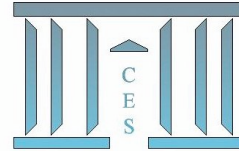




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Land use dynamics and the environment

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Land use dynamics and the environment

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Abstract

We build a benchmark framework to study optimal land use, encompassing land use activities and environmental degradation. We focus on the spatial externalities of land use as drivers of spatial patterns: even if land is immobile by nature, location's actions affect the whole space through pollution, which flows across locations resulting in both local and global damages. In contrast to the previous literature on spatial dynamics, we prove that the social optimum problem is well-posed, *i.e.*, the solution exists and is unique. Taking advantage of this result, we illustrate the richness of our model by means of a numerical analysis. Considering a global dynamic algorithm, we find that our model reproduces a great variety of spatial patterns related to the interaction between land use activities and the environment. In particular, we identify the central role of abatement technology as pollution stabilizer, allowing the economy to achieve stable steady states that are spatially heterogeneous.

Keywords: Land use, Spatial dynamics, Pollution.

Journal of Economic Literature: Q5, C6, R1, R14.

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1 Introduction

Land use activities are usually defined as the transformation of natural landscapes for human use or the change of management practices on human-dominated lands (Foley *et al.*, 2005). It is widely accepted that these activities have greatly transformed the planet's surface, encompassing the existence and evolution of spatial patterns (see, for instance, Plantinga, 1996; Kalnay and Cai, 2003; and Chakir and Madignier, 2006). In this regard, Spatial Economics analyses the allocation of resources over space as well as the location of economic activity and, thus, the formation of spatial patterns. In particular, great effort has been devoted to understanding firms' location, transport costs, trade, and regional and urban development (Duranton, 2007). However, the spatial drivers behind the interaction between land use and the environment are still far from being understood. In this paper we contribute to the theoretical foundations of land use change and the environment by considering the interaction between land use activities and pollution. To this end we will develop a theoretical model that focuses on the spatial externalities of land use as drivers of spatial patterns.

There is an abundant literature on the interaction between land use and pollution. In particular, agricultural research has devoted great attention to the effects of pollution on agricultural land use. For instance, Heck *et al.* (1984), USEPA (1984) and Adams *et al.* (1986) have considered the adverse effects of air pollution on vegetation, including crops. From a climate change perspective, overall a slight benefit to agricultural activities has been predicted (see, among others, Adams, 1989; Deschênes and Greenstone, 2007; and Haim *et al.*, 2011, for the US; and Olesen and Bindi, 2002, for Europe). However, Olesen and Bindi (2002), and Deschênes and Greenstone (2007) point out that there will be considerable spatial heterogeneity across states and countries in this regard. Moreover, these studies also recognize the necessity to consider other effects of climate change, such as human health damages, sea level rise, soil degradation, biodiversity, etc. About the environmental effects of land use Kalnay and Cail (2003) conclude that changes in land use due to urbanization and agriculture may explain the general increase in the minimum and slight decrease in the maximum surface temperature. Houghton *et al.* (1999), Houghton and Hackler (2001), Matson *et al.* (1997), and Tilman *et al.* (2001) have also identified significant environmental impacts of land use. Moreover, Foley *et al.* (2005) point out that the effects of environmental degradation due to land use are global but also regional/local.

Although this literature has been very fruitful, the dominant approach has been

empirical. Therefore, there is a general agreement about the lack of explicit modelling of the spatial drivers behind the interaction between land use and pollution. Closely related to the integrated assessment approach, bottom-up models of agricultural economics (for instance, de Cara and Rozakis, 2004; de Cara *et al.*, 2005; and Havlík *et al.*, 2011) have contributed to the understanding of the spatial drivers of land use. However, these models focus on partial equilibrium (mainly the supply side) and do not completely consider the intertemporal dimension of the problem. In this paper we use an alternative approach based on the Dynamic Spatial Theory (see Desmet and Rossi-Hansberg, 2010, for a survey). Even if this approach was only recently developed, it is based on an old and central question in economic theory: the optimal and market allocation of the economic activity across space (see the seminal works of Hotelling, 1929; and Salop, 1979).

Within this theory, one can identify three distinct sets of models. The natural spatial generalization of the Ramsey model is presented in Brito (2004) and Boucekkine *et al.* (2009). Both include a policy maker who decides the trajectory for consumption at each location. The main feature of these models is the spatial dynamics of capital, which flows in space to meet optimal decisions according to a partial differential equation. Although these sophisticated models are promising, they are *ill-posed* in the sense of Hadamard (1923): one cannot ensure either existence or uniqueness of solutions. To date, there have been two pragmatic approaches. First, one can consider myopic agents. This is the approach followed by Desmet and Rossi-Hansberg (2009 and 2010). While each location solves a static problem, their model is dynamic in time. Indeed, each location decides the optimal amount to consume, how much to invest in R&D, and how much to save, taking land revenues, prices and salaries as given. Finally, all savings are coordinated by a cooperative that invests along the space. Second, one can abstract from physical capital mobility but allow for spatial externalities. In Brock and Xepapadeas (2008b) there is technological diffusion since aggregated neighbouring capital affects the location's production. Although they overlook ill-posedness, they show that diffusion-induced instability may create spatial patterns in infinite horizon optimal control problems. Moreover, they also provide a framework and useful tools to study local stability in a continuum of spatial sites.¹

In contrast to the aforementioned literature, we use the new theory on spatial dy-

¹Brock and Xepapadeas (2008a and 2010) and Xepapadeas (2010) extend the concept of diffusion in an environmental context, focusing on resources that diffuse over the space, such as fisheries and biomass in general.

namics in order to understand the spatial drivers behind land use and the environment. To the best of our knowledge, our paper provides a first analytically tractable general equilibrium framework of land use that encompasses (i) spatial and time dimensions which are presented in a continuous manner, (ii) spatial externalities due to pollution and abatement activities, and (iii) environmental degradation. Our starting point is the Spatial Ramsey model in Boucekine *et al.* (2009). We propose a benchmark framework in continuous time and space to study optimal land use. Each location is endowed with a fixed amount of land, which is allocated among production, pollution abatement, and housing.² Although the unique production input (land) is spatially immobile by nature, this is a model of spatial growth where locations' actions affect the entire space through pollution. Indeed, we assume that the production generates local pollution, which flows across locations.³ In this regard, we illustrate the diffusion mechanism by means of the well-known Gaussian Plume equation (see Sutton, 1947a and 1947b). Finally, we consider that local pollution damages production due to its negative effect on, for instance, individuals' health (among others, Elo and Preston, 1992; Pope, 2000; Pope *et al.*, 2004; and Evans and Smith, 2005) and land productivity. Moreover, we assume that pollution as a whole (global pollution) may also reduce production. This indirect effect of pollution can, for instance, be linked to the negative effect of anthropogenic GHGs on climate change.⁴

In contrast to Boucekine *et al.* (2009), Brock and Xepapadeas (2008a,b and 2010), and Xepapadeas (2010), we prove the existence and unicity of social optimum, *i.e.*, our problem is *well-posed*. In a nutshell, we improve the spatial structure of the social planner problem and this allows us to overcome the ill-posedness of the existent literature. As a consequence, the Pontryagin conditions turn out to be necessary and sufficient. To illustrate the richness of our model, we also undertake numerical simulations. To this end we adapt an algorithm first developed in Camacho *et al.* (2008) to the current problem, where well-posedness guarantees the uniqueness of the simulated trajectories. With this numerical tool in hand, we study the different drivers of spatial heterogeneity.

²In this simplified set-up, the land devoted to abatement may be interpreted as pollution removal due to, for instance, prairies and forests (see de Cara and Rozakis, 2004; de Cara *et al.*, 2005; Nowak *et al.*, 2006; and Ragot and Schubert, 2008). In general, one can also consider that abatement activities require physical space, *i.e.*, land.

³For instance, tropospheric ozone, methane and CO (Akimoto, 2003).

⁴Akimoto (2003) points out methane and CO as examples of contaminants with both local and global effects. Moreover, CO affects the oxidizing capacity of the atmosphere, raising the lifetime of GHGs.

In particular, we find that the abatement technology stands out as a fundamental element to achieve steady state solutions, which are compatible with the emergence of long run spatial patterns. Finally, as an alternative to the linear quadratic approximation of Brock and Xepapadeas (2008a,b and 2010) and Xepapadeas (2010), we would like to underline that our numerical analysis is global: we obtain a simulation of the entire trajectory of the states, controls, and co-states from their initial distributions until they eventually reach (or not) a steady state.

The paper is organized as follows. In section 2 we explain the Gaussian Plume equation that describes the pollution dynamics in our set-up. We present the economic model in section 3. Section 4 provides the Pontryagin conditions as well as the results of existence and unicity of social optimum. In section 5 we consider the numerical exercises. Finally, section 6 concludes.

2 The Gaussian plume

We describe the dynamics of pollution by means of a well-known model in physics called the Gaussian plume. The Gaussian plume is a standard set-up of atmospheric dispersion that introduces a mathematical description of the transport of airborne contaminants. Roberts (1924) and Sutton (1932) were the first to study the atmospheric dispersion problem. Since then great effort has been devoted to provide analytical solutions to the problem (see, for instance, Arya, 1999, Caputo *et al.*, 2003, and Stockie, 2011). The simplest of these solutions is the Gaussian plume, which has been mainly applied to air pollutants. However, it can be also used to study the dispersion of pollutants in aquifers and porous soils and rocks (Freeze and Cherry, 1979, and French *et al.*, 2000), as well as nuclear contaminants (Jeong *et al.*, 2005, and Settles, 2006).

Let us introduce the main equations of a Gaussian plume by means of considering the example of a pollutant emitted by a single source located at $x \in \mathbb{R}^3$. According to this model, the dynamics of the pollution at location x in time t , $p(x, t)$, is given by the following second-order partial differential equation (PDE) of parabolic type:

$$p_t(x, t) + \nabla \cdot J(x, t) = E(x, t), \quad (1)$$

where $p_t(x, t)$ denotes $\partial p(x, t)/\partial t$, $E(x, t)$ are the emissions of the single source in time $t \geq 0$, ∇ is the gradient, and $J(x, t)$ represents the flux of contaminant. This flux usually comprises the effect of diffusion and/or advection. Diffusion describes the spread of a

pollutant through regions of high concentration to regions of low concentration. In this regard, one can assume that the diffusive flux is proportional to the pollution gradient (Fick's law), *i.e.*, $J_D = -D\nabla p$, where D is a parameter that represents the diffusion coefficient of the physical environment (air, water, soils, rock, *etc.*). The second component of the flux is the advection due to wind, which is usually represented by $J_A = pv$, where v is the wind velocity. Therefore, $J = J_A + J_D = pv - D\nabla p$.

As pointed in the introduction, our model is based on the Spatial Ramsey model introduced by Boucekkine *et al.* (2009). Our set-up requires a slightly modified Gaussian plume. In particular, the former plume in (1) considers a single pollution source, where emissions are usually assumed to be exogenous and constant in time. Moreover, these plumes are often studied just at the steady state. In contrast to that, our model assumes a continuum of immobile sources, where emissions may change with time and are part of the policy maker's decisions. Moreover, our analysis studies both the dynamic transition and the steady state.

For the sake of analytical tractability we also consider several simplifications. First, our paper focuses mainly on the case $x \in \mathbb{R}$, *i.e.*, space is unidimensional. Second, we assume that advection is implicitly included in the diffusion effect.⁵ Finally, it is assumed $D = 1$ in order to illustrate the problem. Therefore, the dynamics of pollution at location x is described by the following Gaussian plume:

$$p_t(x, t) - p_{xx}(x, t) = E(x, t), \quad (2)$$

where p_{xx} denotes $\partial^2 p / \partial x^2$.

3 The model

We assume that space is the real line \mathbb{R} so that there exists a continuum of locations. Each location has a unit of land, which can be devoted to three different activities: production, housing and pollution abatement. For simplicity, we shall assume that the space required for housing at each location is equal to its population density. There

⁵For Gaussian plumes that include advection see, for instance, Arya (1999) and Stockie (2011). Our model does not explicitly consider advection because it would require further physical assumptions that are beyond the scope of this paper (*e.g.*, wind velocity and direction, and its spatial and time variability). Moreover, the time horizon usually considered in this kind of problems minimizes this effect. Besides that, the effect of advection is negligible in cases of pollution transportation in soils, rocks, etc.

exists a unique consumption good the production of which only requires land and which we denote by $F(l)$. Finally, the remainder of the land is used to abate pollution $G(1 - l - f(x))$.

Pollution has two dimensions in the model we present. The local dimension $p(t, x)$ (local pollution) comes directly from the production of the consumption good. It damages production due to the negative effect on, for instance, individuals' health and land productivity. Moreover, even if land is spatially immobile, location's decisions affect the whole space since the pollutant travels across space following the Gaussian plume equation described in (2). Additionally, pollution may also harm production as a global pollutant (*e.g.*, anthropogenic GHGs). We then allow for the distinction between local and global pollution, where global pollution is naturally defined as:⁶

$$P(t) = \int_{\mathbb{R}} p(x, t) dx.$$

We introduce pollution damages in production using a damage function $\Omega(p, P, x)$, which represents the share of foregone production due to local and global pollution. If we denote by $A(x, t)$ total factor productivity at location x at time t , we have that this location produces $\Omega(p, P, x)A(x, t)F(l)$ units of final good when it devotes an amount l of land to production. For simplicity reasons we shall assume that the abatement technology is not affected by pollution. In the remaining of the paper we make the following assumptions regarding the production functions:

(A1) Functions F and G are positive, increasing, concave, and their first and second derivatives exist and are positive, that is:

$$\begin{aligned} F(\cdot) \in C^2, \quad F(0) = 0, \quad F'(\cdot) > 0, \quad F''(\cdot) \leq 0, \quad \lim_{s \rightarrow 0} F'(s) = \infty, \quad \lim_{s \rightarrow \infty} F'(s) = 0, \\ G(\cdot) \in C^2, \quad G(0) = 0, \quad G'(\cdot) > 0, \quad G''(\cdot) \leq 0, \quad \lim_{s \rightarrow 0} G'(s) = \infty, \quad \lim_{s \rightarrow \infty} G'(s) = 0. \end{aligned}$$

(A2) $\Omega(p, P, x) \in C^{2,2}$, *i.e.*, it is twice differentiable with respect to p and P , decreasing in each factor $\Omega_1(p, P, x) < 0$, $\Omega_2(p, P, x) < 0$. Function $\Omega(p, P, x)$ is defined on $\mathbb{R}^+ \times \mathbb{R}^+$ and takes values in $[0, 1]$.

We assume that the policy maker collects all production and re-allocates it across locations at no cost:

$$\int_{\mathbb{R}} c(x, t) f(x) dx = \int_{\mathbb{R}} \Omega(p, P, x) A(x, t) F(l) dx, \quad (3)$$

⁶Well-known pollutants (see, among others, Nordhaus, 1977; and Akimoto, 2003) with mostly global effects are CO₂ and stratosphere ozone. Air contaminants in general (including tropospheric ozone, NO_x, and CO₂ plumes) are examples of local pollutants that flow among locations.

where $c(x, t)$ denotes consumption per capita at location x and time t . As we have pointed out in the introduction, this assumption improves the spatial structure of the social optimum problem in Boucekkine *et al.* (2009). This literature indeed assumes that each location produces its own consumption in the social optimum. However, social welfare may still increase under the possibility of spatial reallocation of production. We therefore enlarge the set of feasible abatement and production decisions by allowing for consumption “imports”.

The policy maker chooses consumption per capita and the use of land at each location, which maximize the discounted welfare of the entire population. As in Boucekkine *et al.* (2009), we introduce two discount functions. The spatial discount represents the weight that the policy maker gives to each location. We identify it as the population density function $f(x)$ in order to avoid any subjective spatial preferences. Moreover, as in the standard Ramsey model, we consider the usual temporal discount $g(t) = \exp(-\rho t)$. The policy maker maximizes the lifetime discounted utility

$$\max_{\{c, l\}} \int_0^\infty \int_{\mathbb{R}} u(c(x, t)) f(x) g(t) dx dt \quad (4)$$

subject to

$$\mathcal{P} \left\{ \begin{array}{l} p_t(x, t) - p_{xx}(x, t) = \Omega(p, P, x) A(x, t) F(l(x, t)) - G(1 - l - f(x)), \\ \int_{\mathbb{R}} c(x, t) f(x) dx = \int_{\mathbb{R}} \Omega(p, P, x) A(x, t) F(l) dx, \\ P(t) = \int_{\mathbb{R}} p(x, t) dx, \\ p(x, 0) = p_0(x) \geq 0, \\ \lim_{x \rightarrow \{\pm\infty\}} p_x(x, t) = 0, \end{array} \right. \quad (5)$$

where $(x, t) \in \mathbb{R} \times [0, \infty)$. As in the previous literature, the last expression in (5) is the spatial boundary condition. It considers that the flow of pollution through the very far ends of the space is zero.⁷

⁷Let us point out that this boundary condition is the most general and less constraining possibility. As in Brock and Xepapadeas (2008a) and Boucekkine *et al.* (2010), when the space is finite, it can be replaced by considering that pollution in both ends of the space is the same at each moment in time, *i.e.*, a circular space. However, this alternative can be eventually rewritten as ours with a finite linear space.

4 Pontryagin conditions, existence and uniqueness results

In this section we present the theoretical contributions of our paper. First, we prove the existence of at least one solution to the dynamical system \mathcal{P} . This result is not a straightforward application of existing results (Camacho *et al.*, 2008) because of some special features of \mathcal{P} . In particular, the present model includes a global variable P , defined as the spatial integral of p . Moreover, in contrast to the previous articles, we consider that the policy maker gathers all production to distribute it later, adding the afore mentioned additional integral constraint on consumption. Therefore, we have to transform the integral constraints into partial differential equations in the proof of proposition 1. We then apply theorem 12.1 in chapter 8 in Pao (1992) to close the proof.

Proposition 1 *Under assumption (1), system \mathcal{P} has at least a solution.*

Proof : See appendix A.

We use the method of variations in Raymond and Zidani (1998 and 2000) to obtain the Pontryagin conditions of problem (4)-(5). We write the associated value function V as a function of c , l , p and P as follows:

$$\begin{aligned}
 V(c, l, p, P) = & \int_{\mathbb{R}^+} \int_{\mathbb{R}} u(c(x, t)) f(x) g(t) dx dt - \\
 & - \int_{\mathbb{R}^+} \int_{\mathbb{R}} q(x, t) g(t) [p_t(x, t) - p_{xx}(x, t) - \Omega(p, P, x) A(x, t) F(l(x, t)) + G(1 - l - f(x))] dx dt - \\
 & - \int_{\mathbb{R}^+} m(t) g(t) (P(t) - \int_{\mathbb{R}} p(x, t) dx) dt - \\
 & - \int_{\mathbb{R}^+} n(t) g(t) (\int_{\mathbb{R}} c(x, t) f(x) dx - \int_{\mathbb{R}} \Omega(p, P, x) A(x, t) F(l(x, t)) dx) dt.
 \end{aligned} \tag{6}$$

Functions q , m and n are auxiliary functions. If there exists an optimal solution (c^*, l^*, p^*, P^*) , then any other solution to problem (4)-(5) can be written as a deviation from the optimal solution as

$$\begin{aligned}
 c(x, t) &= c^*(x, t) + \epsilon \kappa(x, t), \\
 l(x, t) &= l^*(x, t) + \epsilon L(x, t), \\
 p(x, t) &= p^*(x, t) + \epsilon \pi(x, t), \\
 P(t) &= P^*(t) + \epsilon \Pi(t).
 \end{aligned} \tag{7}$$

To obtain the Pontryagin conditions, we take the first order derivative of V with respect to ϵ , in the spirit of minimizing the distance to the optimal solution. As a

result, we obtain a reverse time parabolic PDE, which describes the dynamics of the shadow price of pollution, together with a static equation associated with optimal land allocation at each (x, t) . Finally, the set of first order conditions also contains spatial boundary conditions on q and a terminal condition on pq :

Proposition 2 *The Pontryagin conditions of problem (4)-(5) are:*

$$\left\{ \begin{array}{l} p_t(x, t) - p_{xx}(x, t) = \Omega(p, P, x)A(x, t)F(l(x, t)) - G(1 - l - f(x)), \\ q_t(x, t) + q_{xx}(x, t) = \left(\Omega_1(p, P, x) + \frac{1}{f(x)}\Omega_2(p, P, x) \right) A(x, t)F(l) \left[u' \left(\frac{\Omega(p, P, x)A(x, t)F(l)}{f(x)} \right) + q(x, t) \right], \\ [u'(c(x, t)) + q(x, t)] \Omega(p, P, x)A(x, t)F'(l) + q(x, t)G'(1 - l - f(x)) = 0, \\ \int_{\mathbb{R}} c(x, t)f(x)dx = \int_{\mathbb{R}} \Omega(p, P, x)A(x, t)F(l)dx, \\ P(t) = \int_{\mathbb{R}} p(x, t)dx, \\ p(x, 0) = p_0(x) \geq 0, \\ \lim_{x \rightarrow \{\pm\infty\}} p_x(x, t) = 0, \quad \lim_{x \rightarrow \{\pm\infty\}} q_x(x, t) = 0, \\ \lim_{t \rightarrow \infty} p(x, t)q(x, t) = 0, \end{array} \right. \quad (8)$$

for $(x, t) \in \mathbb{R} \times [0, \infty)$.

Proof : See appendix B.

The following corollary shows that consumption per capita is identical across locations. Indeed, this spatial homogeneity is an expected result since the policy maker does not have any location preference and can spatially reallocate production.

Corollary 1 *Consumption per capita is spatially homogeneous, i.e. $c(x, t) = c(t)$.*

Proof : See appendix C.

The next step is to prove that our problem is well-posed, in stark contrast to the previous literature on spatial economics in continuous time and space. In these papers the problem was that of a policy maker maximizing the welfare of a region in a period of time, where the state variable is always physical capital, k , and there is no other production factor and no externality. In the end, the set of Pontryagin conditions was made of the parabolic PDE for k , a reverse parabolic PDE for its shadow price, q , plus transversality conditions in space for k_x , q_x and a terminal condition on pq for all $x \in \mathbb{R}$. As noted in Boucekkine *et al.* (2009), the resulting system is ill-posed. Indeed,

in classical growth models without space there exists a unique relationship between the initial condition and the terminal state of the co-state variable. Hence the terminal condition helps to recover the unique initial condition for the co-state which makes q satisfy the terminal condition. When we deal with spatial models, the solution for q at (x, t) depends on its initial distribution q_0 through an integral plus an integral form dependant on the values of k and q :

$$q(x, t) = \int_{\mathbb{R}} q_0(x) dx + \int_{\mathbb{R}} \int_{\mathbb{R}^+} Q(q, k, c) dx dt.$$

Consequently, there exist infinite possibilities for q_0 that make pq satisfy the terminal condition.

However, this important drawback does not exist in our framework. One can indeed pick a unique initial distribution for q since we have improved the spatial structure of the social optimum problem. In a similar direction, Desmet and Hansberg-Rossi (2010) also overcome ill-posedness in a spatial set-up. On the one hand, their agents are myopic solving a static problem at each moment in time. On the other, they impose more structure to their problem by means of considering a cooperative that manages the aggregated savings.⁸ In our setup the reallocation of production, together with the spatial immobility of the production factor, allows us to show that the solution is unique:

Proposition 3 *The problem (4)-(5) is well posed: its solution exists and is unique.*

Proof: We prove that although the initial distribution for q , $q_0(x) = q(x, 0)$, is not provided by the first order conditions, it is however unique. We begin by exploiting the first order condition obtained in the proof of corollary 1, $u'(c) = n(t)$. From the proof of proposition 2 in appendix B, we know that

$$m(t) = \frac{1}{f(x)} \Omega_2 A F(l) (q + n). \quad (9)$$

Equation (9) implies that $\frac{1}{f(x)} \Omega_2 A F(l) (q + n)$ is independent of x , so that

$$\frac{\partial}{\partial x} \left(\frac{1}{f(x)} \Omega_2 A F(l) (q + u'(c)) \right) = 0.$$

If $A(x) \neq 0, \forall x \in \mathbb{R}$ and $q + u'(c) \neq 0$ for all (x, t) then

$$\Omega_2 A F(q + u'(c)) \left(\frac{-f'}{f} + \frac{\Omega_{2,3}}{\Omega_2} + \frac{\Omega_{2,1}}{\Omega_2} p_x + \frac{A_x}{A} + \frac{F'}{F} l_x + \frac{q_x}{q + u'(c)} \right) = 0.$$

⁸Also notice that Desmet and Hansberg-Rossi (2010) does not consider the social optimum problem.

Hence,

$$\frac{-f'}{f} + \frac{\Omega_{2,3}}{\Omega_2} + \frac{\Omega_{2,1}}{\Omega_2} p_x + \frac{A_x}{A} + \frac{F'}{F} l_x + \frac{q_x}{q + u'(c)} = 0.$$

If $q + u'(c) = 0$, using that $q(\Omega AF' + G') + u'(c)(\Omega AF') = 0$ (see proof of proposition 2 in the appendix B), we find that $qG' = 0$. Given assumption (A1), this last equality implies that $q = 0$, which leads to a corner solution. Let us focus then on the interior solutions and assume that $q + u'(c) \neq 0$.

Our next step is to consider the following couple of equations evaluated at $t = 0$:

$$\begin{cases} \frac{-f'}{f} + \frac{\Omega_{2,3}}{\Omega_2} + \frac{\Omega_{2,1}}{\Omega_2} p_x + \frac{A_x}{A} + \frac{F'}{F} l_x + \frac{q_x}{q + u'(c)} = 0, \\ q(\Omega AF' + G') + u'(c)(\Omega AF') = 0, \end{cases} \quad (10)$$

where $u'(c) = u'(\int_{\mathbb{R}} \Omega(p, P, x)A(x, t)F(l)dx)$. Notice that for any initial distribution for p given, $p_0(x) = \{p(x, 0) : x \in \mathbb{R}\}$, (10) is a two dimensional system of ordinary differential equations with 2 boundary conditions, $\lim_{x \rightarrow \{\pm\infty\}} q_x = 0$, which has a unique solution in (l, q) under the model assumptions ■

Finally, let us observe that the existence of a unique solution actually guarantees that the Pontryagin conditions are not only necessary but also sufficient.

5 Numerical exercises

Due to the complexity of the Pontryagin conditions (8), we illustrate the richness of our model by means of a numerical analysis. Moreover, as observed in the introduction, the uniqueness of the simulated trajectories is ensured since our social optimum problem is well-posed. Appendix D provides a description of the computational setting, together with our global dynamics algorithm to solve (8).

We will focus on the emergence of spatial patterns and the drivers behind this kind of heterogeneity. Even if the main objective of the paper is to provide a benchmark set-up, we will show that our simplified model already reproduces an ample variety of spatial heterogeneity scenarios related to the interaction between land-use and the environment. In particular, we will analyse the persistence in time of spatial heterogeneity. In this regard, we will study if spatial disparities are equally persistent and if they vanish with time. Moreover, we will see if spatial differences may arise in an initially equally endowed world. Finally, we will point out that the abatement technology stands out as

a fundamental ingredient to achieve steady state solutions, which are compatible with the formation of long run spatial patterns.

Our numerical exercise is divided in two parts: sections (5.1)-(5.3) consider that population is uniformly distributed, while section (5.4) assumes a Gaussian distribution in order to study the effect of population agglomeration. The parameter values are provided in Table 1. For illustration purposes we consider that locations' land endowment, L , is equal to 300, and that total population is equal to 110.⁹ We would like to underline that the values provided in this table aim at illustrating our model, and they do not correspond to any specific situation since we shall focus on the qualitative properties of our set-up.

B	Minimum productivity	0.5
A	Max. productivity increase	10
D	Abatement efficiency	0.1
ρ	Time discount rate	0.05
γ_1	P damage	0.005
γ_2	p damage	0.005
α	Cobb-Douglas parameter	0.75
L	Locations' land endowment	300
p_0	Initial pollution at x	100
	Total Population	110

Table 1: Parameter values for the numerical exercises.

We assume that the space is a line of length 5 divided into 500 locations. The time horizon varies from 10 to 40 depending on the convergence speed of the variables. Agents preferences are given by a logarithmic utility function. We have a Cobb-Douglas production function, where the net productivity is $B + A\Omega(p, P, x)$ with $\Omega(p, P, x) = e^{-\gamma_2 p - \gamma_1 P s(x)}$. Following Weitzman (2009), Ω is an exponential damage function, taking values in the interval $[0, 1]$. We consider that local and global pollution harm productivity, where γ_1 and γ_2 are constants: for given a (p, P) , the fraction $1 - \Omega(p, P, x)$ represents the foregone productivity at location x . For the sake of simplicity we assume that A and B are both constant in space and time. Moreover, $s(x)$

⁹Notice that the time horizon and space are both finite in numerical exercises. This implies that total population does not need to be equal to 1 since the convergence of the integral in the objective function is ensured. Therefore, taking advantage of this property, we increase both total population and land endowment in order to enlighten our numerical results.

stands for the sensitivity of location x to global pollution. Assuming a linear abatement technology, we have $G(l) = Dl$.

We consider in all scenarios that initial pollution is uniformly distributed. We believe of no particular interest the case when the only spatial feature is the initial distribution of pollution. Obviously, any difference in the initial endowment of pollution vanishes with time if all other variables are spatially homogeneous.

5.1 The benchmark scenario

We begin our analysis with the benchmark scenario in which population is evenly distributed on space. It is the objective of this benchmark illustration to underline the trade-off between production and abatement. Accordingly, we have reduced the amount of land devoted to housing by means of considering a uniform distribution of population that gives 0.22 people per location. This implies that each location needs 0.22 units of land for housing, which is not critical when total land endowment is 300.¹⁰ We further assume that spatial sensitivity to pollution is constant in space, *i.e.*, $s(x) = 1$ for all x . Figure 1 shows the results.

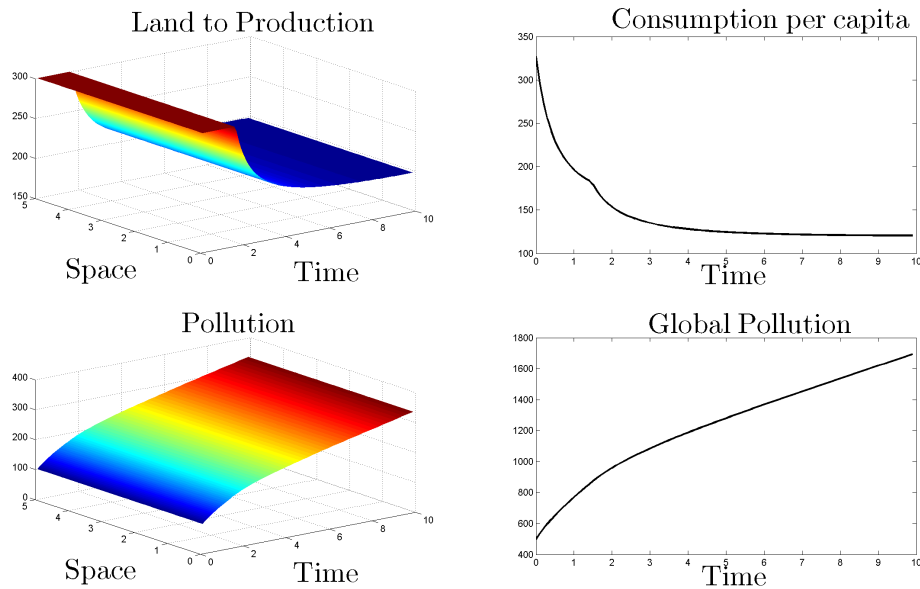


Figure 1: Benchmark scenario.

¹⁰We will consider the effect of population agglomeration and the subsequent accrued need for housing in section 5.4.

Given that there are no spatial disparities, it is not surprising that the optimal trajectories are uniform in space. The allocation of land to production starts at its highest possible level (a corner solution) and it remains at this level until the environmental damage is large enough. At this point, land to production is optimally reduced, thus devoting part of the land endowment to abatement. Consumption observes a decreasing trajectory, due to the pollution damage to production and the replacement of land to production by abatement. It eventually reaches a steady state, while pollution grows steadily.

The optimal land trajectory attains its steady state after 5 periods. Despite of using 2/3 of land to production, the economy cannot keep the initial level of consumption in the long-run due to the damage caused at the beginning. Indeed, both types of pollution cause everlasting and increasing damage that current abatement cannot make disappear completely. Let us study in the next sections the emergency of spatial patterns and the role of the different ingredients of our model in this regard.

5.2 Role of abatement technology

We consider now a simple case of heterogeneous abatement technology, in which there exist two regions of equal size.¹¹ The efficiency of abatement technology in the first region is equal to 0.2, that is to say, twice the abatement efficiency in the second region:

$$D(x) = \begin{cases} 0.2, & \text{if } x \in [0, 2.5], \\ 0.1, & \text{if } x \in [2.5, 5]. \end{cases}$$

The results are displayed in figure 2.

We can observe that the heterogeneity in abatement induces heterogeneity in land allocation from the beginning. Indeed, at time zero land to production is a step function, in which the less advanced region in abatement fully specialises in production (reaching a corner solution). At the same time, the most advanced region devotes around half its endowment to production and half to abatement activities. With time, the spatial heterogeneity intensifies. Indeed, the most advanced region gets more specialised in abatement due to the increasing levels of pollution. Moreover, one should also notice that within the advanced region, spatial differences arise even if all its locations possess the same abatement technology. Indeed, locations close to the border with the less advanced

¹¹For empirical evidence of differences in abatement technology see, for instance, de Cara *et al.* (2005) and Nowak *et al.* (2006).

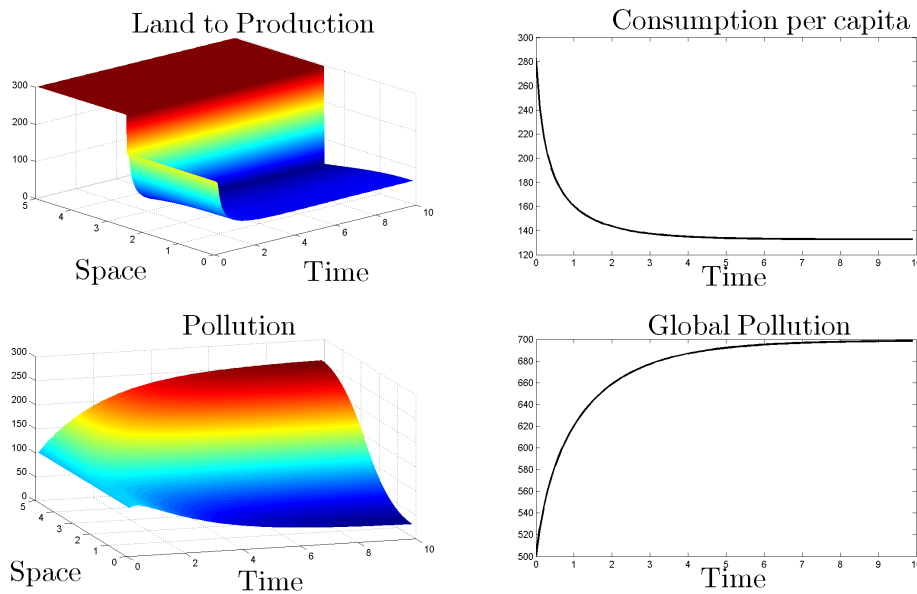


Figure 2: Role of abatement technology.

region need to devote more land to abatement in order to alleviate the pollution inflow coming through the frontier. Obviously, these spatial differences would be magnified if the pollutant under study moved slower.

As a consequence of the allocation of land, local pollution is heterogenous in space. All locations in the less advanced region emit the same. However, the closer the location is to the advanced region the lower their level of local pollution. Indeed, the level of local pollution within the advanced region is also heterogenous due to the inflow of pollution from the less advanced region. Therefore, even if the most advanced locations at the border devote the largest share of land to abatement, they end up with the highest pollution level of their region.

Finally, one should observe that all variables reach a steady state. This is in contrast to the previous scenario, where neither local nor global pollution attained a stable trajectory. Hence, this result points out the role of abatement as a pollution stabilizer. Moreover, since the steady state equilibrium is spatially heterogenous, we can also conclude that permanent differences in abatement technology induce lasting heterogeneity in land allocation and local pollution.

- **Local and global damage**

In the previous scenarios, we have considered that both local and global pollution causes the same damage per unit (*i.e.*, $\gamma_1 = \gamma_2$). However, our model also allows to study the case of contaminants with only local or global effects.¹²

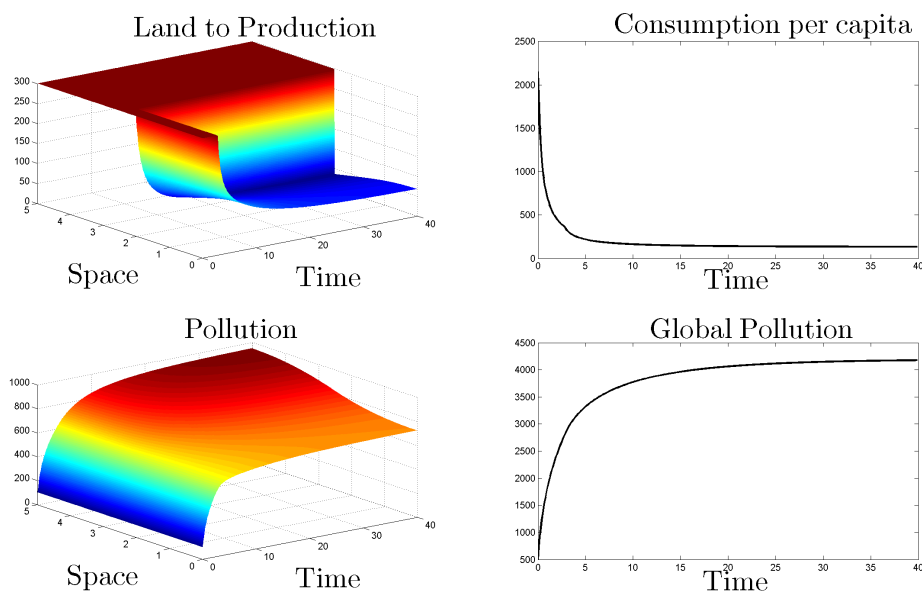


Figure 3: Damage function only depends on local pollution ($\gamma_1 = 0$).

When the damage is only local, $\gamma_1 = 0$ in Ω . Since in this case damage does not depend on global pollution, which is the largest pollutant by definition, the total damage of pollution is lower than in the previous scenario. As a consequence, one can see in figure 3 that locations do not abate at first. Nevertheless, the most advanced regions in abatement start abating when local pollution gets to a high enough level. At the end, the economy reaches a steady state, which is qualitatively identical to the previous case. However, the levels of local (and global) pollution are higher because of the lower damage of pollution. Moreover, one can also observe that the rise of spatial heterogeneity is postponed. We therefore conclude that the absence of global damage can delay the emergence of spatial patterns, due to a lower damage of pollution.

¹²The results of these scenarios are qualitatively equivalent to the case of pollutants with mainly local ($\gamma_2 > \gamma_1$) or global ($\gamma_2 < \gamma_1$) effect. Obviously, if $\gamma_1 = \gamma_2 = 0$ no land will be devoted to abatement since pollution does not damage our economy. Therefore, consumption will stay at its maximum steady state level (after taking housing into consideration, the remaining land will be completely assigned to production), where both local and global pollution increase steadily.

Finally, let us consider the situation where the damage is only global ($\gamma_2 = 0$ in Ω). Due to the greater pollution damage, the abatement specialisation of the most advanced region is faster than in the case of a pollutant with only local damage. Additionally, we also observe a lower level of both local and global pollution. The qualitative behaviour of the optimal trajectories is indeed similar to figure 3.

5.3 Spatially heterogeneous damage

Let us study the situation where some areas of the space are more sensitive to pollution than others. This is the case of, for instance, the negative impact of global warming on coastal zones due to the sea level rise and, in particular, the soil quality degradation (see, among others, Nicholls and Cazenave, 2010; and Nicholls *et al.*, 2011). Another well-known example, also related to global warming, is the desertification of drylands (for instance, Geist, 2005; Reynolds *et al.*, 2007; and Barnett *et al.*, 2008). In both cases, global warming is usually associated with the increase of global pollutants such as the anthropogenic GHGs. Therefore, in our simplified set-up we illustrate this situation by means of assuming that the sensitivity to global pollution $s(x)$ is spatially heterogeneous in the damage function Ω . We specifically consider the case in which $s(x)$ is a step function so that

$$s(x) = \begin{cases} 1, & \text{if } x \in [0, 2.5], \\ 5, & \text{if } x \in [2.5, 5], \end{cases}$$

where the locations in the interval $[2.5, 5]$ represent the most sensitive region to pollution. The numerical results are presented in Figure 4.

One can observe that, at the beginning, production is larger in the less sensitive region. However, soon afterwards, this region reduces the land devoted to production, and the space becomes heterogeneous at the steady-state. This result goes against the *à priori* belief that most sensitive regions would produce less than the others (and “import” most of their consumption) in order to preserve their environmental quality. The explanation is the following. Since pollution flows over space, even the regions with non-existent or little production will experience positive levels of local pollution. Moreover, the pollution as a whole (global pollution) damages production too. Then, the less sensitive locations optimally reduce their production and devote some land to abatement. Moreover, if the most sensitive locations had been endowed with better abatement technology, then they would have devoted more land to abatement relatively to the less sensitive locations.

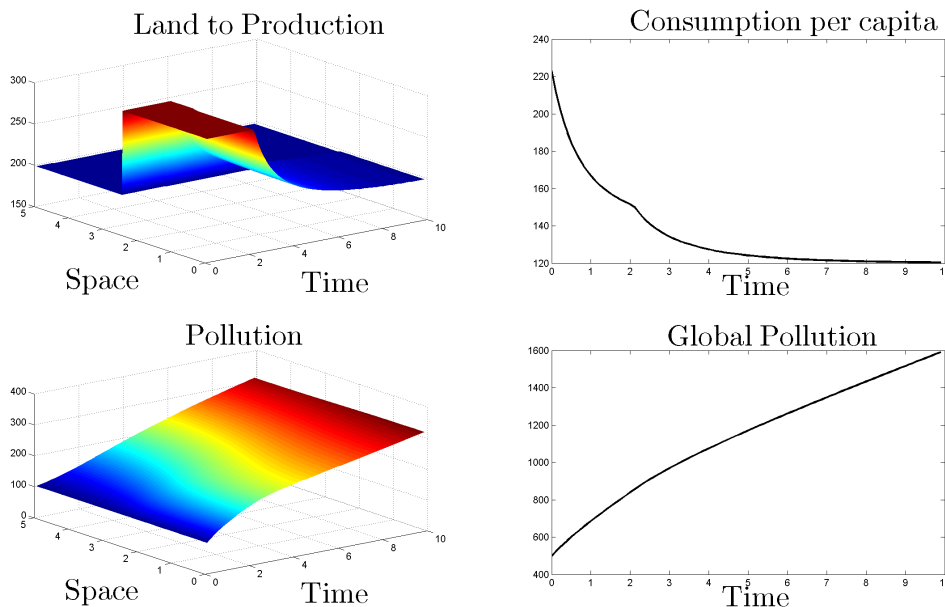


Figure 4: Spatially heterogeneous damage.

Let us finally point out that, in contrast to the previous scenarios, this case provides an additional spatial dynamics feature. Indeed, one can clearly see that our example also illustrates the situation of an initial spatial heterogeneity (in land to production) that vanishes in the long run.

5.4 The effect of population agglomeration

Let us study now the effect of population agglomeration and the resulting housing requirement. We consider that population is distributed according to a Gaussian function over the interval $[0, 5]$, that is, population agglomerates around the center of the space, $x = 2.5$. In order to underline the effect of population agglomeration, we increase total population to 10,500. Population in $x = 2.5$ is indeed almost 130. Moreover, although locations' land endowment is still equal to 300, the part of L devoted to housing in the central area is much larger than in the previous scenarios due to accrued population.¹³ Finally, in the areas far away from the center, the weight of population is similar to that in the benchmark scenario.

¹³Although the increase in total population is sizable, a homogenous distribution of 10,500 people over 500 locations would imply 21 individuals per location. In our simplified model, 21 individuals would need 21 units of land for housing, which is a small figure with respect to the location's land endowment.

In this section we present two exercises. Let us first compare the optimal trajectories under this population distribution with the benchmark scenario. Figure 5 shows that, due to the concentration of population, locations in the central area cannot devote as much land to production as locations at the far ends. This means that agglomerations optimally “import” most of their consumption from the neighbouring areas, which are more specialised in production.

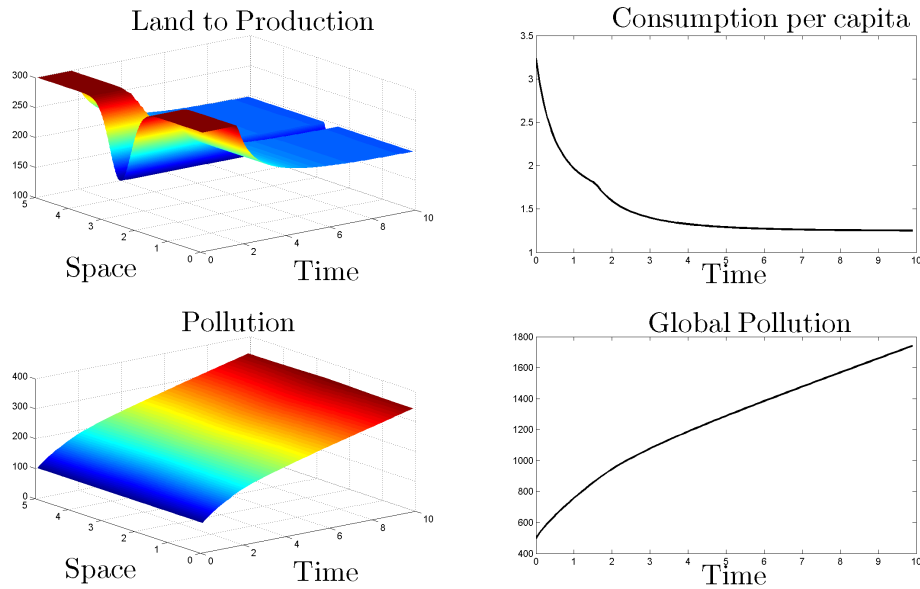


Figure 5: The role of population agglomeration (Gaussian distribution).

One could think then that agglomerations would be less polluted locally because most of their consumption comes from the periphery and housing does not pollute in our simplified framework. However, by the same token, agglomerations also devote less land to abatement than the rest of locations. Consequently, we observe an heterogeneous distribution not only of land, but also of local pollution. Even if local pollution tends to be homogeneous in space with time, slight spatial disparities persist since agglomerations cannot abate pollution coming from neighbouring regions.¹⁴

Our last point above, regarding the spatial mobility of local pollution, is indeed reinforced in figure 6. In this second exercise we have increased abatement efficiency (D) from 0.1 to 0.2 in all locations. In effect, driven by this improvement, all locations

¹⁴Considering pollution due to housing and/or transportation would amplify this effect. These additional sources of pollution may have interesting implications, in particular if labour is a spatially mobile production factor. However, this is beyond the objectives of the paper and we leave it for future research.

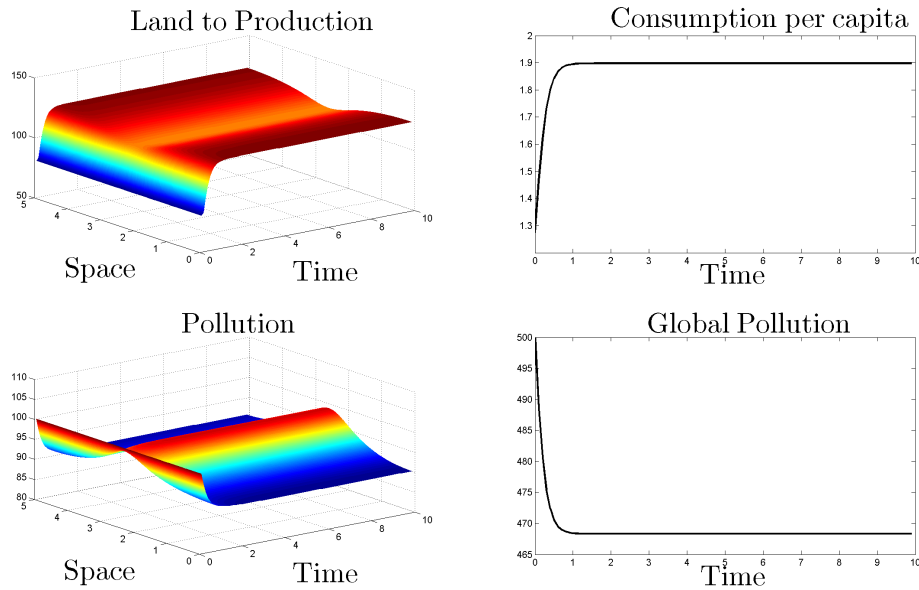


Figure 6: Population agglomeration with abatement efficiency doubling.

devote some land to abatement from the beginning. Consequently, both local and global pollution decrease in levels, allowing for a greater consumption per capita in the long-term. However, spatial disparities are amplified since the central area cannot abate as much because of the housing constraint.

Finally, in contrast to the first exercise, one should observe that all variables reach a steady state, which is characterized by lasting spatial heterogeneity in both land allocation and local pollution. Again, as in section 5.2, this result points out the role of abatement as pollution stabilizer. Abatement efficiency indeed enhances consumption and enables the economy to reach a stable steady state, which is spatially heterogenous.

6 Concluding remarks

The main objective of this paper is to propose a benchmark framework to study optimal land use, encompassing land use activities and pollution. Although land is immobile by nature, location's actions affect the whole space through pollution, which flows across locations resulting in both local and global damages. An important contribution of this paper is that, in contrast to the previous literature, we prove the existence and unicity of social optimum. Well-posedness is indeed ensured by means of improving the spatial structure of the social planner problem, allowing for the spatial reallocation

of production. Therefore, this alternative turns out to be a very promising strategy to overcome ill-posedness. We additionally undertake a numerical analysis. Taking advantage of the well-posedness result, which ensures the uniqueness of the simulated trajectories, we adapt the algorithm developed in Camacho *et al.* (2008). We find that our benchmark model reproduces a great variety of spatial patterns related to the interaction between land use activities and the environment. In particular, we identify the central role of abatement technology as pollution stabilizer, allowing the economy to achieve stable steady states, which can be spatially heterogeneous.

Several remarks can be made with regard to our setup. First, this paper assumes that population is exogenously distributed. However, Papageorgiou and Smith (1983) show that spatial externalities can induce population agglomerations. Therefore, an interesting extension of our framework would consider that population is endogenously distributed. This is the case of, for instance, migration flows induced by environmental degradation (Marchiori and Schumacher, 2011). Second, we do not explicitly model climate change in our paper. However, it is well-known that the damage of global pollutants such as anthropogenic GHGs is closely related to climate change and, in particular, global warming. One could then improve our framework by means of considering a comprehensive statement of this interaction. This would allow us to examine, among other things, the significance of non-monotonicities in the environmental degradation as drivers of spatial heterogeneity (Deschênes and Greenstone, 2007). Let us finally observe that the decentralisation of the social optimum has not been explored yet in this kind of literature. In this regard, a challenging extension could study the possibility of optimal tax/subsidy schemes that will evolve with time but also across the space. This spatial dependence is indeed consistent with numerous papers where optimal corrective policies take spatial information into account (*e.g.*, Tietenberg, 1974; Hochman *et al.*, 1977; Henderson, 1977; and Hochman and Ofek, 1979).

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Appendices

A Proposition 1 proof

We shall prove that the system of partial differential equations constraining the policy maker’s objective function has a unique solution for every choice of feasible functions (c, l) . This proves the existence of at least a solution to the policy maker’s problem. In this regard, we shall begin with converting the set of constraints into a system of parabolic differential equations.

First, notice that we can take the derivative of P with respect to t and we use the law of motion for p in \mathcal{P} to obtain:

$$P_t(t) = \int_{\mathbb{R}} p_t(x, t) dx = \int_{\mathbb{R}} (p_{xx}(x, t) + \Omega(p, P, x)A(x, t)F(l(x, t)) - G(1 - l - f(x))) dx,$$

which implies that

$$P_t(t) = p_x(x, t)|_{x \rightarrow \infty} - p_x(x, t)|_{x \rightarrow -\infty} + \int_{\mathbb{R}} [\Omega(p, P, x)A(x, t)F(l(x, t)) - G(1 - l - f(x))] dx.$$

Since $\lim_{x \rightarrow \pm\infty} p_x(x, t) = 0$, we have that

$$P_t(t) = \int_{\mathbb{R}} [\Omega(p, P, x)A(x, t)F(l(x, t)) - G(1 - l - f(x))] dx. \quad (\text{A.1})$$

Our initial set of constraints can be written as a system of parabolic equations. Indeed, we can interpret (A.1) as a partial differential equation, with the second order operator equal to zero. We would need to artificially transform P into a two dimensional function,

$P(x, t) \equiv P(t), \forall x \in \mathbb{R}$. Then:

$$(P') \left\{ \begin{array}{l} p_t(x, t) - p_{xx}(x, t) = \Omega(p, P, x)A(x, t)F(l(x, t)) - G(1 - l - f(x)), \\ P_t(x, t) = \int_{\mathbb{R}^+} [\Omega(p, P, x)A(x, t)F(l(x, t)) - G(1 - l - f(x))] dx, \\ p(x, 0) = p_0(x) \geq 0, \\ \lim_{x \rightarrow \pm\infty} p_x(x, t) = 0, \\ P(x, 0) = \int_{\mathbb{R}} p_0(x) dx, \\ \lim_{x \rightarrow \pm\infty} P_x(x, t) = 0, \end{array} \right. \quad (\text{A.2})$$

for all $(x, t) \in \mathbb{R} \times \mathbb{R}^+$. As in Camacho *et al.* (2008) and Boucekkine *et al.* (2009), we make use of Pao (1992) to prove the existence of a solution to this kind of equations for any $(x, t) \in \mathbb{R} \times (0, T]$, with $T < \infty$, after transforming the integral term in each dynamic equation. We proceed to the following change of variable $\Pi(x, t) = e^{-\gamma t} P(x, t)$ and we obtain:

$$\Pi(x, t)_t + \gamma \Pi(x, t) = e^{-\gamma t} \int_{\mathbb{R}} [\Omega(p, e^{\gamma t} \Pi, x)A(x, t)F(l(x, t)) - G(1 - l - f(x))] dx.$$

Then, we define function $\pi(t)$ as

$$\pi(t) = e^{-\gamma t} \int_{\mathbb{R}} [\Omega(p, e^{\gamma t} \Pi, x)A(x, t)F(l(x, t)) - G(1 - l - f(x))] dx.$$

Notice that since the integrand is globally Lipschitz continuous, so it is function π . We have to study now the existence of solution of the following system of equations:

$$\left\{ \begin{array}{l} p_t(x, t) - p_{xx}(x, t) = \Omega(p, P, x)A(x, t)F(l(x, t)) - G(1 - l - f(x)), \\ \Pi_t(x, t) + \gamma \Pi(x, t) = \pi(t), \\ p(x, 0) = p_0(x) \geq 0, \quad \lim_{x \rightarrow \pm\infty} \frac{\partial p(x, t)}{\partial x} = 0, \\ \Pi(x, 0) = \int_{\mathbb{R}} p_0(x) dx, \\ \lim_{x \rightarrow \pm\infty} \Pi_x(x, t) = 0. \end{array} \right. \quad (\text{A.3})$$

We can then apply theorem 12.1 in chapter 8 in Pao (1992) to ensure the existence of a unique solution to the system of parabolic equations for every choice of the couple (l, c) .

B Proposition 2 proof

We can take the first order derivative of the value function V with respect to ϵ , the size of the deviation. There is a main difference with the literature in spatial growth

in continuous space which emanates from the diffusion factor. Indeed, in the present model we have that:

$$\int_{\mathbb{R}^+} \int_{\mathbb{R}} q(x, t) p_{xx}(x, t) dx dt = \int_{\mathbb{R}^+} q(x, t) p_x(x, t) \Big|_0^R dt - \int_{\mathbb{R}^+} q_x(x, t) p(x, t) \Big|_{-\infty}^{\infty} dt + \int_{\mathbb{R}^+} \int_{\mathbb{R}} q_{xx}(x, t) p(x, t) dx dt, \quad (\text{A.4})$$

and as usual:

$$\int_{\mathbb{R}^+} \int_{\mathbb{R}} q(x, t) p_t(x, t) dx dt = \int_{\mathbb{R}} p(x, t) q(x, t) \Big|_0^{\infty} - \int_{\mathbb{R}} \int_{\mathbb{R}} p(x, t) q_t(x, t) dx dt.$$

We then obtain:

$$\begin{aligned} \frac{\partial V(c, l, p, P)}{\partial \epsilon} = & \\ = & \int_{\mathbb{R}^+} \int_{\mathbb{R}} u'(c(x, t)) f(x) g(t) \kappa(x, t) dx dt - \int_{\mathbb{R}^+} \int_{\mathbb{R}} g(t) \pi(x, t) [q_t(x, t) + q_{xx}(x, t)] + \\ & + \int_{\mathbb{R}^+} \int_{\mathbb{R}} g(t) q(x, t) \Omega_1(p, P, x) A(x, t) F(l(x, t)) \pi(x, t) dx dt + \\ & + \int_{\mathbb{R}^+} \int_{\mathbb{R}} g(t) q(x, t) [\Omega_2(p, P, x) A(x, t) F(l(x, t)) \Pi(t) + \Omega(p, P, x) A(x, t) F'(l(x, t)) L(x, t)] dx dt + \\ & + \int_{\mathbb{R}^+} \int_{\mathbb{R}} g(t) q(x, t) G'(1 - l - f(x)) L(x, t) dx dt - \\ & - \int_{\mathbb{R}^+} m(t) g(t) (\Pi(t) - \int_{\mathbb{R}} \pi(x, t) dx) dt - \\ & - \int_{\mathbb{R}^+} n(t) g(t) (\int_{\mathbb{R}} \kappa(x, t) f(x) dx) dt + \\ & + \int_{\mathbb{R}^+} n(t) g(t) (\int_{\mathbb{R}} [\Omega_1(p, P, x) A F(l) \pi(x, t) + \Omega_2(p, P, x) A F(l) \Pi(t) + \Omega(p, P, x) A F'(l) L(x, t)] dx) dt. \end{aligned}$$

To get the necessary conditions, we can group the elements multiplying κ , π , L and P , and equate them to zero. If all factors multiplying deviations from optimal values for c , p , P and l are equal to zero, we obtain that the deviation ϵ is optimal, *i.e.*, $\frac{\partial V}{\partial \epsilon} = 0$. We would need then:

$$\begin{cases} \kappa : & u'(c) = n(t), \\ \pi : & q_t + q_{x,x} = (q + n) \Omega_1 A F(l) + m, \\ \Pi : & m(t) = \frac{1}{f(x)} \Omega_2 A F(l) (q + n), \\ L : & q (\Omega A F' + G') + n(t) (\Omega A F') = 0. \end{cases} \quad (\text{A.5})$$

To these conditions, we need to add the following transversality conditions:

$$\begin{cases} \lim_{x \rightarrow \pm\infty} q_x = 0, \\ \lim_{t \rightarrow \infty} p q = 0. \end{cases}$$

We obtain the final version of the first order conditions substituting $m(t)$ by $R \Omega_2 A F(l) (q + n)$ into the dynamic equation for q .

C Corollary 1 proof

We can read in the first equation of the system (A.5) in the previous proof that $u'(c(x, t)) = n(t), \forall(x, t)$. Hence, neither $u'(c(x, t))$ nor $c(x, t)$ depend on space.

D Computational setting

Since the time horizon is finite, we can reverse time in the equation describing the dynamic behaviour of q in time and space in (A.3). Calling $h(x, t) := q(x, T - t)$, we obtain the following system of parabolic differential equations where we have removed the independent variables (x, t) for simplicity reasons, writing $(x, T - t)$ when necessary:

$$\left\{ \begin{array}{l} p_t - p_{xx} = \Omega(p, P, x)AF(l) - G(1 - l - f), \\ h_t - h_{xx} = \\ = - [\Omega_1(p(x, T - t), P(x, T - t), x) + R\Omega_2(p(x, T - t), P(x, T - t), x)] \times \\ \times AF(l) [u'(c(T - t)) + h], \\ [u'(c) + h(x, T - t)] \Omega(p, P, x)AF'(l) + h(x, T - t)G'(1 - l - f) = 0, \\ c(t) = \frac{\int_0^T \int_R \Omega(p, P, x)AF(l)dsdt}{\int_R f(s)ds}, \\ P(t) = \int_R p dx, \\ p(x, 0) = p_0(x) \geq 0, \\ \lim_{x \rightarrow \{0, R\}} p_x(x, t) = 0, \\ \lim_{x \rightarrow \{0, R\}} h_x(x, t) = 0, \\ \lim_{t \rightarrow T} p(x, t)h(x, T - t) = 0, \end{array} \right. \quad (\text{A.6})$$

for $(x, t) \in [0, S] \times [0, T]$. We simulate the system above using a finite difference approximation. The idea of this method is to replace the second derivative with respect to space with a central difference quotient in x , and replace the derivative with respect to time with a forward difference in time. In order to implement this approximation we need to set up a grid in our space $[0, R] \times [0, T]$. The points in this grid are couples $(j\Delta x, n\Delta t)$ for $j = 0, 1, \dots, J$ and $n = 1, 2, \dots, N$, where $J\Delta x = R$ and $N\Delta t = T$. Then, if v is a function defined on the grid, we write $v(j\Delta x, n\Delta t) = v_j^n$.

Let us provide an example. If we want to use a finite difference approximation for the parabolic differential equation $\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}$, we write:¹⁵

¹⁵This method is called the implicit finite difference approximation. Other approximation schemes

$$\frac{v_j^{n+1} - v_j^n}{\Delta t} = \frac{1}{\Delta x^2} (v_{j+1}^{n+1} - 2v_j^{n+1} + v_{j-1}^{n+1}). \quad (\text{A.7})$$

We can write (A.6) as

$$\frac{p_j^{n+1} - p_j^n}{\Delta t} - \frac{1}{\Delta x^2} (p_{j+1}^{n+1} - 2p_j^{n+1} + p_{j-1}^{n+1}) = \Omega(p_j^n, P_j^n, j)AF(l_j^n) - G(1 - l_j^n - f_j^n), \quad (\text{A.8})$$

$$\frac{h_j^{n+1} - h_j^n}{\Delta t} - \frac{1}{\Delta x^2} (h_{j+1}^{n+1} - 2h_j^{n+1} + h_{j-1}^{n+1}) = \quad (\text{A.9})$$

$$= - (\Omega_1(p_j^{T-n}, P_j^{T-n}, j) + R\Omega_2(p_j^{T-n}, P_j^{T-n}, j)) AF(l_j^{T-n}) [u'(c^{T-n}) + h(x, t)], \quad (\text{A.10})$$

$$[u'(c^n) + h_j^{T-n}] \Omega(p_j^n, P_j^n, j)AF'(l_j^n) + h_j^{T-n}G'(1 - l - f_j^n) = 0, \quad (\text{A.11})$$

$$\text{with } P^n = \sum_{j=0}^J p_j^n \text{ and } c^n = \frac{\sum_{j=0}^J (\Omega(p_j^n, P_j^n, j)AF(l_j^n))}{\sum_{j=0}^J f(j)}.$$

To these equations, we add the border conditions $p_{j-1}^n = p_j^n$ and $h_{j-1}^n = h_j^n$, $\forall n = 1, 2, \dots, N$ and the definition of P .

D.1 The algorithm

We adapt the algorithm developed in Camacho *et al.* (2008) to problem (A.6). There are still some differences: we need an initial guess for matrix $\{h_j^n\}_{j=1 \dots N}^{n=1 \dots N}$. Depending on this guess, we obtain a land distribution $\{l_j^n\}_{j=1 \dots N}^{n=1 \dots N}$ and then a first approximation to consumption. To improve the convergence speed we run an intermediate loop to improve the initial guess for c and l .

In order to reduce the computational load, we compute $P^n = \sum_{j=0}^J p_j^{n-1}$. Although this is just an approximation, we underline that the distance between $P(t)$ and $P(t - \Delta t)$ is infinitesimal since P is a continuous function. In the same manner, we compute $c^n = \frac{\sum_{j=0}^J (\Omega(p_j^{n-1}, P_j^n, j)AF(l_j^n))}{\sum_{j=0}^J f(j)}$.

exist but the implicit one is unconditionally stable, meaning that it is stable without restrictions on the relative size of Δt and Δx . It also allows us to use a larger time step and to save this way computational time.

Step 1: *Initialization*

We choose an initial distribution for air pollution $p_0 = \{p_{0,j}\}$, land allocation $l_0 = \{l_{0,j}\}$ and three stopping parameters ϵ_i for $i = 1, 2, 3$. We compute $P^0 = \sum_{j=0}^J p_j^0$. We assume an initial guess for $\{h_j^m\}_{j=1\dots J}^{m=1\dots N}$.

Step 2: *Iteration*

We repeat the following scheme until the euclidean distance between two consecutive matrices q is smaller than ϵ_1 or until the number of iterations equals a fixed number K .

For every $n = 1, \dots, N$ and given $p_{n-1}, l_{n-1}, P_{n-1}$, we compute

$$c_n = \frac{\sum_{j=0}^J (\Omega(p_j^{n-1}, P_j^n, j) AF(l_j^{n-1}))}{\sum_{j=0}^J f(j)}.$$

Step 2.1: *Improvement of the first guess*

With c_n and the guess $\{h_j^m\}_{j=1,\dots,J}$, using (A.11), we obtain a guess for $\{l_j^n\}$. We recompute c_n with $\{l_j^n\}$ instead of $\{l_j^{n-1}\}$. We iterate the process until the euclidean distance between two consecutive outcomes for c_n is smaller than ϵ_2 .

Step 2.2: *Upwind*

At every n we compute p_n^j for $j = 1, \dots, J$ with the resulting c_n and $\{l_j^n\}$, using the upwind algorithm applied to equation (A.10). Then, using (A.10) we compute a new guess for $\{h_j^m\}_{j=1\dots J}^{m=1\dots N}$ and compute its distance to $\{h_j^m\}_{j=1\dots J}^{m=1\dots N}$. If the distance is smaller than ϵ_3 , then STOP. If not, we repeat Step 2.