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ABSTRACT

Tax distortions are presented as a fiscal externality. The interpretation explains why substitution effects and changes in compensated demand are relevant in the calculation of the excess burden and in the common formulation of the "Ramsey Equations."

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1. INTRODUCTION

Normative tax theory stresses the economic cost of a tax structure. An indirect tax creates a distortion, because the consumer price ceases to measure the resource cost. In consequence, the use of an indirect tax structure causes the utility of the representative individual to fall more than it falls if the same tax revenue is collected using a lump-sum levy. The additional utility fall is measured by the excess burden. It is usually associated with the substitution effect. For example, Stiglitz (1988, pages 439 and 441) writes: "The substitution effect of the tax is sometimes called the *distortion* caused by the tax...If there is no substitution effect, then the tax gives rise to no excess burden." However, the use of the substitution effect to explain the excess burden is puzzling: the excess burden is associated with a utility fall, whereas the substitution effect is associated with a change in the consumption bundle along the indifference curve.

In the traditional analysis of Dupuit (1844) and Marshall (1890), income effects are ignored. The use of an indirect tax causes the individual to reduce his consumption of the taxed commodity, and the consequent fall in consumer surplus is in excess of the resources transferred to the government. The excess burden is measured by the Harberger (1964) triangle under the uncompensated demand curve. In the presence of income effects, the excess burden is measured by the triangular area under the compensated demand curve. However, the interpretation is now puzzling: the excess burden is measuring a utility loss associated with a change in quantities, but the relevant quantity changes hold utility constant.

The analytical reason for the utility loss is clear: consumer prices no longer measure resource costs. What is unclear is how utility is lost. The individual is optimizing, so what market failure is causing an inefficient outcome to result? This paper interprets the distortion of the indirect tax as a fiscal externality. If an *individual* buys more of the taxed commodity, there is a tax revenue gain. Because the tax revenue requirement is fixed, the additional tax revenue generated by the increased purchase is rebated to *all* agents, or is a benefit external to the individual. As is normal in the presence of activities which generate positive externalities, the individual chooses a quantity which is less than the efficient quantity. The excess burden is the benefit of setting quantities efficiently. Normative theory values a benefit by the compensating variation. Compensated demand enters naturally because the benefit is measured with the agents' utilities being unchanged.

The "Ramsey Equations" show how the quantity changes induced by a tax structure may be used to evaluate an indirect tax structure. In the absence of income effects and provided the tax revenue requirement is small, Ramsey (1927) showed that the utility loss of the representative individual is minimized when the tax structure causes an equi-proportionate fall in the consumption of all taxed commodities. As intuitively the first-order conditions would be expected to equalize the marginal utility loss associated with the use of each instrument, the reader might be excused for thinking that the Ramsey Equations compare the effect of the induced quantity changes on utility. However, Samuelson (1986) showed that, in the presence of income effects, it is the change in compensated demands which is important. In this context, changes in compensated demands are non-intuitive, as of course utility does not change if all changes are along an indifference curve. I show that the quantity changes which appear in the Ramsey Equations measure the benefit of marginal tax reforms. If the use of one tax instrument is marginally substituted by the use of other tax instruments, the induced quantity changes may cause indirect tax revenue to rise - an external benefit. The revenue gain enables tax instruments to be compared. Tax rates are adjusted so that no feasible tax reform generates a positive benefit.

The contribution of this paper is to provide a better understanding of well-known but puzzling results. It does this by showing how quantity changes matter: quantity changes produce tax revenue, which is a favorable externality. As such, it complements recent work by Triest (1990), Fullerton (1991) and Ballard and Fullerton (1992) who clarify measures of the marginal cost of funds.

2. TRADITIONAL THEORY: PRICE EFFECTS

The analysis focuses on a representative individual. To facilitate the graphical analysis, Sections 2 and 3 presuppose that there are two commodities, x and y, and the individual achieves utility $U(x, y)$. Commodity y is the numeraire, and its price is normalized to unity.[1] The technology of production allows one unit of commodity x to be created from p units of commodity y; p is the competitive producer price of commodity x. The government collects tax revenue by levying an indirect tax rate t on commodity x; the consumer price of x is q, $q=p(1+t)$. Because p is fixed by the production technology, q represents the tax structure. The individual has endowment M (units of numeraire), his indirect utility is $V(q; M)$ and the individual's demand for x is $x(q; M)$. The tax revenue collected from the representative individual is

$$R(q) = (q-p)x(q; M).$$

Traditional analysis has focused on the price distortion. The excess burden is the compensating variation associated with replacing the indirect tax structure with a lump-sum tax of equal revenue.

DEFINITION: *The excess burden EB(q) associated with the tax structure q is defined by the relationship:*

$$V(q;\ M) \equiv V(p;\ M\text{-}R(q)\text{-}EB(q)). \tag{1}$$

The excess burden arises because the commodity's consumer price exceeds its resource cost. If the price is lowered from q to p, the lower price is advantageous to the individual; the price reduction must be combined with a resource withdrawal if utility is unchanged. If the price is Q, $Q < q$, resources T(Q) are withdrawn if the preexisting utility is maintained,[2]

$$V(q;\ M) \equiv V(Q;\ M\text{-}T(Q)).$$

A further lowering of the tax rate from Q to Q+dQ (dQ<0) requires additional resources dT to be simultaneously withdrawn if utility is unchanged,

$$V(Q;\ M\text{-}T(Q)) \equiv V(Q+dQ;\ M\text{-}T(Q)\text{-}dT).$$

Expanding the right-hand side,

$$0 = \frac{\partial V}{\partial Q}\, dQ - \frac{\partial V}{\partial M}\, dT,$$

or $$dT = \left(\frac{\partial V}{\partial Q} \Big/ \frac{\partial V}{\partial M}\right) dQ = -\ x(Q;\ M\text{-}T(Q))\ dQ. \tag{2}$$

The last equality uses Roy's Identity. Resources are withdrawn equal to the price reduction of the pre-existing consumption allocation.

If the consumer price is lowered from q to p in incremental steps, and resources simultaneously withdrawn at each step so that utility is unchanged, the total resource withdrawal is obtained by summing. Using Definition (1),

$$R(q) + EB(q) = \int_0^{T(p)} dT = - \int_q^p x(Q;\ M\text{-}T(Q))\ dQ.$$

The above expression is analytically awkward because T is an implicit function of Q. Write $h(Q;\ \bar{U})$ as the individual's compensated demand for the taxed commodity at price Q and utility $\bar{U}$. By duality, $h(Q;\ V(Q;M\text{-}T)) \equiv x(Q;\ M\text{-}T)$. Hence

$$R(q) + EB(q) = - \int_q^p h(Q;\ V)\ dQ = \int_p^q h(Q;\ V)\ dQ\ , \tag{3}$$

where $V \equiv V(q;\ M) \equiv V(Q;\ M\text{-}T(Q))$. Compensated demand is relevant because each incremental price reduction dQ is accompanied by an additional resource withdrawal (to leave utility unchanged).

3. TAX DISTORTIONS AS A FISCAL EXTERNALITY

The analysis of Dupuit (1844), Marshall (1890) and Harberger (1964) stresses that surplus is lost because of *quantity* changes. Similarly, Mirrlees (1975) notes: "Ramsey's original theorem ...emphasizes that attention should be fastened on demand effects, rather than on tax levels." Section 2 focused on *price* effects, making no mention of quantity changes. Put differently, the analysis of Section 2 is incomplete because it provides no rationale for why the compensating variation is positive if the indirect tax structure q is repalced by a lump-sum tax G. I now turn to the relationship between quantity changes and the excess burden. The analysis is summarized by notes.

NOTE 1: *the use of an indirect tax creates a fiscal externality.*

(FIGURE 1 HERE)

To highlight the externality, I initially consider the Dupuit-Marshall-Harberger approach using consumer surplus changes. Implicitly, demand is insensitive to income, uncompensated and compensated demands are equal, and consumer surplus is a measure of welfare. Figure 1 shows the demand for commodity x of the representative individual. Competitive firms supply the commodity elastically at producer price p.

The government requires tax revenue G from the individual. If the tax is collected by an indirect tax q on commodity x, the representative individual buys $\hat{x}$ units. Using the areas labelled in Figure 1, the individual obtains consumer surplus A. If the indirect tax is replaced by a lump-sum tax G, the consumer price falls to p and the individual's demand increases to x^*. The individual obtains consumer surplus A+B+G from the commodity; he pays lump-sum tax G, and achieves overall surplus A+B. The excess burden B is the surplus gained by the reform.

For each unit of the taxed commodity purchased by the individual, the individual gains private benefit *and* the government gains tax revenue (q-p). Because the government tax requirement is fixed, any change in tax revenue is rebated to *all* agents, or is a fiscal benefit external to the individual. The individual ignores the external benefits of additional purchases; he substitutes the taxed commodity by the untaxed commodity, and buys less than the efficient quantity x^* of the taxed commodity. As is true for traditional externalities, private incentives lead to "too little" of the activity generating the positive externality.

Suppose the individual is made to increase his consumption from $\hat{x}$ to x^* with the indirect tax applied (*i.e., he continues to pay consumer price q for each unit*). He achieves consumer surplus A-C, and the tax revenue of the government is G+B+C. The government is able to pay compensation C to the individual, and still have resources G+B. The excess burden or surplus B is retained as resources by the government - excess to the government's resource requirement and available to be rebated to *all* agents. This suggests an alternative interpretation for the excess burden: the excess burden is the social benefit if the representative individual increases his purchases to the efficient level. This is formalized below.

In contrast to the indirect tax, the use of the lump-sum tax G gives efficiency because changing quantities creates no additional tax revenue, or there is no externality.

NOTE 2: *the excess burden is the social benefit if the representative individual buys the efficient quantity of the taxed commodity.*

If demand is income sensitive, consumer surplus is inappropriate. I view each agent as being atomistic. There are a continuum of agents: a typical agent is indexed by the point a on the unit line [0,1]. The population is normalized to unity, or $\int_0^1 da = 1$. The consumption of agent a is denoted (x^a, y^a) and his utility is $U(x^a, y^a)$. The formal problem of each agent is

$$\max_{x,y} \quad U(x, y) \quad \text{s.t.} \quad qx + y = M + z.$$

z is the rebate received by the agent from the government. It is his share of the tax revenue in excess of the government's tax revenue requirement G,

$$z \equiv \frac{\int_0^1 (q-p)x^a \, da - \int_0^1 G \, da}{\int_0^1 da} = \int_0^1 (q-p)x^a \, da - G. \qquad (4)$$

In the traditional externality, agents are linked because the aggregate activity of all agents enters the utility function of each agent.[3] With a fiscal externality, agents are linked through the budget constraint: if agents buy more of the taxed commodity, the additional tax revenue is rebated to all agents. However, because each agent is atomistic, the rebate z received by the agent is independent of his own consumption choice. From the perspective of the agent, z is a lump-sum transfer. With z being lump-sum to each agent, demands are well defined functions, $x(q; M+z)$ and $y(q; M+z)$. I assume that the tax rate q is chosen such that

$$(q-p)x(q; M) = G, \qquad (5)$$

or, if all agents consume $x(q; M)$, $z=0$.

If an individual increases his purchase of the taxed commodity, the extra tax revenue generated is a benefit external to the individual. The normative tradition measures private benefits and private costs as compensating variations. If an individual is made to take an action with favorable external effects, the private benefit to others is the resources which, if withdrawn from them, maintains their utility at the pre-existing level; the private cost to him is the compensation he requires to maintain his pre-existing utility level. The social benefit is the sum of all private benefits less the private cost. To facilitate the use of calculus, I now focus on the group of agents located between i and i+di. I calculate the social benefit if each member of the group is made to increase his purchases from $x(q; M)$ to x^i. If each member is made to buy x^i at consumer price q, the rebate to each agent in the population is $z(x^i)$, or, using Equations (4) and (5),

$$z(x^i) = (q-p)(x^i - x(q;\ M))\ di.$$

The compensation[4] $c^i(x^i)$ required by each member of the group is defined by[5]

$$V(q;\ M) \equiv [\ U(x^i,\ y^i) \quad \text{where} \quad qx^i + y^i = M + z(x^i) + c^i(x^i)\ ].$$

Other agents outside the group are affected by the rebate. The private benefit to agent a outside the group of the consumption x^i by members in the group is $b^a(x^i)$; $b^a(x^i)$ is defined by

$$V(q;\ M) \equiv V(q;\ M + z(x^i) - b^a(x^i)), \qquad a \notin [i, i+di]. \qquad (6)$$

Hence $b^a(x^i) = z(x^i)$, or the benefit to an agent outside the group is the tax rebate he receives. The social benefit of consumption x^i by members in the group is the benefit to all agents outside the group, less the compensation required by members in the group,

$$[\int_0^i z(x^i)da + \int_{i+di}^1 z(x^i)da] - c^i(x^i)di$$

$$= z(x^i)\int_0^1 da - [z(x^i)+c^i(x^i)]di = (q-p)(x^i - x(q;M))di - [z(x^i)+c^i(x^i)]di.$$

(FIGURE 2 HERE)

Figure 2 shows the indifference curve corresponding to utility V(q; M) of an individual. If there is no tax, the individual's budget line is *ℓm* (of slope p). With tax structure q, the individual's budget line is *ℓn*. Point E corresponds to consumption x(q; M). If the individual's consumption of the taxed commodity is increased to x^i - purchased at price q - he pays additional tax *rt* and requires additional numeraire (or compensation) *st*. His contribution to the social benefit is *rs*.

Each member of the group is buying x^i units of the taxed commodity and is receiving compensation so that he buys y^i units of the numeraire, $U(x^i, y^i) = V(q; M) \equiv V$. If each member is made to buy additional taxed commodity dx^i - paying consumer price q - the increase in the rebate is dz. Using Equation (4)

$$dz = (q-p)dx^i \, di.$$

The additional compensation required by each member of the group is dc^i, defined by[6]

$$U(x^i, y^i) = U(x^i+dx^i, y^i+dy^i)$$

where $q(x^i+dx^i) + (y^i+dy^i) = M + z(x^i) + c^i(x^i) + dz + dc^i.$

Expand $U(x^i+dx^i, y^i+dy^i)$ around (x^i, y^i), and use the definition of $c^i(x^i)$,

$$0 = U_x dx^i + U_y dy^i \quad \text{and} \quad q dx^i + dy^i = dz + dc^i.$$

Eliminate dy^i,

$$dz + dc^i = q dx^i - \frac{U_x}{U_y} dx^i.$$

The resource transfer required by the member is his increased expenditure on x, $q dx^i$, less the resources saved from his reduced consumption of y.

The marginal social benefit of the increased consumption dx^i by members of the group is the marginal benefit to all agents outside the group less the marginal compensation required by members in the group,

$$[\int_{a=0}^{a=i}(dz)da + \int_{a=i+di}^{a=i}(dz)da] - dc^i di = dz - [dz+dc^i]di = [(q-p)dx^i - (q-\frac{U_x}{U_y})dx^i]di.$$

Denote the marginal rate of substitution as Q, $Q \equiv U_x(x^i, y^i)/U_y(x^i, y^i)$. Note that if an agent has a marginal rate of substitution Q and achieves utility V, his consumption of the taxed commodity is the compensated demand h(Q; V), or $x^i = h(Q; V)$.[7] As each member of the group increases his consumption from x(q; M) to x^i, and from x^i to x^i+dx^i, he is being compensated, or he is moving along his indifference curve. Write $dx^i = dh(Q; V)$. The marginal social benefit becomes

$$[(q-p)\ dh(Q; V) - (q-Q)\ dh(Q; V)]\ di = (Q-p)\ dh(Q; V)\ di.$$

The marginal social benefit is the difference between the tax revenue generated by the extra purchases and the resources required as compensation.

The demand curve of Figure 1 is now interpreted as the compensated demand curve (so that the vertical axis measures (U_x/U_y)). If an individual is made to increase his purchase of the taxed commodity by dx^i, the increased tax revenue is area *efgh*, the additional compensation required by the individual is area *efbc*, and the contribution to marginal social benefit is area *cbgh*. Provided $Q > p$, the tax revenue generated (q-p) by an additional purchase exceeds the compensation (q-Q) required by the individual, and the contribution to marginal social benefit is positive. In Figure 2, $Q > p$ implies that the indifference curve is steeper than line ℓm, and an increase in x^i causes tax revenue to increase more than the required compensation, so that distance *rs* increases.

The above analysis is true at any level of compensated purchases x^i. The social benefit achieved if each member of the group increases his purchases from $x(q;\ M){=}h(q;\ V)$ to x^i is obtained by summing the private benefits of the incremental steps, or is

$$[\int_{h(q;V)}^{x^i} (Q-p)\ dh(Q;V)]\ di. \tag{7}$$

The social benefit is maximized if x^i is chosen so that

$$\frac{d}{dx^i}[\int_{h(q;V)}^{x^i} (Q-p)\ dh(Q;\ V)]\ di = (Q(x^i)-p)\ di = 0,$$

where $Q(x^i)$ is the marginal rate of substitution at x^i. As suggested by the figures, the social benefit is maximized if $Q(x^i) = p$ or $x^i = h(p;\ V)$.

Integrating Expression (7) by parts, the maximal social benefit is

$$[\int_{h(q;V)}^{h(p;V)} (Q-p)dh(Q;\ V)]\ di \quad = \quad [-(q-p)x(q;M) - \int_q^p h(Q;\ V)\ dQ\ ]\ di.$$

Using Figure 1, and continuing to interpret the demand curve as compensated demand, the maximal contribution to social benefit by the individual is the area B: it is the contribution created if the individual buys the efficient quantity and is fully compensated. The contribution arises because the tax revenue generated (B+C) exceeds the compensation (C) required by the individual.

The social benefit due to the changed consumption of each member of the group measures the resources which, if withdrawn from the system, leave utilities unchanged. Its calculation presupposes that the rebate $z(x^i)$ is withdrawn from each agent a outside the group by a lump-sum tax $b^a(x^i)$ - as shown in Equation (6). In the calculation of the social benefit, each agent outside the group is unaffected by the increased consumption of agents in the group. Therefore, the maximal aggregate social benefit is calculated by summing over all groups of agents, or is

$$[-(q-p)x(q;M) - \int_q^p h(Q;\ V)\ dQ\ ] \int_0^1 di = EB(q).$$

The last equality follows from Equation (3). The aggregate social benefit is the excess burden, and is the triangular area under the compensated demand curve.

NOTE 3: *the excess burden is a measure of the utility loss associated with the externality.*

The aggregate excess burden is the most resources which can be withdrawn from the system while maintaining utilities at their pre-existing level. An alternative calculation is the maximal utility achieved by each agent if no resources are withdrawn from the system. I continue to think of the consumer price as being q; the choice of x is jointly determined by all agents, and the rebate is $[(q-p)x-G]$. The maximum utility achieved by agents acting cooperatively is

$$\max_{x,y} \quad U(x,\ y) \quad \text{s.t.} \quad qx + y = M + [(q-p)x - G].$$

or

$$\max_{x,y} \quad U(x,\ y) \quad \text{s.t.} \quad px + y = M - G.$$

The externality is internalized: it is "as if" the consumer price is p and there is a lump-sum tax G. The utility achieved is $V(p;\ M-G)$. In Figure 2, each agent - or the representative individual - achieves point E".

The utility loss caused by the externality is

$$V(p;\ M-G) - V(q;\ M) = V(p;\ M-G) - V(p;\ M-G-EB(q)).$$

In Figure 2, the utility loss is the utility fall on moving along the iso-resource line from E" to E. On moving from E to E', utility is unchanged, consumption of the taxed commodity is increased until the marginal rate of substitution is p, and resources equal to the excess burden are withdrawn from the representative individual. Therefore, the excess burden continues the normative tradition of measuring utility changes by resource changes: the excess burden measures the utility loss on moving from E" to E, by being the resource withdrawal which causes an equal utility fall at producer prices p - from E" to E'.

4. THE RAMSEY EQUATIONS

In Section 3, the tax structure is fixed and the benefit of setting quantities efficiently is calculated. In this section, the tax structure is variable and each agent purchases the inefficient quantity. I interpret the "Ramsey Equations," which characterize the optimal tax structure, as an example of the externality developed in the previous section: changes in the tax structure lead to changes in quantities, and to external benefits.

I again focus on the representative individual. To facilitate the graphical analysis, Sections 2 and 3 considered x to be a single taxed commodity; x is now a vector of n commodities which may be taxed, $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. Leisure is the numeraire commodity; the production of one unit of commodity i requires p_i labor units - the competitive producer price vector is $\mathbf{p} = (p_1, \ldots, p_n)$. The government has a resource requirement G, which (by assumption) must be collected using only the indirect taxation of commodities. The concern is about how tax rates should be set, and not about expenditure; the public project, on which the tax revenue is spent, is therefore ignored.

The Ramsey Problem is to choose the tax rates or the consumer price vector $\mathbf{q} = (q_1, \ldots, q_n)$ to maximize the representative individual's utility,[8]

$$\max_{\mathbf{q}} V(\mathbf{q};\ M+z), \qquad \text{s.t.} \quad G + z = (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q};\ M+z) \quad \text{and} \quad 0 \le z.$$

The constraint $0 \le z$ is imposed because the problem presupposes that lump-sum taxes are unavailable as an instrument, but (as in Section 3) any surplus tax revenue is rebated. The Appendix shows that the solution has $z = 0$. Setting $z = 0$, the Lagrangian[9] is $L = V(\mathbf{q};\ M) + \lambda[(\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q};\ M) - G]$. The typical first order condition for q_k is[10]

$$\frac{\partial L}{\partial q_k} = \frac{\partial V}{\partial q_k} + \lambda\Big[\sum_{i=1}^{n}(q_i - p_i)\frac{\partial x_i}{\partial q_k} + x_k\Big] = 0, \qquad k=(1,\ldots,n).$$

Writing $\mathbf{h}(\mathbf{q};\ V)$ as the compensated demand vector and $\alpha \equiv \partial V(\mathbf{q};\ M)/\partial M$ as the marginal utility of income, using Roy's Identity and the Slutsky Equation, the first-order conditions become,

$$-\frac{1}{x_k}\sum_{i=1}^{n}(q_i - p_i)\frac{\partial h_i}{\partial q_k} = 1 - \sum_{i=1}^{n}(q_i - p_i)\frac{\partial x_i}{\partial M} - \frac{\alpha}{\lambda}, \qquad k=(1,\ \ldots,\ n). \quad (8)$$

The right-hand side is the same for all k. This equation occurs in Diamond and Mirrlees (1971, Equation (37)) and many subsequent authors. It is the modern formulation of the "Ramsey Equations," and is the equation I interpret.[11]

My interpretation stresses the tax revenue gained by tax reforms which maintain utilities at their pre-existing levels. This tax revenue is a social benefit - resources which may be returned to individuals as a rebate using the instrument z. To emphasize tax revenue effects, I show that the tax structure $\mathbf{q}^*$, which solves the Ramsey Problem, solves the dual problem of maximizing tax revenue subject to the reservation utility level $V(\mathbf{q}^*, M)$.[12] Denote tax revenue collected using tax structure $\mathbf{q}$ as $R(\mathbf{q})$, $R(\mathbf{q}) = (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q}; M)$.

LEMMA: *If the tax structure* $\mathbf{q}^*$ *solves the Ramsey Problem, then* $\mathbf{q}^*$ *also solves*

$$\max_{\mathbf{q}} \ R(\mathbf{q}) \quad \text{subject to} \quad V(\mathbf{q}^*; M) \leq V(\mathbf{q}; M).$$

PROOF: The proof is by contradiction. Suppose there does exist a tax structure $\mathbf{q}'$ which raises more revenue than $\mathbf{q}^*$, $G < R(\mathbf{q}')$, and achieves at least the same utility, $V(\mathbf{q}^*; M) \leq V(\mathbf{q}'; M)$. Because $\mathbf{q}^*$ solves the Ramsey Problem, $V(\mathbf{q}'; M)$ cannot exceed $V(\mathbf{q}^*; M)$, or $V(\mathbf{q}^*; M) = V(\mathbf{q}'; M)$. Consider a small equiproportionate reduction in all consumer prices from $\mathbf{q}'$ to $\mathbf{q}''$, $\mathbf{q}'' = \theta\mathbf{q}'$, $\theta < 1$. $G < R(\mathbf{q}')$ and hence θ may be chosen sufficiently close to 1 to ensure that $G < R(\theta\mathbf{q}')$. The change in utility on moving from $\mathbf{q}'$ to $\mathbf{q}''$ is,

$$dV = \sum_{i=1}^{n} \frac{\partial V}{\partial q_i} (q_i'' - q_i') = \alpha(\mathbf{q}'; M) \ (1-\theta) \sum_{i=1}^{n} x_i(\mathbf{q}'; M) q_i',$$

where the last equality uses Roy's Identity. Budget balance of the individual implies $\sum_{i=1}^{n} x_i(\mathbf{q}'; M) q_i' > 0$. Hence $dV > 0$, or $V(\theta\mathbf{q}''; M) > V(\mathbf{q}^*; M)$, contradicting that $V(\mathbf{q}^*; M)$ is the solution to the Ramsey Problem.

NOTE 4: *the left-hand side of Equation (8) is the tax revenue gain if an unit lump-sum tax is substituted for the marginal use of instrument* q_k.

I consider the tax revenue gain if (a) a small lump-sum tax dT is levied and if (b) the tax rate on the kth commodity is adjusted by dq_k ($dq_k < 0$) to leave utility unchanged,

$$V(q_1, \ldots, q_k, \ldots, q_n; M) \equiv V(q_1, \ldots, q_k + dq_k, \ldots, q_n; M - dT).$$

Because lump-sum taxation is not a feasible instrument, this is strictly a thought-experiment. Expanding, or using Equation (2), q_k must change such that

$$dT = -\ x_k dq_k.$$

Utility is unchanged. Demand changes are therefore changes in compensated demands, and the total change in tax revenue is

$$dR \quad = \quad dT + d[\sum_{i=1}^{n}(q_i - p_i)h_i] \quad = \quad dT + h_k dq_k + \sum_{i=1}^{n}(q_i - p_i)dh_i.$$

But $dT = -x_k dq_k = -h_k dq_k$, or the increase in the lump-sum levy is exactly offset by the lower tax receipts from the lower tax rate on the pre-existing bundle. Hence the total tax revenue increase if an unit lump-sum tax is levied, and tax rate q_k lowered to keep utility unchanged, is

$$\left.\frac{dR}{dT}\right|_V = -\frac{\sum_{i=1}^{n}(q_i - p_i)dh_i}{x_k dq_k}.$$

The tax revenue gain arises from the external (fiscal) benefit of the induced *quantity* changes.

NOTE 5: *the right-hand side of Equation (8) is the maximal tax revenue gain if an unit lump-sum tax is substituted for the marginal use of all tax instruments.*

To interpret the right-hand side of Equation (8),[13] I consider the tax revenue gain if (a) the lump-sum tax dT is levied, and if (b) *all* tax rates are adjusted so that utility maintains its pre-existing level. The changes are considered to be done as two stages in sequence. If the lump-sum tax dT is levied, tax revenue increases at the first stage by dT less the associated decrease in indirect tax revenue due to the decrease in demands,

$$dT - \sum_{i=1}^{n}(q_i - p_i)\frac{\partial x_i}{\partial M}\, dT.$$

The lump-sum tax causes the utility of the individual to fall αdT; hence tax rates must be adjusted at the second stage so that utility rises αdT. The Lagrangian multiplier measures the change in the objective as the constraint is tightened, $\partial V/\partial G = -\lambda$; hence the minimum tax revenue fall brought about by the need to raise utility αdT in the second stage is $(\alpha dT)/\lambda$. The right-hand side of Equation (8) is therefore the maximal tax revenue gain if an unit lump-sum tax is imposed and all tax instruments are adjusted to leave utility unchanged. The first term is the lump-sum tax, the second term is the fall in indirect tax revenue due to the effect of the lump-sum tax on demand, and the third term is the tax revenue lost by the adjustment of all tax instruments.

NOTE 6: *The Ramsey Equations are interpreted: no tax revenue is gained if the marginal use of instrument q_k is substituted by the marginal adjustment of all tax instruments.*

The left-hand side of Equation (8) is the tax revenue gain if a unit lump-sum tax is imposed and instrument q_k is lowered to leave utility unchanged. The right-hand side is the least tax revenue lost if a unit lump-sum transfer is made, and *all* tax instruments are adjusted to leave utility unchanged. The combined change substitutes the collection of tax revenue from instrument q_k to all other instruments: it does not impose a lump-sum levy and is feasible even if the planner is (by assumption) unable to levy a lump-sum tax. Any tax revenue gain is the external (fiscal) benefit of the application of the pre-existing tax structure to the induced quantity changes. Equation (8) is interpreted: at the optimum tax structure, it is impossible to increase tax revenue by marginally substituting from the use of any tax instrument to others (to leave utility unchanged).

The Ramsey Equations characterize the indirect tax structure which maximizes the utility of the representative individual subject to the government revenue requirement. Intuitively, at the optimum tax structure, the utility lost by the collection of a marginal unit of tax revenue is the same for each instrument. I now show how this condition is contained in the Ramsey Equations.

NOTE 7: *at the tax structure characterized by the Ramsey Equations, the utility loss caused by the collection of marginal unit of tax revenue is the same for each instrument.*

Note 4 considered the tax revenue gain of levying a marginal lump-sum tax and lowering tax rate q_k to leave utility at its pre-existing level. I suppose here that the two changes are made separately and in sequence. At the first stage, the lump-sum tax dT is levied: utility falls αdT and tax revenue increases. At the second stage, tax rate q_k is lowered so that utility rises αdT. The utility gain αdT at the second stage is the same for each instrument q_k and is the utility change over which tax instruments are compared. If a "large" utility loss occurs when a marginal unit of tax revenue is collected by raising tax rate q_k, there is only a "small" tax revenue loss when tax rate q_k is lowered to raise utility αdT. The tax revenue gain at the first stage is the same for all instruments; a "small" tax revenue loss at the second stage implies a "large" tax revenue gain overall. Therefore, the tax revenue gained from the combined stages gets larger as there is a larger utility loss associated with using instrument q_k to collect a marginal unit of tax revenue.

The Ramsey Equations show that the same overall tax revenue results for each instrument, or for all instruments. Therefore the Ramsey Equations imply that the same utility fall occurs if a marginal unit of tax revenue is collected by any or by all instruments.

5. CONCLUSION

This paper seeks to integrate the excess burden into the general theory of externalities. The use of a proportional tax causes the purchase of a private good to have external benefits. Private incentives lead to "too little" of the socially desirable action, and the outcome is inefficient. The excess burden is associated with a change in quantities. The following is often wrongly argued: if demand is price inelastic, the absence of a change in demand for the taxed commodity implies that there is no excess burden. This paper makes clear why this argument is incorrect. Because of the externality created by additional purchases, an external benefit is created if the individual increases his consumption. Therefore, the excess burden arises *because* his consumption does not change.

If the tax structure is chosen optimally, the Ramsey Equations show that social benefit cannot be created or that tax revenue cannot be increased by the marginal substitution of one tax instrument by others. Closely related to the Ramsey Equations of the optimal tax literature are the "Ramsey Pricing Formulae" of public utilities (e.g., Boiteux (1956) and Baumol and Bradford (1970)). In this context, the regulator must choose consumer prices to maximize welfare, subject to the public utility recovering fixed costs. The intuition provided in this paper may be used to explain the optimal mark-up over marginal cost in these regulated industries.

APPENDIX

Consider z to be a fixed parameter, and $(q_1,\ldots,q_n)$ to be the choice variables. The Lagrangian is $L_2 = V(\mathbf{q};\ M+z) + \lambda[(\mathbf{q}-\mathbf{p})\cdot\mathbf{x}(\mathbf{q};\ M+z) - G-z]$. The first order condition for q_k is

$$\frac{\partial L_2}{\partial q_k} = \frac{\partial V}{\partial q_k} + \lambda\Big[\sum_{i=1}^{n}(q_i-p_i)\frac{\partial x_i}{\partial q_k} + x_k\Big] = 0, \qquad k=(1,\ldots,n). \qquad (A.1)$$

The total derivative of the objective with respect to a change in z is

$$\frac{dV}{\partial z} = \frac{dL_2}{dz} = \frac{\partial L_2}{\partial z} = \frac{\partial V}{\partial M} + \lambda\Big[\sum_{i=1}^{n}(q_i-p_i)\frac{\partial x_i}{\partial M} - 1\Big]. \qquad (A.2)$$

The first equality is because the Lagranian function equals the maximal value of the objective, and the second follows from the envelope theorem. In Equation (A.1), use Roy's Identity, substitute for $\partial V/\partial M$ from Equation (A.2) and use the Slutsky Equation,

$$\frac{\partial L_2}{\partial q_k} = -\frac{dV}{dz}x_k + \lambda\sum_{i=1}^{n}(q_i-p_i)\frac{\partial h_i}{\partial q_k} = 0, \qquad k=(1,\ldots,n).$$

Multiply by (q_k-p_k), sum over k, and use the revenue constraint $\sum_{k=1}^{n}(q_k-p_k)x_k = G+z$,

$$-\frac{dV}{dz}(G+z) + \lambda\sum_{k=1}^{n}\sum_{i=1}^{n}(q_i-p_i)\frac{\partial h_i}{\partial q_k}(q_k-p_k) = 0, \qquad k=(1,\ldots,n).$$

But $\lambda = -dV/dG > 0$, and the Slutsky Matrix is negative semidefinite. Therefore, provided $G+z\geq 0$, $dV/dz \leq 0$, or (given $0 \leq z$) $z = 0$ is optimal.

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FIGURES

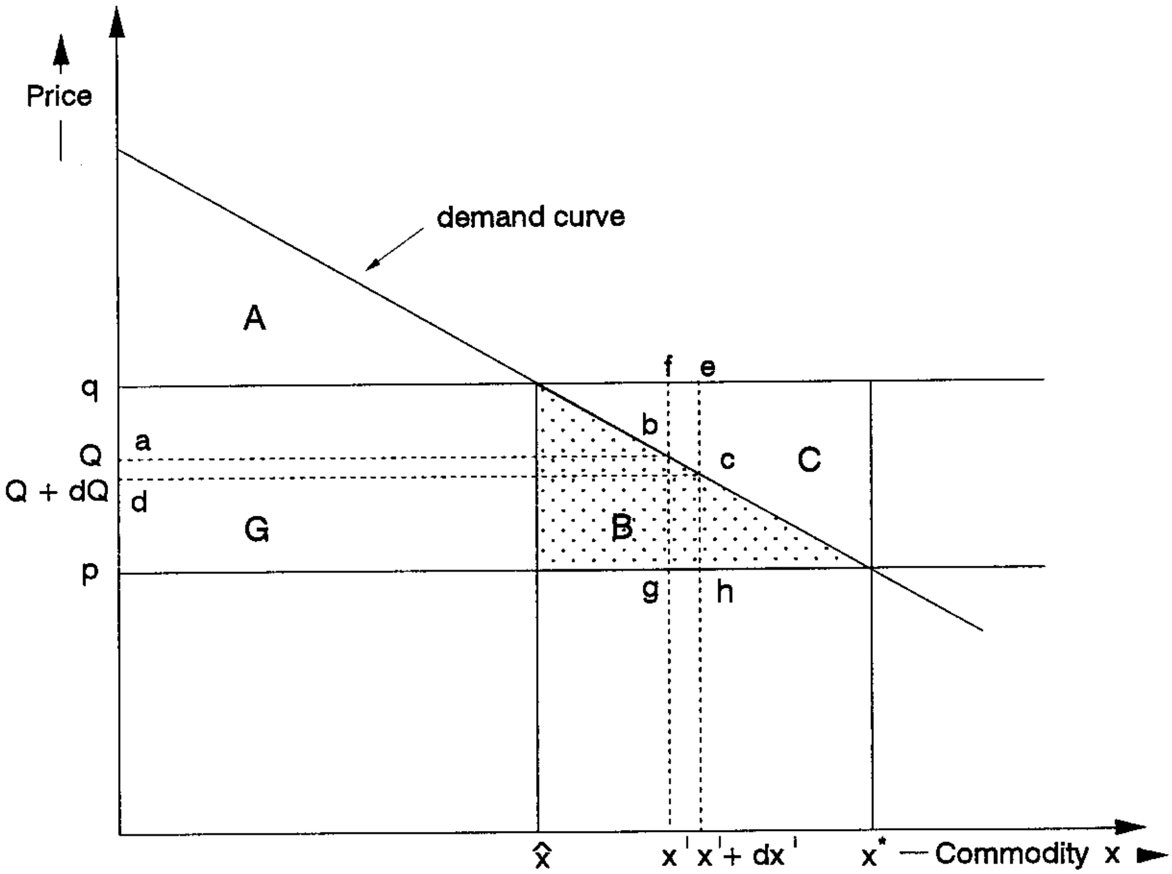

Figure 1: Consumer surplus analysis.

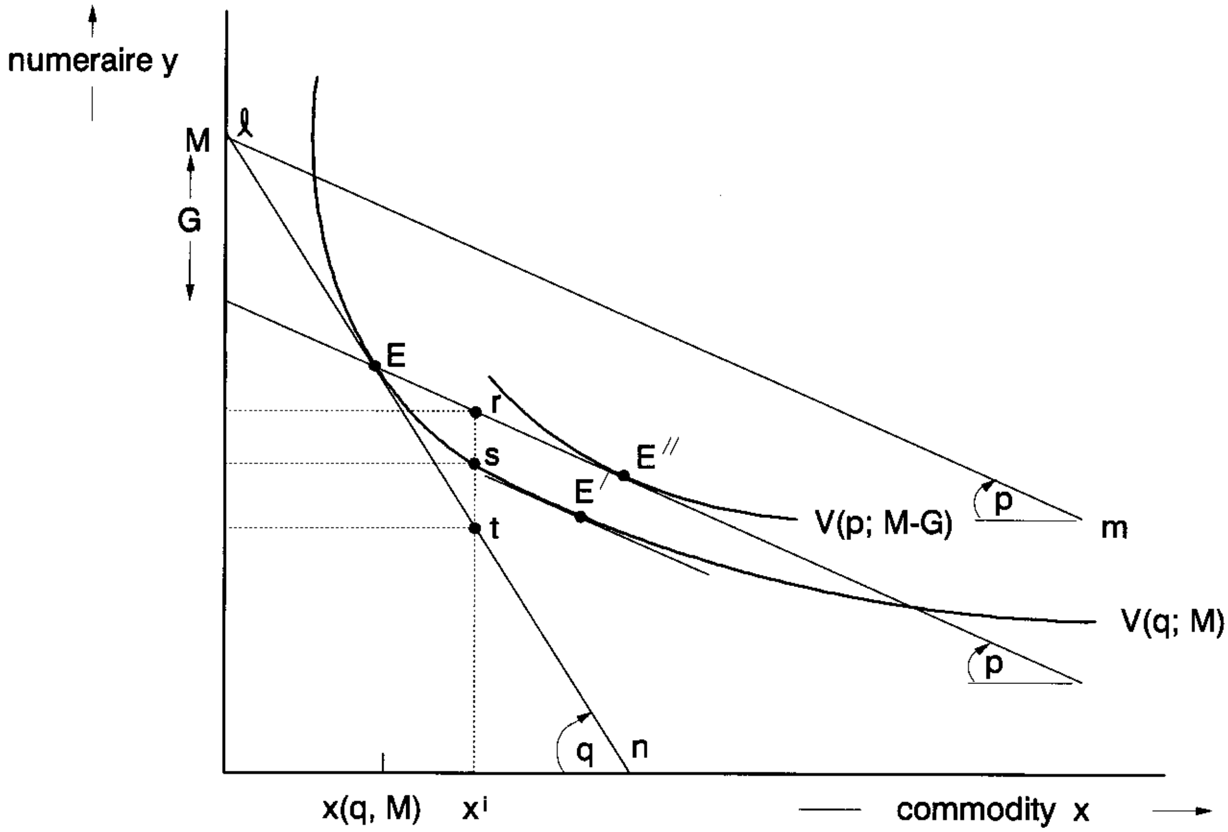

Figure 2: indifference curve analysis.

FOOTNOTES

1 A natural interpretation is: "commodity y" is leisure, the wage is normalized to unity, the endowment M is the leisure endowment, and the production of one unit of x requires p units of labor.

2 Many of the succeeding expressions may be obtained using the expenditure function (Diamond and McFadden (1974)). However, the expenditure function in Note 2 must be constrained because x^i is a pre-determined variable: this complicates the notation and the presentation.

3 E.g., a traditional externality exists if an individual i obtains utility from his own consumption x^i, y^i, and from the aggregate consumption level of good x, $U(x^i,\ y^i;\ \int_0^1 x^a da)$.

4 $z(x^i) = (x^i - x(q;\ M))\ di$. As $di \to 0$, $dz \to 0$, or $z(x^i)$ is of lower order than $c^i(x^i)$. It is the integral $\int_0^1 z(x^i) da$ which is of the same order as $c^i(x^i)$.

5 If y is a vector of untaxed commodities with prices $\mathbf{r}$, $c^i(x^i)$ is defined by

$$V(q;\ M) \equiv [\ \max_{\mathbf{y}^i}\quad U(x^i,\ \mathbf{y}^i)\quad \text{s.t.}\quad qx^i + \mathbf{r}\cdot\mathbf{y}^i = M + z(x^i) + c^i(x^i)].$$

6 If y is a vector of untaxed commodities with prices $\mathbf{r}$, dc^i is defined by

$$V(q;\ M) \equiv [\ \max_{\mathbf{dy}^i}\ U(x^i+dx^i,\ \mathbf{y}^i+\mathbf{dy}^i)\quad \text{s.t.}\quad q(x^i+dx^i)+\mathbf{r}\cdot(\mathbf{y}^i+\mathbf{dy}^i) = M+z(x^i)+c^i(x^i)+dz+dc^i]$$

7 $h(Q;\ V)$ is defined by: $\min_{h,y^c}\ Qh + y^c$ s.t. $U(h,\ y^c) = V$. The FOC is

$$\frac{U_x(h,\ y^c)}{U_y(h,\ y^c)} = Q.$$

Therefore, $Q = U_x(x^i,\ y^i)/U_y(x^i,\ y^i)$ and $U(x^i,\ y^i) = V$, implies $x^i = h(Q,\ V)$.

8 Because $\partial V/\partial M > 0$, choosing $\mathbf{q}$ to solve the Ramsey Problem is equivalent to choosing $\mathbf{q}$ to minimize the excess burden,

$$\min_{\mathbf{q}} \quad EB(\mathbf{q}) \quad \text{s.t.} \quad V(\mathbf{q}; M) = V(\mathbf{p}; M-R(\mathbf{q})-EB(\mathbf{q})) \quad \text{and} \quad R(\mathbf{q}) = G.$$

This is noted by Diamond and McFadden (1974) and Auerbach (1985).

[9] Mirrlees (1986) discusses when the Lagrangian technique is inappropriate.

[10] Using Roy's Identity, and rearranging, this first-order condition may be written as

$$\frac{x_k}{x_k + \sum_{i=1}^{n}(q_i-p_i)\dfrac{\partial x_i}{\partial q_k}} = \frac{\lambda}{\alpha}, \qquad k = (1, \ldots, n).$$

The left-hand side is the Marginal Cost of Funds of instrument q_k. Using the arguments of Dixit (1970), Mayshar (1990) and Triest (1990), the first-order conditions written in this form equalize the utility loss associated with using each instrument to collect a marginal unit of tax revenue. These are the "intuitive" first-order conditions referred to in the Introduction.

[11] Noting that $\partial h_i/\partial q_k \equiv \partial h_k/\partial q_i$, Equation (8) is often written as

$$-\frac{1}{x_k}\sum_{i=1}^{n}(q_i-p_i)\frac{\partial h_k}{\partial q_i} = 1 - \sum_{i=1}^{n}(q_i-p_i)\frac{\partial x_i}{\partial M} - \frac{\alpha}{\lambda}, \qquad k=(1, \ldots, n).$$

For small price changes Δq_i, the change in the compensated demand of the kth commodity is $\Delta h_k = \sum_{i=1}^{n}(\partial h_k/\partial q_i)\,\Delta q_i$. If the tax rate is small, set $\Delta q_i = q_i - p_i$ and the above equation becomes $\Delta h_k/h_k = \text{const.}$. If an optimal tax structure is imposed, and if compensation is paid, there will be an equal percentage change in all taxed commodities (Samuelson (1986)). With no income effects, $\partial h_k/\partial q_i \equiv \partial x_k/\partial q_i$, the optimal tax structure imposes an equi-proportionate fall in the demand of each taxed commodity (Ramsey (1927)).

[12] This is noted in Harris and Wildasin (1985).

[13] This interpretation appears in Atkinson and Stern (1974).