teyssiere (2003), Hobijn, Franses, and Ooms (2004) and Xiao and Lima (2007) have been applied more generally to explore the stationarity properties of various processes. Most of the focus has rested on the first two moments of a process thereby implying that these tests are tests of covariance stationarity. Further, most of these tests make a somewhat restrictive assumption in requiring the existence of fourth moments.

We propose a new test for strict stationarity, that considers the whole distribution of the process rather than just its first two moments, and examine its asymptotic properties. We provide two alternative bootstrap approximations for the exact distribution of the test statistic. A Monte Carlo study illustrates the properties of the new test and an empirical application to the constituents of the S&P 500 illustrates its usefulness.

In terms of future research it is worth noting that the analysis of the current paper focuses on univariate processes. However, it is possible to conceive of situations where the contemporaneous codependence structure of two processes changes whereas their marginal distributions remains fixed. As a result our analysis can be easily extended to multivariate density estimators and this is the topic of currently undertaken research.

8 Appendix

8.1 Proof of Theorem 1

The result of Theorem 1 follows if we show the following:

1. Convergence of the finite dimensional distributions of $\sqrt{nh}\left(\hat{f}_{[nr]}(x) - \hat{f}_n(x))\right)$ to a multivariate normal distribution (Result A)

2. Derivation of the covariance kernel of the limit of the process $\sqrt{nh}\left(\hat{f}_{[nr]}(x) - \hat{f}_n(x))\right)$ (Result B)

3. Stochastic equicontinuity of $\sqrt{nh}\left(\hat{f}_{[nr]}(x) - \hat{f}_n(x))\right)$ (Result C)

Note that Assumption 5 ensures that the estimation error arising from the need to estimate γ is negligible. Hence, we disregard the presence of $\hat{\gamma}$ in the rest of the proofs. Result A follows immediately from assumptions 1-4 and Theorem 2.10 of Pagan and Ullah (2000). Result B is derived in subsection 8.2. Result C can be established using Theorem 1 of Doukhan, Massart, and Rio (1995). In particular, two conditions need to be satisfied: Firstly that $K\left(\frac{x_j - x}{h}\right)$ is β-mixing with summable β-mixing coefficients. This is trivial given Assumption 1. Secondly, that the square root of the logarithm of the L^{2v} bracketing numbers is integrable. We focus on this condition. Let $x \in \Gamma \subset \mathbb{R}$. Then, one can always find a set Γ_N and constant $G < \infty$ such that for all $x \in \Gamma$ there is $x_k \in \Gamma_N$ such that

$$|x - x_k| \leq GN^{-1}$$

Set the bracketing numbers $N(\delta) = GB\delta$. By the boundedness of $K(.)$ it follows that

$$|K\left(\frac{x_j - x}{h}\right) - K\left(\frac{x_j - x_k}{h}\right)| \leq B|x - x_k| \leq BGN^{-1} = \delta$$

Thus, $N(\delta)$ satisfy the definition of the L^{2v} bracketing numbers. Since

$$\int_0^1 \sqrt{\log(N(\delta))} \leq \sqrt{\log(GB)} + \int_0^1 \sqrt{\log(\delta)} < \infty$$

the second condition is satisfied giving Result C and therefore proving Theorem 1.

8.2 Derivation of the covariance kernel of the limit of the process $\sqrt{nh}\left(\hat{f}_{[nr]}(x) - \hat{f}_n(x))\right)$

Throughout this subsection we assume that the null hypothesis holds. Assuming, without loss of generality that $n_2 > n_1$, define $r_{j,n} = \frac{n_j}{n}$, $j = 1, 2$, and $r_j = \lim_{n\to\infty} r_{j,n}$.

Then,

$$\hat{f}_{n_j}(x) \equiv \hat{f}(r_{j,n}) = \frac{1}{n_j h_j} \sum_{i=1}^{n_1} K\left(\frac{x_i - x}{h_j}\right). \tag{3}$$

Then,

$$\hat{f}(r_{1,n}) - \hat{f}(r_{2,n}) = \frac{1}{n_1 h_1} \sum_{i=1}^{n_1} K\left(\frac{x_i - x}{h_1}\right) - \frac{1}{n_2 h_2} \sum_{i=1}^{n_2} K\left(\frac{x_i - x}{h_2}\right) = \tag{4}$$

$$\frac{1}{n_1} \sum_{i=1}^{n_1} \left[\frac{1}{h_1} K\left(\frac{x_i - x}{h_1}\right) - \frac{n_1}{n_2 h_2} K\left(\frac{x_i - x}{h_2}\right)\right] - \frac{1}{n_2 h_2} \sum_{i=n_1+1}^{n_2} K\left(\frac{x_i - x}{h_2}\right). \tag{5}$$

Define

$$w_i = \frac{1}{h_1} K\left(\frac{x_i - x}{h_1}\right) - \frac{n_1}{n_2 h_2} K\left(\frac{x_i - x}{h_2}\right). \tag{6}$$

We need to derive $E(w_i)$ and $Var(w_i)$ and their limits. Define $\psi_j = \frac{x_i - x}{h_j}$, $j = 1, 2$, and $r_n = \frac{n_1}{n_2}$. Then,

$$E(w_i) = \int_{\mathbb{R}} K(\psi_1) f(h_1 \psi_1 + x) d\psi_1 - r_n \int_{\mathbb{R}} K(\psi_2) f(h_2 \psi_2 + x) d\psi_2. \tag{7}$$

Further,

$$\lim_{n \to \infty} E(w_i) = f(x) \int_{\mathbb{R}} K(\psi_1) d\psi_1 - r f(x) \int_{\mathbb{R}} K(\psi_2) d\psi_2 = (1 - r) f(x) \int_{\mathbb{R}} K(\psi) d\psi, \tag{8}$$

where $r = \lim_{n \to \infty} r_n$. Next,

$$Var(w_i) = E(w_i^2) - E(w_i)^2.$$

So,

$$E(w_i^2) = h_1^{-2} E\left(K\left(\frac{x_i - x}{h_1}\right)^2\right) + r_n^2 h_2^{-2} E\left(K\left(\frac{x_i - x}{h_2}\right)^2\right) - \tag{9}$$

$$2 h_1^{-1} h_2^{-1} r_n E\left(K\left(\frac{x_i - x}{h_1}\right) K\left(\frac{x_i - x}{h_2}\right)\right).$$

Taking limits of each term on the RHS of (9) after normalising by h_1 gives

$$\lim_{n \to \infty} h_1^{-1} E\left(K\left(\frac{x_i - x}{h_1}\right)^2\right) = \lim_{n \to \infty} \int_{\mathbb{R}} K(\psi_1)^2 f(h_1 \psi_1 + x) d\psi_1 = \tag{10}$$

$$f(x) \int_{\mathbb{R}} K(\psi_1)^2 d\psi_1.$$

Next

$$\lim_{n \to \infty} r_n^2 h_2^{-2} h_1 E\left(K\left(\frac{x_i - x}{h_2}\right)^2\right) = r^{2+\alpha} f(x) \int_{\mathbb{R}} K(\psi_2)^2 d\psi_2. \tag{11}$$

noting that $h_j = n_j^{\alpha}$, $j = 1, 2$ for some $\alpha < 0$. Further, noting that $\frac{x_i - x}{h_2} = \psi_1 r_n^{\alpha}$, we get

$$\lim_{n\to\infty} 2h_2^{-1} r_n E\left(K\left(\frac{x_i - x}{h_1}\right) K\left(\frac{x_i - x}{h_2}\right)\right) = \tag{12}$$

$$\lim_{n\to\infty} 2h_2^{-1} r_n h_1 \int_{\mathbb{R}} K(\psi_1) K(\psi_1 r_n^{\alpha}) f(h_1\psi_1 + x) d\psi_1 = 2r^{1+\alpha} f(x) \int_{\mathbb{R}} K(\psi_1) K(\psi_1 r^{\alpha}) d\psi_1.$$

Overall,

$$\lim_{n\to\infty} h_1 E(w_i^2) = f(x)\left[\left(1 + r^{2+\alpha}\right)\int_{\mathbb{R}} K(\psi)^2 d\psi - 2r^{1+\alpha}\int_{\mathbb{R}} K(\psi) K(\psi r^{\alpha}) d\psi\right] = \tag{13}$$

$$f(x) K^{(1)}(r),$$

where $K^{(1)}(r) = (1 + r^{2+\alpha}) \int_{\mathbb{R}} K(\psi)^2 d\psi - 2r^{1+\alpha} \int_{\mathbb{R}} K(\psi) K(\psi r^{\alpha}) d\psi$. We next examine the second term of the RHS of (5). We have

$$\frac{1}{n_2 h_2} \sum_{i=n_1+1}^{n_2} K\left(\frac{x_i - x}{h_2}\right) = (1 - r_n)\frac{1}{n^*} \sum_{i=1}^{n^*} w_{i+n_1}^*, \tag{14}$$

where $w_i^* = \frac{1}{h_2} K\left(\frac{x_i - x}{h_2}\right)$, $n^* = n_2 - n_1$ and by similar analysis to that given above

$$\lim_{n\to\infty} E(w_i^*) = f(x) \int_{\mathbb{R}} K(\psi) d\psi \tag{15}$$

and

$$\lim_{n\to\infty} h_1 E\left(w_i^{*2}\right) = r^{\alpha} f(x) \int_{\mathbb{R}} K(\psi)^2 d\psi. \tag{16}$$

Then, under the null hypothesis of strict stationarity,

$$\lim_{n\to\infty} E\left(\hat{f}(r_{1,n}) - \hat{f}(r_{2,n})\right) = 0. \tag{17}$$

Next,

$$nhVar\left(\hat{f}(r_{1,n}) - \hat{f}(r_{2,n})\right) = nhVar\left[\frac{1}{n_1}\sum_{i=1}^{n_1} w_i - \frac{(1 - r_n)}{n^*}\sum_{i=1}^{n^*} w_{i+n_1}^*\right]. \tag{18}$$

But, the two sums in the variance term in the RHS of (18) are made up of different observations in the sample and are therefore independent. So,

$$nhVar\left[\frac{1}{n_1}\sum_{i=1}^{n_1} w_i - \frac{(1 - r_n)}{n^*}\sum_{i=1}^{n^*} w_{i+n_1}^*\right] = nhVar\left[\frac{1}{n_1}\sum_{i=1}^{n_1} w_i\right] + \tag{19}$$

$$nhVar\left[\frac{(1 - r_n)}{n^*}\sum_{i=1}^{n^*} w_{i+n_1}^*\right].$$

Then,

$$nhVar\left[\frac{1}{n_1}\sum_{i=1}^{n_1} w_i\right] = \frac{nh}{n_1}\left(E\left(w_i^2\right) - E(w_i)^2\right). \tag{20}$$

Taking limits with respect to n gives that

$$\lim_{n\to\infty} nhVar\left[\frac{1}{n_1}\sum_{i=1}^{n_1} w_i\right] = r_1^{-1-\alpha} f(x) K^{(1)}, \tag{21}$$

since $\lim_{n\to\infty} \frac{nh}{n_1} E(w_i) = 0$. Next,

$$\lim_{n\to\infty} nhVar\left[\frac{(1-r_n)}{n^*}\sum_{i=1}^{n^*} w_{i+n_1}^*\right] = \lim_{n\to\infty}\left[\left(\frac{nh(1-r_n)^2}{n^* h_1}\right) h_1 \left(E\left(w_i^{*2}\right) - E(w_i^*)^2\right)\right] = \tag{22}$$
$$r_2^{-1} r_1^{-\alpha}(1-r)r^\alpha f(x)\int_{\mathbb{R}} K(\psi)^2 \, d\psi = r_2^{-1} r_1^{-\alpha}(1-r) f(x) K^{(2)},$$

where $K^{(2)} = \int_{\mathbb{R}} K(\psi)^2 \, d\psi$. So, overall

$$\lim_{n\to\infty} nhVar\left(\hat{f}(r_{1,n}) - \hat{f}(r_{2,n})\right) = f(x)\left[r_1^{-1-\alpha} K^{(1)}(r) + r_2^{-1} r_1^{-\alpha}(1-r) r^\alpha K^{(2)}\right]. \tag{23}$$

This specialises in the case where $n_2 = n$ to

$$\lim_{n\to\infty} nhVar\left(\hat{f}(r_{1,n}) - \hat{f}(1)\right) = f(x)\left[r_1^{-1-\alpha} K^{(1)}(r_1) + (1-r_1)K^{(2)}\right] = \tag{24}$$

$$f(x)\left[\left(r_1^{-1-\alpha} + r_1 + 1 - r_1\right)\int_{\mathbb{R}} K(\psi)^2 \, d\psi - 2\int_{\mathbb{R}} K(\psi) K(\psi r^\alpha) \, d\psi\right] \tag{25}$$

As a final ingredient for the determination of the covariance kernel of the process $\hat{f}(r) - \hat{f}(1)$, $r \in (0,1)$ we have to derive

$$\lim_{n\to\infty} nhCov\left[\hat{f}(r_{1,n}) - \hat{f}(1), \hat{f}(r_{2,n}) - \hat{f}(1)\right] = \tag{26}$$
$$\lim_{n\to\infty} nhE\left[\left(\hat{f}(r_{1,n}) - \hat{f}(1)\right)\left(\hat{f}(r_{2,n}) - \hat{f}(1)\right)\right].$$

Define

$$w_i(r_{j,n}) = \frac{1}{h_j} K\left(\frac{x_i - x}{h_j}\right) - \frac{r_{j,n}}{h} K\left(\frac{x_i - x}{h}\right), \quad j = 1, 2, \tag{27}$$

and

$$w_{1,i}^* = \frac{1}{h} K\left(\frac{x_i - x}{h}\right). \tag{28}$$

Then,

$$\hat{f}(r_{j,n}) - \hat{f}(1) = \frac{1}{n_j}\sum_{i=1}^{n_j} w_i(r_{j,n}) - \frac{1}{n}\sum_{i=n_j+1}^{n} w_{1,i}^*, \quad j = 1, 2. \tag{29}$$

So, by independence across i, we have

$$E\left[\left(\hat{f}(r_{1,n})-\hat{f}(1)\right)\left(\hat{f}(r_{2,n})-\hat{f}(1)\right)\right]=\frac{1}{n_1n_2}\sum_{i=1}^{n_1}E\left(w_i(r_{1,n})w_i(r_{2,n})\right)+ \tag{30}$$

$$\frac{1}{n^2}\sum_{i=n_2+1}^{n}E\left(w_{1,i}^{*2}\right)-\frac{1}{n_2n}\sum_{i=n_1+1}^{n_2}E\left(w_{1,i}^{*}w_i(r_{2,n})\right).$$

For the second term of (30) we have from above

$$\lim_{n\to\infty}hE\left(w_{1,i}^{*2}\right)=f(x)K^{(2)}. \tag{31}$$

Looking at the first and third terms in turn gives

$$w_i(r_{1,n})w_i(r_{2,n})=\frac{1}{h_1h_2}K\left(\frac{x_i-x}{h_1}\right)K\left(\frac{x_i-x}{h_2}\right)+\frac{r_{1,n}r_{2,n}}{h^2}K\left(\frac{x_i-x}{h}\right)^2- \tag{32}$$

$$\frac{r_{1,n}}{hh_2}K\left(\frac{x_i-x}{h}\right)K\left(\frac{x_i-x}{h_2}\right)-\frac{r_{2,n}}{hh_1}K\left(\frac{x_i-x}{h}\right)K\left(\frac{x_i-x}{h_1}\right).$$

Taking overall expectations and normalising by h gives

$$\lim_{n\to\infty}hE\left(w_i(r_{1,n})w_i(r_{2,n})\right)=f(x)\left[r_2^{-\alpha}\int_{\mathbb{R}}K\left(\psi\right)K\left(\psi\left(\frac{r_1}{r_2}\right)^{\alpha}\right)d\psi- \tag{33}\right.$$

$$\left.r_1\int_{\mathbb{R}}K\left(\psi_2r_2^{\alpha}\right)K\left(\psi_2\right)d\psi_2-r_2\int_{\mathbb{R}}K\left(\psi_1\right)K\left(\psi_1r_1^{\alpha}\right)d\psi_1+r_1r_2K^{(2)}\right]=$$

$$f(x)K^{(3)}(r_1,r_2),$$

where

$$K^{(3)}(r_1,r_2)=r_2^{-\alpha}\int_{\mathbb{R}}K\left(\psi\right)K\left(\psi\left(\frac{r_1}{r_2}\right)^{\alpha}\right)d\psi- \tag{34}$$

$$r_1\int_{\mathbb{R}}K\left(\psi\right)K\left(\psi r_2^{\alpha}\right)d\psi-r_2\int_{\mathbb{R}}K\left(\psi\right)K\left(\psi r_1^{\alpha}\right)d\psi+r_1r_2K^{(2)}.$$

For the third term of (30) we have

$$w_{1,i}^{*}w_i(r_{2,n})=\frac{1}{hh_2}K\left(\frac{x_i-x}{h}\right)K\left(\frac{x_i-x}{h_2}\right)-\frac{r_{2,n}}{h^2}K\left(\frac{x_i-x}{h}\right)^2. \tag{35}$$

Again taking expectations and normalising by h gives

$$\lim_{n\to\infty}hE\left(w_{1,i}^{*}w_i(r_{2,n})\right)=f(x)\left[\int_{\mathbb{R}}K\left(\psi_2\right)K\left(\psi_2r_2^{\alpha}\right)d\psi_2-r_2K^{(2)}\right]= \tag{36}$$

$$f(x)K^{(4)}(r_2),$$

where

$$K^{(4)}(r_2)=\int_{\mathbb{R}}K\left(\psi\right)K\left(\psi r_2^{\alpha}\right)d\psi-r_2K^{(2)}. \tag{37}$$

Overall,

$$\lim_{n\to\infty} \frac{nh}{n_1 n_2} \sum_{i=1}^{n_1} E\left(w_i(r_{1,n}) w_i(r_{2,n})\right) = r_2^{-1} f(x) K^{(3)}(r_1, r_2), \tag{38}$$

$$\lim_{n\to\infty} \frac{nh}{n^2} \sum_{i=n_2+1}^{n} E\left(w_{1,i}^{*2}\right) = (1 - r_2) f(x) K^{(2)}.$$

and

$$\lim_{n\to\infty} \frac{nh}{n_2 n} \sum_{i=n_1+1}^{n_2} E\left(w_{1,i}^{*} w_i(r_{2,n})\right) = \lim_{n\to\infty} \frac{n^2 h(r_{2,n} - r_{1,n})}{n_2 n} E\left(w_{1,i}^{*} w_i(r_{2,n})\right) = \tag{39}$$

$$\left(1 - \frac{r_1}{r_2}\right) f(x) K^{(4)}(r_2).$$

Thus,

$$\lim_{n\to\infty} nhCov\left[\left(\hat{f}(r_{1,n}) - \hat{f}(1)\right)\left(\hat{f}(r_{2,n}) - \hat{f}(1)\right)\right] = \tag{40}$$

$$f(x)\left[r_2^{-1} K^{(3)}(r_1, r_2) + (1 - r_2) K^{(2)} - \left(1 - \frac{r_1}{r_2}\right) K^{(4)}(r_2)\right]. \tag{41}$$

Overall, the covariance structure of $\sqrt{nh}\left(\hat{f}(r) - \hat{f}(1)\right)$, $r \in (0,1)$ is given by (25) and (41).

8.3 Proof of Theorem 2

Theorem 2 follows if we note the following. Theorem 2.10 of Pagan and Ullah (2000) holds when replacing Assumption 1 with Assumption 6 using the results of Robinson (1983) (see also Pagan and Ullah (2000, Sec. 2.6.3)). The covariance structure of the process $\sqrt{nh}\left(\hat{f}_{[nr]}(x) - \hat{f}_n(x))\right)$ remains the same as in Theorem 1 as long as Assumption 6 holds. Further, assumption 6 satisfies the mixing condition needed for Theorem 1 of Doukhan, Massart, and Rio (1995). Hence, Theorem 2 follows.

8.4 Proof of Theorem 4

In order to prove Theorem 4 we use Theorem 2.2 of Horowitz (2002) which is a restatement of a result in Mammen (1992). Given the normality result of Theorem 1, Mammen's result implies that if RSIB is applied using $f_n(.)$ rather than $\hat{f}_n(.)$ to generate the boostrap sample then

$$\lim_{n\to\infty} P_n\left[\sup_x \left|G_n^{RSIB}(x, T_n) - G_n(x, T_n)\right| > \varepsilon\right] = 0,$$

where $G_n^{RSIB}(x, T_n)$ denotes the estimate of $G_n(x, T_n)$ using the RSIB but with $f_n(.)$ rather than $\hat{f}_n(.)$. But

$$\lim_{n\to\infty} P_n \left[\sup_x \left| \hat{G}_n^{RSIB}(x, T_n) - G_n(x, T_n) \right| > \varepsilon \right] \leq$$

$$\lim_{n\to\infty} P_n \left[\sup_x \left| \hat{G}_n^{RSIB}(x, T_n) - G_n^{RSIB}(x, T_n) \right| > \varepsilon/2 \right] +$$

$$\lim_{n\to\infty} P_n \left[\sup_x \left| G_n^{RSIB}(x, T_n) - G_n(x, T_n) \right| > \varepsilon/2 \right].$$

Thus, the Theorem is proved if we show that

$$\lim_{n\to\infty} P_n \left[\sup_x \left| \hat{G}_n^{RSIB}(x, T_n) - G_n^{RSIB}(x, T_n) \right| > \varepsilon/2 \right] = 0.$$

We have that

$$\hat{G}_n^{RSIB}(x, T_n) = \frac{1}{B} \sum_{s=1}^{B} I(\hat{T}_n^{*,s} \leq x),$$

and

$$G_n^{RSIB}(x, T_n) = \frac{1}{B} \sum_{s=1}^{B} I(T_n^{*,s} \leq x),$$

where $\hat{T}_n^{*,s}$ and $T_n^{*,s}$ denote the RSIB boostrap test statistics from the bootstrap samples $(\hat{x}_1^{*,s}, \ldots, \hat{x}_n^{*,s})'$ and $(x_1^{*,s}, \ldots, x_n^{*,s})'$ obtained using the same uniformly distributed i.i.d. random variables $(u_1^{*,s}, \ldots, u_n^{*,s})'$, and $\hat{f}_n(.)$ and $f_n(.)$ respectively and B is the number of bootstrap replications. Further, let $\hat{f}_i^{*,s}(x)$ and $f_i^{*,s}(x)$ denote the boostrap estimates of $f_i(x)$ used to construct $\hat{T}_n^{*,s}$ and $T_n^{*,s}$ respectively. Then,

$$\lim_{n\to\infty} P_n \left[\sup_x \left| \hat{G}_n^{RSIB}(x, T_n) - G_n^{RSIB}(x, T_n) \right| > \varepsilon/2 \right] =$$

$$\lim_{n\to\infty} P_n \left[\sup_x \left| \frac{1}{B} \sum_{s=1}^{B} I(\hat{T}_n^{*,s} \leq x) - \frac{1}{B} \sum_{s=1}^{B} I(T_n^{*,s} \leq x) \right| > \varepsilon/2 \right] \leq$$

$$\lim_{n\to\infty} P_n \left[\sup_x \left| I(\hat{T}_n^{*,1} \leq x) - I(T_n^{*,1} \leq x) \right| > \varepsilon/2 \right] =$$

$$\lim_{n\to\infty} P_n \left[\sup_x \left| \begin{array}{c} I\left(\sup_{i\in\{1,..,n\}} \sup_{y\in\mathcal{X}} \left(\hat{f}_i^{*,1}(y) - \hat{f}_n^{*,1}(y)\right)\right) \leq x\right) - \\ I\left(\sup_{i\in\{1,..,n\}} \sup_{y\in\mathcal{X}} \left(f_i^{*,1}(y) - f_n^{*,1}(y)\right)\right) \leq x\right) \end{array} \right| > \varepsilon/2 \right], \quad (42)$$

where we have assumed for simplicity that the difference $f_i(x) - f_n(x)$ is not standardised.But, for all $\varepsilon > 0$ there exists some $\varepsilon_1 > 0$ such that (42) is bounded by

$$\lim_{n\to\infty} P_n \left[\sup_{j\in\{1,..,n\}} \sup_x \left| K\left(\frac{\hat{x}_j^{*,1} - x}{h}\right) - K\left(\frac{x_j^{*,1} - x}{h}\right) \right| > \varepsilon_1 \right], \quad (43)$$

In turn, for all $\varepsilon_1 > 0$ there exists some $\varepsilon_2 > 0$ such that (43) is bounded by

$$\lim_{n\to\infty} P_n \left[\sup_{j\in\{1,..,n\}} \left|\hat{x}_j^{*,1} - x_j^{*,1}\right| > \varepsilon_2 \right]. \tag{44}$$

But, (44) is zero as long as

$$\lim_{n\to\infty} P_n \left[\sup_{i\in\{1,...,n\}} \sup_x \left|\hat{f}_i(x) - f_i(x)\right| > \varepsilon_3 \right] = 0, \tag{45}$$

for all $\varepsilon_3 > 0$. (45) is proved under the theorem's assumptions in Theorem 2.8 of Pagan and Ullah (2000). Hence, Theorem 4 is proven.

8.5 Proof of Theorem 5

Let $G(x, T)$ denote the limit of $G_n(x, T_n)$. The subsampling approximation to $G(x, T)$ is given by

$$\hat{G}_{n,b}^{RSDS}(x, T_n) = \frac{1}{n_b} \sum_{s=1}^{n_b} I\left(T_{n,b}^{*,s} \le x\right), \tag{46}$$

where $n_b = n-b+1$ and $T_{n,b}^{*,s}$ is the s-th resampled statistic based on a resampled block of size b. For x_α, where $G(x_\alpha, T) = \alpha$, we first need to prove that $\hat{G}_{n,b}^{RSDS}(x, T_n) \xrightarrow{p} G(x_\alpha, T)$. But, $E(\hat{G}_{n,b}^{RSDS}(x, T_n)) = G_b(x, T_b)$ because the subsample is a sample from the true model. Hence, it suffices to show that $Var(\hat{G}_{n,b}^{RSDS}(x, T_n)) \to 0$ as $N \to \infty$. Let

$$I_{b,s} = I\left(T_{n,b}^{*,s} \le x\right), \tag{47}$$

$$v_{n_b,h} = \frac{1}{n_b} \sum_{s=1}^{n_b} Cov\left(I_{b,s}, I_{b,s+h}\right). \tag{48}$$

Then,

$$Var\left(\hat{G}_{n,b}^{RSDS}(x, T_n)\right) = \frac{1}{n_b}\left(v_{n_b,0} + 2\sum_{h=1}^{n_b-1} v_{n_b,h}\right) = \tag{49}$$

$$\frac{1}{n_b}\left(v_{n_b,0} + 2\sum_{h=1}^{b-1} v_{n_b,h}\right) + \frac{2}{n_b}\sum_{h=b}^{n_b-1} v_{n_b,h} = V_1 + V_2.$$

We first determine the order of magnitude of V_1. By the boundedness of $I_{b,s}$, it follows that $v_{n_b,h}$ is uniformly bounded across h. Hence, $|V_1| \le \frac{b}{n_b} \max_h |v_{n_b,h}|$, from which it follows that $V_1 = O(b/n_b) = o(1)$. We next examine V_2. For this we need to note that

$$|V_2| \le \frac{2}{n_b} \sum_{h=b}^{n_b-1} |v_{n_b,h}|, \tag{50}$$

But, by, e.g., Lemma A.0.2 of Politis, Romano, and Wolf (1999),

$$|Cov\left(I_{b,s}, I_{b,s+h}\right)| \leq 4\alpha_{h-b+1}$$

where α_m denote the α-mixing coefficients of the process x_i. Since $\alpha_m < \beta_m$, and by assumption 6, $\sum_{h=b}^{n_b-1} |v_{n_b,h}|$ is finite and so $|V_2|$ converges to zero, implying that $Var\left(\hat{G}_{n,b}^{RSDS}(x, T_n)\right) \to 0$. To complete the proof of the Theorem, we need to show that the pointwise result $\hat{G}_{n,b}^{RSDS}(x, T_n) \xrightarrow{p} G(x_\alpha, T)$ also holds uniformly. Given any subsequence $n^{(k)}$ of n, we can extract a further subsequence $n^{(k^{(j)})}$ such that $\hat{G}_{n^{(k^{(j)})},b}^{RSDS}(x, T_n) \xrightarrow{a.s.} G(x_\alpha, T)$ for all x in a countable dense set of the real line. Therefore, on a set of probability one, $\hat{G}_{n^{(k^{(j)})},b}^{RSDS}(x, T_n) \xrightarrow{p.} G(x_\alpha, T)$ and this convergence is uniform by Polya's theorem and the continuity of the density of the maximum of a countable set of normally distributed random variables.

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Table 3: Probability Values for S&P 500 Series (ABBOTT LABS.- COMPUTER SCIS.)

Company Name	P. Value	Company Name	P. Value
ABBOTT LABS.	0.544	ADC TELECOM.	0.235
ADOBE SYS.	0.644	**ADVD.MICRO DEVC. (1)**	**0.007**
AES (2)	**0.000**	AFLAC	0.342
AIR PRDS.& CHEMS.	0.685	ALBERTO CULVER 'B'	0.530
ALBERTSONS	0.289	ALCOA	0.174
ALLEGHENY EN.	0.087	ALLEGHENY TECHS.	0.248
ALLERGAN	0.295	ALLIED WASTE INDS.	0.188
ALLTEL (3)	**0.020**	ALTERA	0.584
ALTRIA GP.	0.356	AMBAC FINANCIAL	0.389
AMERADA HESS	0.624	AMER.ELEC.PWR.	0.114
AMERICAN EXPRESS (4)	**0.000**	AMER.GREETINGS 'A'	0.349
AMERICAN INTL.GP. (5)	**0.020**	AMER.POWER CONV.	0.544
AMGEN	0.362	AMSOUTH BANC.	0.268
ANADARKO PETROLEUM	0.752	**ANALOG DEVICES (6)**	**0.020**
ANDREW (7)	**0.020**	ANHEUSER - BUSCH COS.	0.074
AON	0.255	APACHE	0.235
APPLE COMPUTERS	0.879	**APPLERA APPD.BIOS. (8)**	**0.027**
APPLIED MATS.	0.087	ARCHER - DANLS.	0.550
ASHLAND	0.940	**AT & T (9)**	**0.027**
AUTODESK	0.624	**AUTOMATIC DATA PROC. (10)**	**0.027**
AUTONATION	0.201	AUTOZONE	0.403
AVERY DENNISON	0.148	AVON PRODUCTS	0.262
BAKER HUGHES	0.081	BALL	0.094
BANK OF AMERICA	0.289	**BANK OF NEW YORK (11)**	**0.013**
BANK ONE	0.597	BARD C R	0.242
BAUSCH & LOMB	0.128	BAXTER INTL.	0.819
BB & T	0.356	BEAR STEARNS	0.409
BECTON DICKINSON & .CO.	0.262	BED BATH & .BEYOND	0.195
BELLSOUTH (12)	**0.047**	BEMIS	0.503
BEST BUY CO.	0.430	BIG LOTS	0.450
BIOGEN IDEC	0.087	BIOMET	0.060
BJ SVS.	0.322	BLACK & .DECKER	0.745
H & R BLOCK	0.195	BMC SOFTWARE	0.792
BOEING	0.054	BOISE CASCADE	0.906
BOSTON SCIENTIFIC	0.879	**BRISTOL MYERS SQUIBB (13)**	**0.000**
BROWN - FORMAN 'B'	0.221	BRUNSWICK	0.866
BURL.NTHN.SANTA FE C	0.295	BURLINGTON RES.	0.342
CAMPBELL SOUP	0.463	CARDINAL HEALTH	0.973
CARNIVAL	0.275	CATERPILLAR	0.101
CENDANT	0.322	CENTERPOINT EN.	0.060
CENTEX	0.114	CENTURYTEL	0.617
CHARLES SCHWAB (14)	**0.040**	CHARTER ONE FINL.	0.221
CHEVRONTEXACO	0.658	CHIRON CORP	0.067
CHUBB (15)	**0.007**	CIGNA	0.235
CINCINNATI FIN.	0.208	CINTAS	0.852
CIRCUIT CITY STORES	0.107	CISCO SYSTEMS	0.154
CITIGROUP	0.342	CITIZENS COMMS.	0.221
CLEAR CHL.COMMS.	0.161	CLOROX	0.148
CMS ENERGY (16)	**0.040**	**COCA COLA (17)**	**0.047**
COCA COLA ENTS.	0.168	COLGATE - PALM.	0.799
COMCAST 'A'	0.188	**COMERICA (18)**	**0.040**
COMPUTER ASSOCS.INTL.	0.067	COMPUTER SCIS.	0.054

Table 4: Probability Values for S&P 500 Series (COMPUWARE - ITT INDUSTRIES)

Company Name	P. Value	Company Name	P. Value
COMPUWARE	0.617	COMVERSE TECH.	0.161
CONAGRA	0.409	CONCORD EFS	0.356
CONOCOPHILLIPS	0.919	CONS.EDISON	0.671
CONSTELLATION EN.	0.248	COOPER INDS.	0.161
COOPER TIRE RUB.	0.879	ADOLPH COORS 'B'	0.403
CORNING (19)	**0.000**	COUNTRYWIDE FINL.	0.819
CRANE	0.107	CSX	0.154
CUMMINS	0.168	**CVS (20)**	**0.000**
DANA (21)	**0.047**	DANAHER	0.738
DEERE & CO. (22)	**0.007**	DELL	0.054
DELTA AIR LINES (23)	**0.013**	DELUXE	0.154
DILLARDS 'A'	0.148	DOLLAR GENERAL	0.510
DOMINION RES.	0.403	DONNELLEY R R	0.503
DOVER (24)	**0.040**	DOW CHEMICALS	0.336
DOW JONES & .CO	0.711	DTE ENERGY	0.289
DU PONT E I DE NEMOURS	0.181	**DUKE ENERGY (25)**	**0.013**
DYNEGY 'A'	0.121	EASTMAN KODAK	0.081
EATON	0.664	ECOLAB	0.322
EDISON INTL. (26)	**0.020**	EL PASO	0.141
ELECTRONIC ARTS (27)	**0.000**	**ELECTRONIC DATA SYSTEMS(28)**	**0.027**
EMC	0.295	**EMERSON ELECTRIC (29)**	**0.000**
ENGELHARD	0.329	ENTERGY	0.537
EOG RES.	0.973	EQUIFAX	0.383
EXELON	0.557	EXPRESS SCRIPTS 'A'	0.349
EXXON MOBIL (30)	**0.000**	FAMILY $.STRS.	0.148
FANNIE MAE	0.134	FREDDIE MAC	0.087
FEDERATED DEPT.STRS.	0.409	FEDEX	0.577
FIFTH THIRD BANCORP (31)	**0.000**	**FIRST DATA (32)**	**0.047**
FIRST TEN.NAT.	0.174	FIRSTENERGY	0.678
FISERV	0.604	FLEETBOSTON FINL.	0.094
FORD MOTOR	0.369	FOREST LABS.	0.228
FORTUNE BRANDS	0.718	FPL GROUP	0.416
FRANK.RES.	0.631	**GANNETT (33)**	**0.040**
GAP (34)	**0.027**	GEN.DYNAMICS	0.356
GENERAL ELECTRIC (35)	**0.000**	GEN.MILLS	0.550
GENERAL MOTORS	0.872	GENUINE PARTS	0.611
GENZYME	0.711	**GEORGIA PACIFIC (36)**	**0.013**
GILLETTE	0.248	GOLDEN WEST FINL.	0.154
GOODRICH	0.054	**GOODYEAR TIRE (37)**	**0.007**
GRAINGER W W (38)	**0.040**	GT.LAKES CHM.	0.087
HALLIBURTON	0.154	HARLEY - DAVIDSON	0.054
HARRAHS ENTM.	0.235	HASBRO	0.060
HCA	0.678	HEALTH MAN.AS.A	0.859
HEINZ HJ	0.389	**HERCULES (39)**	**0.000**
HERSHEY FOODS	0.302	**HEWLETT - PACKARD (40)**	**0.000**
HILTON HOTELS	0.550	HOME DEPOT	0.060
HONEYWELL INTL. (41)	**0.007**	HUMANA	0.188
HUNTINGTON BCSH.	0.128	ILLINOIS TOOL WKS.	0.289
INGERSOLL - RAND (42)	**0.013**	INTEL	0.101
INTL.BUS.MACH.	0.906	**INTL.FLAV.& FRAG. (43)**	**0.013**
INTL.GAME TECH. (44)	**0.000**	**INTL.PAPER (45)**	**0.040**
INTERPUBLIC GP. (46)	**0.040**	ITT INDUSTRIES	0.570

Table 5: Probability Values for S&P 500 Series (JP MORGAN CHASE - PULTE HOMES)

Company Name	P. Value	Company Name	P. Value
JP MORGAN CHASE & .CO. (47)	**0.020**	JEFFERSON PILOT	0.664
JOHNSON & JOHNSON	0.490	JOHNSON CONTROLS	0.792
JONES APPAREL GROUP	0.591	KB HOME	0.456
KELLOGG	0.356	KERR - MCGEE	0.074
KEYCORP	0.289	KEYSPAN	0.295
KIMBERLY - CLARK	0.698	KINDER MORGAN KANS	0.275
KLA TENCOR	0.074	KNIGHT - RIDDER	0.201
KOHLS	0.597	KROGER	0.725
LEGGETT& PLATT	0.530	LILLY ELI	0.423
LIMITED BRANDS	0.523	**LINCOLN NAT. (48)**	**0.047**
LINEAR TECH. (49)	**0.000**	LIZ CLAIBORNE	0.121
LOEWS (50)	**0.000**	LNA.PACIFIC	0.282
LOWE'S COMPANIES	0.362	**LSI LOGIC (51)**	**0.013**
MANOR CARE	0.054	**MARATHON OIL (52)**	**0.034**
MARSH & MCLENNAN (53)	**0.000**	MARSHALL & ILSLEY	0.282
MASCO	0.745	**MATTEL (54)**	**0.034**
MAXIM INTEGRATED PRDS.	0.779	MAY DEPT.STORES	0.430
MAYTAG	0.215	MBIA	0.295
MBNA	0.772	MCCORMICK & .CO NV.	0.174
MCDONALDS	0.094	**MCGRAW - HILL CO. (55)**	**0.007**
MEADWESTVACO	0.154	**MEDIMMUNE (56)**	**0.027**
MEDTRONIC	0.523	MELLON FINL.	0.188
MERCK & .CO.	0.705	MEREDITH	0.215
MERRILL LYNCH & .CO.	0.101	MGIC INVT	0.060
MICRON TECH. (57)	**0.000**	MICROSOFT	0.141
MILLIPORE	0.087	**MOLEX (58)**	**0.040**
MOTOROLA (59)	**0.007**	**NABORS INDS. (60)**	**0.020**
NAT.CITY (61)	**0.034**	**NATIONAL SEMICON. (62)**	**0.013**
NAVISTAR INTL. (63)	**0.040**	NEW YORK TIMES 'A'	0.477
NEWELL RUBBERMAID	0.369	NEWMONT MINING	0.107
NEXTEL COMMS.A	0.564	NICOR	0.396
NIKE 'B'	0.329	**NISOURCE (64)**	**0.007**
NOBLE	0.510	NORDSTROM	0.564
NORFOLK SOUTHERN (65)	**0.040**	NORTH FORK BANCORP.	0.443
NTHN.TRUST	0.067	NORTHROP GRUMMAN	0.309
NOVELL	0.409	NOVELLUS SYSTEMS	0.550
NUCOR	0.242	OCCIDENTAL PTL.	0.463
OFFICE DEPOT	0.094	OMNICOM GP.	0.081
ORACLE	0.195	PACCAR	0.215
PALL	0.987	PARAMETRIC TECH.	0.134
PARKER - HANNIFIN	0.342	PAYCHEX	0.644
PENNEY JC (66)	**0.000**	PEOPLES ENERGY	0.906
PEOPLESOFT	0.711	PEPSICO	0.443
PERKINELMER (67)	**0.000**	PFIZER	0.268
PG & .E	0.161	PHELPS DODGE	0.101
PINNACLE WEST CAP.	0.141	PITNEY - BOWES	0.060
PLUM CREEK TIMBER (68)	**0.013**	**PMC - SIERRA (69)**	**0.000**
PNC FINL.SVS.GP.	0.718	PPG INDUSTRIES	0.168
PPL (70)	**0.034**	PRAXAIR	0.711
PROCTER & GAMBLE	0.463	PROGRESS EN.	0.577
PROGRESSIVE OHIO	0.846	**PROVIDIAN FINL. (71)**	**0.000**
PUB.SER.ENTER.GP.	0.624	PULTE HOMES	0.342

Table 6: Probability Values for S&P 500 Series (QUALCOMM - 3M)

Company Name	P. Value	Company Name	P. Value
QUALCOMM	0.584	**RADIOSHACK (72)**	**0.034**
RAYTHEON 'B' (73)	**0.000**	REEBOK INTL.	0.107
REGIONS FINL.	0.617	ROBERT HALF INTL.	0.617
ROCKWELL AUTOMATION	0.101	**ROHM & HAAS (74)**	**0.040**
ROWAN COS. (75)	**0.020**	RYDER SYSTEM	0.349
SAFECO	0.201	SAFEWAY	0.705
SARA LEE	0.074	**SBC COMMUNICATIONS (76)**	**0.007**
SCHERING - PLOUGH (77)	**0.013**	**SCHLUMBERGER (78)**	**0.007**
SCIENTIFIC ATLANTA	0.698	SEALED AIR	0.617
SEARS ROEBUCK & .CO.	0.060	**SEMPRA EN. (79)**	**0.040**
SHERWIN - WILLIAMS	0.054	SIGMA ALDRICH	0.799
SLM	0.275	SNAP - ON	0.725
SOLECTRON	0.134	SOUTHERN	0.356
SOUTHTRUST	0.477	SOUTHWEST AIRLINES	0.738
SPRINT	0.067	ST.JUDE MED.	0.503
ST.PAUL	0.094	STANLEY WORKS	0.208
STAPLES	0.168	STARBUCKS	0.906
STARWOOD HTLS.& .RESORTS(80)	**0.000**	STATE STREET	0.242
STRYKER	0.081	SUN MICROSYSTEMS	0.121
SUNGARD DATA SYSTEMS	0.141	SUNOCO	0.221
SUNTRUST BANKS (81)	**0.034**	SUPERVALU	0.074
SYMANTEC	0.423	SYMBOL TECHS.	0.295
SYNOVUS FINL.	0.262	SYSCO	0.403
T ROWE PRICE GP.	0.772	**TARGET (82)**	**0.007**
TECO ENERGY	0.268	TEKTRONIX	0.483
TELLABS	0.242	TEMPLE INLAND	0.228
TENET HLTHCR.	0.805	**TERADYNE (83)**	**0.040**
TEXAS INSTS. (84)	**0.040**	**TEXTRON (85)**	**0.013**
THERMO ELECTRON (86)	**0.027**	**THOMAS & .BETTS (87)**	**0.000**
TIFFANY & CO	0.114	TIME WARNER	0.148
TJX COS.	0.148	TORCHMARK	0.121
TOYS R US HOLDINGS CO. (88)	**0.007**	TRIBUNE	0.188
TXU	0.463	TYCO INTL.	0.564
US BANCORP (89)	**0.000**	UNION PACIFIC	0.631
UNION PLANTERS	0.168	**UNISYS (90)**	**0.047**
UNITEDHEALTH GP.	0.644	US.STEEL	0.624
UNITED TECHNOLOGIES	0.067	UNOCAL	0.812
UNUMPROVIDENT	0.074	UST	0.651
V F	0.268	VERIZON COMMS.	0.315
VIACOM 'B'	0.812	VULCAN MATERIALS	0.369
WACHOVIA (91)	**0.047**	WALGREEN	0.067
WAL MART STORES	0.356	**WALT DISNEY (92)**	**0.047**
WASHINGTON MUTUAL	0.369	WASTE MAN.	0.852
WELLS FARGO & .CO	0.403	WENDY'S INTL.	0.597
WEYERHAEUSER	0.443	WHIRLPOOL	0.530
WILLIAMS COS. (93)	**0.034**	**WINN - DIXIE STRS. (94)**	**0.013**
WORTHINGTON INDS.	0.987	WRIGLEY WILLIAM JR.	0.416
WYETH (95)	**0.000**	XCEL ENERGY	0.181
XEROX (96)	**0.000**	XILINX	0.262
ZIONS BANCORP. (97)	**0.013**	3M	0.154

Figure 2: Recursively Estimated Mean for S&P 500 series that reject the strict stationarity null hypothesis

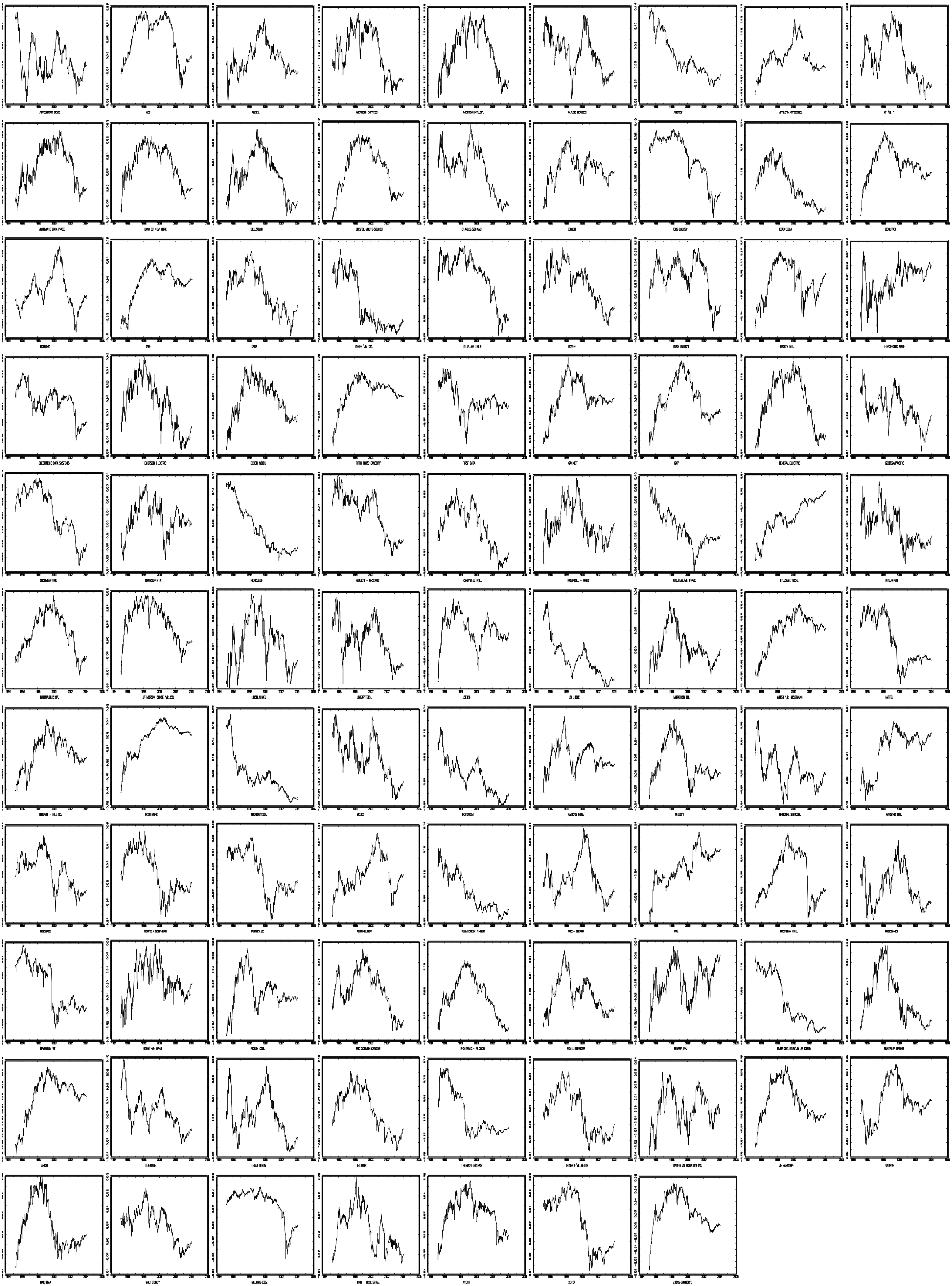


Figure 3: Recursively Estimated Variance for S&P 500 series that reject the strict stationarity null hypothesis

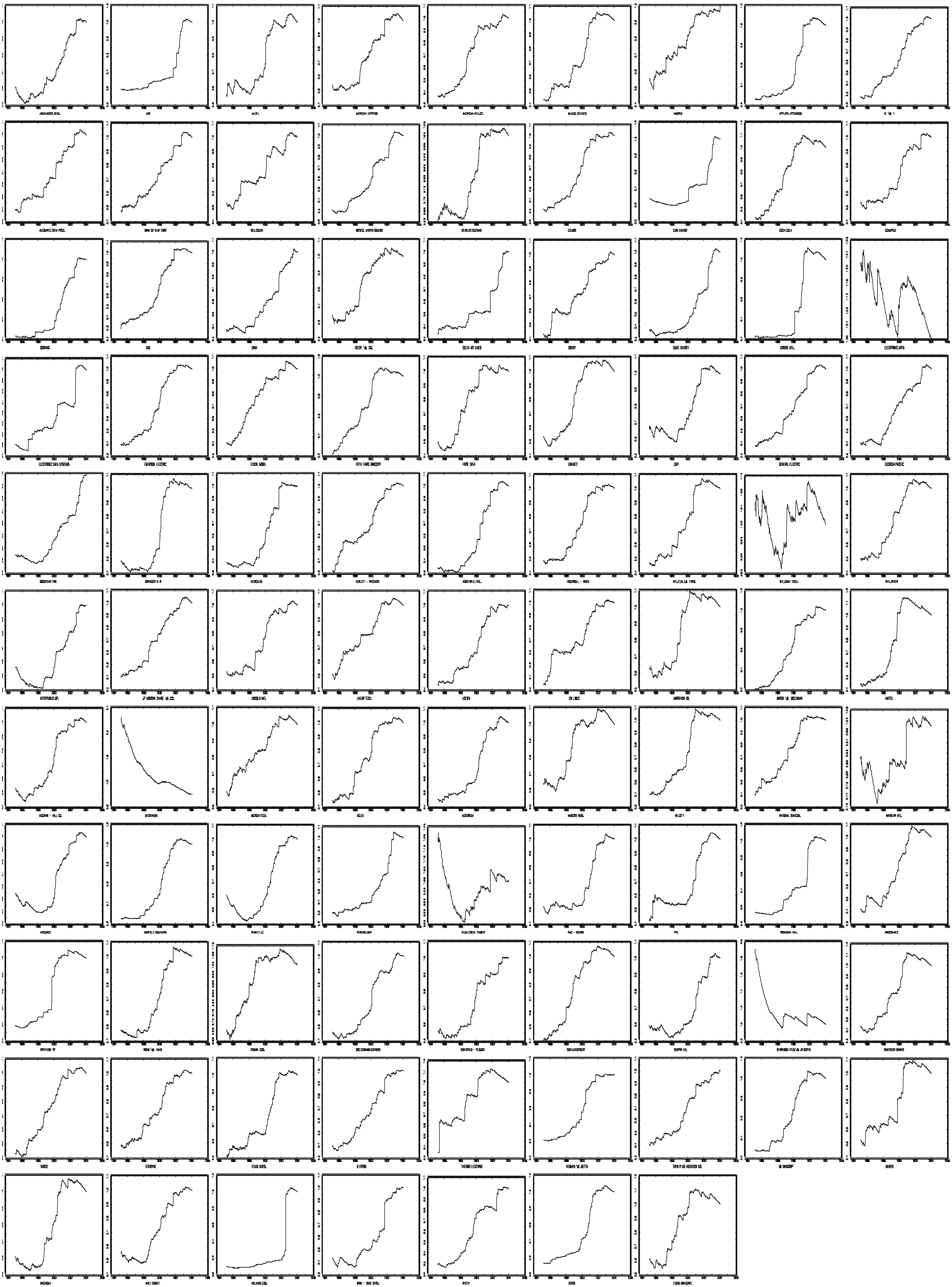


Figure 4: Recursively Estimated Skew for S&P 500 series that reject the strict stationarity null hypothesis

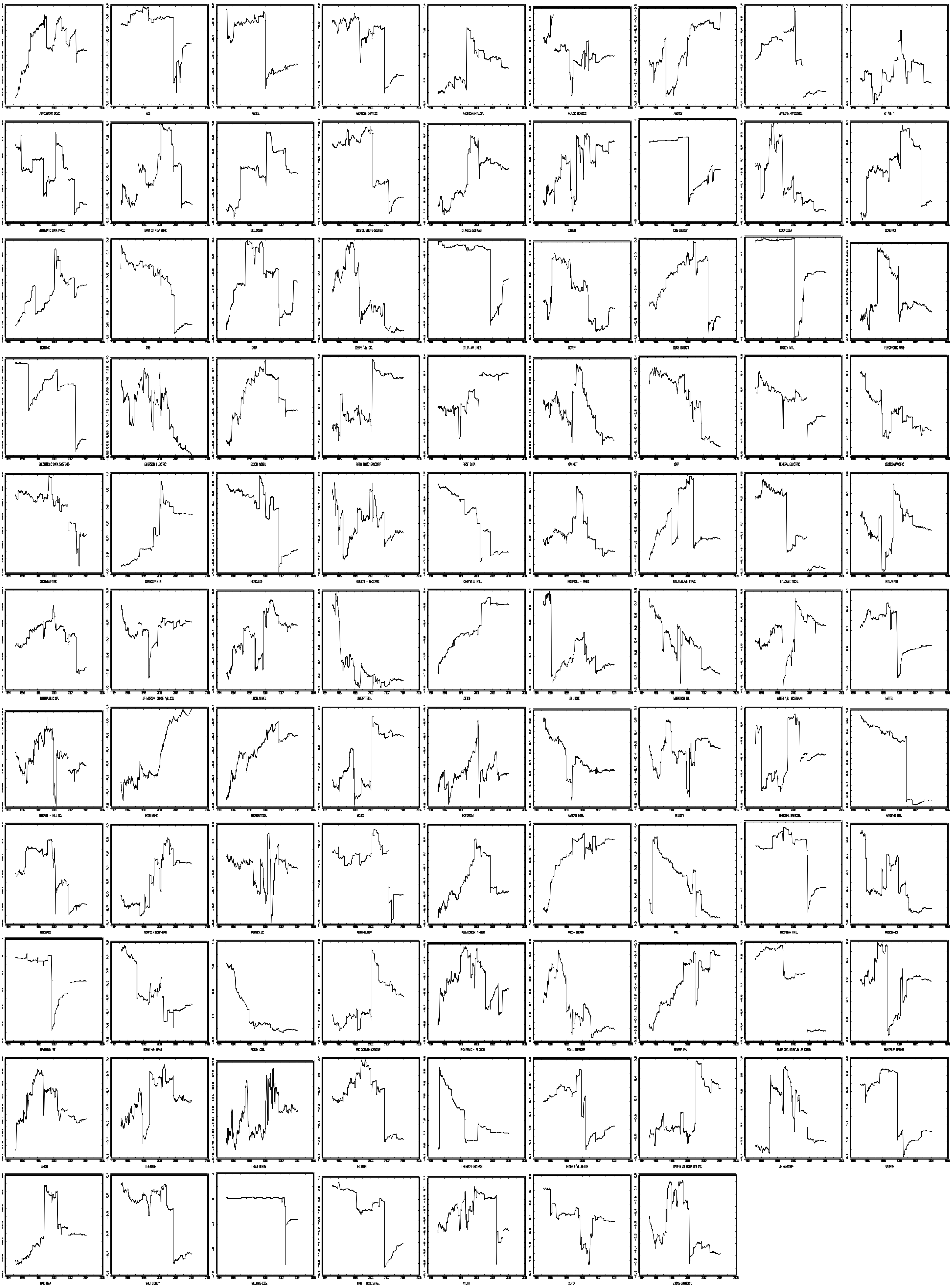
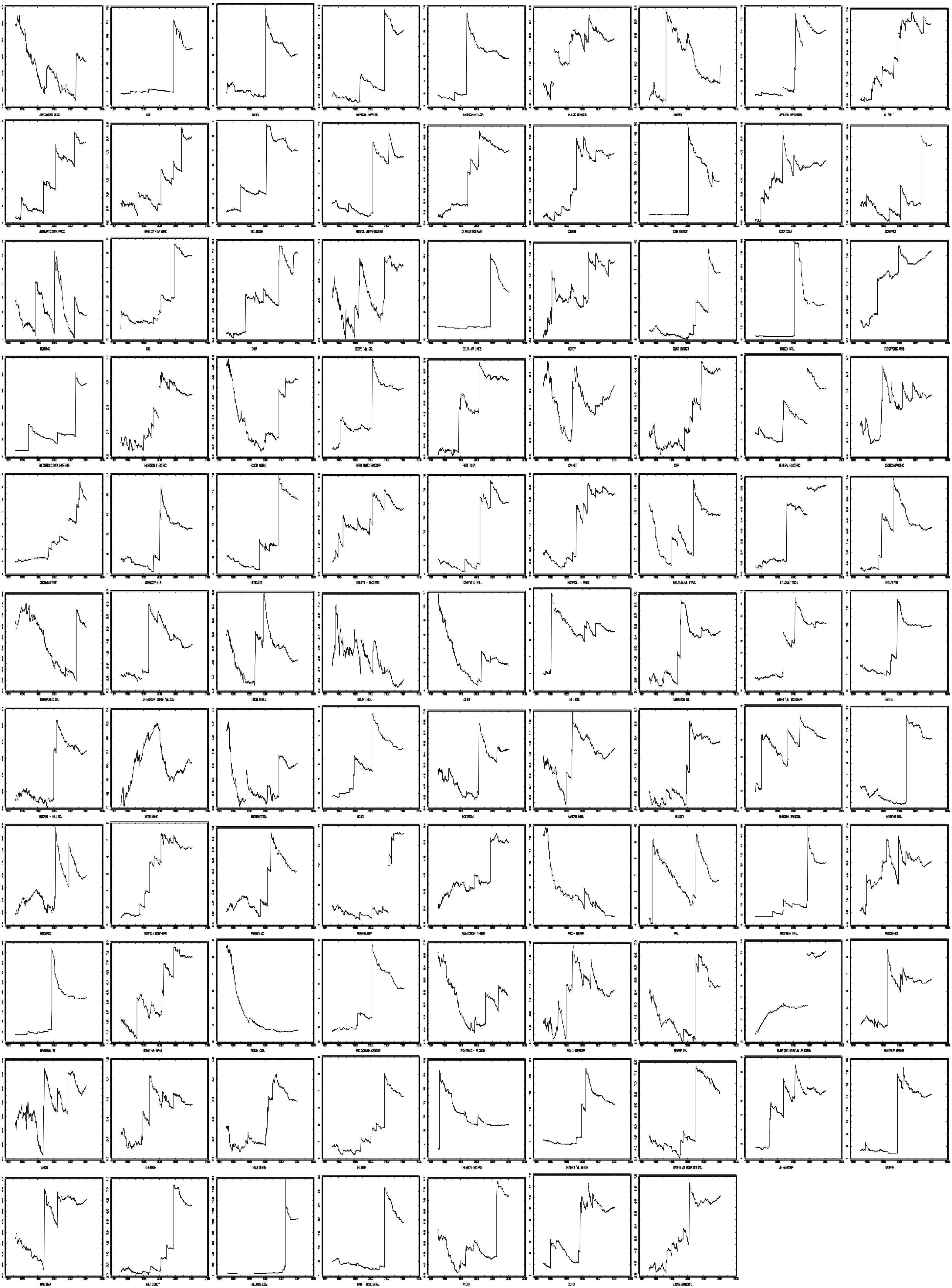


Figure 5: Recursively Estimated Kyrtosis for S&P 500 series that reject the strict stationarity null hypothesis



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