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ABSTRACT

The paper characterizes the optimal policy for a model of inspection and repair when the number of projects is endogenous and the firm maximizes the discounted sum of returns. A current project is either functioning or failed, but its status in any period can be determined only by evaluation. The firm may discard any subset of current projects in each period, but it may evaluate only one current project or one new project per period. We demonstrate that the optimal policy takes one of two forms. A "discard" policy specifies that the firm evaluate a new project in each period and discard current projects at some critical age. An "age inspection" policy specifies that the firm evaluate a new project only if all current projects are sufficiently young.

1. Introduction

In this paper, we analyze a model to determine the optimal allocation of entrepreneurial attention and its relation to the size, growth and innovativeness of a centrally organized firm. A firm consists of a number of projects of different ages. In each period, the manager must make two decisions. First, he must decide which projects to discard, and second, he must choose between evaluating one existing project or evaluating a new project for possible adoption. The expected (observed) return to any project depends only upon its age since its last evaluation, but it can be restored to a "renewed" project of age 1 with some probability each time it is evaluated.

The state of the system can be characterized by the number of current projects of each age. Upon evaluation, a project of age i is restored to a project of age 1 in the next period with probability p_i. If a new project is evaluated, it is adopted in the next period with probability p_0. The objective of the decision maker is to maximize the sum of discounted returns. Denoting the expected return of a project of age i by r_i, the problem is then completely characterized by the discount factor, β, the sequence of expected returns to projects of each age, $(r_i)_{i=1}^{\infty}$, and the sequence of probabilities with which an evaluated project of age i can be restored to a new project, $(p_i)_{i=0}^{\infty}$.

Depending on the values of these parameters, we show that the optimal policy takes one of two forms. If, upon evaluation, the probability of adopting a new project is large relative to probability of restoring any current project, then the optimal decision rule is a "discard" policy. In this case, each adopted project is maintained for a fixed number of periods d and then discarded. A new project is evaluated each period. Otherwise, the optimal decision rule is an "age inspection" policy. In this case, the firm maintains each current project until some age c at which point it is reevaluated. (Because only one project can be evaluated in each period, there will never be more than one project of age c, starting from an initial state with no current projects.) A new project is evaluated only if there is no current project of age c to be evaluated.

Much of the literature on inspection and repair models derives from the work of Barlow, Hunter and Proschan (1963, 1965). The basic model consists of a single project which at any time is either functioning or failed. A functioning may deteriorate in any moment, but a failed project remains failed unless explicitly renewed. In any time, the state of the project can be observed at a cost whereupon a failed project may be renewed at an additional cost. The problem is to determine the age at which a project should be inspected, given the history of past inspections since the last renewal. For the case where failure frequency is a Polya frequency function of order 2 and assuming the objective is to minimize the average costs per unit time, they derive an algorithm for determining the optimal sequence of inspection dates.

Assuming the objective is to minimize the sum of discounted costs, Ross (1971) shows that the optimal policy can be characterized by at most four intervals of time in which the project is either unattended, inspected, or renewed without inspection. Luss (1976) and Rosenfield (1976) further extend the model to allow the project to take on a finite number of states which are ordered so that an unattended project can only move to a worse state. The decision to inspect reveals the current state of the project and the decision to renew restores the project to the best state. Rosenfield shows that the optimal policy is similar to the four region policy defined by Ross except that the regions depend on the state of the system at its last inspection. Wappanapanom and Shaw (1979) further modify the model to allow for inspection to affect the rate of deterioration of the project. Starting with Eckles (1968), a number of authors have analyzed various versions of the model where the current state is imperfectly observed upon inspection (see e.g. Smallwood and Sondik (1973), White (1977), Sondik (1978), Albright (1979), and Lovejoy (1978)).

In the model analyzed in this paper, the inspection and repair decisions are effectively combined. The main point of departure from the literature cited above is that, although only a single project can be evaluated in any period, new projects can be introduced so that the number of projects is endogenous. Consequently, the opportunity cost of evaluating any specific project may depend on the entire menu of currently operating projects. The simplicity of our characterization of the optimal policy derives from the fact that, despite the endogeneity of the opportunity cost of evaluation, the existence of younger projects at any menu actually attained by the optimal policy does not affect the optimal inspection or discard age of a current project.

2. The Problem

Time is discrete and indexed by the nonnegative integers. Projects are indexed by the number of periods since their last evaluation. A project last evaluated i periods ago has **age** i and earns a current return r_i. Future returns are discounted by the factor β per period, $0 < \beta < 1$.

Let Q denote the set of sequences $(q_i)_{i=1}^{\infty}$ defined over $\{0,1\}$ with a finite sum, and let q and q′ denote typical elements. In each period, the agent possesses some **menu** of projects $q \in Q$ where q_i denotes the number of projects of age i. From this menu, he chooses to maintain a submenu of projects $q' \leq q$ from which the firm earns a current return $R(q') \equiv \sum_{i=1}^{\infty} q_i' r_i$. All other projects are permanently discarded from the menu.

Simultaneously, either **one** maintained project or **one** new project is selected for evaluation. If a project of age i is evaluated, then with probability p_i, it is restored and appears in the next period as a restored project of age 1. Otherwise, it is immediately and permanently discarded. If a new project is evaluated it is "restored" with probability p_0.

Let I denote the nonnegative integers. For each $q \in Q$, let $\Lambda(q) \equiv \{(q',i) \in Q \times I\colon q' \leq q \text{ and } i > 0 \text{ implies } q_i' = 1\}$ denote the set of 2-tuples consisting of a retained menu and an evaluation choice. A **history** h is an 2n+1-tuple of an initial menu and n menu-choice pairs for any nonegative integer n. A **policy** μ is a pair of functions (μ_Q,μ_I) defined on the set of histories for which $(\mu_Q(h),\mu_I(h) \in \Lambda(q_n)$ for any history $h = (q_0,q_1,i_1,\ldots,q_n,i_n)$. Given history h, $\mu_Q(h)$ denotes the menu of projects to be retained by the agent and $\mu_I(h)$ denotes the project to be selected for evaluation.

Let e_i denote the menu q for which $q_i = 1$ and $q_j = 0,\ j \neq i$. For $q \in Q$, $i = 1,2,\ldots,$ let q_{-i} denote the menu q' such that $q_i' = 0$ and $q_j' = q_j,\ j \neq i$, and let $q_{-0} \equiv q$. Let $S\colon Q \to Q$ denote the shift operator defined by $(Sq)_1 = 0$ and $(Sq)_i = q_{i-1},\ i = 1,2,\ldots.$ For any menu initial menu q and any policy μ, let $P_\mu^t(q,q')$ denote the probability that menu q' is realized in period t when policy μ is applied to menu q. Similarly, let $R_\mu^t(q) = R(\mu_Q(q,t))$ denote the current return to the retained menu when policy μ is applied to menu q in period t. Then $P_\mu^0(q,q) = 1$, and, if $q' = \mu_Q(q,0)$

and $i = \mu_I(q,0)$, then $R^0_\mu(q) = R(q')$, $P^1_\mu(q,e_1+Sq'_{-i}) = p_i$ and $P^1_\mu(q,Sq'_{-i}) = 1-p_i$.

For any policy μ and any bounded function $G:Q \to R$, let $P^t_\mu(q)G \equiv \sum_{q'\in Q}P^t_\mu(q,q')G(q')$ denote the expected value of G with respect to distribution $P^t_\mu(q)$. Then $W(\mu) \equiv \sum_{t=0}^{\infty}\beta^t P^t_\mu R^t_\mu$ defines the function on Q which gives the value of a firm using policy μ. Letting M denote the set of policies, the maximal value of the firm with an initial menu q is then $V(q) \equiv \sup_{\mu\in M}\{W(\mu)(q)\}$.

For two real valued functions F,G defined on Q, we write $F \le G$ if $F(q) \le G(q)$, $q \in Q$. A policy μ is **optimal** if $W(\mu) \ge W(\mu')$ for all policies μ'. A policy is **stationary** if it depends only on the current menu of projects. We denote a typical stationary policy by $\lambda:Q \to Q\times I$.

We assume that $\sum_{t=0}^{\infty}|r_t| < \infty$ and $\sup_{t\ge1}\{\sum_{i=0}^{t}r_i\beta^i\} > 0$. Then, since $\Lambda(q)$ is finite for any menu q, it follows Theorem 7(b) of Blackwell (1965) that there is an optimal policy λ which is stationary.

3. The Main Result

Our objective is to characterize an optimal stationary policy over the domain of menus which could be realized from an initial menu with no projects when that policy is adopted. As we shall see, this policy may take one of two forms. In one case, a new project is evaluated every period and current projects are discarded without evaluation upon reaching a critical age. In the other case, each current project is evaluated periodically and a new project is evaluated only if no current project requires evaluation. Which policy is adopted depends on whether the benefit of evaluating an existing project at the prescribed date exceeds the benefit from evaluating a new project instead and maintaining the existing project until some later date.

The idea behind the proof is to calculate the value of a project assuming that the only opportunity cost to evaluating a project is the foregone evaluation of a new project. That is, we do not consider the opportunity cost of foregoing the evaluation of some alternative project. Given this assumption, we calculate the optimal age at which a project should either be discarded or evaluated.

To calculate these values, consider a project of age 1. If it is maintained until some age t and then dropped for the following period, its value is $u_1^t \equiv \sum_{i=1}^{t} r_i \beta^{i-1}$. Let $u_1 = \sup_{t\geq 1}\{u_1^t\}$. Alternatively, let v_1^t denote the value of a project of age 1 which is maintained and evaluated after every t periods as long as it is restored. Since evaluating the project at age t implies an opportunity cost of foregoing the evaluation of a new project, we have $v_1^t = \sum_{i=1}^{t} r_i \beta^{i-1} + \beta^{t+1}(p_t - p_0)v_1^t$ which implies that $v_1^t \equiv [\sum_{i=1}^{t} r_i \beta^{i-1}]/[1-\beta^t(p_t-p_0)]$. Let $v_1 \equiv \sup_t\{v_1^t\}$. Then, assuming that it may be discarded immediately, $w_1 \equiv \max\{u_1, v_1, 0\}$ is the maximal value of acquiring a successful project of age 1.

The form of the optimal policy depends on the relative size of u_1 and v_1. Let $u_1 = u_1^d$ and $v_1 = v_1^c$. That is, c denotes the optimal age at which a maintained project should be evaluated and d+1 denotes the age at which a maintained project should be discarded if it is not to be evaluated. Recall that Q_k is the set of menus containing projects no older than k. Then our fundamental characterization may be stated as follows.

THEOREM 1: (a) If $w_1 = u_1$, there is an optimal policy λ such that, for $q \in Q_{d+1}$, (i) $\lambda_Q(q) = q_{-(d+1)}$, and (ii) $\lambda_I(q) = 0$.
(b) If $w_1 = v_1$, there is an optimal policy λ such that, for $q \in Q_c$, (i) $\lambda_Q(q) = q$, and (ii) $\lambda_I(q) = c$ if $q_c = 1$, and $\lambda_I(q) = 0$ otherwise.

Letting a stand for c or d+1 as v_1 is greater or less than u_1, Theorem 1 may be summarized as follows. Assuming that the firm has maintained no projects older than a, the firm should maintain any project of age a or less and drop any project of age a+1. Its evaluation policy depends on the relative size of v_1 and u_1. If v_1 is less than u_1, the firm should evaluate a new project in each period. We will refer to this policy as the **discard** policy. If v_1 exceeds u_1, the firm should evaluate a new project only if there is no maintained project of age a. Otherwise, it should evaluate the project of age a. We will refer to this policy as the **age inspection** policy.

Notice that we have not defined the policy for all possible project menus. However, starting from an initial menu contained in Q_a, all following menus will also be contained in Q_a as long as a policy satisfying the conditions of Theorem 2 is followed. Therefore, for a firm starting with a menu in Q_a, say $q = 0$, Theorem 2 completely specifies the actions an agent should take.

In general, an optimal policy may take a much more complicated form for project menus containing older projects. First, even if a discard policy is optimal, it may be optimal to maintain and perhaps even evaluate projects older than age a if the (r_t) sequence is not monotonic. Second, if an age inspection policy is optimal, it may be optimal to maintain and evaluate projects older than age a, depending on the current menu, even if both the (r_t) and the (p_t) sequences are monotonic decreasing.

4. Proof of the Main Result

Recall that u_1 and v_1 are the values of following respectively the optimal discard and age inspection rules for a project of age 1 assuming that the only opportunity cost of reevaluation is forgoing the evaluation of a new project. In this section, the analogous values are calculated for projects of any age. We then establish that, for any menu q, the sum of the component values forms an upper bound to V(q). Finally, we establish that for $q \in Q_a$, this value is attained by the policies defined in Theorem 1.

Suppose λ is a stationary policy. Then, letting $R_\lambda = R_\lambda^0$ and $P_\lambda = P_\lambda^1$, we let T_λ denote the operator on the set of functions from Q to R defined by $T_\lambda G \equiv R_\lambda + \beta P_\lambda G$. We will use the following monotonicity property.

LEMMA 1: Let G be a bounded real valued function on Q. If, for all stationary policies λ, $T_\lambda G \leq G$, then $V \leq G$.

PROOF: We show first that $G \geq \sum_{i=0}^{t}\beta^i P_\lambda^i R_\lambda + \beta^{t+1}P_\lambda^{t+1}G$ for $t = 1,2,\ldots.$ For $t = 0$, the result follows from the hypothesis that $G \geq T_\lambda G$ and the definition of T_λ. Suppose the property is true for $i = 0,\ldots,t-1$. Then $G \geq \sum_{i=0}^{t-1}\beta^i P_\lambda^i R_\lambda + \beta^t P_\lambda^t G \geq \sum_{i=0}^{t-1}\beta^i P_\lambda^i R_\lambda + \beta^t P_\lambda^t T_\lambda G = \sum_{i=0}^{t}\beta^i P_\lambda^i R_\lambda + \beta^{t+1}P_\lambda^{t+1}G$.

Therefore, since G is bounded and $\beta < 1$, letting $t \to \infty$, we obtain $G \geq \sum_{i=0}^{\infty}\beta^i P_\lambda^i R_\lambda$. In particular, Theorem 1 above implies the existence of an optimal policy λ' such that $G \geq \sum_{i=0}^{\infty}\beta^i P_{\lambda'}^{\,i} R_{\lambda'}$. But Theorem 6(f) of Blackwell (1965) implies that $\sum_{i=0}^{\infty}\beta^i P_{\lambda'}^{\,i} R_{\lambda'} = V$. Q.E.D.

Our next step is to construct an upper bound for V. As before, suppose that there is no opportunity cost to foregoing the evaluation of existing projects so that w_1 is the value of a newly evaluated project. Then the value of a firm with no projects is just the expected discounted value of the firm in the following period after it evaluates a new project, $w_0 = \beta[p_0 w_1 + w_0]$. Solving for w_0, we may define $w_0 \equiv p_0 w_1[\beta/(1-\beta)]$.

Next, consider the value of a project of age k. If it is maintained until age t and dropped for the following period, its value is $u_k^t \equiv \sum_{i=k}^t r_i\beta^{i-k}$. Let $u_k \equiv \sup_t\{u_k^t\}$. Alternatively, if it held until age t and then evaluated, its value is $v_k^t \equiv \sum_{i=k}^t r_i\beta^{i-k} + (p_t-p_0)\beta^{t+1-k}w_1$. Let $v_k \equiv \sup_t\{v_k^t\}$. Then, assuming that it may be discarded immediately, $w_k \equiv \max\{u_k,v_k,0\}$ is the maximal value of a project of age k.

LEMMA 2: (a) $w_k \geq r_k + \beta w_{k+1}$, $k = 0,1,2,\ldots$. (b) If $w_1 = u_1$, then $w_k = \sum_{i=k}^d r_i\beta^{i-k}$, $k = 0,\ldots,d$, and $w_{d+1} = 0$. (c) If $w_1 = v_1$, then $w_k = \sum_{i=k}^c r_i\beta^{i-k} + \beta^{c+1-k}(p_c-p_0)w_1$, $k = 0,\ldots,c$.

PROOF: (a) By definition, either (i) $w_{k+1} = 0$, or there is a $t \in \{k+1,k+2,\ldots,\infty\}$ such that either (ii) $w_{k+1} = \sum_{i=k+1}^t r_i\beta^{i-k-1}$ or (iii) $w_{k+1} = \sum_{i=k+1}^t r_i\beta^{i-k-1} + \beta^{t-k}(p_t-p_0)w_0$. In case (i), we have, by definition of w_k, $w_k \geq r_k = r_k + \beta w_{k+1}$. In case (ii), we have $w_k \geq r_k + \sum_{i=k+1}^t r_i\beta^{i-k} = r_k + \beta w_{k+1}$. In case (iii), we have $w_k \geq r_k + \sum_{i=k+1}^t r_i\beta^{i-k} + \beta^{t-k+1}(p_t-p_0)w_0 = r_k + \beta w_{k+1}$.

(b) Fix $k \in \{1,\ldots,d\}$. By definition, there is a $t \geq k$ such that $w_k = \sum_{i=k}^t r_i\beta^{i-k} + \beta^{t-k+1}w_1\max\{0,p_t-p_0\} \geq \sum_{i=k}^d r_i\beta^{i-k}$. Suppose $w_k > \sum_{i=k}^d r_i\beta^{i-k}$. Then, $w_1 = \sum_{i=1}^d r_i\beta^{i-1} < \sum_{i=1}^t r_i\beta^{i-1} + \beta^t w_1\max\{0,p_t-p_0\}$, contradicting the definition of w_1. Similarly, suppose there is a $t \geq d$ such that $w_{d+1} = \sum_{i=d+1}^t r_i\beta^{i-d-1} + \beta^{t-d}w_1\max\{0,p_t-p_0\} > 0$. Then $w_1 = \sum_{i=1}^d r_i\beta^{i-1} < \sum_{i=1}^t r_i\beta^{i-1} + \beta^t w_1\max\{0,p_t-p_0\}$, again contradicting the definition of w_1.

The proof of Part (c) is similar. Q.E.D.

LEMMA 3: For all $q \in Q$, $V(q) \leq w_0 + \sum_{i=1}^\infty w_iq_i$.

PROOF: For each $q \in Q$, let $U(q) \equiv w_0 + \sum_{i=1}^{\infty} w_i q_i$. If we can show that, for any stationary policy λ, $T_\lambda U \le U$, the result will follow from Lemma 1. Note first that, since $w_i \ge 0$ by definition, it follows immediately that $U(q') \le U(q)$ for $q' \le q$.

Now consider any stationary policy λ and menu q. Let $q' = \lambda_Q(q)$ and $k = \lambda_I(q)$. Then $T_\lambda U(q) = R_\lambda(q) + \beta P_\lambda(q)U = R(q') + \beta[p_k U(e_1+Sq'_{-k}) + (1-p_k)U(Sq'_{-k})]$. If $k = 0$, then, using the definitions of U and w_0 and Lemma 2(a), we obtain

$$T_\lambda U(q) = \sum_{i=1}^{\infty} q_i' r_i + \beta[p_0 w_1 + w_0 + \sum_{i=1}^{\infty} q_i' w_{i+1}]$$

$$\le w_0 + \sum_{i=0}^{\infty} q_i' w_i = U(q') \le U(q).$$

Similarly, if $k > 0$, then, using the fact that $w_k \ge r_k + \beta(p_k - p_0)w_1$, we obtain

$$T_\lambda U(q) = \sum_{i=1}^{\infty} q_i' r_i + \beta[p_k w_1 + w_0 + \sum_{i=1, i\neq k}^{\infty} q_i' w_{i+1}]$$

$$\le r_k + (p_k - p_0)\beta w_1 + w_0 + \sum_{i=1, i\neq k}^{\infty} q_i' w_i \le w_0 + \sum_{i=1}^{\infty} q_i' w_i = U(q') \le U(q).$$

Q.E.D.

PROOF OF THEOREM 1: (a) Consider first the case where $w_1 = u_1$ and the firm adopts a policy λ satisfying Part (a). Then

(1) $$W(\lambda)(q) = R(q_{-(d+1)}) + \beta[p_0 W(\lambda)(e_1+S(q_{-(d+1)})) + (1-p_0)W(\lambda)(S(q_{-(d+1)}))].$$

We will show that, for $q \in Q_{d+1}$, $W(\lambda)(q) = w_0 + \sum_{i=1}^{d} w_i q_i$. The theorem will then follow from Lemma 3. Theorem 5 of Blackwell (1965) implies that $W(\lambda)$ is the unique solution to equation (1). All that remains is to verify that, for $q \in Q_{d+1}$, $W(\lambda)(q) = w_0 + \sum_{i=1}^{d} q_i w_i$ is the solution.

Suppose it is. Then, since Lemma 2(b) implies that $w_k = \sum_{i=k}^{d} r_i \beta^{i-k}$, $k = 1,\ldots,d$, and $w_{d+1} = 0$, it follows that $W(\lambda)(q) = w_0 + \sum_{k=1}^{d} q_k [\sum_{i=k}^{d} r_i \beta^{i-k}]$. Substituting this expression into the right hand side of equation (1) and using the definition of w_0 then yields

$$W(\lambda)(q) = R(q_{-(d+1)}) + \beta[p_0 w_1 + w_0] + \beta \sum_{i=1}^{d} w_{i+1} q_i = \sum_{i=1}^{d} r_i q_i + \beta[p_0 w_1 + w_0]$$
$$+ \sum_{i=1}^{d} [\sum_{k=i+1}^{d} r_k \beta^{k-i}] q_i = w_0 + \sum_{i=1}^{d} [\sum_{k=i}^{d} r_k \beta^{k-i}] q_i = w_0 + \sum_{i=1}^{d} w_i q_i .$$

(b) If $w_1 = v_1$ and a policy λ satisfying the conditions of Part (b) is adopted, then $W(\lambda)$ must satisfy, for $q \in Q_c$,

(2) $$W(\lambda)(q) = R(q) + \beta[p_0 W(\lambda)(e_1 + Sq) + (1-p_0) W(\lambda)(Sq)] \qquad \text{if } q_c = 0.$$
$$W(\lambda)(q) = R(q) + \beta[p_c W(\lambda)(e_1 + S(q_{-c})) + (1-p_c) W(\lambda)(S(q_{-c}))] \qquad \text{if } q_c = 1.$$

Similar arguments then establish that $W(\lambda)(q) = w_0 + \sum_{i=1}^{c} q_i w_i$ is the unique solution over Q_c. Q.E.D.

5. Conclusion

The main result of the paper demonstrates that, over the relevant domain of menus, the optimal policy takes one of two forms. It is either a discard policy in which the firm evaluates a new project in each period and discards any project reaching a critical age d, or it is an age inspection policy in which case the firm evaluates a new project only if all current projects are younger than some critical age c. In general, the form of the optimal policy may depend in a complicated way on the sequences (r_t) and (p_t). However, by suitably parametrizing these sequences, it is possible to establish some detailed comparative statics results as in Gifford (1989).

She uses the model to analyze the factors which determine the optimal size and innovativeness of an entrepreneurial firm. The firm may maintain an arbitrary number of projects. In any period, each current project is either functioning, in which case it earns a positive return g, or is failed, in which case it earns a negative return -b. The actual status of a project cannot be observed, but the manager knows that in each period a good project tends to fail with probability ϕ. Consequently, the expected return to a project of age i is $r_i = (1-\phi)^i g - \phi^i b$. Upon evaluation, a new project is adopted as a "renewed" project with probability p_0. If a current project which has failed is evaluated, it is restored with probability ρ. Consequently, upon evaluation, a project of age i is renewed with probability $p_i = (1-\phi)^i + \phi^i \rho$.

One important insight is that the propensity for a firm to innovate may change discontinuously with small changes in its environment. When p_0 is large relative to ρ, the firm will adopt a discard policy so that the firm is characterized by continual innovation. In this case, the rate of innovation is independent of g/b, ϕ, and the expected size (number of current projects) of the firm. On the other hand, expected firm size increases with g/b and p_0, and decreases with ϕ.

When p_0 is small relative to ρ, the firm will adopt an age inspection policy in which case the firm is characterized by much less innovation. New projects are introduced only when the firm has been unable to restore an existing project at some point in the past. Consequently, in this case, the rate of innovation decreases with firm size. Gifford also shows that it is increasing in g/b and p_0. On the other hand, the bound on firm size, given by the age at which projects are reevaluated, is increasing in g/b and p_0, and decreasing in ρ.

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