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EQUILIBRIUM WAGE AND DISMISSAL PROCESSES

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Equilibrium Wage and Dismissal Processes†

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Abstract

We develop and estimate an equilibrium model of the labor market in which inefficient employees are systematically eliminated from the sector of the market characterized by asymmetric information and moral hazard. Systematic selection on the distribution of productivity characteristics produces unique wage sequences which are increasing in tenure for employees never previously terminated even in the absence of long-term contracting between employees and individual firms. We provide sufficient conditions for there to exist an unique termination-contract type equilibrium, and we estimate the equilibrium model using micro-level data from the National Longitudinal Survey-Youth Cohort panel.

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1. INTRODUCTION

In this paper we propose a new behavioral interpretation of the concavity of age-earnings profiles observed in virtually all cross-sectional and panel data sets. In the model set forth here, experienced workers are paid more than less-experienced workers due to the operation of a dynamic selection process. Over time, incompetent individuals are discovered and dismissed by firms operating in a sector of the economy in which output is imperfectly observed and rents to employees are present. Once an individual has been dismissed from a job in this sector, he or she can only obtain employment in another sector of the economy in which output is perfectly observable and rents to employees are assumed to be zero.[1] The intertemporal selection process on the skill distribution of employees in the sector with imperfectly observed output results in the wages of these workers being bid up over time. We derive conditions under which an unique labor market equilibrium exists in which the wages paid employees in the sector in which output is imperfectly observed are consistent with their decisions regarding the supply of effort on the job.

From an empirical perspective, the model proposed here differs from others capable of generating concave age-earnings profiles in its dependence on a selection process which involves the dismissal of employees. For example, models of human capital accumulation of either the general or specific kind [see, *e.g.*, Becker (1975) and Mincer (1974)] rely on assumptions concerning the production function of human capital to produce concave age-earnings profiles rather than turnover patterns. In fact, human capital explanations of earnings growth imply little about turnover processes, other than that the probability of separation is a decreasing function of the stock of specific human capital [*e.g.*, Oi (1962) and Parsons (1972)].[2]

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[1] Other recently proposed partial equilibrium models which make use of inter-sectoral differences in monitoring and effort requirements to generate implications for wage and employment distributions are those of Bulow and Summers (1986) and Albrecht and Vroman (1990). These models do not include the dynamic selection phenomenon which is the focus of attention in this paper. The model is closest in structure to that of MacLeod and Malcomson (1988), which is described below.

[2] Though most models of human capital investment must be augmented with shocks

The empirical implications of models of worker-firm productivity matching [*e.g.*, Jovanovic (1979), Johnson (1978), Miller (1984), and Flinn (1986)] most closely resemble the implications for observed wage-turnover processes of the model exposited here. The prototypical matching model in which workers and firms incrementally learn the value of their match-specific productivity parameter over time and in which draws are independently and identically distributed across all worker-firm pairings generates upward-sloping age-earnings profiles. This result is due to the systematic elimination of "bad" matches, so that the probability of being in a such a match declines with age. The standard matching model carries the implication that in an expectational sense productivity is constant over the course of each match, though (random) fluctuations in productivity will occur. In contrast, in our selection model productivity varies in a systematic manner within matches. For empirical purposes, an advantage of our model is that wages are completely determined by expected productivity in each period, as opposed to the matching framework in which some mechanism determining how match-specific rents are allocated must be introduced [see Mortensen (1978,1982)]. A disadvantage of the current formulation of the model vis-à-vis matching models is the implication that there would be no variation in wages for individuals of the same labor market age working in the same sector of the economy.

In the matching framework, much has been made of the fact that separations are efficient [given costless renogation of contracts] so that there exists no behavioral distinction between employee- and firm-initiated separations. This is in marked contrast to our model, where only firm-initiated separations matter in the sense of changing the choice-set and future utility flows of individuals who leave a match. While it is notoriously difficult to empirically distinguish employee- and firm-initiated separations, empirical results seem to indicate that employees reporting that they were involuntarily separated from their previous employer have lower wages on their next job than observationally-equivalent employees reporting a voluntary separation [*e.g.*,

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to the firm's demand for labor function in order for separations to occur. An exception is Rosen's (1972) model in which firms offer differential investment opportunities to workers, so that an optimal investment profile implies an optimal sequence of interfirm moves. Mobility in such a case is purely voluntary, as opposed to the involuntary moves produced within our model.

Bartel and Borjas (1981)]. Futhermore, Gibbons and Katz (1990) have found that in the population of individuals who were laid-off by their previous employer, those whose layoff resulted from a plant closing [taken as being exogenous with respect to the employee's unobserved characteristics] earned higher wages than those whose layoff was "discretionary." While the interpretation of such findings is open to question, it does appear that the method of separation has an effect on future labor market outcomes.

Though the model developed here is predicated on the existence of moral hazard in employment relationships, the fact that age-earnings profiles are upward sloping is *not* a result of long-term contracting between employees and individual firms in the primary sector of the economy. Long-term contracts which promise higher wages in the future conditional on satisfactory performance require firm compliance which is impossible to generate within the market and institutional structures assumed below.[3,4] Because of this, primary-sector firms offer employees a sequence of one-period contracts conditional on satisfactory performance. We show that favorable selection [from firms' perspectives] on employee types is required to support an increasing wage equilibrium within the competitive market structure considered.

The model developed here builds on that of Shapiro and Stiglitz (1984) in several ways, though the focus of the analysis is substantially different. First, in our model there is no unemployment, which is central to the model of Shapiro and Stiglitz. The incentive not to shirk is provided by the chance of permanent reputation loss, whereas in Shapiro and Stiglitz the cost of dismissal is associated with the location of a new employer, the difficulty of

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[3] These types of contracts have been considered by Becker and Stigler (1974) and Lazear (1979,1981), among others. An increasing-remuneration contract typically can be offered only when perfect compliance is assumed on the part of the firm or when additional incentive-compatability constraints are imposed in the determination of the contract [*e.g.*, Kuhn (1986)].

[4] Carmichael (1985) and others have argued that the use of various remuneration schemes over the course of the contract as a device to solve agency problems can be dominated by a system in which employees are required to post performance bonds. MacLeod and Malcomson (1989) have shown that when the employer as well as the employee can default on the contract, bonding mechanisms *per se* cannot solve the incentive problem; they merely serve to redistribute the total surplus from the match away from employees to employers. In the present analysis, competition between firms and the existence of moral hazard precludes the posting of bonds by employees.

which is indexed by the unemployment rate. These differences in the modelling of the labor market imply that that dismissal has only transitory effects in the model of Shapiro and Stiglitz [were it to occur], whereas the effects are permanent in the model we analyze.[5]

Second, we consider the case in which employees are heterogeneous with respect to a characteristic unobservable by the firm and which enters the employee's decision rule regarding the supply of effort on the job. Shapiro and Stiglitz consider the case of homogeneous employees, as is true in most models of moral hazard applied to the labor market. The problem with such a formulation from an empirical perspective is that, generally speaking, in equilibrium no employees will shirk and hence no dismissals will be observed. The advantage of our formulation of the problem is that in equilibrium wages and separation rates are both determined within the model, with separations occurring for labor market participants at all [finite] experience levels with positive probability.

A model very similar in spirit to the one developed here appears in MacLeod and Malcomson (1988). Moral hazard and and heterogeneous agents [whose type is private information] are prominently featured in both models. The key difference between the two formulations is the specification of the production process [continuous in effort in MacLeod and Malcomson and discrete here] and the information set of the employer. Even though the distribution of types is continuous, MacLeod and Malcomson derive an equilibrium in which the [representative] firm sorts employees into a finite number of ranks and periodically readjusts their positions in the hierarchy according to a fixed promotion rule which is based on output realizations. Their promotion rule results in only promotions in equilibrium; demotions [analagous to our dismissals] would not be observed. Their formulation has the advantage of being able to generate a "fuller" age-specific distribution of earnings.[6]

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[5]Elsewhere [Flinn (1993)] we have shown that in a model similar to the one analyzed here except for [1] homogeneity of employees and [2] a stochastic production process, dismissals can occur as part of a punishment strategy employed by either competitive or monopsonistic firms. In that model, dismissal effects on welfare are transitory as in Shapiro and Stiglitz.

[6]By "fuller" we refer to the number of points of support of the earnings distribution. In MacLeod and Malcomson the earnings distribution is discrete but the number of points of support could be made arbitrarily large for

The plan of the paper is as follows. In Section 2, the optimization problem of employees in the primary market is described and their decision rule derived regarding the amount of effort to supply on the job given the termination contracts they face. Section 3 contains a description of the problem facing firms operating in the primary sector of the economy [in which moral hazard is present] and provides sufficient conditions for there to exist an unique equilibrium termination contract in the class of such contracts which specify that wages be increasing in tenure. In Section 4 we develop an econometric model which is used to estimate the primitive parameters which characterize the equilibrium. In Section 5, data from the National Longitudinal Survey - Youth Cohort [NLSYC] are used to provide some descriptive evidence on the effects of reported dismissals on wage realizations, and, in conjunction with the econometric model described in Section 4, to obtain estimates of the equilibrium model. Section 6 contains a brief conclusion.

2. CHARACTERIZATION OF THE PRODUCTION PROCESS AND THE EMPLOYEE'S PROBLEM

We will consider the labor market experiences of a cohort of labor market participants, where the cohort is defined in terms of its year of entry into the market [assumed exogenous]. Cohort members are differentiated solely in terms of their productive ability in what we shall refer to as the "primary" sector of the economy.[7] All cohort members share the following lifetime welfare function:

$$[1] \qquad W((w_s, e_s)_{s=1}^{\infty}) = \sum_{t=1}^{\infty} \beta^{t-1}(m(w_t) - e_t),$$

where w_t denotes the wage payment received at time t, e_t is the effort expended on the job at time t, β is a discount factor which takes values in the open unit interval, and m is a monotone increasing function. We note from

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certain choices of the primitive parameters. Our model only produces a single wage for labor market entrants and a wage distribution with two points of support for labor market participants of age greater than one.

[7] A completely equivalent model posits cohort members homogenous in terms of productive efficiency but differentiated by disutility of effort. All the results derived below hold under either interpretation of the source of the heterogeneity.

the outset that both employees and employers are assumed to be risk neutral [in terms of monetary units m(w)], so that insurance motives will play no role in the labor market equilibrium derived below. Without loss of generality, we will exposit the model assuming that $m(x) = x$.[8]

The economy consists of two sectors. In the primary sector, effort expended on the job is only imperfectly observable by the firm. In the secondary sector, effort is extracted from individuals as part of the production process [*i.e.*, effort is not a choice variable for employees in this sector]; furthermore, the level of effort extracted when employed in the secondary sector is the same for all individuals. Within the secondary sector the onerousness of the production task is equal to the price of output which is time invariant. With free entry of firms, bidding for employees will produce a secondary sector wage [w^s] equal to output price. Therefore all firms in this sector will earn profits of zero and all employees will obtain utility flows of zero each period.[9] Since in equilibrium all employees in the primary sector earn rents due to the imperfect observability of effort and population heterogeneity and given inelastic demand for "qualified" labor market participants by primary sector firms, all cohort members will begin their labor market careers in the primary sector. Only dismissal from the primary sector will lead employees to accept employment in the secondary sector, which consequently should be viewed as a punishment.

Labor market participants face deterministic sequences of wages in the primary and secondary sectors of the economy; both sequences are determined under competitive labor market conditions. All labor market participants choose primary sector employment when such a choice is available to them; given participation in the primary sector at time t, the probability of being allowed to choose primary sector employment at time t+1 is a function of the amount of effort the agent supplies at his primary sector job in period t and his productivity type, ξ. Let the probability of being allowed to participate

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[8] When the model is estimated, we will work with ℓn wages instead of wage levels.

[9] The important feature of the utility yield in the alternative sector from the point of view of the subsequent analysis is that it is constant over time and is the same for all population members. If effort is perfectly observable and productivity is constant, then a constant wage contract is to be expected.

in the primary sector in period t+1 given primary sector participation in period t and an effort supply of e_t be given by $p_{t+1|t}(e_t;\xi)$. Given these conditional probabilities, the probability that a type ξ individual who intends to supply a sequence of effort levels in the primary sector given by $\{e_s\}_{s=1}^{\infty}$ will be allowed to participate in this sector in period t is $p_t(e_1,\ldots,e_{t-1};\xi) = \prod_{s=1}^{t} p_{s+1|s}(e_s;\xi)$, $t = 2,3,\ldots$. Then each individual chooses a sequence of effort supplies so as to maximize their expected welfare, or

$$[2] \qquad \{e_s^*\}_{s=1}^{\infty} = \arg\sup_{\{e_s\}} E\, \mathcal{W}(\{w_s,e_s\})$$
$$= \arg\sup_{\{e_s\}} (w_1 - e_1) + \sum_{t=2}^{\infty} \beta^{t-1}\, p_t(\{e_s\}_{s=1}^{t-1};\xi)\, (w_t - e_t) ,$$

since the utility flow associated with secondary sector employment has been set to zero. From [2] it is clear that the continuation probabilities play a key role in determining employee behavior and labor market equilibrium. We now turn to the specification of these functions.

All individuals employed in the primary sector at age t have a choice of whether or not to supply an amount of effort sufficient to produce a unit of output in the period. The type ξ of a labor market participant will be interpreted as his index of *productive inefficiency* in the primary sector. The production function for an individual of type ξ in period t is given by

$$[3] \qquad y(e_t;\xi) = \begin{cases} 1 & \Leftrightarrow \quad e_t \geq \xi \\ 0 & \Leftrightarrow \quad e_t < \xi \end{cases} .$$

Thus if an individual of type ξ supplies at least that much effort in period t one unit of output is produced with probability one; if he supplies less than that much effort, a unit of output is produced with probability zero.

The labor market choices of an individual of age t are only a function of the agent's observable labor market history at time t, which is generated in the following manner.

Assumption 1: All participants in the primary sector have a fixed probability π [> 0] of being monitored each period.

In A1 we essentially consign to the monitoring process a very limited role in the determination of labor market equilibrium by fixing the monitoring probability exogenously at the constant value π.[10]

The next assumption regarding the information sets of employers in the primary sector is made to simplify the form of the termination contracts. Define the three indicator variables $\varphi_t = 1$ iff the individual is monitored in period t, $s_t = 1$ iff the individual is employed in the primary sector in period t, and $\iota_t = \varphi_t[1-y(e_t;\xi)]$.

Assumption 2: The only private information available to a primary sector employee's firm at the conclusion of period t is ι_t. All primary sector firms have access to each labor market participants termination history

[4] $$\mathcal{H}_t = (t_1,\ldots,t_{t-1}),$$

where $t_s = 1$ if the individual was terminated by a primary sector firm in period s and equals 0 otherwise.

A2 is quite critical to the determination of the equilibrium set of contracts in the labor market. Note that by the definition of ι_s, a primary sector employer in period s can only learn if one of its period s employees was *monitored AND produced no output* or not. In particular, no information is revealed to an employer when an employee is monitored and is found to produce a unit of output. If such information were available, presumably optimal termination contracts would be written conditionally upon it if it were permissible to do so.[11]

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[10] Allowing for the determination of an optimal monitoring function within the model would greatly complicate matters though it would provide an interesting extension. For one recent example of the derivation of an optimal monitoring policy in a labor market context see Hosios and Peters (1993), though their model does not include moral hazard. In this paper we view the monitoring technology as a component of the production process, the entirety of which is predetermined.

[11] There may be good institutional reasons for assuming that such information could not be used in setting contracts, however. For example, suppose that two primary market participants of the same age had been monitored different numbers of times, and that each had "passed" all monitoring tests. Since the

Also note that although A2 gives primary sector firms private information regarding their own employees, this informational advantage cannot be utilized by these employers to gain savings in wage payments through the threat of unjust dismissal. Since dismissal by a primary sector firm will result in permanent assignment to the secondary sector under the termination contract specified below, one must consider whether firms could threaten to unjustly dismiss employees who refuse to take a reduction in wages below those specified in equilibrium. The presence of moral hazard ensures that this will not occur, since wage reductions will result in corresponding effort and expected revenue reductions. Thus a primary sector firm will only terminate an employee at the end of period t when $i_t = 1$. No primary sector firm will hire an individual who has been terminated by a primary sector firm at any point in his labor market career. While we shall merely assume no such rehirings for the moment, Proposition 4 demonstrates that this is in fact an equilibrium outcome.

Under our termination contract then, labor market participants will be premanently separted from the primary sector following any period in which $i_t = 1$. We can now complete our desciption of the employees problem by writing

$$[5] \qquad p_{t+1|t}(e_t;\xi) = 1 - p[i_t=1|e_t,\xi] = 1 - \pi\,\chi[e_t < \xi] \ ,$$

where $\chi[A]$ denotes the indicator function which takes the value 1 if logical expression A is true and zero otherwise.

We will describe the optimal sequence of effort choices under the termation contract using a dynamic programming formulation of the choice problem in [2]. First note that since effort yields disutility, no individual of type ξ will supply more effort than ξ in any period. Similarly, since the outcome of not producing a unit of output and being monitored is independent of the amount of effort [less than ξ] supplied, any individual deciding not to

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number of occurences of being monitored is a stochastic exogenous event, differential remuneration based on such a characteristic could be viewed as "statistical discrimination." Therefore, it might be plausibly argued that legal systems of the type found in many countries today constrain employers to utilize the information only in $(i_1,\ldots,i_{t-1})$ when determining period t remuneration even when more information is available.

produce a unit of output in the primary sector will choose to supply an effort level of zero. Then the age t problem of a primary-sector participant is

$$[6] \qquad V_t(\xi) = \max \{w_t - \xi + \beta V_{t+1}(\xi);\ w_t + \beta(1-\pi)V_{t+1}(\xi)\},$$

where the first argument in the max operator corresponds to the value of working and the second argument corresponds to the expected value of shirking.

It is straightforward to show that as long as the wage sequence is bounded, the decision rules of primary sector employees will possess a critical value property in all periods. However, the sequence of critical values is in general an exceedingly complicated function of the wage history. To simplify the analysis, we limit attention to a certain class of wage sequences which include essentially all life-cycle wage processes estimated using micro-level or aggregated data.[12]

Assumption 3: The wage sequence in the primary sector is strictly positive, monotone increasing, and bounded.

Assuming that wages are strictly positive will serve to rule out the trivial equilibrium in which primary sector wages and output levels are set equal to zero each period. The condition that the wage sequence be increasing provides us with the opportunity to greatly simplify the computation of the critical values, since an individual who is indifferent between shirking and working in period t will supply effort in all future periods due to the increased attractiveness of the sequence of primary sector wages he will face. This argument is formalized in the proof of the following proposition, which can be found in Appendix A along with those for most of the other formal results and propositions which follow.

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[12]While it is true that estimated life-cycle wage functions often exhibit negative wage growth for older workers, we would argue that the main relevance of the arguments presented in this paper is to the wage growth process of relatively recent labor market entrants. Estimated life-cycle wage functions typically imply positive wage growth for at least the first half of the labor market career, a period which should comfortably contain most of the learning described in our model.

Proposition 1: The decision rule for an agent of type ξ employed in the first sector in period t is

$$[7] \qquad \begin{array}{lll} \textit{Work} & \textit{if} & \xi \le \xi_t \\ \textit{Shirk} & \textit{if} & \xi > \xi_t \end{array}$$

$$\text{where } \xi_t = MQ_{t+1},\ M = \frac{\beta(1-\beta)\pi}{1-\beta(1-\pi)},\ \text{and } Q_t = \sum_{s=t}^{\infty} \beta^{s-t} w_s .$$

From [7] it follows immediately that an increase in the monitoring rate increases the critical effort ξ_t in each period, since $\partial M/\partial \pi > 0$. It is also not difficult to demonstrate that each critical value ξ_t, $t \in \mathbb{N}$, is increasing in the discount factor β.

3. FIRM BEHAVIOR AND THE DETERMINATION OF EQUILIBRIUM WAGES

In developing the effort rules utilized by primary sector employees, we characterized secondary sector firms as competitive in the input market and as price-takers in the output market. We will also view primary sector firms as competitive in the input market. A cohort consists of a continuum of members each of whom participates in the labor market in each period 1,2,... [each individual is infinitely-lived]. We will associate the set of cohort members with the unit interval and assign subsets of cohort members the Lebesgue measure. Let there be N_f [$< \infty$] firms operating in this sector each period, and let each employ and identical proportion of primary sector employees each period, N_f^{-1}. Each firm's objective is to maximize the discounted sum of expected profits, given by

$$[8] \qquad \mathcal{J}(\{w_s\}_{s=1}^{\infty}) = \sum_{t=1}^{\infty} \beta^{t-1} \{\rho\, H_t(\xi_t(\{w_s\}_{s=t+1}^{\infty})) - w_t\},$$

where the representative firm's discount factor is assumed identical to that of the employees, ρ is product price which is taken as given by the firm, and where the dependence of the period t critical value ξ_t on all future wages in the primary sector has been made explicit. Note that profits are expressed per employee; this is an inconsequential normalization since in equilibrium firms earn zero profits in every period. The expected output of a randomly selected employee of the firm's in period t is given by $H_t(\xi_t)$, where H_t is the cumulative distribution function of types in period t. The distribution function changes over time due to the systematic selection on types induced by termination contracts.

Market structure is defined as follows.

Assumption 4: The market structure of the primary sector of the economy is characterized by:

(4A) *Free entry and exit of firms.*

(4B) *Inability of firms to issue credible mulitiple-period contracts.*

(4C) *Exogenous output price determination.*

Because of free entry and exit, primary sector firms earn zero profits in equilibrium and so are no better off than their secondary sector colleagues. Moreover, such firms cannot issue credible long-term contracts due to the absence of agents or mechanisms which could ensure firm compliance with employment contracts defined over more than one period. If multiple-period contracts are not enforcable, then the [expected] zero profit condition implied by free entry is strengthened to the condition of expected zero profit in each period of operation for each primary sector firm.

Given that primary sector firms earn zero profits in each period, by [8] the wage in period t is

$$[9] \qquad w_t = \rho H_t(\xi_t(\{w_s\}_{s=t+1}^{\infty})).$$

It is apparent that the period t wage depends on all future wages through the critical value function ξ_t. As we will describe, the period t wage in fact depends on the *entire wage sequence* $w_2, w_3, \ldots$ through the distribution function H_t.

Output price ρ is determined exogenously, for example on a world output market. The equilibrium we describe is unique for any output price strictly greater than zero.

Assumption 5: Productive inefficiency in the primary sector is continuously distributed on $\mathbb{R}_+$ [the set of nonnegative real numbers] according to H, which is second-order differentiable and concave.

The assumption of differentiability is helpful in the proof establishing existence and uniqueness of primary sector equilibrium, and seems a practical necessity in any empirical application of the model.[13] The concavity assumption is strong, but distributions commonly used in empirical analysis are contained in this class.[14] For example, among distributions only defined over the nonnegative real line, the exponential is concave.[15] A multitude of concave distributions with support $\mathbb{R}_+$ can be created by truncating some commonly used [mean zero] distributions defined on $\mathbb{R}$ at zero [from below]; a few examples are the Normal, t, and logistic.

Having completed a statement of the model structure, we are now ready to consider the properties of primary sector equilibrium. We begin with the following restriction on any equilibrium wage sequence, which is sufficiently obvious that no formal proof is supplied.

Result 1: Every equilibrium wage sequence $\{w_s^*\}$ is convergent with limit point $\bar{w}^* \in (0,\rho]$.

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[13] This is due to the nonobservability of individual effort expenditures on the part of primary sector employees. Most analysts are likely to have access to but a few periods of observations of primary sector wages and dismissal rates. From this information, it may be possible to determine critical effort levels for each period, which in turn can be used to fit a distribution H. With only a few points of evaluation, however, it is a practical necessity to use a parametric distribution, which will typically be assumed to be continuously differentiable at least through second order.

[14] The concave distribution function condition utilized in the principal-agent literature, which was introduced by Mirrlees (1979) and discussed at length in Grossman and Hart (1983), is employed in a very different context from the one here. The concavity assumption used by these authors applies to distributions of outcomes conditional on (unobserved) actions by the agent, and is applied to yield the implication that remuneration is monotone in outcomes. Such concerns are not of relevance given the structure of our model.

[15] Other distributions commonly used for nonnegative random variables such as the Weibull and gamma are concave only for subsets of their respective parameter spaces.

All wage sequences are increasing by A3 and employees are paid their expected revenue product in each period, a quantity which is never greater than ρ under the production technology [3]. Therefore, any equilibrium wage sequence is bounded from above by the product price and from below by 0. Under A5, the set of employees with productive inefficiency exactly equal to zero has measure zero, so that the limiting wage must be strictly greater than zero.

As in any matching model [*i.e.*, matching "competent" individuals to the primary sector], the process of selection is what produces all the interesting dynamics. Under the dismissal process described above, a proportion π of all shirkers of age t are dismissed. Age t individuals in the primary sector who shirk are those whose productive inefficiency is greater than ξ_t. Thus the upper tail of the productive inefficiency distribution is thinned[16] each period, with the interval subject to thinning changing with the critical value ξ_t. When the sequence $\{w_s\}$ is increasing, by [7] the sequence $\{\xi_t\}$ is also increasing. When the sequence $\{\xi_s\}$ is increasing, the thinning process produces a sequence of distributions which are easily characterized in terms of the initial productive inefficiency distribution H and the monitoring rate π:

$$[10] \qquad H_t(\xi_t) = 1 - \tilde{H}(\xi_t)A_t(\{\xi_s\}_{s=1}^{t-1}) \,,$$

$$\text{where } A_t(\{\xi_s\}_{s=1}^{t-1}) = \left\{ 1 + \frac{\pi}{(1-\pi)^{t-1}} H(\xi_1) + \ldots + \frac{\pi}{1-\pi} H(\xi_{t-1}) \right\}^{-1} ,$$

and $\tilde{H}$ denotes the survivor function, 1-H. The sequence of distribution functions $H, H_2, \ldots$ display an ordering property important in establishing the existence of equilibria over the set of increasing wage sequences.

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[16] Using terminology from the stochastic process literature, see, *e.g.*, Cox and Isham (1985).

Result 2: For any increasing sequence of values $0 < \xi_1 \leq \xi_2 \leq \ldots \leq \xi_t$ *and* $s < t$,

$$[11] \qquad H_t <_{SD} H_s$$

where $<_{SD}$ *denotes the first-order stochastic dominance operator.*

From R2 we know that even if the wage sequence was constant so that $\xi_1 = \xi_2 = \ldots$, the probability of a randomly selected employee in the primary sector producing a unit of output in period t is an increasing function of t; in this statistical sense primary sector employees become "more productive" as they gain experience. This increased productivity results in increased wage payments in a competitive labor market with spot contracts, and supports the increasing wage sequences to which we are limiting our attention.

Equilibria in our model are formally specified as the fixed points of the operator

$$[12] \qquad T(\{w_s\}) \equiv \begin{bmatrix} \rho\ H(\xi_1(\{w_s\}_{s=2}^{\infty})) \\ \rho\ H_2(\xi_2(\{w_s\}_{s=3}^{\infty})) \\ \vdots \\ \rho\ H_\tau(\xi_\tau(\{w_s\}_{s=\tau+1}^{\infty})) \\ \vdots \end{bmatrix} .$$

Recall that while the system appears recursive in that the critical value ξ_s is a function only of wage payments at time s+1,s+2,... , the distribution *functions* $H_2, H_3, \ldots$ are functions of the wage sequence $\{w_s\}_{s=2}^{\infty}$. For example, the distribution function of ξ in the population of primary sector employees in the second period, H_2, is a function of ξ_1, π, and H. The probability that a primary sector employee supplies effort in the second period thus is a function of the primary sector wage in period 2, w_2, even though this wage doesn't affect the effort supply of primary sector employees in the second period. More generally, at any time t, wages in periods 2 through t have purely *compositional* effects on expected productivity in that they [partially] determine the distribution function H_t but do not determine the effort supplies of period t primary sector employees. Wages in periods t+1,t+2,...

have both compositional and *direct* effects in that they [partially] determine the distribution function H_t and completely characterize the effort supply decisions of period t primary sector employees.

We first establish an important property which any solution to [12] must exhibit and which strengthens R1.

Proposition 2: For any equilibrium wage sequence, $\bar{w}^* = \rho$.

By inspection of the distribution function given in [10], it is clear that for any sequence of critical values, the equilibrium wage in period $t < \infty$ is strictly less than the product price; this is due to the fact that for finite t a subset of positive measure of each primary sector firm's set of employees shirks. Only in the limit does this set of employees have measure 0; therefore in the limit each primary sector employee produces a unit of output with probability 1 and therefore is paid their expected revenue product of ρ.

Our main theoretical result is the following.

Proposition 3: There exists an unique sequence $\{w_s^*\}$ *such that*

$$\{w_s^*\} = T(\{w_s^*\}).$$

The uniqueness of the equilibrium wage sequence in the primary sector is of particular importance given our goal of empirical implementation of the model. Moreover, the fact that the operator T is differentiable in the primitive parameters of the model leads to an equilibrium wage sequence which is similarly differentiable. These differentiability properties greatly facilitate estimation of the model, as is shown in the following section.

To this point we have merely asserted that individuals dismissed from the primary sector at any point in their labor market careers would never be subsequently hired by a primary sector firm. We now demonstrate that this is an equilibrium outcome in the following specific sense. Given that a primary sector firm has dismissed a set of employees which were determined not to have produced output in some previous period under the terms of the original termination contract, it has no incentive to offer such individuals a new termination contract of similar form.

Proposition 4: The determination contract offered primary sector employees is renegotiation proof.

The basic intuition for this result is clear. Primary sector firms make zero profits on the "best" labor market participants, *i.e.*, those not previously dismissed. Moral hazard considerations prevent primary sector firms from reducing the compensation of previously dismissed employees, so that primary sector firms employing such individuals must earn negative profits. The renegotiation-proofness of these termination contracts is critically dependent on the competitive labor markets assumption and the presence of moral hazard.

4. ESTIMATION OF THE WAGE-DISMISSAL PROCESS

The stochastic process for wages and dismissals at the individual level is quite simple to describe given the equilibrium wage sequence in the primary sector, which is determined by the discount factor, β, the time-invariant monitoring rate, π, the heterogeneity distribution, H, the product price, ρ, and finally the sequence of wages paid in the secondary sector of the economy, which we have set equal to w^s in each period.

In mapping the behavioral model into an estimable econometric model, provisions must be made for measurement error in both the log wage and dismissal sequences. We assume that the log wage rate in sector Δ at time t is given by

$$[13] \qquad w_t^{\Delta} = \mu_t^{\Delta} + \epsilon_t ,$$

where $\mu_t^{\Delta} = w_t^*$ when $\Delta = p$ [*i.e.*, the wage draw is from the primary sector] and $\mu_t^{\Delta} = w^s$ when $\Delta = s$ [*i.e.*, the wage draw is from the secondary sector], and where ϵ_t is an i.i.d. normal random variable with mean 0 and standard deviation σ_{ϵ}.

It is necessary to include measurement error in the self-reported dismissal information since the model implies that at most one dismissal will occur over the course of a labor market career, while 6% of the sample used in the empirical work reported below report more than one dismissal in their first four years in the labor market. Of course, individuals can be laid-off or dismissed for reasons having little to do with their own actions, for example because of adverse shocks to the demand for their employer's product. On the other hand, reporting to an interviewer that the reason for a job separation was due to dismissal may be unpleasant for some respondents, resulting in some proportion of "true" dismissals going undetected. Let d_t^* denote the true dismissal outcome during period t, where $d_t^* = 1$ if a dismissal occurred in the interval [t,t+1) and $d_t^* = 0$ if one did not. We have assumed that

$$[14] \qquad p(d_t = j \mid d_t^* = j) = \lambda \in (0,1), \quad j = \{0,1\},$$

where d_t is the reported dismissal outcome during period t, with $d_t = 1$ if the agent reported a dismissal occurring in the period [t,t+1) and which is equal to 0 otherwise.[17]

The data used to estimate the model consists of ℓn wage observations for the first T years of labor market experience for a sample of n individuals, along with the set of self-reported dismissal indicator variables $d_1, \ldots, d_{T-1}$. The sampling frequency is one year, and we adopt the convention that the decision frequency is also one year.

With these assumptions regarding the measurement error processes, we can now define the likelihood contribution of sample member i. Since the measurement error processes in the ℓn wage and dismissal sequences are independent, we have

$$[15] \qquad L_i = \sum_{\underset{\sim}{d}^* \in \mathcal{D}^*} g(\ell n\, \underset{\sim}{w}_i, \underset{\sim}{d}_i \mid \underset{\sim}{d}^*) p(\underset{\sim}{d}^*)$$

$$= \sum_{\underset{\sim}{d}^* \in \mathcal{D}^*} g_1(\ell n\, \underset{\sim}{w}_i \mid \underset{\sim}{d}^*) g_2(\underset{\sim}{d}_i \mid \underset{\sim}{d}^*) p(\underset{\sim}{d}^*) \quad ,$$

where the set $\mathcal{D}^*$ contains all latent dismissal sequences with positive probability, g is the joint density of ℓn wages and observed dismissal

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[17] We had originally specified a different measurement error process for each state j, *i.e.* $\lambda_j = p(d_t = j \mid d_t^* = j)$, but experienced numerical problems during the estimation process forcing us to adopt the specification in [14].

outcomes conditional on the true dismissal sequence, g_1 is the conditional density of ℓn wages given the true dismissal sequence, and g_2 is the conditional probability function of observed dismissals given the true dismissal sequence.

In terms of the model, recall that the secondary sector of the economy is an absorbing state, and that in this state no dismissal with informational value can take place. Over T periods, "true" dismissals can take place at most once. Given the fact that the critical effort level sequence is strictly increasing over time, we have

$$[16] \quad p(\sum_{t=1}^{T-1} d_t^* = 0) = H(\xi_1) + (1-\pi)[H(\xi_2)-H(\xi_1)] + \ldots$$
$$+ (1-\pi)^{T-2}[H(\xi_{T-1})-H(\xi_{T-2})] + (1-\pi)^{T-1}[1-H(\xi_{T-1})] \ ,$$
$$p(d_j^* = 1) = \pi(1-\pi)^{j-1}[1-H(\xi_j)] \ , \ j = 1,\ldots,T-1 \ .$$

This probability distribution is determined by all the behavioral parameters of the model, that is, the critical effort level sequence $\{\xi_s\}$ is a function of the parameters β, ρ, H, and π.

The distributions g_1 and g_2 are easily specified. The ℓn wage sequence over the first T periods in the labor market conditional on the true dismissal sequence is

$$[17] \quad g_1(\ell n \ \underset{\sim}{w}_i | \sum_{t=1}^{T-1} d_t^* = 0) = \sigma_\epsilon^{-T} \prod_{t=1}^{T} \phi((\ell n \ w_{it} - w_t^*)/\sigma_\epsilon) \ ,$$
$$g_1(\ell n \ \underset{\sim}{w}_i | d_j^* = 1) = \sigma_\epsilon^{-T} \prod_{t=1}^{j} \phi((\ell n \ w_{it} - w_t^*)/\sigma_\epsilon) \prod_{t=j+1}^{T} \phi((\ell n \ w_{it} - w^s)/\sigma_\epsilon)$$
$$j = 1,\ldots,T-1,$$

where $\ell n \ w_{it}$ denotes the ln wage of sample member i in period t and ϕ denotes the probability density function of a standard normal random variable.

Finally, the conditional probability function of reported dismissals given the true dismissal sequence is

[18] $$g_2(\underset{\sim}{d}_i|\underset{\sim}{d}^*) = \lambda^{A(i)}(1-\lambda)^{T-1-A(i)}$$

where $A(i) = \sum_{t=1}^{T-1} |d_{it}-d_t^*|$.

The log likelihood for the sample, given independence of the measurement error processes across individuals, is simply $\mathcal{L}(\theta) = \sum \ell n(L_i)$, where the parameter vector θ includes β, π, H, ρ, λ, σ_ϵ, and w^s. In proving uniqueness of any nontrivial primary sector equilibrium, we relied heavily on A5 which posits the differentiability and concavity of the heterogeneity distribution function H. To ensure compliance with this assumption, we have estimated the model assuming a parametric form for H, $H(\cdot;\alpha)$, in which A5 is satisfied for all $\alpha \in \Theta_H$, where Θ_H denotes the parameter space for the distribution H. In particular, the estimates reported below are obtained under the assumption that the heterogeneity distribution is half-normal, so that the cumulative distribution function of ξ is given by

[19] $$H(\xi;\alpha) = 2[\Phi(\xi/\alpha)-.5],$$

where Φ denotes the c.d.f. of a standard normal random variable.[18]

Maximum likelihood estimates of model parameters were obtained through the use of a modified method of scoring algorithm using numerical derivatives. At each iteration, current values of the behavioral parameters were utilized to compute the equilibrium termination contract wage sequence and corresponding critical value sequence. The algorithm used in this step of the estimation process is described in Appendix B. Given the differentiablity of the equilibrium termination contract with respect to the primitive parameters, it is straightforward to demonstrate that the m.l. estimator is consistent and asymptotically efficient, though the likelihood function has not been shown to be globally concave and no initial consistent estimator is available for θ. We can however report that the algorithm converged to the same point in the parameter space when started from a variety of initial values for θ.

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[18] We also estimated the model under the assumption that ξ was exponentially distributed in the population. Estimates of parameters other than α were relatively insensitive to this change in distributional assumptions.

5. EMPIRICAL RESULTS

The sample used in the empirical work was drawn from the National Longitudinal Study - Youth Cohort [NLSYC], which is [for the most part] a nationally-representative sample of approximately 12000 individuals who were 14-21 years of age in 1979. These individuals have been reinterveiwed on an annual basis during the 1980's; currently, nine waves of information are available for the years 1979-1987.

In defining our subsample several stringent criteria were imposed. To avoid problems of intermittent labor market participant, only males were included. Initial conditions problems were largely circumvented by requiring each sample member to be engaged in full-time schooling and be out of the labor market in one of the years 1979-1983, and then to be employed at the time of the next four consecutive interviews. The employment condition at the time of the interviews was imposed because no rationale for unemployment is included in the model. Finally, only cases with complete information on wages and reasons for job separations [along with a few other demographic characteristics not included in the present study] were eligible for inclusion in the sample. The sample with which we work includes 198 individuals.

In constructing the dismissal sequence, we examined the reported reason for all job changes occurring over each sample period. If an agent reported that any job held during the period between the interviews ended due to a "dismissal" or a "layoff," the individual was considered to have reported a dismissal from the primary sector during that period.[19] Though this definition of dismissals is very arbitrary, we will see that the dismissal sequence defined in this way negatively impacts ℓn wages, as we would hope to observe. Moreover, the crudeness of the definition is mitigated to some extent by the allowance for measurement error discussed in the previous section.

The sample is described in Table 1, where the average and the standard deviations of ℓn hourly wages are given by period for each of the eight possible dismissal sequences. All wages were first expressed in terms of 1980 dollars. In terms of the occurence of dismissals, more than three-fourths of

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[19] In point of fact, the survey records information on up to five job spells for sample period, though few respondents seem to hold more than five jobs in any one year period.

TABLE 1

Means and Standard Deviations of Ln Wage Sequences Conditional on Reported Dismissal Sequence

Dismissal Sequence			*Ln Hourly Wage Rate*				
d_1	d_2	d_3	$\ell n(w_1)$	$\ell n(w_2)$	$\ell n(w_3)$	$\ell n(w_4)$	*Proportion*
	ALL		1.51 (.42)	1.61 (.42)	1.67 (.47)	1.77 (.47)	
0	0	0	1.52 (.41)	1.65 (.42)	1.74 (.46)	1.84 (.48)	.778
1	0	0	1.44 (.36)	1.36 (.20)	1.50 (.35)	1.64 (.35)	.086
0	1	0	1.50 (.47)	1.34 (.45)	1.27 (.52)	1.25 (.36)	.040
0	0	1	1.16 (.11)	1.38 (.50)	1.38 (.31)	1.55 (.40)	.035
1	1	0	1.68 (.49)	1.86 (.25)	1.50 (.28)	1.68 (.36)	.035
1	0	1	1.39 (.37)	1.39 (.38)	1.43 (.37)	1.68 (.10)	.015
0	1	1	2.23 (1.07)	1.76 (.22)	2.00 (1.18)	1.33 (.38)	.010
1	1	1	-	-	-	-	0

this sample reports no dismissal or layoff over their four years in the labor market, though one must keep in mind that the restriction that all sample members be employed at the time of each interview almost surely leads to downward-biased estimates of population dismissal rates. Approximately 16 percent of the sample reported one dismissal over the entire period, 6 percent reported two, and no sample member reported three. Of those individuals reporting one or two sampling periods with dismissals, dismissal experiences are largely concentrated in the first two years of labor market participation.

Only dismissal sequences with zero or at most one reported dismissal have enough observations to make interpretation of the associated ℓn wage sequences worthwhile. For the group which experiences no dismissals over the sample period, average ℓn wages increase in regular increments; in period four, average ℓn hourly wages are 22 percent greater than their first period level. Conversely, the groups which experience one dismissal exhibit at least one period of no growth in ℓn wages over the four year interval. The largest of these groups, the one in which the dismissal occurs between the first and second wage observation, exhibits a noticeable drop in average ℓn wages from the first to the second period. After the second period, ℓn wages increase in a regular fashion, though in the fourth period the average ℓn wage is still less than the average ℓn wage in the second period for those exhibiting no dismissals. The group defined by having one dismissal between the second and third wage observations has a steadily decreasing pattern of average ℓn wages, and there is a large drop in average ℓn wages between the second and third sampling period. The group with their one dismissal occurring between the third and the fourth sampling period actually experiences an increase in average ℓn wages at this time. The extremely small numbers of individuals in these last two groups make any more detailed examinations of patterns precarious.

In Table 2 we examine the effect of the dismissal history on current ℓn wages both conditionally and unconditionally on the ℓn wage history of the individual. We have estimated the regression function using ordinary least squares, and have reported the Eicker-White heteroskedasticity-consistent standard errors which are asymptotically valid under any pattern of heteroskedasticity in the population.

The regressions of the second period log wage rate on the binary variable

TABLE 2

OLS Regressions of Ln Wages on the Dismissal and Ln Wage History

[Eicker-White Heteroskedasticity-Consistent Standard Errors in Brackets]

	Dependent Variable					
Coefficient	$\ell n(w_2)$		$\ell n(w_3)$		$\ell n(w_4)$	
Constant	1.625	.666	1.714	.185	1.822	.303
d_1	-.132 [.068]	-.123 [.071]	-.170 [.082]	-.063 [.070]	-.058 [.078]	.072 [.058]
d_2			-.192 [.144]	-.271 [.115]	-.340 [.103]	-.283 [.079]
d_3					-.206 [.098]	-.064 [.097]
$\ell n(w_1)$		.635 [.069]		.306 [.096]		.038 [.064]
$\ell n(w_2)$				.659 [.084]		.422 [.083]
$\ell n(w_3)$						.449 [.077]

indicating dismissal between periods one and two reveal (marginally) significant negative effects of this experience whether or not we condition on the period one ℓn wage rate. The absolute size of the coefficient is also unaffected by the inclusion of the ℓn wage history.

Regressions of third period ℓn wage rates on the dismissal history reveal similiar patterns when we do not condition on the ℓn wage history. The effects of a dismissal on the third period ℓn wage are roughly independent of the timing of the dismissal. The size of the dismissal effects is a bit larger than was the case for the regression reported in column 1 of the table. When we condition on the ℓn wage history as well (column 4), dismissals between the first and second wage observations no longer have statistically signficant effects on third period ℓn wages, probably due to the transmission of the consequence of dismissal to second period ℓn wages. However, dismissal between periods 2 and 3 substantially reduces the ℓn wage rate in the third period conditional on the ℓn wage history.

Regressions of ℓn wages in the fourth period yield slightly different patterns of the coefficients. When the regression includes only the dismissal sequence, second period and third period dismissals are found to have large and statistically significant effects on ℓn wages. Experiencing a dismissal in the first period is associated with lower fourth period ℓn wages, though the coefficient is insignificantly different from zero. The effect of a second period dismissal on fourth period ℓn wages is especially notable both for its absolute size and signficance level. When we condition on the ℓn wage history, only the second period dismissal indicator has a coefficient statistically significant from zero. For whatever reason, adjustments in third period ℓn wages apparently are not sufficient to capture the effects of prior dismissals in this regression function.

The descriptive analyses contained in Tables 1 and 2 lend support to the notion that dismissals [even though crudely measured] have negative impacts on subsequent ℓn wage realizations. The high dimensionality of the data make it difficult to summarize these effects in a parsimonious manner. The structural representation does present some help in this direction, besides yielding parameter estimates which are readily interpretable within the context of the behavioral model exposited in Sections 2 and 3.

Table 3 contains the m.l. estimates of the structural model. Before

TABLE 3

Maximum Likelihood Estimates of Structural Parameters

(Asymptotic Standard Errors in Parentheses)

Parameter	*Description*	*All*	*Schooling Group* *Low*	*High*
$\beta^{\dagger}$	*Discount factor*	.95	.95	.95
π	*Monitoring rate*	.370 (.187)	.252 (.311)	.458 (.251)
α (H)	*Parameter of heterogeneity dist.*	1.404 (.281)	1.467 (.824)	1.398 (.278)
ρ	*Ln product price in primary sector*	1.957 (.201)	1.958 (.689)	2.010 (.181)
λ	*Measurement error parm. for dismissals*	.951 (.006)	.945 (.012)	.957 (.007)
σ_{ϵ}	*Measurement error parm. for ℓn wages*	.426 (.008)	.400 (.012)	.421 (.011)
w^{s}	*Secondary sector ℓn wage*	1.340 (.074)	1.247 (.110)	1.482 (.092)
	Log likelihood value	-649.236	-287.458	-330.235

† All estimation was performed conditional on a fixed value of the discount factor.

discussing the results, two comments are in order. First, in order to avoid potential numerical problems, in all estimated equilibrium models we have fixed the value of the discount factor at .95. Second, to determine the extent to which individuals from different schooling groups were segregated in terms of labor market opportunities, we reestimated the structural model using high and low schooling sub-samples. So as to divide the relatively small sample approximately evenly, we defined the low-schooling cut-off value at less than or equal to 13 years.

From the point estimates obtained when using the entire sample, 37% of agents in the primary sector of the economy are monitored each period, however this parameter is not precisely estimated. There is a relatively large difference in the monitoring rates experienced by agents in the primary sector when estimation is performed separately over low- and high-schooling groups. We should not think of these agents as being employed in the *same* primary sector; rather, this indicates that the jobs characterized by the presence of asymmetric information and moral hazard available to individuals with lower levels of schooling are not as highly monitored as are the analagous jobs which are available to individuals with higher levels of schooling.[20]

The point estimate of the heterogeneity distribution parameter α indicates that the mean value of ξ in the population is 1.12 and the standard deviation is .846. The limit point of the critical value sequence is estimated to be 1.7133; given the estimated heterogeneity distribution, 22.2 percent of the population have values of ξ greater than this value and thus will eventually be dismissed from the primary sector with probability one. In fact, because the estimated equilibrium critical value sequence $\{\xi^*_s\}$ converges so quickly to its limit point, only a total of 22.8 percent of the cohort are dismissed from the primary sector over the life cycle. The estimate of the parameter α is somewhat higher in the low-education labor market, but is not precisely estimated.

Product prices in the primary sector are higher in the high-education market compared with the low-education market, though the differences are not

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[20]Our labor market model treats monitoring rates as exogenously determined so not much more can be said when comparing these rates across groups. An interesting extension of the model would involve the optimal determination of wage rates *and* monitoring rates.

as large as one might expect. This is probably due to the fact that only the first four years of labor market ℓn wages are being used.

The measurement error parameter λ is estimated to be approximately .95, indicating that there is a large degree of information content in the reported dismissal sequence. The point estimates of σ_ϵ indicate somewhat more measurement error in ℓn wages among the high-education group, a not unexpected result. Differences between schooling groups in ℓn wage rates in the secondary sector are pronounced, and go in the expected direction.

We conducted a likelihood ratio test to determine whether the six parameters estimated were statistically distinguishable. The value of the test statistic is 63.086, which under the null hypothesis of no group differences is distributed as a chi-square random variable with six degrees of freedom. The null of no differences in the behavioral parameters is thus overwhelmingly rejected, which invites us to consider more carefully in what manner, if any, the labor markets for observationally-distinguishable labor market participants are interrelated.

Finally, in Table 4 we present some implications of the model estimates for ℓn wage sequences in the primary sector of the labor market and for age-earnings profiles not conditional on sector (*i.e.*, marginal predicted distributions), as well as for dismissal rates. For all three sets of estimates, the ℓn wage in the secondary sector is less the the implied ℓn wage rate in the primary sector in the first period. Since ℓn wages in the primary sector are monotone increasing, this implies that observed ℓn wages in period t [unconditional on sector] will always be bounded by the primary sector ℓn wage and the ℓn wage in the secondary sector. By construction, first period ℓn wages are identical to first period primary sector ℓn wages. However, by period four average ℓn wages in the primary sector are 1.82 [using the estimates from the entire sample] as compared with the average of 1.74 in the population, due to the fact that 17% of the population is predicted to be in the secondary market at the outset of period four. The increasing dispersion of the ℓn wages across sectors has resulted in increasing variance in the marginal distribution of predicted ℓn wages. As we can see, this roughly mimics the behavior of the standard deviations of observed ℓn wages over time. The model implies that 8% of the population is dismissed between periods 1 and 2, 7% of those in the primary sector at the start of period 2

TABLE 4

Implied and Observed Ln Wage Sequences
and Primary Sector Proportions

(Standard Deviations in Parentheses)

Dist. Type	$\ell n(w_1)$	$\ell n(w_2)$	$\ell n(w_3)$	$\ell n(w_4)$	$Prop_2$	$Prop_3$	$Prop_4$
<u>*All*</u>							
Primary	1.51 (.43)	1.65 (.43)	1.75 (.43)	1.82 (.43)	1	1	1
Marginal (Predicted)	1.50 (.43)	1.62 (.43)	1.69 (.45)	1.74 (.46)	.92	.86	.83
Marginal (Observed)	1.51 (.42)	1.61 (.42)	1.67 (.47)	1.77 (.47)			
<u>*Low Schooling*</u>							
Primary	1.39 (.40)	1.51 (.40)	1.61 (.40)	1.68 (.40)	1	1	1
Marginal (Predicted)	1.39 (.40)	1.49 (.41)	1.56 (.42)	1.62 (.43)	.93	.87	.83
Marginal (Observed)	1.39 (.39)	1.49 (.41)	1.52 (.43)	1.64 (.43)			
<u>*High Schooling*</u>							
Primary	1.60 (.42)	1.77 (.42)	1.87 (.42)	1.93 (.42)	1	1	1
Marginal (Predicted)	1.60 (.42)	1.74 (.43)	1.82 (.44)	1.86 (.45)	.91	.86	.83
Marginal (Observed)	1.61 (.41)	1.71 (.40)	1.82 (.46)	1.89 (.48)			

are dismissed by period 3, and that 4% of those beginning period 3 in the primary sector are dismissed by period 4. Comparison with the observed marginal distributions of ℓn wages indicates a relatively good fit, with some tendency to over-predict average ℓn wages in period 3 and to under-predict average ℓn wages in period 4. Similiar comments apply to the results computed specific to schooling groups. Growth patterns in wages are broadly similiar for high and low schooling groups, with the main differences appearing to be in initial ℓn wages.

6. CONCLUSION

The model presented in this paper incorporates moral hazard in employment relationships in order to provide a central role for "involuntary" separations in the wage growth process of young labor market participants. As opposed to most models of moral hazard applied to the labor market, increasing wages are not motivated by long-term contracting considerations, but instead result from the operation of a dynamic selection process in which individuals inefficient at production in the primary sector are systematically discovered and dismissed. By primary sector firms competing for an improving quality distribution of employees, a positively-sloped age-earnings profile is produced.

By limiting attention to increasing wage sequences, we have been able to specify conditions sufficient to ensure the existence of equilibria; the most critical of the assumptions employed would appear to involve restrictions on the information set of firms and that of concavity of the distribution of productive inefficiency. While the concavity assumption is very restrictive, we believe that in regards to empirical implementation of the model it is not particularly troublesome. In the empirical work which we reported, we saw that model fit [to the mean and variance of the marginal distributions of ℓn wage rates] was relatively good for one member of this class of distributions. Results obtained using one other distribution from this class [the exponential] suggested that model fit was approximately identical. Apparently, the more serious empirical issue is one of identification of the "true" distribution within the class of concave distribution functions.

While a large proportion of employee-firm separations are reported to be "involuntary," the vast majority of moves are not so characterized by employees. A major deficiency of the model presented here is the exclusion of"quits." If voluntary moves are made for exogenous reasons, little modification of the above model is required if prospective employers are able to correctly distinguish between applicants on the basis of the reason for their separation from their previous employer. If this is not the case, the situation would be much more like the one described in Greenwald (1986). All movers would be treated equivalently, and dismissed individuals would receive a subsidy by being mixed in with those not dismissed. This would in turn reduce the incentive to supply effort on the job. Empirical evidence suggesting a significant difference in post-separation wages by reason for separation supports the contention that potential employers do not treat all applicants in an identical manner.

We believe that our empirical results have suggested that dismissals do have relatively long-term effects on subsequent wage realizations, and that the behavioral model put forth in the paper is capable of parsimoniously summarizing wage and dismissal processes. Our future work along this line will attempt to incorporate voluntary quits into this general framework, so as to provide a more complete story of turnover and wage growth over the life cycle.

APPENDIX

Proofs of Selected Propositions and Results

Proof of Proposition 1:

From [6], an individual employed in the primary sector in period t will choose to work if

$$-\xi + \beta\pi V_{t+1}(\xi) \geq 0$$

$$\Rightarrow \quad V_{t+1}(\xi) \geq \xi/(\beta\pi).$$

The function $V_{t+1}(\xi)$ is decreasing in ξ. To see this, write

$$V_{t+1}(\xi) = w_{t+1} - \delta_{t+1}(\xi)\xi + \sum_{s=t+2}^{\infty} \mathcal{P}^s(\delta_{t+1}(\xi),\ldots,\delta_{s-1}(\xi))\beta^{s-(t+1)}(w_s - \delta_s(\xi)\xi),$$

where $\delta_s(\xi)$ denotes the optimal decision made by a type ξ individual in the primary sector at time s, with $\delta_s(\xi) = 1$ if the type ξ individual works in period s and $\delta_s(\xi) = 0$ otherwise, and $\mathcal{P}^s$ denotes the probability of "surviving" in sector one from period t+1 through period s given the sequence of optimal decisions $\delta_{t+1}(\xi),\ldots,\delta_{s-1}(\xi)$. Because the wage sequence is bounded, $V_k(\xi)$ is finite for all pairs (k,ξ).

For a type $\xi' < \xi$ individual facing the same wage sequence,

$$V_{t+1}(\xi') \geq w_{t+1} - \delta_{t+1}(\xi)\xi' + \sum_{s=t+2}^{\infty} \mathcal{P}^s(\delta_{t+1}(\xi),\ldots,\delta_{s-1}(\xi))\beta^{s-(t+1)}(w_s - \delta_s(\xi)\xi')$$

$$\geq V_{t+1}(\xi),$$

with the inequality being strict if $\max(\delta_{t+1}(\xi),\delta_{t+2}(\xi),\ldots) = 1$, that is, if the type ξ individual would ever choose to work in periods t+1, t+2, ... Since an individual will work in period t if and only if $V_{t+1}(\xi) - \xi/(\beta\pi) \geq 0$, and since $V_{t+1}(0) > 0$, $V_{t+1}(\infty) - \infty = -\infty$, and $V_{t+1}(\xi) - \xi/(\beta\pi)$ is a continuous, strictly decreasing function of ξ, there exists a unique value ξ_t such that $V_{t+1}(\xi_t)-\xi_t/(\beta\pi) = 0$. Then the decision rule has the critical value property in all periods.

Now consider the form of the critical value function when the wage sequence is increasing, so $w_1 \le w_2 \le \ldots$. For a type ξ individual,

$$\begin{aligned} V_{t+1}(\xi) &= w_{t+1} - \delta_{t+1}(\xi)\xi \\ &\quad + \sum_{s=t+2}^{\infty} \mathcal{P}^s(\delta_{t+1}(\xi),\ldots,\delta_{s-1}(\xi))\beta^{s-(t+1)}(w_s-\delta_s(\xi)\xi) \\ &\le w_{t+2} - \delta_{t+1}(\xi)\xi \\ &\quad + \sum_{s=t+2}^{\infty} \mathcal{P}^s(\delta_{t+1}(\xi),\ldots,\delta_{s-1}(\xi))\beta^{s-(t+1)}(w_{s+1}-\delta_s(\xi)\xi) \\ &\le V_{t+2}(\xi). \end{aligned}$$

Thus

$$V_{t+2}(\xi_t) \ge V_{t+1}(\xi_t) = \xi_t/(\beta\pi)$$

$$\Rightarrow \quad \xi_{t+1} \ge \xi_t \quad \text{for all } t.$$

Since an increasing wage sequence implies an increasing critical value sequence, it immediately follows that any individual who prefers or is indifferent between working and shirking in period t will never prefer shirking in any period $t' > t$. Thus the "marginal" worker in period t, namely, that individual with a value of productive inefficiency equal to ξ_t will never shirk in the future. Then

$$\begin{aligned} V_{t+1}(\xi_t) &= \sum_{s=t+1}^{\infty} \beta^{s-(t+1)}(w_s-\xi_t) \\ &= Q_{t+1} - \xi_t/(1-\beta), \end{aligned}$$

so that

$$Q_{t+1} - \xi_t/(1-\beta) - \xi_t/(\beta\pi) = 0$$

or

$$\xi_t = MQ_{t+1}. \qquad \square$$

Proof of Result 2:

Consider

$$H_2(y) = (1-\pi\bar{H}(\xi_1))^{-1}\{\chi[y\leq\xi_1]H(y) + \chi[y>\xi_1][(1-\pi)H(y)+\pi H(\xi_1)]\},$$

where $\bar{H}(x) \equiv 1-H(x)$, and compare the value of this expression with $H(y)$. For $y \leq c$, we have

$$H_2(y) = (1-\pi\bar{H}(\xi_1))^{-1}H(y) > H(y) \qquad \text{for } \pi > 0 \text{ and } \xi_1 > 0.$$

For $y > \xi_1$,

$$[H_2(y)-H(y)|y>\xi_1] = (1-\pi)H(y)+\pi H(\xi_1)-(1-\pi\bar{H}(\xi_1))H(y)$$

$$= \pi H(\xi_1)[1-H(y)]$$

$$\geq 0,$$

and so $H_2 <_{SD} H$.

By induction,

$$H_t <_{SD} H_{t-1}, \quad t = 2,3,\ldots .$$

Transitivity of the first order stochastic dominance operator yields the desired result. $\square$

Proof of Proposition 2:

By R1 every equilibrium increasing wage sequence has a limit point in the half-open interval $(0,\rho]$. This implies that the associated critical value sequence $\{\xi_s(\{w^*_\tau\}_{\tau=s+1}^{\infty})\}$ has a limit point $\bar{\xi}(\{w^*_s\}) \in (0,\rho M/(1-\beta)]$. Using [10], we have that any increasing wage equilibrium must have a limit point

$$\begin{aligned}\ell im_{t\to\infty} w^*_t &= \ell im_{t\to\infty} \rho\, H_t(\xi_t(\{w^*_s\}_{s=t+1}^{\infty})) \\ &= \rho - \rho\, \ell im_{t\to\infty} \bar{H}(\xi_t(\{w^*_s\}_{s=t+1}^{\infty}))\ \ell im_{t\to\infty} A_t(\{w^*_s\}), \\ &= \rho - \rho\, \bar{H}(\ell im_{t\to\infty} \xi_t(\{w^*_s\}_{s=t+1}^{\infty})) \times 0 \\ &= \rho. \end{aligned}$$

□

Proof of Proposition 3:

By P2 all equilibrium wage sequences in the class of increasing sequences taking values in the interval $(0,\rho]$ have limit point ρ. In the class of increasing sequences, the mapping between wage sequences and critical value sequences is one-to-one, so equilibrium can be equivalently characterized in terms of either.

Using P1, we can establish the recursion

[A.1] $$\xi_t = \beta\xi_{t+1} + Mw_{t+1}, \quad t = 1,2,\ldots\ .$$

By the definition of competitive equilibrium in period t+1,

[A.2] $$w_{t+1} = \rho H_{t+1}(\xi_{t+1};\xi_1,\ldots,\xi_t),$$

where we explicitly represent the dependence of the distribution function at time t+1 on previous critical values $\xi_1,\ldots,\xi_t$ for clarity. Then in equilibrium

[A.3] $$\xi_t = \beta\xi_{t+1} + \rho M H_{t+1}(\xi_{t+1};\xi_1,\ldots,\xi_t),$$

and we see that in equilibrium the critical effort levels are [implicitly] recursively determined.

Consider the determination of ξ_1 given an arbitrary value of $\xi_2 = a > 0$.

We have

[A.4] $$\xi_1 = \beta a + \rho M H_2(a;\xi_1),$$

where any solution ξ_1 is constrained to lie in the interval $(0,a]$. At $\xi_1 = 0$, RHS[A.4] is equal to $\beta a + \rho M H(a) > 0$, and when $\xi_1 = a$ RHS[A.4] is equal to $\beta a + \rho M H(a)/(1-\pi\tilde{H}(a))$. RHS[A.4] is concave in ξ_1 on $(0,a]$. Thus if $a \geq \beta a + \rho M H(a)/(1-\pi\tilde{H}(a))$, there exists an unique solution $\tilde{\xi}_1(a)$ which satisfies the requirements of the equilibrium. Let the set of all a for which solutions exist be denoted $\mathcal{A}_2$. Since $\rho M H(a)/(1-\pi\tilde{H}(a))$ is concave in a, this set is connected and is of the form $\mathcal{A}_2 = [a_2,\infty)$, where a_2 is defined by $0 = (1-\beta)a_2 - \rho M H(a_2)/(1-\pi\tilde{H}(a_2))$. For $a \in \mathcal{A}_2$, consider the partial derivative

[A.5a] $$\frac{\partial\tilde{\xi}_1}{\partial a} = \frac{\beta + \rho M h_2(a;\xi_1)}{1-\rho M\{\partial H_2(a;\xi_1)/\partial\xi_1\}} > 0$$

at any solution to [A.4]. Also,

[A.5b] $$\frac{\partial^2\tilde{\xi}_1}{\partial a^2} = \frac{\rho M\{\partial h_2(a;\xi_1)/\partial a\}}{D} + \left\{\frac{\rho M[\beta+\rho M h_2(a;\xi_1)]}{D^2}\right\}\frac{\partial^2 H_2(a;\xi_1)}{\partial\xi_1\partial a} < 0 ,$$

where D refers to the denominator in RHS[A.5a]. Thus on $\mathcal{A}_2$, $\tilde{\xi}_1(a)$ is a concave function.

Now consider the determination of ξ_1 and ξ_2 given $\xi_3 = a > 0$. We have

[A.6] $$\xi_2 = \beta a + \rho M H(a;\tilde{\xi}_1(\xi_2),\xi_2),$$

where we define the function $\tilde{\xi}_1(\xi_2)$ as equal to the non-zero solution to [A.4] if $\xi_2 \in \mathcal{A}_2$ and as equal to 0 if $\xi_2 \notin \mathcal{A}_2$. We look for solutions of [A.6] for which $\xi_2 \in [0,a]$. At $\xi_2 = 0$ [$\Rightarrow \xi_1 = 0$], RHS[A.6] is equal to $\beta a + \rho M H(a)$. At $\xi_2 = a$, RHS[A.6] is equal to $\beta a + \rho M H(a;\tilde{\xi}_1(a),a)$, which is a concave function of a. On the interval $(0,a]$,

[A.7a] $$\frac{\partial \mathrm{RHS}[A.6]}{\partial\xi_2} = \rho M\left\{\frac{\partial H_3(a;\xi_1,\xi_2)}{\partial\xi_2} + \frac{\partial H_3(a;\xi_1,\xi_2)}{\partial\xi_1}\frac{\partial\tilde{\xi}_1}{\partial\xi_2}\right\} > 0$$

and

$$\text{[A.7b]} \qquad \frac{\partial^2 \text{RHS[A.6]}}{\partial \xi_2^2} = \rho M \Bigg\{ \frac{\partial^2 H_3(a;\xi_1,\xi_2)}{\partial \xi_2^2} + \frac{\partial^2 H_3(a;\xi_1,\xi_2)}{\partial \xi_1^2} \left[\frac{\partial \tilde{\xi}_1}{\partial \xi_2}\right]^2$$

$$+ 2 \frac{\partial^2 H_3(a;\xi_1,\xi_2)}{\partial \xi_1 \partial \xi_2} \frac{\partial \tilde{\xi}_1}{\partial \xi_2} + \frac{\partial H_3(a;\xi_1,\xi_2)}{\partial \xi_1} \frac{\partial^2 \tilde{\xi}_1}{\partial \xi_2^2} \Bigg\} < 0,$$

so that RHS[A.6] is a concave function of ξ_2.

Then there exist unique solutions $\tilde{\xi}_1(\tilde{\xi}_2(a))$ and $\tilde{\xi}_2(a)$ with $\tilde{\xi}_2(a) > 0$ for all a such that $a \geq \beta a + \rho MH(a;\tilde{\xi}_1(a),a)$. Denote this connected set by $\mathcal{A}_3 = [a_3,\infty)$. To see that $a \in \mathcal{A}_3 \Rightarrow \tilde{\xi}_1(\tilde{\xi}_2(a)) > 0$, consider a value $\hat{a} \in \mathcal{A}_3$ such that $\tilde{\xi}_1(\tilde{\xi}_2(\hat{a})) = 0$. Then a_3 would have to be such that the following equation yielded a solution for ξ_2:

$$\text{[A.8a]} \qquad \xi_2 = \beta a + \rho MH_3(a;0,\xi_2)$$

At $\hat{a}_3$, the solution to [A.8a] would be $\tilde{\xi}_2(\hat{a}_3) = \hat{a}_3$. From [10], observe that $H_3(a;0,q) = H_2(a;q)$. Then

$$\text{[A.8b]} \qquad \xi_1 = \beta a + \rho MH_2(a;\xi_1)$$

yields a nonzero solution for ξ_1 for $a \geq \hat{a}_3$ ($= a_2$ in this case), which is a contradiction. Then all $a \in \mathcal{A}_3$ produce non-zero unique solutions for ξ_1 and ξ_2 with $\xi_1(\xi_2(a)) \leq \xi_2(a)$, and $a_3 \geq a_2$.

In period t we write

$$\text{[A.9]} \qquad \xi_t = \beta a + \rho MH_{t+1}(a;\xi_1(\xi_t),\ldots,\xi_{t-1}(\xi_t),\xi_t)$$

to determine ξ_t and by implication the entire path of critical values from period 1 through t. By extension of the argument given above, RHS[A.9] is a concave function of ξ_t. Then if $a \in \mathcal{A}_{t+1}$, there exists unique non-zero solutions for all $\xi_1,\ldots,\xi_t$.

Let $\bar{\mathcal{A}} = \lim_{t\to\infty} \mathcal{A}_t = [\lim_{t\to\infty} a_t,\infty)$. Now $a_\infty = \lim_{t\to\infty} a_t = \rho M/(1-\beta)$, so that for any $a \geq a_\infty$ there exists an unique increasing wage sequence with all elements positive. By P2, any nonzero equilibrium wage sequence must converge to ρ in the limit, which implies that the associated critical value sequence must converge to the limit point $\rho M/(1-\beta)$, which is equal to α_∞. Thus there exists an unique equilibrium critical value sequence [which implies an unique wage sequence] in the class of increasing and bounded sequences for all $\rho > 0$, $\beta \in (0,1)$, $\pi \in (0,1]$, and H satisfying A5. □

Proof of Proposition 4:

Consider two distinct distribution functions H and H′ both of which satisfy A5, and let $H <_{SD} H'$. Then for given values of ρ, β, and π, we have that $T(\{w_s\};H) > T(\{w_s\};H')$ for all increasing and bounded wage sequences $\{w_s\}$. By P3, $\{w_s^*(H)\} \geq \{w_s^*(H')\}$, with the inequality being strict for all finite values of s. This implies that $\{\xi_s^*(H)\} \geq \{\xi_s^*(H')\}$.

Now consider a group of dismissed employees. From the viewpoint of primary sector employers, the "best" such group in terms of the distribution of ξ is the one consisting of employees dismissed after their first period in the labor market. Denote the distribution of ξ among this group of individuals by $H_d(H,\xi_1^*)$; the support of this distribution is (ξ_1^*,∞). Since H is first-order stochastically dominated by $H_d(H,\xi_1^*)$, if there exists a termination contract which can be offered to this group of individuals and which earns nonnegative profits it must be the case that the associated equilibrium wage sequence $\{w_s^*(H_d)\} \leq \{w_s^*(H)\}$, which implies that $\xi_1^*(H_d) < \xi_1^*(H)$. Since the set of employees dismissed after one period who have a value of ξ less than $\xi_1^*(H)$ has measure zero, a termination contract which earns nonnegative profits cannot be offered to this set of previously dismissed employees or any other. □

APPENDIX B

Computation of the Equilibrium

In this appendix we describe the computation of the Nash equilibrium termination contract described in the text. The algorithm described here is used in the computation of the maximum likelihood estimates of the behavioral and measurement parameters which completely characterize the distribution of the data.

Computation of the equilibrium wage sequence will engender two types of approximation error. The first source of approximation error is generated by replacing the operator T with the operator T_S in which all rows after row S are replaced with the value ρ. Since agents are assumed to be infinitely-lived and since an equilibrium wage of ρ is only attained asymptotically, it is clear that some approximation error is induced by implicitly setting the probability of shirking equal to 0 at times S+1,S+2,... . The second source of approximation error is more standard, and arises because we only iterate on the operator T_S a finite number of times rather than the infinite number strictly required to define an exact fixed point for this monotone operator.

To compute the finite S approximation to the infinite-horizon equilibrium, our strategy will be as follows. First, define an equilibrium wage sequence for the problem in which the free-entry [zero-profit] assumption is satisfied for periods 1 through S, and after which wages are set to ρ. We establish that for each $S \geq 1$, there exists an unique fixed point for this operator. We show that the fixed point of T_S converges to the fixed point of T uniformly as S gets indefinitely large.

Because it follows directly from the proof of P3, we simply state the following:

Result B.1: Define the map

$$[B.1] \qquad T_S((w_s)) \equiv \begin{bmatrix} \rho\ H(\xi_1((w_s)_{s=2}^{\infty}) \\ \vdots \\ \rho\ H_S(\xi_S((w_s)_{s=S+1}^{\infty}) \\ \rho \\ \rho \\ \vdots \end{bmatrix}$$

on the space of inceasing wage sequences on $(0,\rho]$. T_S has an unique fixed point.

Of course, the fixed point of T_S will have the property that all sequence elements beginning with S+1 are equal to ρ.

For any given value of ρ, β, π, and H, let (w_s^*) denote the fixed point associated with T and let $(w_s^*(S))$ denote the fixed point associated with T_S.

Proposition B.1: $\lim_{S\to\infty} d_\infty((w_s^*(S)),(w_s^*)) = 0.$

Proof: For any (w_s) in the class of increasing sequences on $(0,\rho]$,

$$\left[T_S - T\right]((w_s)) = \begin{bmatrix} \rho\ [H(\xi_1((w_s)_{s=2}^{\infty})) - H(\xi_1((w_s)_{s=2}^{\infty}))] \\ \vdots \\ \rho\ [H_S(\xi_S((w_s)_{s=S+1}^{\infty})) - H_S(\xi_S((w_s)_{s=S+1}^{\infty}))] \\ \rho\ [1 - H_{S+1}(\xi_{S+1}((w_s)_{s=S+2}^{\infty}))] \\ \rho\ [1 - H_{S+2}(\xi_{S+2}((w_s)_{s=S+3}^{\infty}))] \\ \vdots \end{bmatrix}.$$

Since the operators T_S and T coincide for the first S elements of the wage sequence and since

$$\sup_{q>S} \rho \left|1 - H_q(\xi_q(\{w_s\}_{s=q+1}^{\infty}))\right|$$

$$= \rho \; [1 - H_{S+1}(\xi_{S+1}(\{w_s\}_{s=S+2}^{\infty}))],$$

then

$$d_\infty(T_S(\{w_s\}),T(\{w_s\}) = \rho \; [1 - H_{S+1}(\xi_{S+1}(\{w_s\}_{s=S+2}^{\infty}))] \; .$$

From [10],

$$\lim_{S\to\infty} H_{S+1}(\xi_{S+1}(\{w_t\})) = 1$$

for all bounded increasing sequences, so

$$\lim_{S\to\infty} d_\infty(T_S(\{w_s^*\}),T(\{w_s^*\})) = 0$$

$$\Rightarrow \lim_{S\to\infty} d_\infty(\{w_s^*\} - T_S(\{w_s^*\}),\{w_s^*\} - T(\{w_s^*\})) = 0$$

$$\Rightarrow \lim_{S\to\infty} d_\infty(\{w_s^*\} - T_S(\{w_s^*\}),0) = 0$$

$$\Rightarrow \lim_{S\to\infty} d_\infty(\{w_s^*(S)\},\{w_s^*\}) = 0. \qquad \square$$

For computational purposes, we make use of the triangle inequality to determine whether or not a given value of S produces an equilibrium $\{w_s^*(S)\}$ "sufficiently close" to the equilibrium of the infinite horizon problem $\{w_s^*\}$. Now $\{w_s^*(S)\}$ converges uniformly to $\{w_s^*\}$, so for any $\epsilon/2 > 0$ there exists an $S(\epsilon/2)$ such that $d_\infty(\{w_s^*(S(\epsilon/2))\},\{w_s^*\}) < \epsilon/2$. By the triangle inequality,

$$d_\infty(\{w_s^*(S(\epsilon/2)+1)\},\{w_s^*(S(\epsilon/2))\})$$

$$\leq d_\infty(\{w_s^*(S(\epsilon/2)+1)\},\{w_s^*\}) + d_\infty(\{w_s^*(S(\epsilon/2))\},\{w_s^*\})$$

$$\leq \epsilon \; .$$

Then S is "sufficiently large" if $\sup_t \left|w_t^*(S) - w_t^*(S+1)\right| \leq \epsilon$.

The second approximation issue concerns the computation of the equilibrium wage sequence for any given value of S. Using the monotone operator T_S, we solve for the equilibrium wage sequence by iterating on the initial wage vector ρ. By the same argument as that given above regarding the selection of S, we have that

$$d_\infty(\{w_s^{k+1}(S)\},\{w_s^k(S)\}) \leq d_\infty(\{w_s^{k+1}(S)\},\{w_s^*(S)\}) + d_\infty(\{w_s^k(S)\},\langle w_s^*(S)\}),$$

where $\{w_s^r(S)\} = T_S^{r-1}(\{\rho\})$, $r = 2,3,\ldots$,

and $\{\rho\}$ denotes an infinite dimensional sequence with all elements equal to ρ. Because the sequence $\{w_s^r(S)\}$ is uniformly convergent with limit point $\{w_s^*(S)\}$, for any $\nu/2 > 0$ there exists some value $K(\nu/2)$ such that

$$\begin{aligned} & d_\infty(\{w_s^{K(\nu/2)+1}\},\{w_s^{K(\nu/2)}\}) \\ & \quad \leq d_\infty(\{w_s^{K(\nu/2)+1}\},\{w_s^*(S)\}) + d_\infty(\{w_s^{K(\nu/2)}\},\{w_s^*(S)\rangle) \\ & \quad \leq \nu . \end{aligned}$$

Operationally, we stop iteration on T_S after iteration K when $d_\infty(\{w_s^{K+1}\},\{w_s^K\}) \leq \nu$, where ν is some positive constant.

There are two sources of approximation error; the bound on the total approximation error is simply the sum of the upper bounds on the individual sources of error. Thus for a given $(\epsilon/2,\nu/2)$ pair, using a $S(\epsilon/2)$ approximation to the infinite horizon problem and iterating on the T_S operator $K(\nu/2)$ times yields a sequence of primary sector wages having the property that the absolute value of the difference between each element and its corresponding element in the infinite-horizon equilibrium sequence is no greater than $(\epsilon + \nu)/2$.

To recapitulate, we compute the (approximate) equilibrium wage and dismissal profile using the following procedure:

A. Choose positive constants $\epsilon/2$ and $\nu/2$.

B. Set S, beginning with S = 1. Using the operator T_S, iterate until [13] is satisfied, beginning with K = 2. Denote the value of the wage sequence at the final iteration by $\{w_s^*(S)\}$.

$\mathcal{C}$. *Repeat step $\mathcal{B}$ for S+1.*

$\mathcal{D}$. *Compute* $M_S = d_\infty(\{w_s^*(S+1)\},\{w_s^*(S)\})$. *If* $M_S \le \epsilon + \nu$, *the (approximate) equilibrium wage sequence for the infinite horizon problem is* $\{w_s^*(S+1)\}$. *If* $M_S > \epsilon + \nu$, *repeat the operation beginning with step $\mathcal{B}$ using S+1.*

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