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THE DECOMPOSITION OF ECONOMIC RELATIONSHIPS BY TIME SCALE USING WAVELETS

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Abstract

Economists have long known that time scale matters in that the structure of decisions as to the relevant time horizon, degree of time aggregation, strength of relationship, and even the relevant variables differ by time scale. Unfortunately, until recently it was difficult to decompose economic time series into orthogonal time scale components except for short and long run in which the former is dominated by noise. This paper uses wavelets to produce an orthogonal decomposition of some economic variables by time scale over six different time scales. The relationships of interest are the permanent income hypothesis and velocity. We confirm that time scale decomposition is very important for analyzing economic relationships and that a number of anomalies previously noted in the literature are explained by these means. The analysis also indicates the importance of recognizing variations in phase between variables when investigating the relationships between them and throws considerable light on the conflicting results that have been obtained in the literature using Granger causality tests.

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The Decomposition of Economic Relationships by Time Scale Using Wavelets

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1. Introduction

For decades, the idea of "time period"in economic analysis was enshrined in textbooks as the "short run" and as the "long run." Marshall, Edgeworth, Schumpeter, Hicks and others of the period realized that there were more time periods involved in economic decision making, but the pedagogical advantages of just two periods dominated the relevance of many periods. In any event, it was recognized early in the profession that the time period of analysis, or as we would now term the matter, the "time scale" of the analysis is very important for determining those aspects of decision making that are relatively more important and those that are relatively less important. In the physical sciences, the notion of time scale captures the notion of the distinction between "slow" and "fast" variables; the former are variables whose values are changing slowly relative to those of the latter variables. For example, in the analysis of turbulence, there is a cascade of time scales each of which determines a particular mode of behavior. Similarly in economics, one can envisage a cascade of time scales within which different levels

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of decisions are being made at different rates. Some decisions are taken with long horizons, others are taken with short horizons. The choice of time scale determines not only the length of the period over which one requires forecasts of future events, but the very choice of variables that are to be the focus of the decision maker's processing of information.

For example, to use the illustrations of the classical economists, a plant manager planning an extension of the plant, or ordering fuel for the plant machinery, or adjusting the plant output rate to recover from a breakdown, are all decisions that can be formulated in terms of different time scales of decision making, from decades to hours. In each case, not only does the manager focus on a particular time horizon for gathering information, but he or she will "average over shorter" time scale variations. Further, the manager will most likely take into consideration quite different variables in the different cases. In the plant planning process, the key issues are the anticipated long run conditions of demand for the firm's product, which depend in turn on the growth of the industry and the firm's relative position in the industry. For ordering fuel they are only concerned with current inventory stocks and the price of fuel over the next few months at most. And in the recovery decision the manager is probably only concerned with overtime labor rates and the probability of breakdown from running machinery too intensely.

In this paper we will investigate some of these issues empirically, by using wavelet analysis that enables one to separate out different time scales of variation in the data. We investigate the role of time scale in economic relationships in terms of two classic relationships. The first is that of the permanent income hypothesis and the latter is velocity. Recently, both concepts have been under empirical investigation and in both cases there is some controversy about the nature of the relationship and the extent to which the relevant theory, or theories, are supported by the empirical evidence. We seek to reinvestigate these issues in the context of separating the relationship into ones between the variables at different time scales of analysis. By so doing we hope to shed light on a number of apparent anomalies in the literature.

1.1. Consumption and Income

The analysis of the consumption function during the post war period has been extensive. Friedman's original and seminal conception of the permanent income hypothesis, [24], [25], was an attempt at that time to reconcile the Keynesian, linear, short run income hypothesis with the obvious empirical finding that long run, say averaged over decades, consumption was proportional to income. Friedman's solution essentially was to propose an "errors in the variables" model to reconcile the short and long run empirical results. Further work recognized the role of the interest rate in determining "permanent income" and distinctions between the consumption of different types of goods.

The next major wave of research interest grew out of the discovery that economic variables seem to be dominated by trend and random walk components and the importance of the rational expectations hypothesis. Hall, [33], stimulated a substantial literature by "confirming" the revised permanent income hypothesis which stated that the marginal utility of consumption and therefore consumption itself evolves according to a random walk with trend. A major implication is that only lagged consumption has any explanatory role to play for current consumption. In subsequent work, Flavin, [23], rejected the permanent income hypothesis as postulated by Hall, but then Nelson, [41], was able to retrieve the hypothesis in part; the difference in results depends on how one detrends the data.

Deaton, [17], added to the debate by pointing out that while the permanent income hypothesis in its new rational expectations guise with income as a random walk implied that consumption should be relatively volatile, the evidence indicates that consumption is relatively smooth; this is the "Deaton paradox". Quah, [43], rescued the hypothesis by returning to the original Friedman observation that one should split income into permanent and transitory components.

Campbell and Mankiw, [9], generalized the permanent income hypothesis by generating a model that relates expected changes in consumption to expected changes in income. They came to the conclusion that the ratio was as high as 50%. Further, they noted the empirical regularity that changes in the real interest rate do not explain changes in consumption. Christiano et al., [11], generated a model in continuous time in which consumption persistence is due to time aggregation or the effects of exogenous technology shocks. Using quarterly data they found it difficult to distinguish between the two models and weakly accepted the permanent income hypothesis.

Molana, [40], concentrated on the role of wealth and interest rates. Molana modelled the contemporaneous variation in the data in terms of an error corrections model. While the model provides additional reasons for the "empirical failure of the simple random walk model" and reintroduces the role of the interest rate, the empirical results are implausible; for example, that the long run wealth elasticity for consumption is two. Campbell and Mankiw in examining the empirical evidence, [10], argue for a "rule of thumb model of consumer behavior" instead of the permanent income hypothesis and that there is no observed role for interest rates.

In terms of long run trends, Viard's analysis, [57], was pragmatic and examined the impact of the post '73 productivity slowdown which implies a long run decline in real income and that in turn should imply an increase in the saving rate; the empirical evidence is the opposite, the saving rate declined.

In all the papers that have been reviewed above the consumption/income relationship is regarded at most as a two fold distinction between permanent and transitory income. However, it is plausible that consumers have different time horizons for different consumption decisions. Consequently, it is productive to consider that the relationship between consumption and income may depend on the range of time scales involved. That is, different relationships, certainly different coefficient values may apply over different time scales; in one sense, this idea is nothing more than a generalization of Friedman's original idea. In the early research, there were only two "ranges of time scale", long period and short.

An alternative view of the decision making process is that the relationships between variables is in fact between frequency components of the variables. In Engle, [21], the idea was expressed that there might be several ranges of frequencies for consumption and income such that different coefficient values might relate consumption and income at different frequency ranges. Engle used band spectrum estimation techniques to evaluate the relationship between consumption and income. He allowed for different relationships at low frequencies and at high frequencies, but was able to adjust the break between low and high frequencies continuously. Engle's results were remarkably consistent in that the two regression results were virtually identical and adjusting the break between low and high frequency did not alter the results.

Recently, Corbae et al., [15], returned to this question with tools developed since the '74 Engle article. This article extended the Engle results in two important ways. First, they allowed for the trend in both consumption and income by recasting the analysis into a cointegration framework. Secondly, the authors derived a theoretical model that indicates that as the frequency of consumption and income rises the marginal propensity to consume falls from its maximum of one at zero frequency; the (very) long run MPC is one since there is no bequest motive in the model and the representative consumer exhausts his wealth in the limit as time goes to infinity. Corbae et al. used seasonally adjusted quarterly per capita data from 1948 to 1990. The results are that the zero frequency marginal propensity to consume, MPC, is about 0.73, less than one with very high probability, and that the constancy of MPC across higher frequencies is also rejected. However, the direct estimates yield the result that the high frequency MPC is larger than that for zero frequency. Redoing the experiment using first differenced data yields the more theoretically satisfactory result of lower MPC at higher frequencies. In interpreting these results the reader should recall that the model used by Corbae et al. is a simple representative agent model incorporating a quadratic utility function, which clearly cannot be a reasonable representation except in a small region around the "bliss point".

The results from these papers are broadly inconsistent with the permanent income hypothesis and with each other. Further, it would seem that interest rates do not matter. Certainly, one cannot regard the relationship between income and consumption as a closed topic.

However, there are some lessons that have been learned from these exercises. First it is reasonably clear that the permanent income hypothesis is correct in the idea that there is some difference between the MPC in the long run and the MPC at other frequencies, Engle's results notwithstanding. Secondly, not enough is known about the mechanisms generating the data to formulate reliably a specific model such that inference can be reduced to the relatively simple process of estimation within the confines of a known model. Thirdly, over the past thirty to fifty years, evidence has accumulated for nonstationarity in the data series that goes beyond the simple presumption of a trend, or of an integrated process. Fourthly, one might well suspect that economic relationships may well differ over different time scales in that different time horizons are involved. At least one should be prepared to consider that relationships over decades may well involve different levels of evaluation than decisions over the next ten hours. The standard representative agent model, say in the statement of the maximization of consumption over time, assumes that any unit of consumption that is being considered over any time horizon is the same; there is no difference in buying paper clips from buying a house. The distinction here is not on the relative size of the purchases, but the time intervals over which the agent is operating and the focus of the agent when making the decision. In the former case, the decisions are over a time scale of weeks and decade changes in income are irrelevant, the latter is over a time scale of years to decades and temporary fluctuations in income are ignored.

1.2. Money and Income

The interaction between monetary aggregates and income has been used to explore a number of different economic questions, from financial development to the money demand function to the operation of monetary policy. The investigation of these various questions differs with respect to the time period over which money and income are assumed to be related. For example, to solve the basic simultaneity problem encountered in estimating the relationships between money demand and supply, it has been customary to pose the money demand equation as a long run (cointegration) relationship between velocity and interest rates, whereas the short run relationship between money and income is affected by monetary policy and constitutes the money supply relationship.

1.2.1. The "long run" Money Demand Relationship

In recent years, the long run money-income relationship has come under increasing scrutiny. In the U.S., attempts to model the demand for money has progressed from a general agreement about the existence of a stable velocity relationship to concern about what appeared to be systematic over prediction of money balances during the 1970s. Subsequently there emerged further concerns that estimates of the money demand function were tending to under predict balances. Levantakis and Brissinis, [37], provide a thorough survey of these developments.

McMillin, [39], reviewed the evidence for a structural break in M1 velocity widely presumed to have occurred in the early 1980's. He found that the break was due more to changes in the process generating velocity rather than to variability in money and income. Conversely, Bomhoff, [4], using a Kalman filter and annual data for the US over the period 1959-1988 found that estimates using a stochastic trend in M1 are not significantly different from those for a constant trend.

The "institutional hypothesis"[1] on velocity involves an extension of the VAR approach of the error-correction models. Restrictions are placed on the cointegrating vectors in order to identify the structural relationships and to examine parameter stability. These constraints are based on the hypothesis that the long run behavior of velocity depends on the rate of financial development and innovation. Raj, [44], finds that the cointegrating vectors in the countries studied suffer from substantial parameter instability. He argues that "if institutional change

[1] This hypothesis was emphasized by Bordo and Jonung, and builds on Friedman's 1956 formulation of velocity as depending on the the state of the financial sector, see for example, Humphrey's survey.

is a plausible explanation of at least some of the on-going changes in long run velocity,... then it might be impossible to obtain a stable relationship."

Several new approaches are under investigation. Boyle, [6], and Dueker, [19] and [20], explore the idea of variable velocity in the theoretical literature. Barnett and Xu, [3], develop a model in which they explore the behavior of velocity under different models of interest uncertainty. The simulation of their model generates volatile coefficients in the velocity functions. They find that estimates of a random coefficients model with money velocity data produce similar results to their simulations.

A recent article by Serletis, [51], reexamines composite sum and Divisia indices of monthly velocity in a search for chaotic behavior.. He finds that the null hypothesis of a unit root cannot be rejected even after allowing for a break in the level and slope of the trend function. In addition, after removing a unit root and stochastic second order dependence using a GARCH model, he claims to have found weak evidence of chaos in the Divisia L velocity, but see [45], and [46] for a countervailing view of such results.

Artis and colleagues, [2], investigated the stability of money demand functions using spectral techniques. They estimated the evolutionary spectral density using a monthly proxy for M1 velocity by taking the product of industrial production and producer price indices as a substitute for monthly nominal GDP. This procedure was first introduced by Christiano, [12]. Stability tests were performed on the evolutionary coefficients for different frequency levels. For the U.S. they did not find evidence for a structural break in M1 velocity, but did find breaks across a wide band of frequencies for M3 velocity.

1.2.2. The "short run" Supply Relationship

The analysis of the stationary parts of the money and income series is generally understood as an examination of the short to medium run relationship between the two variables. Analysis of "short run" money income relationships have concentrated on testing the strength of Granger causality between various monetary aggregates and nominal or real income.

The seminal piece in this literature is a paper by Sims, [52] who found using quarterly data that money Granger causes real output, where output is measured by the industrial production index. However, when he reestimated this equation in 1980, [53], using the 6 month commercial paper rate, his results changed. The proportion of variance in the real variables attributable to money innovations was much lower than in the original specification. These findings spawned a literature investigating the strength and direction of Granger causality between measures of monetary activity and output. Most of this literature used logarithmic differences with either quarterly or monthly data, in order to obtain short run stationary data.

Stock and Watson, [55], used monthly data from 1960-1985 and found that output was influenced by changes in the growth of M1. Friedman and Kuttner, [26], extended the same data to 1990 and reported that the strength of the Stock and Watson result is weaker using the extended period. The research continued with many papers that tested for improved methods of trend removal, the possibility of cointegrating structures, checked out-of-sample performance, and examined alternative lag structures and the relative strength of other financial variables. Feldstein and Stock, [22], using quarterly data attempted to show that the strongest relationship exists between M2 and nominal GDP.

Spencer, [54], showed that VAR results are very sensitive to trend removal, lag length, and the level of temporal aggregation. Abate and Boldin, [1], concluded that money-output specifications suffer from significant heteroscedasticity and uncorrected serial correlation. Using monthly data on industrial production and alternative monetary aggregates over the period 1960:1-1992:12, Abate and Boldin concluded that "contrary to the claims of some researchers, the money output relationship does not break down in the 1980s and M2 helps forecast into the early 1990s."

1.2.3. The Overall View of the Money Income Relationship

The general conclusion from the literature is that the relationship between money, defined in several ways and output, also defined in several ways, is still not completely understood. This is true both in the long run and in the short. Further, there is disputed evidence on the existence of structural breaks and the possibility for nonlinearity in the relationships. The ambiguity of these results may well be due to the fact that there are several time scales involved in the relationships and that a single dichotomy between "trend" and short run stationary fluctuations may be inadequate to separate out the time scale structured relationships between the variables. There is also the possibility that the overly restrictive dichotomy between trend and stationary components has masked shifting dynamical relationships between the variables. The time scale at which the introduction of checkable interest deposits affect M1 demand is most likely different from the one at which a change in Fed operating procedures affects the money output link, and different from the time scale at which disintermediation might affect M2 velocity.

1.3. The Major Implications for Research

In both streams of literature, the consumption income relationship and that between money and income, several similar strands of difficulty appear. The first has to do with non-stationarity of the data beyond that which might be incorporated in a unit root, or a trend. One does not have to accept, as in the money income relationship, a structural break in order to be concerned about non-stationarity of these data. At both the theoretical and empirical levels, there are many reasons to suspect that the relationships might well have evolved over the decades since the war; nowhere is this more probable than in the money income relationship, especially during the last decade. The evolution and increased efficiency of the capital and the money markets will have affected both relationships. Long run changes in the relative proportions of income spent on various consumption categories will affect the consumption income relationship, as would the obvious effects caused by broad large scale changes in demographics.

The next strand of common problems to the two sets of data involves the partially recognized need for analyzing relationships between variables conditioned on a given time scale of analysis. We have made the case that the choice of time scale will determine not only the time horizon involved in decision making, but also the degree of time aggregation, or averaging, the choice of variables that enter the relationships, and even the direction of causality in that at short time scales the causal relationship may well be from income to money, but that at higher time scales the other way around.

The final strand that links the two sets of data is that in both cases, while there is a core of agreement over some basics in the relationships, there is considerable disagreement over everything else. These disagreements have waxed and waned as different researchers have tried various and ingenious methods for examining the basic relationships. The conjunction of different theories, different data sets, and different econometric technologies, has probably inhibited our ability to learn more rapidly from the observed data. The only matter that is certain is that simple, non-complex, stable relationships between consumption and income and between money and income do not exist, for otherwise they would by now have been revealed and tested many times over.

We conclude this review of the literature by stating that the main requirements of an exploratory analysis involve the recognition that nonstationarity in addition to the so called "unit root" problem are relevant issues; that an allowance for effects that are localized in time is needed, that relationships may well vary by time scale of analysis; and that, given the lack of knowledge about the underlying functional forms involved, nonparametric procedures will be required. This combination of conditions indicates that as an exploratory tool an analysis using wavelets should prove useful and to this topic we now turn.

2. Wavelet Analysis

The discussion to follow is meant to give only an intuitive feel for wavelet analysis and to stress those points that are at the heart of the examination of the statistical properties of the data. For a thorough review of the basics of wavelet analysis Chui, [13], is an excellent reference and Daubechies, [16], contains a detailed analysis of the mathematical properties of wavelets. A useful recent article that helps to link wavelet analysis to more conventional time series analysis is Priestley, [42] and a useful non-technical review article is [50]. An important article that explores the use of the wavelet approach to the estimation of complex signals contaminated by noise is Donoho et al., [18]. Further, Brillinger in [7] develops the relevant distribution theory.

Wavelet analysis has points of comparison and points of contrast to Fourier analysis. Recognizing both is important for understanding what wavelet analysis can bring to the examination of a data series. Both procedures involve the projection of a signal onto an orthonormal set of components, trigonometric in the case of Fourier series representations, "wavelets" that are to be defined below in the case of wavelet analysis. Fourier projections are most naturally defined for functions restricted to $L^2(0, 2\pi)$, since Fourier series have infinite energy, but finite power, when extended to being defined over the entire real line. Intuitively, this is the source of the difficulty that a single disturbance to a signal affects the analysis at all frequencies and that a single disturbance in time is interpreted by Fourier analysis as an event of period T, where T is the length of the observed series. While Fourier analysis allows for the complex superposition of individual harmonics, or "waves", the maintained hypothesis is that over any sub-segment of the observed time series the precise same frequencies hold at the same amplitudes; the signal is as it were "homogeneous over time."

In contrast, the functions that are to be represented by wavelets have finite energy over the entire real line and are naturally defined within $L^2(R)$. This means that the functions involved have narrow support and the functions rapidly converge to zero as the index t approaches $\pm$infinity. The functions allowed in wavelet analysis are not necessarily "homogeneous over time" as is assumed for Fourier analysis. In short, the functions involved need to be represented by "wavelets", that is, "little waves" as opposed to the trigonometric functions that have constant amplitude over the entire real line. However, this approach brings its own difficulty in that functions that do not have narrow support, have to be approximated by a sequence of functions that do. It is this difficulty that leads to the necessity for defining wavelets with respect to specific locations and then considering a sequence of such functions, each indexed by a particular location. By stringing together a sequence of such wavelet functions each localized to a particular position on the time axis, quite complex functions can be approximated.

There is another aspect of wavelets that produces much of their appeal and that aspect derives from the "rescaling" capability of wavelets. Instead of considering a single sequence of functions $g(t, u)$, where u denotes a sequence of positions about which the function $g(.)$ is centered, we consider a double sequence of functions:

$$g(t) = \frac{1}{\sqrt{s}} g(\frac{t-u}{s}) \tag{2.1}$$

where s is a sequence of scales. The term $\frac{1}{\sqrt{s}}$ maintains the norm of $g(.)$ at one. The function $g(.)$ is centered at u with a scale of s; that is, the energy of $g(.)$ is concentrated in a neighborhood of u the size of which is proportional to s. Essentially, as s is increased the length of the support of $g(.)$ in terms of t increases. For example, if the support of $g(.)$ for $s = 1$ is $[-d, d]$ when $u = 0$, then the effect of s is to broaden the support to $[-ds, ds]$. In effect, the rescaling characteristic of wavelets in the time domain is equivalent to the rescaling of frequencies in Fourier analysis. The process is also known as "integral dilation". That is, in Fourier analysis all frequencies are generated by rescaling, or dilating, a single fundamental frequency. If ω_0 is the fundamental frequency, then the function to be approximated is projected onto a sequence of expressions of the form:

$$\{e^{-is\omega_0 t}\} \tag{2.2}$$

In short, 2π-periodic functions are representable by integral dilations of the single function $e^{-i\omega_0 t}$.

Dilation in the time domain has advantages in addition to those stemming from frequency dilations. In the latter case, one is considering projecting the entire signal onto ever lower frequencies. In the former case with wavelets, a localized component of the signal is projected onto an ever broader base. The two important aspects here are that the projection is for a *local* component of the signal and that each projection is onto a wavelet whose support is a function of s. In a sense the choice of a dilation, or scale, level indicates the size of the "packets" used to represent the signal. A broad support wavelet yields information on signal variations on a large scale; a small support wavelet yields information on signal variations on a small scale. One way to visualize the process is in terms of maps in that a large scale map gives the broad picture without details and a small scale map fills in the details. A rapidly oscillating signal might be regarded as a superposition of a sequence of "small packets" of information, whereas a slowly oscillating signal might better be regarded as a superposition of very large packets of information. Projections at a given scale are not affected by features of a signal at scales that require broader support, except for the highest, and ignore features at scales that require narrower support. Resolution is another property of representations which reflects the ability to "resolve" the local signals; that is, to be able to separate nearby frequencies. High resolution requires one to be able to detect small scale variations in the data, but is not to be confused with the concept of "scale" itself.

However, as indicated by Priestley, [42], there is only an intuitive and very indirect connection between frequency and the scale of the analysis. It is only true in the simplest of cases that large scale wavelets are associated with low frequencies in that the detection of low frequencies requires components with very wide support in the time domain. Correspondingly, that the analysis of high frequencies requires a high sampling rate which is provided by components that have narrow support and so provide high frequency samples is also a naive interpretation. Consider for example, a given scale, say 2^3, or eight months, and that the signal contains components that are at that scale. It might still be true that the power of the projections depends in a cyclic manner on the value of u, the index of position; for example, the power of the projections indexed by u might oscillate with a period of ten years, or indeed any period greater than eight months.

More importantly, this intuitive idea of "packets of information" that arises out of the dilation procedure can be used and interpreted, even when a signal cannot usefully be regarded as a superposition of trigonometric components. For example, Priestley, [42], illustrates this idea in the context of a signal with a fixed frequency, but a time varying amplitude; Fourier analysis, of course, incorrectly detects power at all frequencies.

We can therefore consider two ways of examining a given signal. At a fixed point in time, say u, we can examine how the localized signal projects on to wavelets of varying length of support. In effect this procedure examines the signal at the given location for variations in the strength of projections onto wavelets of various scales, or dilations. The other view is to examine how the signal projections vary by position as indicated by the value of u, but at a given fixed scale, or dilation, s.

The literature on wavelets is growing rapidly, but so far in economics the development has been sparse. Goffe, [30], illustrated the application of wavelets to nonstationary data and Gilbert, [29] attempted, using quarterly data, to examine macro relationships for rapid regime shifts. In the two Ramsey and Zhang papers, [47]and [48], an effort was made to use waveform dictionaries to analyze financial data; waveform dictionaries provide a generalization of both wavelets and Fourier analysis. The emphasis in these two papers was on examining the time-frequency (Wigner) distributions. Ramsey et al., [49], pursued an approach that was common in the earlier wavelet literature by querying the statistical self-similarity of financial data. The detection of discontinuities and the occurrence of sharp cusps are explored by Truong and Patil and by Wang in [56], [58], respectively. Further work on financial data and the role of fractional differencing is explored in [32], [34], and [36].

One of the sources of confusion in understanding the role of wavelets is that the wavelet approach can be used in very many different ways, see for example the discussion in [50]. Some of these procedures, or aspects of wavelet analysis, might be useful in economic and financial analysis, but not necessarily all. For example, the early emphasis in using wavelets to explore ideas of self-similarity was not very productive, nor did that analysis lead to very useful insights into economic and financial mechanisms; see for example, Ramsey et al., [49]. Priestley, [42], indicates some of the difficulties involved in shifting from models that are essentially deterministic with low levels of noise to the stochastic processes that are more common in the economics and finance literature, see also for example, the discussion in Donoho et al., [18].

In this paper we are interested in three major facets of wavelet analysis; the ability to handle nonstationary data, localization in time, and the resolution of the signal in terms of the time scale of analysis. The nonstationarity that we are concerned about is a broader notion than the existence of a mere unit root process. Given the discussion above, it is clear that some allowance should be made for variation in the process over time as well as for local effects.

We end this section by summarizing the description of a signal in terms of wavelets and define a few terms that will be used subsequently. There are a variety of functions that have been developed for use as the fundamental wavelet that is to be dilated by s and translated by u. Some examples are:

- Haar, a square wave with compact support;
- Daublets, continuous orthogonal wavelets with compact support;
- Symmlets, a "symmetrical" alternative to Daublets;
- Coiflets, symmetric and with vanishing higher moments.

Depending on normalization rules there are two types of wavelet within a given family, such as the Symmlets; father and mother wavelets.

$$\begin{aligned}\Phi_{j,k} &= 2^{-\frac{j}{2}}\Phi\left(\frac{t-2^j k}{2^j}\right)\\ \Psi_{j,k} &= 2^{-\frac{j}{2}}\Psi\left(\frac{t-2^j k}{2^j}\right)\end{aligned}$$

$$\begin{aligned}Father\ wavelets &: \int \Phi(t)dt = 1 \\ Mother\ wavelets &: \int \Psi(t)dt = 0\end{aligned} \tag{2.3}$$

Father wavelets are used for the "lowest frequency" smooth components, those requiring wavelets with the widest support and mother wavelets are used for the "higher frequency" detail components. In short, Father wavelets in the sequel are used for the "trend components" and the Mother wavelets are used for all the deviations from trend.

Any function $f(t)$ to be represented by a wavelet analysis can be built up as a sequence of projections onto Father and Mother wavelets indexed by both $\{k\}$, $k = \{0, 1, 2, ...\}$ and by $\{s\} = 2^j$, $\{j = 1, 2, 3...\}$. In actual data analysis using discretely sampled data, it is necessary to create a lattice over which the calculations will be made. Mathematically, it is convenient to use a dyadic expansion as illustrated in equation 2.3.

The coefficients in the expansion are given by the projections:

$$\begin{aligned} s_{J,k} &= \int f(t)\Phi_{J,k}(t)dt \qquad (2.4)\\ d_{j,k} &= \int f(t)\Psi_{j,k}(t)dt \\ j &= 1,2,..J \end{aligned}$$

where J is the maximum scale sustainable by the number of data points. The representation of the signal $f(t)$ can now be given by:

$$\begin{aligned} f(t) &= \sum_k s_{J,k}\Phi_{J,k}(t) + \sum_k d_{J,k}\Psi_{J.k}(t) \qquad (2.5)\\ & \sum_k d_{J-1,k}\Psi_{J-1,k}(t) + ... \\ & ... + \sum_k d_{1,k}\Psi_{1,k}(t) \end{aligned}$$

The large J refers to the highest level of dilation that is used for the "low frequency", smooth variation of $f(t)$ and the small j refers to the "higher frequency" detail coefficients. When n the number of observations is divisible by 2^J, then the number of coefficients of each type is given by:

- At the finest scale: $2^1 : \frac{n}{2}$ coefficients $d_{1,k}$;
- At the next scale: $2^2 : \frac{n}{2^2}$coefficients $d_{2,k}$;
- At the coarsest scale: $2^J : \frac{n}{2^J}$ coefficients $d_{J,k}$;
- At the coarsest scale: $2^J : \frac{n}{2^J}$ coefficients $S_{J,k}$;

$$n = \frac{n}{2} + \frac{n}{4} + ... \frac{n}{2^{J-1}} + \frac{n}{2^J} + \frac{n}{2^J} \qquad (2.6)$$

We can summarize the string of coefficients by

$$w = \begin{pmatrix} S_J \\ d_J \\ d_{J-1} \\ . \\ . \\ d_1 \end{pmatrix} \qquad (2.7)$$

However, most of the coefficients of w are zero, or very close to zero; the matrix w is sparse. The wavelet associated with each coefficient is termed an "atom" and each row of w represents the coefficients of a "crystal".

We can define the multiresolution decomposition of a signal by specifying:

- S_J coarsest scale
- $S_{J-1} = S_J + D_J$
- $\vdots$
- $S_{j-1} = S_j + D_j$

$\{S_J, S_{J-1}, ...S_1\}$ is a sequence of multiresolution *approximations* of the function $f(t)$ at ever increasing levels of refinement. The corresponding multiresolution *decomposition* of $f(t)$ is given by:

- $\{S_J, D_J, D_{J-1}, ...D_j, ...D_1\}$.

The sequence of terms: $S_J(t), D_J(t), D_{J-1}(t), ...D_1(t)$ represent a set of *orthogonal* signal components that provide representations of the signal at resolutions 1 to J; each D_{J-k} provides the orthogonal increment to the representation of the function $f(t)$ at the scale, or resolution 2^{J-k}. The sequence of partial sums:

$$S_{j-1}(t) = S_J(t) + D_J(t) + D_{J-1}(t) + ... + D_j(t)$$

provides a multiresolution *approximation* of the signal down to the scale 2^j. Consequently the sequence of terms: $\{S_j(t)\}$, provide a sequence of approximations to the signal that include ever finer scales and ever more detail and so an increasingly closer approximation to the signal; whereas the sequence of $\{D_j(t)\}$ provide the orthogonal increments at each individual scale, or resolution, level.

In addition to the differences in the choice of scale function, Haar, Symmlet, Daublet, and so on, there are other differences that can be fine tuned to suit a particular project. In our case it was felt that symmetry of the scaling function was most important, so that we chose the Symmlet as the basic scaling function. We are able to determine the length of the compact support at the finest time scales; in our case we chose an intermediate value of 11 observations. Another major question is the degree of smoothness of the wavelet and this is in turn determined by the number of vanishing moments; again we chose an intermediate value for the mother wavelets.

The discussion of scale and its variation as reflected in the projections of the data onto the corresponding wavelets merits further discussion. At each scale the strength of that projection determines the magnitude of the corresponding coefficient. In essence, one is extracting at the chosen scale those components of the data that resonate most strongly at that scale. There is, however, only a weak relationship between "scale" and period, or frequency, of oscillation for data that contain spectral power within certain frequency bands. It is true that very small scale projections are best at representing behavior at very high frequencies, or very short periods, and that very long scales are best at representing oscillations with very long periods. But this link between scale and frequency is only interpretable when the data are stationary; the wavelet projections are still meaningful even if there are no oscillations in the data. The view using wavelets is that the total variation of the data centered at any given point in time is obtained by "adding up" the components extracted at each of the admissible scales. An alternative view is that obtained by aggregating the data to various levels using a windowing technique, which can in turn be interpreted as a projection on to the vector $\{1/N, 1/N, ...1/N\}$, where there are N observations selected to be in each projection, $N = \{2, 3, 4, 5...\}$. However, by taking this view one soon recognizes the non-optimality of such a sequence of projections, in that at the very least orthogonality of the projections has been lost, not to mention the fact that one does not have a basis. Consequently, the wavelet approach potentially adds to our understanding of the relationships between variables by enabling us to separate the relevant time scales that may be involved.

3. The Data Used in the Analysis

The details of the data used and their sources are listed in Table 3.1. Monthly data, seasonally unadjusted where available, were used in order to have a sufficiently high sampling rate to carry out the wavelet analysis, to introduce as little preprocessing of the data as possible, and to allow for the effects of short term dynamics on the results. The real interest rate was calculated by subtracting an annual inflation rate from the U.S. Treasury Bill rate for 1 year bonds. The expected inflation rate at time t was proxied by the actual inflation rate from time t to time $t + 1$. The inflation rate is based on the implicit deflator for personal consumption expenditures on durables.

Name	S.A.	Dates	Units	Source	Code
Disp. Pers. Inc. (real)	Y	1960:5-1994:4	Bill. 1987$	Citibase	GNYDQ
Nominal Pers. Inc.	Y	1960:5-1994:4	Bill. $	Citibase	GMPY
Pers. Consump. Exp.	Y	1960:5-1994:4	Bill. 1987$	Citibase	GMCQ
PCE - Durables	Y	1960:5-1994:4	Bill. 1987$	Citibase	GMCDQ
PCE - Non-Durables	Y	1960:5-1994:4	Bill. 1987$	Citibase	GMCNQ
PCE - Services	Y	1960:5-1994:4	Bill. 1987$	Citibase	GMCSQ
M1	N	1960:5-1994:4	Bill. $	Citibase	FMZ1
M2	N	1960:5-1994:4	Bill. $	Citibase	FMZ2
U.S T-bill Maturity-1yr	N	1960:5-1993:12	% per ann.	Citibase	FYGT1
PCE-Imp Defl (Durables)	Y	1960:5-1994:4	1987=100	Citibase	GMDCD

Table 3.1: The Data Used in the Analysis

4. The Methods Used

Our main interest is in the reconstructions of the time series by crystals and the relations between them. The process is exploratory. Our objective is to examine the extent to which an allowance for different effects by scale and for variations in the relationships over time lead to insight into the total variation of the signal over time. Our conclusion is that these topics merit further study and the development of procedures that are designed for the classes of statistical requirements that are indicated by our preliminary analysis.

We used the Wavelets package produced by StatSci of MathSoft that was written by Bruce and Gao, [8]. We choose as the basic wavelet the Symmlet, designated "S12". This wavelet is a compromise between competing requirements. S12 is nearly symmetric, is intermediate in support length with eleven units, has five vanishing moments, and is twice differential. This choice of wavelet is an intermediate choice in that it has reasonably narrow compact support, is fairly smooth, is nearly symmetric, and has a moderate degree of flexibility. We experimented with alternative choices of scaling function and of wavelet, but the qualitative results were very robust to such changes and the initial choice of wavelet seemed to be the best on balance. We used the boundary condition "infinite" to reflect the fact that the data contain a "trend" component.

The empirical results are presented in two parts, the first is for the relationship between consumption and income and interest rates. The second is to provide an examination of velocity in terms of its behavior at different time scales and to examine the composition of the dynamics of velocity with respect to the dynamics of its constituents, money and of income. With each set of data the idea is to examine in some detail the relationship between the variables when the variation in each variable has been restricted to a specific scale. For example, instead of looking at the relationship between consumption and income "averaged" over all time scales, we examine the relationship at each time scale separately. Similarly, we examined the relationship between money and income by individual time scales instead of examining that between income and money averaged over all time scales.

5. The Consumption Income Relationship

Tables 5, 5.2, and 5.3 show the coefficient estimates from running a sequence of least squares regressions of consumption on disposable income using the data described in the previous section. Figures 8.1 to 8.3 illustrate some of the results obtained.

In Figure 8.1 is portrayed the time paths of durable and non-durable goods consumption at the S5 level; that is, this level includes the long term drift in the data with respect to the longest time scale available for these data, $2^6 = 64$ months using the father wavelet coupled with the deviations induced at the scale of 2^6 months using a mother wavelet. Figures 8.2 and 8.3 illustrate the relationship between total consumption and income at scales varying from $2^5 = 32$ months to 2 months using mother wavelets. Recognize that at each scale level one is seeing the "isolated" effect of the given scale; rather at each scale, the graphs indicate the relationship between total consumption and income where the variation in both variables has been restricted to the indicated scale. The graphs indicate clearly that except for the highest scale, there is only a weak relationship between the variables; indeed below D4 there seems to be little of significance. The regression results reveal somewhat more structure, but nevertheless confirm that below S5, the relationship is very noisy.

A review of Table 5.1 indicates that the degree of fit of the regression of total consumption on income that is shown in Figure 8.1 falls as we move to shorter time scales and the slope coefficient declines to zero as the time scale of analysis declines. There is an exception to this pattern in that at the very lowest level of time scale analysis, the degree of fit is greater than the previous level and the slope coefficient, which is statistically significant at even high confidence levels, is commensurate with that obtained at a level between D2 and D3. The intercept terms are very small and operationally insignificant, even where formally significant in a statistical sense.

Tables 5.2 and 5.3 portray these relationships by time scale in terms of a decomposition of total consumption into non-durable goods and services in one table and durable goods in the other table. The general conclusions that were obtained with respect to total consumption are confirmed with respect to the components. In each case the intercept term is insignificant in effect and is statistically insignificant for all regressions past the "trend fit". The slope coefficient for non-durable goods and services has a maximum of 0.78, but the pattern for degree of regression fit and the size of the slope coefficient across time scales reflects the same pattern observed for the total consumption variable.

The pattern of results for durable goods is similar to that for non-durable goods, but with some interesting differences. As expected the slope coefficient for durable goods is much lower than that for non-durable goods at all time scales. However, there is the interesting difference that the slope coefficient at the highest time scale is less than that at the next level down. Or to express the matter another way, the marginal propensity to consume for durable goods has a maximum value at a time scale of $2^5 = 32$ months, which may well be quite plausible in that cars and household appliances dominate the durable goods index and a time horizon of three years seems in this context to be reasonable. Further, examining Figure 8.1 for durable goods there is clear evidence of a shift in the relationship between consumption and disposable income that occurred over the period January 1979 through August 1981; the slope that prevailed before this extraordinary movement was regained by the end of the eighties. The trend for non-durables and services does not exhibit similar behavior.

In the discussion above, we mentioned that the plot for D5 is very different. The explanation is to be found in Figure 8.4. This Figure shows the time series plots of both consumption and income superimposed on each other. The interesting aspect is that from about 1976 on variations in income and consumption at this time scale were in phase. But before this period, income and consumption were out of phase, although moving into phase. We also note that at this time scale, over the period during which the variables were in-phase, consumption was less volatile than income. The reverse was true when the variables were out-of-phase. In Figure 8.5 we relate these events to the official NBER business cycle dates for each of income and consumption; the NBER dates are shown in Table 5.8. It is interesting to note that at this time scale at least, it is consumption that is in phase with the business cycle peaks and troughs and income is in phase only after it has become in phase with consumption.

Crystal	Intercept (std. err.)	Slope Coefficient (std. err.)	R^2
S5	-63.448 (6.206)	0.934 (0.002)	0.997
D5	0.217 (0.539)	0.669 (0.029)	0.573
D4	-0.032 (0.355)	0.338 (0.030)	0.239
D3	0.029 (0.272)	0.224 (0.028)	0.139
D2	0.0173 (0.270)	0.034 (0.027)	0.004
D1	0.001 (0.297)	0.143 (0.029)	0.057

Table 5.1: Regressions of Total Consumption on Income for Individual Crystals

The result obtained above introduces an important generalization to modelling relationships; namely the need to allow for variations in the timing of relationships. Thus, in this example, we might need to consider:

$$C_t = \beta Y_{t-d(X)} + \varepsilon_t$$

where d(X) is variable lag function that might well indicate a lead relationship for suitable values of X. The variable(s) represented by X could include institutional factors, variations in anticipated price changes, or interest rate changes. The idea is that besides having to model the relationship between the levels of variables, we may also have to consider the timing relationship as an added component. For example, during the early period following the first oil price shock in 1973, the then Federal Energy Office announced that in three weeks the price of gasoline would be allowed to rise; supplies fell dramatically and almost instantaneously and demand rose as precipitately, so that excess demand was phenomenally high and the rate of change was very swift. Both sides of the market were trying to rearrange the **timing** of their market actions as well as in this case of wishing to change their level of activity. Further, anticipated changes in interest rates, or credit constraints can lead to a change in timing of relationships.

Crystal	Intercept (std. err.)	Slope Coefficient (std. err.)	R^2
S5	44.72 (3.253)	0.783 (0.001)	0.999
D5	0.195 (0.316)	0.392 (0.017)	0.572
D4	-0.032 (0.249)	0.203 (0.021)	0.188
D3	-0.00 (0.128)	0.128 (0.017)	0.120
D2	0.019 (0.142)	0.020 (0.014)	0.005
D1	-0.01 (0.170)	0.085 (0.016)	0.062

Table 5.2: Regressions of Non-Durable Goods and Services Consumption on Income for Individual Crystals

Crystal	Intercept (std. err.)	Slope Coefficient (std. err.)	R^2
S5	-108.164 (3.754)	0.152 (0.001)	0.965
D5	0.021 (0.284)	0.277 (0.015)	0.453
D4	-0.001 (0.153)	0.135 (0.013)	0.211
D3	0.029 (0.190)	0.096 (0.019)	0.057
D2	-0.001 (0.213)	0.014 (0.021)	.001
D1	0.002 (0.230)	0.058 (0.022)	0.016

Table 5.3: Regressions of Durable Goods Consumption on Income for Individual Crystals

	Intercept (std err)	Income (std err)	Interest Rate (std err)	Interaction (std err)	R^2
S5	-31.3	0.054	-11.65	0.003	0.940
	5.15	0.002	1.76	0.001	
D5	-0.072	0.064	-0.149	-0.000	0.528
	0.056	0.003	0.071	0.002	
D4	-0.020	0.014	-0.585	0.011	0.166
	0.048	0.004	0.075	0.006	
D3	-0.003	0.015	0.275	0.001	0.143
	0.027	0.003	0.048	0.003	
D2	-0.003	0.009	0.171	0.006	0.058
	0.025	0.002	0.052	0.004	
D1	0.002	0.008	-0.071	0.001	0.027
	0.026	0.003	0.087	0.008	

Table 5.4: Regressions of Furniture and Household Equipment on Disposable Income, the Real Interest Rate and an Interaction Term for Individual Crystals

	Intercept (std err)	Income (std err)	Interest Rate (std err)	Interaction (std err)	R^2
S5	13.02	0.048	-16.52	0.005	0.954
	4.41	0.002	1.50	0.001	
D5	-0.188	0.180	0.152	0.030	0.289
	0.266	0.015	0.339	0.010	
D4	-0.001	0.097	-0.151	0.009	0.211
	0.111	0.010	0.175	0.013	
D3	–0.007	0.063	0.565	0.072	0.074
	0.177	0.018	0.314	0.020	
D2	-0.024	0.008	-0.925	0.032	0.016
	0.195	0.019	0.415	0.035	
D1	-0.014	0.036	-1.142	-0.007	0.015
	0.217	0.022	0.739	-0.087	

Table 5.5: Regressions of Motor Vehicles and Parts on Disposable Income, Real Interest Rate and an Interaction Term for Individual Crystals

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	Intercept (std err)	Income (std err)	Interest Rate (std err)	Interaction (std err)	R^2
S5	-19.13	0.028	-2.82	0.001	0.993
	0.80	0.000	0.27	0.000	
D5	-0.018	0.026	0.242	0.001	0.438
	0.033	0.002	0.042	0.001	
D4	-0.005	0.017	-0.124	0.016	0.104
	0.036	0.003	0.057	0.004	
D3	-0.002	0.007	0.044	0.003	0.026
	0.025	0.003	0.044	0.003	
D2	-0.004	-0.004	0.104	0.003	0.011
	0.032	0.003	0.067	0.006	
D1	0.001	0.008	-0.462	0.025	0.068
	0.031	0.003	0.104	0.012	

Table 5.6: Regressions of Other Durables on Income, the Real Interest Rate and an Interaction Term for Individual Crystals

	mean	median	st. dev.
Durable Goods Consumption: Total	274.5	256.1	113.3
Durable Goods Consumption: Furniture et al	99.6	90.45	40.6
Durable Goods Consumption: Motor Veh. et al	126.0	118.3	45.7
Durable Goods Consumption: Other Durables	48.9	50.4	21.3
Real Interest Rate	2.86	3.67	3.11

Table 5.7: Summary Statistics for Durable Goods Consumption and the Real Interest Rate

Trough	Peak
	1960:4
1961:2	1969:12
1970:11	1973:11
1975:3	1980:1
1980:7	1981:7
1982:11	1990:7
1991:3	

Table 5.8: NBER Business Cycle Dates

5.1. Decomposition of Durable Goods Results by Category and the Role of Interest Rates

As was noted above, the relationship between consumption and income shown in Figure 8.1 indicated a dramatic shift in the relationship beginning in about 1979. This observation is worth exploring further and in addition it would be productive to examine durable goods consumption as a function of the real interest rate, notwithstanding the negative results in the current literature. Tables 5.4, 5.5, and 5.6 summarize the regression results. The three sub-categories of durable goods consumption were furniture and household items, automobiles and auto-parts, and the category "other", which includes jewelry, boats and sports equipment; summary statistics on the sub-series are shown in Table 5.7.

The main issue to be addressed is whether the shift in the relationship between income and consumption can be explained by the variation in the real interest rate. A corollary investigation is whether real interest rates play a more important and longer term role in the demand for durable goods as consumer theory would indicate.

Before beginning the main investigation, we observe that the qualitative properties that were observed for all the categories combined are reflected in the three sub-categories. That is, the degree of relationship as measured by R^2 declines with the reduction in scale. The strength of the effect of income on consumption as measured by the magnitude of the regression coefficient also declines with the reduction in scale; all intercept terms are statistically indistinguishable form zero, below the S5 scale. At the S5 scale the intercept terms are both statistically significant and substantial in size relative to the respective mean values. There is some indication that the relative importance of furniture and equipment is increasing relative to motor vehicles and parts.

Figure 8.6 shows the time series plots at the S5 level of the three sub-categories of durable consumption, furniture, autos, and "other durables." with straight line approximations of the pre 1976 trend superimposed as a visual reference for the shift in trend after 1979. One notices immediately that the three categories behave in different ways after 1979. Furniture after a minor slackening in consumption, increases at a faster pace than before. Autos are the most affected and account for much of the "dip" shown in the total durable goods index in Figure 8.1. The "other" category has a more moderate fluctuation.

One common reaction to observing this situation is to rationalize the result as a reaction to an "oil price shock". We are unpersuaded that this is a reasonable first line of attack because the timing of the change is apparently anticipatory of the actual oil price rise. Further, the far greater price rise in real terms that occurred in November 1973 that was even more of an unanticipated shock did not produce a similar reaction to automobile purchases. More convincing is the observation that net of our allowance for the interest rate effect to be discussed below there is very little evidence for any special "shock."

The model that was used to investigate the potential relevance of interest rates at each scale level was a linear model that allowed for an interaction term between interest rates and income. If we define consumption by C_{it}, for each of the three categories of consumption, Y_t as real disposable income, and r_t, as the real interest rate, the regression model assuming an error term e_{it} that is approximately Gaussian in distribution is:

$$\begin{aligned} C_{it} &= \alpha_{0,i} + \alpha_{1,i}Y_t + \alpha_{2,i}r_t + \alpha_{3,i}Y_t r_t + e_{it}, \\ i &= 1, 2, 3. \end{aligned} \tag{5.1}$$

Analysis of the residuals indicated that the assumption of approximate Gaussianity was justified.

Before examining the interest rate coefficients themselves, the pattern of coefficient values for income are very interesting in the case of motor vehicles and parts. The size of the income coefficient for the scale of $2^5 = 32$ months is nearly four times greater than for the trend levels and about twice for the next lower scale level. Furniture and household equipment exhibits this "business cycle " scale concentration to a very limited extent and the "other" category, like non-durable goods, illustrates a monotonic decline in income coefficient values. Consequently, it appears that the anomalous coefficient behavior for durable goods as a whole is explained almost entirely by motor vehicles and parts. This would make some intuitive sense in that one can easily believe that relative to the other components, automobile consumption is most likely best analyzed on a three year time scale.

The interest rate effects, reported in Tables 5.4 to 5.6, were revealing. First of all, the interest rate effect at the highest scale was strongest for automobiles, considerably less for furniture and least for "other", even after allowing for the differences in units of measurement. Not only is the interest rate very highly significant statistically, its effects on the levels of consumption at the top scales is considerable. In all cases the interest rate effect falls off dramatically as the scale is reduced from sixty four months and in most cases the interest coefficient becomes insignificant, this is particularly true for motor vehicles and parts.

"Furniture and household equipment" provides an interesting contrast. The interest rate is statistically significant with a negative sign to the D4 level, or down to a scale of sixteen months. However, for variations within a year, time scales of four and eight months, the interest rate effect is seemingly positive and significant. One might speculate that these results represent timing effects in that with interest rates rising, customers are able to buy at the old interest rate when suppliers in anticipation of a demand downturn attempt to decrease inventory levels.

The only real puzzle is provided by the regressions for "other goods" in which there is a strong positive interest rate effect at business cycle time scales. The remaining interest rate coefficients are statistically insignificant. This result could be interpreted as a substitution effect in which the demand for furniture and motor vehicles rises with a lowering of interest rates so that there is a corresponding decrease in the demand for "other goods".

If we examine the role of the interest rate at *all* time scales using S4, that is, we are examining the relationships at all time scales up to thirty two months, then for every category the interest rate effect is statistically insignificant; never above a t-ratio of 1.4. These results help to explain the non-significance of the real interest rate variable in regressions using the variables at all time scales; the lack of relationship that holds at the shorter scales dominates the theoretically expected results at the very highest scales.

A similar regression using non-durable goods also indicated that at the highest scale there was a strong interest rate effect of the correct sign with a t-ratio of 6.1, but that for all scales up to S4, there was a marginally significant positive interest rate effect with a t-ratio of 2.5. One might hazard the idea that at business cycle frequencies and below, high interest rates by dampening long term durable goods consumption, free up resources for short term non-durable goods consumption. However, the empirical support for this result is not clear, so that these remarks are purely speculative until confirmation can be obtained.

In any event, the overall results unambiguous show that the interest rate effect is a long time scale effect, not a short, and has its maximum impact on those components that intuitively one would expect to be most interest rate sensitive. With respect to income it is still true for automobiles and furniture that the maximum income effect is at level $2^5 = 32$ months, rather than at sixty four months, whereas the "other" category, more closely follows the monotonic decline found for non-durable goods.

6. Money and Income by Time Scale

In this section we use the wavelet decompositions of monetary aggregates and nominal income in order to investigate whether there are distinct differences in the relationship between these variables at different time scales. We also wish to explore the dynamical relationships between the variables at each scale in more detail.

We note a difficulty in that monthly measures of nominal GDP are not available. Christiano [12] and Artis et al [2] both used as a proxy the product of the industrial production and producer price indices. We used an alternative proxy for nominal GDP namely, nominal personal income, NPI, which is available in monthly data. The series used are listed in Table 3.1. Our proxy obviously underestimates nominal GDP because we have left out business income. However, a regression of GDP on our proxy and on the index produced R^2 of 0.996 and 0.978 respectively and a regression of log differenced GDP on log differenced NPI and on the log differenced index produced R^2 values of 0.123 and 0.020 respectively. While NPI as a proxy for the monthly values for GDP leaves much to be desired in terms of the detail variation, the proxy seems to be closer to monthly GDP than the index. Further, if the remarks that are to follow concerning results are restricted to our definition of income little confusion should be occasioned. Our results should be easily interpretable for the definition of income used and should be indicative for the measure of income that we could not use.

Although the following analysis concentrates on wavelet decompositions of the monetary aggregates and nominal income individually, we also examine the wavelet decompositions of velocity measures using the ratio of nominal income to money balances. Figure 8.7 plots the raw data time series for M1, M2, Nominal Personal Income and the two velocity measures in order to remind the reader of the major characteristics of these data.

The remainder of this section is in three subsections; the first provides an overview of the wavelet results and addresses the issue of whether there is any wavelet evidence for a structural break in velocity. The second examines the spectral analysis of the time scale components and the third section examines the effect of time scale on the evidence for Granger causality.

6.1. A Review of Wavelet Multiresolution Decompositions

Figures 8.8 to 8.12 are time series plots of the individual crystals for M1, M2, Nominal Personal Income, V1, and V2; where V1 is velocity defined with respect to M1 and V2 is velocity defined with respect to M2. In order to provide a comparison using the standard analysis, we have included in each set of graphs plots of the log first differences of each variable

The series of plots fall quite naturally into three broad groups; the resolution of "trend" at the very longest scale possible with these data series, namely sixty four months, a second group that involves the scales from D6 to D4, sixty four months (of differences) to sixteen months, and finally the finest scales, D3 to D1.

Beginning our review of results with the first group, we note that as would be anticipated, the natural variables, M1, M2, and nominal personal income (NPI), all have approximate exponential growth paths. The two velocity measures are more interesting in that both indicate strong deviations from steady growth. For V1, velocity reaches a peak in 1981 and has been declining since. There is no evidence at this scale of a structural break, although it is clear that one can no longer support the hypothesis of a steady increase in velocity. Whether the observed time path exhibits a moderately slow change in parameter values, or whether the results indicate a more subtle relationship between M1 and NPI is an open question.

Somewhat more interesting is the observation that at the scale S6 for the V2 variable, there is a very definite cycle in the upward growth of velocity with a period of oscillation of 17 years. In interpreting this result, one should recall that our measure of income does not include corporate income.

The next group of scales include the time scales for deviations around the longest time scale and vary from sixty four months to sixteen months. The reader should recognize that the scales on the axes for all these plots are all very much smaller than was true for the "trend " scale; this is not unexpected given the design of the crystals, but should be noted to facilitate interpretation of the results. One can best summarize the plots for the natural variables by remarking that the variance before 1980 was in all cases very much less than that for the period after 1980. However, for several crystals it is also clear that the increase in variance began before 1980, but was not clearly evident until well after 1980. The evidence for a rapid rise in variance of the time scale paths is best for M1. An hypothesis stimulated by these observations is that there were changes in the time paths of the variables M1, M2, and NPI, that were exacerbated, or at least added to, by the Volker experiment during 1979-1981. In short, the seeds of the change observed so clearly in the post 1981 period seem to have had their beginnings, albeit subtle beginnings, before that time. At least this possibility should be considered. There is no obvious evidence from the distribution of coefficients for a structural break in the data at these time scales.

The last group of scales involves variation in the data over periods of eight months or less. In this group of time scales as well, the evidence for the hypothesis that 1980 marked a transition period, rather than a structural break, is even more clear. One of the most striking features of this group of time scales is the remarkable periodicity in V1 that occurs at the four and two months scales,but that is not apparent in the constituent variables. This periodicity does not occur in V2 at these time scales.

We may also capitalize on the orthogonality of the crystals to decompose the total variance of each variable across the component time scales; this decomposition by variance is shown in Figure 8.13. In all cases the greatest percentage of the total variance is explained by the highest scale, S6. Consequently, we have shown in Figure 8.13 the variance decomposition by time scale only for the crystals D6 and lower scales. We see immediately that the energy decomposition for M1 is quite different than that for M2. For M1 the biggest contribution is at the D6 level, but for M2 it is at the D5 level, and even D4 has a larger contribution than does D6. In short, there is much more relative variation in M2 at the scales D5 to D3, that is, between eight and thirty two months, than is true for M1. The variance decompositions for the velocity measures are different between M1 and M2 as well, but the contrast for velocity is not as sharp as it was for the money supply measures themselves.

6.2. Spectral Analysis of the Crystals

We mentioned on two occasions above the presence of well pronounced oscillations in some of the crystals. In this section we explore this observation more fully and examine some of the dynamical relationships more carefully. Table 6.1 summarizes the major spectral peaks found at each scale level for each variable. We see that M1 and M2 have the same period at all levels, at least approximately, and that they are in phase. However, M1 and NPI have quite different periods at the D6 and D4 levels; but there seems to be a common period at the D5 level. In Table 6.1 the figures listed in bold are those that do **not** appear in a spectral analysis of the log first differenced data. Consequently, we see that decomposing the data by time scale reveals oscillations that are not apparent in the raw data. It is only at the very lowest time scales and hence at the very highest frequencies that we observe the same evidence for periodicity in both the differenced data and in the time scale decomposed data.

An obvious and important question for understanding the relatively short run relationship between M1 and NPI, is to speculate about the mechanism that is producing such precise cyclical outcomes. The question is more interesting than that there are three pronounced cycles in V1 velocity data. The interesting aspect is that these periods do not appear in either M1 or in NPI, so that the observed periodicities are produced by the very short run dynamics linking M1 to NPI.

Another example of interactive dynamics producing interesting results is illustrated by the relationship between M1 and NPI at the time scale of D4. Figure 8.14 has overlaid the plots for the D4 crystals for both M1 and NPI. What is interesting here is that at this time scale the two variables M1 and NPI repeatedly move in and out of phase with respect to each other over the sample period. In part this is due to the fact that at this scale the two series have different spectral peaks; that of NPI is approximately twenty four months and that of M1 is approximately twenty one months. Consequently, the whole series of observations is made up of stages in which the two series are moving into phase and periods when they are moving out of phase. Any stable relationship that might exist between these two variables at this scale will have to have incorporated into it an allowance for this difference in period between the two series. This analysis is too simple in that for each variable the spectral peaks are broadly based because there seems to be a certain amount of frequency variation about the modal frequencies. If in these data there were only a simple difference between two frequencies with periods of twenty one and twenty four months, the period from fully in phase to fully in phase again would be every 189 months, or every fifteen years and nine months. The observed matching of phase seems to be about every twelve years approximately.

If we calculate the coherence and phase relationship between the two variables at this scale, the results indicate very high coherence, almost one, for periods of one hundred months, and for period ranges of 10 to 7.4 months, 4.5 to 3.8 months, 2.86 to 2.6, and 2.1 to 2.0; these high coherence ranges are very precisely demarcated; see Figure 8.15. The phase spectrum increases monotonically over the range from zero frequency to π, rotating about $\pi/4$ radians. We define the group delay, or envelope delay by:

$$d(\omega) = -\{\frac{d\phi_{12}(\omega)}{d\omega}\}$$

where $\phi_{12}(\omega)$ is the phase spectrum between the two series. In the simple situation in which one of the variables is linearly related to the other with a lag of "R" months, the group delay would be a linear function of frequency with a slope coefficient of "R". In the more general situation, as exemplified here, the group delay is no longer a simple constant, but varies with frequency. A group delay of this type can be created by relating two variables with multiple lags, or by allowing the lag structure to vary over time.

We note that the derivative is greatest at low frequencies and is much less at high frequencies. We can approximate the change in phase into two nearly linear segments; a low frequency range and a high frequency range. At the D4 time scale and for a period of ten to fifteen months, or longer, the approximate slope is 67.7 months and for periods less than ten months the approximate slope is 6.6 months. We can roughly summarize this result at the D4 time scale by saying that at relatively low frequencies, or equivalently at relatively long periods, money leads income by about five years and six months and at the shorter periods money leads income by about six months.

As we considered in the consumption income relationship, we should also in this case consider the potentiality that the timing of the relationship between money and income may change over time; that is:

$$m_t = \beta y_{t-d(x)} + \varepsilon_t$$

where, m_t and y_t are real logarithms of money and of income respectively and, as in the discussion on the consumption income relationship, d(x) is a variable lag that for suitable values of "x"could well be a "lead". A potential explanation for such a timing function d(x) in the money income relationship is provided by variations in Federal Reserve policy. For example, the Federal Reserve might change from a reactive to a proactive policy with a consequent change in the timing of the relationship between changes in monetary actions and variations in income. Alternatively the Federal Reserve might have asymmetric strategies between tight and loose monetary policies, so that the lead/lag relationship might well vary over the business cycle. Certainly, these issues are worth exploring more fully and their existence is plausible.

6.3. Granger Causality

As we documented above, Granger causality tests have provided over two decades of debate concerning the possible line of causality between money, however defined, and income. Given the discussion above, two aspects of wavelet analysis might well provide some insight into this matter. First, it is likely given our experience so far that separating money and income into time scales and analyzing the

Series	D6	D5	D4	D3	D2	D1
M1	**81.6**	**58.3**	21.5	13.2	8.5, 7.6	2.9
M2	**81.6**	**58.3**	**20.4**	**13.6**	**8.3**, 7.7, 5.7	3.7, 2.9
Nom. Personal Income	**102.1**	**58.3**, 51.0	**24.0**	15.7, **12.4**	*diffuse*	*diffuse*
V1 = NPI/M1	**82.8**	**59.1**	**23.0**	**11.8**	6.0, 4.0	3.0
V2 = NPI/M2	**81.6**	58.3	**24.0**	15.7	**6.8**	2.7

Table 6.1: Spectral Peaks in Individual Crystals for Each Variable Identified by Period in Months. Bold Entries Indicate Peaks Missing in The Spectrum of Log. Diff. Data

relationships within time scales will provide considerable insight into the mechanism linking money and income. Secondly, the research results quoted in the previous section indicate that another potential complicating factor stems from the variation in phase between the variables at certain time scales. In this section we will investigate each of these aspects.

Table 6.2 summarizes the first set of results. We ran Granger causality tests using the version cited in [31] and restricted attention to M1 and NPI. The joint F tests for the inclusion of lagged values of income in an AR representation of the money equation and for the lagged values of money in the AR representation of the income equation are quoted in the third and fourth columns of Table 6.2. The null hypothesis for each F test is that the added coefficients are zero and therefore that lagged income does not reduce the variance of money forecasts or that lagged money does not reduce the variance of income forecasts. If neither null hypothesis is rejected the results are quoted as "inconclusive" and if both pair of F tests reject, the result is labelled as a "feedback " mechanism. A unique direction of causality is indicated only when one of the pair of F tests rejects and the other accepts the null hypothesis.

The results indicated in Table 6.2 are very interesting. At the very finest scale, D1, the evidence indicates (weakly) that NPI Granger causes M1, but at the intermediate time scales represented by D2 to D4, we observe with varying degrees of strength that M1 Granger causes income. At the highest levels of time scale, the test results indicate that the mechanism is a feedback one. These results are intuitively plausible. At the finest time scale, it is reasonable to suppose that variations in economic activity initiate corresponding changes in checking account balances, so that one concludes that NPI Granger causes M1. At the higher time scales, the monetary authorities are trying to control the money supply so that it is plausible that M1 Granger causes NPI. At the very highest scales it might well be true that M1 and NPI interact in a feedback mechanism. Consequently, we have evidence that not only is it true that the degree of causal relationship varies across scales, but that the *direction* of causality differs by time scale. Examining the log first differences for the historical period under examination, we obtain the unremarkable result that the Granger causality tests are inconclusive. The various outcomes obtained for different time scales provides one explanation of this result in that the differenced data averages over the individual scale effects, so that the change in direction across scales can easily lead to an inconclusive outcome for a time aggregate expressed in differenced form.

A second question that arises from this analysis concerns the dynamical links between the variables and the way in which they may vary over time. We may question how sensitive our results are to the choice of time period over which we perform the tests. For example, using the data in log first difference form, one can show that M1 Granger causes NPI during the period 1970-1979, but that the tests are inconclusive over the period 1980-1994. We also examined in light of this result the Granger causality tests for M1 and NPI at the D1 level with the data separated into two periods; 1970-1979 and the 1980-1994. For the former period, we obtain that M1 Granger causes NPI, but that for the latter period, the situation is reversed in that NPI Granger causes M1. Given these results, we decided to explore the sensitivity of Granger causality tests to variations in phase between any two variables. Table 6.3 presents the results of one such investigation.

Refer to Figure 8.14 in which are plotted the D4 crystals for M1 and NPI. We have mentioned in the previous section the differences in fundamental periods between the two series; Figure 8.14 plots out as a pair of time series the two variables using the D4 crystals. It is immediately clear that the two series at this time scale are moving into and out of phase with each other. Consequently, depending on whether the series are moving into phase, or out of phase, one is likely to obtain different results from a Granger causality test, depending on which part of the "cycle" is included. The idea is to go beyond the usual observation that different causality results can be obtained from different segments of the data. Our point is that a key element in obtaining different causality results in the presence of phase varying series is that different causality results depend on one's selection, implicit, or otherwise, of the sequence of phase relationships. Table 6.3 illustrates this notion. We observe that if the series are moving into phase, Granger causality tests indicate that M1 causes NPI, whereas when the series are moving out of phase, one obtains with the same tests the opposite result, NPI causes M1; and that if one picks a period in which the series move into and out of phase with each other, Granger causality tests can produce an apparently inconclusive result.

One lesson here is that results that are apparently sensitive to the subset of data that are chosen might well be caused by the effects of the variation over time of the phase relationship between the two variables, even after one has allowed for different effects across time scales. With respect to the different results obtained before and after 1980, we note from Figure 8.14 that 1970 to 1979 is a period in which the series moved from out of phase to in phase; whereas the period after 1980 is characterized by a move from in phase to out of phase.

Our results have not yet been thoroughly demonstrated and much more careful analysis will be needed to substantiate the tentative results that we have quoted in this section. However, the ideas that we have raised merit serious consideration, both from the econometric perspective and from a theoretical one. The role of variations in the phase modifying the structural relationship between variables may not only provide more parsimonious models of the relationship, but should also lead to interesting developments in the theory underlying money income relationships. Indeed, in general, the role of phase variation should be examined more carefully in that allowing for such variation may resolve many current anomalies in the literature.

7. Summary and Conclusions

A key element in any set of economic time series of any length is the potential presence of non-stationarity beyond the notion of a unit root. Further, the theoretically imprecise statement of the functional relationships between economic variables has led many researchers to consider non-parametric procedures. An important class of such procedures is that provided by wavelets. More importantly, wavelets have some characteristics that make them particularly suitable as a vehicle for analyzing economic data. Because of the translation and scale properties, non stationarity in the data is not a problem when using wavelets and prefiltering is not needed. Further, because of the flexibility in choice of basis function; that is, choice of wavelet function, and because of the property of "narrow" compact support, wavelets are particularly well suited to handle complex signals that involve cusps, discontinuities, and rapid changes in modeling regime.

For the objectives of this paper, an even more important property of wavelets involves the separation of time scales of variation into a sequence of scales that can

	Results	Null Hypotheses	
		M1$\nrightarrow$NPI	**NPI$\nrightarrow$M1**
D6 (5 lags)	*feedback*	6.398 (0.000)	6.968 (0.000)
D5 (20 lags)	*feedback*	4.491 (0.000)	5.242 (0.000)
D4 (19 lags)	M1$\Longrightarrow$ NPI	3.334 (0.000)	**0.809** (0.695)
D3 (17 lags)	M1 $\Longrightarrow$NPI	1.838 (0.023)	**1.294** (0.193)
D2 (23 lags)	M1$\Longrightarrow$ NPI	5.620 (0.000)	**1.146** (0.293)
D1 (14 lags)	M1$\Longleftarrow$ NPI	**1.558** (0.089)	5.194 (0.000)
log diff. (12 lags)	*inconclusive*	**0.534** (0.892)	**2.063** (0.186)

Table 6.2: Results of Granger Causality Tests on Individual Crystals and Log Differenced Data for M1 and NPI: P Values in Parentheses

Phase shifts	**IN-OUT**	**OUT-IN**	**IN-IN**
Time Period	**1962:4-1967:8**	**1967:8-1974:5**	**1962:4-1974:5**
	M1$\Longrightarrow$ NPI	**M1$\Longleftarrow$ NPI**	*inconclusive*
NPI $\nrightarrow$ M1	6.401 (0.00)	0.997 (0.434)	0.643 (0.696)
M1 $\nrightarrow$ NPI	1.465 (0.208)	2.714 (0.020)	0.933 (0.474)

Table 6.3: F-tests of Granger Causality between M1 and Nominal Personal Income at [D4] across Phase Shifts: P values in parentheses

be decomposed orthogonally. A key premise underlying the research in this paper is that the relationships between economic variables may be better expressed in terms of restrictions to given time scales. The idea of time scales in structural relationships is ubiquitous in other fields including biology and has in fact a long history in economics. Very simple versions of this notion are captured by concepts of the "short run" and the "long run" and by Friedman's formulation of the permanent income hypothesis. For example, the relevant variables, decision horizons, length of window for time averages, and so on will vary considerably between the decision to buy a house, or a car, or dinner, or paper clips.

A separate concept that can be easily investigated in the context of a wavelet decomposition of economic variables is that of the timing of economic relationships and its variation over the business cycle. For example, the timing of the relationship between money and income may vary with Federal Reserve policy, even to the point where monetary changes sometimes lead and sometimes lag income changes. Another example is provided by the timing of consumption income variations which may be affected by expected changes in prices or in interest rates. These variations in timing are recognized as "phase drift" in terms of the spectral analysis of the data, even after restricting attention to a specific time scale. That is, as in the consumption-income relationship, there may be time periods in which the timing of the normal relationship between variables may change; variables that were in phase, may move out of phase, and variables that were out of phase with each other may move further apart in time, or come together. The importance of phase drift is that if ignored in an empirical study, one may easily conclude that the relationship is a complicated nonlinear function, when in fact the relationship can be quite simple, but subject to phase drift.

These general ideas were explored in the context of two famous areas of research; the consumption income relationship and that between money and income.

The first case study was that involving the relationship between consumption and income. We can conclude that an appropriate way to model consumption-income relationships during the post war period is that at the time scale dominated by trend, a simple proportional relationship exists and the marginal propensity to consume is for all goods and services about 0.94. As one reduces the time scale of analysis, the degree of fit and the slope of the consumption-income relationship declines monotonically, except for the very lowest scale, which partially reverses the decline.

The results for the scale analysis of the consumption/income relationship for durables. are even more interesting. While the overall evidence of time scale variation was as important in durables. as elsewhere, there were two other facts that are of interest. The first involves an unexpected drop in durable goods expenditure at the highest scale of analysis between early 1979 and mid 1981 approximately. The second fact involved the maximum value for the MPC, which for durable goods is at the scale of 32 months. In fact, this result is almost entirely due to automobile purchases and there is some evidence that three years is the relevant time horizon for automobile purchases, at least over the historical period studied.

The former fact, the dip in the relationship between durable goods consumption and income, seems to be explained by the strong dependence of the relationship on the real interest rate. Indeed, we discovered that at the longest time scales for both durable goods and for non-durable goods that the real interest rate had the theoretically expected signs, was very highly significant, and explained a substantial proportion of the total variation in consumption. However, at shorter time scales the interest rate effect declined dramatically for durable goods and for non-durable goods there seems to be a moderate positive effect. We speculated that this might represent the effect that when interest rates are high and have their maximum effect on durable goods purchases, resources are released for shorter run non-durable goods purchases.

In almost all cases the residuals from the regressions with interest rates included were close in distribution to Gaussian and seemed to be both identically distributed and uncorrelated.

The reader will recall that because monthly measures of GDP are not available, we used NPI, which is available monthly, as a proxy; this measure while clearly underestimating GDP seems to be closer to the variation in GDP than the index measure that has been used in the past. In any event, all the comments to follow are easily interpretable for NPI and should be indicative for GDP. In terms of the money income relationships, we observe little evidence of a structural break in either velocity measure in 1980, although there is a definite, but smooth change in the rate of increase in velocity using the M1 measure of money at the longest scales. Indeed, there is weak evidence that the changes that are generally presumed to have been induced by the Volker experiment between 1979 and 1981 were to some extent anticipated by subtle changes that were only fully apparent after the end of 1980, or even later. There was however a rather surprising degree of evidence for periodicities at certain time scales for the M1 velocity and a seventeen year cycle for M2 velocity at the very longest time scale. These strongly observed periodicities are not apparent in the constituent series, so that one may well conclude that the observed periodicities are due to the interaction between money and income at specific scales.

While the distribution of energy, that is the distribution of variance, across time scales is quite different between M1 and M2 and their corresponding measures of velocity, the two measures of money are in phase through out the period. But for M1 and income at the D4 scale, the two series move into and out of phase with respect to each other over the whole observed period. A spectral analysis of these two series indicated that over very well pronounced frequency bands M1 and income were very highly coherent and that the phase relationship drifted up in a smooth fashion. This latter finding is consistent with a relationship between money and income in which the timing of the relationship is changing over the observed period.

The results of applying Granger causality tests between money and income at various time scales provided very interesting results. First, at the very lowest time scales we observed that income Granger causes money, but that at business cycle periods that money Granger causes income and that at the longest scales that there is apparently a feedback mechanism at work. However, these conclusions are tentative because of the observed sensitivity of Granger causality tests to phase drift.

At the business cycle scales, the same scales in which there is phase drift between money and income, the observation of Granger causality depends on whether the two series are moving into phase with respect to each other, or are moving out of phase. More precisely, in one case one can obtain the result that money Granger causes income and in the other that income Granger causes money, or neither, or both, depending on the combination of phase changes that are observed in the data used for the Granger causality tests. This is a result that requires considerable further research in that its implications for the nature of the relationship between economic variables and how we discover apparent lines of causality is crucial.

While further work is necessary in order to pursue these tentative findings, they are of interest for those wishing to model macro data successfully. The immediate requirements for further work involve first the confirmation, possibly using data from other countries, of the qualitative results presented in this paper. The second step is to explore the potential causes of the changes that were observed in the time series at the various scales. The empirical results lead us to rethink our theoretical development to include the role of time scale as a characterization of economic data and to recognize that sometimes the "loss of simple relationships between variables" merely indicates the occurrence of a phase shift in the relationship, that is, a change in the timing of the relationship between variables. Of course, the corresponding challenge is to discover why the phase shift occurs and what factors bring the shift to closure. Finally, it is gratifying to be able to indicate how the role of the real interest rate in consumption can be resuscitated.

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8. List of Graphs

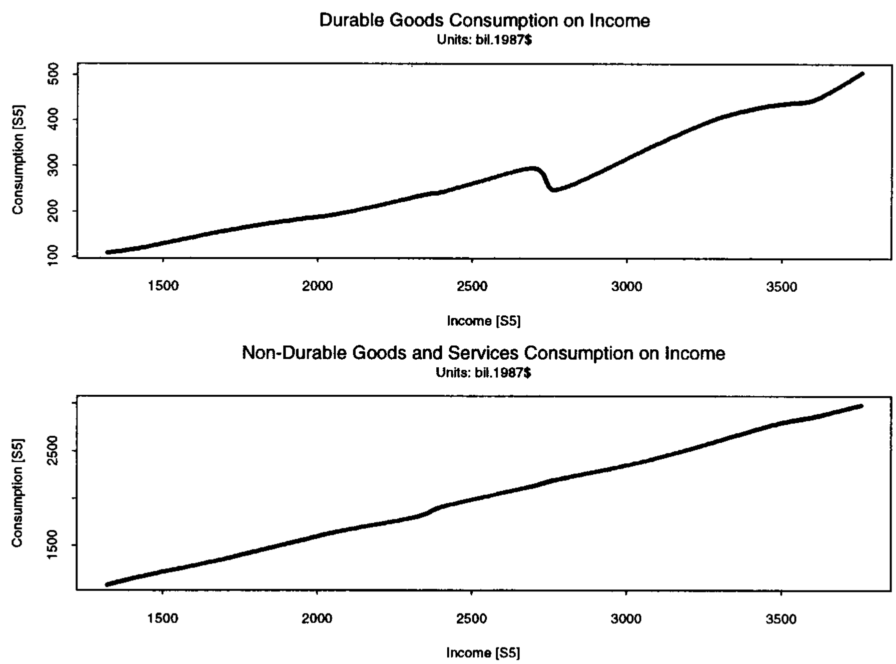

Figure 8.1: Plots of Consumption on Income Using Crystal [S6]. 1960:6-1994:4

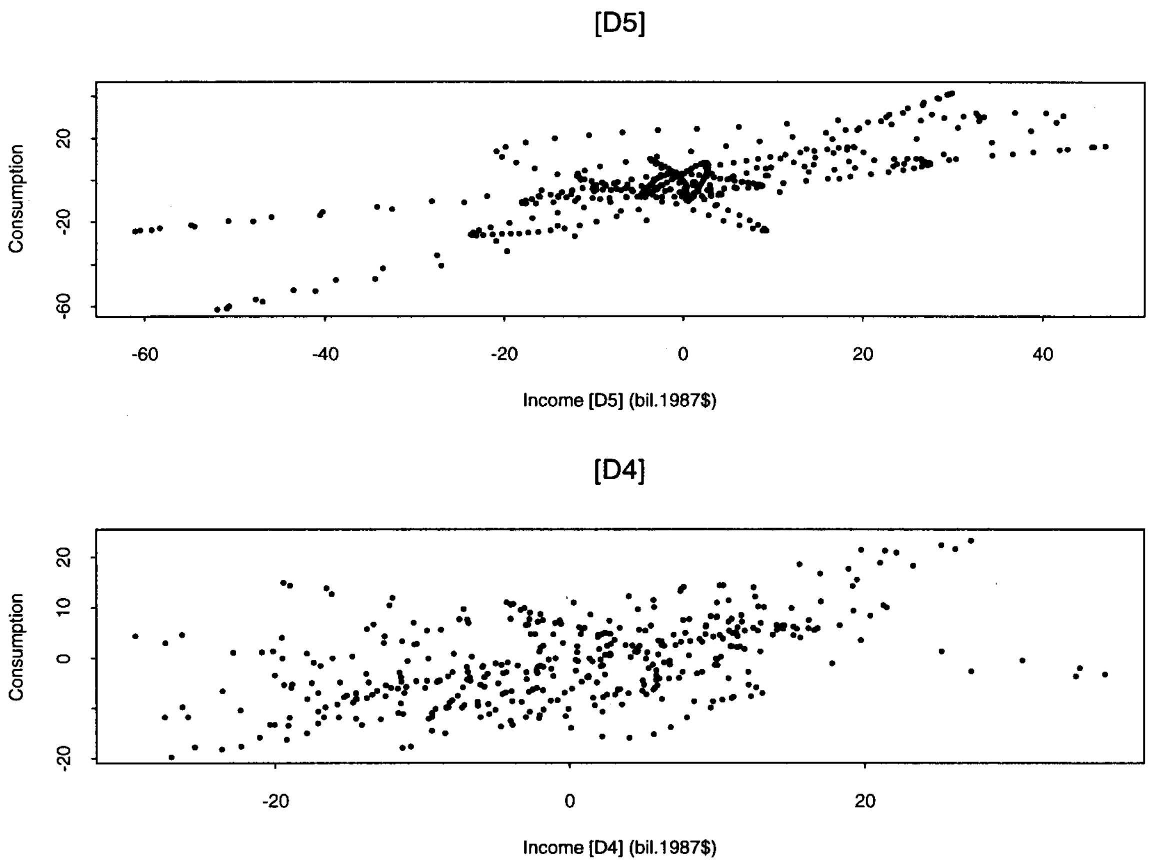

Figure 8.2: Plots of Total Consumption on Income using Crystals [D5] and [D4]. 1960:5-1994:4

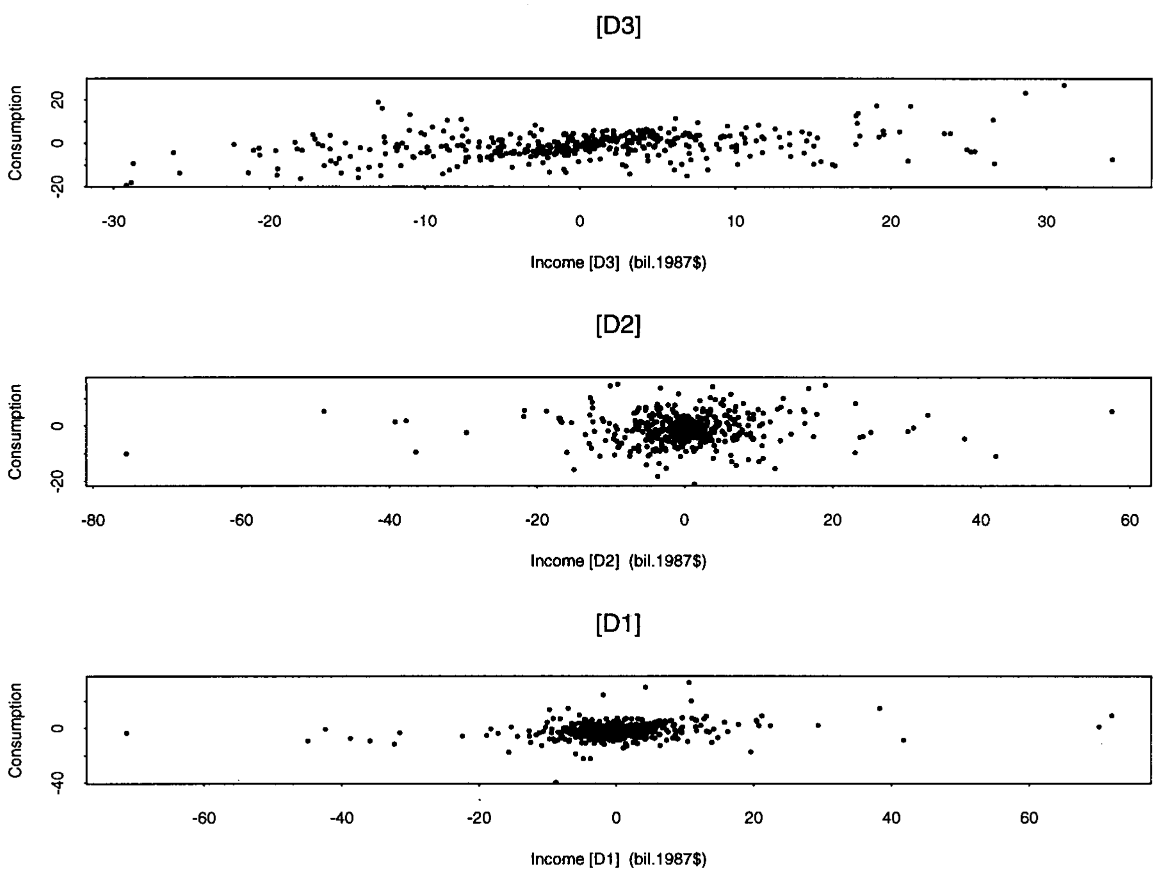

Figure 8.3: Plots of Total Consumption on Income using Crystals [D3], [D2] and [D1]. 1960:5-1994:4

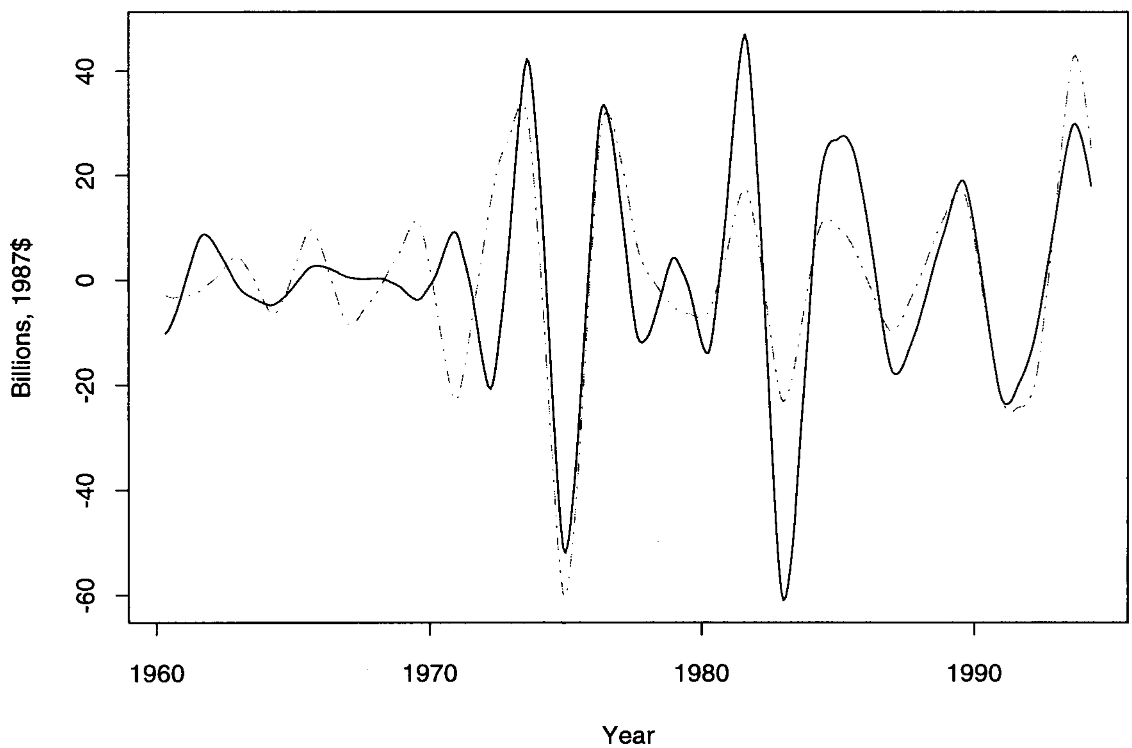

Figure 8.4: Time Series Plots of Total Consumption and Income using Crystal [D5]. (Income: solid line. Consumption: dashed line) 1960:5 - 1994:4

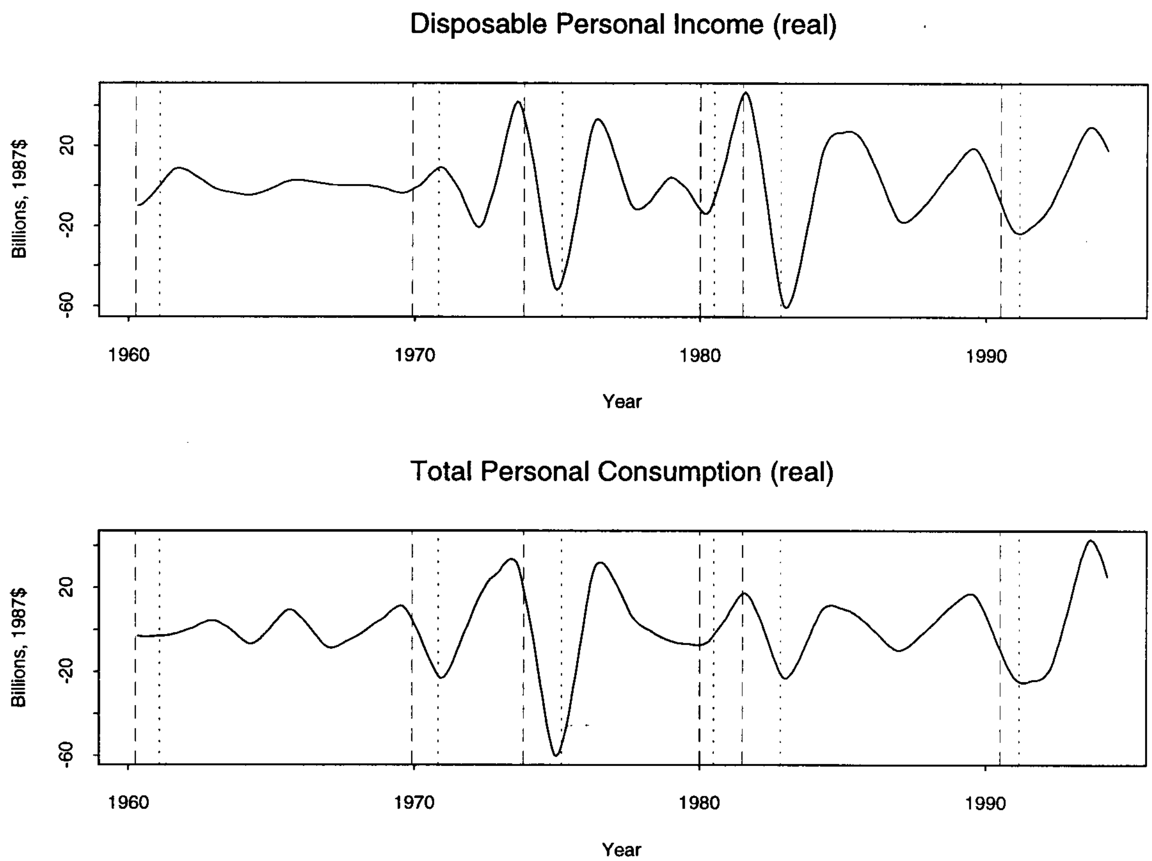

Figure 8.5: Real Disposable Income [D5], Real Personal Consumption [D5], and NBER Business Cycle Dates (Troughs: dotted lines. Peaks: dashed lines) 1960:5-1994:4

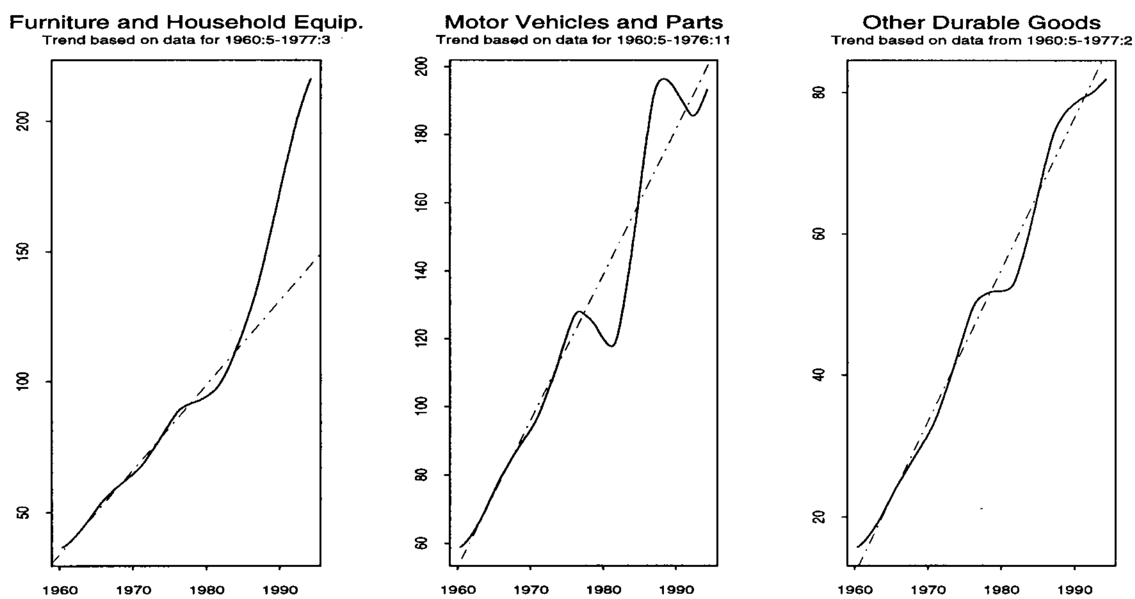

Figure 8.6: [S6] Crystals for Components of Personal Consumption Expenditure on Durable Goods with Trend Lines (dotted) 1960:5-1994:4

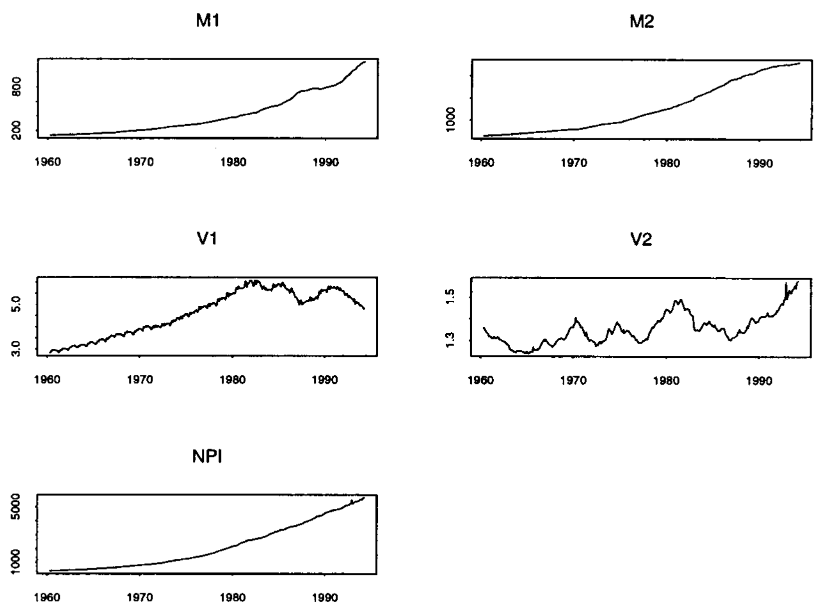

Figure 8.7: Time Series Plots of M1, M2, V1, V2 and Nominal Personal Income 1960:5-1994:4.

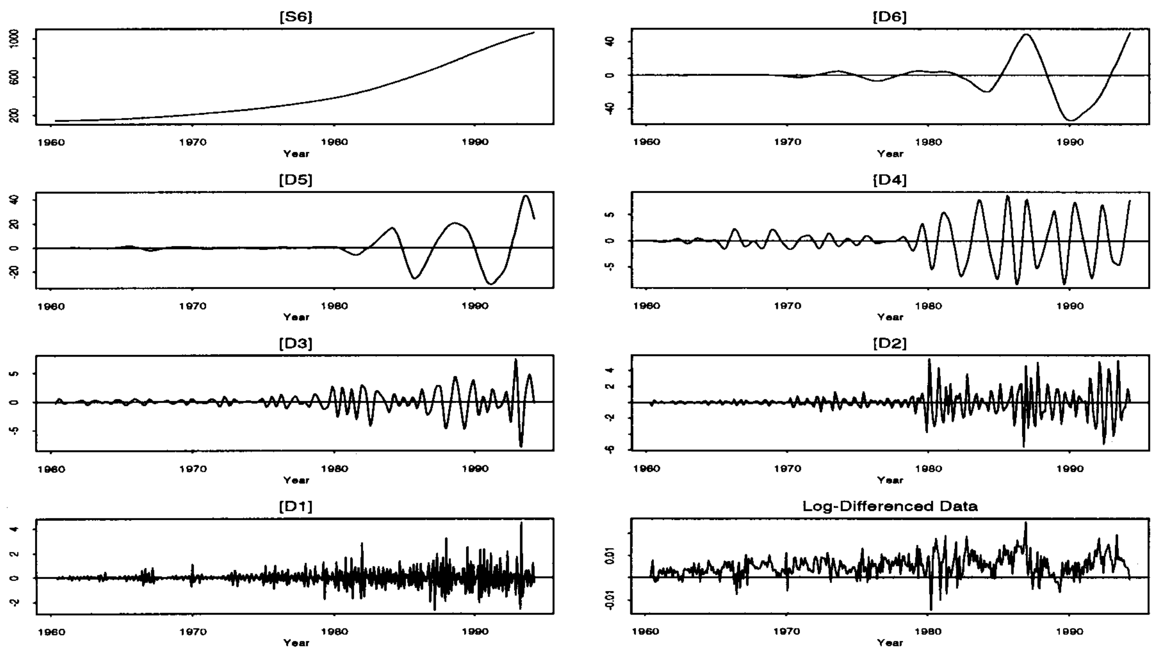

Figure 8.8: Time Series Plots of Individual Crystals, and Log-Differenced Data for M1

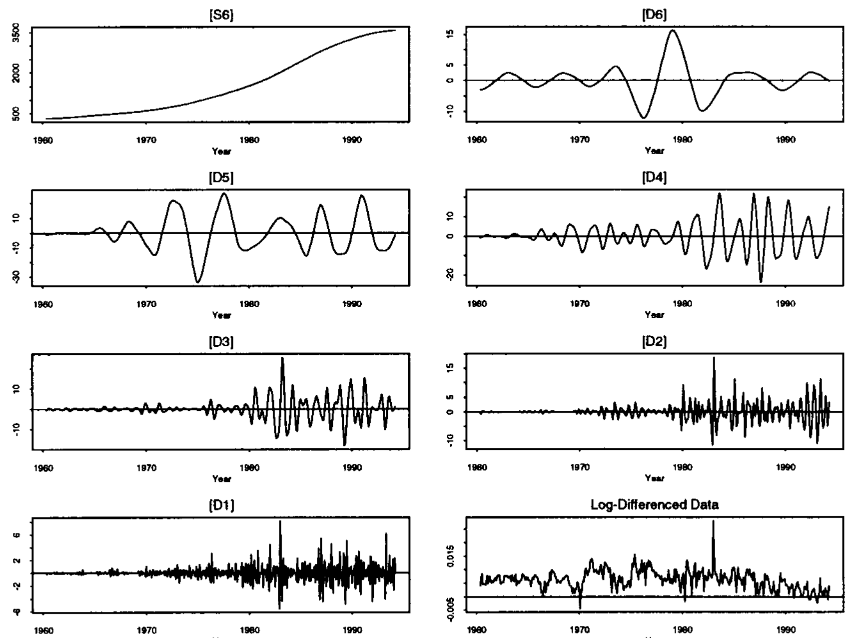

Figure 8.9: Time Series Plots of Individual Crystals, and Log-Differenced Data for M2

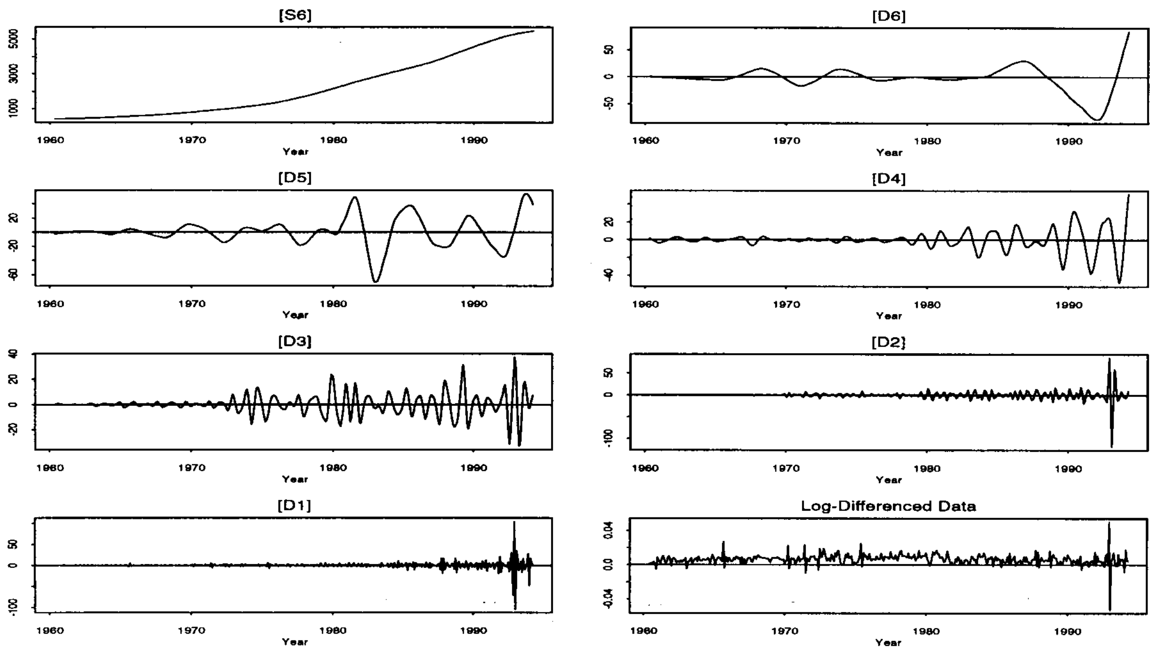

Figure 8.10: Time Series Plots of Individual Crystals, and Log-Differenced Data for Nominal Personal Income

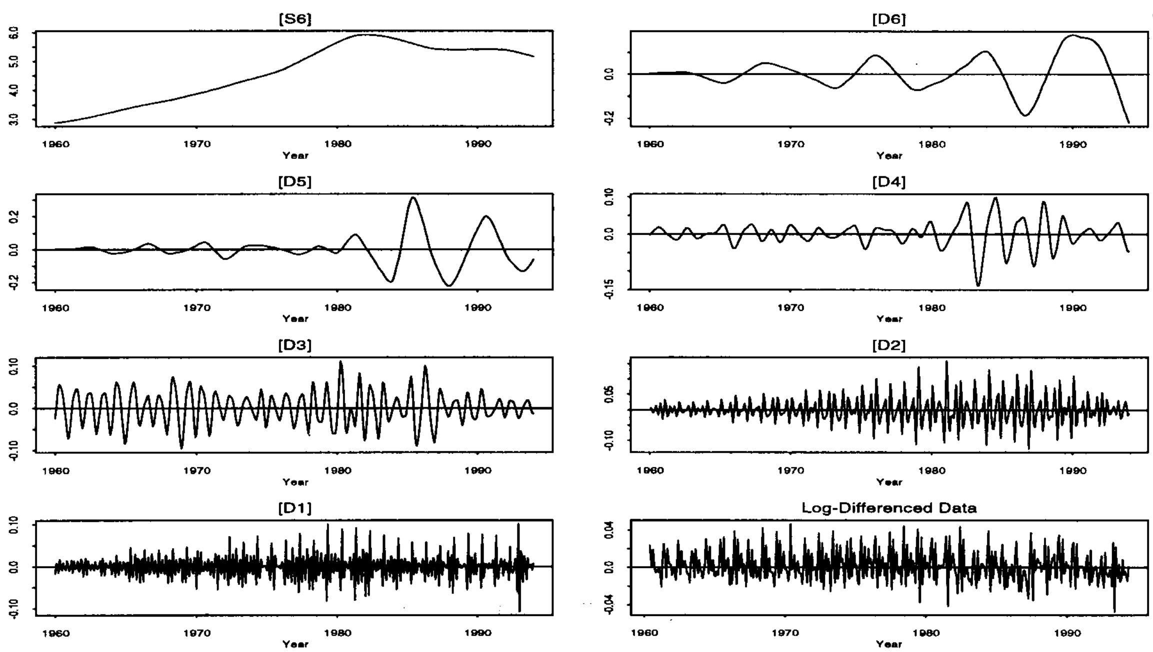

Figure 8.11: Time Series Plots of Individual Crystals, and Log-Differenced Data for M1 Velocity

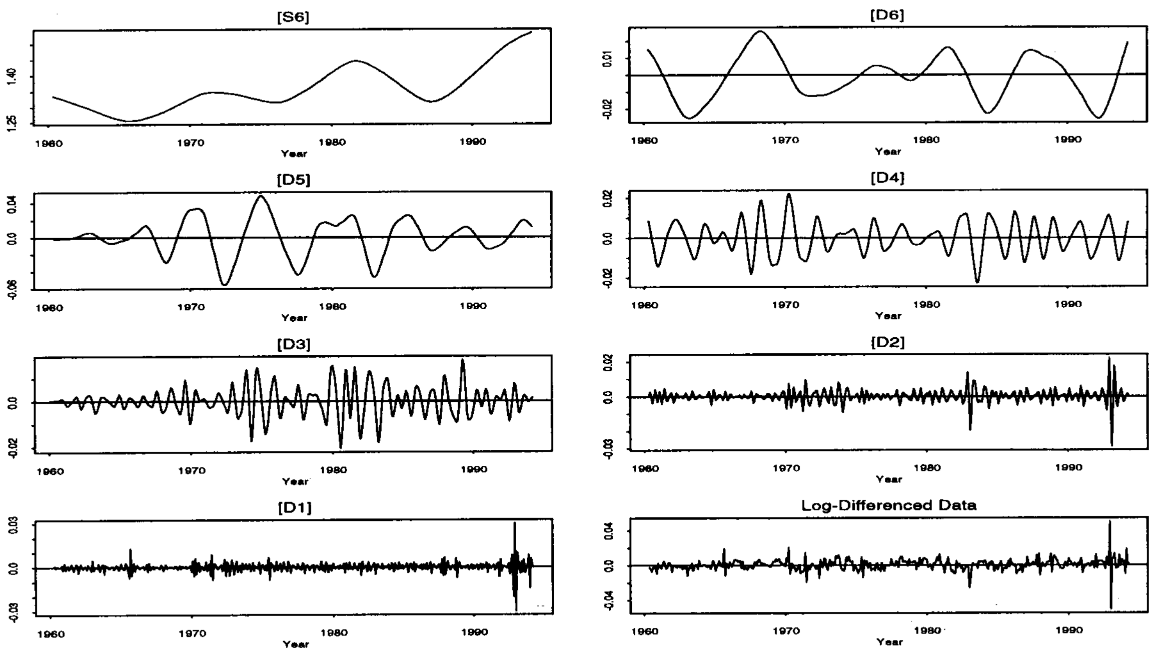

Figure 8.12: Time Series Plots of Individual Crystals, and Log-Differenced Data for M2 Velocity

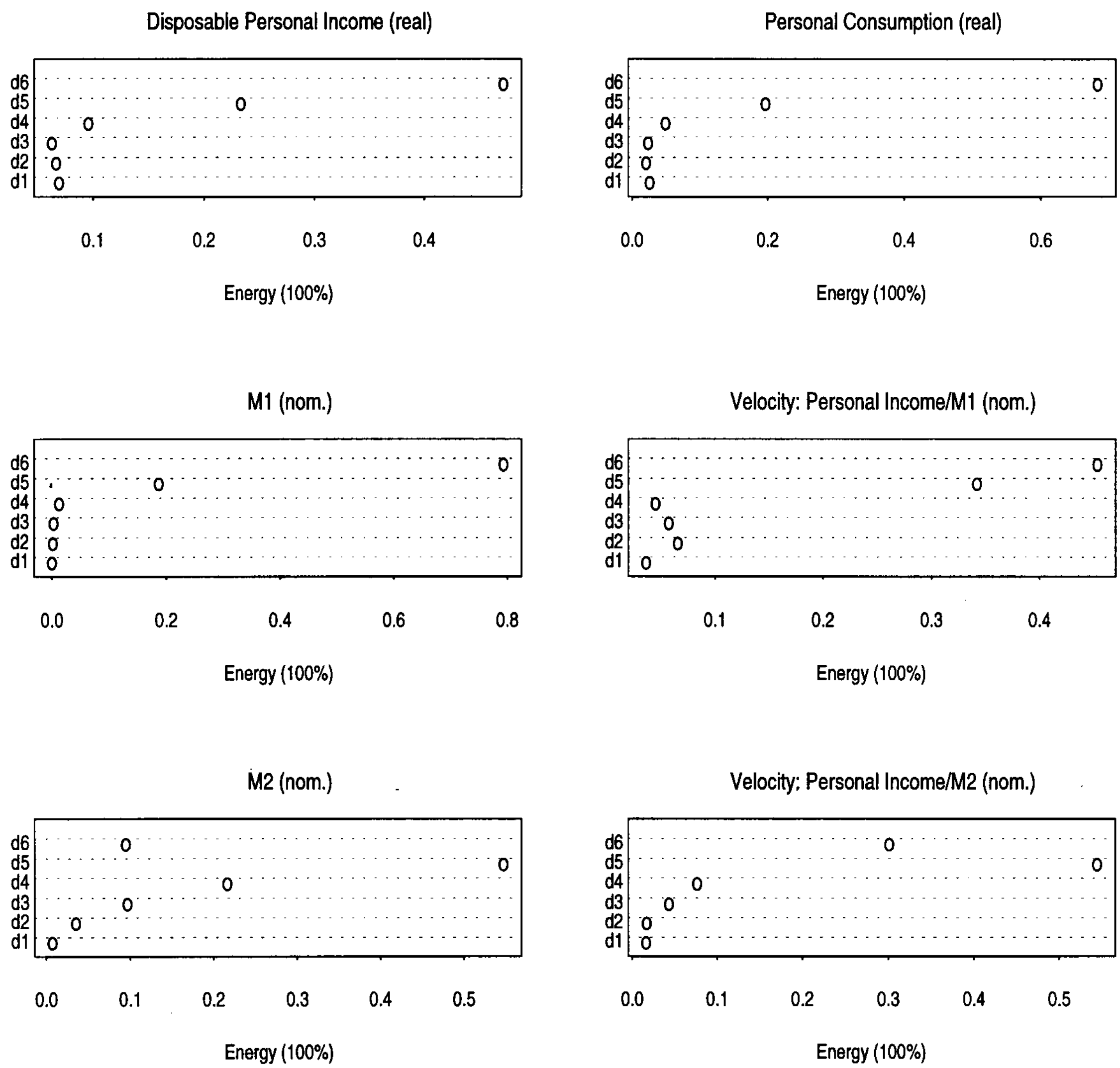

Figure 8.13: Dotcharts for Selected Variables Indicating the Relative Distribution of Energy Across Crystals After Allowing for [S5]

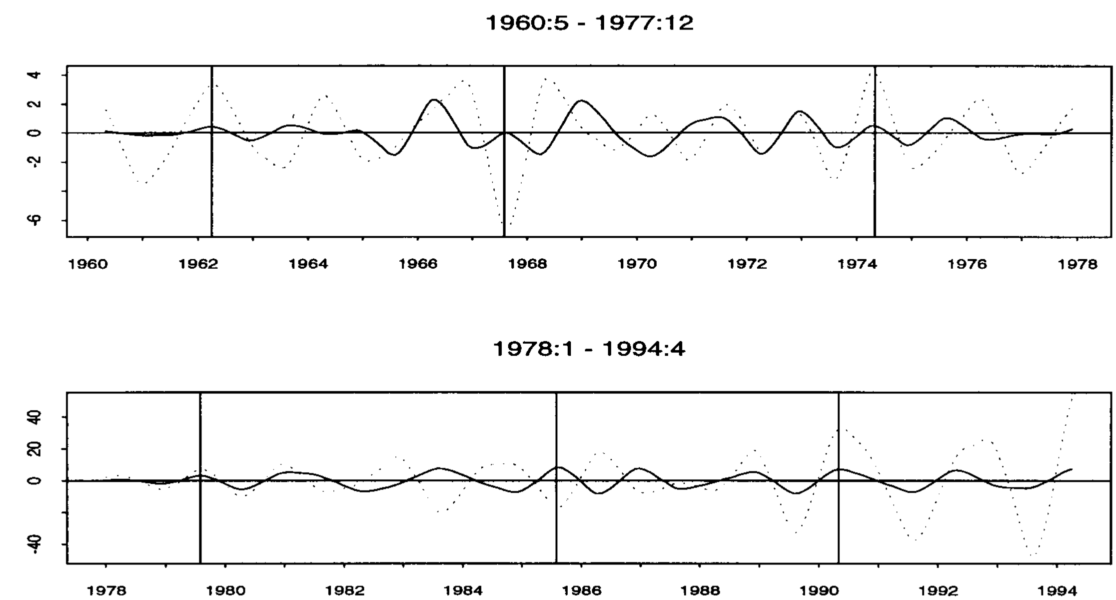

Figure 8.14: Time Series Plots of M1 (solid) and Nominal Personal Income (dots) at Crystal [D4]

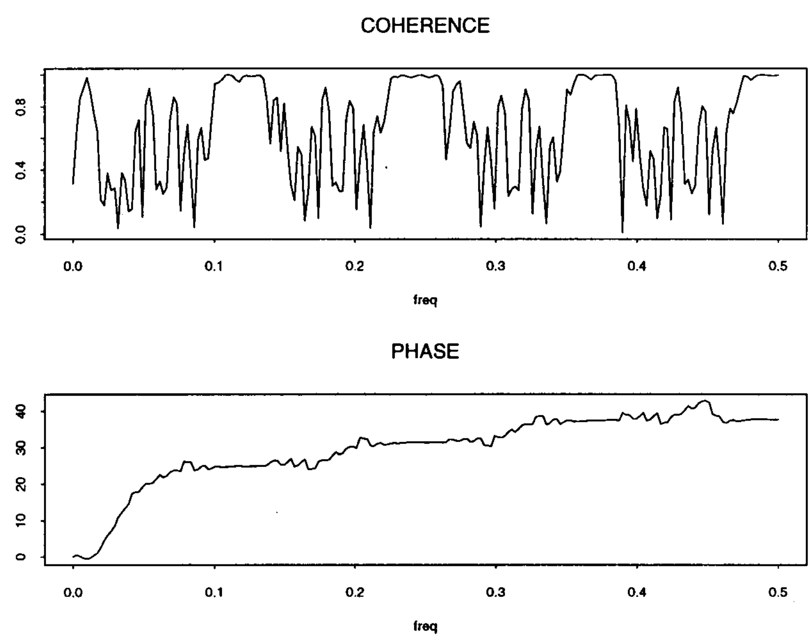

Figure 8.15: Coherency and Phase between M1 and Nominal Personal Income at [D4]