ECONOMIC RESEARCH REPORTS

TESTS OF ALTERNATIVE THEORIES OF FIRM GROWTH

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R.R. #86-36 December 1986

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Revised: September 1986

(Forthcoming in the *Journal of Political Economy*)

This paper is a substantially revised version of a paper that has circulated in various forms since 1984. I would like to thank James Albrecht, Zvi Griliches, John Hause, James Heckman, Ariel Pakes, Bruce Phillips, Herbert Simon, William Whiston, Sidney Winter, two anonymous referees, and seminar participants at Yale University for helpful comments on the previous drafts of this paper. I am especially grateful to Boyan Jovanovic for numerous helpful discussions. Some of my research was completed while I was a Visiting Scholar at New York University's C.V. Starr Center for Applied Economics. My research was supported in part by funding from the Office of Advocacy, U.S. Small Business Administration to CERA Economic Consultants, Inc. under Contract SBA–7186–OA83. The views expressed in this paper do not necessarily reflect the views of the Small Business Administration or its employees or any of my colleagues who have so kindly commented on earlier versions of this paper.

ABSTRACT

The study examines the relationship between firm growth,firm size, and firm age for a sample of manufacturing firms between 1976 and 1982. Firm growth is found to decrease with firm age and firm size. These findings are robust to alternative assumptions concerning the effects of sample censoring and the functional form of the growth relationship. The inverse growth–age relationship is consistent with a theory of firm learning proposed by Jovanovic while the inverse growth–size relationship is inconsistent with a number of theories that assume or imply Gibrat's Law.

I. Introduction

This paper makes several contributions to the empirical literature on firm growth. First, it examines the relationship between firm growth and firm age. This relationship is important because some theories of firm growth predict a particular pattern of growth over the life cycle of the firm.[1] The study finds that firm growth decreases with firm age. This inverse relationship between growth and age is consistent with Jovanovic's (1982) theory of firm growth in which firms uncover their true efficiencies over time with a Bayesian learning process.

Second, it examines the relationship between firm growth and firm size for several alternative samples of firms including the complete size distribution of firms. The precise form of this relationship is important because a number of theories either assume or imply a certain relationship for particular samples. The study finds that firm growth decreases with firm size for all relevant samples. This finding is important because a number of theories, including Simon and Bonini (1958), Lucas (1967), Lucas(1978), and special cases of Jovanovic (1982), either assume or imply that firm growth is independent of firm size, i.e. that Gibrat's Law holds.[2]

Third, in examining the relationship between firm growth and firm size the study address three econometric questions that previous studies have largely ignored. The first question concerns the shape of the firm growth–size relationship. Previous studies have assumed a linear relationship. The study finds that the relationship is highly nonlinear so that the firm growth–firm size relationship varies over the size distribution of firms. The second question concerns the effect of sample selection on the firm growth–

firm size relationship. Mansfield conjectured that the inverse relationship between growth and size was an artifact of the exit of slow–growing firms from the sample. The study finds that the inverse relationship is robust to this kind of sample censoring. The third question concerns the effect of heteroskedasticity on inferences. The study reports results that pass White's specification test (1980a, 1982)and are therefore presumably not afflicted by heteroskedasticity or, for that matter, a host of other possible specification problems.

The study is based on a sample of approximately 20,000 manufacturing firms drawn from a dataset created by the Small Business Administration from information originally collected by Dun and Bradstreet. Firm growth is analyzed between 1976 and 1982 which was the longest time span available at the time the a study was done. Firm size is measured by employment since asset data are not available. Data are pooled across industries.[3] Two related papers examine whether the results are sensitive to the time span considered (which ends in the 1982 recession), the pooling of industries, and the use of employment as a size measure. Evans (1987) reports separate results for 4–digit SIC code industries and examines growth between 1976 and 1980. Evans (1986a) examines the growth of assets, sales, and employment for Fortune 500 companies between 1958 and 1984. The results of these papers are consistent with the results reported below.

The next section of the paper describes the data used in the study. The third section summarizes the assumptions or implications of various theories for the firm growth–size–age relationship and then presents a statistical framework for testing these assumptions or implications. The fourth section reports the estimation and test results. The fifth section discusses the results and makes some suggestions for future research in this area.[4]

II. Data

The sample was drawn from the Small Business Data Base which was constructed by the Office of Advocacy of the U. S. Small Business Administration from information originally collected by Dun and Bradstreet for its credit reports.[5] Data are available on firm age, number of employees, sales, and various aspects of corporate structure for 1976, 1978, 1980, and 1982. Employment data are available for more firms and are more reliable than are sales data.[6] Asset data are not available. Consequently this study uses employment as a measure of firm size. The dataset includes most manufacturing firms with employees. Because Dun and Bradstreet collects data either in response to a credit check on a firm or because a firm wants a credit rating, firms that do not purchase factors on credit are less likely to enter the dataset. The result of this selection is that very small firms are underrepresented. For 1980, the data set contains 4.0 million firms from all industries whereas the Internal Revenue Service lists 13.3 million firms filing tax returns. The breakdowns by sales size are: $1 million or more–0.5 million firms for SBA and 0.5 million firms for IRS; $100,000–$999,999–2.1 vs. 2.3; $25,000–$99,999–1.1 vs. 2.7; and $0–$25,000–0.25 vs. 7.8. Most firms with sales of under $100,000 do not have employees other than the owner; many are not full time concerns. See Executive Office of the President (1983, p. 417) for further details. Thus, while this dataset does not represent the complete population of firms it does represent most full–time enterprises.

The fact that a firm has data on a file for a particular year does not necessarily mean that the data apply to that year. Because of reporting and collecting delays, the data for some firms are older than the data for other firms. In order to ensure that the growth rates apply to roughly the same time period, firms were excluded from the study if their data were more than two years old in either 1976 or 1982.[7] The growth rates used in this study is defined as the annual logarithmic change in employment between the date at which data on the 1976 file applied and the date at which the data on the 1982 file applied. The dependent variable for the growth regressions is therefore defined as

$$\ln S_{t'} - \ln S_t / 4[t' - t]$$

where t and t′ are measured in three–month intervals.

Dun and Bradstreet asks firms when they were started. The SBA used this information to calculate the age of the firm. Unfortunately, age is reported by year only for firms under seven years of age in 1976. It is reported by four age intervals for firms seven years or older: 7–20, 21–45, 46–96, and 96 and older. Separate estimates are obtained for firms with continuous age data and for firms in each of the age intervals. Firms in the 46–95 and 96+ categories were pooled because there are few firms in the latter category.[8] The regressions for firms in each of the age intervals are correctly specified under the null hypothesis that age does not matter. This is sufficient for testing some of the hypotheses discussed in the next section.

There is no information on the dataset concerning mergers or acquisitions. When one firm acquires another firm, the acquiring firm will experience growth and the acquired firm will appear as a dissolution. It is not possible to treat internal growth and growth from merger separately with this dataset.[9] While this treatment of growth and dissolution is not entirely satisfactory, it is a reasonable price to pay for the quantity of data obtained.[10]

As with most data based on surveys, the SBDB data are subject to various errors due to firms reporting bad data, firms misunderstanding questions, or Dun and Bradstreet making various clerical errors. Several studies have examined the quality of these data. MacDonald (1986) finds that Dun and Bradstreet tends to assign more small firms to manufacturing than does Census. Given the ambiguity of industry definitions this is not surprising. This study reduces the seriousness of this problem by undersampling small firms. For the larger firms he examines, MacDonald finds that the SBDB employment data are fairly accurate when checked with the firms themselves or other data sources. Jacobsen (1985) reaches a similar conclusion about the quality of the data from a comparison of SBDB data and unemployment insurance records.[11] Finally, Reynolds and West (1985) find that 7 percent of the supposedly new firms they contacted were in fact older than 8 years. For analyzing growth over fairly long periods of time, the quality of the SBDB appears reasonable compared with other micro datasets used by economists.

The study drew a sample of 27046 firms stratified by firm size in 1976 from the dataset. Larger firms were oversampled. Of 21218 surviving firms data less than two years old were available for 17399.[12] For each age–size category, Table 1 reports the number of surviving firms, the average annual growth rate of employment between 1976 and 1982 for these surviving firms, and the fraction of firms that survived between 1976 and 1982. As one would expect given that the economy was in a severe recession in 1982, there are substantial decreases in employment especially for larger firms. Exit rates generally decline with the age and size of the firm.

III. Empirical Framework

Some Theoretical Considerations

Empirical findings that firm growth is roughly independent of firm size (see Hart and Prais (1956) for the classic study) have lead to the development of a number of industrial organization theories in which Gibrat's Law is taken as an assumption or as a desirable implication. Simon and Bonini (1958) assume that Gibrat's Law holds for firms above the minimum efficient size level. Lucas' (1978) influential model of the size distribution of firms assumes Gibrat's Law in order to prove the existence and uniqueness of an equilibrium. Lucas' (1967) model of capital adjustment implies Gibrat's Law applies to the time series of firm employment, capital and output. Jovanovic develops special cases of his model of firm learning in which Gibrat's Law holds in the limit for mature firms or for firms that entered the industry at the same time.

While many studies do find that Gibrat's Law holds, at least as a first approximation, most of these studies are based on samples of the largest firms in the economy. Yet the theories cited above above apply to the complete size distribution of firms in homogeneous–product industries. It is well known that firm growth decreases with firm size for smaller firms (see Scherer (1982)). But there is a tendency to dismiss this finding because this inverse relationship is possibly an artifact of sample censoring (see Mansfield (1962)). The major contribution made here is to test Gibrat's Law for theoretically relevant samples of firms after controlling for sample censoring.[13]

Recent theoretical work on industry evolution has emphasized the importance of learning on firm growth and changes in market structure. Jovanovic (1982) and Lippman and Rumelt (1982) examine the implications of the assumption that firms can learn about their efficiencies from realizations of costs.[14] Nelson and Winter (1982) develop a model in which boundedly rational firms search for innovations.

Jovanovic's model has a particular rich set of testable predictions concerning the lifecycle patterns of growth. The most general version of his model predicts that firm growth decreases with firm age holding firm size constant.[15] Under certain assumptions concerning technology and the distribution of ability his theory implies that firm growth is independent of firm size for mature firms or for firms in the same age cohort.[16] Nelson and Winter's model predicts that firm growth initially increases but then decrease with firm size for mature firms.[17]

The firm growth relationship is given by

(1) $S_{t+1}=[G(A_t,S_t)]^d[S_t]e_t$

where t denotes time with $t'>t$, $d=t'-t$, and e is a lognormally distributed error term with possibly nonconstant variance. Equation (1) suggests the following regression framework[18]

(2) $[\ln S_{t'}-\ln S_t]/d=\ln G(A_t,S_t)+u_t$

where u_t is normally distributed with mean zero and possibly nonconstant variance and is independent of A and S.

This study tests some of the assumptions or implications of the theories discussed above for firm growth by estimating the relationship between firm growth, firm size, and firm age and evaluating the partial derivatives of growth with respect to age and size. Let

$$g_A = \partial \ln G/\partial \ln A$$

denote the partial derivative of the logarithmic growth rate with respect of logarithmic firm age and

$$g_S = \partial \ln G/\partial \ln S$$

denote the partial derivative of the logarithmic growth rate with respect to firm size. The assumptions or predictions of the above theories for these partial derivatives are reported in Table 2.[19] It is also useful to note at this point that the elasticity of ending–period size with respect to beginning–period size is given by

$$E_S = \frac{\partial Ln(S_{t'})}{\partial Ln(S_t)} = 1+dg_S$$

and that the elasticity of ending–period firm size with respect to beginning–period firm age is given by

$$E_A = \frac{\partial ln(S_{t'})}{\partial ln(A_t)} = dg_A$$

In order to compare these figures across studies based on different periods of time it is useful to normalize d to 10 years in reporting these elasticities.[20]

Empirical Issues

Several statistical issues arise in estimating lnG(A,S) and calculating the partial derivatives of growth with respect to size and age.[21] The first conerns the functional form of G. The study tested successive higher–order logarithmic expansions until there was no evidence of further nonlinearity.

The second concerns heteroskedasticity. Previous studies (e.g. Hymer and Pashigian (1962)) find that the variability of firm growth decreases with firm size which suggests that u_t is not constant across firms. For all the regressions reported here it was possible to find a weighting scheme that purged the heteroskedasticity in the sense that the weighted regression passed White's (1980a) heteroskedasticity test.

The third issue concerns sample censoring. The extent to which the sample is censored varies with age as shown in Table 1. The exit rate decreases from more than a third for young firms to less than a tenth for old firms. Standard methods (see e.g. Amemiya (1984)) are used to test whether sample censoring biases the results. The probability of remaining in the sample is assumed to depend on firm size and firm age. The probit of survival is estimated jointly with the firm growth equation using maximum likelihood.[22] The fourth issue concerns the level of statistical significance to use in the various tests. Because the sample used in the estimation is extremely large by conventional standards, the tests reported here are much more powerful than tests reported in most fields of economics. At any given level of statistical significance, the tests reported here place a greater burden of proof on the null hypothesis than would tests calculated from the sample sizes typically used by economists. In order to offset this bias against the null hypothesis, the study relies on the Bayesian approach suggested by Leamer (1978). He provides a formula for adjusting the critical value of the F-statistic for changes in sample size. The formula leads to rejection of the null hypothesis if the posterior probability of the null hypothesis falls short of the (diffuse) prior probability of the null hypothesis. The formula is given by $[T-k/p][T^{p/T}-1]$ where T is sample size, p is the number of restrictions, and k is the number of variables.

IV. Estimation and Test Results

Separate results are reported for firms between 0 and 6 years, 7–20, 21–45, 46 and older in 1976. The regressions for firms younger than 7 years in 1976 are based on a logarithmic expansion of lnG(A,S) in A and S. The regressions for firms older than 7 years in 1976 are based on a logarithmic expansion of lnG(S|A) in S because continuous age information is not available for these age cohorts. A second–order expansion is taken in both cases. Table 3 reports summary statistics on the variables used in the regression analysis.

White's portmanteau specification test is applied to the ordinary least squares regression. Failure of this test may indicate a number of possible problems. The most likely problems in the present context are heteroskedasticity and nonlinearities. If the test fails——as it does for older firms——alternative specifications that involve weighting to purge the heteroskedasticity and higher–order approximations are considered. All the final specifications reported pass White's test.

The problem of sample selection is then considered. This problem is potentially most serious for young firms which have a high exit rate. For these firms, a probit equation for survival over the period and a regression equation for growth over the period are estimated jointly using maximum likelihood. The study finds that the correlation between the errors terms of the growth and survival equations is small and not statistically significant and that the maximum likelihood estimates of the growth equation are almost identical to the regression estimates. Similar findings are reached for older firms.[23]

Young Firms

Taking a second order expansion of lnG(A,S) yields

$$\ln G = b_0+b_1\ln S+b_2\ln A+b_3[\ln S]^2+b_4[\ln A]^2+b_5[\ln S][\ln A]+u$$

Table 4 reports the estimation and specification–test results. Using White's specification test we cannot reject the hypothesis that the ordinary least squares specification is correct. Thus we can reject heteroskedasticity and additional nonlinearity in the functional form of lnG(A,S).[24] Table 4 also reports maximum–likelihood estimates of the growth and survival equations. The positive sign on the correlation coefficient is consistent with Mansfield's conjecture that exits tend to be slow growing firms. However, the coefficient is small and statistically insignificant. The fact that the ordinary least squares and maximum–likelihood estimates are close indicates that sample–selection bias is not a problem here.[25]

Using the ordinary least squares specification reported in Table 4, F–tests of some of the hypotheses listed in Table 2 are performed. The F–statistic for the hypothesis that $g_S=0$ is 225.53; for the hypothesis that $g_A=0$ is 18.99; and for the joint hypothesis that $g_S=g_A=0$ is 145.75. These F–statistics are well above the Leamer 5 percent critical level of 8.4. We can therefore reject the hypotheses that growth is independent of either size or age. At the sample mean $g_S=-0.0374$ and $g_A=-0.0381$. Moreover, g_S and g_A are both less than zero over most of the sample space. Therefore firm growth decreases with firm size holding firm age constant and decreases with firm age holding firm size constant.

At the sample mean the estimates imply that the elasticity of ending–period size with respect to beginning–period size is .63 for a 10 year period. The elasticity of ending–period firm size with respect to beginning–period age is –.38 for a ten year period. Thus a 1.0 percent increase in beginning–period firm size leads to .63 percent increase in ending–period firm size over a decade. A 1.0 percent increase in beginning–period firm age leads to a .38 percent decrease in ending–period size over a decade.

Preliminary investigation for the older age categories found that the regression equation was nonlinear in size and that the residuals were heteroskedastic. Regressions that pass White's specification test are reported in Table 5. Heteroskedasticity was purged by using weights estimated from a regression of the squared residuals from the ordinary–least squares regression against the squares and crossproducts of the regressors.[26] Fourth–order expansions were necessary for the 7–20 and 21–45 year old age groups.[27] As one would expect given the results for the young firms and given the relatively small exit rates of the old firms, sample–selection bias was not empirically important for the old firms.[28]

The F–statistic for the hypothesis that $g_S=0$ is 103.27 for firms between the ages of 8 and 20, 53.3575 for firms between the ages of 21 and 45, and 39.8736 for firms older than 45 years. The critical level of the F–statistic (based on the Leamer 5–percent level) is approximately 10. Thus we can reject the hypothesis that firm growth is independent of firm size at a high significance level. At the sample mean g_S is –0.0512 for ages 8–20, –0.0231 for ages 21–45, and –0.019 for ages 45+. At the sample mean the elasticity of ending–period size with respect to beginning period size for a ten–year period is .488 for ages 8–20, .769 for ages 21–45, and .806 for ages 45+.[29]

V. Conclusions

Two key empirical findings emerge from these estimates together with the estimates reported in several companion studies. First, firm growth decreases with firm age holding firm size constant for young firms. The companion study reported in Evans (1987) finds that this relationship also holds between 1976 and 1980 for 87 of the 100 4–digit industries examined in that study. The companion study also finds that firm growth decreases with firm age between 1976 and 1980 for a sample that pools the older firms together and uses an estimate of age based on the mean for the age category as a regressor. Thus, the inverse relationship between growth and age is robust to the alternative specifications considered in Appendix C to Evans (1986b), to alternative samples, and to alternative time periods.

Second, firm growth decreases with firm size. Gibrat's Law predicts that E_S=1.00. The study finds at the sample mean that E_S=.63 for 0–6 year old firms, .49 for 7–20 year old firms, .77 for 21–45 year old firms, and .81 for firms more than 45 years old.[30] A companion study based on growth for 1976–1980 (Evans (1987)) finds that E_S is .68 for 0–6 year old firms and .85 firms older than 7 years at the sample mean.[31] That study also finds that firm growth decreases with firm size for 89 out of 100 industries analyzed and that this relationship was statistically significant for 61 out of 100 industries. Studies of large firms by Evans (1986a), Hall (1987), and Kumar (1985) find that Gibrat's Law fails for several different measures of firm size, over longer periods of time than considered here, and for different phases of the business cycle. These studies find that E_S is approximately .90. The growth–size relationship reported here therefore appears robust.[32] The departures from Gibrat's Law tend to decrease with firm size. This relationship is seen from the fact that E_S is larger (and closer to 1.0) for samples with larger firms.[33]

These results help assess the validity of the theories that are either based on some assumption about the growth–size–age relationship or that have some prediction about this relationship. Jovanovic's theory implies as a general matter that firm growth decreases with firm age. This prediction is confirmed at least for the young firms in the sample considered here. Under the special assumption that firm costs are Cobb–Douglas, with decreasing returns to scale, his theory implies that firm growth is independent of firm size for mature firms. This implication fails whether we define mature firms as older than 45, older than 20, or older than 7. Under the special assumption that the distribution of efficiency is lognormal, his theory implies that firm growth is independent of firm size for firms that entered at the same time. This prediction also fails. Firm growth decreases with firm size holding firm age constant according to the young–firm regressions.[34]

Lucas's (1967) capital–adjustment theory predicts (as a consequence of the assumed constant–returns to scale technology) that Gibrat's Law holds for the complete size distribution of firms. His (1978) theory of the size distribution of firms assumes that Gibrat's Law holds for the complete size distribution of firms. The evidence reported here shows that there are strong departures from Gibrat's Law and that these departures are not due to sample–selection bias as conjectured by Mansfield (1962).[35]

Simon and Bonini assume that firm growth is independent of firm size for firms that are larger than the minimum efficient size level. Presumably, most older firms have reached this minimum efficient size level.[36] Otherwise, they could not survive for long periods of time. Therefore, the fact that firm growth is not independent of firm size for firms older than 7, 20, and 45 years is not consistent with the stochastic theory. Recent studies of large firms by Evans (1986), Hall (1987), and Kumar (1985) also find that Gibrat's Law fails, although not as severely as for the mainly small firms considered in this study.

Finally, Nelson and Winter (1982) report simulation results that indicate that firm growth increases and then decreases with firm size for firms 20 years or older.[37] This prediction is not confirmed by the results reported above or in Evans (1987).

Naturally, the growth–size–age relationships documented in this study are only one criterion by which to assess the theories mentioned above. The rejection of the assumptions or implications of these theories does not necessarily repudiate many of the basic insights offered by these theories, just as the acceptance of these assumptions and implications cannot prove these theories. These relationships do, however, show that our theories of firm growth could use some refinement and suggest some avenues to pursue. The inverse growth–size relationship indicates some caution in appealing to Gibrat's Law for theories that are meant to apply to the full size distribution of firms rather than the to largest firms in the economy. It also suggests the development of theories that can explain this relationship.[38] The inverse growth–age relationship suggests that the learning considerations captured by Jovanovic's model are important.

Further theoretical and empirical work on the lifecycle aspects of firms may prove fruitful.

Appendix A

Theories of Firm Growth

The three theories reviewed in this appendix imply or assume alternative versions of the law of proportionate growth. The versions differ in the sample of firms for which the law is predicted to apply. The stochastic theory discussed below predicts that firm growth rates are independent of firm size for firms above the minimum efficient size level. The capital–adjustment theory predicts that this relationship holds for all firms. The learning theory implies that, under certain conditions, this relationship applies to firms of the same vintage.

Stochastic Theory

There are many versions of the stochastic theory of firm growth embodied in Gibrat's Law. Simon and Bonini (1958) present one of the more sophisticated versions, yet one that is not based on any underlying model of profit maximization.[39] They assume that the firm cost curve for industries exhibits constant–returns–to–scale above some critical size level called minimum efficient firm size. They also assume that firm growth is independent of firm size for firms larger than this minimum. Their justification for the latter assumption is that "if ... there exists approximately constant returns to scale (above a critical minimum size of firm) it is natural to expect the firms in each size–class to have the same chance on the average of increasing or decreasing in size in proportion to their present size." (Simon and Bonini, (1958, p. 609.)) Finally, they assume that firms enter the smallest size class above the minimum efficient firm size at a constant rate. They show under these assumptions that the size distribution of firms is given by the Yule distribution

$$f(s)=KB(s,\rho+1)$$

where B is the Beta function, s is firm size, and ρ is a function of the rate of entry into the smallest size class. They produce some estimates that suggest that this distribution provides a satisfactory fit. Thus, this version of the stochastic theory predicts that firm growth is independent of firm size for firms larger than the minimum efficient firm size.[40] As a proxy for firms that are larger than the minimum efficient size level, this study looks at old firms. Because firms could not survive in an industry for a long period of time at an inefficient level, most old firms are presumably efficient firms.

Capital–Adjustment Theory

Lucas (1967) examines the response of firms to changes in industry demand when it is costly to increase capital inputs. Firms produce an homogeneous product with the common production function F(L,K,I) where L denotes labor, K denotes capital, and I denotes the level of investment and where F is homogeneous of degree one.[41] The firm's objective is to choose investment and labor force plans I(t) and L(t) to maximize the present discounted value of future profits given by

$$V(0)=\int e^{-rt}\{pF(L(t),K(t),I(t))-wL(t)-vI(t)\}dt$$

where p denotes the industry product price, w denotes the competitive wage rate, v denotes the competitive price of capital, and r denotes the discount rate. Under certain assumptions concerning the production function, Lucas shows that the time path of firm size is given by

$$d\ln K(t)=g(r,G(\delta),w/p,v/p)-\delta$$

where δ is the capital depreciation rate. A similar relationship holds for employment and output.

Because this model is deterministic, it implies that all firms grow at the same rate. But it is straightforward to introduce probabilistic elements into the model. Firms may make unsystematic errors in calculating the optimal time paths. Alternatively, if future demand is uncertain firms may make unsystematic forecast errors.

A probabilistic version of the capital–adjustment model therefore implies that firm size is independent of firm growth for all firm sizes. Unlike the stochastic model discussed above, the capital–adjustment model makes no distinction between small and large firms. All firms start out at the same size for given factor prices, product prices, and discount rate. Their subsequent rate of growth is independent of this initial size.

Learning Theory

Jovanovic (1982) considers an industry that is small relative to the economy as a whole so that it takes factor prices as given, produces an homogeneous product, and faces a known and deterministic path of demand over time. Random costs for each firm at time t are given by $c(q_t,x_t)=c(q_t)x_t$ where x is a firm–specific random efficiency factor drawn from the density $x_t=\xi(\eta_t)$ where $\eta_t=\theta_t+\epsilon_t$. The term θ represents the deterministic but unknown ability level of individual who manages firm x and the term ϵ represents firm–specific shocks. The distribution of ability across the population of potential managers is normal with mean $\theta.$ and variance γ^2. The distribution of shocks is normal with mean zero and variance σ^2.

At the start of the industry each individual in the population of potential managers estimates that his ability level is average, i.e., $\theta=\theta^*$. An individual who starts a firm observes his costs and revises his estimate of θ accordingly through a Bayesian learning process. New estimates of θ over time are obtained by taking a weighted average of the original estimate of θ and the estimates of θ inferred from observed costs. The estimated θ will converge to the true θ for individuals who remain in business for a long period of time. An individual withdraws from business when the present discounted value of expected future profits, conditional on his most recent estimate of θ, is less than his alternative opportunity cost. Jovanovic shows that a unique perfect foresight equilibrium solution exists for this model under certain technical assumptions concerning the nature of costs and demand.[42]

Jovanovic's model has several interesting implications. First, firm growth decreases with firm age holding firm size constant but is generally not independent of firm size holding firm age constant. Firm growth may increase or decrease with firm size for firms of the same vintage depending upon the shape of the cost function c(q) and the distribution of efficiency in the population. Thus, Gibrat's Law generally does not hold. Second, the probability of firm dissolution decreases with firm age holding firm size constant and with firm size holding firm age constant. Third, the variability of firm growth decreases with firm age holding firm size constant.

It is possible to impose two assumptions on Jovanovic's model under which certain variations of Gibrat's Law do hold. (1) Firm growth is independent of firm size for mature firms (i.e., firms whose estimated θ has converged to its true value) if and only if $c(q)=Aq^b$, $b>1$. I call this special case of Jovanovic's theory the Cobb–Douglas version. (2) Firm growth is independent of firm size for firms of the same age if and only if $c(q)=Aq^b$ and $z_t=x^*_t/x^*_t$ is distributed independently of x^*_t. The latter condition is satisfied if x_t is distributed lognormally. Consequently, I call this special case of Jovanovic's theory the lognormal version.[43]

Before discussing the methodology for testing these theories, several points deserve emphasis. First, most of the previous empirical studies of Gibrat's Law have focused primarily on the evolution of conglomerates.[44] Yet all of the theories of industry evolution focus on firms in homogeneous–product industries. While these theories may have implications for the evolution of conglomerates, conglomerate data are not appropriate for testing these theories. This study concentrates on the complete size distribution of firms to which these theories apply. Second, since the theories above are for industries, the tests should arguably done by industry rather than across industries. This study reports cross–sectional results for all manufacturing industries in order to highlight some of the statistical issues involved in testing Gibrat's Law (especially selection bias) and in order to maintain comparability with previous tests that have also been cross–sectional.[45] A companion study (Evans (1986b)) examines the firm growth–size–age relationship by narrowly defined industries.

Appendix B

Statistical Issues

Several statistical problems arise in estimating the growth–size–age relationship. First, the sample upon which (2) is estimated excludes firms that fail between t and t′. Consequently, the expectation of u conditional on a firm being in the sample in periods t′ and t may be nonzero and may be correlated with A and S. In the empirical study reported below and in Mansfield (1962) this sample selection problem arises primarily because firms fail between t and t′.[46] About a fifth of the firms in the sample fail. In empirical studies based on large firms (Simon and Bonini (1958), Hart and Prais (1956), and Singh and Whittington (1975)) this problem arises because firms fail to meet the sample selection criterion––e.g., being listed on the relevant stock exchange or being one of the largest 500 industrial firms––in both years.

In order to test and correct for sample–selection bias, we must model the process of firm survival. We assume that a firm stays in business if the present–discounted value of its future stream of profits conditional on the information it has accumulated on its ability through time t, V, exceeds its alternative opportunity cost W. V is a function of firm size and age, V=V(A,S), and W is assumed to be constant across firms.[47] Thus

$$V_i=[V(A_i,S_i)-W]+u_{i1}$$

where u_{i1} is a normally distributed disturbance term with mean zero and constant variance $\sigma_{11}=1$.[48] We cannot observe V_i, but we can observe whether a firm survives to another period. Thus we can write

$$S_i = \begin{cases} 1, \text{ if } V_i>0 \\ 0, \text{ if } V_i<0 \end{cases}$$

The conditional expectation of S_i given a_i and s_i is

(3) $$E[S_i|a_i,s_i]=Pr[u_{i1}>-V(a_i,s_i)]=F[V(a_i,s_i)]$$

where F is the cumulative normal distribution function (with variance of one).[49]

Given (3) we could use standard techniques for testing and correcting for sample-selection bias. The standard techniques assume that the distribution of the error term in the selection equation (here the survival equation) and the behavioral equation (here the growth equation) are distributed as a bivariate normal. Write the growth equation as

(4) $$\ln G_i=\ln g(A_i,S_i)+u_{i2}$$

where u_{i2} is normally distributed with mean zero and constant variance σ_{22}. The covariance between u_{i2} and u_{i1} is constant and equal to σ_{12}. Sample–selection bias arises because only firms that have survived––i.e., met the selection criterion (3)––are included in the estimation of (4). If σ_{12} is nonzero, the expectation of u_{i2} conditional on $u_{i1}>-V(a_i,s_i)$ is nonzero and in general will be correlated with the regressors of (4).[50] One method for treating this problem involves estimating (3)–(4) jointly using maximum likelihood.[51]

Second, it is necessary to specify the precise functional form of lnG(A,S). Previous studies have generally taken a first–order logarithmic approximation in s. No one, to my knowledge, has tested the validity of this approximation. The functional form of lnG(A,S) is important for several reasons. Economic theory provides little guidance for selecting any particular functional form for lnG(A,S). But the use of a poor approximation to will lead to biased estimates of the test statistics for the relationships described in Table 2. It is therefore important to test for the appropriateness of whatever approximation to lnG(A,S) we choose. I rely on the approach suggested by White (1980b).[52]

A difficulty, however, arises here. Suppose that the probability of being in the sample is a function of size and age and we use the method proposed by Heckman (1976, 1979) to test for sample–selection bias. From the estimated probit equation we calculate the Mills ratio $\lambda=\lambda(A,S)$.[53] We then include $\lambda(A,S)$ as a regressor in the growth equation. Since $\lambda(A,S)$ is a nonlinear function of lnA and lnS, an asymptotically equivalent way to proceed would be to include higher–order terms of lnA and lnS in the regression. If these higher–order terms are statistically insignificant, we have evidence against sample selection bias and we have evidence against nonlinearity in the functional form of lnG(A,S). If these higher–order terms are statistically significant, we cannot be sure whether they arise because of nonlinearity of lnG(A,S) or because of sample selection. Without imposing ultimately arbitrary restrictions on the functional form of lnG(A,S) or including regressors in the survival equation that are not included in the growth equation, there is no way around this difficulty. Where there is evidence of nonlinearity in the growth equation, I report tests under the assumption that the nonlinearity is intrinsic to the growth equation and under the assumption that the nonlinearity is an artifact of sample selection bias. The evidence suggests that the nonlinearity is not an artifact of sample selection bias.

The third problem concerns heteroskedasticity of the disturbance term in the regression equations for firm growth and in the probit equation for firm survival. Previous studies have found that the variability of firm growth over time decreases with firm size. See Hart and Prais (1956), Mansfield (1962), and Hymer and Pashigian (1962). My studies find mixed evidence in this regard. There is little evidence of heteroskedasticity for the young firms in this study or for the very large firms analyzed in Evans (1986a) based on a White test of the residuals. In Evans (1987), I measure standard deviation of firm growth from three observations on firm growth for each firm and examine the relationship this measure of variability and firm size and age. I find a weak relationship between variability and size and a somewhat stronger relationship between variability and age. This evidence suggests that the disturbance term for the firm growth equation may be heteroskedastic. Because firm growth and firm survival are generated by the same process, a nonconstant variance for firm growth suggests a nonconstant variance for firm survival as well.

The existence of heteroskedasticity in the current context creates several problems. First, the usual estimates of the covariance matrix for the regression equation (2) will be biased if the disturbance term is heteroskedastic. This problem is easily remedied by calculating the heteroskedasticity–consistent covariance matrix suggested by White (1980a). Second, the usual bivariate normal specification of the sample selection model is not correct.[54] If the only source of heteroskedasticity is in the disturbance for the regression equation, parameter estimates will be consistent. If there is heteroskedasticity in the probit equation or in the covariance between the disturbances in the probit and regression equation, parameter estimates will be inconsistent. I investigate this problem below by reporting White's (1980a) test for heteroskedasticity for the regression equation and reporting regular and White (1982) standard errors for all equations. It turns out that the maximum likelihood and White standard errors are almost identical for the probit equations.[55] This similarity suggests that the probit equation is specified correctly. It also turns out that the regression equation for young firms passes White's (1980a) test. Thus, the bivariate normal specification for young firms appears to be appropriate. The ordinary least squares regression equation for old firms does not pass White's test and further investigation shows that the failure is due to heteroskedasticity. However, sample selection bias is not likely to be important for these firms since most——more than 85 percent——survive.

It is important to recognize that the sample selection, nonlinearity, and heteroskedasticity problems are interrelated. The connection between the problems can be seen in the context of applying White's (1980a, 1982) specification test to the growth equation. This test may fail for several reasons.[56] (1) The "true" disturbance term might be heteroskedastic. (2) There might be sample selection which will lead to a dependence between the regressors and the left–out selection term which is absorbed into the regression disturbance. (3) The functional form of the behavioral equation might be misspecified, thereby leading to a dependence between the regressors and the excluded higher order terms which are absorbed in the regression disturbance. (4) Some of the regressors may be correlated with the regression disturbance because of endogeneity, errors–in–the–variables, or omitted variables.

In the present case, there is no good reason for excluding any of these potential problems.[57] Consequently, I proceed as follows. I estimate an n^{th} order logarithmic approximation to the growth function.[58] I use the residuals from this regression to calculate White's (1980a) test.[59] This test involves performing a regression of the squared residuals against the squares and cross–products of the regressors in the behavioral equation and testing whether the coefficients of this regression are jointly zero. If I pass this test I reject the hypotheses that heteroskedasticity or nonlinearity exist. If I fail this test I use the predicted squared residuals to form weights. I use these weights to calculate weighted least squares estimates of the behavioral equation.[60] I then repeat the White (1980a) test of the residuals from this weighted least squares regression.

I report both weighted and unweighted regression results. It is useful to compare the weighted least squares results with the ordinary least squares results for the reasons discussed by White (1980b). Coefficient estimates are consistent under heteroskedasticity but inconsistent when there are excluded higher–order terms. If the weighted and unweighted coefficient estimates are similar, we have some confidence that nonlinearity is not a problem. If they differ, nonlinearity in the functional form is a possible culprit.[61]

Once I have achieved a specification that passes the tests for heteroskedasticity and nonlinearity, I investigate sample selection bias. I estimate a probit equation for survival and calculate λ. I then estimate alternative versions of the equation that differ in the degree of nonlinearity assumed. By comparing these alternative versions we can assess the sensitivity of the results to what is assumed about sample selection or nonlinearity.

Appendix C

Tests of Robustness

This section reports various alternative estimates of the equations reported in the text. Table C1 reported OLS and WLS estimates of the young–firm growth equation. The fact that the equation passes the White test and that the OLS and WLS estimates are almost identical suggests that heteroskedasticity and further nonlinearity are not problems. Table C2 reports maximum likelihood estimates of the growth and survival equations for various assumptions concerning the functional form. The estimates indicate that sample–selection bias is not a problem. The correlation coefficient is statistically significant for all functional–form assumptions. Moreover, the sample–selection corrected ML estimates of the second–order expansion for the growth equation are almost the same as the uncorrected OLS estimates. Table C3 reports OLS and WLS estimates of the growth equations for older firms. The OLS estimates fail the White test. The WLS estimates pass the White test. The passage of the White test together with the fact that the OLS and WLS coefficient estimates are similar (within a standard error of each other) suggest that further nonlinearity is not a problem. We would not expect sample censoring to be a serious problem for the older firms since the exit rate for these firms is less than 15 percent. But as a check, Table C4 reports sample–selection corrected estimates of the growth equation for alternative functional specifications. Heckman's Lambda method was used here for convenience. Two points are notable. First, although Lambda is statistically significant for some of the lower–order expansions, the negative growth–size relationship remains. Second, the statistically significant estimates of Lambda generally imply a growth–survival equation correlation coefficient well in excess of 1.0. That fact suggests that Lambda is at least partly proxying for left–out higher–order terms in the growth relationship.

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FOOTNOTES

[1] See Jovanovic(1982), Nelson and Winter (1982), Klepper and Grady (1985), Hopenhayn (1986) for lifecycle theories of firm growth.

[2] Also see Nelson and Winter (1982) who predict an inverted–U relationship for older firms.

[3] The inclusion of dummies for 2–digit industries was found, in preliminary work, not to affect the general results.

[4] Three appendices are available in Evans (1986b). The first discusses theories of firm growth in greater detail than is desirable here. The second discusses the statistical methodology used to obtain the estimates. The third examines the sensitivity of the results to assumptions about sample–selection and the functional form of the growth relationship.

[5] See Executive Office of the President (1983, pp. 405–438) for further details.

[6] Most firms have employment data on a quarterly basis for FICA reporting whereas sales data are often available on a yearly basis. Consequently, the employment data reported to Dun and Bradstreet interviewers are more likely to be up–to–date and accurate than are the sales data. Estimates for sales are similar to those for employment.

[7] Roughly a fifth of all firms were excluded from the sample. The results of the analysis, however, are basically unchanged when all firms are included.

[8] An alternative method for handling the lack of continuous data on age for older firms is to fit a distribution to the age data, calculate average age for firms in given age intervals, and include imputed age as a regressor. See Brock and Evans (1986) and Evans (1987) for estimates based on this approach.

[9] All previous studies with the exception of the Kumar (1985) have also ignored this issues.

[10] A careful study of mergers of food manufacturing firms with more than 20 employees found that 21.4 percent of firms exited while 3.3 percent were acquired by other firms. The proportion of mergers among all firms is probably smaller. See MacDonald (1985) who also discusses the reliability of the data.

[11] One problem he finds is that many of the firms with no employment in the SBDB had closed according to the unemployment records. This is less likely to be a problem for the eight–year time period considered in this study.

[12] The average length of time between observations was 5.8 years with a standard deviation of 0.5 years.

[13] Evans (1987) reports separate industry results that are broadly consistent with the results reported here.

[14] Also see recent work by Hopenhayn (1986) and Ericson and Pakes (1986).

[15] This relationship is due to the Bayesian learning process assumed.

[16] The first implication holds if technology is Cobb–Douglas with decreasing returns to scale. The second implication holds if the distribution of ability in the population is lognormal.

[17] This prediction is based on simulations of a model that does not have an analytic solution.

[18] For the related work reported in Brock and Evans (1986), alternative functional forms for the lng were tested by regressing the dependent variable below against Box–Cox transforms of the levels, squares, and cross products of age and size. But the likelihood function failed to converge. A semi–log specification (where the exogenous variables are measured in levels) was also tested. The mean square error for the double–log specification was slightly lower than that for the semi–log specification. Apparently, the likelihood function is relatively flat with respect to the Box–Cox transform parameter. Results for the semi–log equation were similar to the results for the double–log equation.

[19] See Appendix A to Evans (1986b) for further discussion.

[20] See e.g. Hart (1975).

[21] These problems are discussed in more detail in Appendix B to Evans (1986b).

[22] It is important to recognize that nonlinearity, heteroskedasticity, and sample censoring are interrelated. A bad approximation to the functional form can manifest itself as heteroskedasticity. Sample censoring can appear as nonlinearity in the growth equation. Appendix B discuss to Evans (1986b) discusses these problems in more detail and Appendix C examines the sensitivity of the results to alternative assumptions concerning functional form, heteroskedasticity, and sample censoring.

[23] Sample selection manifests itself as an omitted variable in the growth regressions. This omitted variable is a nonlinear function of firm age and size which are the key determinants of firm survival. (See e.g. Heckman (1976) for further discussion of sample selection as an omitted variable problem.) It is therefore difficult to identify the effects of sample censoring from nonlinearities in the growth function. This problem is treated in detail in Appendix C to Evans (1986b) which shows that the findings reported below are robust to alternative assumptions concerning sample censoring and nonlinearities.

[24] The following crude test for further nonlinearity suggested by White (1980b) was conducted. Weighted least squares estimates using the inverse square root of employment as a weight were obtained and compared with the ordinary–least squares estimates. The coefficient estimates were within a standard error of each other. This fact indicates that the second–order approximation is probably sufficient.

[25] This result is also true when a first–order approximation to the growth equation is considered.

[26] See Appendix C for further discussion of this procedure which is suggested by White (1980b).

[27] The weighted and unweighted estimates are very close, indicating that further nonlinearity is not a problem. See Appendix C to Evans (1986b) for further details.

[28] Appendix C to Evans (1986b) shows that the findings reported below are robust to alternative treatments of sample censoring and nonlinearities.

[29] The sign of g_s is negative for all observations with the exception of five firms between the ages of 21 and 45.

[30] E_s is normalized to 10 years.

[31] Older firms were pooled.

[32] Hart's (1975) summary of earlier studies suggests that there was a negative relationship before World War II and a neutral or positive relationship between the end of World War II and the early 1960's. Recent studies based on post–1960 data generally find a negative

relationship.

[33] This relationship is also seen from the fact that g_{SS} is positive at the sample mean for all but one of the regressions––that for 20–45 year old firms––reported here.

[34] Jovanovic's predictions that firm failure and the variability of firm growth decrease with firm age are confirmed in Evans (1987).

[35] For further discussion of the importance of the failure of Gibrat's Law see Evans (1987).

[36] The average number of employees is 527 for firms older than 45 years, 73 for firms older than 20 years, and 37 for firms older than 7 years.

[37] Their model does not have an analytic solution.

[38] For work in this direction see Jovanovic and Rob who develop a model in which departures from Gibrat's Law across industries are determined by differences in product heterogeneity.

[39] Also see Ijiri and Simon (1977) for a summary of more elaborate versions of the stochastic theory.

[40] Although Simon and Bonini tested their theory with Fortune 500 firm data their theory applies to all firms above the minimum efficient size level. Moreover, it is not clear that data on extremely large diversified conglomerates are appropriate for testing their theory. Finally, I have found (1986a) that firm growth is not independent of firm size for Fortune 500 companies observed between 1958 and subsequent business cycle peaks of 1960, 1967, 1973, and 1984.

[41] $F-(LI)=0$ by assumption.

[42] For a more discursive discussion of Jovanovic's model see Brock and Evans (1986, Chapter 3).

[43] See Jovanovic (1982) for proof of these two propositions.

[44] The major exception is the excellent study by Mansfield (1962).

[45] Moreover, performing a complete set of tests for a representative sample of narrowly defined industries would have been extremely expensive.

[46] Although some of the "failures" in the sample may reflect mergers.

[47] This assumption could be relaxed by assuming the W is also a function of A and S.

[48] Since the coefficients of the probit model are identified only up to a constant of proportionality, I have made the conventional normalization of the variance to 1.

[49] A more straightforward way to analyze firm surival would be a mortality framework where time to failure is taken as the dependent variable. I plan to look at this in future work.

[50] See Heckman (1976) for a discussion.

[51] See Amemiya (1984).

[52] Nonlinearity of the functional form is especially likely in the application described in this paper because of the wide range of the data. See White (1980b) for discussion on this point.

[53] As discussed below, the evidence suggests that V(A,S) is a function of lnA and Lns. Thus G(*) is a nonlinear function of lna and lns.

[54] Arabmazar and Schmidt (1982) find that this misspecification does not cause serious bias for the Tobit model. However, the model they examine contains only a constant term.

[55] They almost always differ by less than one percent.

[56] See White (1980a, pp. 823–824) for further discussion. The following list is not meant to exhaust the possible reasons for failure.

[57] I would argue that this is generally the case. Economists hardly ever

have grounds for having faith in either the functional forms of the behavioral equations or stochastic processes they assume. Moreover, when we do have theoretical grounds for asserting a particular functional form or stochastic process, we are generally interested in testing whether these assumptions are consistent with the data. We are thereby forced to nest our assumed functions and stochastic processes within more general functional forms and stochastic processes. All of the problems I address below are potentially present in standard applications such as the estimation of a female wage equation.

[58] The degree of n varies from two to four depending upon the specification.

[59] White proposes this test as a test for heteroskedasticity when one has faith that this is the only source of model specification. Since, as I have argued above, heteroskedasticity is seldom our only worry I have eschewed calling this a test for heteroskedasticity.

[60] This procedure is discussed in White (1980a, p. 827.)

[61] Of course, once we have passed the White (1980a) test we can have some faith that nonlinearity is not a problem. For lower–order expansion that do not pass the White test, the ordinary and weighted least squares estimates differ markedly.