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Macroeconomic Forecasting with Mixed Frequency Data:
Forecasting US Output Growth

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Abstract

Many macroeconomic series such as US real output growth are sampled quarterly, although potentially useful predictors are often observed at a higher frequency. We look at whether a mixed data-frequency sampling (MIDAS) approach can improve forecasts of output growth. The MIDAS approach is compared to other ways of making use of monthly data to predict quarterly output growth. The MIDAS specification used in the comparison employs a novel way of including an autoregressive term. We find that the use of monthly data on the current quarter leads to significant improvement in forecasting current and next quarter output growth, and that MIDAS is an effective way of exploiting monthly data compared to alternative methods. We also exploit the best method to use the monthly vintages of the indicators for real-time forecasting.

Keywords: Mixed data frequency, coincident indicators, real-time forecasting, US output growth.

JEL classification: C51, C53.

1 Introduction

The unavailability of key macroeconomic variables such as GDP (or GNP) at frequencies higher than quarterly has led to many macroeconometric models being specified on quarterly data. We consider the usefulness of information available at higher frequencies, such as monthly data, for forecasting output growth. There are a range of leading and coincident indicator variables at a monthly frequency. If all the variables in the model have to be sampled at the same frequency, data available at the monthly frequency has to be converted to the quarterly frequency, for example by averaging the months (or taking the last month in the quarter), and information on the first month (or first two months) of the quarter being forecast is discarded.

In this paper we explore whether the MIDAS (MIxed Data Sampling) approach of Ghysels, Santa-Clara and Valkanov (2004), Ghysels, Sinko and Valkanov (2006b) can be successfully adapted to the modelling and forecasting of a key US macroeconomic variable – US output growth. MIDAS allows the regressand and regressors to be sampled at different frequencies. Typically, the regressand is sampled at the lower frequency. With few exceptions, MIDAS has been used for high-frequency financial data (see, for example, Ghysels, Santa-Clara and Valkanov (2006a)), although Ghysels and Wright (2006) look at survey-based macroeconomic forecasts. We compare the results of using MIDAS to forecast output growth with two other approaches that exploit monthly data. The first is an extension to the models used by Koenig, Dolmas and Piger (2003) and the second is a two-step procedure that firstly generates forecasts of missing monthly indicator values which are then averaged to generate quarterly observations. The MIDAS specification used in the comparison employs a novel way of including an autoregressive term.

Looking ahead to the results, we find that the use of within-quarter information on monthly indicators can result in marked reductions in RMSE compared to quarterly-frequency autoregressive (AR) or autoregressive distributed-lag (ADL) models. Within the set of models that use monthly information, MIDAS fares well across the set of indicators we consider. Coupled with their flexibility and ease of use, we conclude that MIDAS models are an attractive way of exploiting the information in monthly indicators.

These findings are based on a recursive out-of-sample forecasting exercise that uses 'conventional' real-time data. As explained by Koenig *et al.* (2003), the use of real-time data is clearly preferable to the use of final-revised data for estimation and evaluation purposes, as out-of-sample forecasting exercises based on final-revised data may exaggerate the predictive power of explanatory variables relative to what could actually have been achieved at the time (Diebold and Rudebusch (1991), Orphanides (2001), Orphanides and van Norden (2005), Faust, Rogers and Wright (2003)). Koenig *et al.* (2003) note that the way in which real-time data is conventionally used in forecast comparison exercises is based on the use of end-of-sample vintage data. They argue that this approach to real-time estimation and forecasting may be suboptimal: that 'at every date within

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a sample, right-side variables ought to be the most up-to-date estimates available *at that time'* (Koenig et al, p.618, described as Strategy 1 on p.619). We find that Koenig et al's suggestion to use real-time-vintage data to estimate forecasting models improves forecast accuracy, and that our main conclusions are unchanged: the inclusion of monthly data in the forecasting problem can dramatically reduce RMSE; and MIDAS is an effective way of incorporating monthly data.

The plan of the rest of the paper is as follows. In section 2 we briefly review the MIDAS approach of Ghysels *et al.* (2004, 2006b) and propose an extension that facilitates the application of MIDAS to macroeconomic data, namely, the inclusion of an autoregressive component. The two other approaches to using monthly indicator data to generate forecasts of quarterly output growth are also described. Section 3 contains the out-of-sample forecast comparison exercise, split into five parts. The first describes our use of monthly and quarterly vintages to estimate and forecast in real-time. Section 3.2 compares the MIDAS-AR forecasts against the quarterly AR model forecasts, using both a conventional real-time data vintage approach, and using final-revised data, to see whether the incorporation of monthly indicator data in the forecasting model results in significant improvements in forecast accuracy. Section 3.3 compares alternative ways of incorporating monthly data into the forecasting problem. Section 3.4 investigates the use of real-time-vintage data in the forecast comparison exercise. Finally, section 3.5 compares the multiple-indicator model forecasts with combining individual models' forecasts. Section 4 offers some concluding remarks.

2   MIDAS regression approach

The MIDAS models of Ghysels *et al.* (2004, 2006b) are closely related to distributed lag models (see, e.g., Dhrymes (1971) and Sims (1974)). The response of the dependent variable to the higher frequency explanatory variables is modelled using highly parsimonious distributed lag polynomials, as a way of preventing the proliferation of parameters that might otherwise result, and as a way of side-stepping difficult issues to do with lag-order selection. Parameter proliferation could be especially important in financial applications, where say, daily volatility is related to 5-minute interval intraday data (so that a day's worth of observations amounts to 288 data points), but parsimony is also likely to be important in typical macroeconomic applications, where quarterly data are related to monthly data, given the much smaller numbers of observations typically available. Modelling the coefficients on the lagged explanatory variables as a distributed lag function allows for long lags with only a small number of parameters needing to be estimated.

The basic MIDAS model for a single explanatory variable, and h-step ahead forecasting, is given by:

$$y_t = \beta_0 + \beta_1 B\left(L^{1/m}; \boldsymbol{\theta}\right) x_{t-h}^{(m)} + \varepsilon_t \tag{1}$$

where $B\left(L^{1/m}; \boldsymbol{\theta}\right) = \sum_{k=1}^{K} b\left(k; \boldsymbol{\theta}\right) L^{(k-1)/m}$, and $L^{s/m} x_{t-1}^{(m)} = x_{t-1-s/m}^{(m)}$. Here, t indexes the basic

time unit (in our case, quarters), and m is the higher sampling frequency ($m = 3$ when x is monthly and y is quarterly), and as shown $L^{1/m}$ operates at the higher frequency. All the parameters of the MIDAS model depend on the horizon h (although this is suppressed in the notation), and forecasts are computed directly without requiring forecasts of explanatory variables. An 'Exponential Almon Lag' function (Ghysels *et al.* (2004, 2006b)) parameterizes $b(k; \boldsymbol{\theta})$ as:

$$b(k; \boldsymbol{\theta}) = \frac{\exp(\theta_1 k + \theta_2 k^2)}{\sum_{k=1}^{K} \exp(\theta_1 k + \theta_2 k^2)}. \tag{2}$$

As macroeconomic forecasts are often produced a number of times during each quarter, monthly data on relevant indicators for the quarter being forecast will sometimes be available. For example, the staff of the Board of Governors of the Federal Reserve prepare forecasts several times each quarter for the meetings of the Open Market Committee: see Karamouzis and Lombra (1989) and Joutz and Stekler (2000). For illustrative purposes, suppose the value of x in the first month of the quarter is known. The MIDAS framework can exploit this data by simply specifying the regression model as:

$$y_t = \beta_0 + \beta_1 B\left(L^{1/3}; \boldsymbol{\theta}\right) x_{t-2/3}^{(3)} + \varepsilon_t$$

where $h = 2/3$ signifies $1/3$ of the information on the current quarter is employed. Forecasts with $h = 1/3$ are also possible using information on the first two months of the quarter being forecast. So the MIDAS model can incorporate within-quarter monthly observations on the indicator variable in a simple fashion.

2.1 Autoregressive structure

Models to forecast US output growth often include autoregressive terms, as in the ADL models of Stock and Watson (2003). Including autoregressive dynamics in models that sample the explanatory variables at a higher frequency is clearly desirable. As noted by Ghysels *et al.* (2006b), this is not straightforward: consider simply adding a lower frequency lag of y, y_{t-1} to (1) for one-step-ahead forecasting, to give:

$$y_t = \beta_0 + \lambda y_{t-1} + \beta_1 B\left(L^{1/3}; \boldsymbol{\theta}\right) x_{t-1}^{(3)} + \varepsilon_t.$$

This strategy is not in general appropriate, as from writing the model as:

$$y_t = \beta_0 (1 - \lambda)^{-1} + \beta_1 (1 - \lambda L)^{-1} B\left(L^{1/3}; \boldsymbol{\theta}\right) x_{t-1}^{(3)} + \widetilde{\varepsilon}_t,$$

where $\widetilde{\varepsilon}_t = (1 - \lambda L)^{-1} \varepsilon_t$, it is apparent that the polynomial on $x_{t-1}^{(3)}$ is the product of a polynomial in $L^{1/3}$, $B\left(L^{1/3}; \boldsymbol{\theta}\right)$, and a polynomial in L, $\sum \lambda^j L^j$. This mixture generates a 'seasonal' response of y to $x^{(3)}$, irrespective of whether $x^{(3)}$ displays a seasonal pattern.

Our suggested solution is simply to introduce autoregressive dynamics in y_t as a common factor (see, e.g., Hendry and Mizon (1978)):

$$y_t = \beta_0 + \lambda y_{t-1} + \beta_1 B\left(L^{1/3}; \boldsymbol{\theta}\right)(1 - \lambda L) x_{t-1}^{(3)} + \varepsilon_t \tag{3}$$

so that the response of y to $x^{(3)}$ remains non-seasonal. A multi-step analogue can be written as:

$$y_t = \lambda y_{t-d} + \beta_1 + \beta_2 B(L^{1/3}, \boldsymbol{\theta})\left(1 - \lambda L^d\right) x_{t-h}^{(3)} + \epsilon_t, \tag{4}$$

which we term the MIDAS-AR. When the horizon h is an integer, then $d = h$, as in equation (3), where $d = h = 1$. When information is available on the indicator in the current quarter, say, the first two months are known, then $h = 1/3$ while $d = 1$.

The referenced literature establishes that non-linear least squares is a consistent estimator for the standard MIDAS. To estimate the MIDAS-AR model, we take the residuals $(\widehat{\varepsilon}_t)$ of the standard MIDAS, and estimate an initial value for λ, say $\hat{\lambda}_0$, from $\hat{\lambda}_0 = \left(\sum \widehat{\varepsilon}_{t-h}^2\right)^{-1} \sum \widehat{\varepsilon}_t \widehat{\varepsilon}_{t-h}$. We then construct $y_t^* = y_t - \hat{\lambda}_0 y_{t-d}$ and $x_{t-h}^{*(3)} = x_{t-h}^{(3)} - \hat{\lambda}_0 x_{t-h-d}^{(3)}$, and the estimator $\hat{\theta}_1$ is obtained by applying nonlinear least squares to:

$$y_t^* = \beta_1 + \beta_2 B(L^{1/3}, \boldsymbol{\theta}) x_{t-h}^{*(3)} + \varepsilon_t.$$

A new value of λ, $\hat{\lambda}_1$, is obtained from the residuals of this regression. Then using $\hat{\lambda}_1$ and $\hat{\boldsymbol{\theta}}_1$ as initial values, we run BFGS to get the estimates $\hat{\lambda}$ and $\hat{\boldsymbol{\theta}}$ that minimize the sum of squared residuals. The computations are carried out using the constrained ML package of Gauss 5, CML 2.0, and selecting the BFGS algorithm. The restrictions imposed in the estimation are that $\theta_1 \leq 300$ and that $\theta_2 < 0$, and we experiment with a number of initial values for $\boldsymbol{\theta}$ in order to counter any dependence of the optimization routine on the initial values.

2.2 Combining indicators

A M-MIDAS-AR model that combines the information of nl monthly leading indicators to predict output growth, h-steps-ahead, would be written as:

$$y_t = \lambda y_{t-d} + \beta_0 + \sum_{i=1}^{nl} \beta_{1i} B_i(L^{1/m}; \boldsymbol{\theta}_i)\left(1 - \lambda L^d\right) x_{i,t-h}^{(m)} + \varepsilon_t \tag{5}$$

where the component indicators are indexed by i, and $m = 3$. Each leading indicator requires the estimation of only two parameters to describe the lag structure $(\boldsymbol{\theta}_i)$ and one to weight their impact on y_t (β_{1i}). Because the number of parameters required for each additional leading indicator is small, one might anticipate a good forecast performance from MIDAS when multiple indicators are included in the forecasting model, relative to other models where the larger number of indicators is accommodated at the cost of many more parameters to be estimated.

2.3 Alternative methods of exploiting monthly indicator data

In addition to MIDAS, two other methods are used in the empirical forecast comparisons. The forecasting models used by Koenig *et al.* (2003) regress quarterly changes in real GDP on a constant and five lagged monthly changes in the monthly indicator variables. Their forecasting models are similar to the MIDAS approach except that the coefficients on the right-side explanatory variables are estimated unrestrictedly, rather than using a restricted distributed function, and there are no autoregressive terms compared to our MIDAS-AR. Koenig *et al.* (2003) only calculate forecasts when the values of the indicators for all the months in the quarter being forecast are known. We term these models mixed-frequency distributed lag models (MF-DL), and use them to generate forecasts for a number of monthly horizons (in addition to Koenig et al's $h = 0$) when only partial monthly information is available on the quarter being forecast (corresponding to $h = 1/3$ and $h = 2/3$).

The second method is to use a vector autoregression (VAR) consisting of the monthly indicator variables to provide forecasts of the missing monthly values, which are then aggregated to provide estimates of the quarterly values of the indicators. This method resembles the 'bridge equation' approach popular in Central Banks (see, e.g., Rünstler and Sédillot (2003) and Zheng and Rossiter (2006)). As an example, suppose we have data only on the first month of quarter t, i.e., $x_{t-2/3}^{(3)}$ is known, but $x_t^{(3)}$ and $x_{t-1/3}^{(3)}$ are not yet known. Forecasts of $\left\{ x_t^{(3)}, x_{t-1/3}^{(3)} \right\}$ are obtained from the VAR, $\left\{ \widehat{x}_t^{(3)}, \widehat{x}_{t-1/3}^{(3)} \right\}$, and the quarterly estimate of x_t is constructed as $\widehat{x}_t = \frac{1}{3} \left(\widehat{x}_t^{(3)} + \widehat{x}_{t-1/3}^{(3)} + x_{t-2/3}^{(3)} \right)$, which is used in the quarterly-frequency ADL model to forecast y_t. We refer to the approach that uses forecasts of missing monthly observations to augment a quarterly-frequency ADL as ADL-F. When the forecast horizon is an integer number of quarters, forecasts of the monthly indicator are not required, and ADL-F corresponds to the standard quarterly-frequency ADL. When using single-indicator models, we use an AR to compute forecasts for the missing monthly values, rather than a VAR, so that the same indicator information is available to all models.

There are other ways of using monthly information. For example, Miller and Chin (1996) propose combining the forecasts from a monthly model with forecasts from a quarterly model. There are also factor model approaches which make use of mixed-frequency data, such as Schumacher and Breitung (2006), which adapt single-frequency factor models (see Boivin and Ng (2005) for a review). However, for the purpose of evaluating the accuracy of MIDAS models, we choose to use MF-DL and ADL-F because they are simple and popular methods when there is a relatively small number of indicators available.

3 Empirical forecasting comparisons

The relative forecast performance of the models is assessed by comparing RMSEs in a recursive forecasting exercise. Because we also exploit monthly vintages of the indicators while forecasting in real time, we first describe how end-of-sample-vintage data and real-time-vintage data are used for model estimation and forecasting in this context. To highlight the principal findings, the forecast comparisons are then presented in four further sections. The first of these compares the MIDAS-AR forecasts against the quarterly AR(1) and ADL forecasts. The number of lags of the indicator in the ADL is selected using SIC setting the maximum to 5. We compare the relative performances when end-of-sample-vintage data is used and when final-revised data is used to see whether the predictive content of monthly data (via the MIDAS-AR) is sensitive to this issue - recall in the introduction we referenced a number of studies where this issue was key. Section 3.3 compares the MIDAS-AR against the two alternative ways of using monthly data (MF-DL and ADL-F) based on a conventional real-time data forecasting exercise. Section 3.4 investigates whether the result of the forecast comparison changes with the use of real-time-vintage data. Finally, section 3.5 compares the multiple-indicator model forecasts with combinations of individual models' forecasts, motivated by the vast literature that attests to the usefulness of forecast combination (see the recent reviews by Diebold and Lopez (1996), Newbold and Harvey (2002) and Timmermann (2006)).

3.1 Use of vintage-data in the real-time forecasting exercises

Our real-time data consists of quarterly vintages of output growth and monthly vintages of the indicators, obtained from the Philadelphia Fed: see Croushore and Stark (2001). We consider three monthly indicators: industrial production (IP), employment (EMP; payroll, non-farm) and capacity utilization (CU). The first two are components of the Conference Board Coincident Index, and all three series as well as real output were downloaded from `http://www.phil.frb.org`. Before employing the indicator data set for forecasting, we construct approximate monthly growth rates by taking the first difference of the log of each series. Real output growth is the quarterly difference of the log of output. The quarterly real-time data sets of the Philadelphia Fed record the data that were available on the 15th of the middle month of a quarter, so that for output the data sets contain the BEA 'Advance' estimates for the latest quarter (the previous quarter) as well as revised data for earlier quarters. It may in principle be possible to incorporate the different releases of output data that are made during the course of a quarter into the forecast comparisons, but data availability prevented us from considering this option.

The exercise consists of forecasting output growth in the quarters 1985:Q2 to 2005:Q1. For each of these quarters we generate forecasts with horizons from $h = 0$ up to 2 quarters, with monthly steps, $h = 1/3$, $h = 2/3$ etc. in the case of the forecasting models that make use of monthly indicator information, as described more fully below. The model estimation sample begins in 1959:Q1.

The monthly data consists of monthly vintages of the indicators from 1985:M1 to 2005:M1. For expositional purposes, let y_{t+1} denote current quarter output growth, and y_{t+2} next quarter output growth.

The timing of the release of official data on output growth and the monthly indicators is as follows. The data-vintage for output growth for quarter $t+1$ contains data up to quarter t. Before data on output growth for $t+1$ becomes available in the $t+2$ quarterly vintage, we will have data on x up to $t+1/3$ from the $t+2/3$ monthly vintage, data on x up to $t+2/3$ from the $t+1$ monthly vintage, and data on x up to $t+1$ from the $t+4/3$ monthly vintage. (We suppress the $^{(3)}$ superscript on x - it is implicit that x is recorded at the monthly frequency, and y is quarterly). The timing of these three monthly releases relative to the release of the quarterly data gives rise to forecast horizons of $h = 2/3$, $1/3$ and 0. We adopt the notation that $y_{\tau;v}$ denotes the value of y in period τ in the vintage v data set (and similarly for x, where the monthly frequency gives rise to τ and v being measured as fractions of quarters). Following Koenig *et al.* (2003) and others, our aim is to forecast the final output growth numbers, where the 'final' data is taken to be the latest data vintage we have access to (2005:Q2), which we denote by T, so that $y_{t+1;T}$ denotes the estimate of the actual value of y in $t+1$ in the final data vintage.

We use two ways of exploiting the real-time data set for forecasting. The first one is the 'traditional' or end-of-sample-vintage data approach to real-time forecasting. We then outline the proposal of Koenig *et al.* (2003) to use real-time-vintage data.

3.1.1 End-of-sample-vintage data real-time forecasting with monthly data

At each forecast origin, the models are estimated and the forecasts are computed using the data contained in the most recent datasets available at that time. For example, for a one-step forecast of $y_{t+1;T}$ from an AR(1), we regress $\mathbf{y}_{t;t+1}$ on $\mathbf{y}_{t-1;t+1}$ (and a constant), where $\mathbf{y}_{t;t+1} = [y_{2;t+1}, \ldots, y_{t;t+1}]'$ and $\mathbf{y}_{t-1;t+1} = [y_{1;t+1}, \ldots, y_{t-1;t+1}]'$, and use the estimated model coefficients and right-side y-value of $y_{t;t+1}$ to compute the forecast $\hat{y}_{t+1;T}$. When we have monthly vintages of indicator data, then the models that utilise this information (MIDAS-AR, MF-DL and ADL-F) provide four forecasts of current quarter output growth ($y_{t+1;T}$), depending on where we are in the current quarter, as set out above. Suppose we have data on the indicator for all the months in the quarter, but not for the current quarter value of output growth: we designate this a zero-horizon forecast (or 'nowcast'). Using MIDAS as an example, we regress $\mathbf{y}_{t;t+1}$ on $\mathbf{y}_{t-1;t+1}$ and $B(L^{1/3}; \theta)\mathbf{x}_{t;t+4/3}$ (and a constant), where $\mathbf{x}_{t;t+4/3} = \left[x_{2,t+4/3}, \ldots, x_{t-1;t+4/3}, x_{t;t+4/3}\right]'$, and the monthly lags of this vector are all from the $t+4/3$ monthly vintage. Then, using these estimates and the last available information on y and x at vintages $t+1$ and $t+4/3$, namely, $y_{t,t+1}$ and $\mathbf{x}_{t+1;t+4/3}$, the forecasts are obtained.

When indicator information is available on the first two months of the current quarter, the $h =$

1/3 forecast of the current quarter and the $h = 4/3$ forecast of the next quarter $(t+2)$ are constructed as follows. For $h = 1/3$, the MIDAS-AR regresses $\mathbf{y}_{t;t+1}$ on $\mathbf{y}_{t-1;t+1}$ and $B(L^{1/3}; \theta)\mathbf{x}_{t-1/3;t+1}$ (and a constant), where $\mathbf{x}_{t-1/3;t+1} = \left[x_{2-1/3,t+1}, \ldots x_{t-1/3;t+1}\right]'$ with $d = 1$. The forecast is conditioned on $y_{t,t+1}$ and $x_{t+2/3;t+1}$. For $h = 4/3$, the forecast is generated from a regression of $\mathbf{y}_{t;t+1}$ on $\mathbf{y}_{t-2;t+1}$ and $B(L^{1/3}; \theta)\mathbf{x}_{t-4/3;t+1}$, again using the last values in the vintage datasets to compute the forecasts. When only information on the first month in the quarter is available, the forecasts horizons are equal to $2/3$ and $5/3$. We use again data on y from the $t + 1$ vintage-data but the data on the indicator is taken from the $t + 2/3$ data-vintage. Finally, we obtain an $h = 1$ forecast of y_{t+1} and an $h = 2$ forecast of y_{t+2} when the latest data are $\mathbf{y}_{t;t+1}$ and $\mathbf{x}_{t;t+1/3}$.

This is the traditional way ('end-of-sample') of conducting a real-time forecasting exercise, adapted to incorporate monthly vintages of monthly indicator data.

3.1.2   A real-time-vintage data approach to real-time forecasting with monthly data

The difference between the end-of-sample and real-time-vintage approaches can be most easily seen by considering forecasting $y_{t+1;T}$ with a single x, for the case $h = 0$. The end-of-sample-vintage approach estimates:

$$y_{s;t+1} = x_{s;t+4/3}\beta + v_s \tag{6}$$

on $s = 1, \ldots, t$ (where $t + 1 \leq T$), and forecasts y_{t+1} as $\hat{y}_{t+1} = x_{t+1;t+4/3}\hat{\beta}$. Koenig *et al.* (2003) show that under general conditions, $\hat{\beta}$ will be an inconsistent estimator of β_0, where β_0 relates the the true value of y_t to x_t: $y_t = x_t\beta_0 + \varepsilon_t$, as $\hat{\beta}$ will in part reflect the nature of the joint revisions process rather than cleanly capturing the forecasting relationship between y_t and x_t. A consistent estimator of β_0 can be obtained from estimating:

$$y_{s;s+1} = x_{s;s+1/3}\beta_0 + \epsilon_s \tag{7}$$

on $s = 1, \ldots, t$, that is, using real-time-vintage data. Forecasts are computed as $\hat{y}_{t+1} = x_{t+1;t+4/3}\hat{\beta}_0$, as in the end-of-sample-vintage approach.

The approaches exemplified by (6) and (7) condition on the same information in the estimated model to generate forecasts, but as is evident, differ in the way in which the estimation sample is constructed. As we have monthly and quarterly vintages of data, and wish to calculate forecasts with monthly horizons, some care is required in implementing the real-time-vintage scheme of (7) in our context. Firstly, consider the AR(1) benchmark, against which we judge the accuracy of the forecasts from the models with monthly indicators. For the AR(1), the left and right-side vectors of observations are given by $[y_{2;2}, \ldots, y_{t-2;t-1}, y_{t-1;t}, y_{t;t+1}]'$ and $[y_{1;2}, \ldots, y_{t-3;t-1}, y_{t-2;t}, y_{t-1;t+1}]'$. Suppose we have a model with two lags of x. If all the months for the current quarter are available, $h = 0$, the logic of the real-time-vintage approach suggests augmenting the right-side AR(1) data vector with the vectors of observations on x given by $\left[\ldots, x_{t-1;t-2/3}, x_{t;t+1/3}\right]'$ and $\left[\ldots, x_{t-4/3;t-2/3}, x_{t-1/3;t+1/3}\right]'$.

Note that the sequence of vintages does not change with the inclusion of lag values, implying that some data used in the estimation have been partially revised. The last available data is used to compute the forecasts $(y_{t;t+1}, x_{t+1;t+4/3}, x_{t+2/3;t+4/3})$, as in the end-of-sample approach. Although our models use longer lags of x than the two lags we consider, no new issues arise.

For $h = 1/3$, the two x-vectors used in estimation are given by $\left[..., x_{t-4/3;t-1}, x_{t-1/3;t}\right]'$ and $\left[..., x_{t-5/3;t-1}, x_{t-2/3;t}\right]'$, and for $h = 4/3$, $\left[..., x_{t-7/3;t-1}, x_{t-4/3;t}\right]'$ and $\left[..., x_{t-8/3;t-1}, x_{t-5/3;t}\right]'$. For both horizons, the estimated models are used to generate forecasts using $(x_{t+2/3;t+1}, x_{t+1/3;t+1})$. When only first-month information is available, we build the right-side vectors in a similar fashion (so for $h = 2/3$, the two x-vectors are given by $\left[..., x_{t-1-2/3;t-1}, x_{t-2/3;t}\right]'$ and $\left[..., x_{t-2;t-1}, x_{t-1;t}\right]'$, for example). Finally, for $h = 1$ the x-vectors are $\left[..., x_{t-2;t-5/3}, x_{t-1;t-2/3}\right]'$ and $\left[..., x_{t-7/3;t-5/3}, x_{t-4/3;t-2/3}\right]'$.

3.2 Does monthly indicator information help? MIDAS-AR versus AR and ADL.

The first set of results uses end-of-sample-vintage data in a real-time forecasting exercise. However, to establish a benchmark we first discuss results obtained using final-revised vintage data throughout. We compare forecasts of the MIDAS-AR with an AR and an ADL, to determine whether monthly indicator information improves forecast accuracy. Table 1 gives the ratios of the RMSEs of the MIDAS-AR against the AR and ADL models. The table shows that there are sizeable reductions in RMSE using MIDAS when monthly data is available on the current quarter - these are of the order of 20% when industrial production is the indicator for 'nowcasts' and $h = 1/3$ horizons (compared to the quarterly-frequency ADL). The RMSEs are calculated for output growth rates calculated as one hundred times the quarterly difference of the natural log of GDP. So taking the MIDAS RMSE to be 0.4 for illustrative purposes, a RMSE ratio of the MIDAS to the ADL of 0.8 translates into respective RMSEs at annual rates of 1.6% and 2.0%, set against an average annual growth rate over the period of 3.2%.

Sizeable gains are also achieved when the indicator is capacity utilization for these horizons, while the differences between MIDAS and the AR/ADL using employment are smaller. We also report average ratios across horizons which confirm that MIDAS is generally no better, and often worse, when no monthly information on the quarter is available.

In passing we note that a comparison of the quarterly frequency ADL and AR models (ADL/AR ratios can be obtained by dividing the 'MIDAS/AR' ratio by the 'MIDAS/ADL' ratio for each indicator in Table 1) presents a less upbeat picture of the usefulness of the indicators: relative to an AR, the quarterly-frequency ADL only enhances accuracy when employment is the indicator.

The exercise using final-revised vintage data allows us to calculate critical values to determine whether the gains we observe to MIDAS at the shorter horizons are statistically significant. Following in the tradition of testing for equal predictive ability of West (1996) and West and McCracken

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(1998) (and see the review by West (2006)), we employ the test of equal forecast accuracy for multi-step forecasts from nested models of Clark and McCracken (2005). The aim is to compare the forecast performance in population, taking the effect of parameter estimation uncertainty into account. Note that the MIDAS-AR nests both the AR and ADL models: it specialises to the ADL when $\theta_1 = \theta_2 = 0$ in equation (2). The null is that the quarterly-frequency AR (ADL) model forecasts are as accurate on RMSE as those of the MIDAS-AR (the unrestricted model) and the one-sided alternative is that the AR (ADL) model forecasts are less accurate. As the test has a limiting distribution that depends on the data, we adopt a bootstrap implementation of the test (similar to that of Kilian (1999)). The entries in the table in bold indicate that the null of equal RMSE is rejected at the 5% level using the bootstrapped critical values, so that the gains to using MIDAS at the shorter horizons are clearly significant for both industrial production and capacity utilization.

Testing for equal predictive ability is more complicated in real-time forecasting exercises when there are data revisions across the vintages. Clark and McCracken (2007) show that data revisions may affect the asymptotic behaviour of tests of equal forecast accuracy, and suggest a way of proceeding using linear models estimated by least squares. It is unclear how useful these results are in our context. The natural solution of using a bootstrap would require the specification of the (unknown) revisions process. For the forecasting comparisons based on the real-time use of end-of-sample and real-time-vintage data we simply report the relative sizes of the RMSFEs, and largely dispense with the use of formal tests of equal predictive ability.

Consider now the end-of-sample-vintage data. The results clearly indicate that IP and CU help predict output growth in real time when we have access to monthly data on the quarter being forecast. When only information on the previous quarter is employed, the indicators are of no value in real time. These results essentially match those obtained using final-revised data. In real time the gains to MIDAS relative to the AR disappear when EMP is the indicator, suggesting that the apparent gains from using EMP to predict output growth were not attainable in practice. In real time, the gains to using EMP at the quarterly frequency also disappear (the comparison of the ADL against the AR). The entries in the table in bold indicate that the null of equal RMSE is rejected at the 5% level using the bootstrapped critical values calculated for the exercise using the final-revised data, but as noted the use of these critical values is questionable when there are data revisions. Nevertheless, it is apparent that MIDAS results in marked reductions in empirical RMSFEs in real-time for the shorter horizons.

3.3 MIDAS-AR versus other methods of exploiting monthly indicators

We compare the MIDAS-AR with other ways of using monthly indicator data, namely the MF-DL and the ADL-F described in section 2.3, using end-of-sample-vintage data. When all the months

of the quarter being forecast are known, ADL-F corresponds to a standard ADL as data on the current quarter value of the indicator is available. Broadly speaking, the results in table 2 indicate that there is little to choose between the models. The MF-DL and ADL-F are better than MIDAS for nowcasting ($h = 0$), but when only one or two months of indicator data are available MIDAS is generally at least as good (see the averages of the ratios across different horizons). Overall, and except when $h = 0$, the performance of MIDAS is promising in comparison to other simple methods of using monthly indicators to forecast quarterly growth. However, Koenig *et al.* (2003) show that the use of real-time-vintage data improves forecast accuracy of MF-DL, and advocate this approach to forecast accuracy comparisons. The next section reports a comparison using real-time-vintage data.

3.4   MIDAS-AR and other methods of exploiting monthly indicators with real-time-vintage data

Conducting the forecasting exercise using real-time-vintage data strengthens the finding that monthly indicator information helps predict output growth, especially at shorter horizons. This is in part because the forecast performance of the AR worsens relative to when end-of-sample data is used (by just over 3% on RMSE at $h = 1$), while the performance of MIDAS and ADL-F generally improves. See the first panel of Table 3 which shows the ratios of the RMSEs from using real-time-vintage to end-of-sample vintage data for each monthly model and the AR. The net result of these changes is that MIDAS is more accurate than the AR for $h = 0$, 1/3 for all three indicators. The bottom panel shows that for horizons with two months of indicator information available (the Av .2 row) the gains to MIDAS are of the order of a 10% reduction in RMSE for IP and CU, and 3% for EMP.

It is worth remarking that MIDAS is generally as good at the shorter horizons as the popular two-step approach to the incorporation of monthly information exemplified by the ADL-F, except when the indicator is EMP, and is almost always superior to the MF-DL.

3.5   Forecast combination

Table 4 compares models which include all three indicators (these have the prefix 'M-' for multiple) with equal-weighted combinations of the individual indicator models. The results indicate that the MIDAS model is clearly preferred to MF-DL. It also beats ADL-F when the horizon is not an integer multiple of quarters, that is, for those horizons when ADL-F relies on monthly forecasts to construct quarterly observations. Combining forecasts is better than combining indicators within a single model (as is often the case: see e.g., Clements and Galvão (2006)), but the combinations of the MIDAS models' forecasts are generally at least as good as the combinations of the forecasts from the other two models.

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4 Conclusions

In recent years increasing use has been made of monthly indicator information to generate forecasts of quarterly macro aggregates such as GDP growth. We investigate whether the MIDAS approach of Ghysels *et al.* (2004, 2006b) can be successfully adapted to the short-term forecasting of output growth, given that it has hitherto been used for forecasting financial variables with daily observations. A typical feature of quarterly macroeconomic time series is that they can often be reasonably well modelled by autoregressive processes. To capture this characteristic of macro data, we extend the distributed-lag MIDAS specification to include an autoregressive term (the MIDAS-AR) and show how this model can be applied in a forecasting context.

Recent research suggests that the predictive content of indicator information must be assessed in an exercise that mirrors a 'real-time-forecasting environment', and that the use of final-revised data may misleadingly suggest that the indicators are better than what could be achieved with the data available at the time. We conduct a traditional real-time forecasting exercise that exploits the monthly vintages of the indicators and the quarterly vintages of output growth and which is consistent with the timing of the releases of the different data vintages. This permits a comprehensive and valid appraisal of the usefulness of monthly information in real time. We find that the use of monthly indicator information results in sizeable reductions in RMSE for short-horizon forecasts when within-quarter monthly data on industrial production and capacity utilization is used.

We also evaluate the suggestion by Koenig *et al.* (2003) of basing the real-time forecasting exercise on real-time-vintage data, as opposed to end-of-sample-vintage data. We do so for a range of forecast horizons, from the 'nowcasts' considered by those authors up to the two-quarter horizon, in steps of one month. The use of real-time-vintage data serves to strengthen our finding that within-quarter monthly-indicator information can result in marked improvements in forecast performance. MIDAS fares well relative to the other models that use monthly information. Coupled with its flexibility and ease of use relative to methods which involve the generation of forecasts of explanatory variables offline, the MIDAS-AR would appear to be a useful addition to the sets of models and methods that exploit monthly indicators for the short-term forecasting of macro-aggregates.

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Table 1

Comparing the Forecasting Performance of MIDAS-AR with the AR and ADL using real-time end-of-sample-vintage data and final-revised data.

The entries (except the first column of each panel) are ratios of Root Mean Squared Forecast Error (RMSE) of MIDAS-AR to the AR and ADL. The RMSEs are computed using final-revised actual values of output growth for forecasts of 1985:Q2-2005:Q1. The ratios in bold imply that the null of equal RMSE is rejected at the 5% significance level using bootstrapped critical values. Av. 0 is the average RMSE when no information on the indicator in the current quarter is employed for forecasting (h=1,2). Av. 1 is the equivalent when only one month on the current quarter is available (h=2/3, 5/3). Av. 2 is the same measure when two months of information are available (h=1/3, 4/3).

Horizon (h)		Industrial Production (IP)			Employment (EMP)			Capacity Utilization (CU)		
MIDAS	AR, ADL	MIDAS-AR (RMSE)	Ratio to: AR	ADL	MIDAS-AR (RMSE)	Ratio to: AR	ADL	MIDAS-AR (RMSE)	Ratio to: AR	ADL
					Using end-of-sample-vintage data					
0	1	0.444	**0.902**	**0.877**	0.514	1.042	1.014	0.452	**0.916**	**0.895**
1/3	1	0.432	**0.876**	**0.851**	0.487	0.989	0.962	0.439	**0.890**	**0.870**
2/3	1	0.516	1.046	1.017	0.524	1.064	1.035	0.515	1.045	1.021
1	1	0.516	1.048	1.019	0.493	1.001	**0.974**	0.492	0.998	0.975
4/3	2	0.469	**0.937**	**0.940**	0.512	1.024	1.018	0.464	**0.928**	**0.930**
5/3	2	0.512	1.024	1.027	0.499	0.998	0.992	0.513	1.025	1.028
2	2	0.510	1.020	1.023	0.496	0.992	0.987	0.504	1.007	1.010
Av. 2	Av. 0	0.451	0.907	0.896	0.500	1.007	0.991	0.452	0.909	0.900
Av. 1	Av. 0	0.514	1.035	1.022	0.512	1.031	1.014	0.514	1.035	1.024
Av. 0	Av. 0	0.513	1.034	1.021	0.495	0.997	0.980	0.498	1.002	0.992
					Using final-revised data					
0	1	0.425	**0.869**	**0.820**	0.471	**0.963**	1.044	0.440	**0.900**	**0.840**
1/3	1	0.409	**0.837**	**0.790**	0.472	**0.966**	1.047	0.438	**0.897**	**0.837**
2/3	1	0.456	**0.932**	**0.880**	0.474	**0.969**	1.051	0.484	0.991	**0.925**
1	1	0.536	1.098	1.036	0.477	**0.975**	1.057	0.521	1.067	0.996
4/3	2	0.459	**0.965**	**0.964**	0.466	**0.981**	0.995	0.469	**0.987**	**0.984**
5/3	2	0.491	1.033	1.032	0.472	0.994	1.009	0.495	1.043	1.039
2	2	0.487	1.026	1.025	0.479	1.008	1.023	0.490	1.031	1.027
Av. 2	Av. 0	0.435	0.902	0.874	0.469	0.973	1.021	0.454	0.942	0.907
Av. 1	Av. 0	0.473	0.983	0.952	0.473	0.981	1.029	0.490	1.017	0.979
Av. 0	Av. 0	0.512	1.063	1.031	0.478	0.991	1.040	0.506	1.049	1.010

Table 2
Comparing the Forecasting Performance of MIDAS-AR with ADL-F and MF-DL using real-time end-of-sample-vintage data.

The entries are ratios of the MIDAS-AR RMSEs to the RMSEs of the stated models. Av. 0 is the average RMSE when no information on the indicator in the current quarter is employed for forecasting (h=1,2). Av. 1 is the equivalent when only one month on the current quarter is available (h=2/3, 5/3). Av. 2 is the same measure when two months of information are available (h=1/3, 4/3).

h	Industrial Production (IP)		Employment (EMP)		Capacity Utilization (CU)	
	MF-DL	ADL-F	MF-DL	ADF-F	MF-DL	ADL-F
0	1.034	1.032	1.054	1.088	1.017	1.030
1/3	0.981	0.989	0.965	1.040	0.981	1.000
2/3	0.999	1.013	0.978	0.985	0.995	1.023
1	0.993	1.019	0.957	0.974	0.995	0.975
4/3	1.010	0.974	0.966	1.015	1.009	0.978
5/3	1.002	0.947	0.979	0.939	0.996	0.958
2	1.008	1.023	0.999	0.987	0.979	1.010
Av. 2	0.996	0.981	0.965	1.027	0.996	0.988
Av. 1	1.000	0.979	0.979	0.962	0.995	0.989
Av. 0	1.000	1.021	0.977	0.980	0.987	0.992

Table 3
Comparing the Forecasting Performance of MIDAS-AR with MF-DL, ADL-F and AR using real-time-vintage data and end-of-sample vintage data.

The entries in the first panel are RMSE ratios of the stated model estimated with real-time-vintage data to the same model but estimated with end-of-sample-vintage data. The entries in the second panel are ratios of the MIDAS-AR RMSEs to the RMSEs of the stated models using real-time vintage data. Av. 0 is the average RMSE when no information on the indicator in the current quarter is employed for forecasting (h=1,2). Av. 1 is the equivalent when only one month on the current quarter is available (h=2/3, 5/3). Av. 2 is the same measure when two months of information are available (h=1/3, 4/3).

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h	Industrial Production (IP)			Employment (EMP)			Capacity Utilization (CU)			AR
	MIDAS-AR	MF-DL	ADL-F	MIDAS-AR	MF-DL	ADL-F	MIDAS-AR	MF-DL	ADL-F	
Real-time-vintage data / end-of-sample-vintage data										
0	0.971	1.005	0.990	0.961	1.034	0.952	0.996	1.041	0.960	
1/3	0.974	0.971	0.962	0.973	0.988	0.970	1.011	1.034	0.971	
2/3	0.993	0.988	1.124	0.959	0.968	0.937	1.027	1.029	1.066	
1	1.018	1.048	0.984	1.036	1.029	1.008	1.098	1.105	0.991	1.035
4/3	1.028	1.066	1.000	1.001	1.001	0.995	1.019	1.023	0.963	
5/3	0.996	1.024	1.071	1.015	1.018	0.965	1.014	1.042	1.069	
2	1.029	1.050	1.014	1.029	1.065	1.029	1.020	1.021	0.991	1.012
Ratio of MIDAS-AR with real-time-vintage data to:										
	AR	MF-DL	ADL-F	AR	MF-DL	ADL-F	AR	MF-DL	ADL-F	
Av. 2	0.890	0.978	1.001	0.972	0.958	1.031	0.902	0.983	0.995	
Av. 1	1.006	0.989	0.888	0.993	0.973	0.998	1.033	0.981	0.961	
Av. 0	1.034	0.976	1.046	1.006	0.965	0.994	1.038	0.984	1.029	

Table 4

Comparing Combinations of Indicators: M-MIDAS-AR, M-ADL-F, M-MF-DL and Means of Forecasts using real-time-vintage data

The entries in the first panel are RMSEs. The second panel records the RMSE ratios of MIDAS to the stated model. Av. 0 is the average RMSE when no information on the indicator in the current quarter is employed for forecasting (h=1,2). Av. 1 is the equivalent when only one month on the current quarter is available (h=2/3, 5/3). Av. 2 is the same measure when two months of information are available (h=1/3, 4/3).

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h	Multiple indicator models			Combining Forecasts		
	M-MIDAS-AR	M-MF-DL	M-ADL-F	Mean MIDAS-AR	Mean MF-DL	Mean ADL-F
0	0.442	0.450	0.432	0.420	0.426	0.420
1/3	0.425	0.441	0.431	0.412	0.421	0.417
2/3	0.520	0.522	0.543	0.490	0.487	0.507
1	0.547	0.565	0.511	0.513	0.519	0.500
4/3	0.498	0.516	0.486	0.478	0.481	0.478
5/3	0.519	0.573	0.594	0.510	0.518	0.537
2	0.526	0.563	0.513	0.514	0.522	0.509
Av. 2	0.463	0.480	0.459	0.446	0.452	0.448
Av. 1	0.519	0.548	0.569	0.500	0.502	0.522
Av. 0	0.536	0.564	0.512	0.514	0.520	0.504
	Ratio of M-MIDAS-AR to:			Ratio of Mean MIDAS-AR to:		
	M-MF-DL	M-ADL-F	AR	Mean MF-DL	Mean ADL-F	AR
0	0.983	1.023	0.898	0.984	0.999	0.852
1/3	0.965	0.987	0.863	0.979	0.989	0.837
2/3	0.996	0.957	1.054	1.007	0.966	0.995
1	0.967	1.071	1.109	0.989	1.027	1.042
4/3	0.964	1.024	0.995	0.993	1.000	0.954
5/3	0.906	0.874	1.037	0.985	0.950	1.019
2	0.934	1.026	1.051	0.986	1.010	1.028
Av. 2	0.965	1.008	0.932	0.987	0.995	0.898
Av. 1	0.948	0.913	1.046	0.996	0.957	1.007
Av. 0	0.951	1.048	1.080	0.987	1.019	1.035