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THE STRUCTURE OF PRODUCTION, TECHNOLOGICAL CHANGE AND THE RATE OF GROWTH OF TOTAL FACTOR PRODUCTIVITY IN THE BELL SYSTEM

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No. 79-14 April 1979

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* This is a joint paper. The order of the authors' names is arbitrary.

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INTRODUCTION

The objectives of this preliminary study are threefold. The first is to analyze empirically the production structure of the Bell System at the aggregate level. Particular attention is focused on the pattern of substitution among the factor inputs and the degree to which the aggregate production function is characterized by economies of scale. In this connection, we explore the role of research and development in the Bell System as an input in the production process, and its interaction with the traditional inputs. Second, we examine the impact of external technological change on the production structure of the Bell System. The issues here include not only the rate of such technical change , but also the extent to which it alters the optimal level and mix of inputs, that is the factor bias of external technical change. The third objective is to explore the interrelationship between scale economies internal to the Bell System and external technical change in determining the rate of growth of total factor productivity (TFP). Specifically, we propose and illustrate a methodology for assessing the relative contributions of technical change and scale economies to the observed growth in TFP.

The recent introduction of competitive elements in the telecommunications sector has revived interest in the structure of production in the Bell System, particularly in the degree of returns to scale. Whether a policy which is likely to fragment the market is a wise course of action depends, in part, on the magnitude of the scale economies. The available empirical work suggests the presence of scale economies in the Bell System (Vinod [1972], Sudit [1973], AT&T [1976] , but the confidence intervals are not tight. Furthermore, recent developments in the theory of duality between cost and production functions have not been exploited in estimating the production structure of the Bell System. This is especially unfortunate since the available time series data are quite limited, and the statistical precision can be enhanced by using duality theory.

Important issues are also raised by the rate and bias of technical change, and especially by the interaction of technical change and scale economies in determining the growth of TFP. Strong productivity growth in the telecommunications industry is particularly important since communications services are used by every sector in the economy; the repercussions of a significant change in the growth of TFP in the Bell System would be widely felt. If economies of scale are present in the Bell System and if they are a significant determinant of TFP growth, then a policy which sacrifices these economies may have dynamic implications well beyond the boundaries of the communications sector. Yet almost nothing is known about the magnitude and determinants of the growth of TFP in the Bell System.

We address these issues by first estimating an aggregate translog cost function for the Bell System, using annual data for the period 1947-76. The implied estimate of the scale economies is then used to explore the sources of the growth in TFP. The paper is organized into six sections. Section I provides a brief overview of duality theory and the choice between estimating a production or cost function. Section II presents the formulation of the model and the constraints necessary for interpreting the empirical results. In Section III some of the conceptual and econometric problems involved in estimating the translog cost function are described. A brief description of the data is given in Section IV. Section V presents and interprets the empirical results. In Section VI we propose a method for determining the contributions of scale economies and external technical change to the growth in TFP. Using the methodology and the empirical estimates from Section V, we present some preliminary calculations on the sources of aggregate TFP growth in the Bell System during the postwar period. Brief concluding remarks follow.

I. MODELLING THE STRUCTURE OF PRODUCTION

In order to estimate the production structure one can either examine the production function or the cost function associated with it. Recent work in duality theory has established that, under rather weak regularity conditions, there is a unique correspondence between the production and cost function and that all of the information about the underlying technology is contained in both functions (Shephard [1970], McFadden [forthcoming]). The choice between them is a matter of statistical convenience and analytical purpose.

The cost function is expressed in terms of factor prices and the level of output; the production function is in terms of factor inputs. The main statistical issue is whether it is safer to treat the factor prices and level of output, or the use of inputs, as exogenous to the firm (in this case). If the firm is a cost minimizer, its input choice is necessarily endogenous and direct estimation of the production function will yield inconsistent results (Mundlak and Hoch [1965]). However, in an analysis of a firm, it seems plausible to neglect monopsony power and treat the factor prices which it faces as fixed exogenously.

In most regulated industries, including telecommunications, the regulator fixes the output price and requires that the firm satisfy the corresponding demand. The level of output could then be treated as exogenous. Actually, it is only recently that the federal and state regulatory commissions have concerned themselves with the structure of rates in telecommunications. Attention had been confined primarily to ensuring that there be neither a revenue deficit nor surplus, leaving the regulated firm relatively (though not entirely) free to determine the rate structure. Under these circumstances, the levels of the different outputs, and hence the aggregate level of output, would be endogenous. This endogeneity would be troublesome in a study of the production structure which disaggregated outputs (see e.g. Fuss and Waverman [1977]). It is somewhat less problematic in an aggregate study such as the present one, and we therefore ignore it.

Since we treat output and factor prices as given, the cost function is used in this analysis. This approach has two additional advantages. First, it yields direct estimates of the various Allen-Uzawa elasticities of substitution. These parameters are the key to describing the nature of the production structure. If the production function were used to derive estimates of the elasticities of substitution, the matrix of production function coefficients has to be inverted and this exaggerates the estimation errors (Binswanger [1974a]). Second, according to duality theory (Shephard's Lemma), the derivative of the cost function with respect to a factor price yields the derived demand for the input. This implies a set of cost share equations whose parameters are a subset of those in the cost function itself. The implied cost share equations can be formulated and estimated together with the cost function. This greatly increases the degrees of freedom in the empirical work and enhances the statistical precision of the estimates. Constraining the cost shares to add to unity, as we must, does not impose any restriction on the underlying technology. However, if the value shares corresponding to the production function are so constrained and estimated jointly with the production function, constant returns to scale are implicity assumed.[1]

II. TRANSLOG FORMULATION

We have chosen the translog specification of the cost function for the empirical work. The translog can be viewed as a second order logarithmic approximation to any transformation surface (Christensen, Jorgenson and Lau [1973]). In its general form, the translog imposes no prior restrictions on the production structure. It allows the testing of various restrictions--such as homotheticity, homogeneity, unitary elasticities of substitution--and the assessment of the sensitivity of parameters of interest to those restrictions.

Assuming that the firm minimizes costs, we can represent the general for of the aggregate cost function as:

$$C = f(\underline{P}, Q, T) \qquad (1)$$

where $\underline{P}$ is a vector of factor prices, Q is the level of output, and T is time which is designed to capture all technical change produced external to the firm. The translog approximation to (1) can be written:

$$\ln C = \alpha_0 + \alpha_q \ln Q + \frac{1}{2}\gamma_{qq}(\ln Q)^2 + \sum_i \alpha_i \ln P_i + \frac{1}{2}\sum_{ij}\sum \gamma_{ij}\ln P_i \ln P_j + \sum_i \gamma_{iq}\ln P_i \ln Q$$

$$+ \sum_i \theta_{iT}\ln P_i \ln T + \beta_T \ln T + \frac{1}{2}\beta_{TT}(\ln T)^2 \qquad (2)$$

where i = L, K, M, R indexes the factor inputs labor, capital, material, and research, which are defined around some expansion point.[2]

It is more convenient to discuss the characteristics and parametric restrictions on (2) in conjunction with the associated cost share equations. According to a useful theorem from duality theory (Shephard's Lemma), the derived demand for an input, X_i, is obtained by partially differentiating the cost function with respect to the factor (service) price of the input

(Shephard [1970]).

$$\frac{\partial C(\cdot)}{\partial P_i} = X_i$$

Using Shephard's Lemma and partially differentiating the translog (2):

$$\frac{\partial \ln C(\cdot)}{\partial \ln P_i} = S_i = \alpha_i + \frac{1}{2}\sum_i \gamma_{ij} \ln P_j + \gamma_{iq} \ln Q + \theta_{iT} \ln T \qquad (3)$$

$$i = L, K, M, R$$

where $S_i = P_i X_i / C$ is the share of costs accounted for by factor i. Note that the coefficients in the share equations are a subset of those in the cost function. Also, since $\Sigma S_i = 1$, only three of the four share equations are linearly independent.

There are constraints on (2) and (3) implied by duality theory and the translog approximation.[3] First, the cost function must be linearly homogeneous in factor prices. This implies:

$$\Sigma \alpha_i = 1$$
$$\sum_j \gamma_{ij} = 0 \qquad (4)$$
$$\sum_i \gamma_{iq} = 0$$

Second, since the translog is viewed as a second order logarithmic approximation, the following symmetry constraint must hold:

$$\gamma_{ij} = \gamma_{ji} \qquad (5)$$

which together with (4) implies

$$\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0$$

The formulation in (2) allows both for neutral (β_T and β_{TT}) and biased (θ_{iT}) external technical change. If there is biased technical change ($\theta_{iT} \neq 0$), the cost shares in (3) are affected. Technical change is i^{th} factor using (saving) if $\theta_{iT} > 0$ ($\theta_{iT} < 0$).

The cost function is homothetic if it can be written as a separable function of factor prices and output (Shephard [1970], pp. 92-95). Equivalently, homotheticity holds if the optimal factor combination is independent of scale (expansion path is linear). It is clear from the share equations that this implies:

$$\gamma_{iq} = 0 \qquad \text{for all } i \tag{6}$$

The cost function is homogeneous in output if the elasticity of costs with respect to output is a constant. This is a special case of homotheticity and requires:

$$\gamma_{iq} = 0 \quad \text{and} \quad \gamma_{qq} = 0 \qquad \text{for all } i \tag{7}$$

The test for homogeneity is nested within the test for homotheticity.

The Allen-Uzawa elasticities of substitution, σ_{ij}, and the (output constant) own elasticities of factor demand, ε_{ii}, can be computed directly in the following manner (Binswanger [1974b]):

$$\sigma_{ij} = \frac{\gamma_{ij}}{S_i S_j} + 1 \qquad i \neq j$$

$$\varepsilon_{ii} = \frac{\gamma_{ii}}{S_i} + S_i - 1 \tag{8}$$

The special case of unitary elasticities of substitution clearly holds if:

$$\gamma_{ij} = 0 \qquad \text{for all } i \neq j$$

Hanoch [1979] has pointed out that the returns to scale in the production function must be defined along the expansion path rather than along an arbitrary input-mix ray, and these will differ if the production function in non-homothetic. Since the scale elasticity (evaluated around a given point) is identical to the reciprocal of the elasticity of costs with respect to output (Hanoch [1975]), we can use the latter as the measure of returns to scale. It is evaluated along the expansion path automatically, since cost minimization underlies the cost function. From (2) the general form of the scale elasticity is:

$$SE = [\alpha_q + \gamma_{qq}\ln Q + \sum_i \gamma_{iq}\ln P_i]^{-1} \tag{9}$$

which will vary with the level of operations and factor prices unless the cost (production) function is homogeneous in output. At the expansion point, $SE = \alpha_q^{-1}$.

III. ESTIMATION AND TESTING PROCEDURES

There is a variety of separability tests which can be applied to the translog function (Denny and Fuss [1977]), but in this paper, we limit the formal testing to four versions:

I. Unrestricted model

II. No non-neutral technical change: $\theta_{iT} = 0$ for all i

III. Homothetic model:[4] $\gamma_{iq} = 0$ for all i

IV. Homothetic, no non-neutral technical change: $\theta_{iT} = \gamma_{iq} = 0$ for all i

Version IV is nested within either Version II or III. We also discuss informally the pattern of empirical results relating to various types of input separability.

As indicated earlier, exploiting duality theory and estimating the cost share equations jointly with the cost function increases the statistical degrees of freedom, since the cost share parameters are a subset of the cost function parameters. The brevity of the available time series (1947-1976) and the multicollinearity of the regressors make this procedure imperative. Including the cost function itself in the multivariate regression system is important because the neutral time trend and the coefficients needed to evaluate the scale elasticity (α_q and γ_{qq}) do not appear in the share equations.

We estimate a system of equation consisting of the cost function and three of the cost share equations (the fourth is linearly dependent and hence not estimable). We follow the literature in specifying additive disturbances in each share equation and the cost function; the former represent errors in optimizing behavior (e.g., Christensen and Greene [1976]). The disturbances are specified to have a joint normal distribution (as required for formal testing) but contemporaneous correlation across equations is allowed. The system can be estimated by Zellner's [1962] seemingly unrelated regression technique (ZEF) which is a generalized least squares method with an error covariance matrix reflecting contemporaneous correlation.

In general, the parameter estimates are not invariant to the choice of which share equation to delete. An extension of a result by Barten [1969], however, shows that a maximum likelihood procedure asymptotically guarantees such invariance. The maximum likelihood, ZEF and iterative Zellner method (IZEF, in which iteration is performed until the residual covariance matrix converges) all have the same asymptotic properties

(Kmenta and Gilbert [1968]) and hence ensure invariance in large samples. However, this need not hold in small samples. It has been shown that the ML and IZEF methods yield numerically identical results in small samples if they both use the same initial residual covariance matrix (Kmenta and Gilbert [1968], Dhrymes [1970]). However, this is no guarantee that Barten's invariance holds in small samples, for that depends on the similarity between the small and large sample behavior of these procedures. The Monte Carlo study by Kmenta and Gilbert suggests that they may be quite different.[5]

The conclusion we draw is that invariance cannot be assumed, as is typically done in the literature, and that some experimentation with the choice of which share equation to delete should become a common practice. In this paper, the IZEF is employed but we experiment with two versions. In one, the intermediate input share equation is dropped (leaving the labor, capital and research shares); in the other, the research share equation is deleted.

Three other econometric problems should be noted. First, if there is serial correlation in the equations which is left uncorrected, the ZEF and IZEF methods will fail to satisfy Barten's invariance theorem even in large samples. Methods for handling autoregression in the ZEF framework are available (Parks [1967], Chow and Fair [1973]), but the problems raised by the adding-up constraint of the share equations and the relationship between the errors in the share and cost equations are not yet resolved. Some progress, however, has been made. Berndt and Savin [1975] demonstrate that, if there is first order serial correlation in a system of singular equations (e.g. shares), Barten's invariance holds in large samples only if the serial correlation parameter is constrained to be equal in all equations. Of course, this does not ensure invariance in small samples.

The related problem of the cost function error has not been treated in the empirical literature on translog cost (and production) models. The conventional approach argues that since the share equations are obtained by differentiating the cost function, the error in the cost function does not appear. Disturbances are then added to the share equations to represent "errors in optimization" (Berndt and Christensen [1973] Christensen and Greene [1976]). This rationale assumes that the firm is optimizing by choosing the factor <u>shares</u> rather than the <u>quantity</u> of the input. However, errors in optimization should refer to the firm not being on the derived demand for the input - the error is in $\partial C/\partial P$, not in $(\partial C/\partial P)P/C$. In that case, the cost function errors will generally appear in the share equations. The problem can only be solved by making explicit assumptions about the nature of the "error transmission" process (see Mundlak and Hoch [1965], Zellner et.al.[1966] in the production function context).

Finally, no account has been taken of the fact that the dependent variable in each share equation is bounded between zero and unity. This means the disturbances cannot be normally distributed. There is a formal analogy here with probability choice models which suggests alternative econometric approaches to estimating the share equations, such as logit and probit (McFadden [1976]). This problem is common to all the literature on production and cost functions which exploits duality. We do not address it in this paper but flag it for future research.

Putting these problems aside, we treat our parameter estimates as maximum likelihood estimates. The various parameter restrictions are tested using the likelihood ratio test:

$$\lambda = \Lambda(R) - \Lambda(U)$$

where $\Lambda(R)$ and $\Lambda(U)$ are the log likelihood values under the restricted and unconstrained versions, respectively. Then -2λ distributes asymptotically as Chi-squared with degrees of freedom equal to the number of independent restrictions being tested. In view of our earlier discussion, this test should be viewed as approximate.

IV. DESCRIPTION OF THE DATA[6]

The measure of aggregate output is the sum of deflated revenues for four service categories - local service, intrastate toll, interstate toll, and a miscellaneous category. The revenues for each category are deflated by a category-specific chain-linked Paasche price index (normalized in 1967). The price index for aggregate output is obtained as a discrete Divisia index (arithmetic weights) of the separate chain-linked Paasche price indices.

The quantity of labor input is the quality adjusted manhours actually worked (excluding vacations, holidays, sick leave, and labor attributable to construction which is included in the measure of plant and equipment). The manhours worked are cross-classified into six occupational groups and seven categories based on years of experience. The quality adjustment involves weighting the manhours in each of the forty-two categories by the ratio of the average hourly wage for that category in the base year (1967) to the average wage for all groups. An implicit price index for labor services is obtained by dividing the total employee compensation charged to expense (including all fringes and social security taxes) by the quantity of labor input.

The quantity of intermediate input is obtained as a Laspeyres index of current expenditures on six categories of materials, rents, and supplies. Each component is separately deflated with a 1967 base year, and the results are summed; about three-quarters of the current expenditures fall in the miscellaneous category and are deflated by the implicit GNP deflator. However, for the price series on materials we use the Laspeyres price index of intermediate materials and supplies from the Survey of Current Business [1978], normalized in 1967.

The stock of capital is defined as the sum of tangible plant (including land), cash, net accounts receivables, and inventories of materials and supplies. Tangible plant, which consitutes the bulk of capital stock (more than 95 percent), is broken down into twenty-three separate accounts. For each plant account of a particular vintage, the historical depreciation reserve is determined from plant mortality tables. The vintage depreciation reserve for the plant account is subtracted from the gross book value of that vintage to obtain the net surviving plant for that vintage and account. The Bell System Telephone Plant Indexes (Laspeyres price indices) are applied to the net surviving plant of each vintage and account and the results summed to obtain the constant (1967) dollar value of net plant. Net capital stock is obtained by adding to net plant the deflated net accounts receivable (using the chain-linked price index for output), and inventories and cash (using the implicit GNP deflator).

We construct the service price of capital, c_K, for each year as follows:

$$c_K = P_I\left\{\frac{[1 - uz - \omega + \omega \cdot u \cdot z]}{1 - u}(r + \delta) + \tau\right\}$$

where P_I is the investment goods deflator, u is the corporate income tax rate, ω and z are the effective rate of investment tax credit and present value of depreciation allowance, τ is the indirect tax rate, r is the realized after-tax rate of return, and δ is the depreciation rate. These parameters are constructed from Bell System data whenever possible; the details are described in the data appendix.

The stock of research and development, R, is constructed as a geometrically weighted sum of deflated R&D expenditures by the Bell System, lagged four years:

$$R_t = \Sigma(1 - \delta)^i R\&D_{t-4-i}$$

where δ is the geometric rate of obsolescence, taken as $\delta = 0.15$. The cumulation begins in 1925. The four year lag is taken to reflect the gestation period during which R&D has no effect on output.[7] Nominal R&D expenditures are deflated by the Laspeyres price index for R&D proposed by Milton [1972] and updated by Battelle [1976]. The service price of research capital is computed as $c_R = P_R(r + 0.15)$ where P_R is the Milton price index for research. This is analogous to the service price of traditional capital; the tax parameters are absent because R&D is treated as an operating expense rather than capitalized for tax purposes.

The total cost figure used in the estimation of the cost function is the sum of four elements: nominal expenditures on labor, current cost of materials, the product of the service price of capital and the constant dollar value of the net stock of capital, and the service price of research times the deflated value of the stock of research. When research is ignored as a productive input, the last component is excluded from the total cost figure.

V. EMPIRICAL RESULTS

We present parameter estimates for three models: 1. the four factor model deleting the research share equation (Model A), 2. the four factor model deleting the materials share (Model B), 3. a three factor model in which we ignore research as a productive input, and delete the materials share for estimation (Model C). For brevity, we present only the unconstrained versions of Model A - C in Table 1. The goodness of fit statistics are given in Table 2. A comparison of Models A and B highlights the effect of dropping a different share equation, whereas, a comparison of Models B and C illustrates the effect of including research as a productive input on the description of the production structure.

Note first that the models fit the data quite well, both for the cost function and the cost share equations. This is encouraging since translog models often yield relatively poor fits to the cost share equations (Denny and Fuss [1975]).[8] Moreover, the regressions track the turning points closely. Nearly all of the turning points in the cost function (27 of 29) and more than three quarters of those in the share equations are identified by the regressions, and in no case is the divergence of any quantitative significance. Also note that there is strong evidence of serial correlation in the equations.

Table 1

Various Cost Function Parameter Estimates: Unconstrained Version

	4 Factor Model Research Deleted	4 Factor Model Materials Deleted	3 Factor Model Materials Deleted
α_0	5.185 (0.363)	6.059 (0.347)	4.625 (0.464)
α_L	0.465 (0.063)	0.330 (0.086)	0.492 (0.108)
α_K	0.616 (0.082)	0.579 (0.084)	0.497 (0.090)
α_M	0.054 (0.039)	0.111* (0.060)	0.011* (0.066)
α_R	-0.135* (0.031)	-0.020 (0.021)	
γ_{LL}	0.177 (0.015)	0.093 (0.018)	0.068 (0.031)
γ_{KK}	0.139 (0.012)	0.145 (0.012)	0.139 (0.017)
γ_{MM}	-0.022 (0.013)	-0.051* (0.020)	-0.028* (0.022)
γ_{rr}	0.036* (0.006)	0.035 (0.003)	
γ_{LK}	-0.122 (0.009)	-0.128 (0.010)	-0.117 (0.020)
γ_{LM}	0.026 (0.012)	0.078* (0.017)	0.050* (0.022)
γ_{LR}	-0.081* (0.007)	-0.044 (0.006)	
γ_{KM}	-0.033 (0.008)	-0.027* (0.011)	-0.022* (0.012)
γ_{Kr}	0.016* (0.007)	0.009 (0.004)	
γ_{Mr}	0.029* (0.005)	-0.0003* (0.006)	

Table 1 (Continue)

	4 Factor Model Research Deleted	4 Factor Model Materials Deleted	3 Factor Model Materials Deleted
γ_{Lq}	-0.074 (0.011)	-0.034 (0.014)	-0.032 (0.020)
γ_{Kq}	0.018 (0.012)	0.022 (0.013)	0.028 (0.016)
γ_{Mq}	0.010 (0.007)	-0.013* (0.010)	0.004* (0.012)
γ_{rq}	0.046* (90.005)	0.025 (0.004)	
θ_{LT}	-0.009 (0.008)	-0.032 (0.009)	-0.041 (0.009)
θ_{KT}	0.046 (0.011)	0.049 (0.010)	0.047 (0.008)
θ_{MT}	-0.017 (0.004)	-0.001* (0.005)	-0.006* (0.003)
θ_{rT}	-0.020* (0.004)	-0.016 (0.002)	
α_q	0.553 (0.048)	0.425 (0.045)	0.534 (0.066)
β_T	-0.086 (0.033)	-0.092 (0.029)	0.144 (0.023)
β_{TT}	0.165 (0.026)	0.202 (0.024)	0.062 (0.017)

Note: An asterisk denotes a parameter (standard error) which has been derived from the homogeneity constraints (4).

Table 2

Goodness of Fit Statistics: Unconstrained Version
(R^2 / Durbin Watson)

	4 Factor Model Research Deleted	4 Factor Model Materials Deleted	3 Factor Model Materials Deleted
Cost Function	0.99/0.58	0.99/0.55	0.99/0.70
Labor Share	0.94/0.44	0.90/0.37	0.87/0.25
Capital Share	0.80/0.47	0.80/0.43	0.84/0.36
Materials Share	0.92/1.05	NA	NA
Research Share	NA	0.87/1.02	NA

Note: The first number refers to R^2, the second to the Durbin Watson statistic.

We do not adjust for it at this stage but will return to the problem of stochastic specification later. The empirical results are still unbiased but not efficient.

Turning to the parameter estimates in Table 1, Models A and B yield qualitatively similar results. In both models it is evident that the γ_{ij} terms are not zero, which implies that the assumption of unitary elasticities of substitution would be violated. The pattern of the θ's suggests that (time-related) technical change is capital-using but saving of materials, labor and research; the cost shares are affected by technical change. The neutral technical change parameters (β_T and β_{TT}) indicate a downward drift of costs at a decelerating rate.[9] The cost elasticity (at the point of expansion), α_q, is somewhat larger in Model A but the difference is statistically insignificant. The estimates of the γ_{iq} terms indicate that the production structure is not homothetic.

The formal tests of non-neutral technical change and homothecity are presented in the upper half of Table 3. In both Models A and B we decisively reject the null hypothesis that there is only neutral technical change and that the production structure is homothetic; the joint hypothesis, therefore, is also rejected.

The three factor model in which research is neglected (Model C) yields somewhat different results. The pattern of non-neutral technical change indicates a bias in favor of both capital and materials. The neutral time trend has the wrong sign and is statistically significant.

Table 3

Test of Parametric Restrictions

	Non-Neutral Technical Change	Homotheticity	Non-Neutral Technical Change & Homotheticity
Critical $\chi^2_{.05}$, 4 Factor Models	7.82	7.82	12.59
4 Factor Model, Research Deleted	26.4	33.8	89.2
4 Factor Model, Materials Deleted	32.8	14.0	75.8
Critical $\chi^2_{.05}$, 3 Factor Models	5.99	5.99	9.49
3 Factor Model, Materials Deleted	21.4	2.6	46.2

It suggests, implausibly, that the cost curve has been shifting upward at an accelerating rate. Moreover, while the γ_{iq} terms have the same sign as in Models A and B, they are far less precise. The lower half of Table 3 indicates that, while the hypothesis of biased technical change can be rejected, the null hypothesis that the production structure is homothetic cannot be rejected by a wide margin. In short, the chief effects of ignoring research as a productive input are 1) to distort the nature of neutral technical change and 2) to make the production structure appear homothetic even though, in a more complete model, homotheticity is rejected.

A concise description of the production structure is provided by the Allen-Uzawa elasticities of substitution and the elasticities of factor demand. Tables 4 and 5 present the results. Looking first at the elasticities of substitution among the traditional inputs (σ_{LK}, σ_{LM}, σ_{KM}), all three models yield very similar results; the differences are not statistically significant. Each traditional input is a substitute for the other ($\sigma_{ij} > 0$), with (the point estimates of) substitutability being strongest between labor and materials and weakest between labor and capital. The point estimates are all significantly different both from zero and from unity. Therefore, based on our sample the fixed coefficient and Cobb-Douglas models would misrepresent the substitution possibilities among the traditional inputs.

The four factor models indicate that research is strongly complementary with labor but substitutable with materials and capital.[10] The elasticities of substitution are statistically significant and much larger than unity, except for σ_{MR} in Model B. While the results are

Table 4

Partial Elasticities of Substitution: Unconstrained Version

	4 Factor Model, Research Deleted	4 Factor Model, Materials Deleted	3 Factor Model, Materials Deleted
σ_{LK}	0.268 (0.054)	0.232 (0.060)	0.276 (0.113)
σ_{LM}	1.474 (0.219)	2.426* (0.308)	1.778* (0.374)
σ_{KM}	0.498 (0.122)	0.592* (0.170)	0.621* (0.163)
σ_{LR}	-5.348* (0.552)	-2.470 (0.473)	
σ_{KR}	2.053* (0.461)	1.592 (0.263)	
σ_{MR}	6.780* (1.080)	0.932* (1.161)	

Note: Evaluated at the 1967 values of the cost shares, using equation (8). An asterisk denotes an elasticity of substitution based on a derived estimate of γ_{ij} using the parameter constraints (4).

Table 5

Own Elasticities of Factor Demand: Unconstrained Version

	4 Factor Model, Research Deleted	4 Factor Model, Materials Deleted	3 Factor Model, Materials Deleted
ε_{LL}	-0.152 (0.041)	-0.377 (0.049)	-0.445 (0.083)
ε_{KK}	-0.243 (0.028)	-0.228 (0.027)	-0.241 (0.038)
ε_{MM}	-1.001 (0.087)	-1.201* (0.138)	-1.041* (0.148)
ε_{RR}	0.078* (0.168)	0.061 (0.088)	

Note: Evaluated at the 1967 values of the cost shares using equation (8). An asterisk denotes an elasticity band on a derived estimate of γ_{ij} using the parameter constraints (4).

qualitatively similar for the two four factor models, Model A yields stronger complementarity between labor and research and substitutability between materials and research, and the differences are statistically significant. The choice of which share equation to delete does matter here.

The elasticities of factor demand for traditional inputs have the correct sign and are statistically different from zero. The point estimate of the elasticity for research has the wrong sign but it is not significantly different from zero. The results indicate that the demands for labor and capital are highly inelastic, materials is basically unitary elastic, while the demand for research appears to be price insensitive.

The scale elasticity from the unconstrained translog cost function will vary over time, since it depends on the relative factor prices which have not remained constant.[11] The time path of the scale elasticity will depend on where the expansion point for the translog is fixed (see footnote 2), but at the expansion point, the scale elasticity simply equals α_q^{-1}. From Table 1, the point estimates (standard error) of the scale elasticity from Models A-C are computed as 1.81(0.05), 2.35(0.05) and 1.87(0.07), respectively. These indicate substantial, and statistically significant, economies of scale in the Bell System at the aggregate level. It is interesting that the scale elasticity in the three factor model is not substantially smaller than in the four factor models.

The empirical results described above were based on the assumption of serially uncorrelated disturbances in all the equations. This was done for two reasons. First, it makes our work more comparable to the literature; we are aware of no study of production structure in telecommunications which adjusts for serial correlation. Second, the proper methodology for such stochastic specifications in a system of singular share equations and a cost function is not well developed. As noted earlier, the only work on this problem is Berndt and Savin [1975]. They demonstrate that if first order autoregression exists in a singular system of equations, the large sample estimates will be invariant to which equation is deleted only if the serial correlation parameter is constrained to be equal in all equations. The relevance of this theorem to small samples remains problematic, as does the role of the cost function disturbance in such a system.

Nonetheless, we did experiment with corrections for serial correlation, primarily to check whether the magnitude of the scale elasticity is sensitive to such adjustments. Following Berndt and Savin, the serial correlation coefficient was constrained to be the same in all the factor share equations and a search over the grid of values (0.3, 0.9) was performed. The serial correlation coefficient in the cost function was computed from the residuals in the cost function estimated by (serially unadjusted) IZEF. The general pattern of results was qualitatively similar to those reported earlier. The implied scale elasticity was not reduced by the adjustment. In fact, the point estimates increased somewhat. They vary between 1.75 and 2.86, depending on the serial correlation coefficient, with an average of 2.08. The main difference was that the neutral time trend entered with a positive sign and its magnitude was sensitive to the serial correlation adjustment.

VI. THE SOURCES OF TOTAL FACTOR PRODUCTIVITY GROWTH

In this section we propose a method for decomposing the measured growth in total factor productivity (TFP) into two categories: 1) growth due to non-constant returns to scale ("scale effect"), and 2) the effect of external technical change. TFP is basically a measure of output per unit of total factor input. Total factor input is a weighted average of inputs, where the weights depend on the underlying production function. If there are increasing returns to scale, part of the growth in TFP will reflect the change in the scale of operations, while the rest is normally ascribed to a shift in the production frontier itself. Clearly, if there were constant returns to scale, the change in TFP would be identical to the technological shift (assuming other factors are exhaustive and accurately measured). However, some of the growth in total factor input is induced by external technical change, through the shifts in the derived demand functions for inputs which technical change causes.[12] In the presence of increasing returns, this in turn raises the level of TFP. This illustrates one level of interaction between scale economies and external technical change which must be taken into account if a proper attribution of the growth in TFP is to be made. It suggests a distinction between the gross scale effect, which includes the input accumulation induced by technical change, and the net scale effect which does not. In this section, we explore these concepts more fully and suggest a way of measuring them.

The rate of growth of TFP is defined as:

$$\dot{TFP} = \dot{Q} - \dot{F} \tag{9}$$

where Q is output, F is total factor input, and a dot represents a rate of growth. At the aggregate level, there is only one output (by assumption) so that $\dot{Q}$ is defined unambiguously. For measuring $\dot{F}$, the Divisia index has become increasingly popular. The Divisia index is a weighted sum of rates of growth, where the weights are the components' share in total value. Hulten [1973] demonstrated that the Divisia index conserves all the information contained in the components and that no other index can do better. It is well known that the Divisia index is a line integral and that its value may therefore not be path independent. Hulten [1973] has shown that the index will be path independent, however, if and only if the aggregate over which it is defined actually exists. Path independence is therefore an essential element of any acceptable Divisia index.

The conventional Divisia index of $\dot{F}$ is the cost share weighted average of rates of growth of inputs:

$$\dot{F} = \sum_i \left(\frac{P_i X_i}{C}\right)\dot{X}_i \ ; \quad \Sigma P_i X_i = C \qquad (10)$$

where P_i and X_i are the i^{th} factor's price and quantity, and C is total cost. Hulten [1973] has shown that (10) is path independent if and only if F is linearly homogeneous, i.e., if the production function exhibits constant returns to scale. To preserve path independent when F is not linearly homogeneous, we must use the "quasi-Divisia" index (Hulten [1973], pp. 1022-3):

$$\dot{F} = \sum_i \left(\frac{P_i X_i}{PQ}\right)\dot{X}_i \qquad (11)$$

where PQ is the value of output. This is a weighted sum (not average) of the rates of growth of the factor inputs, where the weights are value (not cost) shares. The path independent Divisia index of TFP becomes:

$$\dot{TFP} = \dot{Q} - \sum_i \left(\frac{P_i X_i}{PQ}\right) \dot{X}_i \tag{12}$$

The next step is to relate this measure of $\dot{TFP}$ to external technical change without regard for the induced effect. Consider the production function defined over inputs $\underline{X}$ and time T:

$$Q = F(\underline{X}, T)$$

Differentiating with respect to time and dividing by Q

$$\dot{Q} = \sum_i \left(\frac{\partial F}{\partial X_i} \frac{X_i}{Q}\right) \dot{X}_i + \dot{T} \tag{13}$$

where $\dot{T} = (\partial F/\partial T)/F$. Assuming cost minimization,[13]

$$\frac{\partial F}{\partial X_i} = \frac{P_i}{\partial C/\partial Q} \tag{14}$$

so (13) becomes

$$\dot{Q} = \Sigma \left(\frac{P_i X_i}{Q(\partial C/\partial Q)}\right) \dot{X}_i + \dot{T} = \frac{P}{\partial C/\partial Q} \Sigma \left(\frac{P_i X_i}{PQ}\right) \dot{X}_i + \dot{T} \tag{15}$$

Substituting (15) into the Divisia index (12),

$$\dot{TFP} = \dot{T} - \left(1 - \frac{P}{\partial C/\partial Q}\right) \dot{F} \tag{16}$$

Define the cost elasticity as $\eta = (\partial C/\partial Q)Q/C$ and substitute in (16) to obtain:

$$\dot{TFP} = \dot{T} + [k\eta^{-1} - 1] \dot{F} \tag{17}$$

where $k = PQ/C = P/AC$ is the ratio of output price to average cost.

Equation (17) gives the relationship between measured growth in total factor productivity, $\dot{TFP}$, and the neutral shift in the production function, $\dot{T}$. The two will differ unless there are constant returns to scale ($\eta = 1$) and the output price is set to generate zero profits ($k = 1$). It should be noted that regulation does not ensure that $k = 1$ since the accounting definition of costs may differ from the economic one, and of course because the allowed rate of return may differ from the real cost of capital. As a practical matter, however, k may be close to unity. Then $k\eta^{-1} \approx \eta^{-1}$, which is just the scale elasticity.

Expression (17) does separate the shift in the production function from the remaining TFP growth, but it does not properly identify the contribution of external technical change to TFP growth. This contribution should include the effect that technical change has on the optimal level (and mix) of total factor input.[14] Equation (17) decomposes TFP into a pure technological shift which is unrelated to factor input decisions, and a gross scale effect which shows the effect on TFP of increasing the level of factor input use. However, part of the growth of F is related to technical change, since technical change alters the marginal productivity of inputs (i.e., shifts the derived demands for factors). If there are economies of scale, this effect of technical change makes an indirect contribution to the growth in TFP. Note that the induced effect of technical change requires that there be scale economies, so the two interact. Also, the entrepreneurial role of responding to a changing economic environment is critical if the indirect contribution is to be exploited. The assumption of cost minimization serves that purpose here.

The task is to identify the indirect contribution of technical change. This involves separating the change in $\dot{F}$ which is induced by technical change from that which is related to changes in factor prices and exogenous shifts in demand (output). We begin with the general form of the derived demand function for input i:

$$X_i = f(\underline{P}, Q, T) \tag{18}$$

where $\underline{P}$ is a vector of factor prices, Q is output and T is an index of external technical change (e.g. time). Totally differentiating (18) with respect to T,

$$\frac{dX_i}{dT} = \sum_i \frac{\partial f}{\partial P_j} \frac{\partial P_j}{\partial T} + \frac{\partial f}{\partial Q} \frac{\partial Q}{\partial T} + \frac{\partial f}{\partial T} \tag{19}$$

Dividing (19) by X_i and rearranging, we can express the equation in terms of growth rates:

$$\dot{X}_i = \sum_j \varepsilon_{ij} \dot{P}_j + \eta_{iq} \dot{Q} + \dot{V}_i \tag{20}$$

where a dot denotes a growth rate, ε_{ij} is the output-uncompensated cross elasticity of factor demand, $\eta_{iq} = \partial \ln X_i / \partial \ln Q$ is the elasticity of factor use with respect to output, and $\dot{V}_i = \frac{\partial f}{\partial T} \frac{1}{f}$ is the neutral shift in X_i. For later use, we also note that the output-uncompensated elasticities of factor demand can be written

$$\varepsilon_{ij} = S_j \; (e^d + \sigma_{ij}) \tag{21}$$

where S_j is the cost share of factor j, σ_{ij} is the Allen-Uzawa elasticity of substitution, and e^d is the (uncompensated) elasticity of demand for output.

Equation (20) expresses the observed (equilibrium) rate of growth of factor i as the sum of three effects, each of which involves a shift in the demand curve for the input: 1) the impact of changes in factor prices (both substitution and output effect), 2) a pure expansion effect reflecting changes in the level of output due to factors such as demand growth, and 3) a neutral shift due to technical change. It is the third effect which must be reassigned from the "scale effect" category to technical change in order to measure the full contribution of technical change to TFP.

Substituting (20) into (17) and using (11), we obtain

$$\dot{TFP} = \dot{T} + \dot{Z}_1 + \dot{Z}_2 + \dot{Z}_3 \tag{22}$$

where

$$Z_1 = (k\eta^{-1} - 1) \sum_i S_i^! \dot{V}_i$$

$$Z_2 = (k\eta^{-1} - 1) \sum_i S_i^! \sum_j \varepsilon_{ij} \dot{P}_j$$

$$Z_3 = (k\eta^{-1} - 1) \sum_i S_i^! \eta_{iq} \dot{Q}$$

and $S_i^!$ is the value share of factor i. The full contribution of technical change to TFP growth is given by $\dot{T} + \dot{Z}_1$; the net scale effect is $\dot{Z}_2 + \dot{Z}_3$. Equation (22) also highlights the interaction between economies of scale and technical change. The indirect contribution of technical change, $\dot{Z}_1$, depends on there being non-constant returns to scale ($\eta \neq 1$, assuming that $k \simeq 1$).

The relative contributions of economies of scale and technical change to TFP growth are seen in the growth accounting equation (dividing (22) by $\dot{TFP}$):

$$1 = \frac{(\dot{T} + \dot{Z}_1)}{\dot{TFP}} + \frac{(\dot{Z}_2 + \dot{Z}_3)}{\dot{TFP}} \tag{23}$$

The next task is to compute these contributions in the translog context. We focus on the contribution of scale economies (second term in (23)); the role of technical change follows from (23).

The translog form of the derived demand for input i is:

$$X_i = \frac{C(\cdot)}{P_i} [\alpha_i + \gamma_{iq} \ln Q + \theta_{iT} \ln T + \sum_j \gamma_{ij} \ln P_j] \tag{24}$$

The only new parameter we need is the elasticity of factor use with respect to output, η_{iq}. From (24),

$$\eta_{iq} \equiv \frac{\partial \ln X_i}{\partial \ln Q} = \eta + \frac{\gamma_{iq}}{S_i} \tag{25}$$

where, as before, η is the cost elasticity and S_i is the cost share of factor i.

In the approximation formulae we derive for $\dot{Z}_2$ and $\dot{Z}_3$, it is assumed that k = 1, i.e., that profits are zero. This implies that the value and cost shares of an input are the same, $S_i = S_i'$. With this in mind, consider

$$\dot{Z}_2 = (\eta^{-1} - 1) \sum_i \varepsilon_{ij} \dot{P}_j$$

Using (21),

$$\dot{Z}_2 = (\eta^{-1} - 1) [e^d \sum_j S_j \dot{P}_j \sum_i S_i + \sum_{ij}\sum S_i S_j \sigma_{ij} \dot{P}_j] \tag{26}$$

Since $\Sigma S_i = 1$ and $\sigma_{ij} = \sigma_{ji}$, (26) becomes

$$\dot{Z}_2 = (\eta^{-1} - 1) [e^d \sum_j S_j \dot{P}_j + \sum_j S_j \dot{P}_j \sum_i S_i \sigma_{ji}]$$

or

$$\dot{Z}_2 = (\eta^{-1} - 1) e^d \sum_j S_j \dot{P}_j \tag{27}$$

since production theory implies the constraint $\sum_i S_j \sigma_{ji} = 0$. Equation (27) shows that the only influence changes in factor prices have on TFP is through their output expansion effect, which depends on the price elasticity of demand. The pure input substitution effect washes out precisely because output is held constant in that effect. Note that $\dot{Z}_2 < 0$ if there are increasing returns ($\eta < 1$) and factor prices are rising.

Next consider $\dot{Z}_3$:

$$\dot{Z}_3 = (\eta^{-1} - 1)\dot{Q}\Sigma S_i \eta_{iQ} \tag{28}$$

Using (25), this can be written

$$\dot{Z}_3 = (\eta^{-1} - 1)\ \dot{Q}\ \Sigma S_i (\eta + \gamma_{iq}/S_i)$$

which becomes

$$\dot{Z}_3 = (1 - \eta)\dot{Q} \tag{29}$$

since $\Sigma S_i = 1$ and $\Sigma_i \gamma_{iq} = 0$ by the homogeneity constraints on the translog cost function (see (4) above).

Using (27) and (29), the contribution of scale economies to TFP growth - the net scale effect (NSE) - can be written:

$$\text{NSE} = \frac{(\eta^{-1} - 1)e^{d}\ \Sigma S_j \dot{P}_j + (1 - \eta)\dot{Q}}{\dot{\text{TFP}}} \tag{30}$$

The contribution of technical change is given by (1 - NSE).

We provide an illustrative application of this methodology to TFP growth in the Bell System during the period 1948-76. The terms $\dot{\text{TFP}}$ and $\Sigma S_j \dot{P}_j$ are constructed using the discrete approximation to the Divisia index (Torquist index, Diewert [1976]). The NSE values we report are based on average Divisia growth rates for three sub-periods as well as for the entire period. The scale elasticity, η^{-1}, is taken as 1.81 (from Table 1, Model A). Three values for the aggregate price elasticity of demand are used, -0.5, -0.6 and -0.7. These values are consistent with the estimates reported in the econometric literature on telephone demand (e.g. Dobell et.al.[1972], and Davis et.al.[1973]). Table 6 presents the results, together with the corresponding TFP growth.

Table 6

Fraction of TFP Growth Due to Scale Economies

	$e^d = -0.5$	$e^d = -0.6$	$e^d = -0.7$	$\dot{TFP}$
1948-56	0.95	0.86	0.78	0.0213
1957-65	0.60	0.53	0.45	0.0311
1966-76	0.17	0.05	-0.07	0.0450
1948-76	0.43	0.33	0.23	0.0342

Note: Columns 1 - 3 computed from (30) in the text.

The results suggest that over the entire period 1948-76, economies of scale accounted for roughly between 15 and 30 percent of TFP growth; technical change contributed the other 70 - 85 percent. A comparison of the subperiods indicates that the relative contribution of scale economies to TFP growth declined steadily over the period. It is interesting to note that the (average) growth rate of TFP steadily increased over that same period (column 4).

These computations are only meant to illustrate the usefulness of the methodology. A more refined analysis would require empirical work on the price elasticity of demand and a closer look at the scale elasticity, especially in the framework of a disaggregated (multi-output) production structure. Furthermore, TFP is defined in this analysis as the difference between the growth in output and total factor input, including research. It would be useful to limit the definition of TFP to traditional inputs, and compute the separate contributions of internally-generated and external technical change.

CONCLUDING REMARKS

In this paper we explored the production structure (via the cost function) and the sources of the growth of total factor productivity in the Bell System during the period 1947-1976. Four main conclusions emerge:

1. The cost structure is well approximated by a translog cost function. Despite the limited availability of data, the empirical estimates we obtain are quite precise. This underscores the importance of exploiting duality theory. The evidence indicates that there is some scope for substitution among the three traditional inputs and research, but at least among the traditional inputs, the elasticities of substitution are distinctly smaller than unity. The production function is not homothetic; the optimal input mix is affected by the scale of operations.

2. The results suggest the presence of strong economies of scale at the aggregate level. The estimates of the scale elasticity cluster around 1.8 or higher, with the lower bound of the confidence intervals falling around 1.5. This conclusion is not sensitive to adjustment for serial correlation or to the specification of the model.

3. There is consistent evidence that external (time-related) technical change has exhibited a factor bias, being capital (and possibly materials) using, and labor and research saving. The optimal mix of inputs is therefore related both to technical change and to scale. There also appears to have been some downward shift of the cost curve, but this conclusion is sensitive to corrections for serial correlation.

4. The preliminary decomposition of the growth in TFP suggests that scale economies accounted for about 80 - 95 percent of such growth during the early postwar period (1947-1956), but that external technical change played an increasingly important role during the later years (1956-1976). This shift in relative influence does not reflect the exhaustion of scale economies. It is due to the fact that total factor input grew at a slower rate (relative to TFP) during the 1956-1976 period. Since economies of scale affect the level of TFP through the growth of inputs, there was less room for economies of scale to exert an influence on TFP.

There are several problems that require further investigation. The first is the econometric problem of modelling and estimating the cost function disturbance in conjunction with the share equations, both in the serially uncorrelated and correlated cases. Second, the statistical difficulties posed by the bounded dependent variables in the share equations need to be resolved. Finally, some attempt should be made to disaggregate the production structure of the Bell System and estimate a multi-output cost function. This would help to identify the sources of the economies of scale, test for jointness in production, and provide a more detailed view of the sources of total factor productivity growth.

FOOTNOTES

1. One disadvantage of the cost function approach is that it does not yield direct estimates of the marginal products of factors, an inversion procedure being necessary to retrieve them from the cost function coefficients. This is especially unfortunate in the case of the research input, since the marginal product of R&D is equal to its gross rate of return. In order to obtain an estimate of the marginal product of R&D, one must either derive it indirectly from the cost function coefficients, estimate the production function without the input share equations, or exploit duality but at the same time impose constant returns to scale. We do not derive the marginal product of R&D in this paper.

2. This means that the variables are implicitly normalized around some point, e.g., the sample mean. The normalization affects the interpretation of the coefficients and the time path of any parameter derived from the translog by differentiation, such as the scale elasticity (see (9) in the text). For more on these and related points, see Denny and Fuss [1975]. Also note that the logarithm of time enters the cost function specification in (2). This is a common procedure (Binswanger [1974b]), but it does imply a rate of external technical change which varies over time.

3. In order that the translog be an adequate representation of the cost function, it is necessary that the estimated cost function be monotonically increasing in factor prices and that it be convex over the range of observations. In view of Shephard's Lemma and (2), monotonicity is satisfied if the predicted cost shares are everywhere positive. Convexity is guaranteed if the Hessian of the estimated elasticities of substitution is negative semi-definite.

4. In the empirical work in this paper, we have constrained $\gamma_{qq} = 0$. Therefore, the test for homotheticity is actually a test of homogeneity conditional on $\gamma_{qq} = 0$, and should be interpreted in this way. We experimented with leaving γ_{qq} free, but because of collinearity, the empirical results proved to be non-robust and generally inferior. As is clear from (9), the problem is that the data do not permit us to identify the source of the variation in the scale elasticity--i.e., to distinguish between the level of output and inputs (via factor prices) in this connection.

5. Results obtained by Kmenta and Gilbert indicate that the efficiency of the ZEF and maximum likelihood procedures declines dramatically as the sample size decreases. The estimated standard errors of the of the coefficients more than double when the sample size declines from 100 to 20, and they again double when the sample size falls to 10.

6. The raw data used in this paper were provided to us by the Bell System. These data are proprietary and inquiries regarding them should be addressed to Peter B. Linhart, Director of Regulatory Research, AT&T.

7. We experimented with different rates of obsolescence and gestation periods, but the empirical results are similar to those reported in this paper.

8. The monotonicity of the cost function is satisfied at all sample observations for the three models (the fitted shares are positive). Using the point estimates of $[\sigma_{ij}]$, the convexity condition is satisfied in the three factor model, but it is sensitive to the data in Models A and B. A proper test of convexity would involve evaluating the Hessian of $[\sigma_{ij}]$, not just at the point estimates, but taking into account the joint confidence intervals (ellipsoids) on the various σ's.

9. Recall, though, that the specification in (2) implies a non-constant rate of technical progress (see Footnote 2). Still, the positive coefficient β_{TT} does imply some attenuation in the (varying) rate of technical change.

10. The available measures of traditional capital and labor include research capital and labor. This may impart some bias toward a finding of complementarity, but since the research inputs are small relative to their traditional counterparts, the bias should not be serious.

11. Since we have constrained $\gamma_{qq} = 0$, the level of output does not affect the scale elasticity. The relative factor prices influence the scale elasticity, as can be seen from (9) together with the constraint $\Sigma\gamma_{iq} = 0$.

12. The derived demands for inputs shift outward if the technical change is (Hicks) neutral or factor using, because the marginal product is increased at any given level of input use. Factor saving technical change shifts the derived demand curves inward. This should not be confused with the shift of the input requirement functions (isoquants); any technical progress moves them toward the origin.

13. If there are increasing returns, marginal productivity pricing would over-exhaust the value of output. By assuming cost minimization in (14), we are in effect scaling down the marginal payment to all factors proportionally. This excludes the case where one factor input is a residual claimant on the value of output.

14. See Hulten [1975] and [1979] for a discussion of induced accumulation with constant returns to scale. In that context, the distinction is between the direct and indirect contribution of technical change to the growth in output. With constant returns, additional input accumulation cannot affect the level of TFP.

DATA APPENDIX

The investment goods deflator, P_I, is computed as the ratio of the current cost to constant cost value of net plant; the underlying deflator is the Bell Telephone Plant Index. The corporate income tax rate, u, is the ratio of federal, state and local income taxes plus the investment tax credit to net property compensation. Property compensation is computed as the value of output minus employee compensation, materials and supplies, property and other non-income taxes, and depreciation expense, in current dollars. The direct tax rate, τ, is the ratio of property and other non-income taxes to the value of net plant.

The realized rate of return, r, is computed as the ratio of the value of output minus all taxes, materials and supplies, employee compensation and depreciation expense, to the value of net plant. The depreciation rate, δ, is given by the depreciation expense (in constant 1967 dollars) divided by the 1967 value of net plant.

The investment tax credit applies only to durable equipment. Since the service price of capital computed in this paper is applied to the entire capital stock, the rate of investment tax credit must correspond to the aggregated capital stock. We compute the effective rate on durable equipment as the ratio of the investment tax credit claimed by AT&T to the sum of investment in central office and station equipment. The rate used in the service price of capital is this effective rate times the ratio of investment in central office and station equipment to total gross investment.

The present value of depreciation allowance is taken as a weighted average of the separate values for producers' durables and structures. These values are taken from Christensen and Jorgenson [1969, Table 8]. The weights reflect the fraction of total gross investment accounted for by producers' durables and structures in 1976, constructed from the twenty-three separate plant accounts.

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