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Abstract

Interest in the interface of nonstationarity and nonlinearity has been increasing in the econometric literature. This paper provides a formal method of testing for nonstationary long memory against the alternative of particular forms of nonlinerarity. The nonlinear models we consider are ESTAR and SETAR models. We provide analysis on the asymptotic properties of the tests and carry out a detailed Monte Carlo study. We find that the tests are in most cases able to dinstinguish between the competing models but in a few cases they are unable to do so raising the prospect that long memory and nonlinear processes may have similar characteristics in small samples.

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Key Words: Nonlinearity, Long Memory, ESTAR Models, SETAR Models.

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1 Introduction

Interest in the interface of nonstationarity and nonlinearity has been increasing in the econometric literature. The motivation for this development maybe be traced to the perceived possibility that processes following nonlinear models maybe mistakenly taken to be unit root nonstationary. Further, the inability of standard unit root tests to reject the null hypothesis of unit root nonstationarity for a large number of macreconomic variables which are supposed according to economic theory to be stationary is another reason behind the increased interest. Previous work in this area includes Enders and Granger (2001), Caner and Hansen (1998), Kapetanios, Shin, and Snell (2003) and Kapetanios and Shin (2003).

Another strand of the literature dealing with this and related conundrums has focused on the possibility of long memory with long memory parameter less than 1 in a number of processes being mistaken for unit root nonstationarity. The distinction between unit root nonstationary process and such long memory processes is significant. The persistence properties of shocks in the two cases are radically different.

As a result there exist two strands of the literature attempting to explain the apparent prevalance of unit root behaviour in two distinct ways. Of course, long memory processes nest unit root processes and therefore one can think of two distinct classes of processes with different implications. One is regime dependent nonlinear processes such as exponential smooth transition autoregressive (ESTAR) and self exchiting threshold autoregressive (SETAR) models which may exhibit a high degree of persistence and the other is linear but persistent to a varying degree and nonstationary long memory processes. Being able to distinguish between the two is a problem which has not been discussed in the literature. Nevertheless the distinction is of relevance. For example long memory processes with a long memory parameter d exceeding 0.5 but less than 1 are nonstationary although displaying independence of initial conditions. On the other hand nonlinear process although highly persistent are geometrically ergodic and therefore asymptotically stationary.

This paper provides a formal method of testing for long memory against the alternative of particular forms of nonlinerarity. The nonlinear models we consider are ESTAR and SETAR models. We provide analysis on the asymptotic properties of the tests and carry out a detailed Monte Carlo study. We find that the tests are in most cases able to dinstinguish between the competing models but in a few cases they are unable to do so raising the prospect that long memory and nonlinear processes may have similar characteristics in small samples. This hypothesis has been put forward in the context of Markov switching models by Diebold and Inoue (2001).

The paper is organised as follows Section 2 presents some preliminary material on long memory processes. Section 3 discusses the nonlinear models we consider. Section 4 presents the tests and their asymptotic properties. Section 5 discusses various extensions of the tests. Section 6 presents the Monte Carlo results. Section 7 presents an empirical application. Section 8 concludes. Finally, proofs of the theorems may be found in the Appendix.

2 Long Memory Processes

Let the null hypothesis be given by an ARFI model of the form

$$(1-L)^d y_t = \phi(L)^{-1}\epsilon_t = u_t,\ 1/2 \geq d < 3/2,\ t = 1, \dots, T \tag{1}$$

where ϵ_t is i.i.d. with variance σ^2 and finite fourth moments. This model does not represent a significant restriction on standard ARFIMA models if we allow for the order of the lag polynomial $\phi(L)$ to tend to infinity. Let us give some preliminary standard results for this model that will prove useful in what follows (see also Beran (1994)). The model for y_t can be written as an infinite moving average in terms of u_t

$$y_t = \sum_{i=0}^{\infty} a_i u_{t-i} \tag{2}$$

where $a_i = \frac{\Gamma(d+1)}{\Gamma(i+1)\Gamma(d-i+1)}(-1)^i$ It can equivalently be written as an infinite autoregression given by

$$y_t = \sum_{i=1}^{\infty} b_i y_{t-i} + u_t \tag{3}$$

where $b_i = -\frac{\Gamma(i-d)}{\Gamma(i+1)\Gamma(-d)}$ Defining $\sigma_T^2 = E(y_T^2)$ it can be proved that for a wide variety of processes u_t and for $0 \leq \xi \leq 1$

$$\sigma_T^{-1} y(\xi) \equiv y_{[\xi * T]} \xrightarrow{d} Y_d \tag{4}$$

where Y_d is a fractional Brownian motion given by

$$\frac{1}{\Gamma(d+1)V_d^{1/2}} \left(\int_0^{\xi} (\xi - s)^d dB(s) + \int_{-\infty}^{0} \left[(\xi - s)^d - (-s)^d \right] dB(s) \right) \tag{5}$$

where

$$V_d = \frac{1}{\Gamma(d+1)^2}\left(\frac{1}{2d+1} + \int_0^\infty \left[(1+\tau)^d - \tau^d\right]^2 d\tau\right) \tag{6}$$

$\sigma_T^2 \sim \sigma_u^2 V_d T^{2d-1}$ and B is a standard Brownian motion for $d \geq 1/2$. We wish to test the null hypothesis that the process y_t follows the above long memory process with given long-memory parameter $1/2 \geq d < 1$ against the alternative hypothesis that it follows a stationary nonlinear model.

3 Nonlinear models

We will consider two widely used nonlinear model classes as our alternative hypotheses. These are the ESTAR and SETAR classes of models

3.1 ESTAR Model

The standard ESTAR model is given by

$$y_t = \sum_{i=1}^{p} \alpha_p y_{t-p} + \sum_{i=1}^{q} \gamma_q y_{t-q}\left\{1 - \exp\left(-\theta y_{t-d}^2\right)\right\} + u_t \tag{7}$$

p, q and d are lag parameters to be chosen. This general model allows for lags greater than one. We will concentrate on the simple model

$$y_t = \alpha y_{t-1} + \gamma y_{t-q}\left\{1 - \exp\left(-\theta y_{t-d}^2\right)\right\} + u_t \tag{8}$$

on which to base our test. Longer lags are of course possible but we will construct a test based on a single lag and allow for serial correlation in the error term u_t to cover the case of longer lags.

Standard tests of linearity for ESTAR models revolve around a Taylor expansion of the ESTAR model and consist of testing the significance of the coefficient of the first term of such an expansion in a regression of the form

$$y_t = a_1 y_{t-1} + \delta y_{t-1}^3 + u_t \tag{9}$$

i.e. a test of $\delta = 0$ is carried out. For more details see Granger and Teräsvirta (1993). Our treatment follows closely the existing literature as exemplified by Kapetanios, Shin, and Snell (2003). We will use this setup to construct the test for long memory in the next section.

3.2 SETAR Model

The SETAR model is given by

$$y_t = \sum_{i=1}^{p} \alpha_i y_{t-p} + \sum_{j=1}^{m} \sum_{i=1}^{q} \beta_{ij} y_{t-i} 1_{\{r_{j-1} \leq y_{t-d} \leq r_j\}} + u_t, \tag{10}$$

where $r_0 = -\infty$ r_i, $i = 1, \ldots, m-1$ are constants and $r_m = \infty$. Again p, q and d are lag parameters to be set. Following previous work in testing against threshold nonlinearity in the context of unit root models (see Kapetanios and Shin (2003) and Bec, Guay, and Guerre (2003)) we concentrate on the following three regime threshold model

$$y_t = \alpha y_{t-1} + \beta_1 y_{t-1} 1_{\{y_{t-1} \leq r_1\}} + \beta_2 y_{t-1} 1_{\{y_{t-1} > r_2\}} + u_t, \tag{11}$$

where $y_{t-1} 1_{\{y_{t-1} \leq r_1\}}$ and $y_{t-1} 1_{\{y_{t-1} > r_2\}}$ are orthogonal to each other by construction. Again longer lags will be allowed for by correcting for serial correlation in u_t. For more information on SETAR models see Tong (1990).

4 Testing for Long Memory

By the infinite AR representation of the long memory model we can write

$$u_t = y_t - \sum_{i=0}^{\infty} b_i y_{t-i} \tag{12}$$

Denote $z_t = \sum_{i=1}^{t} b_i y_{t-i}$. We wish to obtain testing regressions similar to those used in standard and nonlinear unit root tests which under the null hypothesis give a long memory model and under the alternative give a nonlinear stationary model. However, the problem is that unlike standard unit root tests the long memory and the nonlinear models are nonnested. However, we can nest the two models if we assume a particular value of d. So the tests we will construct will have a fixed d under the null hypothesis. This may appear restrictive at first but provides tests with tractable asymptotic properties whereas using methods for nonnested models would require simulated critical values and would complicate enormously the analysis. Then under the null hypothesis of a long memory model with long memory parameter d we consider the following setups for testing the alternative hypothesis of the two nonlinear classes of models for the simple case where $u_t = \epsilon_t$.

4.1 ESTAR Model

In analogy to testing for linearity against ESTAR nonlinearity for stationary models we propose the following regressions in which to test for long memory

$$u_t = \alpha_1(y_{t-1}^3 + z_t) + \epsilon_t \tag{13}$$

and

$$u_t = \alpha_1 y_{t-1}^3 + \alpha_2 z_t + \epsilon_t \tag{14}$$

The motivation for these equations are as follows: We consider the general model

$$u_t = \alpha_1(1 - e^{-cy_{t-1}^2})y_{t-1} + \alpha_2 z_t + \epsilon_t \tag{15}$$

Under the null hypothesis $c = \alpha_2 = 0$, the model becomes

$$u_t = \epsilon_t \tag{16}$$

which is a long memory model. Under the hypothesis $c \neq 0$ and $\alpha_2 = 1$ the model becomes a STAR model of the form

$$y_t = \alpha_1(1 - e^{-cy_{t-1}^2})y_{t-1} + \epsilon_t \tag{17}$$

To test the hypothesis $c = \alpha_2 = 0$ which involves the unidentified parameter α_1 we use the standard approach for testing linearity in ESTAR models and take a Taylor expansion of the exponential function to give a testing equation of the form (14). The testing equation (13) is considered because it tests only for the significance of one parameter and may lead to a more powerful test. We will denote the tests based on regressions (13) and (14) by $STAR_1^d$ and $STAR_2^d$ respectively. The distribution of the Wald tests is given by the following theorem

Theorem 1 *The asymptotic distributions of the Wald tests for the null hypothesis of α_1 in (13) and $\alpha_1 = \alpha_2 = 0$ in (14) is given by*

$$\frac{\left(\int_0^1 Q_{1,d}(r)dB(r)\right)^2}{\left(\int_0^1 Q_{1,d}(r)^2 dr\right)} \tag{18}$$

and

$$\left(\int_0^1 Q_{2,d}(r)dB(r)\right)' \left(\int_0^1 Q_{2,d}(r)Q_{2,d}(r)'dr\right)^{-1} \left(\int_0^1 Q_{2,d}(r)dB(r)\right) \tag{19}$$

respectively where $Q_{1,d}(r)$ and $Q_{2,d}(r)$ are defined in the appendix.

4.2 SETAR Model

The testing regressions for the case of the SETAR models are given by

$$u_t = \alpha_1 (y_{t-1} I(|y_{t-1}| > r) + z_t) + \epsilon_t \tag{20}$$

$$u_t = \alpha_1 y_{t-1} I(|y_{t-1}| > r) + \alpha_2 z_t + \epsilon_t \tag{21}$$

$$u_t = \alpha_1 y_{t-1} I(y_{t-1} < r_1) + \alpha_2 y_{t-1} I(y_{t-1} \geq r_2) + \alpha_3 z_t + \epsilon_t \tag{22}$$

Clearly, under the null hypothesis of a long memory model with long memory parameter d the coefficients α_1 in (20), α_1 and α_2 in (21) and α_1 and α_2 in (22) will be zero. Under the alternative hypothesis of a nonlinear model of the SETAR form appropriate choices for the parameter lead to SETAR specification for the above equations. For equation (21) setting α_2 to -1 lead to a SETAR specification for y_t. The same holds for α_3 in equation (22). Note that equations (20) and (21) relate to a symmetric SETAR model whereas (22) relates to an asymmetric, and therefore more general, SETAR model.

To derive the asymptotic null distribution of the Wald statistics, we first begin to consider the simple case that threshold parameters are given. In this case, it will be shown that the asymptotic null distribution of the Wald statistic does not depend on the values of r_1 and r_2. We have the following theorem

Theorem 2 *The asymptotic distribution of the Wald test for the null hypothesis of $\alpha_1 = 0$ in (20), $\alpha_1 = \alpha_2 = 0$ in (21) and $\alpha_1 = \alpha_2 = \alpha_3 = 0$ in (22) are given respectively by*

$$\frac{\left(\int_0^1 Q_{3,d}(r) dB(r)\right)^2}{\int_0^1 Q_{3,d}^2(r) dr} \tag{23}$$

$$\left(\int_0^1 Q_{4,d}(r) dB(r)\right)' \left(\int_0^1 Q_{4,d}(r) Q_{4,d}(r)' dr\right)^{-1} \left(\int_0^1 Q_{4,d}(r) dB(r)\right) \tag{24}$$

and

$$\left(\int_0^1 Q_{5,d}(r) dB(r)\right)' \left(\int_0^1 Q_{5,d}(r) Q_{5,d}(r)' dr\right)^{-1} \left(\int_0^1 Q_{5,d}(r) dB(r)\right) \tag{25}$$

where $Q_{3,d}(r)$, $Q_{4,d}(r)$ and $Q_{5,d}(r)$ are defined in the appendix. These are the distributions of the Wald statistics for $r = r_1 = r_2 = 0$.

We will denote the three tests analysed in Theorem 2 by $SETAR_1^d$, $SETAR_2^d$ and $SETAR_3^d$ respectively. In the following discussion on the threshold we will concentrate on the more general model 22 which contains two threshold partameters. The discussion can be easily modified to accomodate the single threshold models. Asymptotic results are so far derived under the simplifying assumption that threshold parameters are known, and thus we now consider a general case with unknown threshold parameters. In such a case it is well-established that this kind of test suffers from the Davies (1987) problem since unknown threshold parameters are not identified under the null. Most solutions to this problem involve some sort of integrating out unidentified parameters from the test statistics. This is usually achieved by calculating test statistics for a grid of possible values of threshold parameters, r_1 and r_2, and then constructing the summary statistics. For stationary TAR models this problem has been studied in Tong (1990). Following Andrews and Ploberger (1994), we consider the two most commonly used statistics which are the average and the exponential average of the Wald statistic defined respectively by

$$\mathcal{W}_{(r_1,r_2)}^{\text{avg}} = \frac{1}{\#\Gamma} \sum_{i=1}^{\#\Gamma} \mathcal{W}_{(r_1,r_2)}^{(i)},\ \mathcal{W}_{(r_1,r_2)}^{\text{exp}} = \frac{1}{\#\Gamma} \sum_{i=1}^{\#\Gamma} exp\left(\frac{\mathcal{W}_{(r_1,r_2)}^{(i)}}{2} \right), \tag{26}$$

where $\mathcal{W}_{(r_1,r_2)}^{(i)}$ is the Wald statistic obtained from the i-th point of the nuisance parameter grid, Γ and $\#\Gamma$ is the number of elements of Γ. The tests will have the superscript *exp* or *avg* to denote what summary statistic is used in their construction.

Unlike the stationary TAR models, the selection of the grid of threshold parameters needs more attention. The threshold parameters r_1 and r_2 usually take on the values in the interval $(r_1, r_2) \in \Gamma = [r_{\min}, r_{\max}]$ where $r_{\min}$ and $r_{\max}$ are picked so that $\Pr(y_{t-1} < r_1) = \pi_1 > 0$ and $\Pr(y_{t-1} > r_2) = \pi_2 < 1$. The particular choice for π_1 and π_2 is somewhat arbitrary, and in practice must be guided by the consideration that each regime needs to have sufficient observations to identify the underlying regression parameters. Considering that our approach assumes that the coefficient on the lagged dependent variable is set to zero in the corridor regime ($r_1 \leq y_{t-1} < r_2$), however, we could assign arbitrarily small samples (relative to total sample) to the corridor regime since we do not have to estimate any parameters in the corridor regime. Notice also that the threshold parameters exist only under the alternative hypothesis in which the process is stationary and therefore bounded in probability. In this case only a finite grid search is meaningful for further estimation. For a discussion on the construction of the grid in stationary threshold models, that supports our approach, see Chan (1993) and Tong (1990).

This observation leads us to make an assumption that the grid for unknown threshold parameters should be selected such that the chosen corridor regime be of finite width. Under this practically meaningful restriction, we can further establish that the theoretical results obtained in Theorem 2 do hold in the more general case with unknown threshold parameters as shown below. By virtue of the fact that a long memory process is $O_p(T^{d-1/2})$ it will stay within a corridor regime of finite width for $O_p(T^{d-1/2})$ periods only. Therefore, setting $\pi_1 = \bar{\pi} - c/T^{d-1/2}$ and $\pi_2 = \bar{\pi} + c/T^{d-1/2}$ where $\bar{\pi}$ is the sample quantile corresponding to zero guarantees that the grid will be finite under the null hypothesis[1]. In practice, c can be chosen so as to give a reasonable coverage of each regime in samples of sizes usually encountered.

However, the pointwise convergence results obtained in Theorem 2 is not sufficient for establishing the uniform stochastic convergence of the asymptotic distribution of the average and the exponential average of the Wald statistic. In addition, we need to prove the stochastic equicontinuity of $\mathcal{W}^{(i)}_{(r_1,r_2)}$ over the set Γ of finite width. Stochastic equicontinuity as defined by Davidson (1994, p. 336, equation (21.43)) is the condition that for $\forall\epsilon$ there exists $\delta > 0$ such that

$$\operatorname{limsup}_{T\to\infty} \Pr\left[\sup_{\mathbf{r}\in\mathbf{\Gamma}} \sup_{\mathbf{r}'\in\mathbf{S}(\mathbf{r},\delta)} \left|\mathcal{W}^{(i)}_{\mathbf{r}} - \mathcal{W}^{(i)}_{\mathbf{r}'}\right| \geq \epsilon\right] < \epsilon, \tag{27}$$

where $\mathcal{W}^{(i)}_{\mathbf{r}}$ is the Wald statistic obtained from the i-th point of the threshold parameter grid, Γ, and $\mathbf{r}' = (r'_1, r'_2) \in S(\mathbf{r}, \delta)$ is a sphere of radius δ centered on $\mathbf{r} = (r_1, r_2)$. Under the assumption that the set Γ is of finite width, we are able to provide a proof of (27).

Theorem 3 *The test statistic $\mathcal{W}^{(i)}_{(r_1,r_2)}$ is stochastically equicontinuous over Γ.*

The stochastic equicontinuity condition (27) together with pointwise convergence of $\mathcal{W}^{(i)}_{(r_1,r_2)}$ to $\mathcal{W}^{(i)}_{(0,0)}$ established in Theorem 3 now establishes the uniform convergence of $\mathcal{W}^{\sup}_{(r_1,r_2)}$ and $\mathcal{W}^{\text{avg}}_{(r_1,r_2)}$ to $\mathcal{W}_{(0)}$ and of $\mathcal{W}^{\exp}_{(r_1,r_2)}$ to $\exp\left(\mathcal{W}_{(0)}/2\right)$.

[1]Further restrictions on the limits of the grid in the form of a minimum difference between the upper and lower bound may be placed to guarantee that the grid width does not tend to zero asymptotically, under the alternative hypothesis. For example the minimum width of the corridor regime around zero could be set to 4 times the standard deviation of a highly persistent AR(1) process, say with AR coefficient of 0.99, where the error variance could be estimated from a linear autoregression.

5 Extensions

5.1 Constants and Trends

Our approach can deal easily with constants and trends in the model. The simplest solution is to demean and detrend the data prior to applying the tests. It is easy to show that the distributions remain unchanged apart from the fact that the demeaned/detrended fractional Brownian motion, rather than the standard one appears in the asymptotic distributions. The demeaned/detrended fractional Brownian motions are denoted respectively by $\hat{Y}_d(r)$ and $\tilde{Y}_d(r)$ and defined as the continuous residual from a projection $Y_d(r)$ on 1 and $(1, r)$. We present critical values for the tests in Table 1 for the case of no detrending or demeaning, and Tables 2 and 3 for the demeaned and detrended case respectively.

Table 1: Test Critical Values (No constant/Trend) [a]

Sig. Level	Test	$d = 0.6$	$d = 0.7$	$d = 0.8$	$d = 0.9$
90%	$SETAR_1^{avg}$	2.662	2.812	2.723	2.733
	$SETAR_2^{avg}$	4.697	4.836	4.760	4.787
	$SETAR_3^{avg}$	6.931	7.247	7.120	7.370
	$STAR_1$	3.016	3.426	3.422	3.747
	$STAR_2$	5.635	6.159	6.370	6.324
95%	$SETAR_1^{avg}$	3.780	4.028	3.846	3.902
	$SETAR_2^{avg}$	6.004	6.225	6.343	6.063
	$SETAR_3^{avg}$	8.475	8.723	8.742	9.209
	$STAR_1$	4.168	4.635	4.633	4.967
	$STAR_2$	7.123	7.670	7.935	8.022
99%	$SETAR_1^{avg}$	5.438	5.683	5.505	5.560
	$SETAR_2^{avg}$	7.776	7.877	8.085	8.186
	$SETAR_3^{avg}$	10.594	10.792	10.919	11.170
	$STAR_1$	5.731	6.285	6.149	6.976
	$STAR_2$	9.324	9.777	9.651	10.161

[a] For $SETAR_i^{exp}$ the critical values are obtained by taking $exp(cv/2)$ as the critical value where cv is the critical value of the respective avg test.

5.2 Serial Correlation

We now extend our analysis to consider serial correlation. As explained in Section 2, we assume an ARFI model with a possibly infinite AR component.

Table 2: Test Critical Values (Demeaned) [a]

Sig. Level	Test	$d = 0.6$	$d = 0.7$	$d = 0.8$	$d = 0.9$
90%	$SETAR_1^{avg}$	2.743	2.734	2.838	2.809
	$SETAR_2^{avg}$	7.343	7.400	7.647	7.667
	$SETAR_3^{avg}$	8.311	8.796	8.766	8.767
	$STAR_1$	5.171	5.931	6.562	6.721
	$STAR_2$	6.820	7.330	7.781	7.964
95%	$SETAR_1^{avg}$	3.787	3.817	4.084	3.951
	$SETAR_2^{avg}$	8.965	9.160	9.249	9.556
	$SETAR_3^{avg}$	9.978	10.357	10.545	10.566
	$STAR_1$	6.646	7.389	8.048	8.306
	$STAR_2$	8.567	8.953	9.433	9.582
99%	$SETAR_1^{avg}$	5.255	5.401	5.531	5.481
	$SETAR_2^{avg}$	10.986	11.317	11.730	11.370
	$SETAR_3^{avg}$	12.441	12.441	12.887	12.480
	$STAR_1$	8.675	9.356	9.624	10.064
	$STAR_2$	10.406	10.950	11.372	11.669

[a] For $SETAR_i^{exp}$ the critical values are obtained by taking $exp(cv/2)$ as the critical value where cv is the critical value of the respective avg test.

We extend the models presented before to get

$$u_t = \alpha_1(y_{t-1}^3 + z_t) + \sum_{i=1}^{p} \phi_i u_{t-i} + \epsilon_t \tag{28}$$

$$u_t = \alpha_1 y_{t-1}^3 + \alpha_2 z_t + \sum_{i=1}^{p} \phi_i u_{t-i} + \epsilon_t \tag{29}$$

$$u_t = \alpha_1(y_{t-1} I(|y_{t-1}| > r) + z_t) + \sum_{i=1}^{p} \phi_i u_{t-i} + \epsilon_t \tag{30}$$

$$u_t = \alpha_1 y_{t-1} I(|y_{t-1}| > r) + \alpha_2 z_t + \sum_{i=1}^{p} \phi_i u_{t-i} + \epsilon_t \tag{31}$$

$$u_t = \alpha_1 y_{t-1} I(y_{t-1} < r_1) + \alpha_2 y_{t-1} I(y_{t-1} \geq r_2) + \alpha_3 z_t + \sum_{i=1}^{p} \phi_i u_{t-i} + \epsilon_t \tag{32}$$

Intuitively, the addition of the extra stationary regressors does not alter the asymptotic distributions of the test statistics since the tests depend on the coefficients of nonstationary variables. The following theorem formalises this intuition.

Table 3: Test Critical Values (Detrended) [a]

Sig. Level	Test	$d = 0.6$	$d = 0.7$	$d = 0.8$	$d = 0.9$
90%	$SETAR_1^{avg}$	2.704	2.643	2.800	2.709
	$SETAR_2^{avg}$	9.220	9.595	9.979	10.337
	$SETAR_3^{avg}$	9.877	10.251	10.562	10.913
	$STAR_1$	5.939	7.105	8.244	8.850
	$STAR_2$	8.377	9.002	9.735	10.225
95%	$SETAR_1^{avg}$	3.722	3.691	3.964	3.869
	$SETAR_2^{avg}$	10.919	11.437	11.831	12.094
	$SETAR_3^{avg}$	11.739	12.118	12.578	12.586
	$STAR_1$	7.261	8.781	10.004	10.487
	$STAR_2$	10.284	10.850	11.545	12.117
99%	$SETAR_1^{avg}$	5.419	5.381	5.626	5.423
	$SETAR_2^{avg}$	13.028	13.691	14.154	14.477
	$SETAR_3^{avg}$	13.874	14.474	15.056	15.037
	$STAR_1$	9.488	10.837	11.879	12.628
	$STAR_2$	12.443	13.055	13.659	14.331

[a] For $SETAR_i^{exp}$ the critical values are obtained by taking $exp(cv/2)$ as the critical value where cv is the critical value of the respective avg test.

Theorem 4 *The asymptotic distributions presented in Theorems 1 to 4 do not change if the Wald statistics are obtained from equations (30)-(28) rather than equations (20)-(13)*

6 Monte Carlo Study

We carry out a detailed Monte Carlo study of the new tests. For all Tables presented the first experiment is a size experiment where the null hypothesis tested is for a long memory model with the relevant d. We consider four distinct values of $d = 0.6, 0.7, 0.8, 0.9$. These are experiments A (for d=0.6), B (for d=0.7), C (for d=0.8) and D (for d=0.8) respectively. For the symmetric SETAR models the tests presented are for regressions (20) and (21). The following experiments are carried out for the SETAR models:

- Exp 2/8/14: $\beta_1 = 0.95$, $\alpha = 1$, $\beta_2 = 0.95$
- Exp 3/9/15: $\beta_1 = 0.9$, $\alpha = 1$, $\beta_2 = 0.9$
- Exp 4/10/16: $\beta_1 = 0.85$, $\alpha = 1$, $\beta_2 = 0.95$
- Exp 5/11/17: $\beta_1 = 0.95$, $\alpha = 1.2$, $\beta_2 = 0.95$

- Exp 6/12/18: $\beta_1 = 0.9$, $\alpha = 1.2$, $\beta_2 = 0.9$
- Exp 7/13/19: $\beta_1 = 0.95$, $\alpha = 1.2$, $\beta_2 = 0.85$

We have three experiments for each coefficient specification. For the first of the three experiments $r_1 = -0.15, \quad r_2 = 0.15$, for the second $r_1 = -1.65, \quad r_2 = 1.65$ and for the third $r_1 = -3.15, \quad r_2 = 3.15$ The following experiments are carried out for the STAR models:

- Exp 2: $\alpha_1 = 1$, $\gamma = -0.01$, $\theta = 0.01$
- Exp 3: $\alpha_1 = 1$, $\gamma = -0.01$, $\theta = 0.05$
- Exp 4: $\alpha_1 = 1$, $\gamma = -0.01$, $\theta = 0.1$
- Exp 5: $\alpha_1 = 1$, $\gamma = -0.05$, $\theta = 0.01$
- Exp 6: $\alpha_1 = 1$, $\gamma = -0.05$, $\theta = 0.05$
- Exp 7: $\alpha_1 = 1$, $\gamma = -0.05$, $\theta = 0.1$
- Exp 8: $\alpha_1 = 1$, $\gamma = -0.1$, $\theta = 0.01$
- Exp 9: $\alpha_1 = 1$, $\gamma = -0.1$, $\theta = 0.05$
- Exp 10: $\alpha_1 = 1$, $\gamma = -0.1$, $\theta = 0.1$
- Exp 11: $\alpha_1 = 1.3$, $\gamma = -0.4$, $\theta = 0.05$
- Exp 12: $\alpha_1 = 1.3$, $\gamma = -0.4$, $\theta = 0.25$
- Exp 13: $\alpha_1 = 1.5$, $\gamma = -0.6$, $\theta = 0.05$
- Exp 14: $\alpha_1 = 1.5$, $\gamma = -0.6$, $\theta = 0.25$

The variance of the error term is always set to 1. For generating the long memory sample we use the inifinite AR representation of a long memory process truncating at 200 lags. We use 1000 replications for each experiment. The significance level is set to 5%.

The size experiments indicate that the tests are correctly sized with a few cases of underrejection observed. The only exception is the SETAR exp test which overrejects slightly in a number of cases. Going on to discuss the power properties of the test we reach a number of conclusions. Firstly, SETAR mean tests are considerably less powerful than SETAR exp tests as expected both from theory and from previous empirical work in the area of testing against nonlinearity (see Kapetanios and Shin (2003)). Secondly, symmetric tests are much more powerful than asymmetric tests when the true DGP is a symmetric SETAR models. Asymmetric tests are slighlty more powerful when the DGP is asymmetric. The power function of the tests does not seem to be monotonic with respect to the assumed d parameter.So for $d = 0.6$ tests are reasonably powerful. They become less powerful for $d = 0.7$ bur regain power for higher d. This non-monotonicity is much more apparent for STAR models where the tests are powerful for $d = 0.6$, lose most of their power for $d = 0.7, 0.8$ and regain it for $d = 0.9$. This indicates the possible similarity in small samples of long memory processes with $d = 0.7, 0.8$ and nonlinear highly persistent but stationary processes.

7 Empirical Application: Long Memory of Real Exchange Rates

In this section we apply the new tests to investigate the properties of the Yen real exchange rate. Our choice of data set reflects previous work in this area by Cheung and Lai (2001) who investigated the presence of long memory in Yen real exchange rates aiming to explain the puzzle of the inability to reject the null hypothesis of unit root nonstationarity using standard unit root tests.

We construct bilateral real exchange rates against the *i-th* currency at time t ($q_{i,t}$) as $q_{i,t} = s_{i,t} + p_{J,t} - p^*_{i,t}$, where $s_{i,t}$ is the corresponding nominal exchange rate (*i-th* currency per numeraire currency), $p_{J,t}$ the price level in the home country, and $p^*_{i,t}$ the price level of the *i-th* country. Thus, a rise in $q_{i,t}$ implies a real appreciation against the *i-th* currency. The price levels are consumer price indices. All variables are in logs. All data are from the International Monetary Fund's *International Financial Statistics* in CD-ROM. The data are not seasonally adjusted. All data are quarterly, spanning from 1960Q1 to 2000Q4 and the bilateral nominal exchange rates against the currencies other than the US dollar are cross-rates computed using the US dollar rates. We consider a very large sample of countries in an attempt to make the empirical analysis more comprehensive. We also consider a grid of values for d since d needs to be specified under the null hypothesis. Reviewing the tests for different values of d clearly provides a more comprehensive picture of the comparison between long memory and nonlinear models for these series. We use $d = 0.6, 0.7, 0.8, 0.9$. Results are presented for the STAR based tests in Tables 24-25 and for the exponential SETAR based tests in Tables 26-27. Tables 24 and 26 present results for tests with no augmentations to take into account possible serial correlation, whereas Tables 25 and 27 present results for tests with 4 lags to take into account serial correlation in the series. For the case of the SETAR test we present results only for the more general asymmetric test $SETAR_3^d$.

Results make very interesting reading. In general the higher the value of d under the null hypothesis the fewer series reject the null hypothesis for the STAR tests, indicating that there is plenty of evidence against long memory for low values of d but that evidence is reduced for values of d which bring the series close to a unit root. This result indicates the limitations of the test as it cannot distinguish between long memory and nonlinearity for all d. We do not use a previously estimated value of d to test against although that would be the obvious thing to do because the test has been constructed for a given fixed d. The asymptotic distributions have been derived under this assumption and would not be valid in the case of an estimated d.

On the other hand the SETAR tests provide more robust evidence. The test reject the null hypothesis of long memory for more series overall. Further the number of series for which the null hypothesis is rejected does not fall as the assumed value of d rises. This can be construed to provide more robust evidence against long-memory in the direction of SETAR nonlinearity for the Yen real exchange rates.

8 Conclusion

Recently there has been increased focus on the interaction of nonstationarity and nonlinearity as alternative data representations. The motivation for this development maybe be traced to the perceived possibility that processes following nonlinear models maybe mistakenly taken to be unit root nonstationary. In this paper we have extended the investigation of this interplay to nonstationary long memory processes. We have suggested tests that can distinguish effectively between nonstationary long memory processes and stationary nonlinear processes. In the processe we have observed that a number of long memory processes are close to nonlinear stationary ones in small samples in the sense that the proposed tests which can distinguish between the two classes of processes in most cases cannot do so in these particular instances. This finding mirrors the work of Diebold and Inoue (2001) who find similarities between long memory and Markov switching processes and conjecture that such similarities may exist for other nonlinear models such as those investigated in this paper. Further research is needed to evaluate the extend of this similarity.

Appendix

Proof of Theorem 1

From the fractional functional central theorem and the standard functional central limit theorem, we know that

$$\sigma_T^{-1} y_{[Tr]} \Rightarrow Y_d(r) \tag{A.1}$$

and

$$T^{-1/2}\sigma^{-1}\sum_{t=1}^{[Tr]} u_t \Rightarrow B(r) \tag{A.2}$$

We then examine the asymptotic behaviour of z_t. z_t is given by $\sum_{i=0}^{t} b_i y_{t-i}$. Define the function $b(r) = b_{[Tr]}$ for $r \in (0,1)$. Define its cumulative sum as $\beta(r) = \int_0^r b(s)ds$. Define also $\beta_t = \sum_{i=0}^{t} b_i$. Then it can be easily seen that

$$\sigma_T^{-1} z_{[Tr]} = \sum_{i=0}^{t} \sigma_T^{-1} y_{t-i}(\beta_{t-i} - \beta_{t-i-1}) \Rightarrow Z_d(r) \equiv \int_0^r Y_d(s) d\beta(s-r) \tag{A.3}$$

By the continuous mapping theorem we have that

$$\sigma_T^{-3} y_{[Tr]}^3 \Rightarrow Y_d(r)^3, \quad \text{and } \sigma_T^{-3}(y_{[Tr]}^3 + z_{[Tr]}) \Rightarrow Y_d^3(r) \equiv Q_{1,d}(r) \tag{A.4}$$

Note how the term $y_{[Tr]}^3$ dominates the term $z_{[Tr]}$. The Wald test for the null hypothesis $\alpha_1 = 0$ in (13) is given by

$$\hat{\sigma}^{-2}\left(\sum_{t=1}^{T} u_t(y_{t-1}^3 + z_t)\right)^2 \Big/ \left(\sum_{t=1}^{T}(y_{t-1}^3 + z_t)^2\right) \tag{A.5}$$

By the continuous mapping theorem we have that

$$T^{-1}\sigma_T^{-6}\sum_{t=1}^{T}(y_{t-1}^3 + z_t)^2 \Rightarrow \int Q_{1,d}^2(r)dr \tag{A.6}$$

For the numerator of (A.5) we will make use of Theorem 2.2 of Kurtz and Protter (1992). As this Theorem will be used repeatedly we comment on it. This theorem states that for processes $\{X_t\}^T \equiv X_T$ and $\{Y_t\}^T \equiv Y_T$ if

(C1) X_T, Y_T are $\mathcal{F}_t$-adapted for some σ-field $\mathcal{F}_t$,

(C2) $(X_T, Y_T) \Rightarrow (X, Y)$ and

(C3) Y_T is a semimartingale then

$$\int X_T dY_T \Rightarrow \int X dY$$

We will be verifying these conditions repeatedly in the appendix. Continuity of the power function implies (C1), (C2) has been shown above and since $\sigma^{-1}1/\sqrt{T}\sum_{t=1}^{[Tr]} u_t$ is clearly a semimartingale (C3) is satisfied and therefore use of this theorem is justified to derive the asymptotic distribution of the stochastic integral involved in the numerator of (A.5) which is given below

$$T^{-1/2}\sigma_T^{-3}\sum_{t=1}^{T} u_t(y_{t-1}^3 + z_t) \Rightarrow \sigma \int Q_{1,d}(r)dB(r) \tag{A.7}$$

The above also establish consistency of $\hat{\alpha}$ and thereby consistency of $\hat{\sigma}^2$. Thus, we get the asymptotic distribution as stated in (18). The derivation of the asymptotic distribution in (19) follows easily from the above if we note that the Wald test for the null hypothesis of $\alpha_1 = \alpha_2 = 0$ in (14) is given by

$$\hat{\sigma}^{-2}\left(\mathbf{u}'\mathbf{z}_1\right)\left(\mathbf{z}_1'\mathbf{z}_1\right)^{-1}\left(\mathbf{z}_1'\mathbf{u}\right) \tag{A.8}$$

where $\mathbf{u} = (u_1, \ldots, u_T)'$, $\mathbf{z}_1 = [(y_0^3, z_1)', \ldots, (y_{t-1}^3, z_t)']'$, and define

$$Q_{2,d}(r) \equiv (Y_d^3(r), Z_d(r)) \tag{A.9}$$

Note that the terms $Y_d^3(r)$ and $Z_d(r)$ have different rates of convergence.

Proof of Theorem 2

We first consider the test based on (20). We start by establishing that

$$\sigma_T^{-1} y_{t-1} I(|y_{t-1}| > r) \Rightarrow Y_d \tag{A.10}$$

for any finite r. This follows from the fact that

$$\sigma_T^{-1} y_{t-1} I(|y_{t-1}| > r) = \sigma_T^{-1} y_{t-1} I(\sigma_T^{-1}|y_{t-1}| > \sigma_T^{-1} r) \tag{A.11}$$

and so

$$\sigma_T^{-1} y_{t-1} I(|y_{t-1}| > r) - \sigma_T^{-1} y_{t-1} I(|y_{t-1}| > 0) = o_p(1) \tag{A.12}$$

which follows from the fact that $y_{t-1} = O_p(T^{d-1/2})$. But

$$\sigma_T^{-1} y_{t-1} I(|y_{t-1}| > 0) = \sigma_T^{-1} y_{t-1} \tag{A.13}$$

Also

$$\sigma_T^{-1}(y_{t-1} I(|y_{t-1}| > r) + z_t) \Rightarrow Y_d + Z_d \equiv Q_{3,d} \tag{A.14}$$

Define

$$x_{1,r,t} = y_{t-1} I(|y_{t-1}| > r) + z_t, \quad \mathbf{X}_{1,r} = (x_{1,r,1}, \ldots, x_{1,r,T})' \tag{A.15}$$

and $\mathbf{u} = (u_1, \ldots, u_T)'$. Then the Wald test for given r is given by

$$W_{(r)} = \hat{\sigma}^2 \mathbf{u}' \mathbf{X}_{1,r} (\mathbf{X}_{1,r}' \mathbf{X}_{1,r})^{-1} \mathbf{X}_{1,r}' \mathbf{u} \tag{A.16}$$

As discussed above the test statistic exhibits invariance in probability with respect to r. We therefore give the probability distribution for $r = 0$. In this case $y_{t-1} I(|y_{t-1}| > 0) = y_{t-1}$. Using Theorem 2.2 of Kurtz and Protter (1992) on convergence of stochastic integrals, discussed in the previous appendix, we see that conditions C1-C3 are easily established and so the asymptotic distribution of the above is

$$\frac{\left(\int_0^1 Q_{3,d}(r) dB(r)\right)^2}{\int_0^1 Q_{3,d}^2 dr} \tag{A.17}$$

where the continuous mapping theorem has also been used for the convergence of the term $(\mathbf{X}_{1,r}' \mathbf{X}_{1,r})^{-1}$. The above also establish consistency of $\hat{\alpha}_1$ and therefore of $\hat{\sigma}^2$.

We move to consider the test based on model (21). Define

$$\mathbf{x}_{2,r,t} = (y_{t-1} I(|y_{t-1}| > r), z_t), \quad \mathbf{X}_{2,r} = (\mathbf{x}_{2,r,1}, \ldots, \mathbf{x}_{2,r,T})' \tag{A.18}$$

Then, the Wald test for given r is given by

$$W_{(r)} = \hat{\sigma}^{-2} \mathbf{u}' \mathbf{X}_{2,r} \left(\mathbf{X}_{2,r}' \mathbf{X}_{2,r}\right)^{-1} \mathbf{X}_{2,r}' \mathbf{u} \tag{A.19}$$

as usual. By the results obtained above

$$\sigma_T^{-1} T^{-1/2} \mathbf{X}_{2,r}' \mathbf{u} \Rightarrow \sigma \int_0^1 (Y_d(r), Z_d(r))' dB(r) \tag{A.20}$$

$$\sigma_T^{-2} T^{-1} (\mathbf{X}_{2,r}' \mathbf{X}_{2,r}) \xrightarrow{d} \int_0^1 (Y_d(r), Z_d(r))(Y_d(r), Z_d(r))' dr \tag{A.21}$$

This gives the required asymptotic distribution.

We now consider the test based on model (22). Define

$$\mathbf{x}_{3,\boldsymbol{r},t} = (y_{t-1}I(y_{t-1} < r_1), y_{t-1}I(y_{t-1} \geq r_2), z_t), \ \mathbf{r} = (r_1, r_2)', \ \mathbf{X}_{3,\boldsymbol{r}} = (\mathbf{x}_{3,\boldsymbol{r},1}, \ldots, \mathbf{x}_{3,\boldsymbol{r},T})' \tag{A.22}$$

Then, the Wald test for given $\boldsymbol{r} = (r_1, r_2)$ is given by

$$W_{(r_1,r_2)} = \hat{\sigma}^{-2}\mathbf{u}'\mathbf{X}_{3,\boldsymbol{r}}\left(\mathbf{X}_{3,\boldsymbol{r}}'\mathbf{X}_{3,\boldsymbol{r}}\right)^{-1}\mathbf{X}_{3,\boldsymbol{r}}'\mathbf{u} \tag{A.23}$$

Now, by (A.11),

$$\sigma_T^{-1}y_{t-1}I(y_{t-1} < r_1) - \sigma_T^{-1}y_{t-1}I(y_{t-1} < 0) = o_p(1) \tag{A.24}$$

$$\sigma_T^{-1}y_{t-1}I(y_{t-1} \geq r_2) - \sigma_T^{-1}y_{t-1}I(y_{t-1} \geq 0) = o_p(1) \tag{A.25}$$

So we concentrate on the $\sigma_T^{-1}y_{t-1}I(y_{t-1} < 0)$ and $\sigma_T^{-1}y_{t-1}I(y_{t-1} \geq 0)$. We note that $xI(x < 0)$ and $xI(x > 0)$ are continuous functions of x unlike $I(x < 0)$ and $I(x \geq 0)$. Then

$$\sigma_T^{-1}y_{t-1}I(y_{t-1} < 0) \Rightarrow Y_dI(Y_d < 0) \tag{A.26}$$

and

$$\sigma_T^{-1}y_{t-1}I(y_{t-1} \geq 0) \Rightarrow Y_dI(Y_d \geq 0) \tag{A.27}$$

by the continuous mapping theorem. Since also

$$\sigma_T^{-1}z_{[Tr]} \Rightarrow Z_d$$

we get that

$$\sigma_T^{-1}\mathbf{X}_{3,\boldsymbol{r}} \Rightarrow (Y_dI(Y_d < 0), Y_dI(Y_d > 0), Z_d)' \equiv Q_{5,r} \tag{A.28}$$

Using again Theorem 2.2 of Kurtz and Protter (1992) where by continuity of the functions involved conditions C1-C3 are satisfied we have that the asymptotic distribution is

$$\left(\int_0^1 Q_{5,d}(r)dB(r)\right)'\left(\int_0^1 Q_{5,d}(r)Q_{5,d}(r)'dr\right)^{-1}\left(\int_0^1 Q_{5,d}(r)dB(r)\right) \tag{A.29}$$

Proof of Theorem 3

We only consider the stochastic equicontinuity of $T^{-1}\sum_{t=1}^{T} I(y_{t-1} > r)y_{t-1}u_t$ because similar arguments can be applied to other terms. We assume that $r \in [-M, M]$ for some constant M. Following the definition of (weak) stochastic equicontinuity in (27), we have to prove that

$$\operatorname{limsup}_{T\to\infty} \Pr\left[\sup_{r} \sup_{r'\in S(r,\delta)} \left| \frac{1}{T}\sum_{t=1}^{T} I(y_{t-1} > r)y_{t-1}u_t - \frac{1}{T}\sum_{t=1}^{T} I(y_{t-1} > r')y_{t-1}u_t \right| \geq \epsilon \right] < \epsilon, \tag{A.30}$$

where $S(r, \delta)$ is a sphere of radius δ centred at r. Assuming without loss of generality that $r' < r$, then the probability in (A.30) can be written as

$$\operatorname{limsup}_{T\to\infty} \Pr\left[\sup_{r} \sup_{r'\in S(r,\delta)} \left| \frac{1}{T}\sum_{t=1}^{T} I(r' \leq y_{t-1} \leq r)y_{t-1}u_t \right| \geq \epsilon \right] \tag{A.31}$$

$$\leq \operatorname{limsup}_{T\to\infty} \Pr\left[\sup_{r} \sup_{r'\in S(r,\delta)} \frac{1}{T}\sum_{t=1}^{T} |I(r' \leq y_{t-1} \leq r)u_t|\,|y_{t-1}| \geq \epsilon \right] \tag{A.32}$$

By the properties of long memory processes, $I(r' \leq y_{t-1} \leq r)$ will take unity at most $[cT^{\alpha}]$ periods, for some $\alpha < 1$ and for some fixed constant c, where $[.]$ denotes integer part, and zero otherwise. Therefore, only $[cT^{\alpha}]$ terms in the summation in (A.31) are non-zero. In the cases where these terms are non zero, $|y_{t-1}|$ can be at most M. Taking the supremum over r and r' inside the summation in (A.32), it is easily seen that (A.32) holds if

$$\operatorname{limsup}_{T\to\infty} \Pr\left[\frac{M}{T}\sum_{i=1}^{[cT^{\alpha}]} |u_{t_i}| \geq \epsilon \right] < \epsilon, \tag{A.33}$$

where t_i denotes the subsequence of periods when the process lies within the finite corridor band. This is smaller than

$$\operatorname{limsup}_{T\to\infty} \Pr\left[\frac{M}{T}\sum_{i=1}^{[cT^{\alpha}]} \{|u_{t_i}| - E(|u_{t_i}|)\} + \frac{M}{T}\sum_{i=1}^{[cT^{\alpha}]} E(|u_{t_i}|) \geq \epsilon \right]. \tag{A.34}$$

By the finiteness of the second moment of u_t, $\frac{M}{T}\sum_{i=1}^{[cT^{\alpha}]} E(|u_{t_i}|)$ tends to zero. Hence, we concentrate on

$$\operatorname{limsup}_{T\to\infty} \Pr\left[\frac{M}{T}\sum_{i=1}^{[cT^{\alpha}]} \{|u_{t_i}| - E(|u_{t_i}|)\} \geq \epsilon \right]. \tag{A.35}$$

But by the law of large numbers, and using the assumption that u_t's are *iid*, we have

$$\text{limsup}_{T\to\infty} \Pr\left[\frac{\sum_{i=1}^{[cT^{\alpha}]}\left\{|u_{t_i}| - E\left(|u_{t_i}|\right)\right\}}{cT^{\alpha}} \geq \epsilon\right] = 0, \tag{A.36}$$

As the normalisation M/T in (A.35) is smaller than the normalisation $1/T^{\alpha}$ needed for (A.36) to hold, (A.35) holds, which proves (A.30).

A similar analysis provides a proof for stochastic equicontinuity of

$$T^{-1}\sum_{t=1}^{T} I(y_{t-1} > r)y_{t-1}^2$$

Given that $T^{-1}\sum_{t=1}^{T} I(y_{t-1} > r)y_{t-1}^2$ is almost surely bounded away from zero for all finite r, stochastic equicontinuity of

$$\frac{\left(T^{-1}\sum_{t=1}^{T} I(y_{t-1} > r)y_{t-1}u_t\right)^2}{T^{-1}\sum_{t=1}^{T} I(y_{t-1} > r)y_{t-1}^2}$$

is obtained.

Proof of Theorem 4

We will prove this Theorem for the most general case of an infinite order ARFI with the truncated lag order $p = p(T) = o_p(T^{1/3})$. Before continuing we briefly comment on the truncation order $T^{1/3}$. This upper bound follows from the work of Berk (1974) where it is shown that for inifinite stationary AR processes any lag order larger than this leads to a second moment matrix for the regressors (lags of the process) which does not converge to its population moment in norm. Under such a scenario any coefficients of stationary variables would not be estimated consistently. See also Ng and Perron (1995). We therefore impose this upper bound. We also assume that ϕ_i in (28)-(32) are $O_p(\lambda^i)$, $|\lambda| < 1$ and that $p \to \infty$.

Define the $T\times p$ data matrix $\mathbf{U_p} = (\mathbf{u_{-1}}, ..., \mathbf{u_{-p}})$ with $\mathbf{u_{-i}} = (\mathbf{u_{-i+1}}, ..., \mathbf{u_{T-i}})$, the $T \times T$ idempotent matrix $\mathbf{M}_T = \mathbf{I}_T - \mathbf{U_p}\left(\mathbf{U_p}'\mathbf{U_p}\right)^{-1}\mathbf{U_p}'$ and $\epsilon = (\epsilon_1, \ldots, \epsilon_T)'$.

We first prove a preliminary result. Consider the quadratic form

$$\mathbf{a}'\frac{\mathbf{U}_p}{\sqrt{T}}\left(\frac{\mathbf{U}_p'\mathbf{U}_p}{T}\right)^{-1}\frac{\mathbf{U}_p'}{\sqrt{T}}\mathbf{b} = \mathbf{a}^{*\prime}\left(\frac{\mathbf{U}_p'\mathbf{U}_p}{T}\right)^{-1}\mathbf{b}^* \tag{A.37}$$

where $\mathbf{a}$ and $\mathbf{b}$ are $T \times 1$ vectors of observations on processes a_t and b_t respectively. Suppose in addition that $\|\mathbf{a}^*\|$ and $\|\mathbf{b}^*\|$ are both $O_p(p(T))$. Write $\frac{\mathbf{U}_p'\mathbf{U}_p}{T}$ as $\mathbf{V}'\mathbf{\Lambda}\mathbf{V}$ where $\mathbf{V}'\mathbf{V} = \mathbf{V}\mathbf{V}' = \mathbf{I}$ and $\mathbf{\Lambda} = diag(\lambda_i)$. Writing $\mathbf{U}_p^* = \mathbf{V}'\mathbf{U}_p^*$ then we see that $\mathbf{U}_p^*$ is a matrix of observations on $p(T)$ orthogonal stationary variates with sample variances $\lambda_1, \lambda_2, ..\lambda_p$ respectively. As a result, the λ_i are each $O_p(1)$. Now write $\mathbf{a}^{**} = \mathbf{V}\mathbf{a}^*$ and $\mathbf{b}^{**} = \mathbf{V}\mathbf{b}^*$ then it follows that

$$\|\mathbf{a}^*\| = \|\mathbf{a}^{**}\| = \|\mathbf{b}^*\| = \|\mathbf{b}^{**}\| = O_p(p(T))$$

It follows from these developments that

$$\mathbf{a}'\frac{\mathbf{U}_p}{\sqrt{T}}\left(\frac{\mathbf{U}_p'\mathbf{U}_p}{T}\right)^{-1}\frac{\mathbf{U}_p'}{\sqrt{T}}\mathbf{b} = \mathbf{a}^{*\prime}\left(\frac{\mathbf{U}_p'\mathbf{U}_p}{T}\right)^{-1}\mathbf{b}^* = \mathbf{a}^{**\prime}diag\left(\lambda_i^{-1}\right)\mathbf{b}^{**} = \sum_{i=1}^{p(T)}\lambda_i^{-1}a_i^{**}b_i^{**} = O_p(p(T)) \tag{A.38}$$

Then, under the null it is straightforward to show that for all relevant terms in the Wald test statistics for both SETAR and ESTAR models

$$\sigma_T^{-\alpha}T^{-1/2}\mathbf{X}'\mathbf{M}_T\epsilon - \sigma_T^{-\alpha}T^{-1/2}\mathbf{X}'\epsilon = o_p(1) \tag{A.39}$$

and

$$\sigma_T^{-2\alpha}T^{-1}\mathbf{X}'\mathbf{M}_T\mathbf{X} - \sigma_T^{-2\alpha}T^{-1}\mathbf{X}'\mathbf{X} = o_p(1) \tag{A.40}$$

where $\mathbf{X}$ is the relevant regressor matrix in each case, $\alpha = 3$ if the regressor is y_{t-1}^3 and $\alpha = 1$, otherwise. To see this we need to note that since u_t is a stationary process we have, using (A.38)

$$\sigma_T^{-\alpha}\mathbf{X}'\mathbf{U_p}\left(\mathbf{U_p}'\mathbf{U_p}\right)^{-1}\mathbf{U_p}'\epsilon = O_p(p(T)) = o_p(T^{1/3}) \tag{A.41}$$

and

$$\sigma_T^{-2\alpha}\mathbf{X}'\mathbf{U_p}\left(\mathbf{U_p}'\mathbf{U_p}\right)^{-1}\mathbf{U_p}'\mathbf{X} = O_p(p(T)) = o_p(T^{1/3}) \tag{A.42}$$

giving the required result. Finally consistency of $\hat{\sigma}^2$ follows from the fact that for $p = o_p(T^{1/3})$, the second moment matrix $\mathbf{U_p}'\mathbf{U_p}$ converges in the supremum matrix norm to its population value as proven by Berk (1974).

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Table 4: Symmetric SETAR mean tests. d=0.6

Exp	$SETAR_1^{avg,0.6}$			$SETAR_2^{avg,0.6}$		
	100	200	400	100	200	400
A1	0.040	0.036	0.034	0.057	0.054	0.041
A2	0.666	0.811	0.968	0.675	0.875	0.992
A3	0.586	0.837	0.984	0.507	0.790	0.977
A4	0.326	0.452	0.719	0.325	0.549	0.878
A5	0.668	0.815	0.953	0.691	0.870	0.990
A6	0.606	0.828	0.987	0.529	0.770	0.982
A7	0.333	0.445	0.729	0.342	0.570	0.886
A8	0.637	0.809	0.961	0.666	0.876	0.990
A9	0.554	0.785	0.956	0.493	0.756	0.948
A10	0.328	0.424	0.655	0.352	0.532	0.868
A11	0.583	0.750	0.917	0.621	0.838	0.983
A12	0.490	0.662	0.865	0.425	0.641	0.902
A13	0.220	0.268	0.359	0.290	0.454	0.751
A14	0.642	0.807	0.939	0.673	0.873	0.994
A15	0.613	0.804	0.955	0.550	0.803	0.976
A16	0.438	0.555	0.751	0.463	0.687	0.935
A17	0.428	0.621	0.714	0.459	0.710	0.900
A18	0.487	0.707	0.840	0.448	0.692	0.843
A19	0.341	0.401	0.411	0.338	0.545	0.693

Table 5: Symmetric SETAR exp tests. d=0.6

Exp	$SETAR_1^{exp,0.6}$			$SETAR_2^{exp,0.6}$		
	100	200	400	100	200	400
A1	0.068	0.066	0.056	0.074	0.074	0.058
A2	0.783	0.915	0.992	0.771	0.929	0.998
A3	0.740	0.948	0.998	0.678	0.916	0.997
A4	0.514	0.679	0.911	0.494	0.707	0.943
A5	0.777	0.916	0.991	0.773	0.937	0.996
A6	0.755	0.924	0.998	0.694	0.906	0.997
A7	0.516	0.706	0.911	0.487	0.745	0.956
A8	0.757	0.900	0.992	0.756	0.924	0.998
A9	0.692	0.919	0.993	0.664	0.898	0.989
A10	0.522	0.654	0.871	0.505	0.700	0.941
A11	0.700	0.854	0.977	0.712	0.895	0.993
A12	0.647	0.827	0.942	0.601	0.797	0.953
A13	0.403	0.489	0.624	0.424	0.615	0.825
A14	0.749	0.892	0.982	0.755	0.913	0.998
A15	0.756	0.912	0.987	0.701	0.893	0.988
A16	0.595	0.751	0.906	0.603	0.796	0.972
A17	0.493	0.691	0.790	0.504	0.759	0.918
A18	0.591	0.786	0.905	0.558	0.772	0.896
A19	0.444	0.524	0.548	0.439	0.617	0.749

Table 6: Symmetric SETAR mean tests. d=0.7

Exp	$SETAR_1^{avg,0.7}$			$SETAR_2^{avg,0.7}$		
	100	200	400	100	200	400
B1	0.026	0.020	0.021	0.049	0.037	0.035
B2	0.329	0.514	0.740	0.277	0.432	0.686
B3	0.196	0.364	0.720	0.157	0.328	0.655
B4	0.059	0.109	0.243	0.045	0.070	0.139
B5	0.350	0.529	0.726	0.302	0.456	0.692
B6	0.219	0.390	0.720	0.198	0.332	0.640
B7	0.077	0.105	0.253	0.063	0.060	0.161
B8	0.324	0.493	0.701	0.281	0.444	0.664
B9	0.168	0.270	0.602	0.144	0.219	0.519
B10	0.062	0.073	0.136	0.048	0.065	0.089
B11	0.273	0.393	0.543	0.227	0.344	0.521
B12	0.117	0.217	0.400	0.101	0.168	0.330
B13	0.038	0.021	0.023	0.036	0.029	0.022
B14	0.389	0.467	0.679	0.327	0.427	0.639
B15	0.208	0.350	0.652	0.142	0.249	0.514
B16	0.098	0.172	0.237	0.080	0.123	0.154
B17	0.265	0.347	0.491	0.210	0.249	0.420
B18	0.180	0.388	0.758	0.109	0.192	0.533
B19	0.086	0.140	0.271	0.053	0.067	0.139

Table 7: Symmetric SETAR exp tests. d=0.7

Exp	$SETAR_1^{exp,0.7}$			$SETAR_2^{exp,0.7}$		
	100	200	400	100	200	400
B1	0.047	0.035	0.044	0.068	0.058	0.060
B2	0.467	0.675	0.881	0.428	0.648	0.857
B3	0.337	0.561	0.888	0.334	0.573	0.869
B4	0.159	0.258	0.457	0.147	0.216	0.369
B5	0.508	0.689	0.893	0.468	0.656	0.852
B6	0.332	0.574	0.874	0.359	0.598	0.871
B7	0.185	0.260	0.498	0.153	0.191	0.394
B8	0.474	0.677	0.855	0.443	0.644	0.817
B9	0.312	0.508	0.827	0.333	0.517	0.825
B10	0.137	0.193	0.350	0.135	0.166	0.277
B11	0.417	0.566	0.720	0.382	0.540	0.702
B12	0.254	0.454	0.717	0.259	0.450	0.685
B13	0.107	0.116	0.119	0.115	0.115	0.106
B14	0.514	0.646	0.849	0.470	0.611	0.824
B15	0.365	0.618	0.857	0.330	0.550	0.803
B16	0.241	0.367	0.490	0.215	0.317	0.416
B17	0.388	0.515	0.680	0.341	0.461	0.612
B18	0.372	0.695	0.941	0.280	0.586	0.888
B19	0.208	0.344	0.546	0.145	0.267	0.464

Table 8: Symmetric SETAR mean tests. d=0.8

Exp	$SETAR_1^{avg,0.8}$			$SETAR_2^{avg,0.8}$		
	100	200	400	100	200	400
C1	0.027	0.026	0.031	0.048	0.043	0.033
C2	0.078	0.139	0.270	0.069	0.092	0.219
C3	0.032	0.087	0.176	0.077	0.166	0.571
C4	0.029	0.050	0.142	0.047	0.070	0.139
C5	0.083	0.120	0.260	0.055	0.097	0.222
C6	0.035	0.068	0.168	0.075	0.172	0.551
C7	0.035	0.043	0.147	0.037	0.049	0.134
C8	0.063	0.112	0.210	0.052	0.082	0.170
C9	0.025	0.030	0.040	0.051	0.106	0.348
C10	0.018	0.015	0.055	0.031	0.040	0.074
C11	0.059	0.047	0.109	0.041	0.040	0.093
C12	0.011	0.017	0.039	0.020	0.067	0.300
C13	0.005	0.003	0.006	0.019	0.035	0.050
C14	0.078	0.119	0.216	0.057	0.084	0.160
C15	0.029	0.031	0.071	0.038	0.041	0.189
C16	0.008	0.013	0.012	0.012	0.023	0.054
C17	0.046	0.081	0.315	0.031	0.039	0.170
C18	0.057	0.297	0.840	0.084	0.327	0.884
C19	0.016	0.131	0.471	0.017	0.133	0.458

Table 9: symmetric SETAR exp tests. d=0.8

Exp	$SETAR_1^{exp,0.8}$			$SETAR_2^{exp,0.8}$		
	100	200	400	100	200	400
C1	0.047	0.063	0.060	0.070	0.069	0.066
C2	0.159	0.279	0.483	0.168	0.281	0.499
C3	0.075	0.178	0.343	0.148	0.352	0.746
C4	0.064	0.106	0.247	0.079	0.112	0.211
C5	0.145	0.259	0.466	0.151	0.273	0.461
C6	0.104	0.166	0.335	0.189	0.343	0.727
C7	0.073	0.095	0.239	0.084	0.093	0.192
C8	0.140	0.238	0.424	0.149	0.254	0.452
C9	0.068	0.116	0.180	0.155	0.296	0.585
C10	0.041	0.054	0.102	0.072	0.083	0.128
C11	0.130	0.166	0.353	0.128	0.193	0.390
C12	0.047	0.106	0.266	0.115	0.251	0.604
C13	0.034	0.037	0.061	0.064	0.082	0.112
C14	0.157	0.258	0.444	0.157	0.250	0.440
C15	0.068	0.138	0.252	0.123	0.235	0.508
C16	0.045	0.053	0.080	0.057	0.099	0.169
C17	0.168	0.347	0.747	0.136	0.274	0.658
C18	0.274	0.722	0.966	0.342	0.741	0.975
C19	0.148	0.422	0.746	0.147	0.420	0.757

Table 10: symmetric SETAR mean tests. d=0.9

Exp	$SETAR_1^{avg,0.9}$			$SETAR_2^{avg,0.9}$		
	100	200	400	100	200	400
D1	0.022	0.031	0.027	0.053	0.057	0.045
D2	0.019	0.021	0.049	0.076	0.145	0.473
D3	0.037	0.048	0.137	0.231	0.630	0.996
D4	0.042	0.110	0.367	0.149	0.379	0.770
D5	0.016	0.017	0.053	0.056	0.158	0.436
D6	0.034	0.047	0.132	0.227	0.661	0.992
D7	0.042	0.105	0.360	0.164	0.353	0.787
D8	0.005	0.011	0.011	0.060	0.124	0.400
D9	0.008	0.008	0.021	0.165	0.541	0.992
D10	0.019	0.043	0.156	0.141	0.342	0.717
D11	0.013	0.011	0.022	0.044	0.132	0.444
D12	0.008	0.037	0.130	0.162	0.589	0.997
D13	0.017	0.028	0.084	0.107	0.325	0.661
D14	0.011	0.005	0.005	0.043	0.066	0.320
D15	0.003	0.006	0.006	0.075	0.360	0.957
D16	0.008	0.005	0.014	0.066	0.215	0.656
D17	0.020	0.143	0.531	0.063	0.319	0.817
D18	0.086	0.466	0.930	0.371	0.862	0.993
D19	0.048	0.249	0.649	0.183	0.527	0.887

Table 11: symmetric SETAR exp tests. d=0.9

Exp	$SETAR_1^{exp,0.9}$			$SETAR_2^{exp,0.9}$		
	100	200	400	100	200	400
D1	0.047	0.064	0.059	0.089	0.097	0.085
D2	0.050	0.067	0.125	0.129	0.213	0.569
D3	0.079	0.129	0.290	0.287	0.672	0.996
D4	0.085	0.206	0.511	0.194	0.416	0.795
D5	0.042	0.063	0.114	0.117	0.219	0.516
D6	0.069	0.127	0.311	0.289	0.716	0.993
D7	0.080	0.189	0.494	0.204	0.383	0.800
D8	0.024	0.043	0.063	0.106	0.203	0.501
D9	0.041	0.051	0.125	0.245	0.609	0.993
D10	0.057	0.093	0.311	0.185	0.382	0.739
D11	0.046	0.075	0.161	0.099	0.255	0.605
D12	0.074	0.216	0.455	0.296	0.711	0.999
D13	0.060	0.118	0.285	0.186	0.400	0.712
D14	0.034	0.030	0.035	0.092	0.141	0.446
D15	0.019	0.040	0.028	0.153	0.492	0.971
D16	0.029	0.020	0.063	0.129	0.297	0.714
D17	0.129	0.479	0.837	0.233	0.621	0.926
D18	0.358	0.788	0.986	0.665	0.934	0.993
D19	0.200	0.560	0.807	0.381	0.734	0.935

Table 12: Asymmetric SETAR mean tests. d=0.6

Exp	$SETAR_3^{avg,0.6}$		
	100	200	400
A1	0.046	0.035	0.024
A2	0.479	0.784	0.986
A3	0.197	0.496	0.918
A4	0.234	0.540	0.917
A5	0.444	0.782	0.992
A6	0.207	0.489	0.927
A7	0.268	0.538	0.925
A8	0.478	0.774	0.987
A9	0.182	0.442	0.848
A10	0.266	0.537	0.916
A11	0.456	0.721	0.970
A12	0.132	0.302	0.724
A13	0.217	0.455	0.820
A14	0.503	0.789	0.993
A15	0.259	0.517	0.942
A16	0.355	0.621	0.958
A17	0.490	0.753	0.932
A18	0.310	0.511	0.798
A19	0.334	0.585	0.850

Table 13: Asymmetric SETAR exp tests. d=0.6

Exp	$SETAR_3^{avg,0.6}$		
	100	200	400
A1	0.071	0.068	0.050
A2	0.656	0.910	0.997
A3	0.493	0.843	0.998
A4	0.461	0.775	0.975
A5	0.638	0.905	0.998
A6	0.463	0.809	0.989
A7	0.465	0.764	0.983
A8	0.642	0.898	0.995
A9	0.451	0.787	0.983
A10	0.489	0.791	0.981
A11	0.635	0.864	0.994
A12	0.375	0.650	0.945
A13	0.420	0.685	0.954
A14	0.661	0.896	0.999
A15	0.517	0.813	0.990
A16	0.564	0.850	0.993
A17	0.585	0.828	0.961
A18	0.533	0.717	0.910
A19	0.493	0.756	0.928

Table 14: Asymmetric SETAR mean tests. d=0.7

Exp	$SETAR_3^{avg,0.7}$		
	100	200	400
B1	0.029	0.036	0.026
B2	0.055	0.067	0.228
B3	0.017	0.028	0.142
B4	0.019	0.026	0.101
B5	0.044	0.059	0.238
B6	0.016	0.033	0.142
B7	0.013	0.032	0.122
B8	0.050	0.063	0.194
B9	0.010	0.009	0.044
B10	0.014	0.022	0.055
B11	0.037	0.041	0.082
B12	0.005	0.002	0.002
B13	0.018	0.014	0.010
B14	0.048	0.054	0.161
B15	0.005	0.004	0.026
B16	0.017	0.015	0.052
B17	0.068	0.043	0.047
B18	0.025	0.031	0.079
B19	0.036	0.045	0.090

Table 15: Asymmetric SETAR exp tests. d=0.7

Exp	$SETAR_3^{exp,0.7}$		
	100	200	400
B1	0.063	0.063	0.053
B2	0.187	0.353	0.757
B3	0.116	0.284	0.729
B4	0.095	0.171	0.410
B5	0.191	0.341	0.749
B6	0.100	0.288	0.721
B7	0.097	0.193	0.433
B8	0.183	0.328	0.689
B9	0.090	0.212	0.602
B10	0.086	0.165	0.399
B11	0.140	0.195	0.490
B12	0.050	0.128	0.354
B13	0.061	0.077	0.190
B14	0.213	0.307	0.674
B15	0.085	0.231	0.609
B16	0.112	0.202	0.470
B17	0.200	0.234	0.326
B18	0.136	0.244	0.655
B19	0.138	0.219	0.408

Table 16: Asymmetric SETAR mean tests. d=0.8

Exp	$SETAR_3^{avg,0.8}$		
	100	200	400
C1	0.045	0.029	0.024
C2	0.007	0.003	0.008
C3	0.024	0.035	0.139
C4	0.019	0.032	0.083
C5	0.011	0.003	0.003
C6	0.016	0.029	0.159
C7	0.027	0.017	0.076
C8	0.009	0.003	0.004
C9	0.010	0.015	0.048
C10	0.016	0.012	0.045
C11	0.009	0.003	0.002
C12	0.003	0.014	0.043
C13	0.008	0.011	0.022
C14	0.010	0.005	0.000
C15	0.005	0.006	0.007
C16	0.006	0.006	0.003
C17	0.039	0.047	0.052
C18	0.077	0.185	0.628
C19	0.057	0.123	0.309

Table 17: Asymmetric SETAR exp tests. d=0.8

Exp	$SETAR_3^{exp,0.8}$		
	100	200	400
C1	0.096	0.069	0.063
C2	0.030	0.036	0.120
C3	0.052	0.108	0.357
C4	0.040	0.069	0.156
C5	0.031	0.035	0.098
C6	0.041	0.085	0.356
C7	0.045	0.056	0.157
C8	0.031	0.027	0.062
C9	0.030	0.052	0.156
C10	0.035	0.034	0.102
C11	0.022	0.009	0.019
C12	0.025	0.057	0.140
C13	0.040	0.030	0.086
C14	0.025	0.023	0.072
C15	0.027	0.030	0.116
C16	0.025	0.045	0.091
C17	0.132	0.178	0.376
C18	0.291	0.622	0.968
C19	0.227	0.451	0.716

Table 18: Asymmetric SETAR mean tests. d=0.9

Exp	$SETAR_3^{avg,0.9}$		
	100	200	400
D1	0.034	0.027	0.028
D2	0.019	0.048	0.155
D3	0.089	0.293	0.882
D4	0.059	0.246	0.713
D5	0.019	0.037	0.145
D6	0.090	0.312	0.879
D7	0.063	0.202	0.719
D8	0.022	0.033	0.111
D9	0.059	0.218	0.831
D10	0.049	0.164	0.667
D11	0.020	0.032	0.134
D12	0.050	0.268	0.867
D13	0.046	0.179	0.606
D14	0.012	0.016	0.058
D15	0.026	0.094	0.608
D16	0.042	0.069	0.460
D17	0.088	0.162	0.488
D18	0.236	0.672	0.993
D19	0.183	0.441	0.792

Table 19: Asymmetric SETAR exp tests. d=0.9

Exp	$SETAR_3^{exp,0.9}$		
	100	200	400
D1	0.075	0.074	0.067
D2	0.045	0.062	0.192
D3	0.131	0.351	0.900
D4	0.096	0.286	0.741
D5	0.033	0.055	0.193
D6	0.125	0.361	0.904
D7	0.103	0.248	0.747
D8	0.042	0.051	0.145
D9	0.101	0.284	0.860
D10	0.097	0.231	0.696
D11	0.055	0.083	0.243
D12	0.125	0.422	0.919
D13	0.116	0.285	0.696
D14	0.033	0.046	0.119
D15	0.086	0.226	0.741
D16	0.097	0.169	0.588
D17	0.246	0.512	0.872
D18	0.569	0.939	1.000
D19	0.468	0.728	0.928

Table 20: STAR tests. d=0.6

Exp	T_1			T_2		
	100	200	400	100	200	400
A1	0.039	0.038	0.054	0.039	0.043	0.029
A2	0.507	0.784	0.948	0.546	0.793	0.947
A3	0.516	0.789	0.955	0.529	0.806	0.956
A4	0.512	0.771	0.957	0.556	0.807	0.951
A5	0.322	0.668	0.971	0.179	0.420	0.872
A6	0.326	0.649	0.967	0.180	0.410	0.875
A7	0.322	0.669	0.961	0.177	0.419	0.831
A8	0.076	0.223	0.626	0.026	0.060	0.310
A9	0.088	0.231	0.643	0.027	0.072	0.312
A10	0.093	0.241	0.623	0.028	0.073	0.317
A11	0.090	0.244	0.671	0.018	0.076	0.331
A12	0.082	0.251	0.647	0.020	0.053	0.354
A13	0.088	0.263	0.623	0.020	0.073	0.307
A14	0.092	0.263	0.646	0.026	0.062	0.302

Table 21: STAR tests. d=0.7

Exp	T_1			T_2		
	100	200	400	100	200	400
B1	0.041	0.035	0.029	0.034	0.029	0.027
B2	0.149	0.301	0.589	0.178	0.233	0.392
B3	0.126	0.278	0.610	0.169	0.208	0.412
B4	0.140	0.295	0.614	0.166	0.214	0.414
B5	0.018	0.026	0.057	0.008	0.006	0.005
B6	0.023	0.023	0.051	0.015	0.008	0.002
B7	0.014	0.017	0.052	0.010	0.005	0.007
B8	0.007	0.000	0.000	0.003	0.000	0.000
B9	0.008	0.000	0.000	0.002	0.001	0.000
B10	0.004	0.004	0.001	0.001	0.002	0.001
B11	0.003	0.001	0.001	0.000	0.000	0.000
B12	0.000	0.000	0.000	0.000	0.000	0.000
B13	0.008	0.002	0.000	0.000	0.000	0.000
B14	0.004	0.002	0.000	0.001	0.000	0.000

Table 22: STAR tests. d=0.8

Exp	T_1			T_2		
	100	200	400	100	200	400
C1	0.058	0.054	0.045	0.036	0.028	0.033
C2	0.036	0.032	0.011	0.049	0.032	0.011
C3	0.033	0.028	0.015	0.052	0.025	0.011
C4	0.036	0.025	0.017	0.055	0.031	0.012
C5	0.015	0.011	0.004	0.007	0.006	0.000
C6	0.013	0.010	0.008	0.002	0.002	0.001
C7	0.013	0.005	0.007	0.004	0.002	0.001
C8	0.035	0.068	0.132	0.013	0.021	0.065
C9	0.023	0.063	0.127	0.009	0.018	0.064
C10	0.029	0.049	0.137	0.013	0.017	0.065
C11	0.036	0.065	0.132	0.009	0.015	0.040
C12	0.041	0.066	0.153	0.012	0.012	0.045
C13	0.031	0.071	0.172	0.005	0.007	0.043
C14	0.036	0.071	0.172	0.006	0.010	0.040

Table 23: STAR tests. d=0.9

Exp	T_1			T_2		
	100	200	400	100	200	400
D1	0.059	0.047	0.043	0.036	0.047	0.032
D2	0.033	0.024	0.010	0.030	0.014	0.005
D3	0.028	0.028	0.017	0.022	0.016	0.007
D4	0.035	0.026	0.018	0.029	0.012	0.005
D5	0.066	0.109	0.308	0.021	0.039	0.169
D6	0.062	0.123	0.330	0.025	0.048	0.173
D7	0.067	0.136	0.305	0.033	0.049	0.161
D8	0.192	0.437	0.801	0.074	0.298	0.893
D9	0.204	0.436	0.782	0.085	0.285	0.888
D10	0.185	0.442	0.825	0.085	0.304	0.895
D11	0.204	0.484	0.850	0.094	0.280	0.856
D12	0.212	0.470	0.851	0.070	0.254	0.847
D13	0.244	0.506	0.885	0.079	0.255	0.854
D14	0.220	0.514	0.860	0.072	0.265	0.823

Table 24: STAR[a] Tests.

Country	$STAR_1^{0.6}$	$STAR_1^{0.7}$	$STAR_1^{0.8}$	$STAR_1^{0.9}$	$STAR_2^{0.6}$	$STAR_2^{0.7}$	$STAR_2^{0.8}$	$STAR_2^{0.9}$
US	19.797*	8.253*	3.188	1.187	27.056*	10.117*	3.305	1.406
Germany	27.454*	12.482*	5.565*	2.549	28.601*	12.492*	6.528	5.375
France	26.348*	11.965*	5.351*	2.517	29.991*	12.331*	5.487	3.812
Italy	16.253*	5.861*	1.696	0.301	17.038*	5.935	1.850	1.452
UK	16.507*	8.708*	4.697*	2.686	20.047*	9.010*	4.987	4.708
Canada	35.824*	15.793*	6.555*	2.633	48.977*	19.838*	7.306	2.633
Austral	35.743*	17.720*	8.501*	4.067	48.140*	21.076*	8.947*	4.099
Austria	116.036*	58.585*	27.161*	11.433*	288.710*	128.393*	53.054*	20.895*
Belgium	16.312*	11.104*	7.724*	5.822*	37.201*	16.702*	8.413*	5.882
Czech	0.316	0.004	0.318	0.882	1.734	0.544	0.398	0.918
Denmark	13.296*	6.243*	2.905	1.399	24.175*	9.792*	3.416	1.494
Finland	14.808*	6.786*	3.104	1.518	25.269*	9.897*	3.480	1.624
Greece	7.818*	5.532*	3.676	2.572	31.158*	12.948*	4.999	2.573
Hungary	0.376	1.009	1.607	2.066	1.845	1.147	1.690	2.687
Iceland	9.385*	5.420*	3.098	1.776	9.784*	5.431	3.666	3.523
Korea	16.812*	6.879*	2.245	0.440	17.093*	8.335*	5.685	6.568
Mexico	13.993*	6.700*	3.378	1.994	18.103*	7.014	3.768	4.566
Nether	24.886*	11.903*	5.597*	2.690	27.078*	11.926*	6.242	5.003
Zealand	25.967*	12.572*	6.102*	3.090	34.341*	14.999*	6.372	3.196
Norway	30.722*	16.274*	8.623*	4.697	41.044*	18.864*	8.854*	4.826
Poland	8.941*	4.483	2.021	0.784	9.679*	6.019	4.604	4.594
Portugal	6.595*	2.043	0.356	0.000	12.224*	3.302	0.369	0.543
Spain	1.215	0.146	0.143	1.195	4.312	1.032	0.204	1.332
Sweden	40.994*	21.724*	11.445*	6.175*	51.604*	24.060*	11.556*	6.466
Switzer	13.869*	8.011*	4.703*	2.827	18.600*	8.813*	4.737	4.092
Turkey	19.058*	7.975*	2.800	0.750	20.639*	8.011*	3.391	3.488
Hong	6.716*	3.184	1.330	0.429	7.047	4.317	3.628	4.179
Singap	22.136*	10.371*	4.587	1.986	34.981*	14.534*	5.337	1.990
Malaysi	56.622*	29.926*	14.815*	7.098*	89.694*	40.448*	17.496*	7.417
Indones	42.989*	18.980*	7.423*	2.341	52.522*	26.261*	14.159*	9.381*
Thailand	22.366*	15.506*	10.185*	6.627*	54.410*	26.793*	13.321*	7.123
Philipp	6.834*	4.306	3.236	2.902	17.411*	7.127	3.506	3.027
SriLanka	0.745	0.001	0.328	0.808	122.426*	49.043*	16.933*	5.411
Argent	3.917	1.654	0.578	0.153	5.089	2.914	2.163	2.295
Bolivia	2.308	1.261	0.644	0.329	7.511*	2.697	0.703	0.674
Brazil	0.303	0.571	0.777	0.873	3.910	1.951	1.050	0.877
Chile	17.964*	3.558	0.090	0.513	21.296*	3.993	0.139	1.503
Colombia	33.952*	14.999*	5.469*	1.404	35.583*	17.120*	8.399*	5.286
Venezu	22.988*	11.070*	4.195	0.896	23.332*	11.195*	4.238	0.907
No. of Rejections	32	27	16	5	33	25	10	2

[a]Starred entries indicate significance at the 5% significance level

Table 25: Augmented STAR Tests

Country	$STAR_1^{0.6}$	$STAR_1^{0.7}$	$STAR_1^{0.8}$	$STAR_1^{0.9}$	$STAR_2^{0.6}$	$STAR_2^{0.7}$	$STAR_2^{0.8}$	$STAR_2^{0.9}$
US	2.910	2.241	1.562	1.040	5.111	4.710	4.346	4.201
Germany	13.760*	9.682*	5.877*	3.199	19.944*	16.726*	13.638*	11.608*
France	13.681*	10.397*	6.866*	4.134	17.454*	14.903*	12.241*	10.479*
Italy	3.092	1.828	0.814	0.247	7.370*	5.698	4.713	4.523
UK	2.897	2.184	1.545	1.059	5.660	5.056	4.688	4.672
Canada	3.942	3.097	2.231	1.528	5.030	4.622	4.153	3.810
Austral	5.888*	4.871*	3.761	2.783	6.698	6.088	5.368	4.770
Austria	1.383	1.891	2.446	2.576	10.201*	10.574*	9.862*	7.940
Belgium	20.082*	17.357*	13.791*	10.503*	20.238*	18.394*	16.333*	14.780*
Czech	0.033	0.169	0.463	0.881	14.402*	5.866	2.794	2.095
Denmark	4.750*	2.879	1.621	0.904	6.835	5.398	4.722	4.742
Finland	4.904*	3.582	2.463	1.663	5.616	4.697	4.072	3.856
Greece	7.163*	5.903*	4.596	3.591	8.981*	8.071*	7.305	7.029
Hungary	1.273	1.487	1.634	1.733	4.994	5.419	5.745	6.028
Iceland	0.963	0.685	0.481	0.332	7.180*	6.857	6.555	6.383
Korea	1.166	0.514	0.105	0.004	5.701	5.297	5.134	5.362
Mexico	2.624	1.665	0.931	0.492	6.713	6.139	5.989	6.294
Nether	14.826*	10.341*	6.411*	3.665	18.578*	15.701*	13.074*	11.342*
Zealand	4.926*	3.893	2.844	2.010	6.155	5.312	4.464	3.876
Norway	7.341*	5.683*	4.055	2.762	7.349*	5.848	4.506	3.572
Poland	0.285	0.216	0.141	0.066	7.936*	7.443	7.001	6.704
Portugal	1.299	0.603	0.148	0.001	3.120	2.593	2.490	2.884
Spain	1.547	1.534	1.950	2.695	2.772	2.557	2.891	3.641
Sweden	7.520*	5.914*	4.332	3.041	8.319*	7.240	6.191	5.432
Switzer	4.934*	3.298	2.132	1.366	6.243	5.314	4.923	5.013
Turkey	0.448	0.179	0.022	0.011	11.238*	11.500*	11.550*	11.739*
Hong	0.000	0.017	0.049	0.091	6.886	6.767	6.637	6.519
Singap	3.781	3.106	2.409	1.822	7.223*	7.239	7.114	7.026
Malaysi	5.904*	5.322*	4.316	3.199	5.970	5.323	4.418	3.531
Indones	1.687	1.753	1.317	0.699	15.091*	14.020*	12.462*	10.903*
Thailand	8.122*	7.959*	7.335*	6.391*	8.435*	8.040*	7.336	6.513
Philipp	10.412*	7.458*	5.365*	4.153	10.829*	7.474	5.483	4.711
SriLanka	3.521	3.433	3.074	2.676	6.112	6.201	5.308	3.992
Argent	0.002	0.006	0.008	0.004	4.268	5.229	5.687	5.795
Bolivia	0.001	0.002	0.002	0.002	4.906	5.566	5.398	5.107
Brazil	0.667	0.855	1.023	1.148	0.795	1.059	1.329	1.604
Chile	0.030	0.001	0.146	0.487	5.650	3.769	3.686	4.220
Colombia	0.003	0.000	0.000	0.019	8.775*	8.375*	8.001*	7.814
Venezu	0.231	0.288	0.210	0.079	0.520	0.447	0.265	0.088
No. of Rejections	15	11	6	2	18	10	8	6

Table 26: SETAR Tests .

Country	$SETAR_3^{0.6}$	$SETAR_3^{0.7}$	$SETAR_3^{0.8}$	$SETAR_3^{0.9}$
US	280486.2*	157.864*	7.992	3.630
Germany	1242665.*	769.156*	48.000*	25.776*
France	1089052.*	389.915*	18.874*	9.036
Italy	1508.738*	14.686*	2.662	1.883
UK	48550.59*	215.810*	25.235*	17.326*
Canada	23234343*	84311.07*	117.869*	8.293
Austral	11363121*	33693.57*	111.795*	12.717*
Austria	12812612*	18882019*	23023941*	11687.94*
Belgium	21869468*	3510.607*	109.560*	41.704*
Czech	7.951	3.423	2.652	3.050
Denmark	33841.61*	101.450*	7.052	2.797
Finland	52303.51*	103.182*	6.642	2.324
Greece	25302933*	5294.142*	45.359*	8.200
Hungary	4.448	1.981	2.536	5.204
Iceland	145.348*	17.007*	6.322	4.750
Korea	22950.61*	116.068*	11.707*	6.014
Mexico	26348.49*	55.163*	5.019	2.536
Nether	304730.1*	525.257*	45.689*	25.395*
Zealand	75266466*	4434.898*	49.732*	8.356
Norway	67785522*	17857.15*	131.520*	16.184*
Poland	510.539*	54.225*	20.194*	15.688*
Portugal	1123.848*	6.584	1.350	1.261
Spain	16079116*	206414.3*	35.038*	2.124
Sweden	11437514*	242192.7*	479.984*	29.899*
Switzer	9869.922*	116.442*	17.364*	11.285*
Turkey	88995.03*	96.738*	6.022	3.283
Hong	78.293*	16.392*	8.805*	7.892
Singap	29972575*	3633.140*	26.146*	4.955
Malaysi	47444719*	99979747*	13165.07*	102.102*
Indones	15195753*	965796.5*	841.589*	30.457*
Thailand	97102455*	2935729.*	5469.395*	301.114*
Philipp	17946.60*	134.854*	24.020*	18.373*
SriLanka	17835812*	14706307*	10966.50*	31.557*
Argent	13.853*	3.769	2.115	1.790
Bolivia	30.311*	5.021	1.938	1.466
Brazil	216.395*	32.756*	12.829*	8.911
Chile	53182.78*	13.737*	1.539	2.149
Colombia	31283355*	14959.81*	168.245*	32.676*
Venezu	121014.8*	243.176*	7.651	1.567
No. of Rejections	37	34	25	16

Table 27: Augmented SETAR Tests .

Country	$SETAR_3^{0.6}$	$SETAR_3^{0.7}$	$SETAR_3^{0.8}$	$SETAR_3^{0.9}$
US	11.915*	9.591*	8.020	7.554
Germany	134718.3*	24917.40*	4649.278*	1476.403*
France	8087.277*	2038.968*	480.539*	178.462*
Italy	28.455*	11.415*	6.270	4.912
UK	59.364*	44.021*	35.666*	33.272*
Canada	13.387*	9.063*	6.471	5.207
Austral	52.902*	41.666*	28.976*	20.732*
Austria	71.085*	83.838*	59.982*	25.583*
Belgium	21368.17*	7744.575*	2449.980*	1002.836*
Czech	27628347*	62137342*	14022653*	31588.25*
Denmark	19.059*	8.896*	5.979	5.637
Finland	14.049*	7.418	4.516	3.366
Greece	71.267*	45.284*	29.634*	24.080*
Hungary	37.195*	59.856*	83.963*	107.793*
Iceland	8.095	7.627	7.039	6.647
Korea	6.583	4.741	3.957	3.959
Mexico	32.068*	14.273*	8.057	6.121
Nether	27532.10*	5697.943*	1285.456*	455.428*
Zealand	23.222*	14.415*	9.086*	6.550
Norway	70.833*	33.083*	16.426*	9.890*
Poland	2392.948*	1815.955*	1291.343*	996.103*
Portugal	4.056	2.953	2.673	3.141
Spain	49.175*	42.452*	45.638*	63.179*
Sweden	63.016*	31.788*	16.592*	10.121*
Switzer	29.036*	17.892*	14.694*	15.529*
Turkey	14.562*	11.980*	11.428*	12.780*
Hong	1.756	1.818	1.854	1.896
Singap	153.932*	177.498*	174.530*	168.992*
Malaysi	75.473*	53.118*	32.977*	20.825*
Indones	183.410*	100.884*	43.960*	18.905*
Thailand	193.147*	160.494*	116.739*	81.967*
Philipp	266.760*	50.551*	20.170*	15.289*
SriLanka	26.574*	28.787*	19.137*	10.607*
Argent	8.464	7.418	6.334	5.498
Bolivia	3.041	3.225	3.011	2.875
Brazil	83926143*	43345723*	40497292*	60974912*
Chile	13.842*	11.515*	17.035*	32.743*
Colombia	73.880*	61.173*	50.387*	45.075*
Venezu	1.428	1.314	1.198	1.114
No. of Rejections	32	31	26	25

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