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ABSTRACT

The excess burden - measuring the utility loss associated with the use of an indirect tax structure - is often represented as the triangular area beneath the compensated demand curve. Similarly, the "Ramsey Equations," which characterize the indirect tax structure which minimizes the utility loss, are often written using changes in compensated demands. Because utility is constant along a compensated demand curve, the use of compensated demands to measure utility changes appears paradoxical. I interpret the distortion as a fiscal externality, and explain the paradox.

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1. INTRODUCTION

Normative tax theory stresses the economic cost of a tax structure. An indirect tax creates a distortion, because the consumer price ceases to measure the resource cost. In consequence, the use of an indirect tax structure causes the utility of the representative household to fall more than it falls if the same tax revenue is collected using a lump-sum levy. The additional utility fall is measured by the excess burden. It is usually associated with the substitution effect. For example, Stiglitz (1988, pages 439 and 441) writes: "The substitution effect of the tax is sometimes called the *distortion* caused by the tax....If there is no substitution effect, then the tax gives rise to no excess burden." However, the use of the substitution effect to explain the excess burden is puzzling: the excess burden is associated with a utility fall, whereas a substitution effect is associated with a change in the consumption bundle holding utility constant.

As noted by Dupuit (1844) and Marshall (1891), the use of an indirect tax causes the household to change its consumption of the taxed commodity, and the change in quantity causes consumer surplus to fall. The fall in consumer surplus is the excess burden, and is measured by the Harberger (1964) triangle. In the presence of income effects, the excess burden is the triangular area under the compensated demand curve. However, the interpretation is now puzzling: the excess burden is measuring a utility loss, but it is being measured by a function which holds utility constant. The innovation of this paper is to interpet the distortion of the indirect tax as a fiscal externality. The household does not internalize all the potential benefits of additional purchases, which include the tax revenue gain of the government. The surplus loss is in fact potential tax revenue which is lost by the government because quantities are set inefficiently. The substitution effect is relevant because it is the substitution of the taxed commodity by the untaxed commodity which causes tax revenue to fall.

"The Ramsey Equations" show how the induced quantity changes may be used to evaluate the optimality of an indirect tax structure. In the absence of income effects and provided the tax revenue requirement is small, Ramsey (1927) showed that the utility loss of the representative household is minimized when the tax structure causes an equi-proportionate fall in the consumption of all taxed commodities. Because the utility loss is minimized, the reader might be excused for thinking that the Ramsey Equations compare the effect of the induced quantity changes on utility. However, Samuelson (1986) showed that, in the presence of income effects, it is the fall in compensated demands which is important. In this context, the importance of compensated demands is puzzling, as of course compensated changes are made along an indifference curve. My interpretation shows that quantity changes are relevant because of the external (fiscal) effects. If a unit lump-sum tax is substituted for the marginal use of a commodity tax, the induced change in quantities causes indirect tax revenue to rise. The revenue gain enables tax instruments to be compared. I relate the tax revenue gain - holding utility constant - to the utility loss associated with use of the commodity tax to collect marginal tax revenue.

The paper is an interpretation of well-known but puzzling results. Section 2 develops a graphical analysis to stress the fiscal externality, and to relate the substitution effect to the excess burden. The Ramsey Equations are interpreted in Section 3. Section 4 concludes.

2. GRAPHICAL ANALYSIS

Since Dupuit (1844) and Marshall (1891), it has been recognized that the use of an indirect tax - which raises the consumer price above the resource cost - causes a loss of consumer surplus. Use of consumer surplus implicitly assumes that demand is insensitive to income; compensated and uncompensated demands are equal. Figure 1 shows the commodity demand. The production of one unit requires p units of resources - with competitive firms the commodity is elastically supplied at producer price p.

(FIGURE 1 HERE)

With no tax, the representative household buys x^1 units, and achieves consumer surplus equal to A plus G plus B in Figure 1. If an indirect tax is imposed on the commodity, the consumer price rises to q. The higher consumer price causes the household to reduce its demand to x^2 ; the consumer surplus of the household is A. Tax revenue G is raised, or surplus G is transferred to the government. Surplus B is lost -this is "the excess burden" of the tax.

The analysis is summarized by notes.

NOTE 1: *the use of an indirect tax creates a fiscal externality.*

The use of the indirect tax creates a fiscal externality - for each unit of commodity purchased by the individual, the individual gains private benefit *and* the government gains tax revenue (q-p). If the household allocation is increased from x^2 to x^1 (he continues to pay consumer price q for each unit), he achieves consumer surplus A-C, and the tax revenue of the government is G+B+C. The government is able to pay compensation C to the household, and still have resources G+B. The surplus B - lost when demand fell from x^1 to x^2 - is therefore lost because the household does not take account of the external (fiscal) benefit of his purchases.

The externality is removed if the mode of tax collection is shifted from the indirect tax to a lump-sum tax - additional purchases generate no extra tax revenue. Demand is insensitive to income, so the household buys x^1 units. The government lump-sum tax is G: if the household surplus is maintained at A, additional resources B must be withdrawn from the household. The resources B are the (absolute value of the) compensating variation of the change: this is "the benefit" of the change, and defines the excess burden of the indirect tax.[1]

DEFINITION: *the excess burden of an indirect tax structure is the (absolute value of the) compensating variation associated with a change in the mode of tax collection from the indirect tax structure to a lump-sum levy G.*

What happens to the resources B withdrawn from the household? They may be collected by the government - the government imposes a lump-sum levy G+B, and the household achieves surplus A. The "benefit" of the reform accrues to the government: this case is considered in Notes 2 and 3. Alternatively, the resources may be returned to the household - the government maintains the lump-sum levy G and the household achieves surplus A+B. The "benefit" of the reform accrues to the household:[2] this case is considered in Note 4.

If demand is income sensitive, consumer surplus is inappropriate. However, the excess burden is still the possible resource gain of the government if a lump-sum levy replaces the indirect tax (household utility being held constant), and still measures the utility gain if the government revenue is fixed.

(FIGURE 2 HERE)

Figure 2 represents the tax problem, in the general case when demand is a function of income. A representative household purchases a single commodity and supplies labor. The leisure endowment of the household is OH. Leisure is the numeraire resource, and the wage is normalized to unity. The production of the commodity uses only labor, and is competitive. p units of labor are required to produce one unit of commodity, so that the producer price of the commodity is p. Lines of slope p correspond to household allocations (leisure and commodity) which have equal resource content. Indifference curves are drawn and labelled by their associated utility levels, $U^2 < U' < U^1$. If the government has a zero resource requirement, OA is the possibility frontier, and the household achieves utility U^1. The transfer of resources G to the government corresponds to moving the household possibility frontier upwards by G to BC. If the planner is restricted to indirect taxation,[3] he sets the consumer price q so that OD is the budget line (slope q) faced by the household, and the household chooses allocation E^2 on BC. The household achieves utility U^2.

The fiscal externality is shown in Figure 2 by the divergence of the lines OD and BC. If the household buys additional commodity, additional resources are transferred to the government equal to the vertical distance of the allocation above BC. This is an external benefit to the government.

NOTE 2: *by linking a marginal reduction in the indirect tax rate with a lump-sum levy, the government is able to capture part of the surplus lost through the fiscal externality.*

e(Q;U) is the expenditure function, or the endowment required to achieve utility U at consumer price Q, and h(Q; U) is the compensated demand of the commodity. Consider a marginal reduction in the indirect tax from q to q+dq (dq<0). The compensating variation of the change is

$$e(q;\ U^2) - e(q+dq;\ U^2) = -(\partial e/\partial Q)dq = -h(q;\ U^2)\ dq.$$

Utility is unchanged if income is withdrawn equal to the reduction in the price of the pre-existing bundle.

In this sub-section, the benefit of the marginal reform is gained by the government. The reform is therefore accompagnied by a lump-sum levy dT,

$$dT \equiv -\ h(q;\ U^2)\ dq, \tag{1}$$

and the household is moved along its indifference curve from E^2 towards E^3. The change in quantities consumed by the household causes additional tax revenue to be collected through the indirect tax structure. Noting that utility is unchanged, changes in demand are changes in compensated demand h, and the additional indirect tax revenue collected is

$$d(q-p)h = h\ dq + (q-p)\ dh.$$

The first term is the price effect - the lower tax receipts due to the lower tax rate on the pre-existing tax base. The second term is the quantity effect - the higher tax receipts due to the application of the pre-existing tax structure to the expanded tax base. The increase in resources gained by the government (the "benefit" of the reform) is the lump-sum levy plus the increase in the indirect tax revenue,

$$dT + h\ dq + (q-p)\ dh = -\ h\ dq + h\ dq + (q-p)\ dh = (q-p)\ dh. \tag{2}$$

The lump-sum levy is exactly offset by the lower tax receipts on the pre-existing base. The increased resources transferred to the government results from the pre-existing tax wedge being levied on a larger tax base: this is the *external* benefit of the quantity change dh.

Using Equations (1) and (2), and noting $x^2 = h(q; U^2)$, the increased resources gained by the government per unit of lump-sum levy is

$$\frac{(q-p)\ dh}{dT} = \frac{(q-p)\ dh}{-x\ dq} . \qquad (3)$$

This expression is used later. In Figure 1, the lump-sum levy is area abcd and the increase in tax revenue is area cefg. The expression is the initial rate at which potential tax revenue, represented by the excess burden, may be converted into actual tax revenue - holding utility unchanged.

NOTE 3: *the excess burden is the Harberger triangle beneath the compensated demand curve.*

The replacement of the indirect tax by the lump-sum levy may be considered as a series of incremental steps. At each step, the indirect tax is reduced and the lump-sum levy is increased. The household moves a step along its indifference curve from E^2 to E^3, and resources (Q-p)dh are transferred to the government. Summing over the steps, or integrating (2), the resource gain of the government equals the resource loss of the household, or[4]

$$EB = \int_q^p (Q-p)\ dh. \qquad (4)$$

The excess burden EB is still the triangular area B in Figure 1 - where the relevant demand curve is compensated demand. The resources gained by the government are the external (fiscal) benefits of the quantity change at each incremental step. Changes in compensated demand - or substitution effects -

are important, because at each step it is the substitution into the taxed commodity which brings benefit to the government.

NOTE 4: *the excess burden is a measure of the utility loss associated with the use of the indirect tax instrument.*

In Figure 2, the highest utility achievable on BC is U' at E' - the utility loss necessitated by the required transfer of resources G to the government is U^1-U'. At E^2, the household achieves utility U^2. U'-U^2 is therefore the utility loss of the household associated with the use of the indirect tax to effect the resource transfer. How does the excess burden, being a resource change along an indifference curve, measure the utility loss U'-U^2 ?

Normative theory measures utility changes by income changes. If the indirect tax is replaced by a lump-sum levy G, and government revenue is fixed, the "benefit" of the reform is gained by the household.[5] The household moves along the iso-resource line from E^2 to E'. The resources lost by the household on moving from E^2 to E^3 therefore equals the resources required by the household to move from E^3 to E'. Hence

$$EB = e(p;\ U') - e(p;\ U^2).$$

The excess burden measures the utility loss U'-U^2, by being the income loss (at consumer price p) which would cause the utility loss U'-U^2.

3. THE RAMSEY EQUATIONS

The above discussion was confined to a single commodity. I now extend the analysis and consider n commodities which may be taxed; The Ramsey

Problem is to determine the optimal mix of tax rates.[6] Leisure is the numeraire and the wage is normalized to unity. The production of one unit of commodity i requires p_i labor units - the competitive producer price of commodity i is p_i. The government has a resource requirement G, which (by assumption) must be collected using only the indirect taxation of commodities. If the tax rate on commodity i is t_i, the consumer price of commodity i is $q_i = p_i(1+t_i)$. I use vector notation: producer and consumer price vectors are $\mathbf{p} = (p_1, \ldots, p_n)$ and $\mathbf{q} = (q_1, \ldots, q_n)$. Because $\mathbf{p}$ is fixed by the production technology, $\mathbf{q}$ represents the tax structure.

The household has non-labor income M.[7] The household is price-taking: its chooses commodity purchases $\mathbf{x}(\mathbf{q}; M) \equiv (x_1(\mathbf{q}; M), \ldots, x_n(\mathbf{q}; M))$ and labor supply $L(\mathbf{q}; M)$. The concern is about how tax rates should be set, and not about expenditure; the public project, on which the tax revenue is spent, is therefore ignored. The household achieves utility $U(\mathbf{x}, L)$, its indirect utility is $V(\mathbf{q}; M)$, and tax revenue is $R = (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q}; M)$.

"The Ramsey Problem" is to choose the tax structure $\mathbf{q}$ to maximize utility subject to the revenue requirement,[8]

$$\max_{\mathbf{q}} V(\mathbf{q}; M), \qquad \text{s.t.} \quad G \leq (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q}; M).$$

The Lagrangian[9] is $L = V(\mathbf{q}; M) + \lambda[(\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q}; M) - G]$. The typical first order condition is

$$\frac{\partial L}{\partial q_k} = \frac{\partial V}{\partial q_k} + \lambda[\, x_k + \sum_{i=1}^{n}(q_i - p_i)\frac{\partial x_i}{\partial q_k}\,] = 0, \qquad k = (1, \ldots, n).$$

Using Roy's Identity, writing $\alpha \equiv \partial V(\mathbf{q}; M)/\partial M$ as the marginal utility of income, and rearranging,[10]

$$\frac{x_k}{x_k + \sum_{i=1}^{n}(q_i - p_i)\frac{\partial x_i}{\partial q_k}} = \frac{\lambda}{\alpha}, \qquad k = (1, \ldots, n). \quad (6)$$

Writing $h_i(\mathbf{q}; V)$ as the compensated demand of the ith commodity, using the Slutsky Equation, and rearranging, the equation may be rewritten as

$$-\frac{1}{x_k}\sum_{i=1}^{n}(q_i - p_i)\frac{\partial h_i}{\partial q_k} = 1 - \sum_{i=1}^{n}(q_i - p_i)\frac{\partial x_i}{\partial M} - \frac{\alpha}{\lambda}, \qquad k=(1, \ldots, n). \quad (7)$$

The right-hand side is the same for all k. This equation occurs in Diamond and Mirrlees (1971, Equation (37)) and many subsequent authors. It is the modern formulation of the "The Ramsey Equations," and is the equation I interpret.[11]

NOTE 5: *the tax revenue gain, if an unit lump-sum tax is substituted for the marginal use of instrument* q_k, *is a measure of the utility lost if instrument* q_k *is used to collect a marginal unit of tax revenue.*

To elucidate the interpretation, I consider a two stage process. In the first stage, the household is levied an unit lump-sum tax: this lowers its utility, but raises tax revenue. In the second stage, the tax rate q_k is lowered until utility returns to its pre-existing level. The fall in utility at the first stage, and the rise in utility at the second stage, is the same for each instrument q_k; this is the utility change over which tax instruments are compared. If a "large" utility loss occurs when a marginal unit of tax revenue is collected by raising tax rate q_k, there is only a "small" tax revenue loss when tax rate q_k is lowered to raise utility the pre-specified amount. The tax revenue gain at the first stage is the same for all instruments; a "small" tax revenue loss at the second stage implies a "large" revenue gain overall. The overall tax revenue gain is therefore an ordinal measure of the utility loss if instrument q_k is used to collect a marginal unit of tax revenue.

NOTE 6: *the left-hand side of Equation (7) is the tax revenue gain if an unit lump-sum tax is substituted for the marginal use of instrument* q_k.

The lump-sum levy at the first stage is dT and the consumer price of the kth commodity rises dq_k ($dq_k < 0$) at the second stage, to leave utility unchanged. Writing $\mathbf{dq}_k$ as the vector $(0, \ldots, dq_k, \ldots, 0)$, and $e(\mathbf{q}; U)$ as the expenditure function,

$$dT = e(\mathbf{q}; V) - e(\mathbf{q}+\mathbf{dq}_k; V) = - (\partial e/\partial q_k)\, dq_k = - x_k dq_k.$$

Utility is unchanged. Demand changes are therefore changes in compensated demands, and the total change in tax revenue is

$$dR = dT + d[\sum_{i=1}^{n}(q_i - p_i)h_i] = dT + h_k dq_k + \sum_{i=1}^{n}(q_i - p_i)dh_i.$$

But $dT = -x_k dq_k = -h_k dq_k$, or the increase in the lump-sum levy is exactly offset by the lower tax receipts from the lower tax rate on the pre-existing bundle. Hence the total tax revenue increase if an unit lump-sum tax is levied, and instrument q_k adjusted to keep utility unchanged, is

$$\left.\frac{dR}{dT}\right|_V = - \frac{\sum_{i=1}^{n}(q_i - p_i)dh_i}{x_k dq_k}.$$

This is the many-commodity analogue of Expression (3) and interprets the left-hand side of Equation (7). The tax revenue gain is the external (fiscal) benefit of the induced quantity changes.

NOTE 7: *the right-hand side of Equation (7) is the most tax revenue gain if an unit lump-sum tax is substituted for the marginal use of all tax instruments.*

To interpret the right-hand side of Equation (7), the two stages are repeated, but with *all* instruments being adjusted in the second stage to minimize the revenue loss. The increase in the lump-sum tax dT causes tax revenue to increase by the amount of the lump-sum levy less the associated decrease in indirect tax revenue due to the decrease in demands,

$$dT - \sum_{i=1}^{n}(q_i - p_i)\frac{\partial x_i}{\partial M}\, dT.$$

The lump-sum tax in the first stage causes the utility of the household to fall αdT. The Lagrangian multiplier measures the change in the objective as the constraint is tightened, $\partial U/\partial G = -\lambda$; hence the least revenue loss in the second stage - associated with the rise in utility αdT - is $\alpha dT/\lambda$. The right-hand side of Equation (7) is therefore the most tax revenue gain if an unit lump-sum levy is imposed and all tax instruments are adjusted to leave utility unchanged.[12] The first term is the lump-sum tax, the second term is the fall in indirect tax revenue due to the effect of the lump-sum tax on demand, and the third term is the tax revenue lost by the adjustment of all tax instruments.

NOTE 8: *Equation (7) is interpreted: the same tax revenue is gained if an unit lump-sum tax is substituted for the marginal use of each instrument q_k, or for the marginal use of all tax instruments.*

The left-hand side of Equation (7) is the tax revenue gain if a unit lump-sum tax is imposed and instrument q_k is lowered to leave utility unchanged. The right-hand side is the least tax revenue lost if a unit lump-sum transfer is made, and *all* tax instruments are adjusted to leave utility unchanged. The combined change substitutes the collection of tax revenue from instrument q_k to all other instruments: it does not impose a lump-sum levy and is feasible even if the planner is (by assumption) unable to levy a lump-sum tax. Any tax revenue gain is the external (fiscal) benefit of the induced quantity changes. Equation (7) is interpreted: at the optimum tax structure, it is impossible to increase tax revenue by marginally substituting from the use of any tax instrument to others (to leave utility unchanged).[13]

Note 5 explains that the tax revenue gain, if an unit lump-sum tax is substituted for the marginal use of instrument q_k, is an ordinal measure of the utility loss associated with marginal tax collection using instrument q_k. Equation (7) therefore confirms that the use of each tax instrument q_k to collect a marginal unit of tax revenue gives the same utility fall, which equals the least utility fall associated with the use of all tax instruments. This is, of course, exactly the first-order condition expected for The Ramsey Problem.

5. CONCLUSION

In the Introduction, I noted that the use of changes in compensated demand to measure changes in utility is puzzling, as compensated demand changes are made along an indifference curve. This paper interprets the distortion of an indirect tax as a fiscal externality. If the household is moved along its indifference curve from an inefficient to an efficient allocation, the fiscal benefit of the induced quantity change is the excess burden: it is a resource gain of the government. If the government revenue requirement is fixed, these resources are retained by the household, raising its utility. Similarly, the left-hand side of the Ramsey Equations measures the external (fiscal) benefit of the associated quantity changes, if an unit lump-sum tax is marginally substituted for an indirect tax. The associated revenue gain enables tax instruments to be compared. The equations show that tax revenue cannot be increased by the marginal substitution of one tax instrument for others (holding utility unchanged). They also show that the use of each instrument to collect a marginal unit of tax revenue gives the same utility loss.

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FIGURES

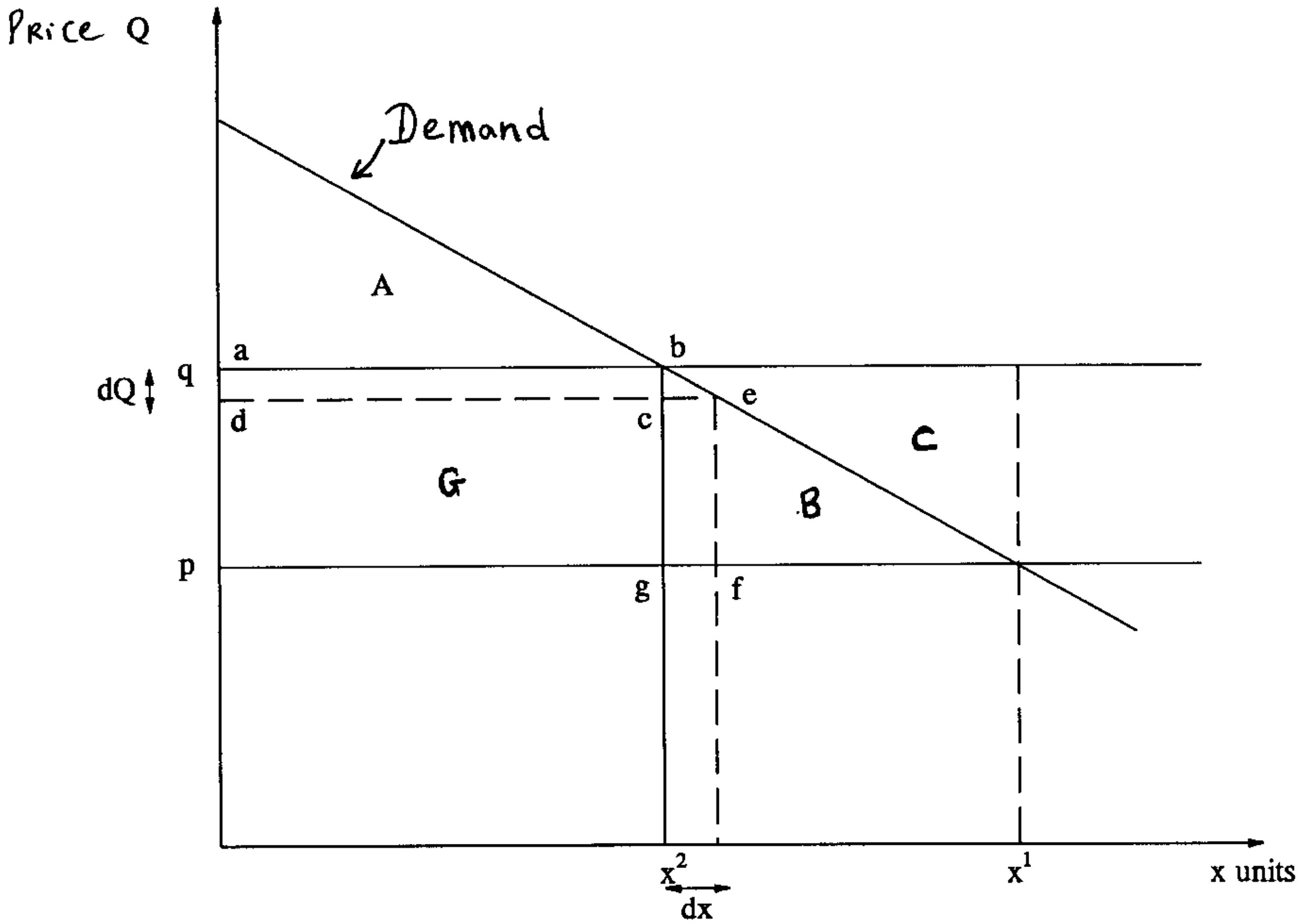

Figure 1: Consumer surplus analysis.

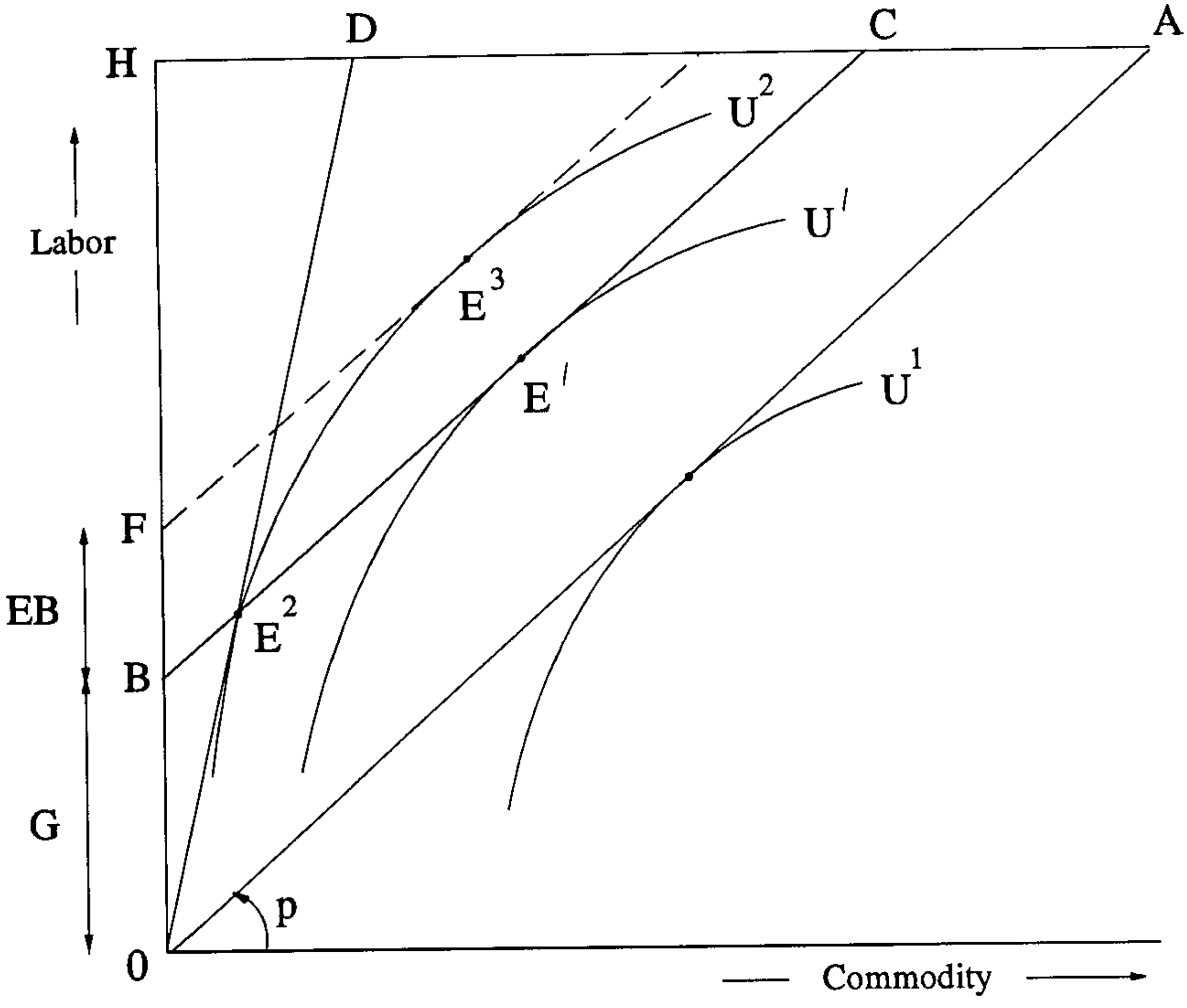

Figure 2: The tax problem.

FOOTNOTES

[1] The change is beneficial and for the remainder of the paper the compensating variation is defined by its absolute value. I.e. If V(Q; M) is the indirect utility of the household at consumer prices Q and with endowment M, the compensating variation of the change from the indirect tax structure q to the lump-sum levy G is defined to be the benefit B: $V(q;\ M) \equiv V(p;\ (M-G)-B)$. In benefit-cost analysis, the "benefit" of a change in government policy is the compensating variation defined similarly.

[2] If any individual household increases his purchases from x^2 to x^1 (at consumer price q), the fiscal gain of the government is returned to *all* households: the benefit is external to the individual household.

[3] If the planner chooses the tax rate t, $q \equiv p(1+t)$.

[4] Integrating Equation (4) by parts gives the more common formulation,

$$EB = \int_p^q h dQ - G.$$

Under the proportional tax, the government collects tax revenue G. If the replacement of the proportional tax by the lump-sum levy is considered as a series of incremental steps, at each step the government collects lump-sum revenue $dT = -hdQ$. Summing over the steps, the final lump-sum levy is $T = \int_0^T dT = -\int_q^p hdQ$. The tax revenue gain of the government is $T-G = \int_p^q hdQ - G$.

[5] This is the analogue of the discussion under Note 1, in which the government revenue was maintained at G, and the household surplus was increased by B.

[6] The analysis is confined to the representative household: the extension to many households is natural.

[7] Evaluation will be made at M=0.

[8] $e(\mathbf{q};\ U)$ is the expenditure function of the household. Using Equation (5)

and noting $e(\mathbf{p};\ U')+G = M$, the excess burden is

$$EB = M - G - e(\mathbf{p};\ V(\mathbf{q};M)).$$

$0 < \partial e/\partial U$. Hence choosing prices $\mathbf{q}$ to minimize the excess burden subject to the tax revenue constraint, $G = (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q};\ M)$, is equivalent to choosing prices $\mathbf{q}$ to maximize the indirect utility function $V(\mathbf{q};\ M)$ subject to the same constraint. This is noted by Diamond and McFadden (1974) and Auerbach (1985).

[9] Mirrlees (1986) discusses when the Lagrangian technique is inappropriate.

[10] Equation (6) is interpreted using the arguments of Dixit (1970), Mayshar (1990), Triest (1990) and Fullerton (1991). If tax instrument q_k is increased to q_k+dq_k, $\mathbf{dq}_k = (0,\ \ldots,\ dq_k,\ \ldots,\ 0)$, the welfare cost is

$$e(\mathbf{q}+\mathbf{dq}_k;\ V(\mathbf{q};\ M)) - e(\mathbf{q};\ V(\mathbf{q};\ M)) = \frac{\partial e(\mathbf{q};\ V)}{\partial q_k}\, dq_k = x_k dq_k.$$

To leave utility unchanged, the household requires an income transfer equal to the increase in the price of the pre-existing consumption bundle. The marginal cost of funds of instrument q_k is the welfare cost per unit of additional revenue raised, or

$$MCF_k = \frac{x_k dq_k}{\dfrac{\partial R}{\partial q_k}\, dq_k} = \frac{x_k}{x_k + \sum_{i=1}^{n}(q_i-p_i)\dfrac{\partial x_i}{\partial q_k}}.$$

This interprets the left-hand side of Equation (6). Many authors estimate the marginal cost of funds for different tax instruments, e.g. Decoster and Schokkaert (1990).

$\alpha = \partial V/\partial M$ and, interpreting the Lagrangian multiplier as the change in the objective as the constraint is tightened, $\lambda = -\partial V/\partial G$. Therefore

$$\frac{\lambda}{\alpha} = -\frac{\partial V/\partial G}{\partial V/\partial M} = \left.\frac{\partial M}{\partial G}\right|_V .$$

λ/α is the least income transfer the household must receive to compensate him for the price changes, which are required for the collection of a marginal unit of tax revenue by the adjustment of *all* tax instruments. This is the marginal cost of funds, MCF, if all tax instruments are simultaneously adjusted. Equation (6) is interpreted: at the optimal tax structure, the marginal cost of funds is the same for each tax instrument, and equals the marginal cost of funds if all tax instruments are optimally adjusted,

$$MCF_1 = MCF_2 = \ldots = MCF.$$

[11] Noting that $\partial h_i/\partial q_k = \partial h_k/\partial q_i$, Equation (7) is often written as

$$-\frac{1}{x_k}\sum_{i=1}^{n}(q_i - p_i)\frac{\partial h_k}{\partial q_i} = 1 - \sum_{i=1}^{n}(q_i - p_i)\frac{\partial x_i}{\partial M} - \frac{\alpha}{\lambda}, \qquad k=(1, \ldots, n).$$

For small price changes Δq_i, the change in the compensated demand of the kth commodity is $\Delta h_k = \sum_{i=1}^{n}(\partial h_k/\partial q_i)\,\Delta q_i$. If the tax rate is small, set $\Delta q_i = q_i - p_i$ and the above equation becomes $\Delta h_k/h_k$ = const.. If an optimal tax structure is imposed, and if compensation is paid, there will be an equal percentage change in all taxed commodities (Samuelson (1986)). With no income effects, $\partial h_k/\partial q_i = \partial x_k/\partial q_i$, the optimal tax structure imposes an equi-proportionate fall in the demand of each taxed commodity (Ramsey (1927)).

[12] This interpretation is made by Atkinson and Stern (1974).

[13] This interpretation is easily understood by noting that Equation (7) is also the first-order condition for the dual problem of maximizing tax revenue subject to a reservation utility: $\max_{\mathbf{q}} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q};\, M)$ subject to $\bar{V} \leq V(\mathbf{q};\, M)$, with $\bar{V}$ suitably chosen. This is noted in Harris and Wildasin (1985).