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# An Optimal Constitution in a Stochastic Environment

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## ABSTRACT

This paper analyzes how political stability depends on economic factors. Fluctuations in groups' economic capacities and in their abilities to engage in rent-seeking or predatory behavior create periodic incentives for those groups to renege on their social obligations. A constitution remains in force so long as no party wishes to defect to the noncooperative situation, and it is reinstated as soon as each party finds it to its advantage to revert to cooperation. Partnerships of equals are easier to sustain than are arrangements in which one party is more powerful in some economic or noneconomic trait. In this sense, inequality is bad for social welfare. Surprisingly, perhaps, it is the rich, and not the poor segments of society who in our model pose the greater threat to the stability of the social order.

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## 1. Introduction

While the idea that political stability and the social order may depend on economic factors is not new, recent empirical findings and historical developments seem to have stimulated a renewed increased interest in this topic (e.g. Alesina and Rodrick 1991). Moreover, recent events in Europe also point to the influence of economic performance on the stability of the political and social order. While some countries, mainly for economic reasons, try to integrate into a bigger political and economic unit (e.g. entrance in the EEC), others instead look to secede from existing blocks (e.g. some Russian and Yugoslav republics).

We shall look at the stability of the constitutional order from this perspective. That is, we will examine the extent to which constitutional stability depends on fluctuations in the economic performance of the parties originally engaged in the constitutional contract. We adopt a contractarian view of the emergence of constitutions and social order. Following Buchanan (1975) we assume that starting from some initial non-cooperative situation, the constitutional contract emerges as a Pareto move for rational utility maximizing parties. The initial pre-constitutional stage may be an anarchic or warfare setting in which case the constitutional contract essentially involves a disarmament agreement; gains from trade are then made possible to all parties through the elimination of socially wasteful outlays on defense and predation. Alternatively we may envisage the initial non-cooperative situation as one of autarky in which trade barriers exist between the parties (say, two countries), or, alternatively, a noncooperative situation in which resources are wasted on rent-seeking behavior. In this case, the constitutional contract involves the elimination of these barriers or wasteful expenditures, and integration into a

larger unit, and efficiency gains stem from the abolition of protectionism or rent-seeking.

Clearly, the post-contract social order and the associated distribution of rights over goods will survive only so long as it represents a Pareto improvement over the noncooperative or autarkic situation. However, in a dynamic setting in which the post-contract economic situation of the parties is subject to shocks due to, say, output fluctuations, this Pareto superiority may be transitory. That is, some party might occasionally be better-off going it alone. It would then consider renegeing on its contractual obligations and, unless the contractual settlement is renegotiated or the costs of regime change are prohibitive, to revert to autarky. In any case, one would expect an increase in social and political instability under those circumstances.

This paper addresses the choice of an optimal constitution that defines an assignment of individual rights over goods when the stochastic effects that bear on the evolution of output are taken into account. The model also allows for the influence of shocks to outcomes that would alternatively prevail under noncooperation.

Our main finding is that a partnership of equals is easier to sustain than one whose members differ significantly in their market skills or in their ability to engage in rent-seeking or even predatory behavior. This is because inequality creates a greater temptation for parties to renege on their contractual obligations. In this respect our conclusions agree with recent theoretical work by Benhabib and Rustichini (1991) and by Persson and Tabellini (1991), but the reasons are quite different. In those two papers, it is the poorer social segments that pose the greater threat to social order by voting higher taxes or by direct expropriation from the rich. Moreover, neither paper deals with income

fluctuations as a possible source of political instability. In our paper, on the other hand, it is the rich groups who may want to go it alone or more generally to find some nonconstitutional means (peaceful or otherwise) to prevent their income from being redistributed to the poor.

## 2. A Diagrammatic Illustration

We start with a diagrammatic illustration of the constitutional problem we intend to discuss in a two-person setting. Figure 1 depicts two utility possibility frontiers. The inner locus is the possibility frontier under noncooperation while the outer locus is the frontier attainable through cooperation. Let  $N$  be some initial allocation of utilities that reflects the noncooperative access to goods and the effort spent in getting them. The two individuals may differ not only in their productive efficiencies, but also in their rent-seeking abilities and even their aggressiveness or physical strength, all of which will influence where on the inner locus  $N$  will be located. Point  $N$  therefore reflects the "natural distribution" which will arise under noncooperation.<sup>1</sup>

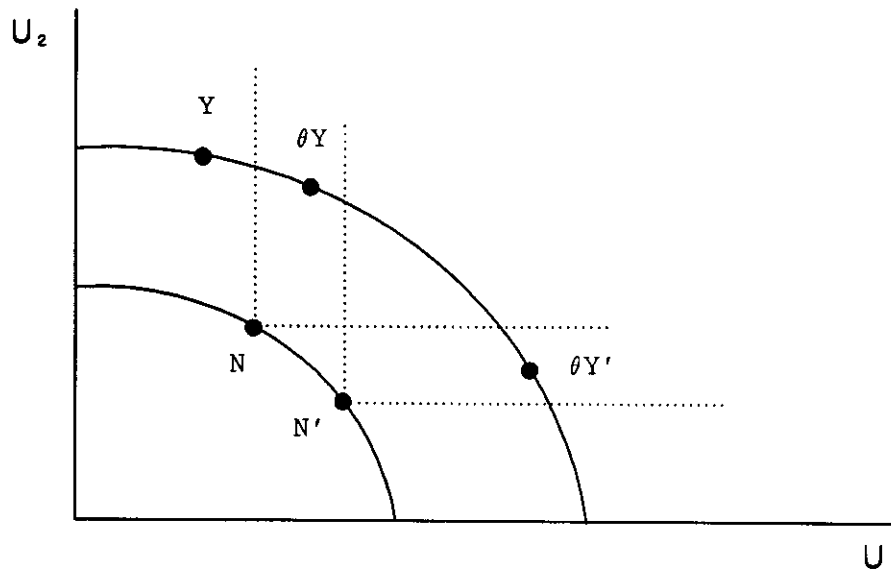


Figure 1: The "Natural Distribution" and the "Direct Production Position".

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<sup>1</sup> For a more complete characterization of this concept see Bush (1972) and Buchanan (1975), and for a treatment that emphasizes players' aggressiveness, see Johansen (1982).

From this initial equilibrium both parties may find that they would be better-off if an agreement were reached. The outer locus describes the opportunity set under cooperation. On this locus, point Y represents the "direct production" position, i.e., the position attained when each individual keeps all that he produces.<sup>2</sup> As illustrated in the diagram, this position need not fall within the region that Pareto dominates the point N. For instance, individual 1 may be a relatively inefficient producer under peace but has a comparative advantage over individual 2 because of superior bargaining power or superior rent-seeking abilities that he can use under noncooperation. If such a situation occurs, the constitutional agreement, if it is to emerge at all, must be accompanied by a transfer of goods from 2 to 1. In other words, the constitutional contract, understood here as a mutually agreed on assignment of property rights over goods, must result in a redistribution of goods. Point  $\theta Y$  indicates a possible post constitutional situation once this redistribution has taken place. Both individuals recognize in  $\theta Y$  that they are better off under the constitutional order.

Assume now that we abandon the static framework above and allow for the passage of time. Suppose, first, that the "natural distribution", N, upon which the constitutional agreement was settled evolves dynamically. Specifically, assume that the individuals' rent-seeking capacities that determine their relative efficiency under noncooperation, change through time in response to some shock. Point N' illustrates a new underlying "natural distribution" reflecting a relative deterioration in individual 2's rent-seeking capacities. The previous constitutional order position,  $\theta Y$ , is now no longer Pareto superior to the underlying "natural distribution". Unless there is a reassignment of

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<sup>2</sup> We again borrow the terminology of Buchanan (1975).

property rights over goods, individual 1 has an incentive to defect and secure point  $N'$  where he is better-off.

Returning to the initial constitutional agreement, consider next the case where there the change is not in the "natural distribution", but in the "direct production" position. That is, suppose that the output obtainable by each individual under cooperation is subject to exogenous shocks. As output fluctuates through time in response to these shocks, the new post-constitutional position may fall outside the Pareto superior region corresponding to point  $N$ . This possibility is illustrated by the point  $\theta Y'$  in the diagram, where we have assumed that the output of individual 2 has experienced a substantial adverse shock. Again we have a situation here where incentives exist (this time on the part of individual 2) to renegotiate the original constitutional contract. In sum, the two types of shock can cause the post-constitutional situation to move outside the region that Pareto dominates the underlying "natural distribution". In such circumstances, as the latter no longer gives support to the agreed-upon distribution of property rights over goods, one would expect an increased instability in the social order.

Figure 1 also makes it clear that regardless of the position of  $N$  and  $Y$ , the region that is Pareto superior to the point  $N$  is never empty, so that a sufficiently flexible redistributive mechanism would always be able to induce everyone to cooperate by giving them an allocation on the outer curve. That is, a social contract that stipulates a redistributive policy that is optimal under each possible contingency would in our model ensure continuous social order.

It is, of course, quite impossible for the social contract to envisage every contingency, just as it is impossible for private contracts to do so, as they would if markets were complete. This is why societies have judicial systems



that interpret the law. But even there, the outcome will often be controversial and imperfect. As Hart (1961, p. 125) argues in the legal literature:

"It is a feature of the human predicament (and so the legislative one) that we labor under two connected handicaps whenever we seek to regulate, unambiguously and in advance, some sphere of conduct by means of general standards to be used without further official direction on particular occasions. The first handicap is our relative ignorance of fact; the second is our relative indeterminacy of aim. If the world in which we live were characterized only by a finite number of features, and these together with all the modes in which they could combine were known to us, then provision could be made in advance for every possibility. We could make rules, the application of which to particular cases never called for a further choice. Everything could be known, and for everything, since it could be known, something could be done and specified in advance by a rule. This would be a world fit 'mechanical' jurisprudence."

A similar view has been put forth in the economics literature; when a decision maker can not calculate the optimal policy for a difficult problem, he will, it is argued, use a simpler rule. As Heiner (1983, p. 585) puts it,

"... agents cannot decipher all of the complexity of the decision problems they face, which literally prevents them from selecting the most preferred alternative. Consequently, the flexibility of behavior to react to information is constrained to smaller behavioral repertoires than can be reasonably administered. Numerous deviations from the resulting behavior patterns are actually superior in certain situations, but they are still ignored because of uncertainty about when to deviate from these regularities."

It thus seems reasonable to assume that the constitutional framers can not foresee every contingency, or even to set up a rule that will best deal with each unforeseen contingency. And so, it may be appropriate to view a constitution as a collection of rules that some group or other may periodically decide to break.<sup>3</sup> This is how we shall look at it in this paper. The next section

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<sup>3</sup> The Civil War, the result of the South's cession from the Union is the only U.S. instance of such a phenomenon. But it is now manifesting itself in many other places.

focuses on the choice of the optimal redistribution to be effected at the constitutional stage under this stochastic environment. Specifically, it analyzes the optimization problem of the constitutional decision-makers who take into account the effect of both types of shocks and who attempt to minimize the instability of the social order.

### 3. The Model

Consider an environment in which individuals do not need to spend resources on warfare or on rent-seeking activities, so that they can keep all they produce. Let  $Y_1$  and  $Y_2$  be the output accruing to individual 1 and 2 under these circumstances, circumstances that we call the direct production position. We assume that output evolves as follows:<sup>4</sup>

$$Y_i = a_i + \lambda_i Y_{i,-1} + v_i \quad 0 < \lambda_i < 1, \quad i = 1, 2. \quad (1)$$

For both individuals output varies through time in response to shocks  $v_i \sim N(0, \sigma_i^2)$  and exhibits persistence or serial correlation through the parameters  $\lambda_i$ . The shocks  $v_i$  are themselves assumed to be mutually and serially uncorrelated.

Under war the output vector  $(Y_1, Y_2)$  is not obtainable as both individuals spend resources in defense and predation. Let  $C_1$  and  $C_2$  denote the cost of the war effort to individuals 1 and 2. We assume that

$$C_1 = C - \epsilon, \quad \text{and} \quad C_2 = C + \epsilon, \quad (2)$$

where  $C > 0$  is a constant, and  $\epsilon$  represents the individuals' relative efficiency in rent-seeking under noncooperation. If  $\epsilon > 0$ , individual 1 is

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<sup>4</sup> Time subscripts are eliminated whenever there is no ambiguity. The treatment will be abstract, so the identity of the parties is not made explicit - they might be different ethnic groups, or they might be the government, unions, trade associations, the military, and so on.

better (i.e. has a lower cost) at this activity than individual 2 is.<sup>5</sup> Over time,  $\epsilon$  evolves as follows:

$$\epsilon = \rho\epsilon_{-1} + u, \quad 0 \leq \rho < 1, \quad (3)$$

where the white noise shock  $u$  follows  $u \sim N(\bar{u}, \sigma_u^2)$ . Some inertia therefore enters the evolution of these attributes. Note that if the mean value  $\bar{u}$  differs from zero, say  $\bar{u} > 0$ , individual 1 has a long-run advantage over individual 2. We assume that  $|\bar{u}/(1-\rho)| < C$  to assure that in the long-run noncooperation is costly to both individuals.

In view of (2) the net noncooperative output, termed the "natural distribution", is

$$Z_1 = Y_1 - C_1, \quad \text{and} \quad Z_2 = Y_2 - C_2. \quad (4)$$

This natural distribution is the starting basis upon which individuals evaluate the potential benefits of a constitutional agreement.

Once in place, the constitutional contract introduces social order into the community as a result of the mutual recognition and acceptance by both individuals of certain property rights on goods produced. We thus assume that, under constitutional order, individuals 1 and 2 get

$$Z_1 = \theta(Y_1 + Y_2), \quad \text{and} \quad Z_2 = (1 - \theta)(Y_1 + Y_2). \quad (5)$$

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<sup>5</sup> One may think of  $C_1 + C_2$  as the "peace dividend".

The redistributive parameter  $\theta$  assigns a fixed share of total output to each individual. Its introduction is needed for two reasons. First, the "direct production" situation described by (1.1) and (1.2) may not Pareto dominate the "natural distribution". This possibility, which we illustrated in Figure 1, may result if for instance one individual, say 1, is relatively very efficient as a rent-seeker so that  $\epsilon > C$ .<sup>6</sup> Second, even if the "direct production" is Pareto superior to the "natural distribution" the parameter  $\theta$  allows for choosing among the many alternative Pareto efficient positions.

We can now formulate the optimization problem faced by the constitutional framers. A Buchanan-Rawls perspective of constitutional design suggests that a representative constitutional decision-maker would choose  $\theta$  optimally without knowing which particular position he will occupy in the social setting, i.e., without knowing whether he will be individual 1 or individual 2. Under this "veil of ignorance"<sup>7</sup> and assuming constant marginal utility of income,<sup>8</sup> the constitutional designer will choose  $\theta$  so as to maximize expected total long-run output  $E(Z_1 + Z_2)$ , where

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<sup>6</sup> This possibility is explicitly discussed by Buchanan (1975): "Some 'redistribution' of goods or endowments may have to take place before a sufficiently acceptable base for property claims can be established... Once any of these transfers takes place, if one is required, and/or behavioral limits are mutually accepted, positive rights of persons in stocks of goods or in resource endowments capable of producing goods may be settled" (p. 64).

<sup>7</sup> For a discussion of the "veil of ignorance" assumption and related concepts see Rawls (1971), Buchanan and Tullock (1962), Buchanan (1967), Harsanyi (1955). The assumption of constant marginal utility is introduced here to rule out some demand for redistribution which, for independent reasons, would typically emerge at the constitutional level when utility functions are concave. See Strotz (1958) and Samuelson (1964).

<sup>8</sup> Although the constitutional arrangement will act to pool the income risk, these risk-neutral agents do not value this function. In spite of this, the constitutional will favor those agents with riskier income streams to offset these agents' greater propensity to defect.

$$Z_1 + Z_2 = \begin{cases} Y_1 + Y_2 & \text{under cooperation} \\ Y_1 + Y_2 - 2C & \text{under noncooperation,} \end{cases}$$

and where the expectation is with respect to the steady-state long-run distribution of  $Y_1$  and  $Y_2$ . But this is the same as solving the problem

$$\min_{\theta} \{ 2C \text{ Prob}(\text{noncooperation}) \}.$$

The constitutional designer will therefore wish to minimize the probability of noncooperation or equivalently to maximize the stability of the political and social order.

To analyze the influence of  $\theta$  on social stability, note that neither party has an incentive to violate the constitution if the following condition holds:

$$Y_1 - C_1 < \theta(Y_1 + Y_2) \quad \text{and} \quad Y_2 - C_2 < (1 - \theta)(Y_1 + Y_2). \quad (6)$$

Let  $X(\theta) \equiv Y_1 - \theta(Y_1 + Y_2)$ . Then (6) can be expressed as  $-C_2 < X(\theta) < C_1$ , or, taking note of (2), as

$$-C < X(\theta) + \epsilon < C \quad (7)$$

When it is positive,  $X(\theta)$  is the redistributive tax individual 1 pays out of his income  $Y_1$ , so as to secure the constitutional outcome. This tax should be

compared with the cost to him of not cooperating, namely  $C_1 = C - \epsilon$ , as given by (2). For the second individual,  $X > 0$  implies a redistributive subsidy under constitutional order over his income  $Y_2$ .

The incentives of the two groups are best explained pictorially. Figure 2 illustrates the three regions implied by eq. (7). It is clear that it is the rich that will tend to want to defect. Holding  $\epsilon$  constant, the richer a group is, or the poorer the other group is, the more likely the group is to defect from the constitutional agreement.<sup>9</sup> The problem that faces the constitutional framer is to choose  $\theta$  so as to maximize the weight that the stationary distribution of the vector  $(Y_1, Y_2, \epsilon)$  assigns to the shaded area. Doing so will maximize the expected value of output in the long run.

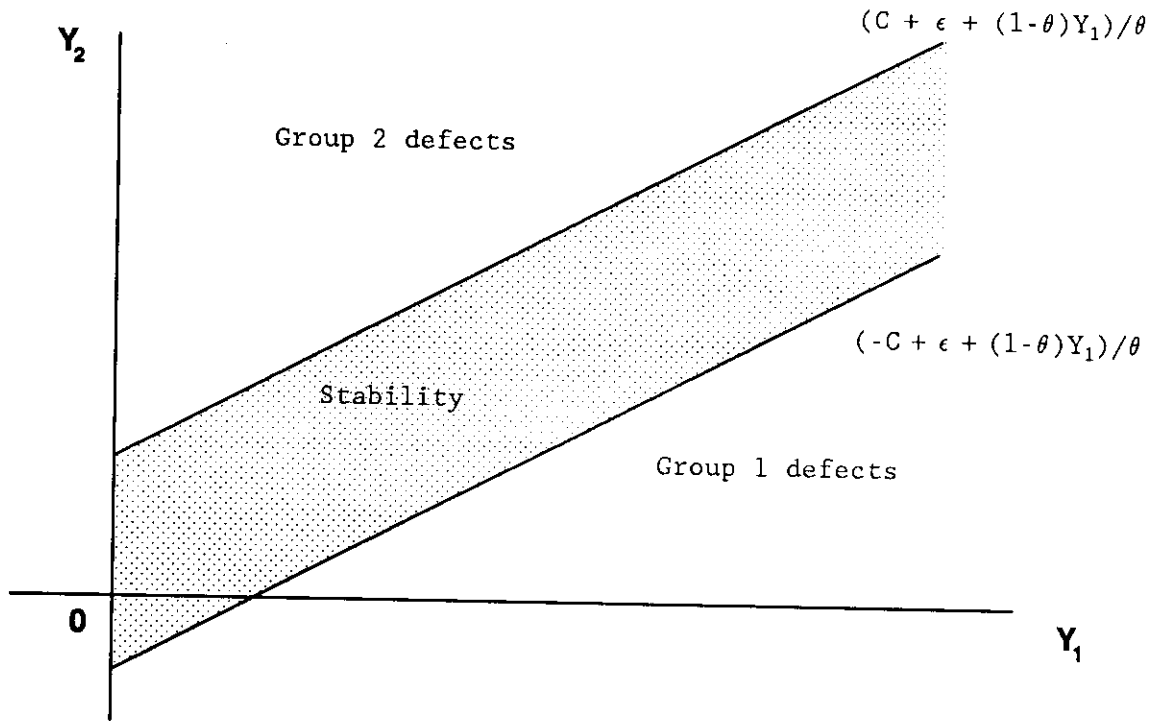


Figure 2: Incentives to Defect as a Function of Income.

<sup>9</sup> The figure becomes symmetric around zero if  $\epsilon = 0$  and if  $\theta = 1/2$ , with the two parallel lines taking on a  $45^\circ$  slope.

The constitutional framer could do better if he could index  $\theta$  on past realizations of the  $Y_i$  and  $C_i$ . Furthermore, he could eliminate defections altogether if he could index  $\theta$  on the current  $Y_i$  and  $C_i$ . Constitutions can be amended and this happens in practice, albeit rarely. Such "midstream" adjustments are costly, and determining their nature would just complicate the analysis without changing its main result, except in the case where constitutional laws can react even to current information. To simplify the analysis, then, we shall focus on the case where the constitutional adjustment costs are infinite, and where the constitutional rules are quite simple -- a fixed sharing rule for total output. We think that the arguments we advanced at the end of the previous section serve to make this special case an interesting one.<sup>10</sup>

The constitutional problem, then, is to choose  $\theta$  to maximize the steady-state probability that the random redistributive tax (subsidy) stays within the limits described by (7). Letting  $w(\theta) \equiv X(\theta) + \epsilon$ , the problem is to maximize  $P \equiv \text{Prob} \{ -C < w(\theta) < C \}$ . The stationary distribution  $w(\theta)$  is normal with mean

$$m(\theta) \equiv \frac{(1-\theta)a_1}{1-\lambda_1} - \frac{\theta a_2}{1-\lambda_2} + \frac{\bar{u}}{1-\rho},$$

and variance

(8)

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<sup>10</sup> Suppose, moreover, that the planner can index  $\theta$  on everything but the current realizations. Then in the iid case ( $\lambda_i = \rho = 0$ ), our solution for  $\theta$  is the same as the reactive solution because the conditional distribution of  $(Y_1, Y_2, \epsilon)$  next period is the same as the stationary distribution of this vector. Thus our solution for  $\theta$  diverges substantially from the reactive one only if the  $\lambda_i$  or  $\rho$  are large.



$$s^2(\theta) \equiv \frac{(1-\theta)^2\sigma_1^2}{1-\lambda_1^2} + \frac{\theta^2\sigma_2^2}{1-\lambda_2^2} + \frac{\sigma_\epsilon^2}{1-\rho^2} .$$

As an expositional device we shall first confront the constitutional framer with two simpler problems. Consider first the influence of  $\theta$  on the mean only. On this sole account, and in view of the symmetry of the limits in (7),  $\theta$  should be set so as to make  $m(\theta) = 0$ . That value is

$$\theta_0 \equiv \frac{a_1/(1-\lambda_1) + \bar{u}/(1-\rho)}{a_1/(1-\lambda_1) + a_2/(1-\lambda_2)} . \quad (9)$$

Since the steady-state "direct production" position of individual  $i$  is  $a_i/(1-\lambda_i)$ , we see that  $\theta_0$  assigns to individual 1 a share in total income which is, except for the term  $\bar{u}/(1-\rho)$ , redistributively neutral. In fact, individual 1 would keep all that he can privately produce in the steady-state under peace. This "direct production" distribution is, however, adjusted by the term  $\bar{u}/(1-\rho)$ . If,  $\bar{u} > 0$ , this term represents a redistributive premium added to individual 1's share so as to compensate him for his noncooperative rent-seeking superiority -- his greater threat-power, so to speak. Thus, the long-run determinants of the underlying "natural distribution" should therefore surface at the constitutional contract level.

Next, consider the influence of  $\theta$  only on the variability of the redistributive tax. According to (7), if we take account of the influence of  $\theta$  solely on  $s^2(\theta)$ , its optimal value should be set so as to reduce  $s^2(\theta)$  to a minimum. That value is

$$\bar{\theta} = \frac{\sigma_1^2/(1-\lambda_1^2)}{\sigma_2^2/(1-\lambda_2^2) + \sigma_1^2/(1-\lambda_1^2)}.$$

We observed in note 6 that, in a sense, the constitutional contract involves a 'pooling of risks' via the associated redistributive function. Suppose that individual 1's output is subject to greater fluctuations so that its steady-state variance is greater than that of individual 2.<sup>11</sup> In this case, individual 1 should, other things equal, receive a comparatively higher share of post-constitutional income (pay a smaller redistributive tax). Political stability therefore requires that risky productive activities should command a premium in the division of post-constitutional social income.

We now turn to the problem that the constitutional framer actually solves: choose  $\theta$  to maximize

$$P = F \left( \frac{c - m(\theta)}{s(\theta)} \right) - F \left( \frac{-c - m(\theta)}{s(\theta)} \right),$$

where  $F$  is the standard normal distribution. This expression is homogeneous of degree zero in  $c$ ,  $m$  and  $s$ , from which it follows that if the  $Y_i$  and  $C_i$  were to be scaled up by some multiple, the optimal  $\theta$  would be unchanged as it should be. The first-order condition is

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<sup>11</sup> Since the steady-state variance of output of individual  $i$  is  $\sigma_i^2/(1-\lambda_i^2)$ , this may come either from a greater variance of impulse shocks ( $\sigma_1 > \sigma_2$ ) and/or from a greater inertia or persistence in its propagation mechanism ( $\lambda_1 > \lambda_2$ ).

$$\left[ F' \left( \frac{-c-m}{s} \right) - F' \left( \frac{c-m}{s} \right) \right] \frac{m'}{s} - \left[ F' \left( \frac{c-m}{s} \right) \left( \frac{c-m}{s^2} \right) + F' \left( \frac{-c-m}{s} \right) \left( \frac{c+m}{s^2} \right) \right] s' = 0. \quad (10)$$

The effects of  $\theta$  on the mean and the variance are in the first and the second term, respectively. As Part 1 of the Appendix shows, the first term is the derivative of a concave function of  $\theta$  which has a zero at  $\theta_0$ ; the second term is the derivative of a concave function with a zero at  $\bar{\theta}$ . This implies that the solution to (10),  $\theta^*$ , is between  $\theta_0$  and  $\bar{\theta}$ :

$$\theta_0 \leq \theta^* \leq \bar{\theta}, \quad \text{if } \theta_0 \leq \bar{\theta}, \quad \text{and} \quad \bar{\theta} \leq \theta^* \leq \theta_0, \quad \text{if } \bar{\theta} \leq \theta_0. \quad (11)$$

We now state some propositions, some of which are proved in the Appendix. The first two help to establish some intuition about the nature of the optimal solution.

Proposition 1: If  $a_1 = a_2 = 0$ , then  $\theta^* = \bar{\theta}$ .

In this case  $m(\theta)$  does not depend on  $\theta$ , so that the only relevant consideration in choosing  $\theta$  should therefore be cyclical in character. Specifically, it should minimize the variability of the tax, and this occurs at  $\bar{\theta}$ .

Proposition 2: If  $\sigma_1^2 = \sigma_2^2 = 0$ , then  $\theta^* = \theta_0$ .

This is the case where output under peace is deterministic, and  $\theta$  can not affect  $s^2(\theta)$ . Only its influence on  $m(\theta)$  matters, and here the optimal solution is  $\theta_0$ .

The next several propositions deal with how the stability of the social order depends on exogenous circumstances. The first result concerns an exogenous decrease in both agents' rent-seeking ability, i.e., a rise in  $C$ . The result is an immediate application of the envelope theorem:

Proposition 3: 
$$\partial P / \partial C = \frac{1}{s} \left[ F' \left( \frac{c-m}{s} \right) + F' \left( \frac{-c-m}{s} \right) \right] > 0 .$$

This is what one would have expected since a rise in  $C$  represents a reduction in the opportunity cost of social order for both individuals.

Fluctuations in the  $Y_i$  and in the  $C_i$  are the cause of instability, and increases in the amplitude of their fluctuations would be expected to destabilize the social order more frequently. This is indeed true:

Proposition 4: 
$$\partial P / \partial \sigma_i < 0 \quad \text{for } i = 1, 2, \epsilon .$$

Technically this result follows because  $P$  is decreasing in  $s(\theta)$  and the latter is an increasing function of the  $\sigma_i$ . A higher  $\sigma_1$  or  $\sigma_2$  imply greater inequality in incomes in the steady state distribution, and also imply greater cyclical instability in incomes. Thus we have established a type of causality that runs from income inequality and cyclical income instability on the one hand to political instability on the other.

The next proposition deals with the effects that an increase in inequality between the average incomes of the two parties will have on the stability of the social order. The results will show, quite consistently, that increases in

inequality are destabilizing. Put differently, a social mechanism that tries to induce cooperation by promising shares ( $\theta$  and  $1-\theta$ ) of total output to the participants will find it harder to do so when those participants are unequal. The possible dimensions of inequality are several, and we shall go through them case by case.

Proposition 5: If  $\theta_0 > \bar{\theta}$ ,  $\partial P/\partial a_1 < 0$  and  $\partial P/\partial a_2 > 0$ .  
 If  $\theta_0 < \bar{\theta}$ ,  $\partial P/\partial a_1 > 0$  and  $\partial P/\partial a_2 < 0$ .

This result is easiest to interpret when  $\sigma_1 = \sigma_2$ ,  $\lambda_1 = \lambda_2$  (so that  $\bar{\theta} = 1/2$ , and so that the two agents' incomes are equally noisy), and  $\bar{u} = 0$ . Then if at the constitutional agreement stage individual 1 is in a relatively stronger position in terms of productivity ( $a_1 > a_2$ ) so that  $\theta_0 > \bar{\theta}$ , then a strengthening of this superiority is destabilizing. In other words, if the groups' income risk and their predatory strength are equal, then income inequality is destabilizing. A similar result applies to changes in a group's predatory strength:

Proposition 6: If  $\theta_0 > \bar{\theta}$ ,  $\partial P/\partial \bar{u} < 0$ . If  $\theta_0 < \bar{\theta}$ ,  $\partial P/\partial \bar{u} > 0$ .

If he already benefits from a relatively stronger position in terms of productivity, an increase in an individuals' rent-seeking ability is destabilizing. The intuition is the same as that of Proposition 5.

When it comes to the effect on stability of changes in the persistence parameters  $\lambda_1$ ,  $\lambda_2$ , or  $\rho$ , two effects are at work. First, an increase in each parameter raises the variance of  $w$  and therefore destabilizes the social order. Second, however, each parameter also affects the mean of steady-state output.

Here the situation is similar to the one analyzed in Proposition 5 and 6. If  $\theta_0 > \bar{\theta}$ , so that individual 1 initially enjoys a superiority of sorts, an increase in  $\lambda_1$  or  $\rho$  or a reduction in  $\lambda_2$ , as it further strengthens his steady-state relative claims on output, represents a destabilizing factor. If  $\theta_0 < \bar{\theta}$ , the converse applies. Sometimes these two effects work in opposite directions, but the following cases produce unambiguous results:

Proposition 7:     If  $\theta_0 > \bar{\theta}$ ,    $\partial P/\partial \lambda_1 < 0$    and  $\partial P/\partial \rho < 0$ .  
                           If  $\theta_0 < \bar{\theta}$ ,    $\partial P/\partial \lambda_2 < 0$ .

All these comparative statics results apply when parameters are varied at the time (or prior to the time) that the constitution is being designed. But the effects that these parameter changes have on  $P$ , as described in the last five propositions, apply in a broader sense as well: the partial derivatives of  $P$  with respect to the parameters were all calculated by holding  $\theta$  fixed -- the envelope theorem allows one to do that at the optimal  $\theta$ . These propositions therefore do tell us the effect that parameter changes would have on the stability of the social order, even if such parameter changes were to occur after the constitution has been designed.

#### 4. Summary and Conclusions.

In this contractarian framework, constitutional order emerges as a Pareto improvement over some alternative state where individuals waste resources. When shocks are present, Pareto dominance of the constitutional order becomes an uncertain outcome. Under these circumstances constitutional framers, acting under a Rawlsian veil of ignorance, will attempt to maximize post-constitutional political stability or, equivalently, to minimize the probability of noncooperation. We have shown that the optimal constitution chosen in this context should reflect long-run attributes of the underlying natural distribution of social income. These deeper parameters should surface at the constitutional level.

We also noted that the constitutional contract typically creates, through its redistributive function, an interdependence between individual pay-offs. In a stochastic framework this interdependence affects the variability of these pay-offs. Account should therefore be taken of this influence on the probability of war.

Having determined the nature of the optimal constitution, we examined how political stability would depend on exogenous events. An increase in the cost of noncooperation or a decrease in the variability of shocks affecting productivity are stabilizing. At the same time, events which strengthen an already existing relative advantage of one individual's income position over the other, reduce stability. Events which increase the persistence or the inertia of the evolutionary processes governing productivity or rent-seeking attributes, are detrimental to stability if they impinge on the individual who already enjoys a relative advantage.

The message of this paper is that if constitutions are constrained to

provide various groups with fixed shares of the total economic pie, they are more likely to survive if the groups in question are roughly similar in their economic and noneconomic capacities. In this respect our message is the same as that of Benhabib and Rustichini (1991) and Persson and Tabellini (1991) who, for very different reasons, also find that inequality is detrimental to the welfare of the society at large.



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Appendix

Part 1: This part substantiates the claims made following equation (10) of the text.

From (11), let  $H(\theta) = \left[ F' \left( \frac{-c-m(\theta)}{s(\theta)} \right) - F' \left( \frac{c-m(\theta)}{s(\theta)} \right) \right] \frac{m'(\theta)}{s(\theta)}$ . To show that  $H$

is the derivative of a concave function in  $\theta$ , note that  $m'(\theta) = -\frac{a_1}{1-\lambda_1} - \frac{a_2}{1-\lambda_2} < 0$ . And since  $c > 0$ ,

$$F' \left( \frac{-c-m}{s} \right) \geq F' \left( \frac{c-m}{s} \right) \quad \text{as } m \geq 0. \quad (1A)$$

Therefore,  $H \geq 0$  as  $m \geq 0$  or, from (13), (9) and since  $m' < 0$ , we have  $H \geq 0$  as  $\theta \geq 0$ . Thus, the function  $H(\theta)$  crosses the abscissa from above which implies that it is the derivative of a concave function in  $\theta$ . Next, let

$$J(\theta) = \left[ F' \left( \frac{c-m(\theta)}{s(\theta)} \right) \left( \frac{c-m(\theta)}{s^2(\theta)} \right) + F' \left( \frac{-c-m(\theta)}{s(\theta)} \right) \left( \frac{c+m(\theta)}{s^2(\theta)} \right) \right] s'(\theta).$$

The term in square brackets is positive since  $-c \leq m \leq c$ . On the other hand,  $s' = \partial s / \partial \theta = (1/s) \left[ \theta \sigma_2^2 / (1-\lambda_2^2) - (1-\theta) \sigma_1^2 / (1-\lambda_1^2) \right]$ , so that,  $s' \geq 0$  as  $\theta \geq \bar{\theta}$ .

Therefore  $J \geq 0$  as  $\theta \geq \bar{\theta}$ , which implies that  $J(\theta)$  is the derivative of a concave function on  $\theta$ .

Part 2: This part proves propositions 5-7.

Proposition 5:

$$\partial P / \partial a_i = (1/s) \left[ -F' \left( \frac{c-m}{s} \right) + F' \left( \frac{-c-m}{s} \right) \right] (\partial m / \partial a_i) \quad i = 1, 2 \quad (2A)$$

where  $\partial m / \partial a_1 = (1 - \theta) / (1 - \lambda_1)$ , and  $\partial m / \partial a_2 = (-\theta) / (1 - \lambda_2)$ . Now using (1A), the term in square brackets in (2A) is negative iff  $m > 0$ , i.e., iff  $\theta^* < \theta_0$ , and therefore, in view of (12), iff  $\bar{\theta} < \theta_0$ .

Proposition 6:  $\partial P/\partial \bar{u} = (1/s) \left[ -F \left( \frac{c-m}{s} \right) + F' \left( \frac{-c-m}{s} \right) \right] (\partial m/\partial \bar{u})$ .

But,  $\partial m/\partial \bar{u} = (1 - \rho)^{-1}$ . Therefore, again using (1A) and (12),  $\partial P/\partial \bar{u} > 0$  iff  $\bar{\theta} > \theta_0$ . Otherwise,  $\partial P/\partial \bar{u} < 0$ .

Proposition 7: We have

$$\frac{\partial P}{\partial \lambda_1} = A \frac{(1-\theta)a_1}{(1-\lambda_1)^2} + B \frac{(1-\theta)^2 \sigma_1^2 2\lambda_1}{(1-\lambda_1^2)^2},$$

$$\frac{\partial P}{\partial \lambda_2} = A \frac{(-\theta)a_2}{(1-\lambda_2)^2} + B \frac{\theta^2 \sigma_2^2 2\lambda_2}{(1-\lambda_2^2)^2},$$

and

$$\frac{\partial P}{\partial \rho} = A \frac{\bar{u}}{(1-\rho)^2} + B \frac{2\sigma_\epsilon^2 \rho}{(1-\rho^2)^2},$$

where

$$A = \frac{1}{s} \left[ -F' \left( \frac{c-m}{s} \right) + F' \left( \frac{-c-m}{s} \right) \right], \quad \text{and} \quad B = \left[ -F' \left( \frac{c-m}{s^2} \right) \frac{c-m}{s^2} + F' \left( \frac{-c-m}{s} \right) \left( \frac{-c-m}{s^2} \right) \right].$$

Each derivative consists of two terms. In each case the second term (the effect on the variance) is, in view of the discussion in part 1 of this Appendix, unambiguously negative. For reasons discussed above, the sign of the first term depends, however, on whether  $\bar{\theta} < \theta_0$ , or  $\bar{\theta} > \theta_0$  in which case it is negative or positive, respectively.