# Department of Economics

## Two-stage Bargaining Solutions

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**Abstract**

We introduce and characterize a new class of bargaining solutions: those which can be obtained by sequentially applying two binary relations to eliminate alternatives. As a by-product we obtain as a particular case a partial characterization result by Zhou (Econometrica, 1997) of an extension of the Nash axioms and solution to domains including non-convex problems, as well as a complete characterizations of solutions that satisfy Pareto optimality, Covariance with positive affine transformations, and Independence of irrelevant alternatives.

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# 1 Introduction

Imagine an arbitrator who can rank alternatives on the basis of a fairness criterion. He chooses an alternative from the feasible set by means of the following procedure. First, he discards all alternatives which are Pareto dominated. Then, among the remaining ones, he picks the fairest alternative. In this paper we introduce and characterize a new class of bargaining solutions that generalizes (to arbitrary criteria) this intuitive two-stage procedure. A two-stage bargaining solution is a solution which can be constructed by sequentially applying two asymmetric binary relations $P_1$ and $P_2$. More precisely, the solution point from each feasible set is the (single-valued) set that $P_2-$dominates all the $P_1-$maximizers.

There are several features of interest in our concept and characterizations. First, the sequential procedure by which we model the arbitrator's decisions is natural. Indeed, Tadenuma [9] has considered the sequential application of exactly the two above criteria, efficiency and fairness, in social choice[1]. Our contribution generalizes and abstracts this idea within an axiomatic bargaining framewok a la Nash ([6])[2]. We provide a complete characterization of two-stage bargaining solutions.

Second, even when the two criteria are well-behaved in the sense of being transitive, they can generate solutions which violate the Independence of Irrelevant Alternatives (IIA) axiom. For example, suppose that $c$ is fairer than $a$ which is fairer than $b$, and that the only possible Pareto comparison is that $b$ Pareto dominates $c$. Then the arbitrator chooses $a$ from $\{a, b, c\}$ (first

[1] Tadenuma studies in particular the effect of the order of application of the two criteria
[2] See Thomson [10] for an overview of axiomatic bargaining theory.

discarding $c$ by Pareto dominance and then $b$ by fairness), but he chooses $c$ from $\{a, c\}$ (by applying fairness). Contrast this with the maximization of a single relation: on a standard domain a bargaining solution maximizes a binary relation if and only if it satisfies IIA (Peters and Wakker [8]), and on many domains including non-convex problems the relation must be transitive when the solution also satisfies Pareto optimality (Denicoló and Mariotti [2]).

Third, we consider solutions on domains that include non-convex problems. In this respect our paper is related to a number of papers on the extension of bargaining solutions satisfying the original Nash axioms (discussed below) on larger domains. Our framework delivers a major by-product by yielding a generalization of a theorem of Zhou [11], which states that any solution that satisfies IIA, Pareto optimality (PAR) and Covariance with positive affine transformations (COV) is a selection from some asymmetric Nash multivalued solution[3]. We show that any solution that satisfies a certain weakening of IIA (discussed below), in addition to PAR and COV, sequentially maximizes two relations that are invariant with affine transformations (that is $xP_iy$ if and only if $\tau(x)\,P_i\tau(y)$ for any positive affine transformation $\tau$). If the solution satisfies IIA in full, then the two relations collapse into a single transitive relation: this provides a complete characterization of PAR, COV and IIA bargaining solutions. Zhou's partial characterization result then follows easily.

The weakening of IIA is achieved through two consistency axioms that, together with PAR, characterize two-stage bargaining solutions. The first

[3]Zhou's theorem is also obtained with different techniques by Denicolo and Mariotti [2] and in a recent paper by Peters and Vermeulen [7].

is Expansion (EXP): if $x$ is the solution point of each problem in a class of bargaining problems then it is the solution of their union. The second axiom is Weak IIA (WIIA): if $x$ is the solution point of two nested bargaining problems $R$ and $T$ which both contain $y$, then $y$ is not the solution point of any 'intermediate' problem $S$. WIIA allows some 'menu effects' excluded by IIA. EXP and WIIA are both implied by IIA while the converse is not true (as the Pareto/fairness example in the opening paragraph illustrates).

# 2 Preliminaries

A *(bargaining) problem* is a pair $(S, d)$, where $S \subset \mathcal{R}^n$ and $d \in S$. The set $S$ is interpreted as the set of feasible alternatives (welfare or utility vectors for $n$ agents) from which an arbitrator must choose, and $d$ is a distinguished point relevant for the arbitrator's decision. Following usage, we call $d$ the *disagreement point.*

$\mathcal{R}^n$ is viewed as a vector space, with the origin and the unit vector denoted 0 and $e$, respectively. The vector inequalities are: $s > t$ iff $s_i > t_i$ for all $i$; $s \geq t$ iff $s_i \geq t_i$ for all $i$. A positive affine transformation is a function $\tau : \mathcal{R}^n \rightarrow \mathcal{R}^n$ such that, for some real numbers $\alpha_i > 0$ and $\beta_i$, $i = 1, ..., n$, $\tau_i(s) = \alpha_i s_i + \beta_i$ for all $i$. Given $S \subset \mathcal{R}^n$ and and a positive affine transformation $\tau$, denote $\tau(S) = \{t \in \mathcal{R}^n | t = \tau(s) \text{ for some } s \in S\}$. For a bargaining problem $(S, d)$, denote $\tau(S, d) = (\tau(S), \tau(d))$.

Given a domain of bargaining problems $\Sigma$, a *solution on* $\Sigma$ is a function $\gamma : \Sigma \rightarrow \mathcal{R}^n$ such that $\gamma(S, d) \in S$ for all $(S, d) \in \Sigma$. Sometimes we will refer to *multi-solutions*, for which $\gamma$ is allowed to be a correspondence.

We consider a very general class of domains of bargaining problems[4]. Say that a domain $\Sigma$ of bargaining problems is *admissible* if the following assumptions hold:

D1: For all $(S,d) \in \Sigma$: $S$ is compact and there exists $s \in S$ such that $s > d$.

D2: For all $d \in \mathcal{R}^n$, for all $s,t \in \{u \in \mathcal{R}^n | u > d\}$, there exists a unique $(M(s,t), d) \in \Sigma$ such that: (a) $s,t \in M(s,t)$ and for all $u \in S$ with $u \neq s,t$, $s \geq u$, or $t \geq u$, or both. (b) for any $(S,d) \in \Sigma$ with $s,t \in S$, $M(s,t) \subseteq S$.

D3: For any class $\{S^k, d\} \in \Sigma$ of problems, $\left(\bigcup_k S^k, d\right) \in \Sigma$.

All domains of non-convex problems considered in the literature are particular cases of admissible domains. For example the set of comprehensive problems (Zhou [11], Peters and Vermeulen [7]), the set of finite problems (Mariotti [4], Peters and Vermeulen [7]), the set of all problems satisfying D1 (Kaneko [3]), the set of d-star shaped problems[5]. While D1 is standard and D3 straightforward, D2 deserves a special note. Its role in the analysis is to guarantee the existence of a 'minimal' problem containing any two given alternatives, and such that the solution point is (under PAR) one of those two alternatives.

From now on, unless specified otherwise, fix an admissible domain $\Sigma$. We consider the following axioms on solutions, intended for all $(S,d), (R,d) \in \Sigma$:

COV: For any positive affine transformation $\tau$, $\gamma(\tau(S,d)) = \tau(\gamma(S,d))$.

PAR: For all $s \in \mathcal{R}^n$ with $s \geq \gamma(S,d)$ and $s \neq \gamma(S,d)$: $s \notin S$.

IIA: If $\gamma(S,d) \in R \subset S$, then $\gamma(R,d) = \gamma(S,d)$.

[4] This class was was essentially introduced in Denicolo and Mariotti [2].

[5] That is, those problems $(S,d)$ for which the convex hull of $\{d,s\}$ is in $S$ for all $s \in S$.

EXP: Given a class of problems $\{S^k, d\}$, if $s = \gamma(S^k, d)$ for all $k$ and $(\bigcup_k S^k, d) \in \Sigma$, then $s = \gamma(\bigcup_k S^k, d)$.

WIIA: If $\gamma(R, d) = \gamma(T, d) = s$ and $t \in R \subset S \subset T$, then $\gamma(S, d) \neq t$.

The first theree axioms are standard in bargaining theory. EXP is standard in choice theory. Only the last axiom is relatively new[6]. It can be interpreted as follows. Start with IIA: one way of reading it is that if new alternatives are added to the problem, then either the solution point is unchanged, or the new solution point is one the the new alternatives. In other words, there are no 'menu-effects': the effect of a new alternative cannot be to change the solution point to one of the 'old' alternatives. By contrast, WIIA allows for some such menu effects. However, *suppose* that adding a large set of new alternatives does not produce any effect. Then adding a smaller set of new alternatives does not make an old alternative a solution point.

# 3 Two-stage bargaining solutions

As standard, we consider only solutions that satisfy translation-invariance (as most known solutions do)[7]. This permits to simplify notation by normalising the disagreement point of all problems to the origin. A bargaining problem can then be defined simply as a subset of $\mathcal{R}^n$ containing the origin, and a bargaining solution can be denoted accordingly. The main new definition of this paper is the following (where $\max(S, P)$ denotes the set of maximal

[6]This axiom was suggested by Michele Lombardi.

[7]In obvious notation, translation invariance means $\gamma(S + t, d + t) = \gamma(S, d) + t$ for all $t \in \mathcal{R}^n$.

elements of the relation $P$ in the set $S$).

**Definition 1** *A solution $\gamma$ is a two-stage solution if there exist two asymmetric relations $P_1$ and $P_2$ on $\mathcal{R}^n_{++}$ such that, for all $S \in \Sigma$,*

$$\{\gamma(S)\} = \{s \in \max(S, P_1) \,|\, sP_2t \text{ for all } t \in \max(S, P_1),\ t \neq s\}$$

*In this case we say that $P_1$ and $P_2$ rationalize $\gamma$. If $P_1$ and $P_2$ can be chosen so that $P_1 = P_2 = P$ we say that the solution is a degenerate two-stage solution, rationalized by $P$.*

We have discussed the idea behind this definition in the introduction, so we shall not repeat it here.

**Example 1 (Nash solutions)**: The symmetric Nash bargaining multi-solution $\nu$ is defined by the correspondence which associates with each problem the maximizers of the symmetric Nash product, namely

$$\nu(S) = \arg\max_{S \cap \mathcal{R}^n_+} \prod_i s_i$$

for all $S \in \Sigma$. A symmetric Nash selection is a solution that coincides with a selection from $\nu$.[8] Some symmetric Nash selections (e.g. those satisfying IIA) can be naturally thought of as a two-stage solution for which $sP_1t$ iff $\prod_i s_i > \prod_i t_i$ and the relation $P_2$ is used to break the ties between Nash product maximizers within each set. For a specific case, consider $n = 2$ and $sP_2t$ iff $s_1 > t_1$. However this is a degenerate two-stage solution, as by taking the union of $P_1$ and $P_2$ one can rationalize the solution in one stage.

[8] Note that the adjective 'symmetric' refers to the objective function to be maximized, obviously not to the selection itself, which on the usual domains will not be symmetric.

**Example 2 (first efficiency, then equality)**: let $sP_1t$ iff $s \geq t$ and $s \neq t$. Let $sP_2t$ iff

$$\sum \left( \frac{s_i}{\sum s_i} \right)^2 > \sum \left( \frac{t_i}{\sum t_i} \right)^2$$

or

$$\sum \left( \frac{s_i}{\sum s_i} \right)^2 = \sum \left( \frac{t_i}{\sum t_i} \right)^2$$

and a suitable tie-breaking rule, left undefined here, is met. The resulting two-stage solution picks the alternative that maximizes a measure of equality over the set of strongly Pareto optimal alternatives. As we have seen in the introduction, this procedure can generate cycles. Together with this observation, our theorem 3 below shows that this two-stage solution is not degenerate.

**Example 3 (first goodness, then efficiency)**: fix a set $G \subset \mathcal{R}^n_+$ of 'good' alternatives. Let $sP_1t$ iff $s \in G$ and $t \in \mathcal{R}^n \backslash G$. Let $sP_2t$ iff $s_1 > t_1$ or $s_1 = t_1$ and $s_2 > t_2$ etcetera. The resulting two-stage solution first eliminates all alternatives which are not 'good', provided there are some good alternatives which are feasible, and then it lexicographically maximizes the welfare of the agents. If there are no good alternatives, one moves directly to the lexicographic maximization stage. As a specific example, let $G = \left\{ s \in \mathcal{R}^n_+ | s = \lambda e \text{ for some scalar } \lambda > 0 \right\}$. In this case goodness is equality: if the feasible set intersects the $45^0$ line, the solution is egalitarian (and whether it is Pareto optimal or not depends, of course, on the domain). If equality is not achievable a specific form of efficiency is sought.

The main result of this section is a complete characterization of two-stage solutions. Say that a relation $P$ on $\mathcal{R}^n$ is *Pareto consistent* if it contains the strong Pareto relation:

**Theorem 2** *A solution is a two-stage solution, which can be rationalized by Pareto consistent* $P_1$ *and* $P_2$*, if and only if it satisfies PAR, EXP and WIIA.*

**Proof:** Sufficiency. Let $\gamma$ be a solution that satisfies the axioms. Note first that by D2 and PAR, given $s, t \in \mathcal{R}^n$ there exist a minimal (in the order of set inclusion) problem $M(s,t) \in \Sigma$ with the property that either $\gamma(M(s,t)) = s$ or $\gamma(M(s,t)) = t$.

Now we explictly construct the relations $P_1$ and $P_2$. Define $sP_1t$ iff there is no $S \in \Sigma$ such that $t = \gamma(S)$ and $s \in S$. Define $sP_2t$ iff $s = \gamma(M(s,t))$. The relation $P_1$ is asymmetric since $sP_1t$ and $tP_1s$ could be both true only if both $s \neq \gamma(S)$ for all $S \in \Sigma$ with $t \in S$ and $t \neq \gamma(S)$ for all $S \in \Sigma$ with $s \in S$: But, as observed, $\gamma(M(s,t)) \in \{s,t\}$. The asymmetry of $P_2$ is guaranteed by the single valuedness of $\gamma$. That $P_1$ and $P_2$ are Pareto consistent follows immediately from PAR.

For any $S \in \Sigma$, obviously there exists no $s \in S$ for which $sP_1\gamma(S)$. Take any $s \in S$ for which $sP_2\gamma(S)$: we show that then $s$ is eliminated in the first round. Suppose to the contrary that there is no $t \in S$ with $tP_1s$. Therefore, by the definition of $P_1$, for all $t \in S \backslash s$ there exists $T_t \in \Sigma$ such that $t \in T_t$ and $s = \gamma(T_t)$. By D3, $\bigcup_t T_t \in \Sigma$. By EXP, $s = \gamma\left(\bigcup_t T_t\right)$. Since $sP_2\gamma(S)$, $s = \gamma(M(s,\gamma(S)))$. Since we have $M(s,\gamma(S)) \subseteq S \subset \bigcup_t T_t \in \Sigma$, WIIA is contradicted. We can conclude that there exists $t \in S$ such that $tP_1s$. Observe finally that $\gamma(S)P_2s$ for any $s \in \max(S, P_1)$: in fact, by D2 and PAR, $P_2$ is a complete relation, and by the previous argument it cannot be $s = \gamma(M(s,\gamma(S)))$ for any $s \in \max(S, P_1)$.

Necessity. Let $\gamma$ be a two-stage solution that satisfies PAR, rationalized by $P_1$ and $P_2$. Let $\left\{S^k\right\}$ be a class of problems. Suppose that $s = \gamma\left(S^k\right)$ for all $k$ and $\bigcup_k S^k \in \Sigma$. Then $tP_1 s$ for no $t \in \bigcup_k S^k$. Moreover $sP_2 t$ for all $t \in \max\left(S^k, P_1\right)$, with$t \neq s$, for all $k$. Therefore $sP_2 t$ for all $t \in \max\left(\bigcup_k S^k, P_1\right)$ (since $\max\left(\bigcup_k S^k, P_1\right) \subseteq \bigcup_k \max\left(S^k, P_1\right)$), and EXP is satisfied. Next, suppose that $s = \gamma(R) = \gamma(T)$ with $t \in R \subset T$. Suppose by contradiction that $t = \gamma(S)$ for some $S \in \Sigma$ with $R \subset S \subset T$. Since $t = \gamma(S)$ there cannot exist $u \in R \subset S$ for which $uP_1 t$. Then $sP_2 t$ (as $s = \gamma(R)$), and there exists $u \in S$ such that $uP_1 s$. But this contradicts $s = \gamma(T)$. QED

The result that follows makes precise the difference betwen the combination of WIIA and EXP on the one hand, and IIA on the other.

**Theorem 3** *A solution is a degenerate two-stage solution, which can be rationalized by a complete*[9]*, asymmetric, transitive and Pareto consistent relation $P$, if and only if it satisfies PAR and IIA.*

**Proof**: Let $\gamma$ be a solution that satisfies the axioms. Since IIA is easily seen to imply WIIA and EXP, asymmetric relations $P_1$ and $P_2$ can be constructed as in the proof of the previous theorem. Note that for any $s, t > 0$, if $sP_1 t$ then (by D2 and PAR) $s = \gamma(M(s,t))$ so that $sP_2 t$. And using IIA and the definition of $M(s,t)$, if $sP_2 t$ then $sP_1 t$: otherwise, $t = \gamma(T)$ for some $T \in \Sigma$ with $s \in T$ would imply $t = \gamma(M(s,t))$, that is $tP_2 s$. Therefore $P_1 = P_2 = P$.

To see that $P$ is transitive, suppose that $sPtPu$. This means (viewing $P$ as $P_2$) that $s = \gamma(M(s,t))$ and $t = \gamma(M(t,u))$. Suppose by contradiction that it is not the case that $sPu$, so that $u = \gamma(M(s,u))$. Let $T = M(s,t) \cup M(t,u) \cup M(s,u)$. By D3, $T \in \Sigma$. By PAR, $\gamma(T) \in \{s,t,u\}$, so that IIA applied to $T$ and one of the sets $M(s,t)$, $M(t,u)$ or $M(s,u)$ is contradicted.

[9]By complete we mean here that either $sPt$ or $tPs$ for any *distinct* $s$ and $t$.

$P$ is clearly complete and Pareto consistent by D2 and PAR. To conclude, it is easy to show that a solution that can be rationalized as in the statement satisfies the axioms. QED

We can see, then, that weakening IIA to the combination of WIIA and EXP has *two* distinct effects: first, it permits solutions that are rationalized by two relations rather than a single one. Second, it permits to relax the transitivity of the rationalizing relations.

Finally, we consider a property that merges WIIA and EXP into a single axiom:

WIIA*: Let $\{S^k, d\}$ be a class of problems . If $s = \gamma(R, d) = \gamma(S^k, d)$ for all $k$, and $t \in R \subset S \subset \bigcup_k (S^k, d)$, then $\gamma(S, d) \neq t$.

By using arguments essentially identical to those in the proof of theorem 2, one can obtain an extension of that theorem:to even more general domains:

**Theorem 4** *Consider a domain $\Sigma$ that satisfies D1 and D2. A solution on $\Sigma$ is a two-stage solution, which can be rationalized by Pareto consistent $P_1$ and $P_2$, if and only if it satisfies PAR, and WIIA*.*

# 4 Covariant solutions

In this section we consider solutions that satisfy COV. To this end, we need to make two further domain assumptions (formulated for normalized problems:

D4: For all $S \in \Sigma$, for all positive affine transformation $\tau$: $\tau(S) \in \Sigma$.

D5: For all $s, t \in \mathcal{R}^n_{++}$, for all positive affine transformation $\tau$: $\tau(M(s,t)) = M(\tau(s), \tau(t))$

As before, fix an admissible domain $\Sigma$, which satisfies in addition D4 and D5. A relation $P$ on $\mathcal{R}^n_{++}$ is invariant with positive affine transformations (or *pat-invariant* in short) iff, for all positive affine transformations $\tau$, $\tau(s)\,P\tau(t)$ whenever $xPt$.

We can now provide a characterization of two-stage and degenerate two-stage COV solutions:

**Theorem 5** *(i) A solution is a two-stage solution, which can be rationalized by $P_1$ and $P_2$ that are Pareto consistent and pat-invariant, if and only if it satisfies WIIA, EXP and COV.*

*(ii) A solution is a degenerate two-stage solution, which can be rationalized by a complete*[10]*, asymmetric, transitive, Pareto consistent and pat-invariant $P$, if and only if it satisfies PAR, IIA and COV.*

**Proof**: Let $\gamma$ be a two-stage solution that satisfies COV, and define $P_1$ and $P_2$ as in the proof of theorem 2. Let $\tau$ be a positive affine transformation. Let $sP_1t$. Suppose by contradiction that it is not the case that $\tau(s)\,P_1\tau(t)$. Then there exists $S \in \Sigma$ such that $\tau(t) = \gamma(S)$ and $\tau(s) \in S$. By COV, $t = \gamma(\tau^{-1}(S))$ (where $\gamma(\tau^{-1}(S))$ is well-defined by D4), so that (since $s \in \tau^{-1}(S) \in \Sigma$) $sP_1t$ is contradicted. Next, let $sP_2t$. By the definition of $P_2$, D5 and COV it is immediate that $\tau(s)\,P_2\tau(t)$.

The statement now follows from theorems 2 and 3. QED

The interest and novelty of part (ii) of the theorem is that it is a *complete* characterization of PAR, COV and IIA solutions. In the literature only partial characterizations are stated for such solutions.

[10]By complete we mean here that either $sPt$ or $tPs$ for any *distinct* $s$ and $t$.

The theorem yields as an easy corollary a generalization (to different domains) of a partial characterization theorem by Zhou [11] given by Denicoló and Mariotti [2] and more recently by Peters and Vermeulen [7]. Define the asymmetric, *$\alpha$-weighted Nash multi-solution*, by

$$\nu^{\alpha}(S) = \arg\max_{S \cap \mathcal{R}^n_+} \prod_i s_i^{\alpha_i}$$

for all $S \in \Sigma$, for some vector of non-negative weights $\alpha = (\alpha_1, ..., \alpha_n) \in \mathcal{R}^n_+$.

**Corollary 6** *A solution that satisfies IIA, PAR and COV is a selection from some asymmetric Nash multisolution.*

**Proof:** It follows from theorems 3.3.3. in d'Aspremont [1], reformulated transforming the variables in logs as in Moulin [5], that for any relation $P$ on $\mathcal{R}^n_{++}$ that is complete, transitive, Pareto consistent and pat-invariant the following holds: If $\sum_i \alpha_i \log s_i > \sum_i \alpha_i \log t_i$, then $sPt$. QED

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