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SUBJECTIVE PROBABILITY AND SUBJECTIVE ECONOMICS

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> Any attempt rigorously to eliminate our human perspective from our picture of the world must lead to absurdity.
>
> -- Michael Polanyi

> The equanimity of your average tosser of coins depends upon a law, or rather a tendency, or let us say a probability, or at any rate a mathematically calculable chance, which ensures that he will not upset himself by losing too much nor upset his opponent by winning too often. This made for a kind of harmony and a kind of confidence. It related the fortuitous and the ordained into a reassuring union which we recognized as nature. The sun came up about as often as it went down, in the long run, and a coin showed heads as often as it showed tails. Then a messenger arrived.
>
> -- Tom Stoppard
> Rosencranz and Guildenstern are Dead

I.

The role of uncertainty in economic life is a concern central to and characteristic of the work of many eminent theorists. Frank Knight's name comes instantly to mind, as does that of G.L.S. Shackle, who has written eloquently on the implications of an unknowable future. And to Ludwig von Mises, a principal progenitor of the "modern Austrian School" of economics,[1] "uncertainty of the future is already implied in the very notion of action."[2]

For all these economists, this concern with uncertainty is closely tied to a belief in methodological "subjectivism," the view that theories involving human beings should be built not around "objective facts" but around the beliefs, perceptions, and expectations of those humans.[3] One might thus expect that, in the long-standing debate over the probabilistic representation of uncertainty, these economists would locate themselves firmly on the side of the "subjective" probabilists. But, perhaps oddly, this is not at all the case.

The nature of probability and its role in human decision-making is a topic that has absorbed scholarly attention for centuries. And a proper examination of the appropriate connection between subjectivism in economics and subjectivism in probability theory would require an essay of at least dissertation length. My purposes here are more limited. First of all, I hope to suggest that the "subjectivist" view of probability might indeed prove more congenial to "subjectivist" followers of Knight, Shackle, or Mises than many suspect. Whatever shortcomings the modern combination of subjective probability and decision theory may have for the modeling of economic behavior, subjective probability _qua_ probability theory -- I will argue -- should be appealing to those who uphold a subjectivist economic philosophy. But I will also suggest that the decision theory formalism may indeed have its uses in economic theory.

For one thing, it presents a self-consistent extention of what F.A. Hayek has called the Pure Logic of Choice,[4] and thus helps illustrate the important concept of opportunity cost in situations of uncertainty. Quite apart from this, the subjective probability/decision theory approach helps make clear that uncertainty is always defined strictly within a specified framework of means and ends. This should be of particular interest to writers of the "Austrian" school, who have stressed the economic importance of entrepreneurial change in means/ends frameworks, an activity to be distinguished from the mere bearing of uncertainty. Indeed, I go so far as to suggest that the failure by these writers to embrace subjective probability has permitted confusion between uncertainty within a framework and uncertainty about (or ignorance of) the framework itself, which has obscured the very distinction they most wish to champion.

II.

All of the writers I've mentioned are fascinated with what we might call the "one-shot" decision situation. This is the situation of what Mises called "case probability," what Knight called "uncertainty," and what Shackle calls, among other things, "self-destructive, non-seriable" decisions.[5] Such situations contrast with those involving "class probability" (Mises) or "risk" (Knight). In the latter circumstances, the occurrence of a random event can be grouped in an actuarial way as an instance of a class of such events; the probability of that event can thus be determined as the relative frequency of occurrence of the event within the population. Situations of "case probability" or "uncertainty," by contrast, involve one-of-a-kind events that cannot be identified as instances of a larger class. And it is in these unique, "non-actuarial" situations that most economic decision-making takes place.

This risk/uncertainty distinction is closely tied to one's understanding of the calculus of probability. Those who view the distinction as fundamentally relevant to the nature of probability normally hold that probability -- or numerically representable probability, at any rate -- is defined only in situations of "risk" or "class probability." To these writers, a probability is a statement about the observed relative frequency of occurrence of the event in question. To say, for example, that the probability of a coin turning up heads is one-half is to say that, as the number of tosses of the coin approaches infinity, the number of heads observed will approach one-half the total number of tosses. Situations of "risk" or "class probability" are those in which there exists some analogous experimental basis for determining such relative frequencies empirically.

This is called the frequency approach to probability, and represents one brand of objective probability theory. One of the most important figures in the development of this approach was Richard von Mises, the brother of Ludwig.

Objectivism in probability theory -- if I may stretch an analogy a little -- is not unlike objectivism in economic theory. In both cases, the objectivist holds that our theories are not logical, rigorous, or scientific unless they exclude all subjective elements in favor of the tangible and measurable. In this regard, the frequency approach may well qualify as the "behaviorism" of probability theory.

Ludwig von Mises's attitude toward the frequency view seems simultaneously antagonistic and approving. On the one hand, he seems to believe that probability has some meaning in one-shot decision situations -- he calls these situations of case "probability," after all. And he seems also to reject the view that the frequency definition of probability circumscribes what we mean by probability: "Only preoccupation with the mathematical treatment could result in the prejudice that probability always means frequency," he asserts.[6]

On the other hand, he appears to believe that the frequency view of probability (which he sees as identical with his own "class probability") is a valid approach to probability theory; indeed, he regards his own formulation of its definition as "the only logically satisfactory one."[7] More to the point, he views frequency probability as the only proper domain for "the calculus of probability," even while considering such a calculus to be little more than a form of charlatanry typical of mathematicians. Jack High, a modern Austrian theorist who goes down the line with

Mises on probability, finds even more to like in the frequency theory of brother Richard, choosing indeed to "defend the frequency view against some attacks by subjective probability theorists."[8]

In sum, the view of these economists seems to be that there exist two different kinds of probability. One of them -- frequency probability -- may be represented in mathematical terms but is irrelevant for decision-making; the other -- "case probability" -- is relevant to the uncertainty faced in decision-making, but cannot be expressed mathematically or accorded the same status of rigor as can the frequency view.

By contrast, the subjectivist view of probability does not see the risk/uncertainty distinction as fundamentally relevant to probability theory *per se*. The distinction, this school holds, is not between two different *kinds of probability* but between two different *information structures*.[9] To the subjective probabilist, all situations are situations of "case probability," and a rigorous (and in principle mathematical) theory of such case probablity is in fact possible. Circumstances of "class probability" -- what Buchanan and Di Pierro have usefully called situations of "potential cognitive certainty"[10] -- are merely special cases in which a particular kind of information may be brought to bear on a decision-maker's probability assessment.

The best way to distinguish the objectivist view of probability from the subjectivist view is as follows. In an objectivist theory, probability is seen as an attribute of the world. For example, the probability of drawing a ball of a particular color from the omnipresent urn containing balls of various colors is, to the objectivist, a property of the balls-in-the-urn themselves. In the subjectivist theory, probability is not an attribute of things but an indication of a particular individual's state of knowledge about uncertain events. Thus, the probability one assigns to picking an ultramarine ball from the urn reflects his estimation of all the factors that might influence the selection of such a ball -- including but not necessarily limited to the color-mix of balls in the urn.

Subjectivists are often described somewhat misleadingly as Bayesians, in reference to a formula called Bayes's rule that is congenial to a subjectivist interpretation. But, although subjectivists may be more inclined than "classical statisticians" to stress the importance of this rule in probability theory, Bayes's rule is a perfectly well-defined part of frequency theory, and does not serve as the distinguishing mark of subjectivism. One occasionally hears suggested that the nineteenth-century marginalist revolution in economics was significant less for its marginalism than for its subjectivism;[11] analogously, to arch-subjective probabilist Ronald Howard, "the most significant part of the [Bayesian] revolution is not Bayes's theorem or conjugate distributions but rather the concept of probability as a state of mind..."[12]

One implication of the subjective view, then, is that it is not meaningful to talk about "knowing" a probability or a probability distribution. A probability assessment reflects one's state of information about an event; it is not something ontologically separate whose value can be determined objectively.

In places, Mises actually articulates the subjectivist attitude quite well. "We may assume," he says, "that the outcome of all events and changes is uniquely determined by eternal unchangeable laws governing becoming and development in the whole universe. We may consider the necessary connection ad interdependence of all phenomena, i.e., their causal concatenation, as the ultimate fact. We may entirely discard the notion of undetermined chance. But however that may be, or appear to the mind of a perfect intelligence, the fact remains that to acting man the future is hidden."[13] And Mises's definition of probability is itself quite congenial to the subjectivist view. "A statement is probable [i.e., probabilistic]," he says, "if our knowledge concerning its content is deficient. We do not know everything which would be required for a definite decision between true and not true. But, on the other hand, we know something about it; we are in a position to say more than simply <u>non liquet</u> or <u>ignoramus</u>."[14]

But no sooner has he said this than, in criticizing "the calculus of probability," he effectively contradicts himself in a significant way. "For this defective knowledge the calculus of probability provides a presentation in symbols of the mathematical terminology. It neither expands nor deepens nor complements our knowledge. It translates it into mathematical language. Its calculations repeat in algebraic form what we knew beforehand. They do not lead to results that would tell us anything about the actual singular events."[15] In fact, numerical probabilities, whatever their source, do tell us something (albeit not something we did not already know) about actual singular events. They indicate that, while we do not have certain knowledge of what will occur, we do have some knowledge relevant to the matter.

"When a man speaks of probability," says Shackle, in expressing a view apparently near to that of Mises, "he pushes ignorance as far as he can into concealment and an inconspicuous role..."[16] But one might just as easily say that, in expressing a probability assessment over an uncertain event, one is making one's ignorance explicit. Indeed, much of the matter here hinges on our use of words. "Knowledge and uncertainty are mutually exclusive...," says Shackle.[17] But it is fairer, it seems to me, to say that, while knowledge and ignorance may be mutually exclusive, knowledge and uncertainty are not; for uncertainty is partial knowledge, part knowledge and part ignorance. And it is precisely because we do have some knowledge -- because we can say more than non liquet or ignoramus -- that subjective probability makes sense.

III.

The information structure of a stationary stochastic process -- that is to say, the information structure of a situation of "risk" or "class probability" -- is a special case that is at once important and beguiling.

It is important because, as Mises correctly points out, it provides the metaphor on which subjective probability theory relies for its practical intelligibility. If I say -- to use Mises's example -- that a candidate has a nine-in-ten chance of being elected, I am saying that, upon careful consideration, I view his or her election to be as likely as the event that the spin of an unbiased wheel-of-fortune calibrated from one to ten will result in a number other than ten. A comparison of the one-shot event with a "canonical experiment" of this sort establishes the probability measure for the one-shot event, a practice known among the cognoscenti of decision analysis as "spinning the wheel" for the decision-maker.

That the assessment of the probability of a one-shot event is drawn with the help of the analogy of a stationary stochastic process (like the repeated spinning of a wheel of fortune) need not be viewed, however, as somehow invalidating the subjectivist approach. Mises's criticism, repeated by High without much amplification, is that the relationship between the likelihood of a candidate's election and the likelihood of a wheel's spin attaining a particular range of values is merely a metaphor. "Analogies and metaphors are always defective and logically unsatisfactory," sniffs Mises.[18] Whatever the status of metaphor, though, there is a strong case that the connection is in fact a logical one -- that probability in the one-shot case and in the canonical experiment can be linked axiomatically.[19]

The information structure of "class probability" is beguiling because of its peculiar properties: it involves, as it were, a radical discontinuity in our state of information. We can often gain partial knowledge relevant to the occurrence of an event by studying the structural properties of the situation of which the event is a part; for instance, we can gain information about the likelihood of drawing an ultramarine ball from an urn by considering the relative frequency of ultramarine balls drawn after a large number of trials with replacement (or perhaps by computing the fraction of such balls in the urn). This provides us with definite knowledge about the stochastic structure itself but with only partial knowledge about the outcome of any particular drawing. To go beyond this level of partial knowledge would be incomparably more difficult, requiring a super-mind capable of predicting the outcome of the draw from, say, the physical laws of the drawing apparatus and the initial positions of the balls.

Getting to the first state of knowledge is easy; to any higher state seems virtually impossible. Thus we are easily led to think that the only knowledge we need consider as relevant to probability is that contained in the objective stochastic structure itself.

The discontinuity becomes far less acute, however, once we leave the realm of Gedankenexperiments and stylized games of chance.[20] Consider an example suggested by Mises. Seven out of ten people who come down with a particular disease survive. If I know nothing about a patient except that he has the disease, then the statistical frequency of survival in this "class" of patients is the relevant probability assessment. If, on the other hand, a physician examines the patient, he may find that the patient is young and strong, and may therefore the chance of survival is not 70 per cent but 90 per cent.

To High, the two probabilities are both meaningful; they are different "kinds" of probability that "refer to two different aspects of the patient."[21] But what the two probability assessments in fact refer to is two different states of information about the patient. The first assessment is associated with the information contained in the structure of the "class," conceived in strict analogy with the balls-in-the-urn problem.

Unlike the balls-in-the-urn problem, though, there is in the medical example no necessary gap between this state of knowledge and the state of perfect information: intermediate states are possible. And, once the doctor has examined the patient, he has jumped to such an intermediate state, obliterating his old state of information and altering his probability assessment. The new assessment _is_ in contradiction with the "class probablity" assessment; although the "class" properties of the problem certainly informed the new assessment, the old assessment based only on those properties has now been entirely superseded.[22] New information has arrived; and the physician can never retreat to his previous "objective" state of knowledge.

IV.

There are, it seems to me, three levels on which one should examine the relationship between probability theory and economic theory. The failure to see these levels as distinct is, I believe, a cause of much confusion on this topic.

The first level is that of probability theory _per se_, of its logic as a way of expressing information. The second level is the role of probability in a Pure Logic of Choice. And the third is the role of this Pure Logic of Choice _cum_ probability in economic theory. Let's take them in order.

V.

Perhaps the best way to begin considering the logic of information in probability theory is by parsing an example provided by Mises.

> Two football teams, the Blues and the Yellows, will play tomorrow. In the past the Blues have always defeated the Yellows. This knowledge is not knowledge about a class of events. If we were to consider it as such, we would have to conclude that the Blues are always victorious and the Yellows are always defeated. We would not be uncertain with regard to the outcome of the game. We would know for certain that the Blues will win again.[23]

The mode of reasoning Mises denounces here -- and quite properly so -- is that of so-called classical statistical inference. A version of the frequency view, this method seeks to infer the real, true, underlying probability distribution of a system using solely the information that can be gained empirically through "sampling."

In this case, the only permissable basis under this view for establishing the probability that the Blues will again defeat the hapless Yellows is the data reporting past trouncings. One implicitly casts the meetings between the two clubs as instances of a seriable experiment; and he would somehow calculate an "estimate" of the true probability, perhaps using a simple maximum-likelihood estimator. That is, the classical statistician would choose as his estimate the probability that would maximize the likelihood function.

If all we know about the meetings between the teams is that the Blues defeated the Yellows six times in the last six games, if p is the real underlying probability that the Blues will win tomorrow, and if D stands for the datum of the six straight Blue victories, then the likelihood function is $\{D|p\}$; that is, it is the probability, as a function of p, that we would have seen the six wins in six tries had the "real" probability underlying the class of Blue/Yellow conflicts been p. The maximum likelihood estimator $\hat{p}$ is the value of p that maximizes the likelihood function. Since the probability of six wins in six independent trials is p^6, the likelihood function is thus p^6, which is maximized when $\hat{p} = 1$. So, as Mises says, our estimate for the probability of a Blue victory tomorrow is unity. "The mere fact that we consider our forecast about tomorrow's game as only probable shows that we do not reason this way."[24]

"On the other hand," he continues,

> we believe that the fact that the Blues were victorious in the past is not immaterial with regard to the outcome of tommorrow's game. We consider it as a favorable prognosis for the repeated success of the Blues. If we were to argue correctly according to the reasoning appropriate to class probability, we would not attach any importance to this fact. If we were not to resist the erroneous conclusion of the "gambler's fallacy," we would, on the contrary, argue that tomorrow's game will result in the success of the Yellows.[25]

I must confess that I find this paragraph difficult to understand.[26] But the first two sentences are perfectly intelligible (and correct).

One reason that we find the conclusions of classical statistics absurd in this football case is that we already have knowledge about football teams in general, and perhaps these teams in particular, that we are actually bringing to bear when we think about tomorrow's game. Bayes's rule provides a way of bringing together this prior information about the Yellows with the "not immaterial" fact of their repeated failures. Without leaving the confines of the frequency view, we can form a Bayesian estimate of the probability of the Blues winning yet again:

$$\{p|D\} = \frac{\{D|p\}\{p\}}{\{D\}}$$

The symbols p and D are as before; $\{p|D\}$ is the called the posterior distribution, and represents my probability distribution over p given the fact of the six Blue victories in six games; $\{D|p\}$ is the likelihood function as before; $\{D\}$ is the unconditional probability of seeing the datum D; and $\{p\}$, the "prior," represents my probability distribution over p before I learned D.

With the forebearance of those uninterested in mathematics, we can quickly illustrate how this would work. $\{D\}$ can be expanded as

$$\{D\} = \int_p \{D|p\}\{p\}.$$

The likelihood function remains p^6. Thus

$$\{D\} = \int_0^1 p^6\{p\}dp.$$

Let's now let {p}, the prior, be uniform between 0 and 1. This is a relatively uninformative prior, suggesting that, before hearing the results of their previous contests, we had really very little idea as to the true underlying probability of the Blues winning. Calculating the integral, this gives {D} = 1/7. And:

$$\{p|D\} = 7\{D|p\}\{p\} = 7p^6\{p\}.$$

In other words, the posterior distribution -- the distribution after considering the datum of six Blue triumphs -- is the prior augmented by a scale factor. To get an "estimator" of the probabality that the Blues will again emerge victorious, we can take the mean of the posterior:

$$\langle p|D\rangle = \int_p p\{p|D\} = \int_0^1 p(7p^6\{p\}) = 7\int_0^1 p^7\{p\}dp = 7/8.$$

This is quite a favorable prognosis, albeit one not quite as favorable as suggested by classical statistics.

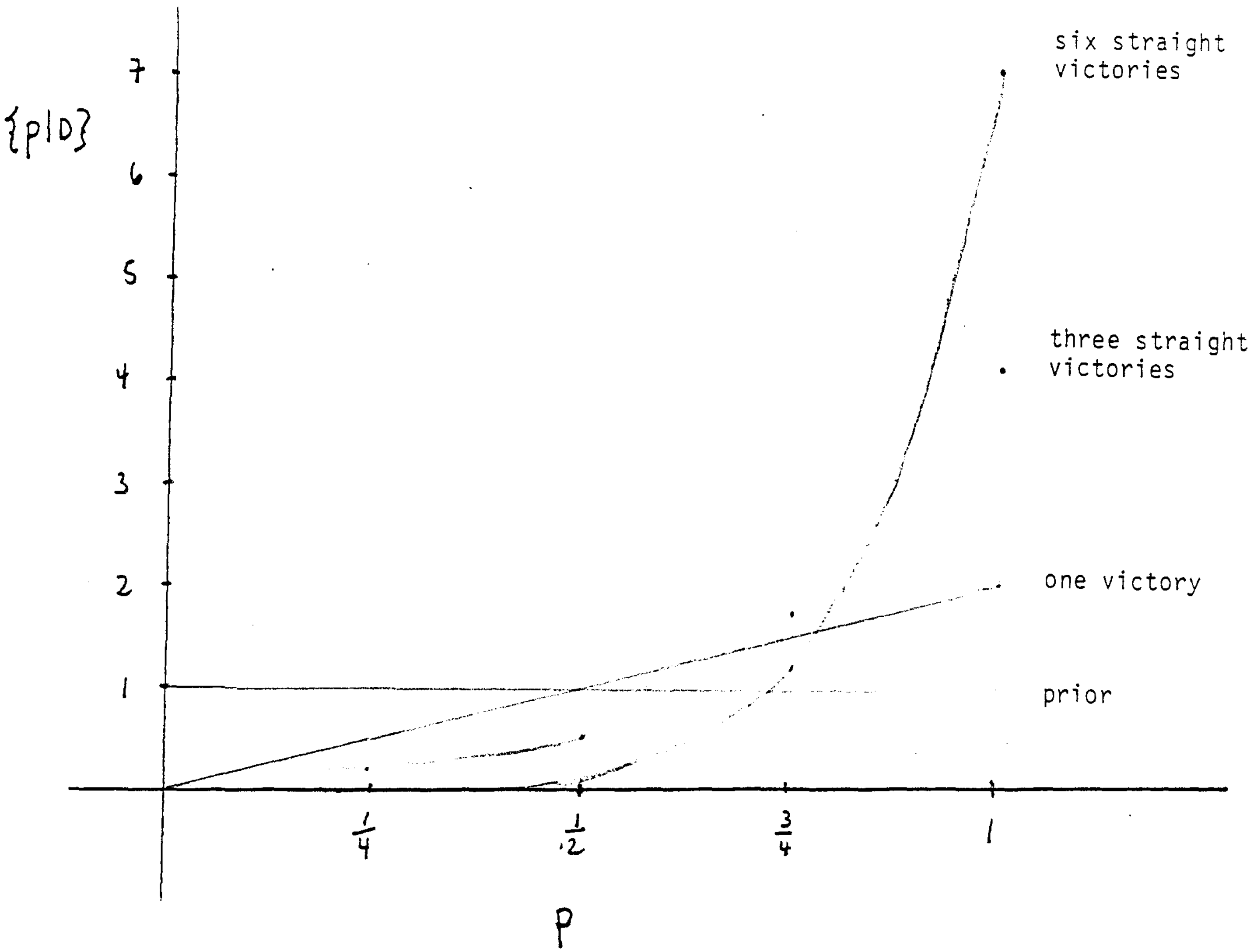


Figure 1.

The effect of posterior information on a distribution.

interpretation of the football problem, it might be useful to digress briefly here, since this "logical" approach not only has a place in subjectivist theory but also serves to illustrate the informational characteristics of probability theory.

To a frequency theorist, the probability of drawing an ultramarine ball from the urn is the relative frequency of such balls drawn as the number of trials approaches infinity. We would also expect that this probability would also be related to percentage of ultramarine balls the urn contains: if the long-run frequency does not turn out to be the same as this percentage, we would consider the drawing "biased." The idea of logical probability (as I'm using the term) is that we can use the information contained in the *a priori* structure of the problem. In this case, the percentage of ultramarine balls in the urn suggests a prior: for, even if the drawing is biased, the urn-structure gives me no idea by itself *which way* it is biased. Laplace's example was that of a coin; if I believe the coin is biased but I don't know which way, I should assign a probability of one-half to the event heads.[30]

This is still, in a sense, an "objective" approach to probability, since it restricts the knowledge admissable in a prior to what can be gained from the logic of the (objective) information structure of the problem. In particular, this approach uses *symmetries* in our uncertainty to gain probability information -- much as theoretical physics uses symmetries to deduce the properties of elementary particles.

Very often, this method is invoked in an effort to capture the notion that "I have no prior information" or "I am completely ignorant" of the prior probability. But the belief that this -- rather than the desire to exploit symmetries -- is the centerpiece of the deductive approach has led to some serious misconceptions. What this method teaches is that, as I've already stressed, to express a probability is in a sense to express a non-zero level of information about an uncertain situation. It is not in fact an easy task to represent ignorance in a prior; and the "ignorant," "diffuse," or "uninformative" priors we do cnstruct are usually "ignorant" only in a relative sense. But this is not a criticism of the deductive method.

The view that the deductive construction of priors is about ignorance has led to a number of misconceived attacks on this approach (or, more charitably, a number of well-aimed attacks on a naive version of the deductive method). These attacks start out by equating this approach with the proclivity indiscriminately to attach equal probabilities to events or to assign uniform distributions to random variables. Thus, the argument goes, if I am told there are two possible events and I am "completely ignorant" about them, I must assign equal probabilities of one-half to each. If I am now given three events, about which I am also "completely ignorant," I must assign them equal probabilities of one-third. Surely this is a contradiction: my standard of "complete ignorance" is inconsistent. In fact, of course, I am not entirely ignorant in either case; the fact that there are two (and only two) events is a datum -- and it's a different datum from the fact that there are three events. To put the matter in subjectivist terms, the two problem formulations provide me with different states of information; and it's no inconsistency to assign a different probability in one state than in the other.

The problem is even clearer when we consider criticisms based on the assignment of uniform priors to random variables. Richard von Mises offers a good example in what he calls Bertrand's paradox.

> Consider the following simple problem: We are given a glass containing a mixture of water and wine. All that is known about the proportions of the liquids is that the mixture contains at least as much water as wine, and at most, twice as much water as wine. The range of our assumptions concerning the ratio of water to wine is thus the interval 1 to 2. Assuming that nothing more is known about the mixture, the indifference or symmetry principle or any other similar form of the classical theory tells us to assume that equal parts of this interval have equal probabilities. The probability of the ratio lying between 1 and 1.5 is thus 50%, and the other 50% corresponds to the probability range 1.5 to 2.
>
> But there is an alternative method of treating the same problem. Instead of the ratio water/wine, we consider the inverse ratio, wine/water; this we know lies between 1/2 and 1. We are again told to assume that the two halves of the total interval, i.e., the intervals 1/2 to 3/4 and 3/4 to 1, have equal probabilities (50% each); yet the

> wine/water ratio 3/4 is equal to the water/wine ratio 4/3. Thus, according to our second calculation, 50% probability corresponds to the water/wine range 1 to 4/3 and the remaining 50% to the range 4/3 to 2. According to the first calculation, the corresponding intervals were 1 to 3/2 and 3/2 to 2. The two results are obviously incompatible.[31]

The two results are indeed incompatible. Which shows only that the uniform distribution is not the universal ignorant prior it is often thought to be. Harold Jeffreys traces this fascination with the uniform back to Bayes and Laplace: "and the weight of their authority seems to have led to the idea that the uniform distribution of the prior probability was a final statement for all problems whatever..."[32]

Bertrand's paradox rests on the fact that the uniform is not invariant with respect to the change of variable involved in the water-wine/wine-water inversion. But this fact implies, in effect, that there is information contained in the assumptions connecting the proportions of water and wine. And, to take advantage of this information, we need only apply the proper symmetry principle: i.e., the appropriate prior should be precisely one that is invariant to the change of variables. As Jeffreys has shown,[33] the least informative prior for a problem like this is $\{w\} \propto dw/w$, where w represents the water/wine ratio. (See figure 2.) There are a number of techniques for constructing suitably "ignorant" priors in various situations, including another idea pushed by Jeffreys, the "improper" prior whose integral is greater than unity.

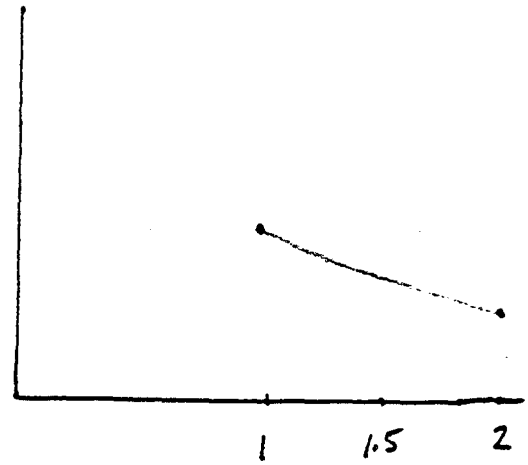
Figure 2.

The solution to Bertrand's paradox.

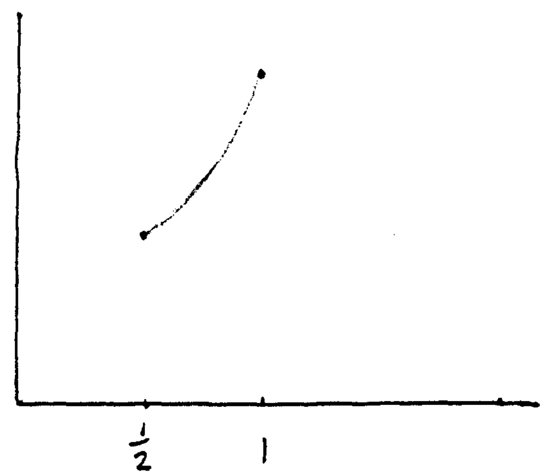
Let w = water/wine ratio

v = wine/water ratio

$$\{w\} = K\frac{dw}{w} = \frac{dw/w}{\int_1^2 dw/w} = \frac{dw/w}{\ln 2} = \frac{dw/w}{.693}$$



$$\{v\} = K\frac{dv}{v} = \frac{dv/v}{\int_{\frac{1}{2}}^{1} dv/v} = \frac{dv/v}{-\ln\frac{1}{2}} = \frac{dv/v}{.693}$$



Equal intervals now have the same probability

$$\int_1^{1.5} \{w\}\, dw = \int_1^{1.5} \frac{dw}{.693w} = \left.\frac{\ln w}{.693}\right]_1^{1.5} = \frac{\ln 1.5}{.693} = \frac{.405}{.693}$$

$$\int_{\frac{1}{2}}^{\frac{3}{4}} \{v\}\, dv = \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{dv}{.693v} = \left.\frac{\ln v}{.693}\right]_{\frac{1}{2}}^{\frac{3}{4}} = \frac{(\ln\frac{3}{4} - \ln\frac{1}{2})}{.693} = \frac{.405}{.693}$$

Let me address one final -- and perhaps insignificant -- point before leaving this topic. In his attack on the deductive approach (he derides its adherents as "subjectivists"), Richard von Mises asserts that, according to the logic of symmetry (the "Principle of Indifference"), "[i]f we know nothing about the stature of six men, we may presume that they are all of equal height."[34] (High repeats this idea,[35] although it's not clear in context exactly what he means by it.) In fact, of course, the Principle of Indifference suggests not that I assign the men equal heights but that I apply the same probability distribution to the random variable describing their heights. Since these would be relatively diffuse priors, the probability that the heights of the six men are within (say) a half-inch of one another[36] would be quite small: thus to say that I know "nothing" about the heights of the men is to say that I know consider it quite <u>unlikely</u> that they are the same height.

Subjectivists differ from "a priorists," of course, in that the former do not feel constrained always to use the least informative prior possible. In many cases, we <u>do</u> have some prior information that we should bring to bear.

In the football case, for example, the procedure of forming a probability estimate for tomorrow's game based on a formal updating of a diffuse prior using only information on previous defeats or victories borders on the ridiculous. And the reason we would intuitively view it as absurd is that we have more knowledge about football games than this process is permitting us to employ. In considering tomorrow's game, we would not, for one thing, view the game as a replication of the "experiments" conducted in previous meetings of the teams. A team changes after every game, sometimes in dramatic ways -- injuries, new personnel, new strategies. And, even if the two teams were to play an infinite number of games each season (an idea NFL schedule-makers are often suspected of supporting), we could still not define an "objective" probability from relative frequencies.

Although we would consider the past performances as relevant, we would also consider the upcoming situation afresh. We may perhaps use the approach taken by Jimmy "the Greek" on the pre-game show: explicitly comparing quarterbacks; the kicking games; various offensive and defensive match-ups; the home-field advantage; and the "intangibles." We may know that the Blue's star quarterback, the linch-pin of the team's slick, pass-oriented offense, suffered a cracked rib in the previous game and will be encased in a "flak jacket" tomorrow -- whereas the Yellow's all-pro running back will return tomorrow from injuries that had kept him out of the previous six contests. To the subjectivist, probability assessments are acts of human judgment; they are not the result of mechanical "probability calculations."

In the end, it seems to me, the objection to externalizing one's judgment in a probability is ultimately traceable to a confusion of uncertainty -- partial knowledge -- with ignorance.

High criticizes probability assessment "because it does not allow for _ignorance of the probability_. Although a person may be unsure about the outcome of a single event, he is completely sure about whether it is more or less likely than another event."[37] He gives an example.

> An investor who is considering the purchase of stock in two companies naturally wants to invest in that company whose stock has the greatest likelihood of rising, assuming he believes that both stocks will rise by the same amount. Now, the investor might decide that company A's stock is more likely to rise than company B's, or vice-versa. Or he may decide that they have an equal chance of rising. Or it may be that he _does not know_ which has a better chance of rising. This last state of knowledge is ruled out by assumption... Although subjective probability is supposed to be a theory of uncertainty, certainty lurks just below the surface.[38]

As a logical matter, this criticism rests on a spurious ontology that violates High's own injunction against confusing case and class probability.

In situations of "case probability," one is uncertain about _events_ not _probabilities_. In "class probability" it may be logically possible to be uncertain about a probability, since a probability is _defined_ as an event: the event that the long-run relative frequency of occurrence equals p. But in a situation of "case probability," as in a stock-price movement that is not an instance of a repeatable experiment, what precisely is this "probability" that we are ignorant of? I may indeed have no idea whether A or B or both will rise. But this doesn't mean that I don't "know" the probability; it means, as a logical matter, that I consider them equally likely to rise.

But surely, you say, it must matter whether one's assignment of a probability is based on the observation of relative frequency or is merely a subjective assessment. In the former case, he has facts, truth, objective knowledge; in the latter case he is surely "more uncertain." Well, let's see. Consider the following situation.

For some reason, you find yourself locked into the following circumstance. There are two lights, a red one and a green one, set to flash by a random process of some sort. You know that one and only one light will flash; if it is the red one, you must pay $100; if the green one, you are awarded $100. But you know nothing about the workings of the process that will cause one of the lights to flash: the lights, for all you know, are connected to banks of complex electronic equipment operated by a bearded, grizzled fellow in flowing robes and conical cap decorated with moons and stars. The flashing of the light took place yesterday in a sealed room under the supervision of a reputable firm of accountants, one of whose number, a sober fellow in dark suit, now stands before you with a sealed envelope containing the result.

Before you open the envelope and acquiesce to your fate, I offer you the following proposition. Upon payment of a fee, I will release you from the lottery you face (i.e., I will pay or receive the $100, as the case may be, in your stead) if you agree to play this game with me: we'll flip a coin which you have observed in several million tosses and which you are persuaded is impeccably fair. If the result is heads, I'll pay you $100; if it's tails, you pay me $100. The original lottery is one in which you don't "know" the probability. In the one I'm offering, by contrast, you know the probability (the long-run relative frequency) to a nicety. Is there some positive amount you would give me to trade lotteries with you? If so, I have some swamp-land in Florida I'd like to talk to you about.

VI.

It's important to be clear on what I'm _not_ saying. I'm not saying that, as a descriptive or "behavioral" matter, we might not in fact observe people paying to switch lotteries. The actual responses of economic agents to various kinds of "cognitive uncertainty" is an economic question, not a question for probability theory.

In fact, the behavioral issue is not necessarily directly relevant to the application of probability theory to the Pure Logic of Choice. It may well be that much of the animus among

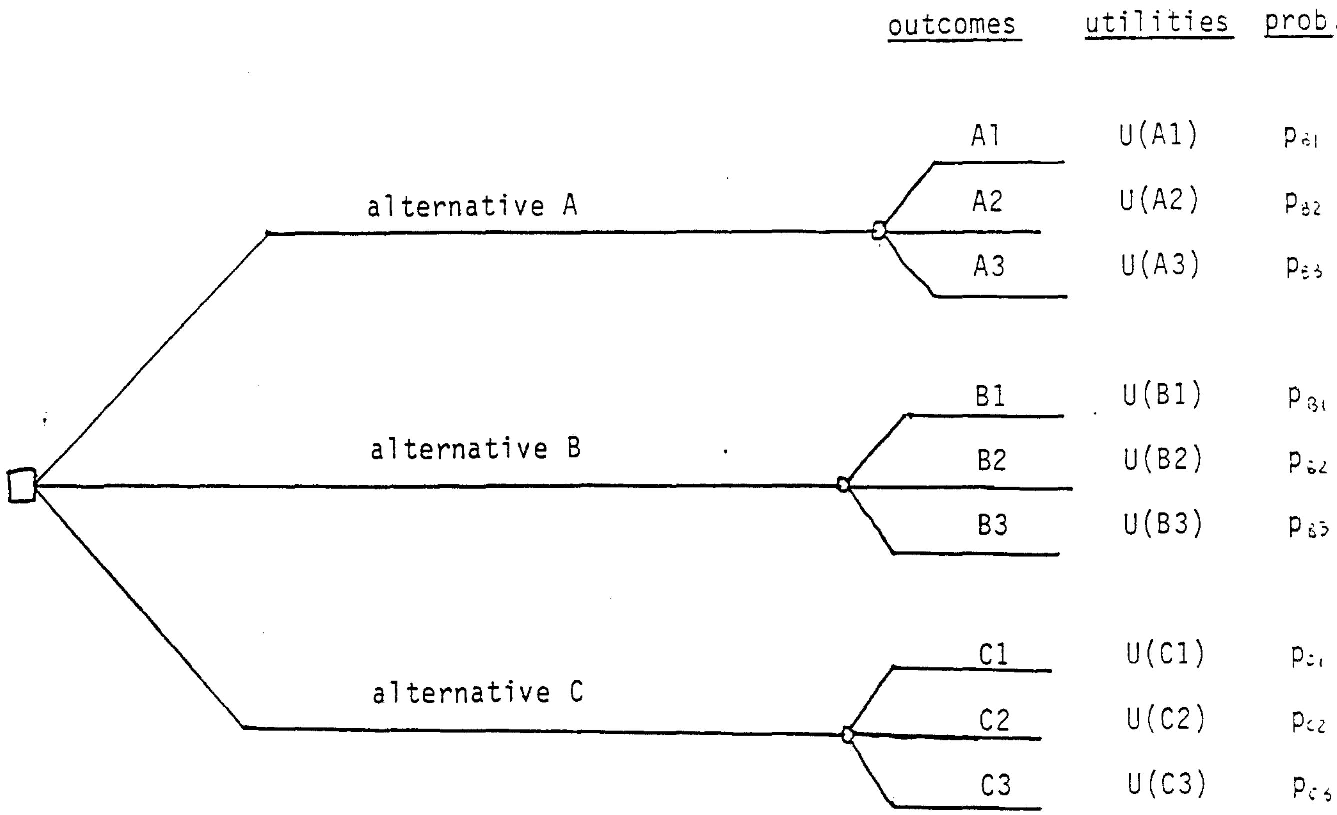
"Austrians" against subjective probability and decision theory -- despite the importance of both subjective judgment and decision-making in Austrian theory -- is related to the fact that, in their view, these tools have been illegitimately applied, by the Chicago School in particular,[39] to descriptive economic theory. The force of Austrian attacks on the "rational maximization" school of modeling may accidentally have spilled over onto subjective probability and decision theory.

A probabilistic Pure Logic of Choice, like its deterministic counterpart, can have two roles. In its normative role, it can be a tool that recommends what one ought, in some well-defined sense, to do. In its economic role, a Pure Logic of Choice has some useful conceptual functions, notably the elucidation of the concept of opportunity cost. In both cases, it is a tool -- not an all-encompassing imperative. And, in both cases, the basic ideas can best be seen using the conceptual apparatus of decision analysis.

We can think of several stages to the taking of a choice. First comes the generation of options, the isolation of alternatives, the formation of the basic means/ends framework for the decision. This part of the process is prior to choice; it is not a mechanical computation. "Alternative generation," to Howard, "is the most creative part of the decision analysis procedure."[40]

The next step is the attaching of values or utilities to the outcomes attendant on each decision alternative. If there were no uncertainty, the optimal choice would follow immediately from the valuation process, and, in fact, all that would be necessary is an ordinal ranking of the outcomes. But in a world of uncertainty -- as Austrians are always stressing -- the outcomes do not follow so easily from the alternatives. When, in such a situation, one judges one alternative superior to another, he is not saying he prefers the possible outcomes of that alternative to the outcomes possible from other alternatives; rather, he is saying that he judges the *lottery* on outcomes of the preferred alternative to be superior to the *lottery* on outcomes of other alternatives. One may thus pass up an alternative rich in valuable outcomes in favor of one full of relatively mediocre outcomes because he sees it as improbable that the valuable outcomes would actually materialize. The opportunity cost of the selection, therefore, is not the utility of outcomes foregone but some foregone convolution of utility and probability.[41] I see no way around this conclusion. And I am not persuaded that subjectivist cost theory that would be done irreparable violence by the separation -- for analytic purposes -- of the outcomes lottery into a component of subjective utility and a component of subjective probability. (This is illustrated schematically in figure 3.)

Do people make decisions this way? *Should* people make decisions this way? These are distinct questions. And the answer to both, I suspect, is yes-and-no.



Let $\bar{A} = p_{A1}U(A1) + p_{A2}U(A2) + p_{A3}U(A3)$

$\bar{B} = p_{B1}U(B1) + p_{B2}U(B2) + p_{B3}U(B3)$

$\bar{C} = p_{C1}U(C1) + p_{C2}U(C2) + p_{C3}U(C3)$

Robbinsian maximizer chooses max $\{\bar{A}, \bar{B}, \bar{C}\}$

If the ranking is $\bar{A}$, $\bar{B}$, $\bar{C}$, then the opportunity cost of the choice of A is $\bar{B}$.

Figure 3.

Let's consider the normative side first. Here I see two main issues: the "unlistability" issue and the "tacit knowledge" issue.

Unlistability, as proposed and described by Shackle, involves the incontrovertible notion that we cannot always -- or perhaps can never -- think of all the possible options we face or all the outcomes possible from those options. Thus, to Shackle, any set of possible states of the world we write down with intent to construct probabilities will be incomplete: we can't list all the states possible.

I believe this has behavioral (i.e., descriptive) implications, which I'll turn to before long. But -- while I can't give full consideration to Shackle's arguments here -- I believe they are fatal neither to normative decision theory nor to decision theory as a Pure Logic of Choice. (They may be fatal to the certain aspects of neoclassical literature, but that's a different matter.)

Normative decision analysis is driven, we might say, by the existential pressure of the decision-situation. Various possibilities or exigencies have presented themselves; I have to make a decision regarding them, for, even if I fail to act, I will by implication have made a decision anyway. If I am to make this decision consciously, I must cast the situation in terms of a framework of means and ends -- a "decision tree." This framework will of necessity be an approximation to reality; but such approximation is inherent in all decision-making. The best I can do for the moment is account for the unforeseen by including various "black box" outcomes (what Shackle calls a "residual hypothesis") to capture "all the other things" that may happen.

Shackle wishes to substitute something called "surprise" for probability, and his "surprise" variables seem to differ from subjective probability only in that they neglect to sum to one. The result of this substitution is to throw away most of the information content of probabilities -- which is what Shackle wants, since he insists upon equating uncertainty with ignorance. But to do so is, it seems to me, to deny probability its most useful function: to characterize partial knowledge within a framework of means and ends.

The critique from tacit knowledge is related and perhaps more fundamental. The issue it raises is: is conscious rationality always the best way to make decisions? Michael Polanyi[42] has stressed the importance of unconscious processes -- tacit processes -- in guiding action, suggesting, in fact, that creative behavior is guided by vague undetermined "potentialities" rather than by explicit premises and rules of choice.

What this suggests to me is that we don't have a theory of when to be consciously rational. But this doesn't mean that conscious rationality of the decision analysis sort is never desirable. As Howard suggests, there are times when these "intuitive" tacit processes fail us and we need conscious devices -- as when a smart pilot trusts his instruments not his otherwise well-honed intuitions in bad weather.[43] Indeed, in most circumstances involving probability, people's intuitions may not be too trustworthy. This is the message of work by psychologists like Tversky and Kahneman,[44] who have shown that people tend to have systematic biases in their intuitive assessments and updates of probability.

VII.

What is the relevance of this probabilistic Pure Logic of Choice to descriptive economics? One important role for the subjective probability-decision theory structure in economics is as an anlaytic or conceptual device. As such, I would argue, it can help to clarify some "Austrian" ideas; and, conversely, the rejection of this structure can serve to obscure -- to draw attention away from -- the really central Austrian insights.

It is indeed true, and quite central to economics, that man does not live in a world of given means and ends, of given alternatives, of given categories of action. Man is ignorant in an important sense: there are things he doesn't know -- and, more importantly, things he doesn't know he doesn't know.[45] One may be able to recognize <u>ex post</u> when an event falls into the category of the black-box "residual hypothesis," but one can't predict the nature or qualitative features of the event. We can call this by a number of names: "strong uncertainty" or "Shackelian uncertainty," perhaps. I have myself gone on at length in a recent paper about this sort of "qualitative uncertainty" (which I refer to contentiously as "extra-neoclassical") in reexamining the Knight-Coase literature on the nature of the firm.[46]

For present purposes, though, I think it best to talk in this connection not of "uncertainty" at all but of ignorance. In the end, the basic insights of Austrian theory have far more to do with the latter than the former. If I don't know what it is I don't know, then I'm ignorant -- not uncertain.

The issue may become clearer if we look to the theory of entrepreneurship. To Kirzner,[47] for example, there are two aspects to economic behavior that, although merged in action, can be separated analytically. One is the component of what Kirzner calls "Robbinsian maximizing," the logical selection of the best alternative from given data. The other component is what he calls entrepreneurship, the act of perceiving various data as relevant, of establishing the means/ends framework for the choice. The entrepreneurial act is not itself a choice -- it is something prior to choice, an act of perception rather than decision. Emerging from this distinction is the analytical category of the entrepreneur, whose role it is to perceive new opportunities, to convert into economic data the previously unknown, unforeseen, and unrecognized.

Kirzner stresses that the entrepreneurial role does not have to do with uncertainty, either in the naive "entrepreneurial profit is a factor payment for bearing risk" sense or in the Knightian sense of the entrepreneur as residual claimant in a world of "uncertainty" (read: "case probability").[48] To Kirzner, in effect, the domain of the entrepreneur is ignorance, not uncertainty in a sense relevant to probability. He talks of situations of "widespread market ignorance" that bring the entrepreneur's talents into play, stimulating the coordinating energies of the market process. And he insists that the ignorance with which the entrepreneur deals is not merely partial knowledge within the "Robbinsian" world of known means and ends; for, although a "framework need not express ends and means known with certainty ..., the framework is a <u>given</u> framework, already containing <u>all</u> the information, fragmentary though it may be, to be used in selecting the best course of action."[49]

This may appear to limit the extent to which the economic process can be represented as a probabilistic decision system. And, in fact, High's attack on subjective probability theory is comingled with a critique of mathematical search theory. All search theories, he argues, depend upon the assumption of a stationary stochastic process of some sort in the economy; and, he rightly concludes, this assumption is not even a proper caricature of what the entrepreneurial market process is all about. But, as High eventually concedes,[50] nothing in his critique depends at all on one's ability or inability to quantify subjective probability.

My reason for objecting to High's attack on subjective probability theory is not merely its apparent gratuitousness in view of the objectives of his exposition. Rather, I hope to suggest the extent to which an insistence that uncertainty is "unmeasurable" can lead -- as it seems in High's case to have led -- to what we may call the Knightian red-herring.

High ends his chapter on search theory with a <u>paean</u> to Knight and a ringing endorsement of Knight's view "that only this unmeasurable uncertainty, i.e., case probability, can account for economic profit."[51] It is certainly true that one-shot situations of "case probability" are the important ones for economists; and in a world containing only insurable risk, there would be no profit. But the <u>Austrian</u> (or at any rate Kirznerian) insight is not the distinction between situations of insurable risk within known and fixed categories and situations of subjective, "unmeasurable" probability within known and fixed categories; rather, it is a distinction between action within known and fixed categories and action involving the perception and introduction of new categories.

To see what this means, consider a world of known and given means and ends that nonetheless contains "case probability." A good example of this might be agricultural speculation, in which everyone buys and sells futures contracts for sow-bellies within a fixed set of rules. In such markets, there is no "class probability" subject to actuarial determination, no insurable risk; the market operates because of divergent probability estimates (judgments, if you wish) about the future price of sow-bellies. There _are_ profits in such a world: each speculator is a residual claimant who picks up the _ex post_ profit (or loss) when the contracts are due.

The first thing to notice about this is that what matters is the uninsurability of the risk involved, not the unquantifiability of probability in any technical sense: the speculators have what are in effect divergent probability estimates not because subjective probability is logically undefined but because they may have different states of information and perhaps different systematic biases in their assessments (not to mention different risk-tolerances). The more important point here, of course, is that the _ex ante_ interpretation of the speculative function in this world is uncertainty-bearing, not entrepreneurship.[52] In fact, the theory of profit appropriate to this world is surely some variant of "naive" profit theory, probably including a supply and demand for speculators.[53]

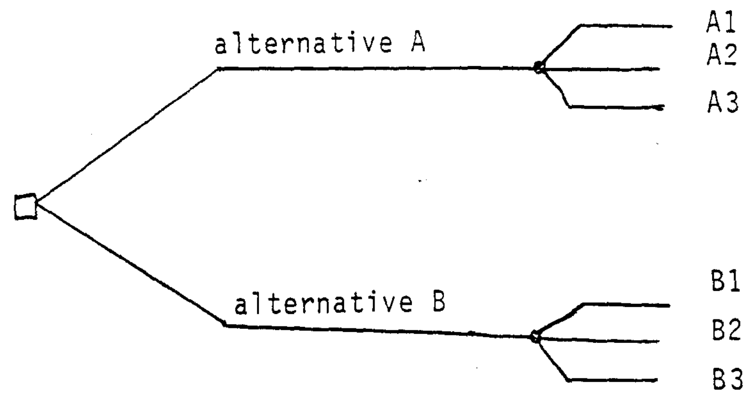
The important insight lies elsewhere: entrepreneurship as the _ex ante_ perception of new means and ends. All entrepreneurial action, we might say, involves "case probability"; but all "case probability" does not involve entrepreneurial action. And a bad case of probability anxiety can easily draw attention away from this important Austrian insight, as, in Kirzner's view, it may have done for Knight himself, whose vision of the _ex ante_ role of the entrepreneur as perceiver was obscured by "the very emphasis on uncertainty in the Knightian system..."[54]

Let me close by illustrating how the subjective probability-decision theory formalism can, I think, be integrated with the theory of entrepreneurship. The following schematic example should illustrate (see figure 4).

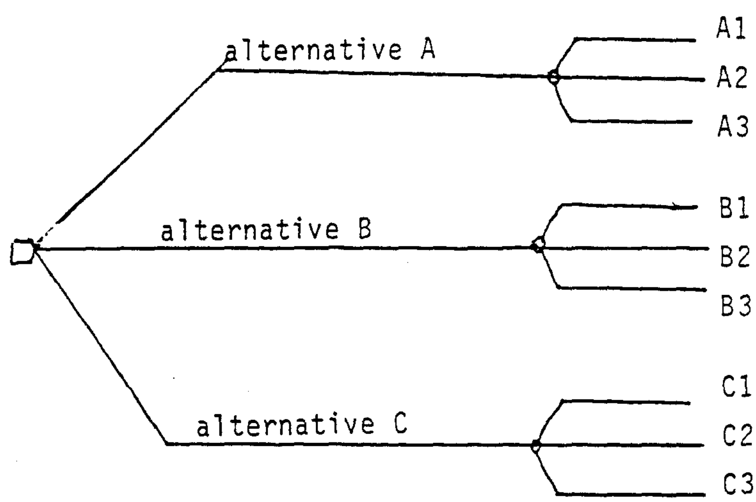
An investor believes himself faced with a choice of two business opportunities, A and B. If he chooses A, he believes, three possible outcomes may ensue, yielding profit levels A1, A2, and A3 with probabilities p_{A1}, p_{A2}, and p_{A3}. Similarly, opportunity B, he feels, might lead to outcomes B1, B2, or B3 with probability p_{B1}, p_{B2}, and p_{B3} respectively. The optimal choice in this situation is to pick the option with the highest expected value (or expected utility if you prefer). The opportunity cost of the decision would be the expected utility of the option not chosen.[55]

Entrepreneurship can occur in one of two ways. In one case -- call it "type I" -- the investor perceives a third opportunity he hadn't seen before: option C, with outcomes C1, C2, and C3 and probabilities p_{C1}, p_{C2}, and p_{C3}. Adapting Kirzner's definition, the investor is an entrepreneur -- i.e., he perceived an entrepreneurial opportunity -- if the perceived new alternative is in fact superior to that of the old alternatives. There is, as Kirzner insists, no cost to entrepreneurial action; that is

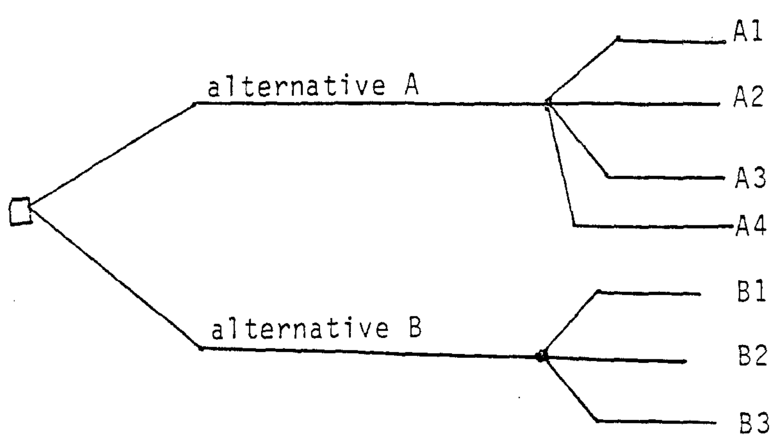
Figure 4.



"Type I" entrepreneurship



"Type II" entrepreneurship



because the entrepreneurial act is not a decision (decisions are always costly since they imply opportunity costs) but an act of perception prior to decision. But we can talk about the entrepreneurial profit of the combined perception-plus-decision, perhaps, as the difference in expected utility between the optimal selection in the "old" means/ends framework and that of the best choice in the "new" framework.

The other way entrepreneurship can occur -- "type II" -- is for the investor to perceive not a new option but a new outcome from an existing option. Perhaps he now sees the possibility of outcome A4 -- that the fortunes of the firm in question may well change in a way had not previously conceived of. Our investor quickly reassesses the probabilities, yielding p_{A1}', p_{A2}', p_{A3}', and p_{A4}'. If the new expected utility is higher than that of option B (which had been his choice before, let us say), then the investor has committed an entrepreneurial act. His expected entrepreneurial profit for having seen this new possibility is again the difference between the highest expected utility before the entrepreneurial perception and the now-higher expected utility after the perception (assuming, of course, that the new perception made him alter his selection).

We could make this more detailed. But the important point is that entrepreneurship involves setting up the decision tree itself, not assessing probabilities once the tree is established. Assessing probabilities is certainly a matter of judgment; and, as

Knight and Mises have stressed, information structures lacking "potential cognitive certainty" allow for divergent probability assessments. But too close an attention to the "unmeasurability" -- or even the subjectivity -- of probability can distract attention from the subjectivity of the means/ends framework itself. It is at this level that the critique from "unlistability" is applicable. In the end, what lies behind the view that subjective probability cannot meaningfully be quantified may very well be this lack of a distinction between the subjectivity of probability and the perceptual quality of the framework within which the probability is applied. And the belief that a calculus of subjective probability should be abandoned -- or replaced with non-distributional "surprise" variables -- might be thought of in the end as an attempt to inject the notion of entrepreneurship into economic theory through the back door.

Notes.

* This paper was presented at the Austrian Economics Seminar of NYU in April, 1981. I am indebted to its participants for helpful comments. I would like particularly to thank Scott Olmsted, Israel Kirzner, Gerald O'Driscoll, Mario Rizzo, and Roger Garrison for helpful ideas and discussions. There remains, however, a nine-in-ten chance that all errors are my own.

1. On which see generally Israel Kirzner, "The 'Austrian' Perspective on the Crisis," in D. Bell and I. Kristol, eds., The Crisis in Economic Theory, New York: Basic Books, 1981, pp. 111-122.

2. Human Action, New Haven: Yale University Press, 1949, p. 105.

3. On the "subjectivism" of these economists and others, see James Buchanan, Cost and Choice, Chicago: The University of Chicago Press, 1969. For a methodological discussion, see also F.A. Hayek, The Counter-Revolution of Science, New York: The Free Press, 1952.

4. "Economics and Knowledge," in Individualism and Economic Order, Chicago: The University of Chicago Press, 1948 (Gateway edition, 1972), p. 35.

5. See Frank H. Knight, Risk, Uncertainty, and Profit, Chicago: The University of Chicago Press, 1969; and

G.L.S. Shackle, _Decision Order and Time in Human Affairs_, Cambridge: Cambridge University Press, 1961.

6. _Human Action_, p. 107.

7. _Op. Cit._, p. 109.

8. Jack C. High, _Maximizing, Action, and Market Adjustment: An Inquiry into the Theory of Economic Disequilibrium_, Ph.D. Dissertation, Department of Economics, University of California (Los Angeles), 1980, p.74.

9. As Buchanan and Di Pierro have recently pointed out, Knight intended his version of the distinction to relate not to the analysis of individual decision-making but to the actuarial properties (or lack thereof) inherent in various _structures_ -- various stochastic processes, one might say -- within the economic system. See James Buchanan and Alberto Di Pierro, "Cognition, Choice, and Entrepreneurship," _Southern Economic Journal_, vol. 46, no. 3, January 1980, p. 693.

10. _Op. Cit._, p. 695.

11. See, e.g., Ludwig Lachmann, "An Austrian Stocktaking," in L.M. Spadaro, ed., _New Directions in Austrian Economics_, Kansas City: Sheed, Andrews, and McMeel, 1978, p. 3, and Buchanan, _op. cit_, p. 6.

12. "Decision Analysis: Applied Decision Theory," in D.B. Hertz and J. Melese, eds., _Proceedings of the Fourth_

<u>International Conference on Operational Research</u>, New York: Wiley-Interscience, 1966, p. 55, reprinted in <u>Readings in Decision Analysis</u>, Menlo Park, Ca.: SRI International, second edition, 1976, p. 85.

13. <u>Human Action</u>, p. 105.

14. <u>Op. Cit.</u>, p. 107.

15. <u>Op. Cit.</u>, p. 108.

16. Shackle, p. 92.

17. <u>Op. Cit.</u>, p. 60.

18. <u>Human Action</u>, p. 114.

19. For an interesting exposition of this argument, see Scott M. Olmsted, "What Is Probability?" Stanford University, photocopy, January, 1982.

20. Even in the balls-in-the-urn case, though, it is conceptually possible to have relevant prior knowledge not contained in the objective stochastic structure -- as in the case of a player who has reason to suspect that the game is rigged.

21. High, p. 101.

22. Mises's analysis of the doctor's new assessment is as follows. "The logical approach remains the same, although it may be based not on a collection of

statistical data, but simply on a more or less exact resume of the doctor's own experience with previous cases. What the doctor knows is always only the behavior of classes." (p. 111.) This seems to me to approximate an analogue to what Karl Popper has identified in science as the "Baconian myth" that induction is the source of a scientific theory. Just as a theory is not a literal digest of experience, a probability assessment is not merely a recapitulation of experience. Such an assessment should, I think, be viewed in Polanyiesque terms as an act of personal knowledge. (Cf. note 42 *infra*.)

23. *Human Action*, p. 111.

24. *Loc. cit.*

25. *Loc. cit.*

26. The gambler's fallacy is the popular misconception that an event is more likely in a given trial if it has failed to appear in a string of previous independent trials (as when the sports broadcaster suggets that a known home-run hitter is "due" because he failed to produce in several previous at-bats). Mises's reference to it here is probably unobjectionable, although it has no necessary connection with this example. But he mentions it again on p. 115 in a way that, read in conjunction with this passage on p. 11, suggests to me that Mises somehow views

the use of any prior information as an instance of the gambler's fallacy. He asserts that if, in assessing Roosevelt's chance of winning the election at 9:1 we believe that Roosevelt is in the position of a man who owns 90 per cent of all the tickets in a lottery, we are committing the gambler's fallacy. This just ain't so; and this is precisely what one should mean when he says Roosevelt's chances are 9:1. This suggests that a possible misconception on Mises's part about the use of prior information (whether from a frequentist or a subjectivist point of view) may be a contributing factor in his rejection of subjective probability.

27. Cf. High, p. 92.

28. *Op. cit.*, p. 91.

29. *Ibid.*

30. Cf. Pierre Simon, Marquis de Laplace, *A Philosophical Essay on Probability*, New York: John Wiley, first English edition, 1902, chapter VII, pp. 56-57.

31. Richard von Mises, *Probability, Statistics, and Truth*, London: George Allen and Unwin, 1957, p 77.

32. Harold Jeffreys, *Theory of Probability*, Oxford: Clarendon Press, Second Edition, 1948, p. 103.

33. *Ibid.*

34. Probability, Statistics, and Truth, p. 75.

35. High, p. 105.

36. In order to avoid measure-theoretic problems, we have talk of continuous random variables as being within a range of one another rather than being equal.

37. High, p. 103, emphasis added.

38. Ibid, emphasis original.

39. Milton Friedman and L. J. Savage, "The Utility Analysis of Choices Involving Risk," Journal of Political Economy, vol. LLV, no. 4, August 1948, p. 279.

40. Howard (1966), p. 87.

41. Professor Kirzner has suggested to me that it was precisely a comingling of utility and probability that was envisaged in Mises's view of utility, i.e., his "utility" already possessed the flavor of expected utility.

42. Personal Knowledge, Chicago: The University of Chicago Press, 1958.

43. "An Assessment of Decision Analysis," Operations Research, vol. 28, no. 1, January/February, 1980, p. 4.

44. For the most recent example of their work, see Amos Tversky and Daniel Kahneman, "The Framing of Decisions

and the Psychology of Choice," Science, vol. 211, 30 January 1981, p. 453.

45. Cf. Israel Kirzner, Perception, Opportunity, and Profit,, Chicago: The University of Chicago Press, 1979, p. 139.

46. "Why Are There Firms?" C.V. Starr Center for Applied Economics Discussion Paper 81-30, New York University, March, 1981.

47. Israel Kirzner, Competition and Entrepreneurship, Chicago: The University of Chicago Press, 1973.

48. Op. cit., p. 75 ff.

49. Op. cit., p. 94.

50. High, p. 112.

51. Op. cit., p. 115.

52. Let me be careful here and suggest that, to Professor Kirzner (as he himself remarked to me), speculation and other forms of arbitrage do technically involve entrepreneurship by his definition. I find this a tiny bit troubling, and I may as a consequence tend to lean toward Schumpeter's view on this point. (Cf. Schumpeter, The Theory of Economic Development, New York: Oxford University Press, 1961.) Schumpeter would rule out such routine activities as non-entrepreneurial because

they do not diverge from conventional channels of means and ends. My own view is that there is room for a conception of entrepreneurship half-way, as it were, between Kirzner and Schumpeter; and I am in the process [February, 1982] of working out some of my own ideas on this point.

53. Cf. Martin Bronfenbrenner, "A Reformulation of Naive Profit Theory," Southern Economic Journal, April, 1960, reprinted in Breitt and Hochman, eds., Readings in Microeconomics, Hinsdale, Ill.: Dryden Press, second edition, 1971, p. 411. As Bronfenbrenner himself suggests, the world of naive profit theory is one of "uncertainty without innovation."

54. Competition and Entrepreneurship, p. 82.

55. Of course, the utilities involved are cardinal in the well-known von Neumann-Morgenstern sense. And this is sure to trouble Austrian purists. But it is important to remember that these are *ad hoc* utilities derived from and defined by the particular decision-situation the agent faces; they are not the cardinal utilities of the early neoclassical economists -- the true focus of modern Austrian and ordinalist attacks -- which envisaged a complete cardinal ranking of all possible commodities. Similarly, in an analogy suggested by Professor Kirzner, the subjective probabilities here are perhaps not the sort of quantification of uncertainty to which Mises was

objecting; rather, they bear to that sort of quantification the same relationship that von Neumann-Morgenstern utility bears to the older form of cardinal utility.