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Competitive Equilibria with Asymmetric Information: Existence with Entry Fees*

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Abstract

This paper studies competitive equilibria in economies characterized by the presence of asymmetric information, where non-exclusive contracts with payoffs dependent on the agents' private information are traded on competitive markets. For such economies competitive equilibria may not exist with linear prices. We show that (non-trivial) competitive equilibria exist, under general conditions, with two part tariffs, i.e. if the cost of trading each contract consists of an entry fee and a linear component in the quantity traded.

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1. Introduction

This paper studies competitive equilibria in economies characterized by the presence of asymmetric information. We consider economies where standardized, non-exclusive contracts, whose payoff depends on the agents' private information, are traded on competitive markets.[1] Contracts are standardized as their terms (price and payoff) are the same across many relationships involving different agents. In addition, they are non-exclusive as the terms of each contract do not depend on the level of transactions made by an agent in other markets.[2]

In the presence of asymmetric information, when prices are linear in the quantity traded, competitive equilibria may not exist, and non existence is a robust phenomenon.[3] The payoff of a contract in fact typically depends on the characteristics of the agent trading the contract (for contracts which provide insurance against individual sources of risk, for instance, the payoff depends on the probability distribution of this risk). Hence when the same (type of, standardized) contract is entered by different agents, this is effectively a different contract. However, when the agents' characteristics are only privately observed these different contracts cannot be separated and are traded together in a single market. As a consequence a problem may arise in ensuring feasibility in such markets since there may not be 'enough prices' to clear these markets. Equivalently, an uninformed intermediary trading with informed agents at linear prices may make negative profits at all prices.

Some non-linearity in price schedules is thus needed to ensure the existence of competitive equilibria with asymmetric information. In particular, we are interested here in characterizing the forms of non-linearity which guarantee existence and can be implemented *with minimal requirements on the observability of agents'*

[1] In the presence of exclusive contracts, the existence (and the efficiency) of competitive equilibria with moral hazard was first shown by Prescott and Townsend (1984) (see also Bennardo (1997), Bennardo and Chiappori (1998), Citanna and Villanacci (1997), Kehoe, Levine and Prescott (1998), Lisboa (1997), Magill and Quinzii (1997)).

[2] Though no explicit assumption is made over the observability of agents' trades, our analysis allows then for the possibility that the transactions of an agent in all markets are not fully observable, so that exclusive contracts cannot be implemented. See Hellwig (1983), Arnott and Stiglitz (1993), Dubey, Geanakoplos and Shubik (1995), Bisin and Guaitoli (1998), Bisin and Gottardi (1998) and Kahn and Mookerjee (1998) for other analyses of economies where non exclusive contracts are traded.

[3] This was shown by Helpman and Laffont (1975) for economies with moral hazard (hidden action) and by Bisin and Gottardi (1998) for economies with adverse selection and other types of informational asymmetries.

trades. The main role of some form of non-linearity in the price schedule is to provide a mechanism according to which the losses arising from the gains agents make by trading on the basis of their private information are re-distributed in the economy, or more generally to ensure the consistency between the different amounts delivered and claimed on contracts traded.

In this paper we show that competitive equilibria exist, for general economies with asymmetric information, if the unit price of contracts is independent of the quantity traded but agents also have to pay, to be able to transact in the market for a contract, a fixed entry fee (i.e. with a two-part tariff scheme). With sufficiently high entry fees the existence of 'trivial' equilibria, without trade, can be easily shown. We show however that there always are also 'non-trivial' equilibria, characterized by the fact that the entry fee is zero in all non-active markets (where no agent wishes to trade): hence at such equilibria there will generically be a non-zero level of trade in all markets. In each active market the entry fee is then set at a level which allows to fully recover the losses made by an uninformed market maker trading with informed traders, so that there is no cross-subsidization across markets at equilibrium.

The presence of an entry fee introduces a non-convexity in the agents' choice problem. As we consider a large economy, with infinitely many agents, of finitely many types, the aggregate excess demand can still be shown to be convex-valued, and the general existence of competitive equilibria can be established. The elements of novelty in the proof come from the fact that, to ensure that only the agents actively trading in a market will pay the fee, and that its level satisfies a no cross-subsidization condition, a different argument from the usual one, based on Caratheodory Theorem, is applied.

Our result shows that price schedules characterized by two components, the linear unit price and the entry fee, are sufficient to clear markets in economies with asymmetric information, whatever the nature of the set of contracts available for trade, and independently of the 'dimension' of the sources of asymmetric information in the economy (i.e. of the cardinality of the set of unobservable possible types or actions of the agents trading the contracts).

The information required to implement two-part price schedules is clearly minimal: one only needs to be able to distinguish agents who trade in a market from agents who don't trade. Two-part price schedules are quite common in financial markets. Examples include Over-The-Counter (OTC) markets for derivative securities (like e.g. options, futures, interest rate swaps), or derivatives exchanges. In these markets traders may be able to take both short and long positions at the same price, and dealers or exchanges compensate the different positions in the securities and guarantee against different traders' default risks upon charging them an entry fee.

In a related paper (Bisin and Gottardi (1998)) we have shown that the existence of competitive equilibria with asymmetric information is also guaranteed, without entry fee, if the buying and selling prices of contracts are possibly different (but are otherwise still a linear function of the quantity traded). The spread between bid and ask prices plays a similar role to the entry fee, as it allows to cover the average losses, per unit traded, made by uninformed agents; the problems in showing existence are however different from the ones encountered in the set-up of this paper and hence the argument of the proof is also rather different. Also in the case of a bid-ask spread a very limited information over agents' trades is needed to implement it: we simply have to be able to separate buyers and sellers, i.e. only the sign of the agents' transactions needs to be observed.

Combining these two results we obtain that competitive equilibria with arbitrary piecewise linear price schedules (and hence, effectively, with any nonlinear price schedule) also exist, under general conditions. These, more general, forms of nonlinearities are clearly informationally more demanding. On the other hand, in economies with asymmetric information, general nonlinearities in the price schedules typically enhance incentives and allow to obtain equilibria with better welfare properties.

The analysis is developed in the framework of a two-period, pure exchange economy with adverse selection.[4] The structure of the economy and the equilibrium notion are presented in section 2. There is both aggregate and idiosyncratic uncertainty. Aggregate shocks are commonly observed. On the other hand agents may have private information over the realization of their idiosyncratic shocks, and trade contracts whose payoff depends on the realization of the aggregate as well as the idiosyncratic shocks affecting them. The existence of 'non-trivial' competitive equilibria is then shown in the following section.

2. The Economy

We consider a two-period pure exchange economy. There are L commodities, labelled by $l \in L = \{1, ..., L\}$, available for consumption both at date 0 and at

[4] All our results extend to economies with moral hazard. As shown by Bisin and Gottardi (1998), the existence problem with linear prices and the conditions under which competitive equilibria exist are essentially the same in economies with moral hazard and with adverse selection.

date 1; commodity 1 is the designated numeraire in every spot. The agents in the economy are of finitely many types, indexed by $h \in H = \{1, ..., H\}$, and there are countably many agents of each type. An agent is then identified by a pair (h, n), where $n \in N$ and N is the set of natural numbers. Let λ^h be the fraction of the total population made of agents of type h.

Uncertainty and Information Structure. Uncertainty is described by the collection of random variables $\tilde{\sigma}$, $(\tilde{s}^n)_{n \in N}$, with support, respectively Σ and S (the same for all n). Both Σ and S are assumed to be finite sets, $\Sigma = \{1, ..., \Sigma\}$ and $S = \{1, ..., S\}$, with generic element, respectively, σ and s.

The random variable $\tilde{\sigma}$ represents the economy's aggregate uncertainty, affecting all agents in the economy, while $(\tilde{s}^n)_{n \in N}$ describes purely idiosyncratic sources of uncertainty: the realization of $\tilde{s}^n$ only affects the (finitely many) agents of index n. The idiosyncratic shocks $(\tilde{s}^n)_{n \in N}$ are identically and independently distributed across n, and are independent of $\tilde{\sigma}$; on the other hand, the idiosyncratic uncertainty affecting an agent (h, n) is possibly correlated with the shocks affecting the other $(H - 1)$ agents with his same index n.

The aggregate shock $\tilde{\sigma}$ is realized at date 1, and is observed by all agents. The realization of $(\tilde{s}^n)_{n \in N}$ also becomes commonly known at date 1; however, at the beginning of date 0 each agent (h, n) privately observes a signal $(\tilde{s}^{h,n})$ over the realization of his idiosyncratic shock $(\tilde{s}^n)$. We allow signals to be correlated across agents of same index n : $\tilde{s}^{h,n}$ may be correlated with $\tilde{s}^{h',n}$, for any n and $h' \neq h$. Let S^h be the support of $(\tilde{s}^{h,n})$, and s^h its generic element.

With no loss of generality we consider here the case where the information revealed by $(\tilde{s}^{h,n})$ over the idiosyncratic shock $(\tilde{s}^n)$ has a partitional structure and the collection of signals received by agents with same index n fully reveals $(\tilde{s}^n)$: $S = \times_{h \in H} S^h$ and $\tilde{s}^n = (\tilde{s}^{h,n})_{h \in H}$.

Let π denote the probability distribution of $(\tilde{\sigma}, \tilde{s}^n)$, for all $n \in N$, and $\pi(s/\sigma) \equiv \pi((\tilde{s}^{1,n} = s^1, .., \tilde{s}^{H,n} = s^H)/\sigma)$. The above conditions are formally stated in the following:

Assumption 1. The probability distribution of $((\tilde{s}^{h,n})_{h \in H}, \tilde{\sigma})$ satisfies:

- $\pi(s/\sigma) = \pi(s), \ \forall s, \sigma$
- $\pi(\tilde{s}^{h,n} = s^h) = \pi(\tilde{s}^{h,n'} = s^h), \ \forall n, n' \in N, s^h \in S^h$
- $\pi(\tilde{s}^{h,n} = s^h, \tilde{s}^{h',n'} = s^{h'}) = \pi(\tilde{s}^{h,n} = s^h)\pi(\tilde{s}^{h',n'} = s^{h'}), \ \forall n \neq n' \in N, h, h' \in H; s^h, s^{h'} \in S^h$

Consumers. Uncertainty enters the economy via the level of the agents' date 1 endowment. Each agent $(h, n) \in H \times N$ has an endowment w_0^h at date 0, and his date 1 endowment, $w_1^h(\tilde{\sigma}, \tilde{s}^n)$, depends upon the realization of his idiosyncratic $(\tilde{s}^n)$ and the aggregate shock $(\tilde{\sigma})$. We assume:

Assumption 2. $w_0^h \in \Re_{++}^L, w_1^h \equiv (w_1^h(\sigma, s); \sigma \in \Sigma, s \in S) \in \Re_{++}^{L(\Sigma S)}$.

A consumption plan for an arbitrary agent (h, n) specifies the level of his consumption of the L commodities at date 0 and in every state at date 1. The consumption set is the non-negative orthant of the Euclidean space. Agents are assumed to have Von Neumann - Morgenstern preferences over consumption plans and the utility index of agent (h, n) is given by a function $u^h : \Re_+^{2L} \to \Re$ satisfying:

Assumption 3. u^h is continuous, strictly increasing and strictly concave.

Market Structure. At date 0 the L commodities are traded on spot markets. At the same date, markets for contingent contracts also open, where agents can trade to insure against their aggregate as well as their idiosyncratic shocks. Markets are complete with respect to the aggregate uncertainty: there are Σ 'Arrow' securities[5] all agents can freely trade, with security $\sigma \in \Sigma$ paying one unit of numeraire if state σ is realized, and zero otherwise. In addition, for each n there are J contracts, with payoff contingent on the realization of $((\tilde{s}^n), \tilde{\sigma})$, which agents of same index n can trade and allow them to get some insurance also against their idiosyncratic shock $(\tilde{s}^n)$: each unit of contract $j \in J$ pays $r_j(s, \sigma)$ units of the numeraire commodity when $\tilde{s}^n = s, \tilde{\sigma} = \sigma$. These are standardized contracts in the sense that ex-ante all security of a given type $j \in J$ are identical, i.e. their payoff has the same distribution for all n; ex-post however their payoff will be different, as it will vary with the specific realization of $(\tilde{s}^n)$ across n.

Given the information structure of the economy, securities with payoff only contingent on σ are traded under conditions of symmetric information and the state is fully contractible; no restriction is imposed then on the agents' ability to insure against σ. On the other hand, the markets for the J types of contracts whose payoff is also contingent on the idiosyncratic uncertainty are characterized by the presence of adverse selection: each agent (h, n) chooses his level of trade after having received a signal $\tilde{s}^{h,n}$ over the payoff of these contracts. No assumption is made on the structure of these contracts, described by the arbitrary $S\Sigma \times J$ payoff

[5] In what follows we will use somewhat interchangeably the terms contract and security.

matrix R, with generic element $r_j(s,\sigma)$: as we will see the insurance possibilities against the idiosyncratic shocks are partly endogenously determined and reflect the agents' information structure and the consequences of the presence of adverse selection in these markets.

At date 1, after the realization of $((\tilde{s}^n)_{n\in N},\tilde{\sigma})$ becomes known to all agents, securities liquidate their payoff and the L commodities are again traded on spot markets.

Markets are perfectly competitive, i.e. agents acts as price-takers in all markets. Moreover, all contracts of the same type j (which only differ for the index n) sell at the same price: these contracts are in fact equivalent to the eyes of uninformed investors and can then be viewed as being traded together within a single, large market. The level of trade of each agent has a negligible impact on aggregate trades in these markets, so that no information agents have over their idiosyncratic shocks is revealed by the level of date 0 prices at a competitive equilibrium (differently from Radner (1979)). Agents retain some specific and exclusive private source of information, though remaining 'small' in terms of the level of their trades (so that their price-taking behavior is justified).

Let p_0 and $p_1(\sigma)$ be the commodity spot prices at, respectively, date 0 and date 1 in state σ; ρ_σ is then the price of Arrow security (or state price) σ; $p_1\equiv(..,p_1(\sigma),..)$ and $\rho\equiv(..,\rho_\sigma,..)$. As we mentioned in the Introduction, some degree of non-linearity in prices in markets with asymmetric information is needed to ensure the existence of competitive equilibria. Let $q_j(\theta_j)$ be then the price of θ_j units of a contract of type j, where $q_j:\Re\rightarrow\Re$ is an arbitrary map; $q(\theta)\equiv(q_j(\theta_j))_{j\in J}$.

In addition, to rule out the possibility that agents, by trading in contracts on the basis of some private information over their payoff, have unlimited arbitrage opportunities, we impose the condition that trades in these contracts cannot be unboundedly large.[6] The trades of each agent (h,n) in the J contracts characterized by the possible presence of asymmetric information are required to lie in the set

$$\Theta\subset\Re^J,\ \text{compact, convex and such that } 0\in\Theta \tag{2.1}$$

Given the assumed information structure, each agent (h,n) chooses at date 0

[6] Alternatively, restrictions could have been imposed on the payoffs of the contract (on R), so as to ensure that the agents' private information is not over the support of the contracts' payoff, but only over their probability distribution.

the present and future level of his trades after learning the realization s^h of $\tilde{s}^{h,n}$; his choice will then be contingent on s^h. Let $S^{-h} \equiv \Pi_{h' \neq h} S^h$, $s^{-h} \equiv (s^h)_{h' \neq h}$, and $\pi(s^{-h}/s^h) \equiv \pi((\tilde{s}^{h',n} = s^{h'})_{h' \neq h}/s^h)$. The agent has to choose: (i) his portfolio of Arrow securities $\zeta^{h,n}(s^h) = (.., \zeta^{h,n}_\sigma(s^h), ..) \in \Re^\Sigma$ and contracts contingent on the agent's idiosyncratic shocks $\theta^{h,n}(s^h) = (.., \theta^h_j(s^h), ..) \in \Theta$, (ii) his consumption plan $c^{h,n}(s^h) = (c^{h,n}_0(s^h);\ c^{h,n}_1(s^h) = c^{h,n}_1(s^{-h}, \sigma; s^h), s^{-h} \in S^{-h}, \sigma \in \Sigma)) \in \Re^{L(1+S^{-h}\Sigma)}_+$. The agent's consumption plan specifies his consumption level at date 0 and his date 1 consumption for every possible value of s^{-h} and σ (the realization of s^{-h} and σ constitutes in fact the residual uncertainty for this agent).

Formally, agent (h, n) has then to solve the following problem:

$$\max_{[c^{h,n}(s^h), \theta^{h,n}(s^h), \zeta^{h,n}(s^h)]} \sum_{s^{-h}, \sigma} \pi(\sigma)\pi(s^{-h}/s^h) u^h(c^{h,n}_0(s^h), c^{h,n}_1(s^{-h}, \sigma, s^h)) \qquad (P^h(s^h))$$

s.t.

$$p_0 \cdot (c^{h,n}_0(s^h) - w^h_0) + \sum_j q_j(\theta^{h,n}_j(s^h)) + \rho \cdot \zeta^{h,n}(s^h) \leq 0$$

$$p_1(\sigma) \cdot (c^{h,n}_1(s^{-h}, \sigma; s^h) - w^h_1(s, \sigma)) \leq \sum_j \theta^{h,n}_j(s^h) r_j\ (s, \sigma) + \zeta^{h,n}_\sigma(s^h), \quad (s, \sigma) \in S \times \Sigma$$

$$c^{h,n}(s^h) \in \Re^{L(1+S^{-h}\Sigma)}_+, \theta^{h,n}(s^h) \in \Theta, \zeta^{h,n}(s^h) \in \Re^\Sigma$$

Given the possible non-linearity of the price function $q(.)$, the above problem may fail to be convex. We will show that it is possible to overcome this difficulty by exploiting the large number of agents to 'convexify' the economy. This requires that, even though the choice problem is the same for all agents of the same type h, they may make different choices at equilibrium. In particular, we will show that it is enough to consider the case in which all agents of a given type h make at most a finite number V of different choices at equilibrium. We will denote by $c^{h,\nu}(s^h), \theta^{h,\nu}(s^h), \zeta^{h,v}(s^h)$ the ν-th choice of the agents of type h who observed the signal s^h, and by $\gamma^{h,\nu}(s^h)$ the fraction of agents of this type making such choice, for $\nu = 1, .., V$. Let then $\gamma^h(s^h) \equiv (\gamma^{h,v}(s^h))_{v \in V}$ and Δ^{V-1} be the $(V-1)-$dimensional simplex.

A *Competitive Equilibrium with Adverse Selection* is then defined by an array of prices $(p_0, p_1, \rho, q(.))$ and, for every h, by a collection of consumption and portfolio plans for agents of type h, contingent on the signal received, together with their relative frequency in the population of agents of that type $\left(c^{h,\nu}(s^h), \theta^{h,\nu}(s^h), \zeta^{h,v}(s^h); \gamma^{h,\nu}(s^h)\right)_{s^h \in S^h, v \in V}$, such that:

- for all h, s^h, the plans $\left(c^{h,\nu}(s^h), \theta^{h,\nu}(s^h), \zeta^{h,v}(s^h)\right)_{v \in V}$ solve $(P^h(s^h))$ at the prices
$(p_0, p_1, \rho, q(.))$;
- for all h, s^h, $\gamma^h(s^h) \in \Delta^{V-1}$;[7]
- commodity markets clear:

$$\sum_h \lambda^h \left(\sum_{s^h, v} \gamma^{h,\nu}(s^h) \pi(s^h)(c_0^{h,v}(s^h) - w_0^h) \right) \leq 0 \tag{2.2}$$

$$\sum_h \lambda^h \sum_{s,v} \gamma^{h,\nu}(s^h) \pi(s) \left(c_1^{h,v}(s^{-h}, \sigma; s^h) - w_1^h(s, \sigma) \right) \leq 0, \ \sigma \in \Sigma \tag{2.3}$$

We have used here the Law of Large Numbers, since in the situation we consider there are always countably many agents making the same choice at equilibrium, in writing the feasibility conditions for date 0 and date 1, in (2.2) and (2.3), as conditional expectations over the idiosyncratic uncertainty.

Note in addition that we have not imposed here the additional requirement that financial markets clear. Equations (2.2), (2.3) only impose the condition that the aggregate payoff of contracts at date 1, as well as the total value at date 0 of the agents' portfolios, obtained summing across agents *and* types of contract, are both non-negative. However this does not imply that the same is true for each type of contract (the aggregate payoff of different contracts may in fact be linearly dependent even though their individual payoffs are not): we may have so some cross-subsidization across contracts taking place at equilibrium.

[7] By allowing $\gamma^h(s^h)$ to be any real vector in the simplex Δ^{V-1}, even though, with countably many agents we should limit our attention to rational numbers, our definition effectively characterizes an approximate equilibrium. Note that, with appropriate assumptions on the probability space ensuring the validity of the implications of the Law of Large Numbers we are using (see, e.g. Al-Najjar (1995) and Yeneng (1998)), our analysis extends to economies with a continuum of agents, in which case no approximation is required.

3. Existence of Competitive Equilibria with Entry Fees

We will show in this section that competitive equilibria always exist, for the economy we described, when the price schedule for each contract $j \in J$ is given by a two-part tariff: there is a constant fee F_j each agent has to pay to be able to trade a non-zero amount in the market for this contract, and by a linear component in the quantity traded with factor q_j, i.e., when

$$q_j(\theta_j) = \begin{cases} F_j + q_j\theta_j, \text{ if } \theta_j \neq 0 \\ 0, \text{ if } \theta_j = 0 \end{cases} \quad j = 1, ..., J \qquad (TPP)$$

As we already argued, such pricing scheme can be implemented with very little information over the agents' trades: it is sufficient to know which agents are trading in each market.

Remark 1. *In Bisin and Gottardi (1998) the existence of competitive equilibria was established when pricing schedules are of the following form:* $q_j(\theta_j) = \{q_j^b\theta_j$ *if* $\theta_j > 0$; $q_j^s\theta_j$ *if* $\theta_j < 0\}$, *i.e. with bid-ask spreads. The two results together then imply the existence of competitive equilibria with pricing schedules which exhibit both discontinuities as well as nonlinearities. The implementation of such, more general, pricing schedules clearly requires the availability of more information over the agents' trades. However, in economies with asymmetric information, it may allow to obtain, at a competitive equilibrium, higher level of welfare. In particular, exclusive contracts - which as we argued allow the decentralization of incentive efficient allocations - need a very general form of nonlinearity to be sustained: the price of each contract has to depend on the trades in all markets.*

With no further restriction on the pricing schedule a large set of competitive equilibria, with quite different features, may exist.[8] We will show in what follows the existence of competitive equilibria with two-part tariff schemes (as in TPP) where:

1. The level of the entry fee F_j, for every $j \in J$, satisfies the following properties:

[8] An explicit modelling of the market-making activity of intermediaries in these markets would clearly help in reducing such indeterminacy, and the conditions we impose below in the text can be viewed as based on such considerations.

- If the market for contract j is active, F_j is set at a level which ensures the equality between the present value of total net payments made to agents at date 1 on this contract and the total net amount paid by these agents at date 0, or the validity of a zero-profit condition in the market for contract j.
  More precisely, the total amount (of the numeraire commodity) which in aggregate state σ is due to - or owed by - all agents who traded contracts of type j, is $\sum_h \lambda^h \sum_{s,v} \gamma^{h,\nu}(s^h)\pi(s)r_j(s,\sigma)\theta_j^{h,\nu}(s^h)$; letting then σ vary, the present value at date 0 of all these amounts, since markets have been assumed to be complete with respect to the aggregate uncertainty[9], can be uniquely determined using the state price vector ρ. The total value of the payments made - or received - by agents at date 0 for trading contract j at the constant unit price q_j is then $-\sum_h \lambda^h \sum_s \gamma^{h,\nu}(s^h)q_j\theta_j^{h,\nu}(s^h)$. The level of the entry fee is set to be equal to the difference among these two values, divided by fraction of the total population who is active in market j.
- If no agent trades in the market for contract j, F_j is set equal to zero.

  Formally, we require (F_j, q_j) to satisfy the following condition, for all $j \in J$:

$$F_j = \begin{cases} \frac{\sum_h \lambda^h \sum_{s,v} \gamma^{h,\nu}(s^h)\pi(s)\{\sum_\sigma \rho_\sigma r_j(s,\sigma)\theta_j^{h,\nu}(s^h) - q_j\theta_j^{h,\nu}(s^h)\}}{\sum_{h,s^h} \lambda^h \pi(s^h) \sum_{\nu:\theta_j^{h,\nu}(s^h)\neq 0} \gamma^{h,\nu}(s^h)}, \\ \qquad \text{if } \sum_{h,s^h} \lambda^h \pi(s^h) \sum_{\nu:\theta_j^{h,\nu}(s^h)\neq 0} \gamma^{h,\nu}(s^h) \neq 0 \\ 0, \quad \text{if } \sum_{h,s^h} \lambda^h \pi(s^h) \sum_{\nu:\theta_j^{h,\nu}(s^h)\neq 0} \gamma^{h,\nu}(s^h) = 0 \end{cases} \qquad (ZP)$$

2. The linear component of the pricing schedule satisfies the following 'fairness' property: for every $j \in J$, q_j is set equal to the unconditional expectation, over the idiosyncratic component of the uncertainty, of the payoff of contract j, while the component of the payoff dependent on the aggregate uncertainty is evaluated according to the state prices ρ :

$$q_j = \sum_\sigma \rho_\sigma \sum_s \pi(s) r_j(s,\sigma), \quad j \in J \qquad (F)$$

[9]Under a proper reformulation of this (zero-profit) condition, however our argument extends also to the case in which markets fail to be complete with respect to the aggregate uncertainty.

Remark 2. *Condition* (ZP) *excludes the possibility of 'trivial' equilibria in which high entry fees support a situation with a zero level of trade in the markets for contracts: it ensures in fact that generically there will be a nonzero level of trades in all markets. In addition,* (ZP) *implies that, at equilibrium, there is no cross-subsidization across markets for different contracts: in each market the value of the total net payments made at date* 0 *by agents who traded the contract equals the present value of the total net payments which will be made at date* 1 *to such agents*[10]*, i.e. a zero profit condition holds for each market.*
Given the characterization of the entry fee in (ZP)*, it is immediate to show that a competitive equilibrium exists for every possible choice of the level of the linear component of the price schedule (which in fact could even be set equal to zero).*
In (F) *we imposed the condition that* q_j *values contract* j *'fairly'. It is easy to see that if we consider the case where agents received no signal over the realization of their idiosyncratic shocks, i.e. with symmetric information, the equilibrium we obtain, under* (ZP) *and* (F)*, is characterized by a zero level of the entry fee in all markets and is Pareto efficient. Thus, under* (F) *the level of the entry fee can be viewed as a measure of the 'cost', imposed by the presence of asymmetric information, by the presence of adverse selection in the markets.*

Let $q \equiv (q_j)_{j \in J}$, and $F \equiv (F_j)_{j \in J}$.

Resolving the date 1 budget equations in the agent's problem $(P^h(s^h))$ for every σ for the portfolio of the associated Arrow security, $\zeta^{h,n}(s^h)$, and substituting the expression obtained into the date 0 budget equation, we find that the budget set of the agents of type h can be rewritten as follows:

$$B^h(p_0, p_1, \rho, q, F; s^h) = \Big\{ c^h(s^h) \in \Re_+^{L(1+\Sigma S^{-h})}, \theta^h(s^h) \in \Theta^h :$$

$$\begin{aligned} & p_0 \cdot (c^h(s^h) - w_0^h) + \textstyle\sum_j q_j(\theta_j^h(s^h)) + \\ & + \textstyle\sum_\sigma \rho_\sigma [p_1(\sigma) \cdot (c_1^h(s, \sigma; s^h) - w_1^h(s, \sigma)) - \textstyle\sum_{j \in J} r_j(s, \sigma)\theta_j^h(s^h)] \leq 0; \; \sigma \in \Sigma \Big\} \end{aligned} \tag{3.1}$$

where, for all j, $q_j(.)$ satisfies (TPP), (ZP) and (F).

We are now ready to define the notion of competitive equilibrium which obtains, from the one defined in the previous section, under the above characterization of the pricing schedule:

[10] Strictly speaking, we should also require that the total revenue from the entry fee is invested at date 0 in claims contingent on σ, so as to perfectly offset the total net payments due to agents the next period in the various states $\sigma \in \Sigma$.

A *Competitive Equilibrium with Adverse Selection and Entry Fees* is defined by an array of prices and entry fees $((p_0, p_1), \rho, (q, F))$ and a collection of conditional consumption and portfolio plans for agents of type h, together with their relative frequency in the population of agents of that type $\left(c^{h,\nu}(s^h), \theta^{h,\nu}(s^h); \gamma^{h,\nu}(s^h)\right)_{s^h \in S^h, v \in V}$, for all h, such that:

- for every h, s^h, all plans $\left(c^{h,\nu}(s^h), \theta^{h,\nu}(s^h)\right)_{v \in V}$ solve:

$$\max \sum_{s^{-h},\sigma} \pi(\sigma)\pi(s^{-h}/s^h)u^h(c_0^h(s^h), c_1^h(s^{-h}, \sigma, s^h)) \qquad (P_{EF}^h(s^h))$$

  subject to

$$\left(c^h(s^h), \theta^h(s^h)\right) \in B^h(p_0, p_1, \rho, q, F; s^h),$$

  at prices (p_0, p_1, ρ, q, F);

- for all $j \in J$, (F_j, q_j) satisfy conditions (ZP), (F);
- for all h, s^h, $\gamma^h(s^h) \in \Delta^{V-1}$;
- commodity markets clear:

$$\sum_h \lambda^h \left(\sum_{s^h,v} \gamma^{h,\nu}(s^h)\pi(s^h)(c_0^{h,v}(s^h) - w_0^h) \right) \leq 0 \tag{3.2}$$

$$\sum_h \lambda^h \sum_{s,v} \gamma^{h,\nu}(s^h)\pi(s) \left(c_1^{h,v}(s^{-h}, \sigma; s^h) - w_1^h(s, \sigma) \right) \leq 0, \ \sigma \in \Sigma \tag{3.3}$$

Let us denote by $\left(c^h(s^h), \theta^h(s^h)\right)(p_0, p_1, \rho, q, F)$ the demand correspondence of agents of type h who received signal s^h, given by the solutions of problem $(P_{EF}^h(s^h))$, for all prices. We can show that $\left(c^h(s^h), \theta^h(s^h)\right)(.)$ is well-behaved and exhibits standard properties, with the only exception of convex-valuedness (with a non-zero entry fee in some market the agent's choice problem is in fact clearly non-convex):

Lemma 3.1. *Under Assumptions 1-3, for all h, s^h the individual demand correspondence $\left(c^h(s^h), \theta^h(s^h)\right)(p_0, p_1, \rho, q, F)$, is non-empty and upper-hemicontinuous for all $((p_0, p_1), \rho, (q, F)) \in \Re_{++}^{L(1+\Sigma)} \times \Re_{++}^{\Sigma} \times \Re^{2J}$; in addition, it exhibits the following boundary behavior:*
for any sequence $\left\{p_0^{(\tau)}, p_1^{(\tau)}, \rho^{(\tau)}, q^{(\tau)}, F^{(\tau)}\right\} \in \Re_{++}^{L(1+\Sigma)} \times \Re_{++}^{\Sigma} \times \Re^{2J}$,
converging to $(p_0, p_1, \rho, q, F) \in \partial(\Re_{++}^{L(1+\Sigma)} \times \Re_{++}^{\Sigma}) \times \Re^{2J}$ as $\tau \to \infty$,

$$\inf\{\|c^h(s^h), \theta^h(s^h)\| : (c^h(s^h), \theta^h(s^h)) \in \left(c^h(s^h), \theta^h(s^h)\right)\left(p_0^{(\tau)}, p_1^{(\tau)}, \rho^{(\tau)}, q_2^{(\tau)}, F^{(\tau)}\right)\} \to \infty$$

The argument of the proof is quite standard and is only sketched below.

Proof. Consider problem $(P_{EF}^h(s^h))$. Under Assumption 2, $B^h(p_0, p_1, \rho, q, F; s^h)$ has clearly a non-empty interior, and is closed and compact for all $((p_0, p_1), \rho, (q, F)) \in \Re_{++}^{L(1+\Sigma)} \times \Re_{++}^{\Sigma} \times \Re^{2J}$. Moreover, $B^h(p_0, p_1, \rho, q, F; s^h)$ is defined by the intersection of hyperplanes, and both choice variables $(c^h(s^h), \theta^h(s^h))$ are, by assumption, bounded below. Therefore, by a standard argument, the correspondence defined by $B^h(p_0, p_1, \rho, q, F; s^h)$ is also continuous. Upper-hemicontinuity of demand then follows from the continuity of the agents' utility function (ensured by Assumption 3).

It is immediate to see that, under Assumption 2, $B^h(p_0, p_1, \rho, q, F; s^h)$ has a non-empty interior also at prices $(p_0, p_1, \rho, q, F) \in \partial(\Re_{++}^{L(1+\Sigma)} \times \Re_{++}^{\Sigma}) \times \Re^{2J}$, so that the boundary behavior property of demand holds. ■

The following is then the main result of this paper :

Theorem 3.2. *Under Assumptions $1-3$, a competitive equilibrium with adverse selection and entry fees satisfying (ZP) and (F) always exists.*

Proof. The proof is organized in various steps.

Step 1. We show first that, for every h and s^h, the image of the individual demand correspondence $\left(c^h(s^h), \theta^h(s^h)\right)(p_0, p_1, \rho, q, F)$ has at most finitely many (say V) values, and this is true for all $((p_0, p_1), \rho, (q, F)) \in \Re_{++}^{L(1+\Sigma)} \times \Re_{++}^{\Sigma} \times \Re^{2J}$.

For any $\mathcal{J} \subseteq J$, define $w_0^h(\mathcal{J}) = \begin{bmatrix} w_{01}^h - \sum_{j\in\mathcal{J}} F_j \\ w_{02}^h \\ \vdots \\ w_{0L}^h \end{bmatrix}$. The set

$$B_{\mathcal{J}}^h(p_0, p_1, \rho, q, F; s^h) \equiv \Big\{ c^h(s^h) \in \Re_+^{L(1+\Sigma S^{-h})}, \theta^h(s^h) \in \Theta^h :$$

$$p_0 \cdot (c_0^h(s^h) - w_0^h(\mathcal{J})) + \sum_{j\in J} q_j \theta_j^h(s^h) +$$
$$+ \sum_\sigma \rho_\sigma [p_1(\sigma) \cdot (c_1^h(s, \sigma; s^h) - w_1^h(s, \sigma)) - \sum_{j\in J} r_j(s, \sigma)\theta_j^h(s^h)] \leq 0;$$

$$\sigma \in \Sigma, \quad \theta_j = 0 \text{ for all } j \in J \backslash \mathcal{J} \}$$

describes then the budget set of an agent h with signal s^h who has chosen to trade (and pay the entry fee) in the subset $\mathcal{J} \subseteq J$ of the set of existing markets for contracts. It is easy to see that $B_{\mathcal{J}}^h(p_0, p_1, \rho, q, F; s^h)$ is closed, compact and, in addition, convex, though it may be empty for some pair $\mathcal{J}$, (p_0, p_1, ρ, q, F). If we let then $\left(c^h(s^h), \theta^h(s^h)\right)_{\mathcal{J}}(p_0, p_1, \rho, q, F)$ be the solution of the problem of maximizing the agent's utility, subject to $(c^h(s^h), \theta^h(s^h)) \in B_{\mathcal{J}}^h(p_0, p_1, \rho, q, F)$, it follows that $\left(c^h(s^h), \theta^h(s^h)\right)_{\mathcal{J}}(p_0, p_1, \rho, q, F)$ will be either empty or single-valued.

It is easy to see that $\left(c^h(s^h), \theta^h(s^h)\right)(p_0, p_1, \rho, q, F) \subset \left\{ \cup_{\mathcal{J}} \left(c^h(s^h), \theta^h(s^h)\right)_{\mathcal{J}}(p_0, p_1, \rho, q, F) \right\}$, where the union in the term on the right hand side is taken over all the subsets of J. As there are finitely many of those, and $\left(c^h(s^h), \theta^h(s^h)\right)_{\mathcal{J}}(p_0, p_1, \rho, q, F)$ contains at most a single element, the claimed result follows.

Step 2. We derive here a convenient representation of the correspondence describing, for every h, s^h, the aggregate (per capita) demand and participation rate in the various markets of all agents of type h who observed signal s^h.

First, we construct finitely many selections from the individual demand correspondence, according to the value of the norm of demand. Define $\left(c^{h,1}(s^h), \theta^{h,1}(s^h)\right)(p_0, p_1, \rho, q, F)$ by associating, to all (p_0, p_1, ρ, q, F), the value (c^h, θ^h) in the set $\left(c^h(s^h), \theta^h(s^h)\right)(p_0, p_1, \rho, q, F)$ which has minimal norm: i.e.

$$\| \left(c^h, \theta^h\right) \| \leq \| \left(\bar{c}^h, \bar{\theta}^h\right) \| \text{ for all } \left(\bar{c}^h, \bar{\theta}^h\right) \in \left(c^h(s^h), \theta^h(s^h)\right)(p_0, p_1, \rho, q, F).$$

Proceed then iteratively to construct the other maps $\left(c^{h,\nu}(s^h), \theta^{h,\nu}(s^h)\right)(p_0, p_1, \rho, q, F)$, $\nu = 2, .., V$, by associating for all prices the value in $\left(c^h(s^h),\theta^h(s^h)\right)(p_0,p_1,\rho,q,F)$ with $v-th$ minimal norm. More precisely, $\left(c^{h,\nu}(s^h),\theta^{h,\nu}(s^h)\right)(p_0,p_1,\rho,q,F)=(c^h,\theta^h)\in\left(c^h(s^h),\theta^h(s^h)\right)(p_0,p_1,\rho,q,F)$ such that:

$$(c^h,\theta^h)\neq\left(c^{h,i}(s^h),\theta^{h,i}(s^h)\right)(p_0,p_1,\rho,q,F) \text{ for } i=1,..,v-1,$$

$$\parallel\left(\bar{c}^h,\bar{\theta}^h\right)\parallel\geqslant\parallel\left(c^h,\theta^h\right)\parallel \text{ for all } \left(\bar{c}^h,\bar{\theta}^h\right)\in\left(c^h(s^h),\theta^h(s^h)\right)(p_0,p_1,\rho,q,F): \left(\bar{c}^h,\bar{\theta}^h\right)\neq\left(c^{h,i}(s^h),\theta^{h,i}(s^h)\right),\ i=1,..,v-1.$$

Evidently, $\left(c^h(s^h),\theta^h(s^h)\right)(p_0,p_1,\rho,q,F)=\cup_\nu\left(c^{h,\nu}(s^h),\theta^{h,\nu}(s^h),I^{h,\nu}(s^h)\right)(p_0,p_1,\rho,q,F)$.

For all h,s^h, for each selection v we define the map:

$$I_j^{h,\nu}(s^h)(p_0,p_1,\rho,q,F)\equiv\begin{cases}1 \text{ if } \theta_j^{h,\nu}(s^h)(p_0,p_1,\rho,q,F)\neq 0\\ 0 \text{ if } \theta_j^{h,\nu}(s^h)(p_0,p_1,\rho,q,F)=0\end{cases}$$

an indicator function describing whether or not the agent is trading in market j, for all j; $I^{h,\nu}(s^h)\equiv\left(I_j^{h,\nu}(s^h)\right)_{j\in J}$.

For every h,s^h, the correspondence describing the (per capita) demand and associated participation rate of all agents of type h who received signal s^h can be conveniently written in terms of the maps defined above:

$$\begin{aligned}&\left(\hat{c}^h(s^h),\hat{\theta}^h(s^h),\hat{I}^h(s^h)\right)(p_0,p_1,\rho,q,F)=\\&co\left[\cup_\nu\left(c^{h,\nu}(s^h),\theta^{h,\nu}(s^h),I^{h,\nu}(s^h)\right)(p_0,p_1,\rho,q,F)\right]=\\&\left\{\sum_\nu\gamma^{h,\nu}(s^h)\left(c^{h,\nu}(s^h),\theta^{h,\nu}(s^h),I^{h,\nu}(s^h)\right)(p_0,p_1,\rho,q,F),\ \forall\gamma^h(s^h)\in\Delta^{V-1}\right\}\end{aligned}$$

where $co[.]$ denotes the convex hull of a set.

Step 3. The representation we obtained of the (per capita) demand and participation correspondence of the agents with the same type and signal is used here to find the correspondence yielding the level of the entry fee, as in condition (ZP), for all (p_0,p_1,ρ,q,F) such that $\sum_{h,s^h}\lambda^h\pi(s^h)\sum_{\nu:\theta_j^{h,\nu}(s^h)\neq 0}\gamma^{h,\nu}(s^h)\neq 0$.

Let $\gamma\equiv\left(\gamma^h(s^h)\right)$, $\Delta\equiv\times_{h\in H,s^h\in S^h}\Delta^{V-1}$, and define:

$F_j\left(p_0,p_1,\rho,q,F\right)=$

$$=\left\{\frac{\sum_h \lambda^h \sum_{s,\sigma}\pi(s)\left(\rho_\sigma r_j(s,\sigma)-q_j\right)\sum_\nu \gamma^{h,\nu}(s^h)\theta_j^{h,\nu}(s^h)\left(p_0,p_1,\rho,q,F\right)}{\sum_{h,s^h}\lambda^h\pi(s^h)\sum_{\nu:\theta_j^{h,\nu}(s^h)\neq 0}\gamma^{h,\nu}(s^h)},\ \forall\gamma\in\Delta\right\}=$$

$$=\left\{\frac{\sum_h \lambda^h \sum_{s,\sigma}\pi(s)\left(\rho_\sigma r_j(s,\sigma)-q_j\right)\sum_\nu \gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)\theta_j^{h,\nu}(s^h)(.)}{\sum_{h,s^h}\lambda^h\pi(s^h)\sum_v \gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)},\ \forall\gamma\in\Delta\right\}=$$

$$=\left\{\sum_h\sum_{s,\sigma}\pi(s^{-h}/s^h)\left(\rho_\sigma r_j(s,\sigma)-q_j\right)\left(\frac{\lambda^h\pi(s^h)\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)\theta_j^{h,\nu}(s^h)(.)}{\lambda^h\pi(s^h)\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)}\right)\cdot\right.$$
$$\left.\cdot\left(\frac{\lambda^h\pi(s^h)\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h))(.)}{\sum_{h,s^h}\lambda^h\pi(s^h)\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)}\right),\ \forall\gamma\in\Delta\right\}=$$

$$=\left\{\begin{array}{c}\sum_h\ \lambda^h\sum_{s,\sigma}\ \pi(s)\left(\rho_\sigma r_j(s,\sigma)-q_j\right)\hat{\theta}_j^h(s^h)(.)\left(\frac{\lambda^h\pi(s^h)\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)}{\sum_{h,s^h}\lambda^h\pi(s^h)\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)}\right),\\ \forall\gamma\in\Delta\end{array}\right\}$$

where the last equality follows from the fact that the weights

$$\frac{\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)}{\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)}$$

are nonnegative and their sum over ν equals 1 (thus have the same properties as $\gamma^{h,v}(s^h)$).

Since the terms $\left(\frac{\lambda^h\pi(s^h)\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)}{\sum_{h,s^h}\lambda^h\pi(s^h)\sum_\nu\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)(.)}\right)$ are also nonnegative and add to 1 when summed over ν, we get:

$$F_j\left(p_0,p_1,\rho,q,F\right)\subset co\{\sum_h\sum_{s,\sigma}\pi(s^{-h}/s^h)\left(\rho_\sigma r_j(s,\sigma)-q_j\right)\hat{\theta}_j^h(s^h)\left(p_0,p_1,\rho,q,F\right);$$
$$\forall\ s^h\in S^h,\ h\in H\}$$

for all (p_0,p_1,ρ,q,F) such that $\sum_{h,s^h}\lambda^h\pi(s^h)\sum_v\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)\left(p_0,p_1,\rho,q,F\right)$ $\neq 0$. Furthermore, recalling that in our set-up the correspondence $\hat{\theta}_j^h(s^h)\left(p_0,p_1,\rho,q,F\right)$ is always well defined and takes values in a compact set (Θ), we can find a subset F_{EF} of $\Re$, compact and convex, such that $F_j\left(p_0,p_1,\rho,q,F\right)\subseteq F_{EF}$ for all (p_0,p_1,ρ,q,F) such that $\sum_{h,s^h}\lambda^h\pi(s^h)\sum_v\gamma^{h,\nu}(s^h)I_j^{h,\nu}(s^h)\ (p_0,p_1,\rho,q,F)\neq 0$.

Step 4. We construct here the map to which a fixed point argument can be applied to show the existence of a competitive equilibrium.

Normalize date 0 prices and date 1 prices in every aggregate state σ on the simplex. Consider then the following truncated price sets:
$\Delta_\delta^{L+\Sigma-1} \equiv \{(p_0, \rho) \in \Re_+^{L+\Sigma} : \sum_l p_{0,l} + \sum_\sigma \rho_\sigma = 1; p_{0,l}, \rho_\sigma \geqslant \delta \ \forall l, \sigma\}$,
$\Delta_\delta^{L-1} \equiv \{p_1(\sigma) \in \Re_+^L : \sum_l p_{1,l}(\sigma) = 1; p_{1,l}(\sigma) \geqslant \delta \ \forall l\}$.
Also, let $Q_j \equiv$ co $\left(\sum_{\sigma\in\Sigma} \rho_\sigma \sum_s \pi(s) r_j(s,\sigma);\ \rho \in \Delta^{\Sigma-1}\right)$, a compact convex set; $Q \equiv \times_j Q_j$.

The aggregate (per capita) excess demand for the whole economy is readily obtained by summing over h, s^h the expressions we obtained for the (per capita) excess demand of all agents with the same type and signal.

$$
\begin{aligned}
z_0\left(p_0, p_1, \rho, q, F\right) &= \sum_h \lambda^h \left(\sum_{s^h, v} \gamma^{h,\nu}(s^h) \pi(s^h) (c_0^{h,v}(s^h)(.) - w_0^h) \right) \\
z_1(\sigma)\left(p_0, p_1, \rho, q, F\right) &= \sum_h \lambda^h \left(\sum_{s^h, v} \gamma^{h,\nu}(s^h) \pi(s) (c_1^{h,v}(s^{-h}, \sigma; s^h)(.) - w_1^h(s,\sigma) \right);\sigma \in \Sigma
\end{aligned}
$$

Summing across agents the budget constraints in (3.1), and using the specification of the entry fee in (ZP), we find that the following expression of Walras law holds for the economy under consideration:

$$
\begin{aligned}
&p_0 \cdot z_0(p_0, p_1, \rho, q, F) + \textstyle\sum_\sigma \rho_\sigma(.., p_1(\sigma) \cdot z_1(\sigma)(p_0, p_1, \rho, q, F), ..) + \\
&+ \textstyle\sum_j [F_j(\sum_{h,s^h} \lambda^h \pi(s^h) \sum_v \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)(.)) - \\
&- \textstyle\sum_h \lambda^h \sum_{s,\sigma} \pi(s) \left(\rho_\sigma r_j(s,\sigma) - q_j\right) \sum_\nu \gamma^{h,\nu}(s^h) \theta_j^{h,\nu}(s^h)(.)] = \\
&= p_0 \cdot z_0(p_0, p_1, \rho, q, F) + \textstyle\sum_\sigma \rho_\sigma(.., p_1(\sigma) \cdot z_1(\sigma)(p_0, p_1, \rho, q, F), ..) = 0
\end{aligned}
\tag{3.4}
$$

for all p_0, p_1, ρ, q, F . Let K_δ be a convex, compact set containing the image of the aggregate demand map at all prices in $\Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma$.

Consider then the map: $(z_0, (.., z_1(\sigma), ..), F, p_0, p_1, \rho, q)$ from the set $K_\delta \times F_{EF} \times \Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma \times Q$ into itself, defined by:

$$
\text{(i)} \quad \begin{bmatrix} z_0 \\ (.., z_1(\sigma), ..) \\ F \end{bmatrix} =
$$

$$\left\{\begin{array}{l} \sum_h \lambda^h \left(\sum_{s^h,v} \gamma^{h,\nu}(s^h)\pi(s^h)(c_0^{h,v}(s^h)(p_0,p_1,\rho,q,F) - w_0^h)\right) \\ \vdots \\ \sum_h \lambda^h \left(\sum_{s^h,v} \gamma^{h,\nu}(s^h)\pi(s)(c_1^{h,v}(s^{-h},\sigma;s^h)(p_0,p_1,\rho,q,F) - w_1^h(s,\sigma)\right) \\ \vdots \\ \left\{\begin{array}{l} = \sum_h \sum_{s,\sigma} \pi(s^{-h}/s^h)(\rho_\sigma r_j(s,\sigma) - q_j) \left(\frac{\lambda^h \pi(s^h) \sum_\nu \gamma^{h,\nu}(s^h)\theta_j^{h,\nu}(s^h)(p_0,p_1,\rho,q,F)}{\sum_{h,s^h} \lambda^h \pi(s^h) \sum_\nu \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)(p_0,p_1,\rho,q,F)}\right), \\ \text{if } \sum_{h,s^h} \lambda^h \pi(s^h) \sum_v \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)\,(p_0,p_1,\rho,q,F) \neq 0 \\ \\ 0, \text{ otherwise} \end{array}\right. \end{array} \quad \forall\gamma \in \Delta \right\}$$

(ii)

$$(p_0, p_1, \rho) \in \arg\max\{p_0 \cdot z_0 + \rho \cdot (.., p_1(\sigma) \cdot z_1(\sigma), ..)\}$$

(iii)

$$q = \sum_\sigma \rho_\sigma \sum_s \pi(s) r_j(s,\sigma)$$

The domain of the above map is clearly compact, convex. We will now show it is also upper-hemicontinuous and convex-valued. The aggregate excess demand correspondence was obtained by taking the sum of correspondences which by Lemma 3.1 are upper-hemicontinuous and, as established in Step 2, also convex-valued. Upper-hemi-continuity and convex-valuedness of the maps defining (p_0, p_1, ρ, q) then follows by a standard argument. Finally, the map defining F, for (p_0, p_1, ρ, q, F) such that $\sum_{h,s^h} \lambda^h \pi(s^h) \sum_v \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)\,(p_0, p_1, \rho, q, F) \neq 0$, is a well-defined continuous function of upper-hemicontinuous, convex-valued correspondences, and hence is also upper-hemicontinuous and convex-valued. It remains to show that upper-hemicontinuity also holds at points where $\sum_{h,s^h} \lambda^h \pi(s^h) \sum_v \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)\,(.) = 0$ (convex-valuedness is clearly satisfied at such points). For this, note that along any sequence $(p_0, p_1, \rho, q, F)^\tau$ such that

$\sum_{h,s^h} \lambda^h \pi(s^h) \sum_v \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h) \left((p_0, p_1, \rho, q, F)^\tau\right)$ converges to 0, as $\tau \to \infty$, the expression

$$\sum_h \sum_{s,\sigma} \pi(s^{-h}/s^h)(\rho_\sigma r_j(s,\sigma) - q_j) \left(\frac{\lambda^h \pi(s^h) \sum_\nu \gamma^{h,\nu}(s^h) \theta_j^{h,\nu}(s^h)((p_0, p_1, \rho, q, F)^\tau)}{\sum_{h,s^h} \lambda^h \pi(s^h) \sum_\nu \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)((p_0, p_1, \rho, q, F)^\tau)} \right)$$

also converges to 0, since as shown in Step 3, it equals:

$$\sum_h \lambda^h \sum_{s,\sigma} \pi(s) \left(\rho_\sigma r_j(s,\sigma) - q_j\right) \hat{\theta}_j^h(s^h)((p_0, p_1, \rho, q, F)^\tau) \cdot$$
$$\cdot \left(\frac{\lambda^h \pi(s^h) \sum_\nu \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)((p_0, p_1, \rho, q, F)^\tau)}{\sum_{h,s^h} \lambda^h \pi(s^h) \sum_\nu \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)((p_0, p_1, \rho, q, F)^\tau)} \right),$$

and $\hat{\theta}_j^h(s^h)((p_0, p_1, \rho, q, F)^\tau)$ converges to zero, while the terms $\left(\frac{\lambda^h \pi(s^h) \sum_\nu \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)(.)}{\sum_{h,s^h} \lambda^h \pi(s^h) \sum_\nu \gamma^{h,\nu}(s^h) I_j^{h,\nu}(s^h)(.)} \right)$, as we argued, always lie in the simplex.

Kakutani's Theorem can then be applied to show that the map has a fixed point. Recalling the expression of Walras law we obtained ((3.4)), it is immediate to see that if, at the fixed point, $(\rho, p_0, p_1)_\delta \in int\{\Delta_\delta^{L+\Sigma-1} \times (\Delta_\delta^{L-1})^\Sigma\}$, we have $[(z_0), (.., z_1(\sigma), ..)]_\delta = 0$, i.e. an equilibrium. If not, let $\delta \to 0$ and consider the associated sequence of fixed points. By a standard argument (see, e.g., Werner (1985)) we can show that this sequence is convergent and, given the boundary behavior property of excess demand, the limit value $(\rho, p_0, p_1)^* \in int\{\Delta^{L+\Sigma-1} \times (\Delta^{L-1})^\Sigma\}$. ■

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