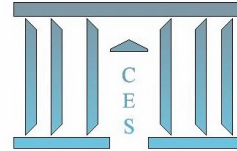




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## The Dollar Squeeze of the Financial Crisis

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2012.09



## The Dollar Squeeze of the Financial Crisis $\diamond$

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**Abstract.** By Covered Interest rate Parity (CIP), the FX swap implied currency interest rates should coincide with actual interest rates. When a difference occurs, the residual is referred to as the *cross currency basis*. We link the Euro-Dollar currency basis (e.g. in 2008) to shadow prices of dollar funding constraints and interpret the basis as the relative physical possession value of the scarcer currency, or the “convenience yield” associated with that currency. This is similar to specialness in repo markets, expressing the physical possession value of a security. We examine how the coordinated central banks intervention can reduce the currency basis.

**JEL Classification:** D52, D53, G12, G14, G15. G18.

**Keywords:** FX swaps; repo; Euro-Dollar currency basis; the 2008 dollar squeeze; possession

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# 1 Introduction

## 1.1 Goal and structure of the paper

The goal of our paper is to establish a theoretical link between the cross-currency basis and the binding funding constraints in one of the currencies. We use our theoretical framework as a lens to better understand the determinants of the 2008 dollar squeeze. Finally, we analyze the effect of the interventions by the Fed and the ECB on the currency basis. To our knowledge, this paper is the first to analyze cross currency basis from a theoretical perspective and draw a parallel between the underlying determinants of this crisis and the “convenience yield” of physically possessing the scarcer currency. In this sense, our paper provides a direct link between the cross-currency basis puzzle and the literature on collateral value of securities and “repo specialness”.

The paper is organized as follows. In the rest of this Introduction we motivate and outline our results. In Section 2 we present our possession approach to the theory of currency basis. Sections 3.1 and 3.2 focus on the 2008 crisis, describing a simple model of the 2008 dollar squeeze and the subsequent central banks intervention. Finally, in Sections 3.3 and 3.4 we establish our results on the behavior of basis during the crisis and how it changed with central bank intervention.

We begin by briefly reviewing covered interest rate parity and by formally quantifying the cross-currency basis.

## 1.2 Covered Interest Rate Parity (CIP) “arbitrage”

We propose in this section a sparse setting to identify the potential incremental value of physically owning an asset. In our setting there are two dates (dates 1 and 2) and two assets. The two assets have the same value at date 1, which implies that buying one asset and selling the other is a self-financing or zero-investment strategy. In what we call a *possession swap*, owners of assets of similar value can agree to exchange their physical possession for a while with the agreement to pass back cash flows to original owners. Because the arrangement does not alter cash-flows received, doing it should have no pricing impact under cash flow replication theory. Well we will see that it does. Let us look at the special case of currencies.

We will provide two interpretations: first in the context of an FX swap, and then in the context of CIP. Let us first consider the FX swap as it closely follows the scheme above: agent 1 can invest the domestic currency by himself and earn interest, or get the same cash flow from agent 2 but by exchanging the domestic currency for foreign currency at the beginning of the period at date 1, and then claiming back the domestic currency from agent 2 at date 2 through FX swap. This is the same amount of currency both in the front leg of the trade (at date 1) and in the back leg (at date 2). In both cases agent 1 has exactly the same currency that he started with plus interest earned over the period. The difference is the fact that agent 1 did not have possession of domestic currency between the two dates in FX swap. So we have an implementation of a possession swap for currencies.

We also have the CIP variation: owner of the first asset can invest the asset (say, domestic currency), earn the spot rate of interest (in domestic currency) and receive the principal plus interest at date 2. Alternatively, the owner can sell asset 1 in the spot market and thus give up the physical ownership, invest in the alternative asset (foreign currency), which can be sold at date 1 in the forward markets so that the proceeds at date 2 are converted back in the original asset (in domestic currency). With the latter, a canonical buy-sell (*that is, buying one asset and selling the other, at zero costs*), the owner of asset 1 is generating cash flows in asset 1 (domestic currency) at date 2, but does not physically own asset 1 between the two dates: instead, the original owner is relying on the counterparty in the forward market to deliver asset 1 at date 2.

The canonical buy-sell entails the *exchange at date 2 of the FX proceeds and the domestic currency*. This means that for an original holder of asset 1 he gets the same cash flows at date 2 in either of the two strategies. Thus, according to *the* cash-flow replication theory, the value of being long in one strategy and short in the other should be self-financing. The only difference between holding asset 1 and engaging into the buy-sell is that the asset is physically possessed between the two periods.

As an example, say the two assets are dollar (U) and euro (E). One policy is to invest the currency to earn interest between dates 1 and 2. If the canonical Buy-Sell has zero value, then exchanging one euro for dollars at the spot rate  $X_1$  (dollars per euro) and investing these  $X_1$  dollars for one period at the interest rate  $r_U$  must be the same as investing one euro during one period at the interest

rate  $r_E$  and then exchange the proceeds  $1 + r_E$  at the forward exchange rate  $\chi$ . Then, if there is no arbitrage, the forward FX rate at date 2 is  $\chi = X_1 \frac{1+r_U}{1+r_E}$ . This formula is also known as *the covered interest rate parity*. A deviation from such formula is referred by *cross currency basis*.

Formally, let us fix the interval between date 1 and 2 (e.g. consider a period of 3 months) for the rest of the paper, and define the cross currency basis as  $\alpha$

$$\chi = X_1 \frac{1 + r_U}{1 + r_E + \alpha}. \quad (\text{Basis definition foreign leg})$$

Equivalently one can put the spread on the dollar leg

$$\chi = X_1 \frac{1 + r_U + \beta}{1 + r_E}. \quad (\text{Basis definition dollar leg})$$

The market convention is more in terms of placing the spread on the foreign leg ( $\alpha$ ). After the initial example, which uses the market convention, for the rest of the paper we will actually use  $\beta$ . Both approaches are clearly equivalent and  $\alpha = -\beta \frac{1+r_E}{1+r_U+\beta}$ .

The economic interpretation of  $\alpha$  and  $\beta$  is as follows: if dollar is the currency in shortage, then the convenience yield associated with the physical ownership of dollars is reflected by the fact that  $\beta > 0$ . The owners of dollars at date 1 will only part with their physical holdings of dollars and agree to a forward transaction if they are compensated at date 2 with the effective interest rate  $r_U + \beta > r_U$ . Later, we develop a theory of funding constraints that will *endogenously derive an expression for the cross-currency basis,  $\beta$ , as a function of underlying constraints and frictions in the markets*. We next develop the intuition as to why cross-currency basis might exist, but not necessarily present an arbitrage opportunity.

A non-zero cross currency basis goes against the typical cash-flow replication arbitrage argument of the buy-sell. For instance, a positive basis ( $\beta > 0$ ) should not survive an arbitrage consisting in borrowing dollars at the rate  $r_U$ , exchanging them for euros at the spot rate ( $X_1^{-1}$ ), investing at the rate  $r_E$  and later exchanging back into dollars at the forward rate ( $\chi$ ). Such combination of borrowing dollars at a rate and lending them through a FX swap would give a profit of  $\chi X_1^{-1}(1 + r_E) > (1 + r_U)$  if  $\beta > 0$ . In the absence of borrowing or funding constraints, it would be scaled up arbitrarily. In reality, there are impediments to enforcing arbitrage: besides transactions costs and commissions, a key impediment is scalability, as doing the arbitrage directly commits funding capability in

the scarcer currency. Such a capability is not unbounded and is shared by many other bank activities<sup>1</sup>.

The inability of one bank to precisely identify the default risk associated with the bank with which it must engage in forward rate agreements may *not* be the core of the problem here, as the counterparty can be chosen to trade the basis at market level. What happens is that funding in one currency can be, in *aggregate*, in high demand. Therefore, agents who possess that currency will be price sensitive when allocating their scarce resource. The market where such funding abilities (one currency vs. another) is exchanged is the basis, and the basis itself can be seen as the market clearing price for the exchanging funding ability in one currency vs another. It was clear that in 2008, right after the failure of Lehman Brothers, it was extremely costly (if not impossible) to carry out the arbitrage described above, as documented by Goldberg, Kennedy and Miu (2009).<sup>2</sup>

Banks were extremely reluctant to lend the scarcer currency for any term for reasons cited earlier: since this is a pre-requisite for enforcing arbitrage, the basis did not converge to zero in the immediate aftermath of credit crisis. To sum up, we make two observations: first, a premium for physical possession of scarcer currency leads to the existence of cross-currency basis. Second, the resulting reluctance to lend scarcer currency can perpetuate the continuation of cross currency basis. It stands to reason that any intervention by central banks to reduce this reluctance, should alleviate the cross currency basis, and push it towards zero.

To motivate the issues involved, we present in Figure 1 the euro-dollar cross currency basis. The basis  $\alpha$  is plotted as an annualized rate. For interest rates  $r_U$  and  $r_E$ , the respective OIS (overnight interest swap) rates are used to proxy secured borrowing rates. LIBOR is used for the unsecured case.<sup>3</sup>

<sup>1</sup>Lack of scalability may arise from the inability of some banks to raise dollar funding as bond investors may place excessive risk premium for buying dollar denominated bonds. This risk premium may be related to the lack of high quality dollar assets, FX risk, etc.

<sup>2</sup>For unsecured borrowing the argument goes as follows: “Normally, arbitrage would drive the basis to zero. For example, if the FX basis is greater than zero, arbitrageurs could borrow dollars unsecured at a relatively low interest rate, and then lend the dollars via an FX swap at a relatively higher implied interest rate. Yet, with the dollar shortage during the crisis, arbitrageurs were unable to borrow sufficient dollars in the unsecured market to take advantage of this opportunity.” (Goldberg, Kennedy and Miu (2009))

<sup>3</sup>Several points should be noted here. LIBOR is “fixed” from the quotes supplied by the panel of banks. LIBOR fixing involves “re-freshing” of the panel whereby weak banks are

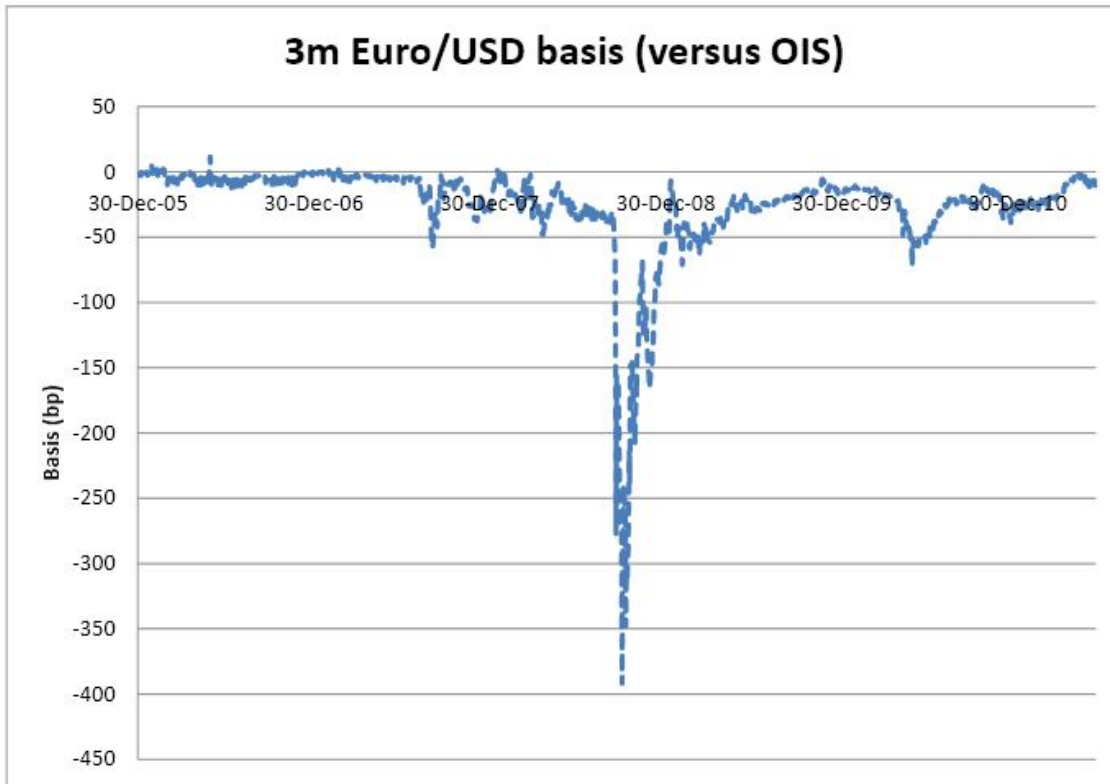


Figure 1: **Euro-dollar currency basis (in bps) relative to OIS 3 months rate (foreign leg).** *Source:* JP Morgan

The empirical facts observed from Figure 1 are: 1) the basis is not close to zero for extended periods of time, and is in fact negative most of the sample period. 2) the basis  $\alpha$  becomes very negative just after Lehman Brothers' shock. We thus need to explain what is wrong with the arbitrage argument, and in particular what happened in 2008, especially after the bankruptcy of Lehman Brothers on September 15, 2008.

Finally we note that it is easy to relate LIBOR-based basis and OIS-based basis. LIBOR rate are given by the following  $R_U = r_U + s_U$  (for dollar) and  $R_E = r_E + s_E$  (for euro), where  $s_U$  and  $s_E$  are the spreads between forward rate agreements (FRA) and the OIS rates, often referred to as the FRA-OIS spread in dollar and euro respectively. If  $A$  is the cross currency basis with respect to

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replaced by strong banks to keep the credit quality of the banks in the panel AA. In addition, only inter-quartile range is used in averaging to fix LIBOR. In all their arbitrage transactions, however, banks must use their own funding costs, which can be different from the "fixed" LIBOR. Also, OIS rates track effective fed funds rates or policy rates. Banks can secure their funding at repo rates, which can differ from OIS rates, especially during a crisis period.

LIBOR, one can relate it to the OIS based equivalent market basis  $\alpha$  :

$$A = \alpha + [s_U \frac{1 + r_E}{1 + r_U} - s_E] + \frac{s_U \alpha}{1 + r_U}.$$

Empirically, in a crisis period, the value of treasury as a collateral raises the most relative to the collateral value of other securities. This in turns influences General Collateral rate (GC) versus LIBOR and FRA-OIS. This is why the most sensitive spread from the above formula – and the one often used by practitioners in the short end of the term structure is basis versus OIS.

### 1.3 Why would the cross currency basis be under pressure?

We start with the theory first (in Section 2). What went wrong with the theory of CIP in the case of euro versus dollar in 2008?

The problem is that for the CIP argument to work, it must be possible to start with arbitrarily large long positions in dollar – the ability to enter into physical possession of the dollar is the difference. Note that in the buy-sell even though a holder of dollars gets the same cash flow holding dollars or doing a buy-sell, in terms of physical possession of asset in his account things are quite different. The financial statement will show a dollar asset balance all along if the dollar is managed according to a policy of physical possession, while in the buy-sell the agent will be in possession of euros. This difference lies at the heart of our analysis.

The basis is the value attributed by the market to the possession desirability of one currency relative to the other. In subsections 2.3 and 2.4, we show how this premium, not captured by naive foreign exchange (FX) “arbitrage” formulas (with no funding constraint), can be directly linked to the prevailing shadow values of binding dollar funding constraints.

Next we offer a description of the cross currency basis during the crisis period of 2008 (in Section 3).

In terms of asset positions, after 2000 European banks built up large net US dollar positions. By mid-2007 European banks’ combined long US dollar positions grew to more than \$800 billion, which were funded by short positions in euros<sup>4</sup>. In 2008 the European banks holding large positions in American Asset

<sup>4</sup>See graph 5 of McGuire and von Peter (2009).



Backed Securities (ABS), particularly in sub-prime asset backed securities, faced serious difficulties in rolling maturing funding in dollars. Usually, European banks funded these American ABS by borrowing dollars, which had to be paid back with an interest at the maturity of these loans. As shown by McGuire and von Peter (2009), already by mid-2007 the major European banks' funding needs were \$1.1-1.3 trillion. These banks could hardly raise the large amount of dollars they needed through usual forms of dollar funding, given the distressed market conditions - in particular, the assets held by European banks lost their ability to be pledged to raise dollar funding. Borrowing, using asset backed commercial paper (ABCP) or repo on ABS, was actually shut down for many market segments - in particular for the high yielding kind of asset typically held by those European institutions. The combination of low quality of the dollar assets and the inability to "roll over" in the ABCP markets resulted in the currency squeeze.

In this currency squeeze, the high value placed on possessing dollars led to an increase in the currency basis (see Section 3.3): the spot rate (dollar per euro), relative to the forward rate, dramatically decreased even adjusting for interest rate moves. This is because not being able to roll over funding, some agents would face a major immediate consequence: "distress selling" (and, ultimately, bankruptcy). Dollars were immediately needed and such a pressure is evident in the cross-currency basis. Baba and Packer (2009) and Goldberg, Kennedy and Miu (2010) are good references on how the premium paid for dollars in the FX swap market, which rose dramatically (up to 400 basis points) in October 2008, is linked to the high dollar funding costs that non-US banks experienced at the end of the year 2008.

#### **1.4 Central banks coordinated intervention**

A hallmark of the present crisis is the active and often aggressive interventions by central banks in private capital markets. While many empirical papers have documented the effects of the actions of the central banks on asset prices, relatively few theoretical papers are available to judge the actions of central banks. To this end, we model in Section 3 the central bank interventions led by the Fed that helped reducing the dollar squeeze in Europe. In a coordinated action, the Fed and the ECB swapped dollars for euros, in order to let the ECB meet some of the high demand for dollars by the European banks. The ECB was then able

to provide dollar funding to the member banks by accepting as collateral euro denominated covered bonds. Such an action, with more aggressive pricing of the swap lines was also taken in December 2011 by the Fed. In addition, the menu of collateral was expanded to include dollar-denominated collateral recently. We offer a theoretical perspective for examining the consequences of such actions by the central banks on cross currency basis.

While the main effect of such dollar repo financing is to fund ABS holdings (which were hard to sell or fund), European banks pledged assets in euro to issue a euro covered bond, which served as collateral to raise dollar. Such a cross currency dollar repo rate for euro covered bond has disadvantages : notably a big haircut to reflect FX risk. Nevertheless, this proved to be the best funding option for some European banks at the time, as the cross currency basis in the free market reached levels of several hundred basis points (see Figure 1 above).

In this paper we shall not make the distinction among the different flavors of collateral accepted by the central banks, which we think is the anchor of funding relevant for FX market. Our framework is readily extended to deal with different collateral, different rates and repo rates obtained pledging them. See Bartolini et al. (2011) for a comparison of repo rates of different collateral (Treasuries, mortgage-based and Federal agency).

The remainder of the paper is structured as follows. In Section 2 we present our theory of currency possession in two different scenarios. One is when funding is done by trading bonds. The other is when funding is obtained by lending securities. For simplicity, we assume that there is no funding by trading bonds in this second scenario. In Section 3 we look at a simple two regions (US and EU) and two central banks (Fed and ECB) economy, and show how in the context of our model, the Fed-ECB coordinated intervention in late 2008 and the first half of 2009 could contribute to the decrease of the currency basis. Through a simple calibration exercise, we show that the intervention was effective in reducing the basis.

## 2 Cross-currency basis: the theory

While the model that we propose below is quite simple, our results are fairly robust. On the other hand, we do not explicitly specify the objective functions of agents. For all our purposes, as we will see, private agents could be maximizing

utility or the present value of discounted profits.

## 2.1 Currency possession in analogy with repo specialness

In this paper we consider a simple model with two dates,  $t = 1, 2$ . We start by looking at an agent in the private sector who has a funding capacity in both dollar and euro, expressed by  $e_U$  and  $e_E$  respectively - modeled here as short term bond holdings that can be sold or lent as a collateral.<sup>5</sup> The corresponding action variables are the traded amounts  $b_U$  and  $b_E$ , respectively.<sup>6</sup> Additionally, we take into account two other markets:

1) the *FX spot market* with action variable  $s$ , where  $s$  denotes the amount of euros at date 1 that can be exchanged against  $X_1s$  amount of dollars at date 1,

2) The *FX swap market* with action variable  $f$ , the amount of euro sold against  $X_1f$  dollar at date 1. Then, at date 2 the same amount of euro  $f$ , is bought back against  $\chi f$  dollars, where  $\chi$  is the rate that can be locked in at date 1 to trade euro against dollar at date 2 (the forward FX rate).

We think that the FX swaps (combined short sale with forward purchase of currency) are to currencies what repos are to securities. A repo transaction exchanges possession of a security against possession of a currency for the duration of the repo transaction. A FX swap exchanges the possession of one currency against the possession of another currency for the duration of the FX swap. Such transactions are naturally collateralized. This feature makes it natural for us to examine the cross currency basis in terms of the scarcity value of a currency, using as a theoretical basis for such an interpretation the analogue of what happens in securities markets.

For FX swaps, similar to repo markets, there is a terminology that is used in practice. We say that one agent buy-sells when that agent goes long in repo, and sell-buys when that agent takes a short in repo. For buy-sell, we buy first

<sup>5</sup>The credit reputation of the agent, capital adequacy, and access to markets such as credit lines are some of the variables, which will inform on the endowments  $e_U$  and  $e_E$ , which reflects his funding capacity. Also deposits base will be a key factor for funding capacity of banks. For simplicity and abstraction, we sum up such funding capacity as an initial supply of bond in the given currency.

<sup>6</sup>We will deal later with a version where the collateral is lent to raise funding, which is probably the most relevant funding market. In such a case, action variables are the respective traded amounts  $z_U$  and  $z_E$  in the repo market of such collateral.

and sell later, acquiring possession of the instrument bought over the duration of the repo transaction or the FX swap.

In repo, an important concept is the *specialness* of a security, which occurs when the security repo rate is below the General Collateral (GC) rate, the highest repo rate for those securities with similar maturity and asset class. See Bottazzi, Luque and Páscoa (2011), Brunnermeier and Pedersen (2009), Duffie, Garleanu, and Pedersen (2002), Duffie (1996), Geanakoplos (2003), and Vayanos and Weill (2008) for the role played by securities as collateral and the determinants of specialness. See also Jordan and Jordan (1997), Longstaff (2004) and Bartolini et al. (2011) for empirical support of these models. Also, see Bech, Klee and Stebunovs (2010) on the relationship between the federal funds rate (unsecured lending) and the repo rate (secured lending) before, during, and after the financial crisis that began in August 2007.

There is an equivalent to specialness that we will introduce in this section: *cross-currency basis*. That such a basis may not be trivial is evident from Figure 1. *Specialness* for security is the correction that lowers the loan rate as a compensation for the value of possession of the security compared to the currency. Cross-currency basis is such a correction to the value of interest rate earned by each currency. Most of the time possession of currency is a trivial matter for the domestic player, and it is much more common for a security to attract possession value through specialness than for a currency. Nevertheless, and especially from an international perspective, currencies also can be in relative scarcity. This is mostly because, for some of those international agents, the link between their assets and the capacity to raise the currency they need is not clear, as they sometimes cannot pledge such assets to raise foreign funding.

Note that the front exchange of the FX swap can easily be neutralized using a simple spot transaction. Note also that each currency earns a different interest when invested in debt, but we shall see that this approach (i.e. CIP) alone is not enough to explain the difference in value between  $X_1$  and  $\chi$ . To try to explain such a difference is one of our purposes in this paper.

We introduce next a term known as “box constraint.” Box constraint means that each agent can possess currencies and securities in non-negative quantities, but overdrafts are not allowed in currencies and security balances. Securities can be shorted and loans in each currency can be arranged, but non-negative possession of such securities and currencies have to be monitored and enforced

all along. Each agent has a box constraint for each currency and asset.

Position and possession should not be confused. The former can be negative, the latter cannot (negative positions have to be compensated in some way, as will be seen in detail). The box “no overdraft” constraint for euros of an European bank can be thought as a standard budget constraint, and the same applies for an American bank. We will now present these box constraints.

## 2.2 Basic Constraints

We look at an agent having potentially a consumption in the dollar and euro good markets. However, most arguments go through if agents are profit-maximizers rather than consumers.

The Buy-Sell example works with many combinations, in the case one asset is a security and the other is a currency we have the situation of the repo market, but we can allow for both to be currencies. To properly model the economics of the problem, as in the case of repo (Bottazzi, Luque and Páscoa (2011)), we need to keep track of balance sheet and title balance of assets carefully. We call this “the assets in the box”. The key is to understand that there are no negative title balances. For currencies this means no-overdraft, but the constraint is adapted to all assets.

We get the dollar no-overdraft box at date 1:

$$p_{U,1}(\omega_{U,1} - x_{U,1}) + X_1(s + f) - b_U \geq 0, \quad (\$.1)$$

where  $X_1$  is the dollar price of one euro at date 1 (spot rate),  $b_U$  is the date 1 amount of a dollar bond transacted at date 1 and paying 1 at date 2. We denote by  $\omega_{U,1}$ ,  $p_{U,1}$  and  $x_{U,1}$  initial endowments, price and demand of US goods respectively.

The euro no-overdraft box at date 1:

$$p_{E,1}(\omega_{E,1} - x_{E,1}) - (s + f) - b_E \geq 0 \quad (e.1)$$

We get the two funding constraints as the box on US and euro bonds

$$b_U + e_U \geq 0 \quad (\text{Funding.U})$$

$$b_E + e_E \geq 0 \quad (\text{Funding.E})$$

Finally, we introduce the date 2 dollar and euro no-overdraft boxes:

$$p_{U,2}(\omega_{U,2} - x_{U,2}) - \chi f + (1 + r_U)(b_U + e_U) \geq 0 \quad (\$.2)$$

$$p_{E,2}(\omega_{E,2} - x_{E,2}) + f + (1 + r_E)(b_E + e_E) \geq 0 \quad (e.2)$$

The agent can obtain dollars at date 1 by (1) exchanging euros for dollars at spot rate  $X_1$ , (2) swapping euros buy dollars at  $X_1$ , giving them back at date 2 at the FX forward rate  $\chi$ , and (3) selling American bonds or the American good. Items (1) and (2) is subject to the box constraint for euros, whereas the amount of bond selling in item (3) is subject the box constraint of the American bond.

### 2.3 Currency basis

We start by assuming agents maximize a utility function defined on the two date bundle  $(x_{E,1}, x_{U,1}, x_{E,2}, x_{U,2})$ . Hereafter, we use the notation  $\mu$  for the Lagrange multiplier of the corresponding constraint.<sup>7</sup> In particular,  $\mu_{U,1}$  is the dollar debt shadow price of the European bank and  $\mu_{\$,2}$  measures how the bank values a marginal increase in its dollars holdings at the second date. With these notations in place, we establish the following result, which derives the cross currency basis  $\beta$  endogenously as a function of the shadow prices of funding constraints.

**Result 1: Endogenous Cross Currency Basis** *In the context of funding by trading bonds, the forward FX rate is:*

$$\chi = X_1 \frac{1 + r_U}{1 + r_E} \left( 1 + \frac{\mu_U - \frac{\mu_E}{X_1}}{(1 + r_U)\mu_{\$,2}} \right)$$

*The forward FX rate  $\chi$  deviates from the frictionless  $X_1 \frac{1+r_U}{1+r_E}$ . Recall that we defined the basis  $\beta$  such that*

$$\chi = X_1 \frac{1 + r_U + \beta}{1 + r_E}. \quad (\text{Basis definition})$$

*With this definition the cross currency basis can be characterized as follows*

$$\beta = \frac{\mu_U - \frac{\mu_E}{X_1}}{\mu_{\$,2}}. \quad (\text{Basis Result 1})$$

<sup>7</sup>Since all constraints are linear, Kuhn-Tucker conditions hold as necessary conditions for the constrained maximization of utility (or of present value profit)

□

The endogenous cross currency basis  $\beta$  can be interpreted as follows. We first set  $\mu_E = 0$  in the basis equation in Result 1. With this simplification, we see that  $\beta = \frac{\mu_U}{\mu_{S,2}}$ . Thus, in the absence of a binding funding constraint in euros, cross currency basis is the marginal value of possessing the dollar denominated bond today, relative to tomorrow's solvency shadow value: this result is very intuitive and says that in an economy with no euro funding constraints, cross currency basis is a transfer pricing mechanism for intertemporal transfer of the scarcer currency. Furthermore, the impact on the basis is the highest in the presence of agents having short term difficulty to raise dollars while clearly staying long term solvent.

When  $\mu_E > 0$ , we see that our model predicts that the cross currency basis should decline. This prediction is also intuitive from an economic perspective: if euros also become scarce, the cross currency basis should decline as there are now convenience yields in *both* currencies.

To see why a possession value comes in the forward, one has to compare a spot transaction with forward transaction. For example, selling euros at date 1 versus locking in this sale at date 1 to be executed at date 2. The difference is not only the prevailing interest rate on both currencies. In the case of spot transaction for the interim period, one possesses dollars instead of euros. The value is thus adjusted for the relative possession value of both currencies.

For dollar such a possession value is driven by *the multiplier on the box of the dollar denominated bond*  $\mu_U$ , which corresponds to the desirability to issue debt in dollars immediately at date 1 (relative to the desirability of date 2 dollars and discounted back to date 1). Notice that  $\mu_U$  is the shadow value of increasing  $e_U$ , that is, of being able to issue bonds. Equivalently, the same additional debt could be incurred by taking a negative position on the bond, doing a short-sale (in a naked way, as we are not modeling repo on bonds yet), so  $\mu_U$  can also be seen as the value attached to violating the no-short-sales constraint (Funding.U).

Comparing the role of bond box multipliers here and in Bottazzi, Luque and Páscua (2011), we see that even now when, for simplifying reasons, we initially do not allow for repo on bonds, the multipliers still capture the desire to possess the bond. In a repo context (as we will show in the next subsection) such a desire is linked to repo specialness, whereas here it is just the desire to issue more debt.

**Remark 1.** Notice that if the agents are profit maximizers instead of utility maximizers, Result 1 would still hold. For instance, let the present value profit function in euro<sup>8</sup> be as follows for a certain discount factor  $\delta$  :

$$p_{E,1}(\omega_{E,1} - x_{E,1}) - (s + f) - b_E + \delta(p_{E,2}(\omega_{E,2} - x_{E,2}) + f + (1 + r_E)(b_E + e_E)) \quad (1)$$

Let  $\tilde{\mu}_{e,1} = \mu_{e,1} + 1$  and  $\tilde{\mu}_{e,2} = \mu_{e,1} + \delta$ . Then it is easy to see that the first order condition on  $b_E$ ,  $s$ , and  $f$  (see equations (4), (5), and (6) in the Appendix, which held for utility maximization will now hold for profit maximization once we replace  $\mu_{e,1}$  by  $\tilde{\mu}_{e,1}$ , and  $\mu_{e,2}$  by  $\tilde{\mu}_{e,2}$ . Hence, Result 1 also applies to the case of profit maximization (the proof is exactly the same as the proof of Result 1).

## 2.4 Funding through repo: the role of haircuts

Previously and by design, we abstracted from repo and short selling of bonds. In practice, however, central bank eligible collateral repo market dominates the short end market in volume and position. In this subsection we examine the consequences if funding were just done through repo, ignoring now bond trades - thus exploring the impact of haircut on the basis. In the next sections we will bring together repo and bond trades (thus allowing for short sales), which will enable us to relate the basis to *both* the unsecured interest rate and the repo rate.

We modify the dollar box no-overdraft equation above as follows:

$$p_{U,1}(\omega_{E,1} - x_{E,1}) + X_1(s + f) - h_U z_U \geq 0 \quad (\text{Repo}.\$.1)$$

where  $z_U$  is the repo pledged out of the collateral initially endowed, and  $1 - h_U > 0$  is the corresponding haircut. To simplify matters, we assume that the haircuts are specified exogenously. We will review the implications of this assumption on our results later. The euro no-overdraft box at date 1:

$$p_{E,1}(\omega_{E,1} - x_{E,1}) - (s + f) - h_E z_E \geq 0 \quad (\text{Repo.e.1})$$

We get the 2 funding constraints as the box on American and European bonds

$$z_U + e_U \geq 0 \quad (\text{Repo.Funding.U})$$

$$z_E + e_E \geq 0 \quad (\text{Repo.Funding.E})$$

<sup>8</sup>We assume that European Banks were the marginal agents under funding pressure in the 2008 crisis, which we address later. Hence looking ahead, we look at profit maximization in Euro.



Finally, we introduce the date 2 dollar and euro no-overdraft boxes - with the repo rate on eligible collateral being  $\rho_U$  and  $\rho_E$  explicitly

$$p_{U,2}(\omega_{U,2} - x_{U,2}) - \chi f + (1 + r_U)e_U + (1 + \rho_U)h_U z_U \geq 0 \quad (\text{Repo.}\$.2)$$

$$p_{E,2}(\omega_{E,2} - x_{E,2}) + f + (1 + r_E)e_E + (1 + \rho_E)h_E z_E \geq 0 \quad (\text{Repo.e.2})$$

With these constraints in place, we are ready to state Result 2.

**Result 2: Effects of haircuts on Cross Currency Basis** *In this simple set up the forward FX rate is:*

$$\chi = X_1 \frac{1 + \rho_U}{1 + \rho_E} \left( 1 + \frac{\frac{\mu_U}{h_U} - \frac{\mu_E}{X_1 h_E}}{\mu_{\$,2}(1 + \rho_U)} \right).$$

and therefore

$$\beta = \frac{1 + r_E}{1 + \rho_E} \left( 1 + \rho_U + \frac{\frac{\mu_U}{h_U} - \frac{\mu_E}{X_1 h_E}}{\mu_{\$,2}} \right) - (1 + r_U), \quad (\text{Basis } \beta \text{ Result 2})$$

and to the first order<sup>9</sup>, the basis is approximately given by

$$\tilde{\beta} = (\rho_U - r_U) - (\rho_E - r_E) + \frac{\frac{\mu_U}{h_U} - \frac{\mu_E}{X_1 h_E}}{\mu_{\$,2}}. \quad (\text{Approximate Basis } \tilde{\beta} \text{ Result 2})$$

□

As in Result 1, the currency basis is driven by the multiplier  $\mu_U$  of the box on bonds denominated in dollars. This shadow value measures now the desire to violate this box constraint by lending more of the bond than one is endowed with. Equivalently, in this no-trade context,  $\mu_U$  is the marginal value of possessing more of the bond.

Observe that when there is no haircut ( $h_U = h_E = 1$ ) the basis is the same as in Result 1 (but using repo rates instead of OIS rates). In fact, in this special case, we have the result that the general collateral rate coincides with the interest rate of the bond (i.e.,  $\rho_E = r_E$  and  $\rho_U = r_U$ , for the European and American bond, respectively)<sup>10</sup>. This implication also rationalizes why investors prefer to quote the basis relative to OIS, which is closer to General Collateral repo rates than relative to Libor. Also notice that if the haircut  $1 - h_U$  increases, so that  $h_U$  decreases (a higher borrowing friction), the basis increases (as dollars become more scarce) - the effect is however opposite for euros - so that the basis represents the relative funding pressure in both currencies.

<sup>9</sup>The exact basis is  $\beta = \tilde{\beta} + \frac{r_E - \rho_E}{1 + \rho_E} (\rho_U - \rho_E + \frac{\frac{\mu_U}{h_U} - \frac{\mu_E}{X_1 h_E}}{\mu_{\$,2}})$ .

<sup>10</sup>It should be emphasized here that the term of the repo funding and the term of the bond are identical in our example.

## 2.5 On the relative funding pressure in the two currencies

We further explain below why the currency basis is driven by the relative funding pressure in the two currencies.

It is generally the case that OIS rates are, on average, closer to General Collateral repo rates than to other rates such as Libor. However, at the time of we wrote this paper - end of 2011 - the difference between the two rates,  $\rho_E - r_E$ , happens to be very dependent on the quality of collateral accessible to the agent in order to avail of repo funding. General euro operations of the ECB (including LTRO) are done at a rate of 1% and this is where the repo of the higher rate countries is also (naturally) trading, as such debt is eligible collateral for the ECB and this is currently their best source of funding. At the same time repo on German government debt is around 0.20%, OIS is around 0.50%. Therefore, whenever the basis is quoted vs rates that are distinct from actual funding rates, the spreads terms seen in the previous formula will be present.

Notice that, fixing the reference basis quoting rate  $r_E$  to OIS, if access to domestic funding is very good (low  $\rho_E - r_E$ , low haircut, and low  $\mu_E$ ) as was generally the case at the end of 2011 for German and to a lesser extent for French banks, then the cross currency basis is pushed higher, as it captures a higher relative funding pressure. Anecdotal evidence shows the involvement of such banks in cross border holdings has been actually very high, and this feeds through the relative importance of dollar operations done by the ECB compared to other central banks. This insight tends to counter the credit-risk interpretation of the basis. In fact, it is when the relative funding pressure *difference* is highest that the basis impact is the highest: a good domestic credit increases the basis. The beauty of the first dollar operation, conducted by the ECB in 2008, was to actually recognize the situation and accept euro collateral to inject dollars.

## 2.6 Currency scarcity and bond scarcity

*In a trade and repo context*, combining subsections 2.3 and 2.4, the box constraint becomes  $b_U + e_U + z_U \geq 0$  and  $\mu_U$  would capture the desire of doing a short sale not covered by borrowing the bond or of lending the bond without having it. More precisely, it would measure the marginal value of being in possession of an additional amount of the bond, that could then be short sold (without having to borrow it through reverse repo) or lent in repo. Hence, a positive  $\mu_U$  means that

the agent wants to have immediate funding in dollars and attaches a value to possessing the bond that provides this funding. Such scarcity of the bond brings the repo rate on the bond below the risk-free rate (as shown in Bottazzi, Luque and Páscua (2011)), that we can take to be OIS.

In our model we have only one class of dollar denominated bonds, so we cannot distinguish the repo rates of different bonds and talk about specialness. In a model with several bonds, if the general collateral rate coincides with OIS, the repo-OIS differential is the repo specialness of the bond. However, a positive  $\mu_U$  is compatible with no specialness, when the general collateral rate is already below OIS. That is, a positive currency basis  $\beta$  is driven by a funding difficulty in dollar denominated bonds but can coexist with these bonds not being on special in repo markets. This is because GC is defined as the highest repo rate within a class of bonds - but the overall class itself can become scarce - it would be reflected in GC vs OIS.

### 3 Focusing on the 2008 dollar squeeze

#### 3.1 Overview of the crisis

We review below the main features of the 2008 dollar squeeze, in order to provide a perspective for modeling this crisis.

As documented before, between 2001 and 2008 European banks increased their holdings of US dollar denominated ABS (mostly residential and commercial mortgages). Originally, such dollar funding was raised through various avenues:

1. Asset Back Commercial Paper (ABCP),
2. Repo, where the ABS is a security with a good repo market, or
3. Direct funding, possibly unsecured, where the bank is lent dollar against euro in the open market.

McGuire and von Peter (2009) estimate that, “until the onset of the crisis, European banks had met their dollar funding needs through the interbank market (\$400 billion), borrowing from central banks (\$800 billion), and using FX swaps (\$800 billion) to convert domestic currency funding into dollars”.

All three avenues (1), (2) and (3) came under significant stress during the 2008 crisis. Vast market segments in (1) and (2) got shut down, as some European banks had been relying on such funding markets to fund these assets, sometimes through vehicles, such as structured investment vehicles or SIVs that got brought back on balance-sheet. In (3) private counter-parties were reluctant to lend their dollars because they were worried about the financial health of their counter-parties and they themselves were hoarding dollars. In fact, as shown by Imakubo et al. (2008), the LIBOR-OIS<sup>11</sup> spread (an indicator of credit risk and liquidity premium) significantly increased between August 2007 and April 2008. European banks could not afford to realize the loss of selling their American ABS long positions at the fire-sale price (“distress selling”). And sometimes the market for their ABS market did not even exist. The bottom line was that they needed to maintain their US dollar funding or face bankruptcy. So the situation in the crisis was one of acute dollar funding need - we want our framework to capture such a pressure and be able to model the policy response of central banks in this new situation.

In view of above market developments, central banks had to step in and coordinate and, as much as possible, try to expand the supply of dollars wherever needed. The Fed could not do it alone because in the end the most critical need for dollars was outside its jurisdiction, i.e., the marginal class of players facing the dollar shortage were European banks. The foreign central banks could not do it alone either as they cannot create dollars. This meant that other foreign central banks had to channel those dollars, and against these the foreign central bank had to accept a collateral in a foreign currency (the foreign central bank’s own). The way Fed worked with the ECB to handle such a dollar demand was to do a spot foreign exchange (FX), giving them dollars at a spot rate  $X$  (dollars per euro), combined with a forward sale of the same amount of euros (in period 2 in our model) at rate  $\chi$  (dollars per euro).<sup>12</sup>

The dollars that ECB borrowed were given to European banks through cross-currency repo. Such repos were actually quite different from repos on government

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<sup>11</sup>LIBOR is the rate on unsecured inter-bank lending, whereas the OIS, in practice, it is used as a GC rate proxy and seen as the expected overnight interest rate, with limited credit and liquidity risk. See Imakubo et al. (2008) for the relationship of LIBOR and OIS with credit and liquidity risk.

<sup>12</sup>The amount of dollar swaps that the Federal Reserve made with the ECB had a peak in December 2008 (see Figure 2 on other months and also on swaps with other central banks).

bonds. European banks issued new euro denominated bonds,<sup>13</sup> backed by non-related European assets (for which the ECB has some expertise). The ECB took those euro covered bonds together with already existing bonds as a collateral against dollar loans.

The holdings of American assets (e.g., subprime ABS, newly illiquid MBS and CMBS) that need funding could not directly be pledged unless the bank can access the American Term Auction Facility (TAF) program through its US affiliates. However, as Goldberg, Kennedy and Miu (2010) point out, the TAF facility was not enough, by itself, to ease the strains in money markets after the Lehman Brothers bankruptcy episode. The central banks swap facilities were crucial for the “normalization” of the LIBOR. See also McAndrews, Sarker and Wang (2008) on the effectiveness of the TAF program on the LIBOR rate during the crisis period.

In the model below, we aggregate all European and American eligible collateral with interest rates  $r_E$  and  $r_U$ , respectively (also referred here as the euro rate and the USD rate). These rates correspond to relevant funding rates of each bank through its central bank. In practice, it will be very close to the repo rate in domestic repo operation of the central banks using high grade domestic collateral. It is a short rate in its nature and usually is very close to short term OIS and GC rates, but such rates can of course differ.<sup>14</sup> See Bech, Klee and Stebunovs (2010) for a characterization of the relationship between the Treasury GC repo rate and the federal funds rate in three different periods: before the crisis (from 2002 to 2007), the early stage of the crisis (from August 2007 to December 2008), and after the Fed intervention. As shown by these authors, the federal funds rate communicated policy to the repo market quite well in the pre-crisis period, but after December 2008 this relationship deteriorated.

### 3.2 Modeling the 2008 dollar squeeze and the central banks intervention

In our simple model with two dates we need to indicate the different type of economic agents. We will introduce two central banks, the European Central

<sup>13</sup>See ECB document “The implementation of monetary policy in the Euro area”, February 2011: <http://www.ecb.int/pub/pdf/other/gendoc2011en.pdf>, page 50.

<sup>14</sup>In fact, funding can be thought of as happening in the repo market of central bank eligible collateral. Generalization to various kinds of available collateral and rates is direct.

Bank (ECB) and the Federal Reserve (Fed). Additionally, we shall distinguish European banks (labeled by  $i$ ) and American banks (labeled by  $j$ ). In our set-up, each European (American) bank starts with a relatively larger amount of euro (dollar) collateral. To simplify, we chose to think of these private banks as being banks and consumers at the same time. But arguments go through if they are profit maximizers.

For the sake of simplicity, we ignore international trade in commodities. Trade thus occurs within the two areas, US and EU, and not between them. We denote by  $p_{E,t}$  and  $p_{U,t}$  the commodity price vectors in the European and US markets at date  $t$ . Our framework is competitive for all effects, and therefore both European and American banks take commodity prices as given.

European and American bonds trades (and initial holdings) are denoted by  $b_E$  and  $b_U$ , respectively ( $e_E$  and  $e_U$ ). Formally, we assume to the same effect that European banks have a large endowment in European government bond and the American banks in US Treasuries. To simplify, we ignore all differences between eligible collateral, in particular the differences among government bonds for different euro-members – in essence we can focus on the German government case as the base case.<sup>15</sup>

In this simple model we identify covered bonds with other regular eligible euro denominated bonds, and look at the introduction of cross currency repo by the ECB (using dollars from the Fed), using all eligible bonds as an abstract representation of the overall funding capability of the European banks in euro.

We formulate the resource constraints or box constraints as before: precise details are shown in the appendix. We directly move to results below.

### 3.3 Cross currency basis in the 2008 crisis

#### 3.3.1 Before the Central Banks intervention

**Result 3: Basis in the Absence of central bank dollar operations** *In the absence of dollar operations, with European banks starving for dollars, the FX forward and FX spot rates are related as in Result 1. The relative possession*

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<sup>15</sup>All European banks hold different member debt, and all are eligible collateral with the central bank. The friction comes from the cross currency funding anyhow.

value of dollar Vs. euro is given by  $\beta/(1+r_E)$ , where  $\beta$  is the basis found in Result 1.

The proof of Result 3 follows the proof of Result 2.

As before, the forward FX rate  $\chi$  modifies the frictionless term  $X_1(1+r_U)/(1+r_E)$ . The possession value of one dollar between dates 1 and 2 is

$$\$Pos = \frac{1}{(1+r_E)} \frac{\mu_U^i}{\mu_{\$,2}^i}, \quad (\$Pos)$$

and for 1 euro the possession value is:

$$EPos = \frac{X_1}{(1+r_E)} \frac{\mu_{E,1}^i}{\mu_{\$,2}^i}. \quad (EPos)$$

□

We discuss the implications of Result 3 next by offering the following comments on funding needs and solvency:

A positive multiplier  $\mu_U$  driving the currency basis signals that European banks have an immediate funding need in dollars. In a trade-only set up (of Result 1), the higher is  $\mu_U$ , the more they wish they could issue debt at the dollar denominated bonds rate  $r_U$ . If we were to allow for both bond trades and bond loans (like in subsection 2.6) the multiplier signals the desire to obtain dollar funding by short-selling or by lending the bond (that is, a value to possessing it for these purposes). In any case, there is a scarcity of the bond.

In a set-up with several bond categories, the basis would be driven by positive multipliers  $\mu_U$  of any scarce bond (any bond whose box constraint is binding and with a positive shadow value for some agent). Following Lehman Brothers' bankruptcy and until the Fed increased the supply of treasuries in October 2008, scarcity coexisted with specialness (repo rate below GC) in many bond categories (and with a peak in repo fails, due to the difficulty in getting a hold on them). However, after the Fed stepped in, specialness was gone but the currency basis persisted. That is,  $\mu_U$  remained positive, which is equivalent to saying that the general collateral rate remained below OIS.

The funding problem signaled by  $\mu_U > 0$  is also related to the solvency problem signaled by another shadow price, the multiplier  $\mu_{\$,1}$  of the no-overdraft constraint (10) in dollars at date 1. In fact, the first order condition on bond

trades requires  $\mu_{\$,1} = \mu_U + \mu_{\$,2}(1+r_U)$  and, therefore, the marginal rates  $\mu_{\$,1}/\mu_{\$,2}$  and  $\mu_U/\mu_{\$,2}$  move together, as long as  $r_U$  stays the same. Hence, a funding need is concomitant with a solvency difficulty, with being on the verge of defaulting. To avoid expanding the model and losing the focus on the basis issue, we chose not to model defaults, but the potential default is there, captured by the desire  $\mu_{\$,1}$  to do it.

### 3.3.2 Dollar scarcity, euro plenty

To get some intuition for the 2008 crisis, one can simplify the above formulas by assuming there existed plenty of European bonds. As a consequence, the euro denominated bonds box constraint was non binding (as  $e_E^i$  is very big). In this case, the possession value of a euro goes down to 0 and one gets:

$$\chi = X_1 \frac{1+r_U}{1+r_E} \left(1 + \frac{\mu_U^i}{\mu_{\$,2}^i(1+r_U)}\right).$$

Thus, the cross currency basis is driven by the possession value of dollars between date 1 and date 2.

## 3.4 Intervention of Central Banks

### 3.4.1 FX Swap Lines

The situation now is one where the dollar funding market is frozen for the European banks. In this case the European banks will turn to ECB to borrow dollars through repo in exchange of euro covered bonds. This implies that the collateral has to be taken into account in the box constraints of the euro covered bond, whereas the cash loans will appear in the dollar no-overdraft box constraints of dates 1 and 2 multiplied in both cases by the spot rate  $X_1$ . We denote by  $\rho$  the repo rate chosen by ECB. The first order condition with respect to  $z_E^i$  is

$$\mu_{\$,1}^i X_1 h = \mu_E^i + X_1 \mu_{\$,2}^i (1 + \rho) h \quad (2)$$

As pointed out by Baba, McCauley, and Ramaswamy (2009) and Coffey, Hrungrung and Sarkar (2009), the cost of borrowing euros in unsecured markets (at the euro LIBOR) and swapping these euros for dollars was higher than borrowing dollars directly in the unsecured market (at the dollar LIBOR), in turn higher than borrowing dollars using the ECB repo facility.



**Result 4: Effect of ECB intervention on basis** *If the best option to raise dollars is to use the dollar repo facility of the ECB, then the forward FX rate is:*

$$\chi = X_1 \frac{1 + \rho}{1 + r_E} \left( 1 + \frac{(1 - h)\mu_E^i}{h(1 + \rho)X_1\mu_{\$,2}^i} \right)$$

□

It is interesting to observe that when the haircut  $(1 - h)$  increases, so  $h$  decreases,  $\chi$  increases. This is because the friction of borrowing dollars through the ECB facility is higher the higher the haircut.

In the post-intervention the repo facility is used, so the FX forward rate  $\chi$  can be obtained according to both Results 3 and 4, which implies the following equation

$$\beta = \rho - r_U + \frac{(1 - h)\mu_E^i}{hX_1\mu_{\$,2}^i}.$$

Thus the cross currency basis can be decomposed into a spread that is in direct control of the coordinating central banks  $\rho - r_U$ . This spread has been recently lowered to promote supply of dollar. Our model clearly implies that such an action will serve to lower the basis. But also clearly, the second term shows the importance of the pool of eligible collateral and the haircut, which is yet another policy lever available to the central banks. Our model suggests the following: if the euro collateral is scarce for the European bank, then  $\mu_E^i > 0$  increases the dollar basis even in the presence of dollar supply operations. It is also clear that requiring a higher haircut increases the basis when the eligible collateral is scarce.

Our result 4 makes it clear why it is effective for the central bank to make the pool of eligible collateral as wide as possible – in the limit we will look at the case of a collateral that is abundant for users of the dollar operations of the central bank. In this case  $\mu_E^i = 0$ .

When  $\mu_E^i = 0$ , the cross currency basis  $\beta$  applied on the dollar side becomes driven by the following formula:

$$\beta = \frac{\mu_U^i}{\mu_{\$,2}^i} = \rho - r_U$$

**Result 5: Effect of ECB supply of dollar when eligible euro collateral is abundant** *Assuming there is plenty of euro collateral, under the central banks' intervention, the basis  $\beta$  becomes, to the first order,  $\rho - r_U$ .*

□

Result 5 shows that, to a first approximation, cross currency basis is in fact equal to the spread between the general collateral repo rates and the OIS rates, from the perspective of the marginal agent.

We now briefly review evidence on the quantity of dollar provided to the market. Note that the policy was put in place in December 2007 and the balances borrowed by the ECB rose quickly to \$450 billion in December 2008. It took about a year to bring those balances down marking a normalization of the market in 2009.

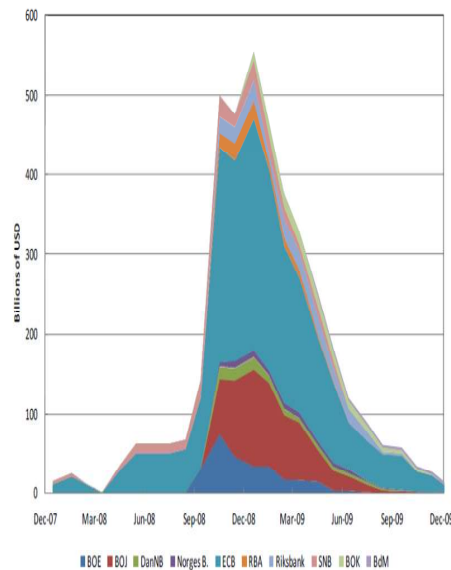


Figure 2: **CB Dollar Swap Amounts Outstanding by Foreign Central Bank.** *Source:* Golberg et al. (2010) based on Federal Reserve System's Monthly Reports on Credit and Liquidity Programs and the Balance Sheet.

### 3.4.2 Calibration

Observe that after the intervention  $\beta$ , as measured by the spread,  $\rho - r_U$ , by virtue of Result 5, is empirically much lower than the 400 basis points peak level. This result is consistent with the empirical evidence shown by Goldberg, Kennedy, and Miu (2010) that the dollar swap lines among central banks were effective in reducing the dollar funding pressures outside the U.S. and other stresses in the money markets.

We can calibrate Result 5 to data to assess the effects of ECB policy on cross currency basis as follows. The repo rate at which the ECB provided dollars to European banks was, for most of the last quarter of 2008 and the first half of 2009, set at a fixed level of approximately 100 basis points above the dollar OIS rate. As the latter (taken here to be  $r_U$ ) was around 20 bps in December 2008, the repo rate  $\rho$  was approximately 120 bps in at this time (this is the rate referred to in Figure 4 as Foreign Central Bank rate). Notice that this rate is significantly higher than the TAF rate at which US banks could obtain dollars (see Figure 4, which compares TAF stop-out rates to OIS and Libor for one month term).

The basis  $\beta$  implied by the main result above is 100 bps, in December 2008, relative to OIS. Relative to LIBOR rates, the basis  $\beta$  is 130 basis points<sup>16</sup>, at this time, which is very close to the data reported for that month as may be seen from Figure 3 below.

It implies also a basis  $\alpha$  on the foreign leg of -100 basis points, for December 2008, both taking  $r_E$  and  $r_U$  as the OIS rates (1.57% and 0.28%, respectively) or as the Euribor and LIBOR rates (3.452% and 1.829%, respectively). In the data,  $\alpha$  in December 2008, relative to OIS, ranges from -150 early in the month to -40 at the end of the month.

<sup>16</sup>Notice that  $\frac{1+r_U^{OIS}+\beta^{OIS}}{1+r_E^{OIS}} = \frac{1+r_U^{LIBOR}+\beta^{LIBOR}}{1+r_E^{LIBOR}}$

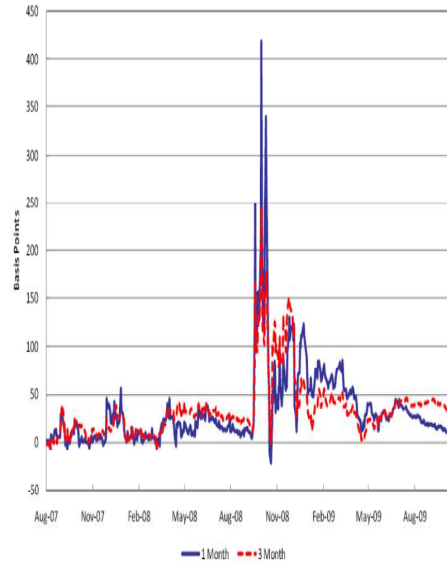


Figure 3: **Euro - U.S. Dollar Implied Swap Basis Spread relative to LIBOR (dollar leg.** *Source: Golberg et al. (2010) based on Reuters, Federal Reserve Bank of New York staff calculations.*

### 3.4.3 Collateral Menu

As we discussed earlier, the funding options available on the euro side are important to insure that we have a “euro-plenty” situation with  $\mu_E = 0$ . Expanding eligible collateral on the euro side certainly has helped in this direction. Also ECB’s recent LTRO is also a step in this direction: as European banks hoard funding through LTROs for the funding of euro assets, the less precious collateral for the dollar operations is tied in euro operations.

Moreover recent policy action shows the central bank going back to the root of the problem. Originally the dollar funding pressure has been created because European bank could not fund a dollar asset (asset backed) in the market. A natural idea is for the ECB to provide such funds accepting the dollar collateral on repo when the market ceases to accept it. Essentially the ECB is doing a dollar repo better than the market could provide.

Using similar analysis as in the open market repo section, and denoting by  $(1 - \tilde{h}_u)$  the haircut chosen by the ECB, and by  $\tilde{\mu}_U^i$  the European bank’s box multiplier for this closed repo operation, we obtain:

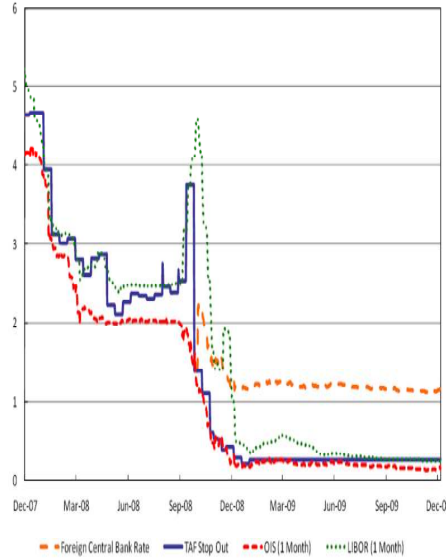


Figure 4: **TAF Stop Out, dollar OIS (1m), and dollar LIBOR (1m).**  
*Source: Golberg et al. (2010) based on Bloomberg (OIS, LIBOR), Federal Reserve (TAF stop out).*

**Result 6: Accepting dollar collateral in the dollar operations.** *When ECB accepts dollar collateral when lending dollars and if this is the best dollar funding option for a European bank, then the basis depends on the ECB's policy repo rate  $\tilde{\rho}_u$  in the following way:*

$$\beta = \tilde{\rho}_U - r_U + \frac{\frac{\tilde{\mu}_U^i}{h_U} - \frac{\mu_E^i}{X_1}}{\mu_{\$2}^i} \quad (\text{Basis Result 6})$$

where  $(1 - \tilde{h}_u)$  is the haircut and has an effect when the collateral becomes scarce.

So overall if there is abundant unused eligible collateral – like for dollar operation using euro collateral – the main driver of the basis becomes the spread, here  $\tilde{\rho}_u - r_u$ . Moreover the same principles apply for the impact of the haircut: a higher required haircut  $1 - \tilde{h}_U$  will decrease the policy impact of the operation unless unpledged dollar collateral is abundant<sup>17</sup>.

<sup>17</sup>Result 6 is formally close to Result 2, with two differences: (1) the relevant repo rate on dollar funding is a policy variable of the ECB and (2) by allowing for both trades and repo on european funding, we could use the interest rate  $r_E$  and ignore the haircut factor  $h_E$ .

## 4 Conclusion

Our paper offers a theoretical framework to better understand the widening of the cross currency basis during the credit crisis of 2008, and the activation of dollar swap lines by the Federal Reserve in coordination with other central banks such as the ECB. The main insight of the paper was that there was a significant “convenience yield” or physical possession value in the scarcer currency, namely, dollar. Our theory shows that this is priced into the cross currency basis. The model proposed here links the cross currency basis to shadow costs associated with funding constraints in the spot markets and on the forward maturity date. Cross currency basis is the rent paid for possession of the scarce currency for one period as a fraction of the long term value of the scarce currency. The model allows us to examine the ameliorating effects of ECB’s repo facility in a quantitative manner. The paper also establishes a conceptual link between the “repo specialness” in securities markets and the cross currency basis in FX swap markets. Finally, our model offers a framework for examining the channels through which the tools of central banks, such as a) price of liquidity, b) volume of liquidity, c) menu of collateral, and d) haircuts, influence the cross currency basis.

## 5 Appendix

### 5.1 Proof of Result 1

The necessary first order optimality conditions (FOC) with respect to  $b_U$  (funding dollar),  $b_E$  (funding euro),  $s$  (spot), and  $f$  (FX swap) are, respectively,

$$\mu_{\$,1} = \mu_U + \mu_{\$,2}(1 + r_U) \quad (3)$$

$$\mu_{e,1} = \mu_E + \mu_{e,2}(1 + r_E) \quad (4)$$

$$X_1 \mu_{\$,1} = \mu_{e,1} \quad (5)$$

$$\chi \mu_{\$,2} + \mu_{e,1} = \mu_{e,2} + X_1 \mu_{\$,1} \quad (6)$$

By using (5) we can simplify the initial form of (6) and get:

$$\chi \mu_{\$,2} = \mu_{e,2} \quad (7)$$

Then,

$$\chi \stackrel{(7)}{=} \frac{\mu_{e,2}}{\mu_{\$,2}} \stackrel{(4)}{=} \frac{\mu_{e,1} - \mu_E}{(1+r_E)\mu_{\$,2}} \stackrel{(5)}{=} \frac{X_1\mu_{\$,1} - \mu_E}{(1+r_E)\mu_{\$,2}} \stackrel{(3)}{=} X_1 \frac{1+r_U}{1+r_E} \left( 1 + \frac{X_1\mu_U - \mu_E}{X_1\mu_{\$,2}(1+r_U)} \right)$$

## 5.2 Proof of Result 2:

In this case the FOC with respect to  $z_U$  (repo dollar funding), and  $z_E$  (repo euro funding) are, respectively,

$$h_U\mu_{\$,1} = \mu_U + \mu_{\$,2}h_U(1+\rho_U) \quad (8)$$

$$h_E\mu_{e,1} = \mu_E + \mu_{e,2}h_E(1+\rho_E) \quad (9)$$

The first order conditions with respect to  $s$  and  $f$  are as (5) and (6). Here again, we can simplify the initial form of (6) and get (7). Now,

$$\begin{aligned} \chi &\stackrel{(6)}{=} \frac{\mu_{e,2}}{\mu_{\$,2}} \stackrel{(9)}{=} \frac{h_E\mu_{e,1} - \mu_E}{h_E(1+\rho_E)\mu_{\$,2}} \stackrel{(5)}{=} \frac{X_1h_E\mu_{\$,1} - \mu_E}{h_E(1+\rho_E)\mu_{\$,2}} \\ &\stackrel{(8)}{=} X_1 \frac{1+\rho_U}{1+\rho_E} \left( 1 + \frac{\mu_U}{h_U\mu_{\$,2}(1+\rho_U)} - \frac{\mu_E}{h_EX_1\mu_{\$,2}(1+\rho_U)} \right) \end{aligned}$$

## 5.3 Relevant institutions, their constraints and objectives

Recall that the repo facility provided by the ECB is a cross currency repo: the collateral (whose positions are  $z_E^{ECB}$  and  $z_E^i$  for the ECB and European bank  $i$ , respectively) is denominated in euros but the cash loan is in dollars, so the haircutted cash loans appear in the dollar box constraints, but the collateral has to be taken into account in the boxes for euro denominated bonds.

### 5.3.1 ECB

We denote by  $s$  spot trades (euro sold independently of the FX swap). For ECB at date 1, the dollar box constraint is :

$$X_1(s^{ECB} + f^{ECB} - hz_E^{ECB}) - b_U^{ECB} \geq 0 \quad (ECB.\$.1)$$

The haircut  $1 - h$  ranged from 10% for 1 day to 17 % for one month, significantly above TAF haircuts that never exceeded 5 %. In equilibrium we expect  $z_E^{ECB} > 0$  (long in repo) and  $f^{ECB} > 0$ , as the ECB only accepts collateral, and is expected to buy-sell dollars (purchaser of dollar in the first leg of the swap).

The ECB's box for euros is:

$$W_{e,1}^{ECB} \equiv W_{e,0}^{ECB} - (s^{ECB} + f_{ECB}) + I_{e,1}^{ECB} - b_E^{ECB} \geq 0 \quad (\text{ECB.e.1})$$

where  $W_{e,0}^{ECB} \geq 0$  represents the ECB initial euro balance, whereas  $I_{e,1}^{ECB}$  stands for the variation of economy-wide euros balances printed or withdrawn at date 1. Notice that the central bank may have to issue currency in order to have non-negative currency balances. Also the ECB may have to inject euro when it purchases foreign currency or European government bonds.

The ECB's boxes for American and European bonds are, respectively:

$$b_U^{ECB} + e_U^{ECB} \geq 0 \quad (\text{ECB.U})$$

$$b_E^{ECB} + e_E^{ECB} + z_E^{ECB} \geq 0 \quad (\text{ECB.E})$$

For simplicity, we assume that the ECB only does dollar repo on euro bonds.

At date 2, ECB's dollar and euro box constraints are, respectively:

$$-\chi f^{ECB} + (1 + \rho)X_1 h z_E^{ECB} + (1 + r_U)(b_U^{ECB} + e_U^{ECB}) \geq 0 \quad (\text{ECB}.\$.2)$$

$$W_{e,2}^{ECB} \equiv W_{e,1}^{ECB} + f^{ECB} + I_{e,2}^{ECB} + (1 + r_E)(b_E^{ECB} + e_E^{ECB}) \geq 0 \quad (\text{ECB.e.2})$$

### 5.3.2 European banks

When it comes to the 2008 financial crisis, European banks because of their large cross border funding exposure have been the relevant marginal agent. Here we will deploy their constraints in the context of the dollar operations of the Fed and ECB. Essentially the dollars are provided through a cross currency repo between the ECB and the European bank. At date 1 a European bank has a dollar box constraint:

$$X_1(s^i + f^i - h z_E^i) - b_U^i - L^i \geq 0 \quad (10)$$

Here, in equilibrium we expect  $z_E^i < 0$ . Some maturing funding that cannot be rolled, here represented as a liability  $L^i > 0$ . Such maturing funding can take the form of a FX forward, a repo settlement or another form of debt.

Finally, bank  $i$ 's box for euros is:

$$e_1(\cdot) \equiv p_{E,1}(\omega_{E,1}^i - x_{E,1}^i) - (s^i + f^i) - b_E^i \geq 0 \quad (11)$$

where  $\omega_{E,1}^i$  is bank  $i$ 's initial endowment of commodities, sold in the European markets.



Bank  $i$ 's box for the US and European bonds are, respectively:

$$b_U^i + e_U^i \geq 0 \quad (12)$$

$$b_E^i + e_E^i + z_E^i \geq 0 \quad (13)$$

At the following date, the European bank  $i$  has dollar and euro boxes, given by, respectively:

$$(1 + \rho)X_1 h z_E^i - \chi f^i + (1 + r_U)(b_U^i + e_U^i) \geq 0 \quad (14)$$

$$e_2(\cdot) \equiv p_{E,2}(\omega_{E,2}^i - x_{E,2}^i) + f^i + (1 + r_E)(b_E^i + e_E^i) \geq 0 \quad (15)$$

Recall that just as in Remark 1 of Section 2 for the objective function of European bank we can use euro profit in the form of  $e_1 + \delta e_2$  for a given discount factor  $\delta$ . Note that one may prefer a utility as it can allow for a risk aversion attitude on the bank side.

Note that all results derive from first order condition of either a generic agent or European banks - thus those are the only agents for whom objective functions should be specified. Results will hold for any objectives of other institutions (namely the central banks) as long as the constraints of these institutions are satisfied.

### 5.3.3 Fed

At the initial date the Fed has the following box constraint for dollars:

$$W_{\$,1}^{Fed} \equiv W_{\$,0}^{Fed} + I_{\$,1}^{Fed} + X_1(s^{Fed} + f^{Fed}) - b_U^{Fed} \geq 0 \quad (16)$$

where we expect  $s^{Fed} < 0$  in equilibrium (as the Fed is a spot seller of dollars), compensated by issuing currency ( $I_{\$,1}^{Fed} > 0$ ) or by making  $b_U$  sufficiently negative (possibly issuing debt at the same time, i.e., increasing  $e_U^i$ ), so that  $W_{U,1}^{Fed} = W_{U,0}^{Fed}$ . The Fed no overdraft box constraint for Euros is:

$$-(s^{Fed} + f^{Fed}) - b_E^{Fed} \geq 0 \quad (17)$$

Box constraints of American and European bonds are, respectively:

$$b_U^{Fed} + e_U^{Fed} \geq 0 \quad (18)$$

$$b_E^{Fed} + e_E^{Fed} \geq 0 \quad (19)$$

At the following date the Fed's box constraints of dollars and Euros are, respectively:

$$W_{\$,2}^{Fed} \equiv W_{\$,1}^{Fed} + I_{\$,2}^{Fed} - \chi f^{Fed} + (1 + r_U)(b_U^{Fed} + e_U^{Fed}) \geq 0 \quad (20)$$

$$f^{Fed} + (1 + r_E)(b_E^{Fed} + e_E^{Fed}) \geq 0 \quad (21)$$

where  $f^{Fed} > 0$  in equilibrium.

We have decided to omit the modeling of American banks constraints as they do not play a role in our results.

## 5.4 Proof of Result 4

The first order conditions with respect to  $s$  and  $f$  are again (5) and (6), respectively. Also here, we can simplify the initial form of (6) and get (7). Now, using the FOC with respect to  $b_E^i$ , we have

$$\chi = \frac{\mu_{e,1}^i - \mu_E^i}{(1 + r_E)\mu_{\$,2}^i}$$

Using the FOC with respect to  $s$ , we have

$$\chi = \frac{X_1\mu_{\$,1}^i - \mu_E^i}{(1 + r_E)\mu_{\$,2}^i}$$

Finally, using (2) we get

$$\chi = X_1 \frac{1 + \rho}{1 + r_E} \left( 1 + \frac{(1 - h)\mu_E^i}{h\mu_{\$,2}^i(1 + \rho)X_1} \right)$$

■

## 5.5 Proof of Result 6

Similar to the proof of Result 1, we can use the FOC wrt  $s$ ,  $f$ , and  $b_E$ , to get:

$$\chi \stackrel{(7)}{=} \frac{\mu_{e,2}^i}{\mu_{\$,2}^i} \stackrel{(4)}{=} \frac{\mu_{e,1}^i - \mu_E^i}{(1 + r_E)\mu_{\$,2}^i} \stackrel{(5)}{=} \frac{X_1\mu_{\$,1}^i - \mu_E^i}{(1 + r_E)\mu_{\$,2}^i}$$

Recall that now the collateral is in dollars. Hence, in the dollar no-overdraft constraints,  $X_1$  does not multiply the haircuted repo trade  $\tilde{h}_U z_U^i$ . So, the FOC with respect to  $z_U^i$  is:

$$\mu_{\$,1}^i \tilde{h}_U = \tilde{\mu}_U^i + \mu_{\$,2}^i(1 + \tilde{\rho}_U)\tilde{h}_U$$

Then

$$\chi = \frac{X_1 \frac{\tilde{\mu}_U^i}{h_U} + X_1 \mu_{\$,2}^i (1 + \tilde{\rho}_U) - \mu_E^i}{(1 + r_E) \mu_{\$,2}^i}$$

and therefore

$$\chi = X_1 \frac{1 + \tilde{\rho}_U}{1 + r_E} \left( 1 + \frac{\tilde{\mu}_U^i}{\mu_{\$,2}^i \tilde{h}_U (1 + \tilde{\rho}_U)} - \frac{\mu_E^i}{\mu_{\$,2}^i X_1 (1 + \tilde{\rho}_U)} \right)$$

Comparing the above expression with

$$\chi = X_1 \frac{1 + r_U + \beta}{1 + r_E}$$

we get

$$\beta = \tilde{\rho}_U - r_U + \frac{\frac{\tilde{\mu}_U^i}{h_U} - \frac{\mu_E^i}{X_1}}{\mu_{\$,2}^i}$$

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