DISTRIBUTIVE JUSTICE AND OPTIMAL TAXATION OF WAGES AND INTEREST IN A GROWING ECONOMY

by

Janusz A. Ordover

March 1974

No. 74-03

DISTRIBUTIVE JUSTICE AND OPTIMAL TAXATION OF WAGES AND INTEREST IN A GROWING ECONOMY

J. A. Ordover*

October 1973

1. INTRODUCTION

All recent models of optimal redistribution without lump-sum or poll taxes can be divided into two categories: those in which all or part of the revenue from taxation is distributed to households as a transfer and those in which both negative and positive transfers are disallowed. In the first category of models, the fiscal authority is endowed with only one tax.[1] In the second category of models, the fiscal authority is empowered to use an optimal mix of taxes to achieve socially desired redistributions of income.[2]

The main purpose of this paper is to remedy at least one deficiency of the models of the first type: By introducing the second tax-- the tax on the interest income-- into the set of available policy instruments we enable the taxing authority to select both the optimal tax schedule _and_ the optimal mix

---

*New York University. This paper is a revised version of a chapter from my dissertation, _Three Essays in Economic Theory_, Columbia University, 1973. Professor Edmund S. Phelps interested me in the questions of distributive justice. He also offered invaluable help in writing this paper. I would like to thank also Professors Elizabeth E. Bailey and William J. Baumol for helpful comments.

1. J.A. Mirrlees, "An Exploration in the Theory of Optimum Taxation," _Review of Economic Studies_, vol. 38, April 1971, 175-208; E.S. Phelps, "Taxation of Wage Income for Economic Justice,"_Quarterly Journal of Economics_, August 1973, vol. 87, 331-54.

2. P.A. Diamond and J.A. Mirrlees, "Optimal Taxation and Production, _American Economic Review_, vol. 61, March-June 1971, 8-27, 261-278, is the most rigorous example of this type of models; see also W.J. Baumol and D.F. Bradford, "Optimal Departures from Marginal Cost Pricing," _American Economic Review_, vol. 60, June 1970, 265-283.

of taxes. The analytical importance of the extension proposed here can easily be demonstrated. First, as is well known, the arbitrary exclusion of certain policy instruments may lead to otherwise avoidable losses in allocative efficiency.[3] Second, such exclusions imply that the income distribution attained is not the best that can be achieved even without recourse to lump-sum taxes.[4] Of couse we do not argue that from the standpoint of efficiency and equity a proliferation of taxing and pricing schemes is always desirable. Administrative considerations, neglected here, must be viewed as an important constraint on the number of feasible tax systems.[5]

Ultimately we would like to study optimal redistribution in a model in which the set of policy instruments is determined endogenously. Such a truly general approach is far beyond the scope of this paper, however. Consequently we restrict our analysis to an economy which is growing at a steady rate, $\underline{n}$, and in which the government uses proportionate taxes on wage income and on interest income to finance redistributive transfers.[6] The problem as formulated here is then that of the optimal mix of proportionate taxes, taking account both of justice and efficiency considerations.

Our analysis builds on the Rawlsian or "maximin" conception of

---

3.F.H. Hahn, "On Optimum Taxation," Journal of Economic Theory, vol. 6, February 1973, 96-106, contains an extensive discussion of allocative efficiency in models with distortionary taxes.

4.See Part II of P.A. Diamond and J.A. Mirrlees, op. cit.

5.W.P. Heller and K. Shell, "On Optimal Taxation with Costly Administration," Paper presented at the 86th Annual Meeting of the American Economic Association, New York, December 1973.

6.We allow in our analysis for governmental borrowing (or lending) but as will be seen governmental borrowing is not an independent policy instrument. At least not in the model formulated in this paper.

intragenerational justice.[7] According to this criterion of social welfare "...income and wealth...are to be distributed equally <u>unless</u> an unequal distribution...is to the advantage of the least favored."[8] Intragenerational maximin justice therefore requires that subject to all relevant constraints the government maximize the social welfare function, $W(\ ) = \min_{\{m\}} u^m(\ )$, where u^m denotes utility of the <u>mth</u> person.

Intergenerational justice, hence the evolution of the economy, is governed by the just savings principle.[9] We leave for future research the analysis of the optimal maximin growth path. Now we simply assume that the society is already endowed with a just capital/worker ratio, whatever this ratio might be. We then interpret the just savings principle as requiring that the "just" capital/worker ratio be preserved by the current generation for the use of future generations. The savings principle acts, therefore, as a constraint on the use of fiscal instruments for the realization of <u>intra</u>generational justice.

Our interest is in the theory of optimal taxation under capitalism and market (decentralized) socialism. We neglect the important issue of <u>production efficiency of economies</u> with distorting taxes[1] and concentrate on

7 J. Rawls, "A Theory of Justice, Harvard University Press: Cambridge, Massachusetts, 1971. See also E.S. Phelps, <u>op. cit.</u>, pp. 332-37; K.J. Arrow, "Some Ordinalist-Utalitarian Notes on Rawls's <u>Theory of Justice</u>," Discussion Paper No. 287, Harvard University, Cambridge, Massachusetts, 1972.

8 J. Rawls, <u>op. cit.</u>, p. 303.

9 J. Rawls, <u>op. cit.</u>, sections 44,45, 46; see also B. Barry, "John Rawls and the Priority of Liberty," <u>Philosophy and Public Affairs</u>, vol. 2, Spring 1973, 274-290; K.J. Arrow, "Rawls's Principle of Just Saving," IMSS, Technical Report No. 106, Stanford University, September 1973.

1 See for example Part I of P.A. Diamond and J.A. Mirrlees, <u>op. cit.</u>; F.H. Hahn, <u>op. cit.</u>; J. Mirrlees, "On Producer Taxation," <u>Review of Economic Studies</u>, vol. 39, January 1972, 105-111; see also J.E. Stiglitz and P. Dasgupta, "On Optimal Taxation and Public Production, "<u>Review of Economic Studies</u>, vol. 39, January 1972, 87-104.

several other basic questions of tax policy: Should a tax be levied on interest earnings? Should the net rate of return to wealth owners be positive? How are the optimal rates of the wage tax and the interest tax related? Should the government use up taxable capacity with respect to one or both these tax instruments?

The main result of our study is that the optimal rate of the interest tax is quite high. In fact in most circumstances it equals one hundred percent or is very close to it. More specifically we show that in a stationary or a slow-growing economy all interest income should be taxed away. Indeed, in such economies, it might be socially desirable to "expropriate" through taxation at least some portion of the privately held physical capital.[2]

The analysis also contributes to a discussion on the desirability of full utilization of taxable capacity for redistributive purposes.[3] Phelps (_op. cit._) had argued that the maximin social welfare criterion calls for maximization of tax revenue, that is for a full utilization of taxable capacity. Atkinson (_op.cit._) has shown that the result does not hold if the least favored are employed. We show that it does not hold if they save _unless_ the net rate of return on saving is zero, that is unless the rate of the interest tax equals one. If this tax rate is lessthan one, maximization of the maximin social welfare function requires underutilization of taxable capacity with respect to both taxes.[4]

---

2. J.A. Ordover and E.S. Phelps, "Proportionate Steady-State Taxation of Wealth and Wages for Lifetime Economic Justice," Discussion Paper, Columbia University, November 1973.

3. E.S. Phelps, _op. cit._; A.B. Atkinson, "'Maximin' and Optimal Taxation," _Review of Economic Studies_, forthcoming.

4. The Phelps' rule is vindicated in J.A. Ordover and E.S. Phelps, _op. cit._, but only in a Special Case when the poor do not save.

Surprisingly, we have not been able to determine unambiguously whether in an economy with an interest tax the wage tax has a lower rate than an economy without such a tax. That is we have not been able to prove that the two tax rates are always substitutes. If they are then the rates of the wage tax as calculated by other authors should be revised downward.[5]

We present the model in Section II. First we discuss the economic behavior of the household then we study the impact of changes in the tax rates on before tax wages and interest. Section III contains the analysis of the model. Of special interest in Section III is the discussion of a quasi-socialist economy. We define a quasi-socialist economy as an economy with a one hundred percent interest tax. We derive sufficient and necessary conditions for the optimality of quasi-socialism and give an explicit solution for the optimal rate of the wage tax. We show also how the wage tax depends on the rate of growth of the economy.

Finally we investigate the applicability of the Golden Rule to the maximin society. The unmodified Golden Rule theorem states that social welfare is maximized when the rate of interest equals the economy's rate of growth. We show that the theorem does hold in our model: If the society could select its capital stock costlessly it would choose a capital stock such that at the optimal constellation of taxes the gross rate of interest equals the economy's rate of growth. Our result differs from that of Hamada.[6] Hamada showed that in the utilitarian economy the Golden Rule need not hold. Unfortunately it is not obvious to us whether the difference in results derives from differences in the respective models or whether it would disappear if Hamada were to introduce the interest tax into his model.

---

5. See A.B. Atkinson, *op. cit.*, for sample calculations of maximin wage tax schedules.

6. K. Hamada, "Lifetime Equity and Dynamic Efficiency on the Balanced Growth Path," *Journal of Public Economics*, vol. 1, November 1972, 379-96.

II. THE MODEL

The building blocks for our model come from the earlier literature on optimal taxation: Following Mirrlees we assume that workers differ with respect to innate ability and that the function which describes the cumulative distribution of ability is continous.[1] The assumption of the proportionate tax we adopt from the work of Sheshinski.[2] From Sheshinski also we borrow the assumption that the supply of labor depends on the amount of investment in human capital and not on the number of hours worked.[3] We follow Atkinson and Phelps in employing the *maximin* criterion of social welfare.[4] Finally, with Hamada, we adopt the Diamond version of Samuelson's intergeneration model of neoclassical growth.[4]

The main objective of this Section is to determine the impact of taxation on the before-tax real wage and on the before-tax rate of interest. This we do in subsection 2 after we have studied the economic behavior of households. In the last part of this Section we briefly describe the budgetary constraint facing the taxing authority.[6]

1. Consumer Behavior.

We assume that persons live for two periods of equal but arbitrary length. They work, if at all, during the first period and are retired during the second period. Thus at any time two generations are alive. We

---

1. J. Mirrlees, "An Exploration in the Theory of Optimum Taxation," *op.cit.*.

2. E. Sheshinski, "The Optimal Linear Income Tax,"*Review of Economic Studies*, vol. 39, July 1972, 297-302.

3. E. Sheshinski, "On the Theory of Optimal Income Taxation, "*Journal of Public Economics*, forthcoming.

4. A.B. Atkinson, *op. cit.*; E.S. Phelps, *op. cit.*.

5. K. Hamada, *op. cit.*; P.A. Diamond, "National Debt in a Neoclassical Growth Model," *American Economic Review*, vol. 55, December 1965, 1126-1150.

6. An interested reader will find a detailed discussion of market organization, prices and social accounting in J.A. Ordover and E.S. Phelps, *op. cit.*.

define a household of type m as a household headed by a worker of type m.[7] A worker of type m is a worker whose ability index is m, $m \in [0,M]$.

At the beginning of the first period every worker (household) is faced with a two-stage optimization problem: First (s)he must determine the optimal life cycle of consumption, given the amount of educational expenditures. Second (s)he must determine the optimal amount of education. Let us introduce the following notation for a worker of type m: x_1^m is the first period consumption; x_2^m is the second period consumption; x_3^m is the amount of education and $j(x_3)$ is the education cost function. Finally we denote the vector of consumer prices by $\{q\} = (q_1, q_2, q_3)$.

We can write the first of those optimization problems as

$$(1) \qquad q_1 x_1^m + q_2 x_2^m + q_3 j(x_3^m) = y^m$$

where y^m is defined as disposable income in the first period. We assume that the net earnings function takes the simple multiplicative form. Hence

$$(2) \qquad y^m = mw(1 - t_w)\, x_3^m + g = m\omega x_3^m + g$$

In equation (2) t_w is the marginal tax rate on wage income and $w(\omega)$ is the gross (net) wage paid to worker of type 1 who has acquired one unit of education. The multiplicative form of the net earnings function can be justified if we assume that efficiency/ability differences among workers are purely labor augmenting; that is the marginal product of a worker of type m is always equal to m times the marginal product of a worker of type 1.

In the Lagrangian formulation the first optimization problem is

$$(3) \qquad \mathcal{L}(\ ) = \text{Max} \ \{ u(x_1^m, x_2^m) - \lambda \,[q_1 x_1^m + q_2 x_2^m + q_3 j(x_3^m) - m\omega x_3^m - g\ ] \}$$

Let us select x_1 to serve as a numeraire then we can set $q_1 = 1$. Since educational cost is defined in terms of the first period consumables it absorbs then we must have $q_3 = q_1 = 1$. The price of second

7. For convenience we assume that each household has only one worker and that non-working members of all households are alike.

period consumption in terms of first period consumption, q_2, must equal $1/(1 + \rho)$, where ρ is the net rate of interest. We write the first order conditions as

$$\text{(3)} \qquad \begin{aligned} u_1^m - \lambda &= 0, \\ u_2^m - \lambda/(1 + \rho) &= 0, \end{aligned}$$

where $u_i^m = \partial u/\partial x_i^m$, $u_i > 0$, $x_i \geq 0$, $\lambda > 0$. Combining the two first order conditions we get

$$\text{(4)} \qquad u_2^m (1 + \rho) = u_1^m .$$

Equation (4) determines the optimal life-cycle consumption for any given disposable income. A worker's disposable income depends on his/her education and ability-- ω and g being outside her/his control. The utility maximizing education must satisfy[8]

$$\text{(5)} \qquad \frac{\partial u}{\partial x_3} \equiv u_3 = u_1 \frac{\partial x_1}{\partial x_3} + u_2 \frac{\partial x_2}{\partial x_3} .$$

From the life-time budget constraint (2) we know that

$$\text{(6)} \qquad m\omega - j'(x_3) \quad (1 + \rho) = \frac{\partial x_1}{\partial x_3} (1 + \rho) \quad + \frac{\partial x_2}{\partial x_3} .$$

Using (4), (5) and (6) we obtain

$$\text{(7)} \qquad \begin{aligned} u_3 &= u_2(1 + \rho) \quad [m\omega - j'(x_3)] \\ &= 0 \end{aligned}$$

Since $u_2 > 0$

---

8. We omit the subscript m whenever possible in order to simplify the notation.

(8) $m\omega - j'(x_3) = 0.$

Equation (8) states that in equilibrium the marginal revenue from an additional unit of education must be equal to the marginal cost of acquiring that additional unit. If we also assume that $j'(x_3) > 0$ then the second order condition for a maximum is also fulfilled.

The most important implication of equation (8) is that the supply of labor does not depend on the net rate of return on savings, ρ, thus changes in the rate of the interest tax, t_r, will have no impact on the labor supply. Another implication of equation (8) is that the labor supply does not depend on the amount of the transfer payment, g.[9]

From the first order condition (8) we can derive for future use

$$\frac{\partial x_3}{\partial m} = w(1-t_w)/j''(x_3) > 0,$$

$$\frac{\partial x_3}{\partial w} = m(1-t_w)/j''(x_3) > 0, \qquad (9)$$

$$\frac{\partial x_3}{\partial t_w} = -mw/j''(x_3) < 0.$$

Note finally that if $j(0) = 0$, $j'(0) = 0$, and if $j'(x_3) > 0$ whenever $x_3 > 0$ then from equation (8) we deduce that for any $t_w < 1$ all workers of type <u>m</u>, <u>m</u> > 0, will work, acquire education and earn some income. Workers who have no ability--whose ability index is zero--earn no income. We shall regard them, therefore, as the least advantaged members of the society.

We now turn to the economics of retired workers. During their working life

---

9. For a model in which ρ and g influence the labor supply see J.A. Ordover and E.S. Phelps, <u>op</u>. <u>cit.</u>.

they were saving for their retirement and acquired the existing stock of capital, K, and of public debt, D.[1] When they enter their retirement period they first collect interest on their holdings of wealth and then dispose of their wealth by selling it to the members of the working generation. Simple accounting tells us that if there is no bequest, retired persons' consumption, C_2, equals the market value of their wealth, K + D, plus gross interest earned, r(K+D), r being the gross rate of interest, minus any tax payments, $rt_r(K+D)$.[2] Formally, we have

$$(10) \qquad C_2 = (1+\rho)\ (K+D),$$

where $\qquad \rho = r(1-t_r)$.

On the steady state growth path consumption grows at a constant rate equal to the population growth rate, $\underline{n}$. Denoting the second period consumption of <u>current</u> workers by X_2 we obtain upon using equation (10) and dividing by the number of workers, N,

$$(11) \qquad x_2 = (1+n)\ (1+\rho)\ (k+d)$$

where $\qquad x_2 = X_2/N$, $k = K/D$ and $d = D/N$. If we denote total first period consumption of current workers by X_1 and total absorption of output for educational purposes by J( ) then total claims per worker on current output,

---

1. If the government is a net creditor private wealth consists of a part of capital stock.

2. Of course workers could avoid paying any interest taxes by entering into the futures contracts with producers. They would have no savings and no interest income. We must disallow trading in futures of this sort, therefore.

other than investment, are[3]

(12) $a(\cdot) = x_1 + x_2/(1+n) + j(x_3)$.

The remaining output is used for investment. Thus we can write the total demand in any period _t_ as

(13) $\text{Demand}_t = a_t + (k_{t+1} - k_t)$

This completes the discussion of household behavior.

2. Production and the Labor Market.

The main purpose of this Section is to study the impact of taxes on employment, gross wage rates and the before-tax rate of interest. These general equilibrium effects of changes in tax rates will depend on the economic behavior of households and on the character of the production process here described by its technology. The economic behavior of households was analyzed in subsection 1. We now turn to the production side of the model.

Output of the economy is produced through cooperation of capital and labor. The supply of labor is measured in standardized efficiency units, and is denoted by L. To obtain the expression for L we assume that the proportion of the workforce having an efficiency index _m or less_ is given by a nondecreasing, cumulative-distribution-function, Φ, with the properties

---

3. It seems appropriate to offer a description, however brief, of the educational system which is implicit in our model. Let us assume that during most of his/her working life, that is during the first period, the worker is fully productive, educational process being completed early in that period. Assume also that those early working days can be divided into three parts of not necessarily equal length. During the first part of the day the worker educates herself/himself by consuming either the already existing capital or the current output. (The very first amount of education must come from consuming the already existing capital, there being no current output to consume.) During the second part of the day (s)he contributes to the disposable output which can be either invested or consumed, $x_1 + x_2/(1 + n)$. In our interpretation $j(x_3)$ represents the output which could be had for consumption, $x_1 + x_2/(1+n)$, or investment if workers were to arrive miraculously at the factory gate optimally educated. In general education is acquired _ex machina_ rather than _deux ex machina_.

(15) $$L = N\int_0^M \ell^m \, d\Phi(m) = N\int_0^M m x_3^m \, d\Phi(m).$$

In per worker terms we have[4]

(16) $$\ell = L/N = \sum_m m x_3^m.$$

We shall assume that the production function, F( ), is everywhere differentiable and is linear homogeneous. That is

(17) $$F(\lambda K, \lambda L) = \lambda F(K,L) > 0 \text{ iff } K,L > 0 \text{ and } \lambda > 0.$$

The above assumptions regarding the F( ) function were made to simplify the analysis. The only technological requirement which we think is necessary is that there exists a functional relationship between the wage rate, w, and the rate of interest, r, such that

(18) $$w = h(r), \qquad dw/dr = h'(r) = -K/L.$$

Bruno defines this relationship which is known as the factor-price frontier as "the locus of maximal (or Pareto-optimal) combinations of the real wage and the rate of interest."[5]

Final output is distributed among consumption, educational expenditures, and capital investment.

(19) $$F(k,\ell) = x_1 + x_2/(\,1 + n\,) + j(x_3) + (k_{t+1} - k_t).$$

---

4. To simplify the notation we replace the integration sign by the summation sign.

5. M. Bruno, "Fundamental Duality Relations in the Pure Theory of Capital and Growth, " _Review of Economic Studies_, vol. 36, January 1969, 39-55.

On the steady-state growth path we must have

$$(20)\qquad k_{t+1} - k_t = nk.$$

In our model the capital/worker ratio, k, is a given constant: this is how we interpreted the just savings principle. Consequently

$$(21)\qquad nk = F(k,1) - x_1 - x_2(1+n)^{-1} - j(x_3)$$

is a constraint upon the use of policy instruments for intragenerational justice.[4]

We now turn to a brief analysis of the labor market. Let us assume that the labor market is perfectly competitive. Again this is only a simplifying assumption: By using an extended set of policy instruments the government can nullify any monopoly/monopsony distortions. In a perfectly competitive economy the wage rate equals the marginal product of labor, i.e.,

$$(22)\qquad w = \partial F(\cdot)/\partial \ell \equiv F_\ell(\cdot) > 0 \ , \ F_{\ell\ell}(\cdot) < 0$$

Hence, by inverting equation (22) we obtain the labor demand function

$$(23)\qquad \ell^d = \ell^d(w,k), \qquad \partial\ell^d/\partial w = \ell^d_w < 0, \ \ \partial\ell^d/\partial k = \ell^d_k > 0.$$

Equation (15) provides the labor supply function, with labor supply dependent upon the gross wage rate, w, the wage tax rate, t_w, and the distribution of abilities. Equilibrium on the labor market implies that

$$(24)\qquad \ell^d(w,k) - \Sigma m z^m = 0.$$

Differentiation of equation (24) with respect to w and t_w yields

(25) $$\frac{\partial w}{\partial t_w} = \frac{\Sigma m(\partial z^m/\partial t_w)}{\ell_w(w,k) - \Sigma m(\partial z^m/\partial w)} .$$

The sign of (25) is unambigously positive since the numerator and the denominator are both unambiguously negative. Thus, given the value of K and the ability - distribution function, the real wage is a unique and increasing function of the wage tax rate, i.e. $w = w(t_w)$, $w'(t_w) > 0$.[6] From equation (25) it also follows that the equilibrium labor/worker ratio, ℓ, and the equilibrium rate of interest, r, decline when t_w increases. The equilibrium ℓ must decline because, higher real wage implies a higher capital/labor ratio, k/ℓ. But in our analysis, k is kept constant. Consequently there must be a reduction in ℓ. That r goes down when w goes up is easily verified from the factor-price frontier equation, [equation (18)].

The propositions pertaining to the labor market can be illustrated by two simple diagrams. In the left panel of Figure 1 we plot the factor price frontier corresponding to equation (18). In the right panel we plot the labor demand function, DD, and the labor supply function, SS equation (15) . An increase in t_w shifts the SS schedule upward and to the left. Alternatively, as in Figure 2, we may plot in the right panel the wage-tax rate function, $w = w(t_w)$. Either of these diagrams then shows immediately how the interest rate and the wage rate are uniquely determined by the wage tax rate, t_w. We recall from Section II. 1. that neither the interest tax rate nor the transfer payment influence the supply of labor and hence the wage rate and the interest rate.

---

6. We should mention the possibility of a discontinuity of the real wage function, $w = w(t_w)$, in the neighborhood of $t_w = 1$; that is when all wages are taxed supply of labor falls to zero.

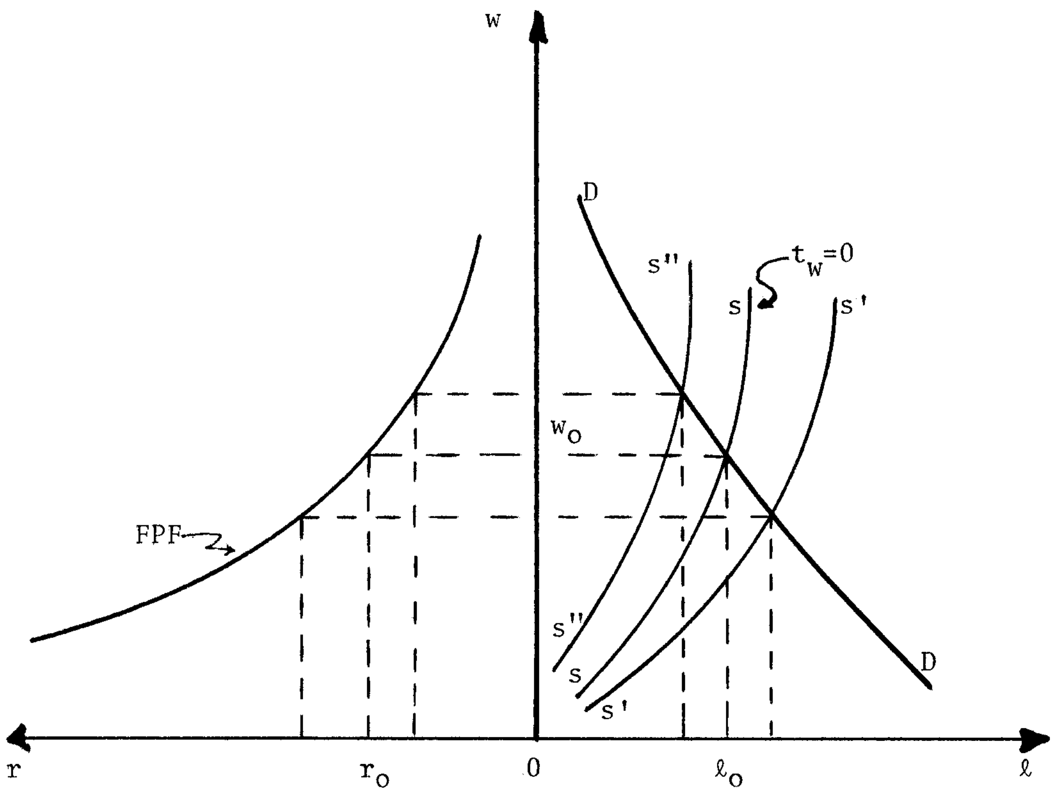

Figure 1

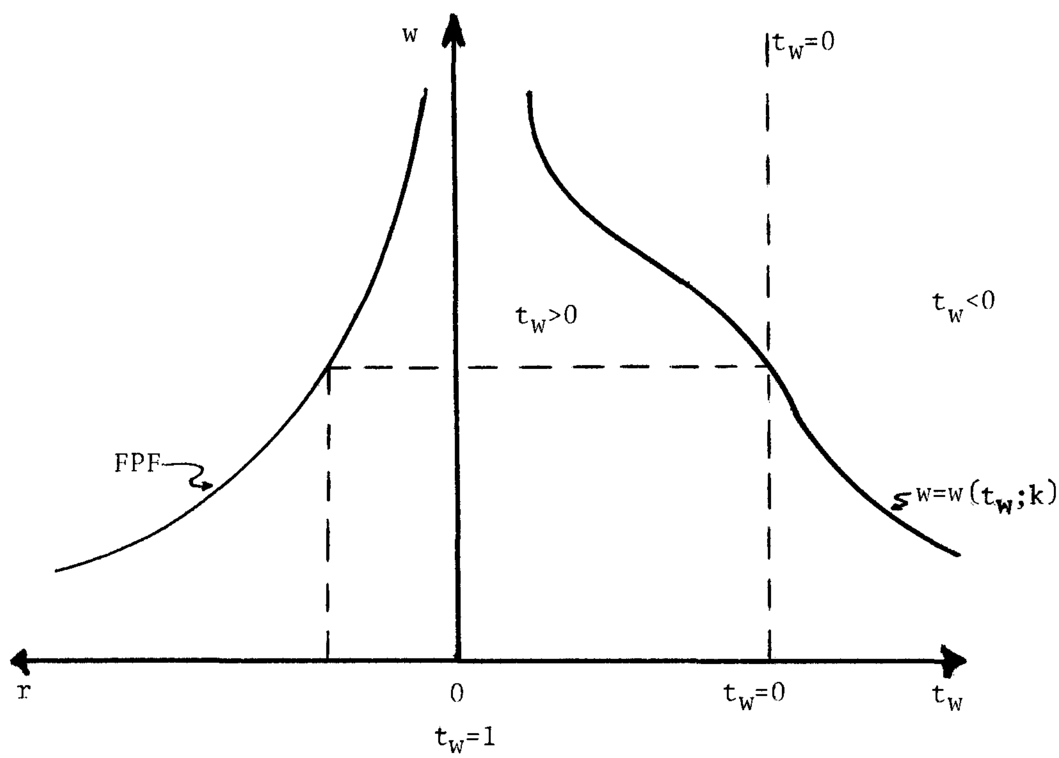

Figure 2

3. The Government Account.

In our model the government has three instruments at its disposal: the tax rate on wages, the tax rate on interest, and the transfer payment per worker.[7] Of these three instruments only two are in fact independent: Once the values of any two have been selected the value of the third is automatically determined from the investment constraint, equation (20). We take the two tax rates to be the independent instruments and assign the transfer payment to the investment constraint. In addition we assume that $t_r \leq 1$. If $t_r > 1$ then we are taxing not only the interest income but also wealth.

There is no requirement in our model that the government balances its budget. If the government incurs a deficit, it issues a one-period bond which sells for one unit of current consumption and pays an interest rate equal to the gross rate of return on capital, r. If the government incurs a surplus, it buys claims to physical capital (that is, private bonds) from the public. The value of the debt per worker, d, is bounded from below since the government can own at most all of the existing stock. On the other hand we do not expect d to go to plus infinity: at very large values of d available transfer may become very large and the investment constraint may be violated.[8]

We can calculate governmental budgetary deficit per worker as

$$(26) \qquad def_0 = g + rd_{-1} - rt_r(d_{-1} + k_0) - wt_w\ell$$

where d_{-1} denotes debt issued in period -1 per current worker and k_0

---

7. The uses of taxes for redistributive and other purposes may create certain problems for the fiscal agency. Those problems arise if the optimal tax vector can support more than one equilibrium allocation only one of which maximizes social welfare. This so called control problem does not arise in our model as we have shown in Section II. 2. See also P.A. Diamond and J.A. Mirrlees, *op. cit.*.

8. The optimal value of debt per worker, d*, can be obtained from equation (11), that is,

$$(11') \quad d^* = [\sum_m x_2^{m*}(g^*, t_r^*, t_w^*)][(1+\rho)(1+n)]^{-1} - k.$$

denotes capital per current worker at the beginning of period 0. "def_o" is the quantity of outlays financed by net borrowing. Consequently,

(27) $$def_o = d_o - d_{-1} \cdot$$

On the steady-state growth path the deficit per worker must be kept constant. Thus

(28) $$def_o = nd_{-1}.$$

In view of (28) we rewrite (26) as

(29) $$(\rho - n)d = rt_r k + wt_w \ell - g.$$

The value of debt per worker in equation (29) is the optimal value of debt. We already indicated in Footnote 8 how to calculate that value of debt.

It remains only to show that in fact the value of debt per worker which is consistent with the just savings principle, equation (21), and with the private savings equation, equation (11), is also consistent with equation (29). That is, we must show that the constraint given by equation (29) is a linear combination of equations (11) and (21). For that purpose we rewrite equation (29) as

(30) $$-nd = F(k,\ell) - \rho(k + d) - \omega\ell - g.$$

Next we substitute the lifetime budget constraint, equation (1), into (29) and obtain

(31) $$-nd = F(k,\ell) - \rho(k + d) - x_1 - x_2/(1 + \rho) - j(x_3)$$

We use now equation (11) to substitute for d

(32) $$-nx_2(1 + \rho)^{-1}(1 + n)^{-1} + nk = F(\ ) - \rho k - \rho\{x_2(1 + \rho)^{-1}(1 + n)^{-1} - k\}$$
$$-x_1 - x_2(1 + \rho)^{-1} - j(x_3)$$

After appropriate cancelations we obtain

$$nk = F(\cdot) - x_1 - x_2(1 + n)^{-1} = j(x_3)$$

which is identical with equation (21). Hence the vector of optimal values, $\{ g^*, t_r^*, t_w^*, d^* \}$, is consistent with equations (11), (21), and (28).

III. THE ANALYSIS

We are prepared now to study the problem posed at the beginning of the paper. Namely to calculate the optimal values of the two tax rates, t_w and t_r, when the government is maximizing the lifetime welfare of the least favored, (A),[1] subject to the investment (just savings) constraint, (B), and subject to the prohibition on taxation of wealth, (C). The maximization problem is

(A) $$\max_{\{t_w, t_r, g\}} u(x_1^0, x_2^0) \quad \text{subject to}$$

(B) $$nk = F(k,\ell) - x_1 - x_2(1+n)^{-1} - j(x_3),$$

(C) $$t_r \leq 1.$$

Our analysis proceeds as follows. In Section III. 1 we analyze the properties of the utility function in the instrument space. For this purpose we define

(33) $$v(t_w,t_r,g;m) = u\,[\,x_1^m(t_w,t_r,g;m),\ x_2^m(t_w,t_r,g;m)\,]\,.$$

We note that with any vector $\{t_w,t_r,g\}$ we can associate a unique value of the $v(\cdot;m)$ function precisely because that vector determines a unique vector of consumer prices, $\{m\omega, q_2\}$.

In Section III. 2 we study the properties of the investment constraint, equation (21). For that purpose we shall introduce a new function, g( ),

---

1. We know before we solve the maximization problem that households of type zero are the least favored. Why? Because, as one would expect, the rate of the wage tax will never exceed one. Consequently all other households will have income which is higher than the income of the zero type household. Besides if the tax rate were to exceed one then all households of type $m > 0$ would simply pretend to be of type zero.

(34) $$g = g\ [t_w, t_r; k, n; \Phi(\ )]\ .$$

This function defines for any pair of t and t the maximum admissible amount of the transfer payment consistent with the just savings principle. Conceivably the values of the parameters of the problem, k, n, and Φ, may be such that g is always negative. Intragenerational maximin justice cannot be realized in such an economy. Consequently, we shall assume that there exists at least one pair of values of the tax rates for which g is positive.

In Section III. 3 we provide the solution to the maximization problem. In Section III. 4 we ask what is the value of capital per worker to which a maximin society would like to be constrained.

1. An Indifference Map in the Instrument Space.

Equation (33) states that utility is a function of several policy variables. We would like to know, therefore, what combinations of t_w, t_r, and g leave a household equally well-off. Since our interest is only in the welfare of the least favored we derive the indifference map for a representative family of type zero. First, using equations (1), (3), and (8) we obtain

(35a) $$v_g^0 = (1 + \rho)\ u > 0,$$

(35b) $$v_r^0 = s^0(1 - t_r)\ u_2 \geq 0 \quad \text{if } t \leq 1,$$

(35c) $$v_{t_r}^0 = -s^0 r u_2 < 0,$$

where savings of the least favored household, s^0, equal the value of the transfer payment less the value of the first period consumption, $s^0 = g - x_1^0$. It would appear that because the least favored do not work they should be indifferent to the rate of the wage tax. But in a general equilibrium context the poor are not indifferent to changes in t_w: An increase in t_w reduces the supply of labor, see Figure 1; and reduces the gross rate of return on capital. If t_r is kept constant an increase in t_w reduces ρ since it lowers the value of the gross rate of return. A drop in the net rate of return, ρ, reduces utility.

Thus we have

$$(35d) \qquad v_{t_w} = v_r \frac{dr}{dw} \frac{dw}{dt_w} \lesseqgtr 0 \quad \text{as} \quad t_r \lesseqgtr 1.$$

Hence along a given indifference curve

$$(36a) \qquad \left.\frac{\partial g}{\partial t_r}\right|_{u,t_w} = sr(1 + \rho)^{-1} > 0,$$

$$(36b) \qquad \left.\frac{\partial g}{\partial t_w}\right|_{u,t_r} = -s(1 - t_r)(1 + \rho)^{-1} \frac{dr}{dt_w} \geq 0 \qquad \text{if } t_r \leq 1,$$

$$(36c) \qquad \left.\frac{\partial t_r}{\partial t_w}\right|_{u,g} = \frac{1}{r} (1 - t_r) \frac{dr}{dt_w} < 0.$$

Equation (36b) states that the poorest households become increasingly "indifferent" to changes in the rate of the wage tax as the interest tax rate becomes higher and higher. In fact, when $t_r = 1$ then $\partial g/\partial t_w = 0$.

We will need one more result for future analysis. Namely we must show that

$$(37) \qquad \frac{\partial^2 g}{\partial t_r^2} > 0.$$

The proof is rather trivial and follows from the properties of the indifference curves in the <u>commodity</u> space. In the commodity space the indifference curves

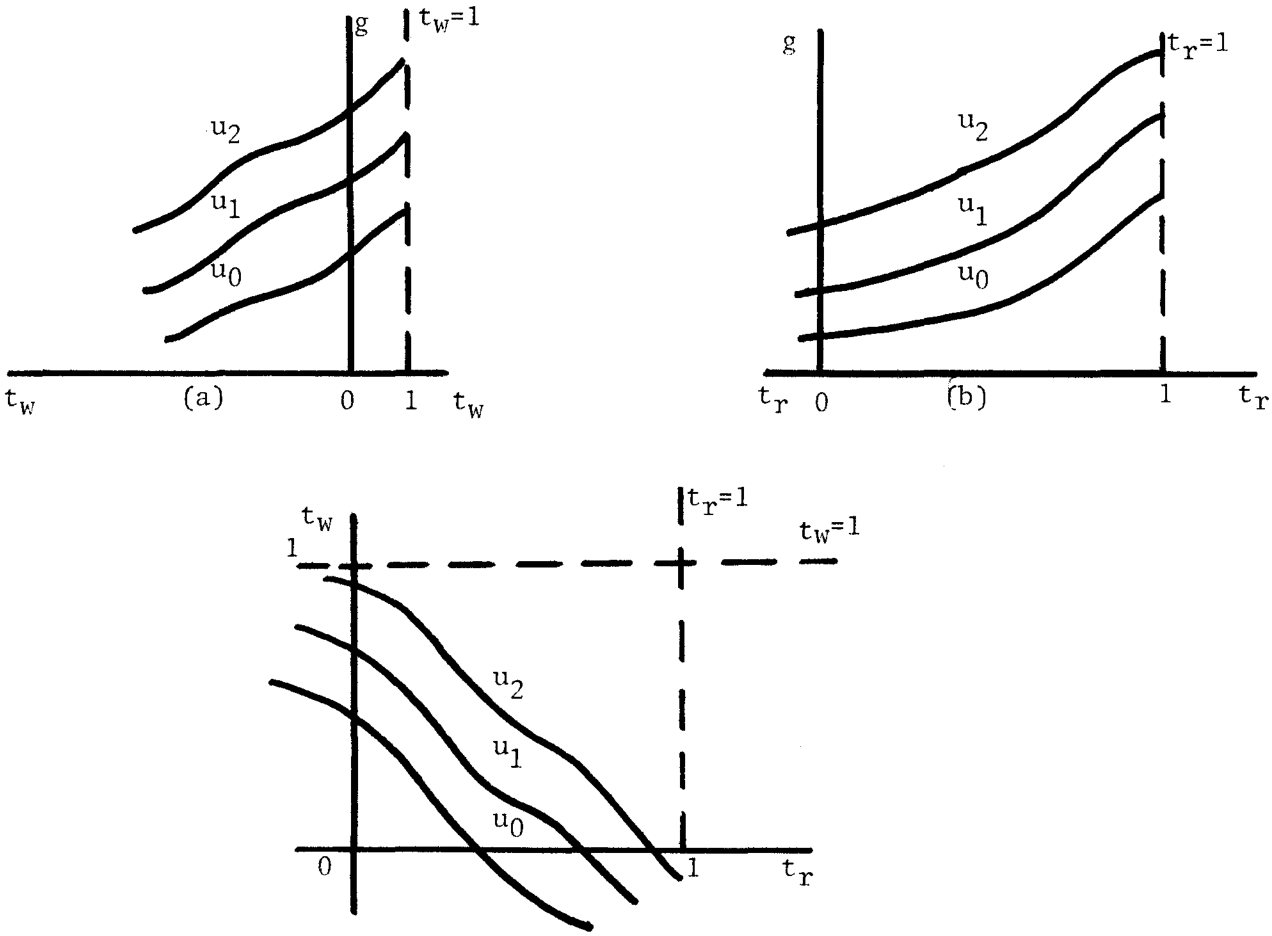

Figure 3: Indifference Curves in Instrument Spaces for a Representative Household of Type 0.

are convex to the origin. Consequently a reduction in the net rate of interest, ρ, due to an increase in t_r will require a compensating variation in g which is the smaller the bigger the value of ρ, that is the lower the value of t_r.

2. <u>The Investment Constraint in the Instrument Space.</u>

In the last Section we discussed some of the properties of the utility in the instrument space. We shall investigate now the properties of the investment constraint, equation (21), which we write in the reduced form as

(38) $$g = g\,[t_w, t_r; k, n; \Phi(\;)]\ .$$

Equation (38) describes the value of transfers, g, as a function of the other decision variables.[2]

All triplets, $\{g, t_w, t_r\}$, which solve equation (21) are said to lie on the isocapital surface. Our first task is to study the behavior of the isocapital surface in the (g,t_r)-space. Let us differentiate partially equation (21) holding t_w constant. We obtain

(39) $$\frac{\partial g}{\partial t_r} = - \frac{\dfrac{\partial q_2}{\partial t_r}\,[x_{12} + x_{12}\,(1+n)^{-1}]}{x_{1g} + x_{2g}(1+n)^{-1}}$$

where $q_2 = (1+\rho)^{-1}$, $x_{12} = \partial x_i/\partial q_2$, and $x_{ig} = \partial x_i/\partial g$. We take the sign of the denominator of (39) to be positive. The denominator is unambiguously positive if neither good is inferior. In what follows we shall assume that both goods are normal.[3] We now prove

<u>Proposition P1.</u> The value of the transfer payment, g, increases with the rate of the interest tax, t_r, if the net rate of return, ρ, is greater than or equal to the rate of population growth, <u>n</u>.

---

2. We write the investment constraint, equation (21), a long-run or a steady-state constraint. In the short-run interpretation of the constraint the value of the debt per worker, d, is treated as a historical datum. In the long-run formulation the value of the debt is the best(optimal) value which can be associated with a given set of values of policy instruments. (See Note 8 on p.15).

Differentiation and summation over all households of the lifetime budget constraint, equation (1), yields

(39) $$x_{12} + q_2 x_{22} + x_2 = 0.$$

Since $x_2 > 0$ we must have $x_{12} + q_2 x_{22} < 0$. If $\rho \geq n$ then $q_2 = (1+\rho)^{-1} \leq (1+n)^{-1}$. Consequently, for all $\rho \geq n$ we must have $x_{12} + x_{22}(1+n)^{-1} < 0$. Which, together with the assumption on the sign of the denominator, implies that equation (38) and thus $\partial g/\partial t_r$ is positive.

It is often assumed in the literature on optimal redistribution that the utility function is Cobb-Douglas.[4] For such a function $x_{12} = \partial x_1/\partial q_2 = 0$ and equation (38) is always positive. Thus

> Proposition P2. If all households have a Cobb-Douglas utility function then the value of the transfer payment always increases with the value of the rate of the interest tax.

The proof follows directly from our observation that $x_{12} = 0$ and $x_{22} < 0$.

The impact of a variation in the rate of the interest tax, t_r, falls only on the "discounted" price of the second period consumption, $(1+\rho)^{-1}$. It does not affect either the first period earnings or labor supply. It affects only the structure of consumption. This explains the simplicity of the analysis of the sign of $\partial g/\partial t_r$. We turn now to a more difficult problem: the analysis of the sign of $\partial g/\partial t_w$.

---

3. We would expect that even if x_2 is weakly inferior our results remain unchanged.

4. See for example J.A. Mirrlees, "An Exploration in the Theory of Optimum Taxation," op. cit., and A.B. Atkinson, op. cit..

Before we proceed with a formal analysis of the expression for $\partial g/\partial t_w$ let us note that the adverse effects of (positive) wage taxation on output place an upper limit on admissible rates of the wage tax. When t_w becomes "very large," close to one, output becomes very small. Demand for x_1, x_2, and x_3, does not decline to zero, however. Thus a negative transfer, a lump-sum taxation, may be required to reduce absorption even further. Otherwise contractual investment obligations, nk, which remain unchanged in the face of declining output, will not be fulfilled. Indeed it can be shown that $g = g(t_w, t_r; \cdot)$ becomes hegative as t_w approaches one.

Differentiation of equation (21) with respect to g and t_w yields

(40)
$$\frac{\partial g}{\partial t_w} = -D^{-1} \{ [x_{12} + x_{22}(1+n)^{-1}] \frac{\partial q_2}{\partial r} \cdot \frac{dr}{dt_w} + \sum_m [\frac{\partial x_1^m}{\partial t_w} \frac{\partial x_2^m}{\partial t_w} \cdot \frac{1}{(1+\rho)} + j'(x_3^m) \frac{dx_3^m}{dt_w}] + (\rho-n) [(1+\rho) (1+n)]^{-1} \sum_m \frac{\partial x_2^m}{\partial t_w} - F_\ell \frac{d\ell}{dt_w} \} .$$

where $D = x_{1g} + (1+n)^{-1} x_{2g}$ and y_m is defined by equation (2). The first square bracket in (40) is negative whenever $\partial g/\partial t_r$ is positive, see equation (38). The expression in the second square bracket measures the change in lifetime absorption of output, at constant commodity prices, resulting from a change in t_w. Thus it must be that

(41)
$$\sum_m [\frac{\partial x_1}{\partial t_w} + \frac{1}{1+\rho} \frac{\partial x_2^m}{\partial t_w} + j'(x_3^m) \frac{dx_3^m}{dt_w}] = \Sigma \frac{d(m\omega x_3^m)}{dt_w} .$$

It is easily shown that, as one would expect, the sum on the left hand side is negative. To see this we prove the following

> Proposition P3. An increase in the rate of the wage tax reduces the wage income of every household of type m, $m > 0$.

For a representative household of type m we have

$$(42)\qquad \frac{d(m\omega x_3^m)}{dt_w} = m\omega \frac{dx_3^m}{dt_w} - mwx_3^m + mx_3^m (1-t_w)\frac{dw}{dt_w} .$$

Equation (42) is evaluated at household's equilibrium allocation. Hence we can make use of equations (8) and (9) and write (42) as

$$(43)\qquad \frac{d(m\omega x_3^m)}{dt_w} = [j'(x_3^m) + x_3^m j''(x_3^m)] \frac{dx_3^m}{dt_w} .$$

Equation (43) is negative because the expression in square brackets is positive and dx_3^m/dt_w is unambiguously negative.

Returning to our examination of equation (40) we note that the third term of that equation is nonpositive if $\rho \geq n$. Unfortunately we cannot prove for the sign of $\partial g/\partial t_w$ a proposition which is analogous to the Proposition P_1. However, we can prove the following

> Proposition P4. The value of the transfer payment, g, increases with the rate of the wage tax, t_w, if (i) $\rho \geq n$, and (ii) $t_w \geq \hat{t}_w$ where $\hat{t}_w = \min_{\{m\}} \varepsilon^m(1+\varepsilon^m)^{-1}$ and $\varepsilon^m = x_3^m j''(x_3^m)\,[j'(x_3^m)]^{-1}$, that is ε is the elasticity of the marginal cost of education.

Part (i) of the Proposition follows from Proposition 1 and from the assumption that all good are strictly noninferior. To prove part (ii) of P4 we subtract from equation (43) the last term of equation (40), $F_\ell \cdot \Sigma d\ell^m/dt_w$, and obtain

$$(44) \qquad \sum_m \left[\frac{d(m\omega x_3^m)}{dt_w} - w \cdot \frac{d\ell^m}{dt_w} \right] = \sum_m \left[j'(x_3^m) + x_3^m j''(x_3^m) - j'(x_3^m)(1-t_w)^{-1} \right] \frac{dx_3^m}{dt_w}$$

$$= \sum_m j'(x_3^m) \left[x_3^m j''(x_3^m)/j'(x_3^m) - t_w(1-t_w)^{-1} \right] \frac{dx_3^m}{dt_w}$$

Equation (44) will be negative if for every $m > 0$

$$(45) \qquad \varepsilon^m - t_w(1-t_w)^{-1} > 0$$

where ε^m is the previously defined elasticity of the marginal cost of education. Let us define now

$$(46) \qquad \hat{t}_w = \min_{\{m\}} \varepsilon^m (1+\varepsilon^m)^{-1} > 0.$$

Then for all $t_w \leq \hat{t}_w$ expression (45) is positive and equation (46) is negative. This proves part (ii) of P4 and completes the proof of Proposition P4.

We turn now to our main task namely to finding the optimal mix of tax rates.

3. Optimal Tax Formulae.

The optimal tax mix is obtained by solving the following Lagrangian

$$(47) \qquad \max_{\{t_r,t_w,g,\lambda\}} \mathcal{L} = v^0(t_r,t_w,g) - \lambda \left[-F(k,\ell) + nk + x_1 + x_2(1+n)^{-1} + j(x_3)\right]$$

which yields the first-order conditions for a maximum

$$(48) \qquad \begin{aligned} \partial v^0/\partial t_r &- \lambda\partial I/\partial t_r = 0 \\ \partial v^0/\partial t_w &- \lambda\partial I/\partial t_w = 0 \\ \partial v^0/\partial g &- \lambda\partial I/\partial g = 0 \end{aligned}$$

where

$$(49) \qquad -nk = x_1 + x_2(1+n)^{-1} + j(x_3) - F(k,\ell) \equiv I(t_r,t_w,g;n,k).$$

Let us assign g to theinvestment constraint. Instead of solving a constrained maximization problem, equation (47), we can now solve an unconstrained maximization problem in which we treat g as a function of t_r and t_w, given the values of n and k. See equations (34), (38), and (40). In the modified version the first-order conditions for a maximum are

(50a) $\partial v^0/\partial t_r + (\partial v^0/\partial g)(\partial g/\partial t_r) \geq 0,$

(50b) $\partial v^0/\partial t_w + (\partial v^0/\partial g)(\partial g/\partial t_w) = 0.$

The inequality in (50a) derives from the side condition that $t_r \leq 1$. We use (50a) and (50b) together with the results of Section III. 1 to prove the following proposition

> Proposition P5. If the least advantaged cannot earn income in the market, the maximin rule calls for the realization of taxable capacity only (i) if they do not save or (ii) if the optimal rate of interest on the interest tax is one hundred percent, i.e. $t_r = 1$.

Part (i) of P5 is discussed in Ordover and Phelps (op. cit.). Consequently, we shall prove only part (ii) of P5. From expressions (35c) and (35d) we know that $\partial v^0/\partial t_i$, $i = r,w$, is always negative if $t_r < 1$; $\partial v^0/\partial g$ is positive, see (35a). Thus $\partial g/\partial t_i$ must be always positive if the optimal value of the interest tax rate, t_r^*, is less than one. If $t_r^* = 1$ then $\partial v^0/\partial t_w = 0$ which implies that atthe optimum $\partial g/\partial t_w = 0$, that is that atthe optimum taxable capacity with respect to the wage tax, t_w, is realized. Further, if $t_r^* = 1$ then taxable capacity with respect to the interet tax is also realized even if $\partial g/\partial t_r > 0$ around $t_r = 1$: Clearly if $\partial g/\partial t_r = 0$ at $t_r^* > 1$ then the interest tax and the wealth tax are imposed, a possibility excluded in this paper.[5]

The question which is prior to the question of full utilization of taxable

5. The reader will also note that if $t_r^* > 1$ then the taxable capacity with respect to t_w is overutilized, i.e. at the optimum t_w $\partial g/\partial t_w < 0$ if $t_r^* > 1$.

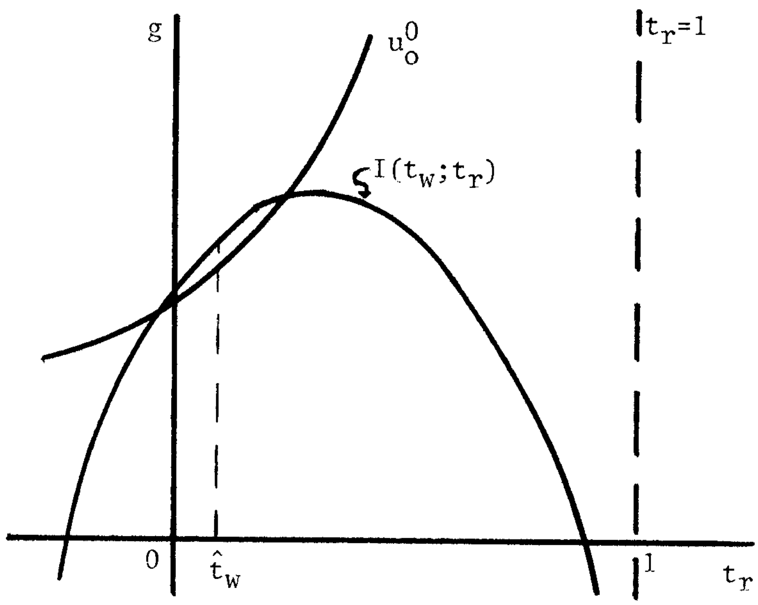

Figure 4: The Investment Constraint in the $(g,\ t_w)$-space.

capacity is whether at an optimum anything should be taxed (or subsidized). To answer this question we first prove

Proposition P6. Define the shadow interest tax, t_n, as $t_n = (n-\rho)q_2(1+n)^{-1}$, then atthe optimum the shadow tax, t_n, is positive.

Using equation (35a), (35c), (38), and (39) we rewrite (50a) as

(51) $$- s^0 - D^{-1} q_2 [-x_2 + x_{22}(\rho-n)q_2(1+n)^{-1}] \geq 0.$$

Assume that per chance the value of ρ is such that $\rho = n$, then (51) reduces to

(52) $$-s^0 + s \geq 0$$

where s^0 is the saving of the least advantaged worker and s is the average saving. Since saving increases with ability expression (52) must be positive. Assume now that there exists the rate of the interest tax, denoted by t_r', such that $r(1-t_r') = \rho = n$. Then from (52) we conclude that the indifference curve passing through the point $(g(t_r';t_w),t_r')$ must have a lesser slope than the isocapital curve passing through the same point. As we move leftward along the indifference curve it becomes flatter. As we move leftward along the isocapital curve it becomes steeper. Clearly then the optimal rate of the interest tax, t_r^*, is greater than t_r'. Which gives $\rho^* = r(1-t_r^*) < n$. A similar argument applies for t_w. Our proposition is now proved.

To provide rationale for Proposition P6 we note that when $\rho = n$ the weight of the second period consumption in the investment constraint is the same as the weight of the second period consumption in the household's life-time budget constraint. Thus the first "dose" of taxation equalizes the Marginal Rate of Substitution, $1/(1+\rho)$, with the Marginal Rate of Transformation, $1/(1+n)$. The second "dose" of taxation drives a required wedge between the MRS and the MRT, however.

The $\rho^* < n$ rule important in itself leads also to

Proposition P7. If the gross rate of interest , r, is greater or equal to the rate of growth, n, when there is no wage taxation, then either $t_r > 0$, or $t_w > 0$, or both.

The proof is trivial and follows from the definition of the net rate of return and from the observation that $dr/dt_w < 0$.

Quasi-Socialism. We have established then that under some conditions at the optimum at least one (nonshadow) tax will be positive. We shall determine now under what conditions the optimal rate of the interest tax, t_r^*, is not only positive but equal to one. Let us first define a quasi-socialist economy as such an economy, in which $t_r^* = 1$. We observe immediately that

Proposition P8. If the gross rate of return is strictly nonnegative then at the optimum a stationary economy is always quasi-socialist.

Assume first that, given the value of k, $r > 0$ for all t_w. Proposition P6 then implies that $t_r^* > 1$. But in our model $t_r \leq 1$. Hence $t_r^* = 1$ and $\rho^* = 0 = n$. Clearly if $r^* = 0$ for t_w^* then at such an r there is no interest income on which to impose the tax and the interest tax becomes a redundant instrument.

The maximum rate of growth which still yields the quasi-socialist solution can be calculated from

(53) $$(s - s^0) + n(s^0 x_{2g} + x_{22})(1+n)^{-1} = 0$$

which gives

(54) $$n(1+n)^{-1} = -(s - s^0)(s^0 x_{2g} + x_{22})^{-1}.$$

We can use the familiar Slutsky equation

(55) $$x_{22} = \sum_m (h_{22}^m - x_2^m x_{2g}^m)$$

where $h^m_{22} < 0$ is the derivative of the ms household compensated demand curve for the second period's consumption with respect to $q_2 = (1+\rho)^{-1}$. We can state the following Tax Rule: For all n such that

$$(56) \qquad \frac{n}{1+n} \leq - \frac{\Sigma\,(x^m_2 - x_2)}{\Sigma h^m_{22} - \Sigma\, x^m_2 - x_2\; x^m_{2g}} > 0$$

the optimal interest tax, t^*_r, is one hundred percent. Unfortunately we have not been able to determine whether the upper-bound on n is above or below current rates of growth.

We shall calculate now the optimal rates of the wage tax when $t^*_r = 1$. The first-order condition, (50b), can be written as

$$(57) \qquad \frac{\partial x_1}{\partial t_w} + \frac{\partial x_2}{\partial t_w} + j'(x_3)\frac{dx_3}{dt_w} - \frac{n}{1+n}\frac{\partial x_2}{\partial t_w} - w\frac{dx_3}{dt_w} = 0.$$

Let us express the $\partial x_2/\partial t_w$ term as

$$(58) \qquad \frac{\partial x_2}{\partial t_w} = \sum_m [\, x^m_{2y} \cdot y^m (x^m_2)^{-1}\, (x^m_2/y^m)\,]\, \frac{dy_m}{dt_w}$$

Where $x^m_{2y} = \partial x^m_2/\partial y^m$ and $dy_m/dt_w = d(\omega m x^m_3)/dt_w$. That is x^m_{2y} is the pure income effect and equals x^m_{2g}; dy_m/dt_w is a total increase in earned income in terms of first-period consumption due to a fall in the tax rate, t_w. Let us define now $\eta^m_2 = x^m_{2y} \cdot y^m \cdot (x^m_2)^{-1}$ and $\alpha^m_2 = x^m_2(1+\rho)^{-1}/y_m$ then,

$$(59) \qquad \frac{\partial x_2}{\partial t_w} = \sum_m (\alpha^m_2 \eta^m_2)\, \frac{d(m\omega x^m_3)}{dt_w}\,(1+\rho).$$

Using equation (44) and the definition of ε^m we rewrite (57) as

$$(60) \qquad \sum_m j'(x^m_3)\{1 - n(1+n)^{-1}\,\alpha^m_2\eta^m_2\}\varepsilon^m\,\frac{dx^m_3}{dt_w} = 0.$$

Equation (60) finally yields the optimal wage tax rate, t_w^*,

$$(61) \qquad \frac{t_w^*}{1-t_w^*} = \frac{\Sigma j'(x_3^m)\,[1 - n(1+n)^{-1}\,\alpha_2^m \eta_2^m]\,\varepsilon^m \frac{dx_3^m}{dt_w}}{\Sigma j'(x_3^m)\,\frac{dx_3^m}{dt_w}} .$$

A much simpler tax formula for t_w^* can be obtained if we assume that the utility function is homothetic and that the elasticity of the marginal cost of education is constant and equal to ε:

$$(62) \qquad \frac{t_w^*}{1 - t_w^*} = \varepsilon\,[1 - n(1+n)^{-1}\,\alpha_2]$$

or

$$(63) \qquad t_w^* = \frac{\varepsilon\,[1 - n(1+n)^{-1}\alpha_2\,]}{1 + \varepsilon\,[1 - n(1+n)^{-1}\alpha_2]} .$$

Two important "qualititative" results can be derived from equations (61) and (63).

> Proposition P9. In a quasi-socialist economy the optimal wage tax is positive.

Because $\alpha_2, \eta_2 \leq 1$ then for each m the expression in the square bracket in (61) is positive. Consequently, equation (61) is positive. And so is equation (63).

> Proposition P10 In a growing quasi-socialist economy the optimal wage tax is lower than that in a stationary state.

For $n = 0$ equation (63) becomes $t_w^* = \varepsilon(1+\varepsilon)^{-1} < \varepsilon A(1+\varepsilon A)^{-1}$ where $A = 1 - n(1+n)^{-1}\alpha_2 < 1$. We should not interpret P10 as akin to the well-known growth dividend result. On the contrary. The optimal rate of the wage tax is not lower in a growing economy than in a stationary one because in the former the same amount of redistribution can be accomplished with lower tax rates. It is lower because the previously optimal redistribution is no longer attainable. In a growing economy consumption must be restrained to leave some output avilable for investment. In order to minimize the social cost of such necessary contraction in noninvestment demand the government must lower the wage tax rate which in turn stimulates the labor supply and increases output. Unfortunately, at least for the poor, the value of the transfer payment must be reduced also. In a sense, economic growth is a burden on the poor irrespective of whether it is a burden on anybody else.

<u>Optimal Taxes when $t_r^* < 1$.</u> In a general case $t_r^* < 1$ and the first-order condition, (50a) is an equality. We shall use this observation to rewrite the first-order condition for a maximum with respect to t_w, equation (50b). First using equations (35a),(35b), and (35c) we express $\partial v^0/\partial t_w$ as

$$(64) \qquad \frac{\partial v^0}{\partial t_w} = -\frac{\partial v^0}{\partial t_r} \cdot \frac{1-t_r}{r} \frac{dr}{dt_w} .$$

Next we note that

$$(65) \qquad \frac{dq_2}{dr} = -\frac{1-t_r}{r} \frac{dq_2}{dt_r} .$$

We substitute (64) and (65) into (40) and into (496) and obtain

$$(66) \quad -\frac{\partial v^0}{\partial t_r} \frac{1-t_r}{r} \frac{dr}{dt_w} - \frac{\partial v^0}{\partial g} D^{-1} [x_{12} + x_{22}(1+n)^{-1}] [-\frac{1-t_r}{r} \frac{\partial q_2}{\partial t_r}] \frac{dr}{dt_w}$$

$$- \frac{\partial v^0}{\partial g} D^{-1} [\Sigma \frac{d(m\omega x_3^m)}{dt_w} + \frac{\rho-n}{(1+\rho)(1+n)} \Sigma \frac{\partial x_2^m}{\partial t_w} - F_\ell \frac{d\ell}{dt_w}] = 0.$$

We immediately notice that the at the optimum the first two terms of (66) cancel each other out. They simply are equal to

$$(67) \qquad -\frac{1-t_r}{r} \frac{dr}{dt_w} [\frac{\partial v^0}{\partial t_r} + \frac{\partial v^0}{\partial g} \cdot \frac{\partial g}{\partial t_r}] = 0.$$

The wage tax division of the fiscal agency must set, therefore,

$$(68) \qquad \Sigma \frac{d(m\omega x_3^m)}{dt_w} + \frac{\rho - n}{(1+\rho)(1+n)} \Sigma \frac{\partial x_2}{\partial t_w} - F_\ell \frac{d\ell}{dt_w} = 0.$$

Equation (68) can be obtained also by a somewhat different method. Assume for the moment that the government can control directly the after-tax prices, ρ and ω, rather than having to do so indirectly through the manipulation of the tax rates, t_w and t_r. The first-order condition for a maximum with respect to ω is

$$(69) \qquad x_{1\omega} + q_2 x_{2\omega} + j'(x_3)x_{3\omega} + q_2(\rho-n)(1+n)^{-1} x_{2\omega} - F_\ell \ell_3 = 0$$

since $\partial v^0/\partial\omega = 0$ from the assumption that the least-favored do not work. Multiplication of equation (69) by $d\omega/dt_w \neq 0$ yields equation (68). At a risk of doing some violence to terminology we can state the following

Proposition P11. If the interest tax division of the fiscal authority is optimally controlling the commodity prices facing the consumer, (q_2,q_2), then the wage tax division should maximize revenue with respect to the after-tax wage, ω.

It is obvious now that unless the optimal interest tax rate equals zero Proposition P11 cannot be realized; because equation (67) generally will not hold with $t_r = 0$. If $t_r^* \neq 0$ then an economy without the interest tax will have income distribution which is not the best that can be attained even without lump-sum taxes. Almost surely it will also have the wage tax rate which is higher than in an economy with interest taxation.

We end this section by showing how to calculate the optimal wage tax. To simplify the analysis we assume that the utility function is homothetic and that $\varepsilon^m = \varepsilon$, all $m\varepsilon(0,M)$. Using equation (59) and (60) we reduce equation (68) to

$$(70) \qquad \frac{t_w}{1-t_w} = \varepsilon\,[1 - (1+\rho)t_n\alpha_2\,]$$

where t_n was defined previously as $t_n = (n-\rho)q_2(1+n)^{-1}$. The reader can verify that the expression in the brace is positive even for very large rates of growth. In fact t_w^* will become negative only if the rate of population growth is much greater than one hundred percent, an absurdity.

The shadow tax rate t_n is calculated from

$$t_n = - [\Sigma (x_2^m - x_2^0)][\Sigma h_{22}^m - \Sigma(x_2^m - x_2^0)x_{2g}^m]^{-1} . \tag{71}$$

The optimal tax formulas, (70) and (71), do not involve the gross rate of return, r, which can be calculated, however, once the optimal value of t_w is known. It is perhaps somewhat surprising that the gross rate of interest has been relatively unimportant in our analysis. This relative unimportance can be attributed to the availability of the interest tax. Indeed we could have neglected the gross rate of return altogether if we were to assume that the fiscal authority uses (ρ,ω) as redistributive tools rather than the two tax rates, t_w and t_r. Still it is interesting to know what gross rate of return the maximin society would like to have if it could select both the tax rates and the capital stock.

The Optimal Capital Stock. We define the optimal capital stock as the capital stock which a just society would like to have and reproduce if that capital stock could be obtained costlessly. Unfortunately there is no assurance that a society following a just savings rule could ever reach such a capital stock starting from some arbitrary initial capital stock. (With free disposal, an overendowed society can always reach the optimal capital stock.)

It is simpler to determine the optimal capital/worker ratio if we adopt the "centralized socialist" interpretation of the model in which the government is assumed to control ω and ρ. The object is to maximize the Langrangian given by equation (47) with respect to k, that is

(72) $$\frac{d\mathcal{L}}{dk} = \frac{\partial\mathcal{L}}{\partial k} + \frac{\partial\mathcal{L}}{\partial\omega} \cdot \frac{\partial\omega}{\partial k} + \frac{\partial\mathcal{L}}{\partial\rho} \cdot \frac{\partial\rho}{\partial k} .$$

But both ω and ρ were already optimized. Thus

(73) $$\partial\mathcal{L}/\partial\omega = \partial\mathcal{L}/\partial\rho = 0$$

Consequently,

(74) $$\frac{d\mathcal{L}}{dk} = \frac{\partial\mathcal{L}}{\partial k} = r - n = 0.$$

Equation (74) leads to

Proposition P12. If the fiscal authority can control both the after-tax wage rate and the after-tax rate of interest then the optimal capital-worker ratio is such that at the optimal values of t_w and t_r the gross rate of return equals the rate of growth.

Our result is only slightly surprising. At the Golden Rule rate of interest the absorbable surplus, $F(k,\ell)-nk$, is maximized. Consequently, the value of the transfer which increases with the value of the surplus is also maximized at the Golden Rule rate of interest. Yet the Golden Rule capital/worker ratio, K/N, which obtains in the maximin economy with distortionary taxes is lower than in an economy with lump-sum taxes even though the capital/labor ratios, K/L, are identical in both economies. Distortionary taxation leads to a reduction in the labor supply. It must lead therefore to an equiproportional reduction in the capital/worker ratio. As might have been expected institutional arrangements leave their trace both on the size of the labor force and on the size of the capital stock.

IV. CONCLUDING REMARKS

In his note "On Optimum Taxation" Hahn remarks that "it is a mistake to import unexplained second-best constraints into a model which leaves no room for their justification."[1] Our model removes one such constraint by allowing the government to tax both wages and interest. We have shown that a simple extension of the set of policy instruments to two types of income taxes leads to gains in equity and in efficiency.

The interest tax is useful because it can enable the decision-maker to introduce a desired wedge between the gross, r, and the net rate of return, ρ. Without the interest tax the rate of return is determined from the factor-price frontier once the rate of the wage tax has been decided. There is no reason to assume, however, that the rate of return so obtained equals that rate of return which from the standpoint of efficiency and equity should be associated with a preselected net wage.

Our analysis indicates that the optimal rate of the interest tax is high. In many or perhaps most instances it equals one hundred percent. One would like to know how general this result is. Specifically, it is important to study the effects on optimal tax rules of tax evasion, administrative costs, natural resources, uncertainty, and so on. However, the most important question which future researchers will have to consider may well be, in essence, a political-economic matter: What limits the existing political and economic institutions impose on both the scope and on the method of redistributive policy? Ahistorical analyses, such as this, can at best tell us whether some steady-states are worthwhile getting to and what to do once we get them. Unfortunately, they are of little help in ascertaining whether these optimal steady-states are attainable from some particular initial conditions--economic and political.

---

1. E.H. Hahn, op. cit., p. 106.

V. BIBLIOGRAPHY

Arrow, K.J., " Some Ordinalist-Utilitarian Notes on Rawls's *Theory of Justice*, " Discussion Paper No 287, Harvard University, 1972.

Arrow, K.J., "Rawls's Principle of Just Saving," IMSS, Technical Report No. 106, StanfordUniversity, September 1973.

Atkinson, A.B., "Maximin' and Optimal Income Taxation," *Review of Economic Studies*, forthcoming.

Barry, B.," John Rawls and the Priority of Liberty," *Philosophy and Public Affairs*, Vol. 2, Spring 1973, pp. 274-290.

Baumol, W.J., and Bradford, D.F., "Optimal Departures from Marginal Cost Pricing," *American Economic Review*, Vol. 60, June 1970, pp. 265-283.

Bruno, M., "Fundamental Duality Relations in the Pure Theory of Capital and Growth," *Review of Economic Studies*, January 1969, Vol. 36, pp. 39-55.

Diamond, P.A., "National Debt in a Neoclassical Growth Model," *American Economic Review*, December 1965, Vol. 55, pp. 1125-50.

Diamond, P.A., and Mirrlees, J.A., "Optimal Taxation and Public Production," *American Economic Review*, March-June 1971 , Vol. 61, pp. 8-27, 261-78.

Hahn F. H., "On Optimum Taxation," *Journal of Economic Theory*,Vol. 6, February 1973, pp. 96-106.

Hamada, K., "Lifetime Equity and Dynamic Efficiency on the Balanced Growth Path," *Journal of Public Economics*, November 1972, Vol. 1, pp. 379-96.

Heller, W.P. and Shell K., "On Optimal Taxation with Costly Administration," Paper presented at the 86th Annual Meeting of the American Economic Association, New York, December 1973.

Mirrlees, J.A., "An Exploration in the Theory of Optimal Income Taxation," *Review of Economic Studies*, April 1971, Vol. 38, pp. 175-208.

Mirrlees, J.A, "On Producer Taxation," *Review of Economic Studies*, Vol. 39, January 1972,pp. 105-111.

Ordover, J.A. and Phelps, E.S., "Proportionate Steady-State Taxation of Wealth and Wages for Lifetime Economic Justice, " Discussion Paper, Department of Economics, Columbia University, September 1973.

Phelps, E.S., "Taxation of Wage Income for Economic Justice," *Quarterly Journal of Economics*, August 1973, Vol. 87, pp. 331-54.

Rawls, J., *A Theory of Justice*, Harvard University Press: Cambridge, Massachusetts, 1971.

Sheshinski, E., "On the Theory of Optimal Income Taxation," *Journal of Public Economics*, forthcoming.

Sheshinski, E., "The Optimal Linear Income Tax," Review of Economic Studies, July 1972, Vol. 39, pp. 297-302.

Stiglitz, J.E. and Dasgupta, P.," On Optimal Taxation and Public Production," Review of Economic Studies, Vol. 39, January 1972, pp. 87-104.