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Option Pricing with a Dividend General Equilibrium Model

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Option pricing with a dividend general equilibrium model

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Abstract

This paper derives a general equilibrium option pricing model for a European call assuming that the economy is exogenously driven by a dividend process following Hamilton's (1989) Markov regime switching model. The derived formula is used to investigate if the European call option prices are consistently priced with the stock market prices. This is done by obtaining the implied risk aversion coefficient of the model, for constant relative risk aversion preferences, based on traded option prices data.

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1 Introduction

There is a voluminous amount of work investigating if the general equilibrium model of Lucas (1978), assuming that the economy is exogenously driven by a dividend process, can explain the variations of stock market prices [for a survey see Cochrane (1997)]. In this paper, we study such a model can account, in a satisfactory manner, for the variability of the European option market prices, across different levels of moneyness or time to maturity. Studying this can indicate whether fundamental economic variables, such as the dividend, can consistently explain the variations of stock and option market prices, or not. In order to show this, we adopt a variant of Lucas' model assuming that the dividend growth rate follows a process which incorporates Markov Chain type of shifts in its mean and volatility according to Hamilton's (1989) Markov Regime Switching (MRS) model.[1]

The choice of the MRS model to represent the exogenous process driving the economy can be justified by recent evidence suggesting that this model can satisfactorily represent distributional features of dividend and stock returns, such as negative skewness and excess kurtosis (leptokurtosis) [see Ceccheti, Lam and Mark (1990, 1993), and Bonomo and Garcia (1996), *inter alia*]. These characteristics of stock returns are found to be necessary in interpreting the volatility smile implied by the BS model, when fitted into actual option price data [see Ghysels, Harvey and Renault (1996), Bakshi, Cao and Chen (1997), and Ait-Sahalia and Lo (1998), *inter alia*].

[1]Note that the adoption of a general equilibrium framework to price the European call option enable us to derive an analytic solution of the call option price which considers for the risk of a regime shift in the economy [see also Naik and Lee (1990), Naik (1993), Amin and Ng (1993), and Ma (1998), *inter alia*]. To derive such solutions, models of option prices which apply Black and Scholes's (BS) arbitrage (risk neutral) arguments in the presence of continuous or discontinuous shifts (or jumps) of the underlying stock return make the restrictive assumption that the price of risk of the shifts is zero [see Merton (1976), and Hull and White (1987), *inter alia*].

The analytic formula of the European call option price that the paper derives for the MRS dividend general equilibrium model, can be though of as a weighted average of equilibrium option prices conditional on all different paths of the regime changes of the economy, over the entire life of the option until its expiration date, and the state of the economy at the expiration date, itself. The weights attached to the conditional option prices are given by the joint probabilities of the above events to occur. The dependence of the option price on the particular regime of the economy at the expiration date of the option reflects the influence of the consumption smoothing motive effect of the regime changes on the stock and option prices, for that date [see Ceccheti, Lam and Mark (1990)].

The ability of the introduced option pricing model to explain the variations of European call option prices is examined by investigating the following: First, its potential to interpret the BS volatility smile, across different moneyness areas, and second, its consistency in pricing traded option data across different maturity and moneyness levels. Addressing the above two questions is important in appraising the influence of risk aversion on option pricing with regime (or discontinuous) shifts in the conditional mean and volatility of the underlying stock. If the price of risk of such type shifts is large, then option pricing models which assume that this source of risk is zero may lead to serious pricing and hedging biases [see Naik and Lee (1990), Cao (1998), and Ait-Sahalia and Lo (2000)].

The paper provides a number of useful results which may have important implications in asset pricing or hedging theory. First, it shows that the degree of risk aversion of a regime shift or an innovation in dividend process does not seems to seriously affect the shape of the BS volatility smile. This exists even for risk neutral preferences. Its shape is mainly determined by the existence of regime switches in the dividend or stock return process. For option pricing models based on risk neutral, local-arbitrage arguments (see fn 1), this result would imply that the empirical failure of the BS model may be attributed to the misspecification of the stochastic process driving the stock price underlying the option, and not to the risk premium effects associated with discontinuous changes (shifts) of stock returns.

Second, the paper provides strong evidence that there may exist apparent inconsistencies in pricing stock and option market data with the dividend general equilibrium model. This is shown using as metric of comparison between the two markets the risk aversion coefficient, for constant relative risk aversion preferences. In particular, the paper shows that the estimate of the risk aversion coefficient implied by the option price data is much smaller than that replicating basic statistics (moments) of the stock data, namely the mean of the price/dividend ratio, and the volatility, skewness and kurtosis coefficients of the one-period stock log-returns of the S&P500 index. This happens even though the model can interpret some of the main stylized facts of the stock and option prices markets, such as the BS volatility smile and evidence of negative skewness and excess kurtosis of stock returns. One explanation of the above inconsistencies of the model across the two markets might be that agents of different degree of risk aversion enter into each or across the two markets.

The paper is organized as follows. Section 2 lays out the assumptions of the MRS dividend model and gives the analytical solution of a European call option price. Section 3 examines the ability of the model to interpret the volatility smile of the BS model and derives estimates of the risk aversion coefficient implied by the model based on stock and option prices data. Finally, Section 4 presents a brief summary and overall conclusions.

2 The structure of the economy.

2.1 The stochastic environment of the economy

Consider an economy of the structure of Lucas (1978) where there is a representative infinitely lived agent who can competitively trade the following assets: a single risky stock, a finite number of pure discount bonds, and a finite number of European call and put options. In each period, t, the stock pays a dividend, D_t, in the form of a perishable consumption good, C_t. The bonds constitute claims of the consumer on one-unit of consumption good at a future date $t+\tau$. The call (or put) options entitle the consumer to buy (or sell) the stock in a future date at a prespecified value, known as strike price, K. In equilibrium, we assume that the consumption good market clears so that consumption equals dividends $[C_t = D_t]$, for all t, and the that representative agent holds only the stock. The other assets are in zero net supply.

The equilibrium prices of the above assets can be obtained by solving representative agent's utility maximization problem after making some assumptions about the stochastic process driving the exogenous variable D_t [see, Bailey and Stulz (1989), Naik and Lee (1990), Amin and Ng (1993), and Cao (1998), *inter alia*]. In so doing, we postulate that the dividend growth rate, $(D_{t+1}/D_t) - 1$, follows the Markov Regime Switching (MRS) model:

$$\frac{D_{t+1} - D_t}{D_t} = \mu_{t+1} + \sigma_{t+1}\epsilon_{t+1}, \tag{1}$$

with $\mu_{t+1} = \mu_0 + \mu_1 S_t$, $\sigma^2_{t+1} = \sigma^2_0 + \sigma^2_1 S_t$ and $\epsilon_t \sim \mathcal{N}iid(0,1)$, where S_t represents the state (regime) that the economy lies in, at each period of time t. The algebraic equivalent of (1) that we will use to derive analytic solutions of the asset prices is

$$\Delta\delta_{t+1} = \tilde{\mu}_{t+1} + \sigma_{t+1}\epsilon_{t+1}, \tag{2}$$

where δ_t is the logarithm of D_t and $\tilde{\mu}_{t+1} = \tilde{\mu}_0 + \tilde{\mu}_1 S_t$, where $\tilde{\mu}_0 = \mu_0 - \frac{1}{2}\sigma^2_0$ and $\tilde{\mu}_1 = \mu_1 - \frac{1}{2}\sigma^2_1$.

The timing of the state variable S_t in (1), or (2), is chosen in order to reflect the fact that in real economies dividend payments are announced in advance [see Cecchetti, Lam and Mark (1990)]. Following recent evidence by Cecchetti, Lam and Mark (1990), and Bonomo and Garcia (1996), *inter alia*, we consider that the economy switches between two regimes: "0" and "1" following a Markov Chain. This means that S_t takes two values: 0 and 1, i.e. $S_t \in \{0, 1\}$. Regime "0", which is restricted by (2) to have the lower volatility, σ_0^2, will be identified by our data as the expansion regime of the economy, implying that $\tilde{\mu}_0 > 0$. Regime "1", which has the higher volatility, $\sigma_0^2 + \sigma_1^2$, will be identified as the recession regime, meaning that $\tilde{\mu}_1 < 0$.

The movements between the two values of S_t are dictated by the transition matrix of probability, $\mathbf{P}$, given by

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{10} = 1 - p_{11} \\ p_{01} = 1 - p_{00} & p_{11} \end{bmatrix}, \tag{3}$$

where $p_{ij} = \Pr[S_{t+1} = j | S_t = i]$ declares the transition probability of moving from state i to state j. By the Markov property, equation (3) implies that the probability that after τ periods of time the economy will lie in a given state i, i.e. $S_{t+\tau} = i$, with $i \in \{0, 1\}$ will be given by the i-th entry of the vector

$$\boldsymbol{\xi}_{t+\tau|t} = \mathbf{P}^{\tau} \boldsymbol{\xi}_{t|t}, \tag{4}$$

where $\boldsymbol{\xi}_{t|t} = (\Pr[S_t = i | \mathcal{I}_t])_{i \in \{0,1\}}$ is a (2×1) dimension vector with elements the starting probabilities that the economy lies in regimes "0" or "1", respectively, at time t, given the information set of the agent, $\mathcal{I}_t$. Note that when S_t is known, at time t, $\boldsymbol{\xi}_{t|t}$ will be given by $\boldsymbol{\xi}_{t|t} = (0, 1)'$, or $\boldsymbol{\xi}_{t|t} = (1, 0)'$. When it is unknown, its elements, $\Pr[S_t = i | \mathcal{I}_t]$, can be inferred by the current and past realizations of δ_t using Hamilton's (1989) filter, assuming that the information set becomes $\mathcal{I}_t = \{\delta_t, \delta_{t-1}, \dots\}$. The long run values of $\boldsymbol{\xi}_{t+\tau|t}$, as τ goes to infinity, known as the ergodic probabilities, are independent of $\Pr[S_t = i | \mathcal{I}_t]$, and are given by the (2×1) vector $\boldsymbol{\pi} = (\pi_i)_{i \in \{0,1\}}$, with elements $\pi_0 = \frac{1-p_{11}}{2-p_{00}-p_{11}}$ and $\pi_1 = \frac{1-p_{00}}{2-p_{00}-p_{11}}$, respectively [see Hamilton (1989)].

In order to obtain analytic forms of bond and option prices, for different maturity intervals, τ, based on the above assumptions of the economy, it will be proved necessary to calculate the probabilities of the expected log-dividend change $\Delta\delta_{t+\tau}$, over the interval τ, conditional on the current information set, $\mathcal{I}_t$. As we will see later, these probabilities will be jointly determined by the probabilities of the values of the state variable $S_{t+\tau}$, at time $t+\tau$, and the sojourn time variable measuring the number of periods that the economy spends in regime "1", over τ, defined as

$$Z_{t,\tau} = \sum_{i=0}^{\tau} S_{t+i}. \tag{5}$$

The sojourn time variable, $Z_{t,\tau}$, takes values, ζ, in the set $\{0, ..., \tau\}$, since S_t is a binary time process, with values 0 and 1. The conditional on $\mathcal{I}_t$ probabilities of the values of $Z_{t,\tau}$, defined $\Pr[Z_{t,\tau} = \zeta | \mathcal{I}_t]$, can be calculated as follows. When the current regime is known, they are the same with the probabilities $\Pr[Z_{t,\tau} = \zeta | S_t = i]$, since $\mathcal{I}_t$ contains S_t. The functional form of the latter has been derived by Darroch and Morris (1968) and Pedler (1971), *inter alia*.[2]

[2] In particular, Pedler shows that $\Pr[Z_{t+\tau} = \zeta | S_t = i]$ is given by

$$\begin{aligned}\Pr[Z_{t+\tau} = \zeta | S_t = i] &= p_{00}^{\zeta} p_{11}^{\tau-\zeta} \{F(-\tau+\zeta, -\zeta; 1; l) \\ &\quad -dp_{11}^{-1} F(-\tau+\zeta+i, -\zeta+1-i; 1; l)\},\end{aligned}$$

$i = \{0, 1\}$, where $d = p_{00}p_{11} - p_{01}p_{10}$, $l = \frac{p_{01}p_{10}}{p_{00}p_{11}}$, and $F(a, b; c; l)$ denotes the hypergeometric function

$$F(a, b; c; l) = \sum_{\kappa=0}^{\infty} \frac{(a)_\kappa (b)_\kappa}{(c)_\kappa} \frac{l^\kappa}{\kappa!},$$

where $(a)_\kappa$, $(b)_\kappa$, and $(c)_\kappa$ are the Pochhammer terms, [e.g. $(a)_\kappa = a(a+1)\cdots(a+\kappa-1)$]. An algorithm calculating $\Pr[Z_{t+\tau} = \zeta | S_t = i]$ has been

When S_t is unknown, S_t is not part of $\mathcal{I}_t$. Then, $\Pr[Z_{t,\tau} = \zeta|\mathcal{I}_t]$ can be calculated by Bayes' rule as $\Pr[Z_{t,\tau} = \zeta|\mathcal{I}_t] = \sum_i \Pr[Z_{t,\tau} = \zeta|S_t = i] \Pr[S_t = i|\mathcal{I}_t]$, where the information set now becomes $\mathcal{I}_t = \{\delta_t, \delta_{t-1}, \dots\}$.

2.2 Equilibrium asset prices

Assume that the representative consumer maximizes his/her lifetime utility and specify the within-period constant relative risk aversion (CRRA) utility function:

$$\mathcal{U}(C_t) = \frac{C_t^{1+\gamma} - 1}{1+\gamma}, \tag{6}$$

where $\gamma \leq 0$ is the coefficient of relative risk aversion. With $\gamma = 0$, one obtains a risk neutral agent and with $\gamma \to -1$, one obtains an agent with logarithmic preferences.[3] Given (6), the equilibrium prices of the stock, denoted P_t, a discount bond of maturity τ, denoted $B_{t,\tau}$, and a European call option of the same maturity, denoted $G_{t,\tau}$, should satisfy the following Euler equations:

$$P_t = D_t^{-\gamma} \sum_{j=1}^{\infty} \beta^j \mathbb{E}_t \left(D_{t+j}^{\gamma+1} \right), \tag{7}$$

$$B_{t,\tau} = D_t^{-\gamma} \beta^\tau \mathbb{E}_t \left(D_{t+\tau}^{\gamma} \right), \tag{8}$$

and,

$$G_{t,\tau} = D_t^{-\gamma} \beta^\tau \mathbb{E}_t \left[D_{t+\tau}^{\gamma} \left(P_{t+\tau} - K \right)^+ \right], \tag{9}$$

suggested by Kedem (1980).

[3]Log-utility has been adopted by many authors in providing closed form solutions of asset pricing models [e.g. Cox, Ingersoll and Ross (1985)].

respectively, where $\mathbb{E}_t(\cdot)$ is used for the conditional on $\mathcal{I}_t$ mathematical expectation, $\mathbb{E}(\cdot|\mathcal{I}_t)$, and $(P_{t+\tau} - K)^+ = \max\{P_{t+\tau} - K, 0\}$. In our analysis, $\mathcal{I}_t$ will be assumed to contain the current and past values of dividends, $\{\delta_t, \delta_{t-1}, \ldots\}$, as well as the current and past realizations of S_t, $\{S_t, S_{t-1}, ...\}$. When is necessary, results will also be given for the case that S_t is considered as unknown.[4] In this case, S_t will be assumed to be independent of the innovation term ϵ_t, for all t.[5]

2.2.1 Stock and Bond prices

For S_t known, Cecchetti, Lam and Mark (1990), and Moore and Schaller (1996) show that the solution of (7) is given by the following proposition.

Proposition 1 *Assume that the log-dividend, δ_t, follows the stochastic process (2), with its underlying assumptions, and that the Euler equation (7) holds. Then, the stock price, P_t, is given by*

$$P_t = \frac{1}{g}\left[\beta e^{g_0+g_1 S_t} - \beta^2 e^{2g_0+g_1}\left(p_{00}+p_{11}-1\right)\right] D_t, \tag{10}$$

where

$$\begin{aligned} g &= 1+\beta e^{2g_0+g_1}\left(p_{00}+p_{11}-1\right)-\beta e^{g_0}\left(p_{11}e^{g_1}+p_{00}\right), \\ g_0 &= (1+\gamma)\,\tilde{\mu}_0+\frac{1}{2}(1+\gamma)^2\sigma_0^2, \\ g_1 &= (1+\gamma)\,\tilde{\mu}_1+\frac{1}{2}(1+\gamma)^2\sigma_1^2, \end{aligned}$$

and D_t declares the [observed] dividend value at time t.

Equation (10) indicates that the price/dividend ratio is not constant, but depends on the value of the state variable, S_t. There are however some values

[4]This assumption is often made when estimating (2) [see Hamilton (1989)].

[5]Note that the independence of S_t from ϵ_t means that the economic agent is surprised by changes in S_t [see Turner, Startz and Nelson (1989)].

of the risk aversion coefficient, γ, where this ratio becomes constant, and thus is independent from S_t. This happens when $g_1 = 0$ which is satisfied when $\gamma = -1$ or $\gamma = -(1 + \frac{2\tilde{\mu}_1}{\sigma_1^2})$ hold.[6] Using the definition of $\tilde{\mu}_1 = \mu_1 - \frac{1}{2}\sigma_1^2$, the last condition on γ can be also written as $\gamma = -\frac{2\mu_1}{\sigma_1^2}$.

Following analogous economic reasoning with Ceccheti, Lam and Mark (1990), the above two conditions on γ, rendering the price/dividend ratio independent from the state variable, can be attributed to the fact that the consumption smoothing effect of the regime changes of the dividend on the stock price is entirely offset by their intertemporal effects on the price of consumption good. The first type of effect arises from the risk aversion attitude of the agent to smooth his/her current and future paths of consumption, over all possible realizations of the dividend process, while the second is due to the effect of dividend changes on the supply of the consumption good.[7]

[6]Note that the first condition, $\gamma = -1$, is due to the shift in the mean, while the second, $\gamma = -(1 + \frac{2\tilde{\mu}_1}{\sigma_1^2})$, is due to the shift in the volatility.

[7]To see how these the above two effects operate, consider that the agent expects that the economy will be in the expansion regime, at the future date $t+\tau$, identified by the higher mean and lower volatility of the dividend growth rate process. Then, the consumption smoothing motive effect will lead the agent to substitute out future for current consumption, due to his/her risk averse attitude to avoid , over time. This will have a negative effect on the stock price, since the agent will sell off part of his/her stock holdings to finance current consumption. The effects of a future expansion of the economy in the future on the current price of the consumption good will have as a consequence a lower price of the asset, because of the higher (and less volatile) supply of the consumption good, at date $t + \tau$. This will lead to extra savings which, in contrast to the consumption smoothing motive effect, will increase the current demand of the stock and, hence, its price.

Note that for $\gamma = -1$, the cancelling out of the consumption smoothing motive and the intertemporal price effect happens independently of whether the economy is in the recession or expansion regime, at date $t + \tau$. For $\gamma = -\frac{2\mu_1}{\sigma_1^2}$, it requires the higher volatility regime, "1", to be the expansion regime, to compensate the representative agent for bearing the risk of very big fluctuations in his/her consumption.

The bond price $B_{t,\tau}$, satisfying the Euler equation (8), can be calculated as shown in the following proposition.

Proposition 2 *Assume that δ_t follows the stochastic process (2), with its underlying assumptions, and that the Euler equation (8) holds. Then, $B_{t,\tau}$ is given by*

$$B_{t,\tau} = \beta^{\tau} e^{\tau\left(\gamma^2\sigma_0^2/2+\gamma\tilde{\mu}_0\right)} \sum_{\zeta=0}^{\tau} e^{\zeta\left(\gamma^2\sigma_1^2/2+\gamma\tilde{\mu}_1\right)} \Pr\left[Z_{t+\tau} = \zeta|\mathcal{I}_t\right]. \qquad (11)$$

PROOF: The proof of the proposition is given in the appendix.

Equation (11) indicates that $B_{t,\tau}$ can be thought of as the weighted average of bond prices corresponding to all possible, different values of the sojourn time variable, $Z_{t,\tau}$, over the maturity horizon τ. The attached weights are given by the conditional on $\mathcal{I}_t$ probabilities of these values. Note that when there is no regime shift, i.e. let us say that the economy lies in regime "0" with probability one, then $B_{t,\tau}$ is constant, given by $B_{t,\tau} = \beta^{\tau} \exp\{\tau\left[\gamma^2\sigma_0^2/2 + \gamma\tilde{\mu}_0\right]\}$, for all t.

2.2.2 European call option prices

To obtain an analytic formula of the European call option price, $G_{t,\tau}$, implied by the Euler equation (9), we first need to derive the conditional on $\mathcal{I}_t$ probability density function of the τ-period log-return of the stock, defined as $q_{t,\tau} = p_{t+\tau} - p_t$, where $p_t = \log P_t$. This function, denoted by $f\left(q_{t,\tau}|\mathcal{I}_t\right)$, will be referred to as the Risk Averse Density [RAD] function, henceforth, since γ is assumed to be different than zero. The RAD function can be derived by writing (10), after taking logarithms of (2) and iterating the resulting equation forwards, as

$$p_{t+\tau} = \rho\left(S_{t+\tau}\right) + \delta_t + \tau\tilde{\mu}_0 + \tilde{\mu}_1 Z_{t,\tau} + \omega_{t,\tau}, \qquad (12)$$

where $\rho(S_{t+\tau}) = \log\left\{\frac{1}{g}\left[\beta e^{g_0+g_1 S_{t+\tau}} - \beta^2 e^{2g_0+g_1}(p_{00}+p_{11}-1)\right]\right\}$ takes values $\rho(i)$, $i \in \{0,1\}$, and $\omega_{t,\tau} = \sum_{i=1}^{\tau} \sigma_{t+i}\epsilon_{t+i}$. Equation (12) implies that the τ-period return is given by

$$\begin{aligned} q_{t,\tau} &= p_{t+\tau} - p_t \\ &= \rho(S_{t+\tau}) - \rho(S_t) + \tau\tilde{\mu}_0 + \tilde{\mu}_1 Z_{t,\tau} + \omega_{t,\tau}. \end{aligned} \tag{13}$$

An analytic formula of the RAD function can be obtained from (13) by defining the following two extensions of the information set, $\mathcal{I}_t$:

$$\mathcal{H}_{t,\tau}(\zeta) = \mathcal{I}_t \cup \{Z_{t,\tau} = \zeta\}, \text{ and} \tag{14}$$

$$\mathcal{G}_{t,\tau}(\zeta, i) = \mathcal{H}_{t,\tau}(\zeta) \cup \{S_{t+\tau} = i\}, \text{ with } \zeta \in \{0, ..., \tau\} \text{ and } i \in \{0,1\} \tag{15}$$

$\mathcal{H}_{t,\tau}(\zeta)$ extends $\mathcal{I}_t$ with the sojourn time variable values, ζ, and $\mathcal{G}_{t,\tau}(\zeta, i)$ extends $\mathcal{H}_{t,\tau}(\zeta)$ with the values of $S_{t+\tau}$. Given the above extensions of $\mathcal{I}_t$, we can calculate $f(q_{t,\tau}|\mathcal{I}_t)$ by applying Bayes' probability rule, as

$$\begin{aligned} f(q_{t,\tau}|\mathcal{I}_t) = {} & \sum_{i=0}^{1}\sum_{\zeta=0}^{\tau} f(q_{t,\tau}, \zeta, \rho(i) | \mathcal{G}_{t,\tau}(\zeta, i)) \\ & \times \Pr[Z_{t,\tau} = \zeta, S_{t+\tau} = i | \mathcal{I}_t], \end{aligned} \tag{16}$$

where

$$\begin{aligned} & f(q_{t,\tau}, \zeta, \rho(i) | \mathcal{G}_{t,\tau}(\zeta, i)) \\ = {} & \left(2\pi\sigma^2(\zeta)\right)^{-\frac{1}{2}} \exp\left\{-\frac{\left[q_{t,\tau} - \tilde{\mu}(\zeta, \rho(i), S_t)\right]^2}{2\sigma^2(\zeta)}\right\}, \end{aligned} \tag{17}$$

with

$$\begin{aligned} \tilde{\mu}(\zeta, \rho(i), S_t) &= \rho(i) - \rho(S_t) + \tau\tilde{\mu}_0 + \tilde{\mu}_1\zeta, \qquad \text{and} \\ \sigma^2(\zeta) &= \tau\sigma_0^2 + \sigma_1^2\zeta. \end{aligned}$$

Equation (17) indicates that the RAD function of $q_{t,\tau}$ is a mixture of the $2 \times (\tau + 1)$ normal density functions $f\left(q_{t,\tau}, \zeta, \rho\left(i\right) | \mathcal{G}_{t,\tau}(\zeta, i)\right)$. These functions calculate the probabilities of $q_{t,\tau}$ for given values of the sojourn time variable, $Z_{t,\tau}$, and the future state variable $S_{t+\tau}$, at time $t+\tau$. The weights attached to each of these densities, $f\left(q_{t,\tau}, \zeta, \rho\left(i\right) | \mathcal{G}_{t,\tau}(\zeta, i)\right)$, are given by the corresponding joint probabilities of the values of $Z_{t,\tau}$ and $S_{t+\tau}$, denoted by $\Pr\left[Z_{t,\tau} = \zeta, S_{t+\tau} = i | \mathcal{I}_t\right]$. The values of the RAD function can be calculated given values of the vector of parameters of the MSR model (2), $\boldsymbol{\theta}' = (\tilde{\mu}_0, \tilde{\mu}_1, \sigma_0, \sigma_1, p_{00}, p_{11})$, the risk aversion and the time preferences coefficients, γ and β, respectively, and the joint probabilities $\Pr\left[Z_{t,\tau} = \zeta, S_{t+\tau} = i | \mathcal{I}_t\right]$. The latter can be obtained by using Kedem's (1980) algorithm.

Note that for the values of γ: $\gamma = -1$ or $\gamma = -(1 + \frac{2\tilde{\mu}_1}{\sigma_1^2})$, the RAD function (17) reduces to

$$f\left(q_{t,\tau} | \mathcal{I}_t\right) = \sum_{\zeta=0}^{\tau} f\left(q_{t,\tau}, \zeta, \rho | \mathcal{H}_{t,\tau}(\zeta)\right) \Pr\left[Z_{t,\tau} = \zeta | \mathcal{I}_t\right], \tag{18}$$

where $f\left(q_{t,\tau}, \zeta, \rho | \mathcal{H}_{t,\tau}(\zeta)\right) = \left(2\pi\sigma^2\left(\zeta\right)\right)^{-\frac{1}{2}} \exp\left\{-\frac{[q_{t,\tau} - \tilde{\mu}(\zeta)]^2}{2\sigma^2(\zeta)}\right\}$, with $\tilde{\mu}\left(\zeta\right) = \tau\tilde{\mu}_0 + \tilde{\mu}_1\zeta$, since $\rho\left(i\right) - \rho\left(S_t\right) = 0$. The functional form of the RAD given by (18) does no longer depend on the particular regime of the economy at the future date, $t + \tau$, alone. It only depends on the realizations of the sojourn time variable, $Z_{t,\tau}$, over τ. This happens since the price/dividend ratio becomes independent of the state variable, for the above values of γ. In this case, the distribution properties of the log-return $q_{t,\tau}$ will be solely determined by those of the log-dividend change $\Delta\delta_{t+\tau}$.

Having obtained the RAD function of the τ-period return $q_{t,\tau}$, we next derive an analytic solution of the European call price, $G_{t,\tau}$, implied by the Euler equation (9). This is given by the following proposition.

Proposition 3 *Assume that the log-dividend, δ_t, follows the stochastic process (2), with its underlying assumptions, and that the Euler equation (9)* *holds. Then, the European call price,* $G_{t,\tau}$*, is given by*

$$G_{t,\tau} = \beta^{\tau} \sum_{i=0}^{1} \sum_{\zeta=0}^{\tau} G_{t,\tau}(i,\zeta) \Pr\left[Z_{t,\tau} = \zeta, S_{t+\tau} = i | \mathcal{I}_t\right], \tag{19}$$

where

$$\begin{aligned} G_{t,\tau}(i,j) &= \beta^{\tau} \Big\{ \rho(i) \, D_t e^{(\gamma+1)^2 \left(\tau\sigma_0^2 + \zeta\sigma_1^2\right)/2 + (\gamma+1)(\tilde{\mu}_0\tau + \tilde{\mu}_1\zeta)} \Phi\left[d_1\right] \\ &\quad - K e^{\gamma^2 \left(\tau\sigma_0^2 + \zeta\sigma_1^2\right)/2 + \gamma(\tilde{\mu}_0\tau + \tilde{\mu}_1\zeta)} \Phi\left[d_2\right] \Big\}, \end{aligned}$$

$$\begin{aligned} d_1 &= \frac{(\gamma+1)\left(\tau\sigma_0^2 + \zeta\sigma_1^2\right) + \tilde{\mu}_0\tau + \tilde{\mu}_1\zeta + \ln\left(\frac{D_t\rho(i)}{K}\right)}{\sqrt{\tau\sigma_0^2 + \zeta\sigma_1^2}}, \\ d_2 &= d_1 - \sqrt{\tau\sigma_0^2 + \zeta\sigma_1^2}, \end{aligned}$$

and $\Phi[\cdot]$ *is the cumulative normal distribution.*

Proof: The proof is given in the Appendix.

Equation (19) implies that the European call price, $G_{t,\tau}$, in an economy where the log-dividend growth rate follows Markov type of regime changes can be thought of as a weighted average of equilibrium option prices conditional on all possible values of the sojourn time variable ζ until the expiration date of the option, $t+\tau$, and the values of the state variable $S_{t+\tau}$, at $t+\tau$, itself. The result of the proposition is a consequence of the functional form of the RAD function (17). The weights attached to the conditional option prices are therefore similar with those attached to the RAD function, and they are given by the joint probabilities $\Pr\left[Z_{t,\tau} = \zeta, S_{t+\tau} = i | \mathcal{I}_t\right]$.[8]

[8] Note that the MRS option pricing model, given by (19), has an analogous formula to that of Naik and Lee's (1990) model, which assumes that the dividend process allows for independent, Poisson driven jumps in the mean. The Naik and Lee model implies an option pricing formula which is average of option prices conditional on the number of all possible dividend jumps between t and $t+\tau$.

There are two interesting cases of (19) which can be simplified. The first is when $\gamma = -1$ (logarithmic preferences) or $\gamma = -(1+\frac{2\tilde{\mu}_1}{\sigma_1^2})$, and corresponds to the RAD function (18). Then, $G_{t,\tau}$ is given by the following corollary.

Corollary 4 *When $\gamma = -1$ or $\gamma = -(1+\frac{2\tilde{\mu}_1}{\sigma_1^2})$, then equation (19) becomes*

$$G_{t,\tau} = \sum\nolimits_{\zeta=0}^{\tau} G_{t,\tau}(\zeta) \Pr\left[Z_{t,\tau} = \zeta | \mathcal{I}_t\right] \tag{20}$$

where

$$G_{t,\tau}(\zeta) = \beta^{\tau} \left\{ P_t \Phi\left[d_1\right] - K e^{\left(\tau\sigma_0^2 + \zeta\sigma_1^2\right)/2 + \tilde{\mu}_0\tau + \tilde{\mu}_1\zeta} \Phi\left[d_2\right] \right\},$$

$$\begin{aligned} d_1 &= \frac{\tilde{\mu}_0\tau + \tilde{\mu}_1\zeta + \ln\left(\frac{P_t}{K}\right)}{\sqrt{\tau\sigma_0^2 + \zeta\sigma_1^2}}, \\ d_2 &= d_1 - \sqrt{\tau\sigma_0^2 + \zeta\sigma_1^2}. \end{aligned}$$

Equation (20) indicates that the European call price, $G_{t,\tau}$, does no longer separately depend on the particular state of the economy at date, $t+\tau$, as in (19), when γ becomes $\gamma = -1$ or $\gamma = -(1+\frac{2\tilde{\mu}_1}{\sigma_1^2})$. As mentioned before, this is due to the fact that the RAD function does no longer depend on $S_{t+\tau}$, separately, for these values of γ.

The second special case of (19) is when there is no regime shift in the economy. Let say $S_t = 0$, with $\Pr[S_t = 0] = 1$, for all t. Then, equation (19) reduces to the formula given in the following corollary.

Corollary 5 *If $\Pr[S_t = 0] = 1$, then equation (19) yields*

$$G_{t,\tau} = B_{t,\tau} \left\{ e^{\tau\gamma\left((\gamma+\frac{1}{2})\sigma_0^2 + \tilde{\mu}_0\right)} P_t \Phi\left[d_1\right] - K\Phi\left[d_2\right] \right\}, \tag{21}$$

where $B_{t,\tau}$ is the τ-maturity bond price when $\Pr[S_t = 0] = 1$, and $d_1 = \frac{\log(P_t/K) + \tau\tilde{\mu}_0}{\sigma_0\sqrt{\tau}} + (\gamma+1)\sigma_0\sqrt{\tau}$ and $d_2 = d_1 - \sigma_0\sqrt{\tau}$.

Equation (21) implies that the underlying RAD function of the log-return will be normal when there is no regime shift in the dividend process. In this case, both the log-dividend and log-stock price follow a random walk with constant drift and volatility parameters. As the BS formula, the option formula given by (21) will therefore imply a constant volatility parameter, when is fitted into traded option price data. Note that for $\gamma = 0$, $\mu_0 = r$ and equation (21) reduces to the familiar BS formula.

3 An evaluation of the general equilibrium MRS option pricing model

This section has two mains goals. The first is to calibrate the MRS dividend general equilibrium model, introduced in the previous section, with the aim of investigating if it can explain some well known, stylized facts of the stock and option pricing literature, such as evidence of negative skewness and leptokurtosis of stock returns and the shape of the BS volatility smile. This is done based on maximum likelihood (ML) estimates of the parameters of the MRS process (2) using as aggregate dividend the series implied by the S&P500 composite monthly index between January 1, 1980 and December 31, 1993. This dividend series is deflated using the CPI index. The second objective of the section is to examine whether the MRS model can consistently explain the variation of option price data, across different moneyness areas and maturity, with that of stock price data. In order to see this, we are based on Ait-Sahalia and Lo's (1998, 2000) option price data set. This set covers the period between January 1 to December 31, 1993. To obtain real values, the option prices are appropriately deflated by the CPI.

3.1 Fitting the MRS process into the S&P500 aggregate dividend

The ML estimates of the parameters of the MRS model can be found in Table 1. The estimates reported in the table assume that S_t is unknown, as in Hamilton (1989).[9] The probability of $S_t = 1$, i.e. $\Pr[S_t = 1|\mathcal{I}_t]$, is given in Figure 1.

The results of both the table and figure clearly show that there are two distinct shifts in the mean and volatility parameters of the real log-dividend change.[10] The pairs of these parameters are: $(\tilde{\mu}_0, \sigma_0) = (0.05, 0.24)$, for the expansion regime, and $(\tilde{\mu}_0 + \tilde{\mu}_1 = -0.05, \sqrt{\sigma_0^2 + \sigma_1^2} = 0.49)$, for the recession regime, respectively. Note that the mean of the recession regime, $\tilde{\mu}_0 + \tilde{\mu}_1$, is not significantly different from zero.

The estimates of the transition probabilities, p_{01} and p_{10}, imply a frequent number of switches between the two regimes and a low degree of persistency of each regime.[11] This can be also confirmed by the graph of $\Pr[S_t = 1|\mathcal{I}_t]$, giving the probability that the economy stays in regime "1", at each point of time.

To see whether the MRS dividend model (2) can replicate the sample moments of the data, in Table 2 we report estimates of the first four uncentered moments of the one-period dividend growth rate implied by the MRS model (2) and their sample counterparts.[12] The standard deviations of the sample

[9] This means that S_t is not considered as part of the information set, $\mathcal{I}_t$. Estimation of the model treating S_t as known is also performed, but this did not significantly change the main results of the paper. This estimation was done along the lines suggested by Moore and Schaller (1996).

[10] Analogous results were obtained for the dividend growth rate.

[11] The persistency coefficient, defined as $\lambda = 1 - p_{01} - p_{10}$, is $\lambda = 38.26\%$. This value of λ is much smaller than one, which indicates a low degree of persistency of each regime.

[12] The theoretical momments are calculated by using Chourdakis and Tzavalis (1999) formulas of the first four centered moments of the MRS process (2) [see also Timmermann (2000)].

moments can be found in parentheses. The result of the table demonstrate that the MRS model can satisfactorily match both the signs and magnitudes of the sample moments. Specifically, the reported standard deviations indicate that the point estimates of the sample moments do not differ more than two standard deviations from their corresponding theoretical values, implied by the model.

Summing up, the results of this subsection suggest that the MRS can adequately describe the real dividend series, implied by the S&P500 index. Given this, we next investigate if the MRS model can explain certain distribution features of stock returns, such as negative skewness and excess (over the normal) kurtosis.

3.2 Properties of the RAD function

Evidence of negative skewness and excess kurtosis of stock returns is often offered in the option pricing literature as an explanation of the BS volatility smile, implied by option price data [see Ait-Sahalia and Lo (1998), *inter alia*]. In this subsection, we investigate whether the MRS general equilibrium model can generate stock log-returns with the above distribution characteristics and analyze the influence of the risk aversion coefficient, γ, on them.

To show this, we calculate the following unconditional (evaluated at the ergodic levels of probability) statistics: the mean of the price/dividend ratio, P_t/D_t, the standard deviation (known as volatility), and the skewness and kurtosis coefficients of the one-period log-return, $q_{t,1}$.[13] This is done for different values of β and γ, and based on the ML estimates of the vector of parameters $\boldsymbol{\theta}$. To calculate P_t/D_t we use the weighted average of (10), over the values of S_t: 0 and 1, using as weights the ergodic probabilities of these values to occur. To estimate the other statistics we are based on the

[13]Note that the last three statistics can also indicate the generating sources of excess volatility, negative skewness and leptokurtosis of the τ-period stock return, $q_{t,\tau}$, since $q_{t,\tau}$ consists of one-period returns.

following second, third and fourth unconditional moments of $q_{t,1}$:

$$\begin{aligned}
M_1^{(2)} &= \pi_0\pi_1\left[A^2+(A+\tilde{\mu}_1)^2\right]+\sigma_0^2+\pi_1\sigma_1^2, \\
M_1^{(3)} &= \pi_0\pi_1(\pi_0-\pi_1)\tilde{\mu}_1\left[\tilde{\mu}_1^2+3(A+\tilde{\mu}_1)A\right] \\
&\quad +3\pi_0\pi_1(A+\tilde{\mu}_1)^2\sigma_1^2, \text{ and} \\
M_1^{(4)} &= \pi_0^2\pi_1^2\tilde{\mu}_1^4\left(\pi_0^2+\pi_1^2\right)+\pi_0\pi_1\left\{[A+\tilde{\mu}_1\pi_1]^4+[A+\tilde{\mu}_1\pi_0]^4\right\} \\
&\quad +\pi_0\pi_1\left\{[A+\tilde{\mu}_1\pi_1]^2\sigma_0^2+[A+\tilde{\mu}_1\pi_0]^2\left(\sigma_0^2+\sigma_1^2\right)\right\} \\
&\quad +\pi_0^2\pi_1^2\tilde{\mu}_1^2\left(2\sigma_0^2+\sigma_1^2\right)+3\left\{\pi_0\sigma_0^4+\pi_1\left(\sigma_0^2+\sigma_1^2\right)^2\right\}, \qquad (22)
\end{aligned}$$

where $A=\rho(0)-\rho(1)$.[14] The proof of the moment functional forms given by (22) can be found in the Appendix.[15]

Figures 2(a)-(d) graphically present the values the statistics of interest with respect to γ, for $\gamma \in (0,-50)$.[16] The horizontal lines of the graphs give the sample counterparts of the statistics, based on the stock price data. In all graphs, we consider that $\beta = 0.985$.[17] As it will be shown later, the MRS model fits better into the option prices data for this value of β. Inspection of the graphs leads to the following conclusions:

(i) There is an inverse relationship between the stock price and the risk aversion coefficient. This relationship is due to the fact that, as risk aver-

[14] We derive the unconditional moments for comparison reasons with the sample moments of the data.

[15] Note that for $\gamma=-1$ or $\gamma=-(1+\frac{2\tilde{\mu}_1}{\sigma_1^2})$ (implying that $A=0$), the moment functions given by (22) reduce to those of the one-period change of the log-dividend [see Chourdakis and Tzavalis (1999)]. This happens since the price/dividend ratio does no longer depend on the state variable, for the above values of γ. Then, as mentioned before, the distribution features of the stock price resemble those of the dividend.

[16] Note that this interval of the values of γ is broad enough to accomodate for the very high degrees of risk aversion, which have been derived in similar studies.

[17] Graphs with different values of β are also produced, but they did not significantly change the pattern of the statistics with γ. These graphs are not presented in the paper for reasons of space. They are available upon request.

sion increases, the consumption smoothing motive effect of dividend changes on the stock price dominates the intertemporal consumption good price effect. As a consequence of this, the stock price will decline with γ.

(ii) There is a positive relationship between the volatility of the return $q_{t,1}$ and the risk aversion, stating that an increase of the degree of risk aversion will increase the volatility of $q_{t,1}$. This implies that small changes in the dividend process will lead to substantial changes in $q_{t,1}$, when γ takes large negative values. These changes in the stock return are required to compensate the representative agent for any large intertemporal consumption loses that arise from changes in the dividend regime.

(iii) In contrast to the relationships of the price/dividend ratio and volatility with γ, the relationship between the skewness coefficient of the return $q_{t,1}$ and γ is less straightforward to be interpreted. The skewness coefficient, defined as $SK_1 = M_1^{(3)} / \left(M_1^{(2)}\right)^{3/2}$, is a $\cup$-shaped function of γ which takes both positive and negative values. For our estimates of $\boldsymbol{\theta}$, the minimum value of this function is located at $\gamma = -4.80$. For $\gamma < -4.80$, SK_1 exponentially declines with the size of γ, and converges to an asymptote.[18]

The negative value of SK_1 means that the probability of obtaining a value of the stock return $q_{t,1}$ which is less than its expected value is smaller than 50%. This can be attributed to the risk averse attitude of the representative agent which requires a compensation for a possible shift to the recession regime, in the next period. If there is no such a shift (i.e. $\tilde{\mu}_1 = 0$), then equation (22) implies that the skewness coefficient will become zero.

The increase of the size of SK_1 with risk aversion, documented in the graph of SK_1 for $\gamma > -4.80$, can be attributed to the fact that the negative degree of skewness which is required by the agent to be compensated for a possible switch to the recession regime becomes higher with the degree of risk aversion. If the risk aversion attitude is not strong enough, then SK_1

[18] Analogous shapes of SK_1 were observed for different combinations of values of the transition probabilities, p_{ij}, or the conditional mean parameters, μ_0 and μ_1.

can be positive, even though the distribution of the log-dividend growth is skewed to the left.

The decrease of the absolute size of SK_1 with the degree of risk aversion can not be given an economic explanation. It is due to the fact that the denominator of SK_1, given by $\left(M_1^{(2)}\right)^{3/2}$, increases faster with the size of γ rather than the numerator, $M_1^{(3)}$. This means that the variance of the stock return $q_{t,1}$ will dominate the effects of risk aversion on $M_1^{(3)}$, as γ increases.

(iv) In contrast to the skewness, the kurtosis coefficient, defined $KU_1 = M_1^{(4)}/\left(M_1^{(2)}\right)^2$, seems to have an almost $\cap$-shaped relationship with γ. For our estimates of $\boldsymbol{\theta}$, KU_1 achieves its maximum value at $\gamma = -3.0$, and then starts declining to an asymptote. As in (iii), we can attribute the declining part of KU_1 to the different speed of convergence of the numerator and denominator of KU_1 with γ. The increasing part of KU_1, for $\gamma > -3.0$, can be attributed to the risk averse attitude of the agent to require extreme changes in stock prices as a compensation for large (or extreme) negative changes in the dividend process.

Overall, the results of this section indicate that the MRS model can generate stock returns with negatively skewed and excess kurtotic distributions which are necessary for the interpretation of the BS volatility smile. The value of the risk aversion coefficient seems to affect both the sign and magnitude of the skewness and excess kurtosis coefficients.

3.3 Can the MRS model interpret the BS volatility smile?

As mentioned before, the negative skewness and excess kurtosis of the stock return has been offered as an explanation of the volatility smile implied by the BS model. This constitutes a $\cup$-shaped relationship between the volatility implied by the BS model and the ratio of stock/strike prices, P_t/K, known as moneyness, something which the BS model predicts to be flat.

Evidence suggests that the smile tends to be stronger as we move from far-in-the money (far-ITM) or far-out-of-the-money (far-OTM) options to at-the-money (ATM) or ITM options [and Bakshi, Cao and Chen (1997), *inter alia*]. These features of the smile mean that the BS model will overprice the ATM or ITM options and underprice the far-ITM or far-OTM call options.

The goal of this subsection is to examine if the MRS option pricing model can generate shapes of BS volatility smile similar with those found in the literature of option pricing and to investigate the influence of risk aversion on them. This is a worthy exercise since it can reveal what is more responsible for the shape of the BS volatility smile: a MRS type of misspecification of the data generating process assumed by the BS model or the influence of risk aversion.

To examine the above questions, in Figure 3 we graphically present the volatility smile implied by the BS model, for $\tau = 1$, when fitted into option price data generated by (19). The figure presents graphs of the BS volatility smile for values of γ in the set $\{0, -10, -15, -30\}$, $K \in (0.90, 1.10)$ and $\beta = 0.985$. The chosen values of γ span a broader set of theoretically plausible values of it. They also enable us to investigate the influence of risk aversion on option pricing with regime switches by comparing the shape of the volatility smile for $\gamma = 0$ (risk neutrality) with that when $\gamma \neq 0$. Note that the values of γ: -10, -15 and -30 can also indicate whether the shape of the volatility smile is consistent with the values of γ replicating the sample statistics of the return $q_{t,1}$, given by the horizontal lines of the graphs of Figure 2(a)-(d).

We compute option prices, using for $\boldsymbol{\theta}$ the ML estimates of the MRS model (2), reported in Table 1. Since S_t is treated as unknown in estimation, the option prices generated by the MRS model are taken to be the weighted average of the conditional option prices, given by (19), over the values of S_t: 0 and 1, using as weights the filtered by the information set $I_t = \{\delta_t, \delta_{t-1}, \dots\}$ probabilities of these values to occur. To fit the BS formula into the generated data, we use the one-period interest rate implied by the bond pricing formula

(11), for $\tau = 1$.

The graphs of the figure clearly indicate that the MRS option pricing model can explain the behavior of BS volatility smile, across different moneyness levels. The graphs exhibit a $\cup$-shape pattern across moneyness, as evidence suggests. The risk aversion coefficient does not seem to have an important influence on the shape of the smile. When the magnitude of γ increases, the smile shifts slightly to the left. This shift can be attributed to the joint effect of the price/dividend ratio and skewness changes with γ. Its economic intuition could be that the risk averse agent wish to pay a higher call option premium to the writer of the option in order to hedge his/her portfolio against a possible consumption loss due to a switch to the recession regime, as the degree of risk aversion increases.

These results support the view that the shape of the BS volatility smile can be mainly attributed to a MRS type of misspecification of the stock price process underlying the option. The influence of risk aversion on the pattern of the smile does not seem to be very important.

3.4 Testing the MRS option pricing model- Implied γ

Having established that the MRS option pricing model can explain some of the main stylized facts of the stock pricing literature and the shape of the BS volatility smile, in this subsection we evaluate the empirical performance of the MRS dividend equilibrium model by testing if it can price traded European call option data, across different values of moneyness and maturity, consistently with the stock market data. To see this, we estimate the risk aversion coefficient, γ, implied by the option pricing formula (19), for $\frac{P_t}{K} \in (0.80, 1.40)$ and $\tau = \{1, 2, 6, 12\}$ months. If the model is correctly specified, then the estimates of γ will be flat across all values of $\frac{P_t}{K}$ and τ.

The estimates of γ are graphically presented in Figures 4(a)-(b). They are retrieved by fitting the option pricing formula (19) into the traded option price data. We use the ML estimates of $\boldsymbol{\theta}$, given in Table 1, as inputs for the formula and we assume that $\beta = 0.985$. Assuming β as known considerably facilitates the estimation procedure. This value of β is chosen since it is found to give on average the minimum function value of the estimation method, across all different values of $\frac{P_t}{K}$ and τ considered.

Inspection of the graphs of the figure clearly indicate that there exists some type of misspecification of the MRS dividend option pricing model (19) in fitting the traded option price data. The implied estimates of γ are not flat, across $\frac{P_t}{K}$ or τ, as the model predicts, but vary between the value $-.8$ and 0. In fact, the absolute value of γ becomes smaller as we move from the ITM to the OTM areas, and it tends to zero, as τ increases. On average, the estimates of γ seem to be close to those found by Ait-Sahalia and Lo (2000) by employing a non-parametric estimation method of the risk aversion coefficient, without making any assumptions about the form of agent's utility function.

The above estimates of γ substantially differ from those replicating the basic distributional characteristics of the log-return $q_{t,1}$ [see Figures 2(a)-(d)]. This can be also thought of as an empirical failure of the MRS dividend asset pricing model. The failure of the model can be attributed to the fact that there is not a common value of γ which can match basic statistics of stock returns, as shown in Figures 2(a)-(d), with those of the option prices. As a consequence of this, the RAD function implied by stock market data will always differ from the one based on option price data.

4 Conclusions

In this paper, we introduce a general equilibrium pricing model of a European call option. The model is based on the assumption that the economy is exogenously driven by a dividend process following Markov Chain types of shifts in the mean and volatility. The paper derives an analytic formula of the European call option price which shows that the European call price can be though of as a weighted average of equilibrium option prices conditional on all different paths of regime changes of the economy, over the life of the option, and the state of the economy at the expiration date of the option. The derived option pricing formula and its implied probability density function are used to evaluate the ability of the Markov regime switching model to explain some of the main stylized facts of the stock and option pricing literature and to adequately price European call option data.

The paper provides a number of new results which may have important implications for asset pricing theory. It shows that the MRS extension of Lucas' dividend general equilibrium model can not consistently price stock and European option market price data. This is shown using as a metric of comparison between the two markets the risk aversion coefficient implied by the model, when fitted into actual data. The paper provides strong evidence that the risk aversion coefficient of the model implied by option price data is much smaller than that matching basic statistics of stock market data. These differences can be translated into serious asset pricing and hedging biases. They can be attributed to different type of investors entering into or across the two markets, with the options market attracting agents that are more risk neutral. It can be argued that the smaller values of the risk aversion coefficient implied by the option data (especially for OTM options) are due to the fact that a higher number of risk neutral investors enter into this market, who are driven by speculative motives of regime changes in the stock market.

A Proofs of Propositions

In this appendix, we prove Propositions 2 and 3, and equation (22), given in the main text. The proofs of the above propositions are based on the following Lemma.

Lemma 6 *Assume that the log-dividend, δ_t, follows the stochastic process (2), then $\mathbb{E}_t\left\{D_{t+\tau}^{\gamma}\right\}$ can be calculated by*

$$\begin{aligned}&\mathbb{E}_t\left\{D_{t+\tau}^{\gamma}\right\}\\ =\;& D_t^{\gamma}e^{\tau\left(\gamma^2\sigma_0^2/2+\gamma\tilde{\mu}_0\right)}\sum_{\zeta=0}^{\tau}e^{\zeta\left(\gamma^2\sigma_1^2/2+\gamma\tilde{\mu}_1\right)}\Pr\left[Z_{t,\tau}=\zeta|\mathcal{I}_t\right].\end{aligned} \tag{23}$$

PROOF: Iterate forwards (2). This yields

$$\delta_{t+\tau}=\delta_t+\tau\tilde{\mu}_0+\tilde{\mu}_1\sum_{i=0}^{\tau-1}S_{t+i}+\omega_{t,\tau}, \tag{24}$$

where $\omega_{t,\tau}=\sum_{i=1}^{\tau}\sigma_{t+i}\epsilon_{t+i}$. From equation (24), we can find the conditional on $\mathcal{I}_t$ probability density function of $\delta_{t+\tau}$, denoted $f\left(\delta_{t+\tau}|\mathcal{I}_t\right)$, as follows.

First, notice that the conditional on $\mathcal{H}_{t,\tau}(\zeta)$ distribution of $\omega_{t,\tau}$ is normal,

$$\omega_{t+\tau}|\mathcal{H}_{t,\tau}(\zeta)\sim\mathcal{N}\left(0,\tau\sigma_0^2+\zeta\sigma_1^2\right), \tag{25}$$

due the assumption that ϵ_{t+i} and S_{t+i} are independent, for all i. Using Bayes' rule and the result of equation (25), $f\left(\delta_{t+\tau}|\mathcal{I}_t\right)$ can be calculated by

$$f\left(\delta_{t+\tau}|\mathcal{I}_t\right)=\sum_{\zeta=0}^{\tau}f\left(\delta_{t+\tau}|\mathcal{H}_{t,\tau}(\zeta)\right)\Pr\left[Z_{t+\tau}=\zeta|\mathcal{I}_t\right], \tag{26}$$

where

$$
\begin{aligned}
f\left(\delta_{t+\tau}|\mathcal{H}_{t,\tau}(\zeta)\right) &= \left[2\pi\sigma^2\left(\zeta\right)\right]^{-\frac{1}{2}}\exp\left\{-\frac{\left[\delta_{t+\tau}-\tilde{\mu}\left(\zeta\right)\right]^2}{2\sigma^2\left(\zeta\right)}\right\}, \qquad (27a)\\
\tilde{\mu}\left(\zeta\right) &= \delta_t+\tau\tilde{\mu}_0+\tilde{\mu}_1\zeta,\\
\text{and} \qquad \sigma^2\left(\zeta\right) &= \tau\sigma_0^2+\sigma_1^2\zeta.
\end{aligned}
$$

Given the functional form of $f\left(\delta_{t+\tau}|\mathcal{I}_t\right)$, the expectation $\mathbb{E}_t\left(D_{t+\tau}^{\gamma}\right)$ can be calculated as follows,

$$
\begin{aligned}
&\mathbb{E}_t\left\{D_{t+\tau}^{\gamma}\right\}\\
=& \int_{-\infty}^{+\infty} e^{\gamma\delta_{t+\tau}} f_{\delta}\left(\delta_{t+\tau}|\mathcal{I}_t\right)d\delta_{t+\tau}\\
=& \sum_{\zeta=0}^{\tau}\left\{\int_{-\infty}^{+\infty} e^{\gamma\delta_{t+\tau}} f\left(\delta_{t+\tau}|\mathcal{H}_{t,\tau}(\zeta)\right)d\delta_{t+\tau}\right\}\\
&\times\Pr\left[Z_{t,\tau}=\zeta|\mathcal{I}_t\right]. \qquad (28)
\end{aligned}
$$

Substituting the solution of the integral $\int_{-\infty}^{+\infty} e^{\gamma\delta_{t+\tau}} f\left(\delta_t|\mathcal{H}_{t,\tau}(\zeta)\right)d\delta_{t+\tau}$, given by

$$
\begin{aligned}
&\int_{-\infty}^{+\infty} e^{\gamma\delta_{t+\tau}} f\left(\delta_{t+\tau}|\mathcal{H}_{t,\tau}(\zeta)\right)d\delta_{t+\tau}\\
=& \int_{-\infty}^{+\infty}\left[2\pi\left(\tau\sigma_0^2+\zeta\sigma_1^2\right)\right]^{-1/2} e^{\gamma\delta_{t+\tau}-\frac{\left[\delta_{t+\tau}-\delta_t-\left(\tau\tilde{\mu}_0+\zeta\tilde{\mu}_1\right)\right]^2}{2\left(\tau\sigma_0^2+\zeta\sigma_1^2\right)}} d\delta_{t+\tau}\\
=& D_t^{\gamma}e^{\frac{\gamma^2}{2}\left(\tau\sigma_0^2+\zeta\sigma_1^2\right)+\gamma\left(\tau\tilde{\mu}_0+\zeta\tilde{\mu}_1\right)}, \qquad (29)
\end{aligned}
$$

into equation (28) yields (23).

A.1 Proof of Proposition 2

The result of Proposition 2 follows, immediately, by substituting equation (24) into (8).

A.2 Proof of Proposition 3

To prove the proposition write $\mathbb{E}_t\left[D_{t+\tau}^{\gamma}\left(P_{t+\tau}-K\right)^{+}\right]$, after taking logarithms of (10), as

$$\begin{aligned}
& \mathbb{E}_t\left[D_{t+\tau}^{\gamma}\left(P_{t+\tau}-K\right)^{+}\right] \\
= & \sum_{i=0}^{1}\sum_{\zeta=0}^{\tau}\mathbb{E}\left[D_{t+\tau}^{\gamma}\left(P_{t+\tau}-K\right)^{+}|\mathcal{G}_{t,\tau}(\zeta,i)\right]\Pr\left[Z_{t,\tau}=\zeta,S_{t+\tau}=i|\mathcal{I}_t\right] \\
= & \sum_{i=0}^{1}\sum_{\zeta=0}^{\tau}\mathbb{E}\left[e^{\gamma\delta_{t+\tau}}\left(\rho\left(i\right)e^{\delta_{t+\tau}}-K\right)^{+}|\mathcal{G}_{t,\tau}(\zeta,i)\right]\Pr\left[Z_{t,\tau}=\zeta,S_{t+\tau}=i|\mathcal{I}_t\right] \\
= & \sum_{i=0}^{1}\sum_{\zeta=0}^{\tau}\int_{\ln[K/\rho(i)]}^{+\infty}e^{\gamma\delta_{t+\tau}}\left(\rho\left(i\right)e^{\delta_{t+\tau}}-K\right)f\left(\delta_{t+\tau}|\mathcal{G}_{t,\tau}(\zeta,i)\right)d\delta_{t+\tau} \\
& \times\Pr\left[Z_{t,\tau}=\zeta,S_{t+\tau}=i|\mathcal{I}_t\right] \\
= & \sum_{i=0}^{1}\sum_{\zeta=0}^{\tau}\left[I_A\left(\zeta,i\right)-KI_B\left(\zeta,i\right)\right]\Pr\left[Z_{t,\tau}=\zeta,S_{t+\tau}=i|\mathcal{I}_t\right], \qquad (30)
\end{aligned}$$

$I_A\left(\zeta,i\right)$ is given by

$$\begin{aligned}
I_A\left(\zeta,i\right) &= \int_{\ln\left(\frac{K}{\rho(i)}\right)}^{+\infty}\rho\left(i\right)e^{(\gamma+1)\delta_{t+\tau}}f\left(\delta_{t+\tau}|\mathcal{G}_{t,\tau}(\zeta,i)\right)d\delta_{t+\tau} \qquad (31) \\
&= \rho\left(i\right)\int_{\ln\left(\frac{K}{\rho(i)}\right)}^{+\infty}\frac{e^{(\gamma+1)\delta_{t+\tau}-\frac{\left[\delta_{t+\tau}-\tilde{\mu}(\zeta)\right]^2}{2\sigma^2(\zeta)}}}{\sqrt{2\pi\sigma^2\left(\zeta\right)}}d\delta_{t+\tau},
\end{aligned}$$

where $\tilde{\mu}(\zeta)=\delta_t+\tau\tilde{\mu}_0+\zeta\tilde{\mu}_1$. Its solution is

$$I_A\left(\zeta,i\right)=\rho\left(i\right)e^{\frac{\sigma^2(\zeta)(\gamma+1)^2+2(\gamma+1)\tilde{\mu}(\zeta)}{2}}\Phi\left[\frac{\left[\tilde{\mu}\left(\zeta\right)+\ln\left(\frac{\rho(i)D_t}{K}\right)+\sigma^2\left(\zeta\right)\left(\gamma+1\right)\right]}{\sigma\left(\zeta\right)}\right]. \qquad (32)$$

The definition and solution of $I_B(\zeta, i)$ is as follows:

$$
\begin{aligned}
I_B(\zeta, i) &= \int_{\ln\left(\frac{K}{\rho(i)}\right)}^{+\infty} e^{\gamma\delta_{t+\tau}} f\left(\delta_{t+\tau}|\mathcal{G}_t(\zeta, i)\right) d\delta_{t+\tau} \\
&= e^{\frac{\gamma^2\sigma^2(\zeta)}{2}+\gamma\tilde{\mu}(\zeta)}\Phi\left[\frac{\tilde{\mu}(\zeta)+\ln\left(\frac{\rho(i)D_t}{K}\right)+\gamma\left(\tau\sigma_0^2+\zeta\sigma_1^2\right)}{\sigma(\zeta)}\right]. \quad (33)
\end{aligned}
$$

Substituting (32) and (33) into proves (30).

A.3 Proof of equation (22)

The unconditional moments of one-period return, $q_{t,1}$, can be derived by writing $q_{t,1}$ as

$$
q_{t,1} = \rho(S_{t+1}) - \rho(S_t) + \tilde{\mu}_0 + \tilde{\mu}_1 S_t + \left(\sigma_0^2 + \sigma_1^2 S_t\right)^{1/2} \varepsilon_{t+1}. \quad (34)
$$

The unconditional expectation of (34) is

$$
\mathbb{E}q_{t,1} = \tilde{\mu}_0 + \tilde{\mu}_1\pi_1. \quad (35)
$$

To derive the higher centered moments of $q_{t,1}$, subtract (34) from (35). This yields

$$
q_{t,1} - \mathbb{E}q_{t,1} = \rho(S_{t+1}) - \rho(S_t) + \tilde{\mu}_1[S_t - \pi_1] + \left(\sigma_0^2 + \sigma_1^2 S_t\right)^{1/2} \varepsilon_{t+1}. \quad (36)
$$

From (36), we can derive the following centered moments:
$M_1^{(2)}$ as:

$$
\begin{aligned}
M_1^{(2)} &= \mathbb{E}\left[q_{t,1} - \mathbb{E}q_{t,1}\right]^2 \\
&= \mathbb{E}\left[\rho(S_{t+1}) - \rho(S_t) + \tilde{\mu}_1[S_t - \pi_1] + \left(\sigma_0^2 + \sigma_1^2 S_t\right)^{1/2} \varepsilon_{t+1}\right]^2
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{i=0}^{1}\sum_{j=0}^{1}\pi_i\pi_j\left[\rho\left(j\right)-\rho\left(i\right)+\tilde{\mu}_1\left(i-\pi_1\right)\right]^2+\sum_{i=0}^{1}\pi_i\left(\sigma_0^2+\sigma_1^2 i\right)\\
&= \pi_0\pi_1\left[A^2+\left(A+\tilde{\mu}_1\right)^2\right]+\sigma_0^2+\pi_1\sigma_1^2,
\end{aligned}
$$

$M_1^{(3)}$ as:

$$
\begin{aligned}
\mathbb{E}\left[q_{t,1}-\mathbb{E}q_{t,1}\right]^3 &= \sum_{i=0}^{1}\sum_{j=0}^{1}\pi_i\pi_j\left[\rho\left(j\right)-\rho\left(i\right)+\tilde{\mu}_1\left(i-\pi_1\right)\right]^3+\\
&\quad +3\sum_{i=0}^{1}\sum_{j=0}^{1}\pi_i\pi_j\left[\rho\left(j\right)-\rho\left(i\right)+\tilde{\mu}_1\left(i-\pi_1\right)\right]\left(\sigma_0^2+\sigma_1^2 i\right)
\end{aligned}
$$

$$
=\pi_0\pi_1\left(\pi_0-\pi_1\right)\tilde{\mu}_1\left[\tilde{\mu}_1^2+3\left(A+\tilde{\mu}_1\right)A\right]+3\pi_0\pi_1\left(A+\tilde{\mu}_1\right)^2\sigma_1^2,
$$

and $M_1^{(4)}$ as:

$$
\begin{aligned}
\mathbb{E}\left[q_{t,1}-\mathbb{E}q_{t,1}\right]^4 &= \sum_{i=0}^{1}\sum_{j=0}^{1}\pi_i\pi_j\left[\rho\left(j\right)-\rho\left(i\right)+\tilde{\mu}_1\left(i-\pi_1\right)\right]^4+\\
&\quad +6\sum_{i=0}^{1}\sum_{j=0}^{1}\pi_i\pi_j\left[\rho\left(j\right)-\rho\left(i\right)+\tilde{\mu}_1\left(i-\pi_1\right)\right]^2\left(\sigma_0^2+\sigma_1^2 i\right)\\
&\quad +3\sum_{i=0}^{1}\pi_i\left(\sigma_0^2+\sigma_1^2 i\right)^2
\end{aligned}
$$

$$
\begin{aligned}
&=\pi_0^2\pi_1^2\tilde{\mu}_1^4\left(\pi_0^2+\pi_1^2\right)+\pi_0\pi_1\left\{\left[A+\tilde{\mu}_1\pi_1\right]^4+\left[A+\tilde{\mu}_1\pi_0\right]^4\right\}\\
&\quad+\pi_0^2\pi_1^2\tilde{\mu}_1^2\left(2\sigma_0^2+\sigma_1^2\right)+\pi_0\pi_1\left\{\left[A+\tilde{\mu}_1\pi_1\right]^2\sigma_0^2+\left[A+\tilde{\mu}_1\pi_0\right]^2\left(\sigma_0^2+\sigma_1^2\right)\right\}\\
&\quad+3\left\{\pi_0\sigma_0^4+\pi_1\left(\sigma_0^2+\sigma_1^2\right)^2\right\}.
\end{aligned}
$$

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Table 1: Estimates of the real dividend MSR process parameters

$\tilde{\mu}_0$	$\tilde{\mu}_0 + \tilde{\mu}_1$	σ_0	$\sqrt{\sigma_0^2 + \sigma_1^2}$	p_{01}	p_{10}
0.05	−0.05	0.24	0.49	12.74	49.00
(0.002)	(0.09)	(0.01)	(0.09)	(5.32)	(21.55)
Log-likelihood: 737.72					

Notes: Standard errors are in parentheses. The values of the coefficients $\tilde{\mu}_0$, $\tilde{\mu}_0+\tilde{\mu}_1$, σ_0, $\sqrt{\sigma_0^2 + \sigma_1^2}$, as well as their standard errors are monthly rates in percentage terms. The transition probabilities p_{01} and p_{101} are also in percentage terms.

Table 2: Theoretical and actual moments

Moments	Actual data	MSR data
First ($\times 10^2$)	0.018 (0.02)	0.050
Second ($\times 10^4$)	0.097 (0.16)	0.101
Third ($\times 10^6$)	−0.0030 (0.01)	−0.0097
Fourth ($\times 10^8$)	0.037 (0.01)	0.052

Notes: Standard deviations of the sample moments are in parentheses.

Figure 1: Filter Probabilities $\Pr\left[S_t = 1 | I_t\right]$

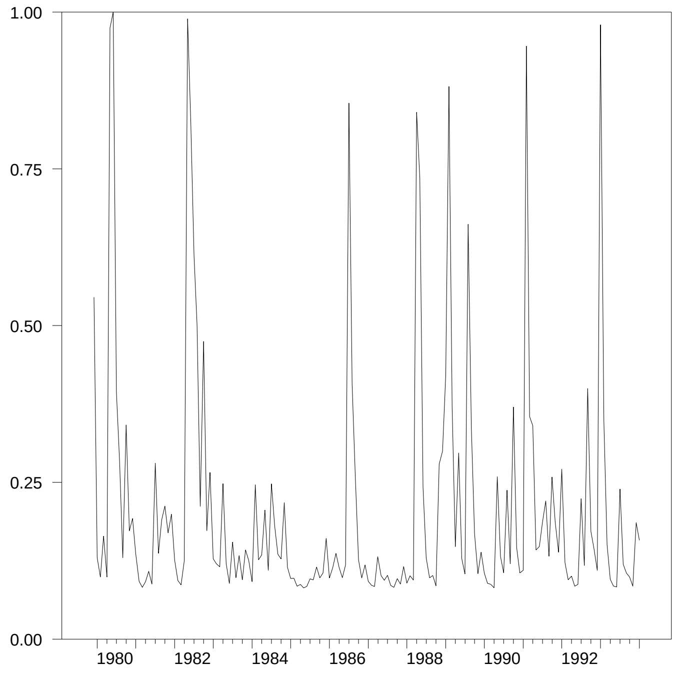
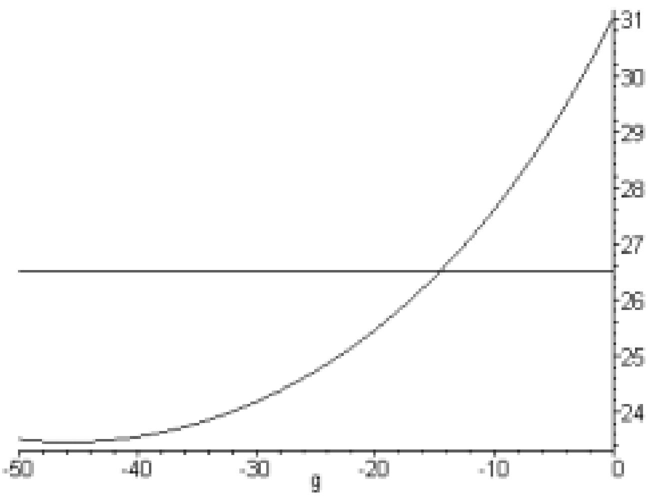
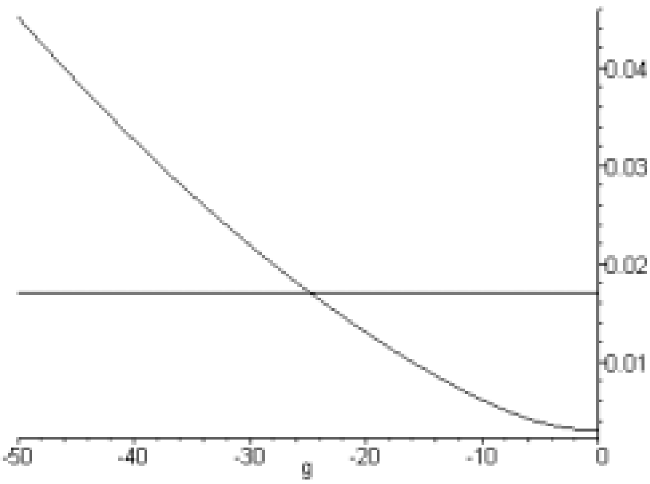


Figure 2: Descriptive Statistics

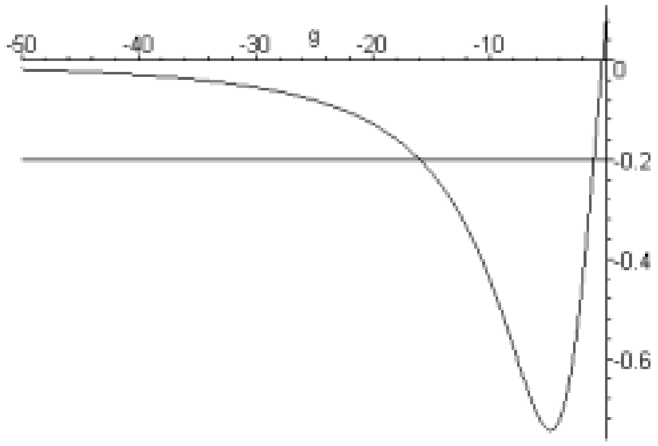


(a) Mean of $\frac{P_t}{D_t}$

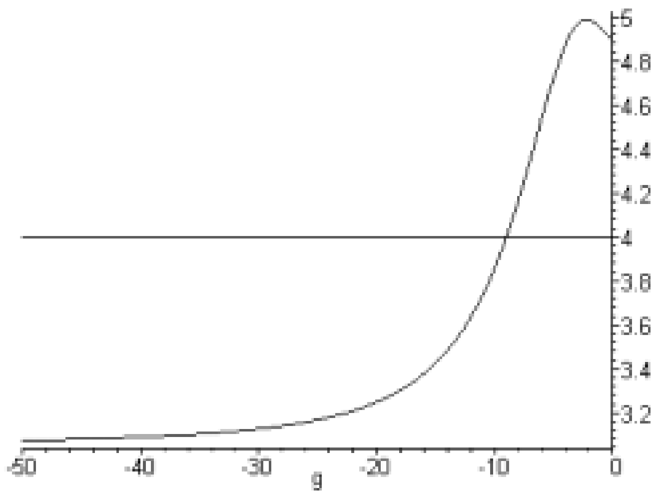


(b) Volatility of $q_{t,1}$

Figure 2: Descriptive Statistics (continued)



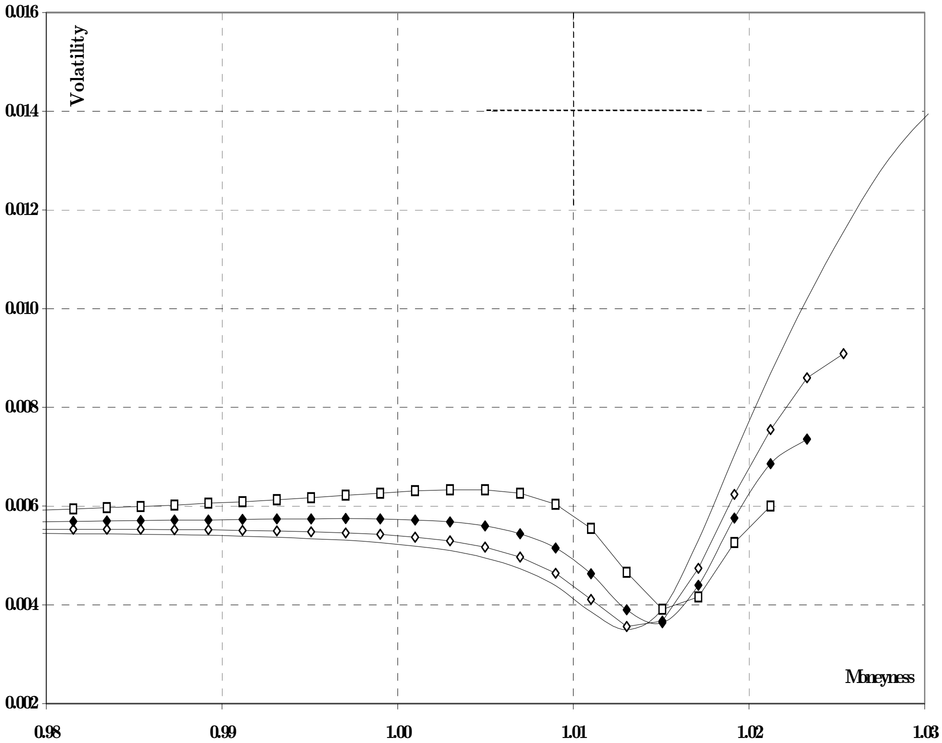
(c) Skewness coefficient of $q_{t,1}$



(d) Excess kurtosis coefficient of $q_{t,1}$

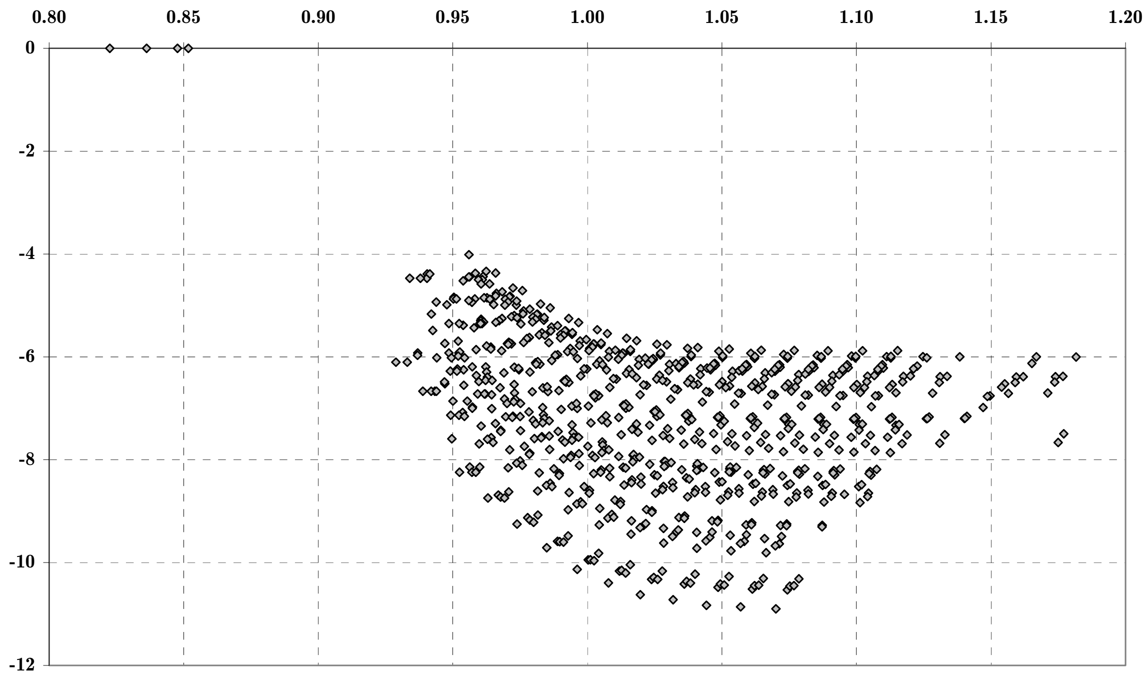
Notes: The horizontal lines give the values of the corresponding statistics based on the sample data.

Figure 3: Implied volatility smiles, for $\tau = 1$



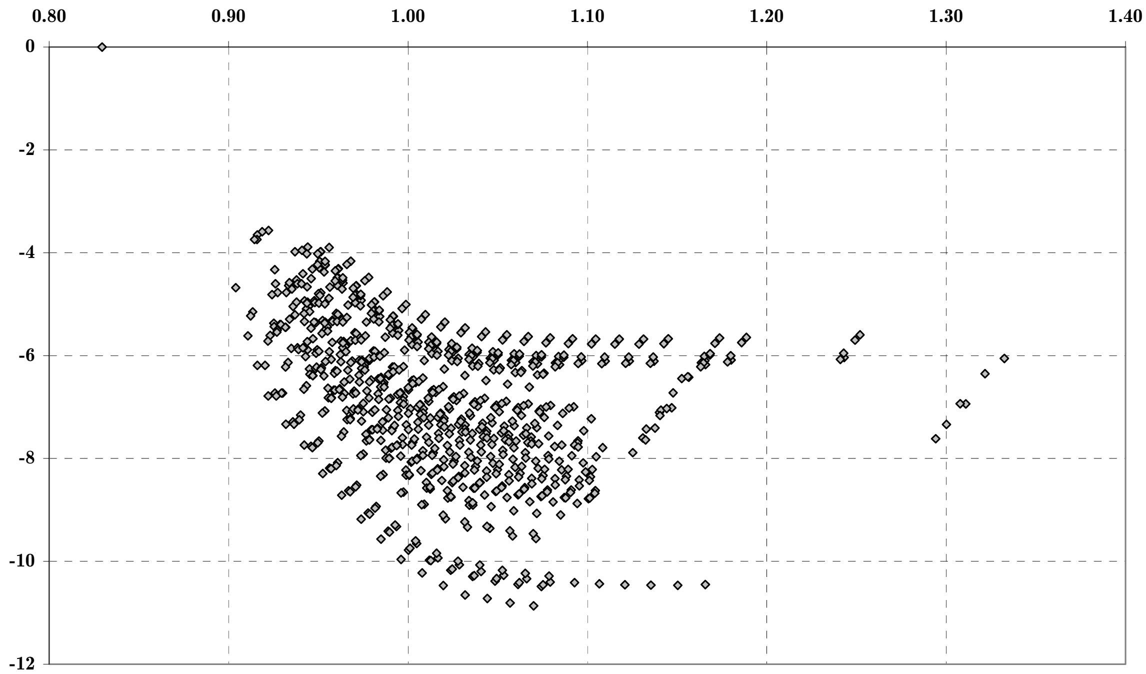
Notes: Solid line [—]: $\gamma = 0$; Empty diamonds [-◇-]: $\gamma = -10$; Full diamonds: [-◆-]: $\gamma = -15$; and Full squares [-■-]:$\gamma = -20$. $\tau = 1$ means one-period, which is a month for our data set.

Figure 4: Implied γ



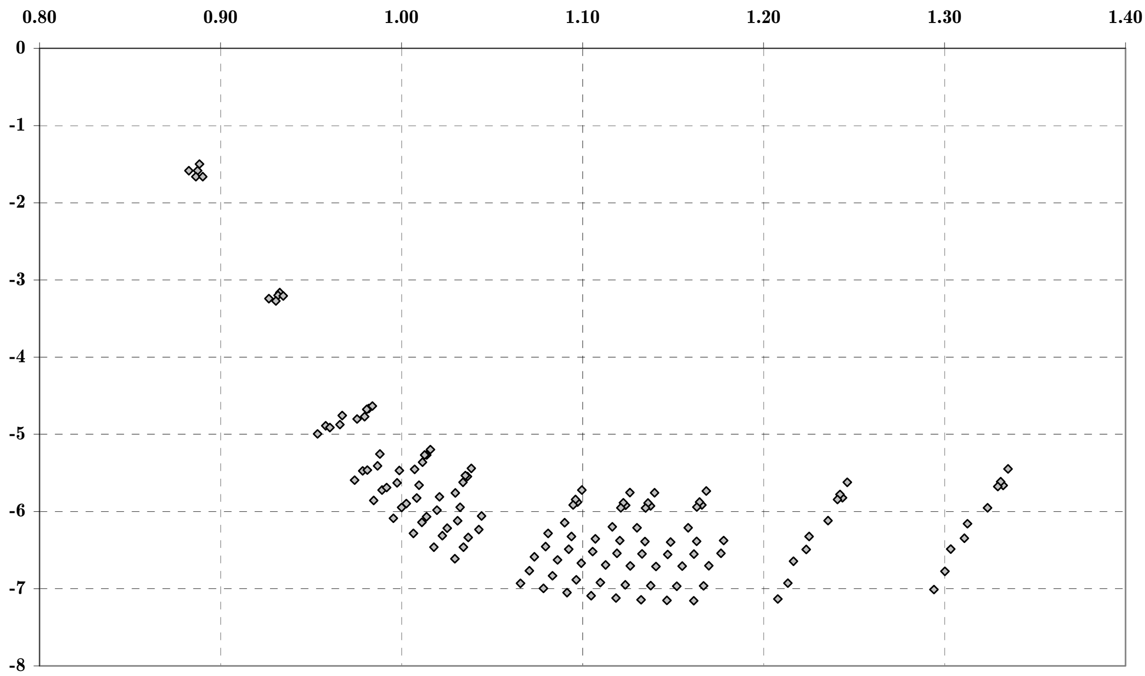
$\tau = 1$

Figure 4: Implied γ (continued)



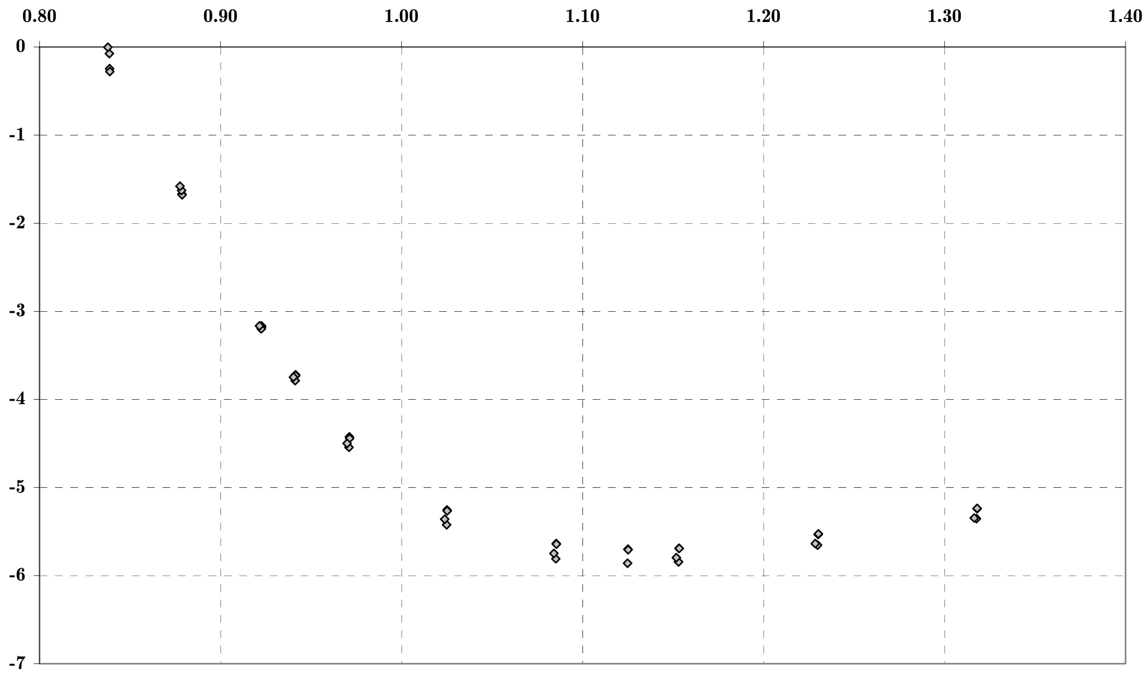
$\tau = 2$

Figure 4: Implied γ (continued)



$\tau = 6$

Figure 4: Implied γ (continued)



$\tau = 12$

Notes: τ stands for monthly periods.

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