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Cohort Size and Schooling Choice*

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Abstract

We develop a perfect-foresight overlapping generations model to investigate the effects of cohort size on schooling decisions and cohort-specific welfare measures. A set of sufficient conditions are presented which ensure the existence of an unique sequence of human capital rental rates and schooling choices for any sequence of cohort sizes. We perform a small simulation exercise in which we examine schooling and welfare elasticities defined with respect to own cohort size and the sizes of neighboring cohorts; the welfare elasticities are decomposed into a part attributable to the direct effect of cohort size change [*i.e.*, ignoring changes in schooling choices] and an indirect, or equilibrium, effect attributable to changes in schooling choices. For the range of structural parameters utilized, the absolute size of the direct cohort size effect is typically an order of magnitude greater than that of the equilibrium effect.

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1. Introduction

The question of the nature and magnitude of the effect of cohort size on schooling choices (*e.g.* Ahlburg *et al* (1981); Connelly (1986); Nothaft (1985); Siow (1984); Stapleton and Young (1988); Wachter and Wascher (1984); Falaris and Peters (1985)), on-the-job investment in human capital (*e.g.* Dooley and Gottschalk (1984)), earnings (*e.g.* Freeman (1976,1979); Welch (1979); Berger (1983,1984); Stapleton and Young (1984); Tan and Ward (1985)), and welfare more generally construed (*e.g.* Easterlin (1980)) has prompted a great deal of research over the past fifteen years, no doubt in large part due to the concommitent entry of the "baby boom" generation into the labor market. While some rigorous theoretical investigations of the question have been presented, most of the analysis of this issue has been empirically-oriented.

The empirical results on cohort size effects on these various outcome measures are mixed, which is partially attributable to the difficulty of operationally defining a "cohort" and specifying the quantitative nature of substitution relationships between cohorts in aggregate production.[1] However, several empirical studies have found that there does appear to be a significant statistical relationship between cohort size, defined in terms of fairly narrow age bands, and measures of the educational attainment of the cohort (see, in particular, Ahlburg *et al* (1981); Falaris and Peters (1985); Wachter and Kim (1982)). Unfortunately, the estimated sign of the relationship is not consistent across studies, though it is probably fair to say that most researchers have found it to be negative.

The theoretical models which have been brought to bear on this issue also lead to conflicting implications regarding the cohort size-schooling relationship. The models which have been developed primarily differ in terms of assumptions regarding: (1) the information sets of agents; (2) the degree of rationality of the agents' decision rules; and (3) the degree of substitutability across cohorts in aggregate production. Virtually all studies assume expected discounted income maximization as the goal of agents making human capital investment choices. The more elaborate theoretical analyses of the problem typically have been carried out using the device of perturbing the population growth rate of an economy on a steady-state growth

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[1] On this point last point see especially the discussions in Connelly (1986) and Stapleton and Young (1988).

path (*e.g.* Stapleton and Young (1988), van Imhoff (1989)).

It appears that a concensus regarding the answer to the intellectually-intriguing and policy-relevant problem of the relationship between cohort size and schooling has been slow to form for [at least] two reasons. First, and probably most importantly, there exist the difficulties inherent in implementing intergenerational models using modern individual-level data sets such as the Current Population Survey or the Panel Study of Income Dynamics in the United States. These data sets contain rich detail on individuals over a very short historical epoch. Theoretical models of the cohort size-schooling relationship necessarily abstract from all individual differences except for a few demographic ones, and in principle would require a larger variance in cohort sizes for estimator precision than is to be observed over the recent past.[2] Since no empirical work is even attempted in this paper, we will offer nothing new on this point.

A second problem seems to be the different approaches taken in modelling schooling choice. As has been recognized by all researchers who have seriously thought about the question, a change in the size of one cohort generally changes the schooling decisions and welfare levels of the members of all cohorts which ever exist, both those born before and those born after the reference cohort. Within such a complex system, it is natural to expect difficulties in conducting simple comparative statics or dynamics exercises which will lead to unambiguous predictions. What has not been so commonly appreciated is the difficulty in constructing a model for which there can be shown to exist a partial equilibrium in which an exogenously given cohort size sequence uniquely determines a rental rate sequence for human capital and [simultaneously] a sequence of schooling choices. One of the contributions of this paper is the construction of such a model.

We develop a framework which is used to investigate fully rational schooling decisions in an overlapping generations economy which exists for some finite but arbitrarily large number of periods. In particular, we will examine properties of the mapping from the cohort size vector $n = (n_1, \ldots, n_m)'$ to the schooling choice vector $s = (s_1, \ldots, s_m)'$, where n_j and s_j denote the size of the j^{th} cohort and the proportion of her first

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[2]This problem is well-noted by Siow (1984), whose study is probably the most ambituous of any to date in terms of attempting to obtain structural parameter estimates of a model of schooling choice and cohort size consistent (for the most part) with optimizing behavior of agents under uncertainty.

"period" of life which the representative member of the j^{th} cohort spends in school. The mapping from n to s is defined by a set of market-clearing conditions for human capital. We derive a set of sufficient conditions on the market demand for human capital, the human capital production functions, and worker's preferences and their information sets, under which there is shown to exist a unique function which maps cohort sizes into rental rates for human capital and schooling choices.

In terms of previous theoretical work on the subject, our main innovations are to investigate the effects of cohort size on schooling outside of a steady-state framework, and, as mentioned above, to empower agents with the ability to utilize fully rational decision rules in equilibrium. We should also quickly point out several restrictive features of our analysis which do not appear in all previous treatments of the problem. We assume that agents from different cohorts alive at the same point in time are perfect substitutes for one another in aggregate production; these agents will only differ in their productive efficiency, which is a concave function of the amount of time they spent in school in their first period of life if they are more than one "period" old, and which is equal to one otherwise. Futhermore, we restrict schooling investment to the first period of life; a more general and substantially more complex analysis would be required if such an assumption were to be relaxed. We have also assumed perfect foresight in the model below; this assumption is shared by many theoretical treatments of the problem. Analyses in which uncertainty concerning future cohort sizes is posited typically utilize strong stationary assumptions on the cohort size process and/or non-rational expectation mechanisms to close the model. Thus, from a modelling perspective, it is difficult to say which type of assumption regarding the information sets of agents is least objectionable.

The other contribution of the paper is to suggest some measures of measuring cohort size effects on schooling. When working within a partial equilibrium framework with even the limited degree of generality of the one considered here, the succinct description of cohort size effects on welfare and schooling outcomes is problematic; we suggest summary measures with some intuitive appeal, at least when the cohort size sequence is generated by a covariance stationary process. We also conduct a small simulation exercise to assess the relative magnitudes and signs of the direct [*i.e.* holding schooling choices constant] and indirect [*i.e.* attributable to schooling choice change] effects of changes in the cohort size sequence on intergenerational welfare patterns. For the small simulation study performed, we are not able to substantiate the claim made by Stapleton and Young (1988) on the basis of their empirical analysis of the problem that optimal educational choice [our "indirect" effect] would almost entirely eliminate the deliterious "direct" effect of being in a large cohort. The indirect effects we compute [with regard to changes in "own" cohort size] typically only eliminate about one-seventh of the negative direct effect on lifetime welfare associated with being a member of a large cohort.

The model developed for the analysis of the cohort size-schooling relationship is presented in Section 2, which also contains the proof of the existence and uniqueness of the mapping from any cohort size sequence to both rental rate and schooling choice sequences. In Section 3 we develop some measures of the effect of cohort size on schooling choice. The measures derived reflect the fact that, in general, the size of any given cohort affects the welfare levels and schooling choices made by <u>all</u> other cohorts which exist over the period for which the economy is defined. As mentioned above, the measures also distinguish between the direct and indirect effects of cohort size changes on welfare. Section 4 contains a small simulation exercise in which we provide example values for the measures of cohort size effects developed in Section 3. We offer a brief conclusion in Section 5.

2. Model Structure and the Characterization of Equilibrium

In the economy, there exist a sequence of m cohorts, $1 \le m < \infty$, each of which contains members who live for $\ell+1$ periods, where $1 \le \ell < \infty$; thus the economy is in existence for a total of $m + \ell$ periods, where $2 \le m+\ell < \infty$. Cohort members are assumed to be wealth maximizers. Within each period of her life, each population member is endowed with one unit of time. In the first period of her life, a cohort member spends some portion of her time endowment attending school, which produces human capital to be supplied inelastically to the labor market in periods 2 through $\ell+1$ of her life.[3] The rest of her time

[3] As alluded to in the Introduction, it would be a major undertaking to generalize the model so as to allow the possibility of investment in every period, at least in so far as demonstrating uniqueness of equilibrium is concerned. With sufficiently high fixed costs of reentering school, sufficiently "wide" definitions of periods, and given the range of cohort sizes observed historically, neglecting the possiblity of schooling reentry may not be too serious.

endowment in the first period of her life is spent in the labor market.

All individuals are initially endowed with one unit of human capital. The amount of human capital they possess in the second through $\ell+1^{st}$ periods of their life is strictly a function of the time spent in school in their first period of life. We assume a kind of implicit depreciation of the human capital endowment, so that if the individual spends no time in school in the first period of her life, she will possess no human capital in the remaining periods of her life. The maximum amount of human capital which can be produced by the agent for use after her first period of life is $\bar{h}$ [$< \infty$]. Following the first period of life, there is no further depreciation of the human capital stock until death.[4] The assumption that human capital in periods 2 through $\ell+1$ belongs to the interval $[0,\bar{h}]$ is important in establishing uniqueness of the equilibrium rental rate sequence; it is not essential for proving existence. Since our main interest is in the empirical ramifications of full-rationality in the choice of schooling level and the returns to schooling, uniqueness would seem to be highly desirable.

In each period, there exists an inverse demand function for human capital. This function is essentially constant over time, except for an allowance for exogenous time dependent multiplicative shifts. These shifts are explicitly assumed not to be a function of the cohort size process. One rationalization for such an assumption could be that labor services are a non-mobile factor to indigenous firms producing for a world market. If each country has an independent cohort size process, then there may be no shifts in world-wide demand for the product; in this case the demand shifters may be thought of as reflecting productivity variability over time which is not related to the population size process. The constant portion of the inverse demand function will be assumed to be strictly decreasing over the positive real line. We now lay out the assumptions of the model in more specificity.

Assumption 1 The cohort size sequence $\{n_1,\ldots,n_m\}$ is known by all members of all cohorts. The sequence will be denoted by the vector $n \equiv (n_1,\ldots,n_m)'$, where $n \in \mathbb{R}^m_+$.

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[4]There is no problem in altering this assumption to allow for standard forms of multiplicative depreciation of the "age" two human capital stock throughout the remaining periods of life in terms of establishing the result reported in Theorem 1 except for the cost of additional notation.

The perfect foresight assumption is crucial in establishing the existence and uniqueness of competitive equilibrium in the labor market in the method of proof used below. Given perfect certainty and the finiteness of life, we are able to prove uniqueness for any cohort size sequence, no matter what the nature of the process generating it. The very generality of this result may make it difficult to argue that the decision rules generated under a perfect foresight assumption are good approximations to those used by agents living in a world of uncertain future cohort sizes. The quality of such an approximation will often hinge on whether or not the future is sufficiently predictable with respect to the past. Obviously there will be a continuum of processes which could generate a legitimate cohort size sequence [from the point of view of establishing uniqueness] for which this will not be the case [*e.g.* those for which the expected value of certain functions of future values of the process with respect to its history are not even defined]. Of course, whether perfect foresight is a "good" assumption or not is ultimately an empirical question.

In terms of the assumption that the economy is of finite duration, it is made to simplify the handling of the endpoint conditions. The finiteness of the economy may not actually necessitate its extinction in a literal sense; rather it may be taken to apply to a given production technology for which agents in cohorts 1 through m are trained. Say that a new technology for which the particular type of schooling investment modelled here is unproductive comes into existence between periods m and $m+\ell$ of the economy. Each cohort must select a production technology for which to acquire training. Then cohort m is the last cohort which acquires training for the old production technology - all cohorts after m acquire training for the new production technology. It must be also be the case that all cohorts stay with the production technology for which they were originally trained throughout their lifetimes (*i.e.* retraining costs are prohibitive) to support this interpretation of the assumption..

Assumption 2 Each agent in each cohort takes the human capital rental rate vector $r = (r_1,\ldots,r_{m+\ell})' \in \mathbb{R}_+^{m+\ell}$ as given when making her schooling investment decision.

Given the large size of a cohort, each agent's investment decision has an infinitesimal effect on the price of human capital over the agent's lifetime. Furthermore, workers are not able to collude effectively so as to withhold human capital from the market, thus increasing aggregate earnings. The perfect foresight assumption regarding cohort sizes (A1) along with (A3-A5) below allow each member of each cohort to solve for the competitive equilibrium rental rates that they and all other population members will face.

Assumption 3 Each cohort member is endowed with one unit of human capital in her first period of life. There exists a human capital production function h which maps time spent in school during her first period of life into her human capital stock when she is age $2,\ldots,\ell+1$. This function has the following properties:

[1] $h: [0,1] \rightarrow [0,\bar{h}],\ \bar{h} < \infty$;

h is continuously differentiable through the second order, with

$h'(s) > 0,\ h''(s) < 0,\ s \in [0,1]$,

and $h'(0) = \infty$ and $h'(1) = 0$.

Human capital stock in ages 2 through $\ell+1$ is a constant concave function of the proportion of the first period of life spent in schooling. The limiting conditions on the first partial derivates of the human capital production function will ensure the existence of an interior solution to the schooling investment function for any cohort facing a future rental rate process which is bounded. While a model along the lines of the one developed here could be built in which schooling investment can take place over a number of periods [*i.e.*, corner solutions are allowed within periods], the assumptions necessary to establish a result like that contained in the Theorem presented below would be substantially different from those utilized here. Finally, as mentioned above, allowing for a deterministic form of depreciation of human capital over periods 3 through $\ell+1$ of a worker's life is straightforward.

Assumption 4 Each member of cohort j has the following value of her schooling choice problem

[2] $$V(r_j,\ldots,r_{j+\ell}) = \max_s \{r_j(1-s) + Q_{j+1}h(s)\},$$

$$\text{where } Q_{j+1} \equiv \beta r_{j+1} + \beta^2 r_{j+2} + \ldots + \beta^{\ell} r_{j+\ell},$$

$$\beta \in (0,1] .$$

Thus the objective of the agent is simply the maximization of the present value of income over her lifetime. All cohort size effects are reflected solely in the rental rate sequence she faces, which in general is determined by the cohort sizes and schooling decisions made by the members of all of the cohorts which will be alive over the duration of the economy.[5] We could allow the agent's objective to be utility maximization (defined over her sequence of earnings) rather than income maximization, though under our perfect foresight assumptions there is little insight to be gained from such an endeavor.

Assumption 5 There exists an inverse demand function for human capital in period t which is of the form

[3] $$r_t = \lambda(t)R(H_t), \quad t = 1,\ldots,m+\ell;$$

where $R\colon \mathbb{R}_+ \to \mathbb{R}_+$,

R is first-order continuously differentiable with $R'(x) < 0$, $x \in \mathbb{R}_+$,

H_t denotes the supply of human capital to the market in period t,

and $\lambda(t) > 0$ is an exogenous constant.

With this basic set of assumptions, we are ready to analyze the model. First, we must characterize the decision rule utilized by all cohort members, which is a straightforward task. Under A1-A5, the optimal schooling choice of a member of cohort j is the solution to

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[5] Or at least all cohorts which will participate in the phase of an infinitely-lived economy characterized by the current production technology, under the interpretation of the finite duration assumption given following A1.

[4] $$0 = -r_j + \beta Q_{j+1} h'(s_j^*)$$

$$\Rightarrow s_j^* = s^*(r_j, Q_{j+1}) = g(r_j/\beta Q_{j+1}),$$

where $g \equiv (h')^{-1}$.

By A3, g has the following properties:

[5] $$g\colon \mathbb{R}_+ \to [0,1)$$

g is first-order continuously differentiable with $g'(x) < 0$, $x \in \mathbb{R}_+$.

Now consider the determination of the equilibrium rental rate sequence. First, some formal definitions are required. Because human capital is inelastically supplied by all cohorts alive in period t other than the one born in period t (which supplies $(1-g_t)n_t$ of human capital to the market), the total amount of human capital supplied in period t is $H_t = (1-g_t)n_t + h_{t-1}n_{t-1} + \ldots + h_{t-\ell}n_{t-\ell}$. This quantity can be written in terms of the rental rate sequence as follows.

Definition 1 The total amount of human capital supplied to the market at time t is

[6] $$H_t = \begin{cases} (1-s^*(r_1,Q_2))n_1 \equiv \zeta_1((r_1,\ldots,r_{\ell+1})'), & t = 1\ ; \\ \vdots & \\ (1-s^*(r_\ell,Q_{\ell+1}))n_\ell + h(s^*(r_{\ell-1},Q_\ell))n_{\ell-1} + \cdots & \\ \qquad + h(s^*(r_1,Q_2))n_1 \equiv \zeta_\ell((r_1,\ldots,r_{2\ell})'), & t = \ell\ ; \\ (1-s^*(r_t,Q_{t+1}))n_t + h(s^*(r_{t-1},Q_t))n_{t-1} + \cdots & \\ \qquad + h(s^*(r_{t-\ell},Q_{t-\ell+1}))n_{t-\ell} \equiv \xi((r_{t-\ell},\ldots r_{t+\ell})'), & \\ \qquad\qquad t = \ell+1,\ldots,m\ ; & \\ h(s^*(r_m,Q_{m+1}))n_m + h(s^*(r_{m-1},Q_m))n_{m-1} + \cdots & \\ \qquad + h(s^*(r_{m-\ell+1},Q_{m-\ell+2}))n_{m-\ell+1} \equiv \chi_\ell((r_{m-\ell+1},\ldots,r_{m+\ell})'), & \\ \qquad\qquad t = m+1\ ; & \\ \vdots & \\ h(s^*(r_m,Q_{m+1}))n_m \equiv \chi_1((r_m,\ldots,r_{m+\ell})'), & t = m+\ell\ . \end{cases}$$

The functions $\zeta_1,\ldots,\zeta_\ell$, ξ, $\chi_\ell,\ldots,\chi_1$ are also functions of the cohort sizes $\{n_1\},\ldots,\{n_1,\ldots,n_\ell\}$, $\{n_{t-\ell},\ldots,n_t\}$, $\{n_{m-\ell+1},\ldots,n_m\},\ldots,\{n_m\}$, respectively, but we have omitted these arguments for notational simplicity.

Now, given the total amount of human capital supplied in period t, from A5 we have that

$$[7] \qquad r_t = \lambda(t)R(H_t)$$

$$= \lambda(t) \times \begin{cases} R(\zeta_1((r_1,\ldots,r_{\ell+1})')), & t = 1 ; \\ \vdots & \\ R(\zeta_\ell((r_1,\ldots,r_{2\ell})')), & t = \ell ; \\ R(\xi((r_{t-\ell},\ldots,r_{t+\ell})')), & t = \ell+1,\ldots,m ; \\ R(\chi_\ell((r_{m-\ell+1},\ldots,r_{m+\ell})')), & t = m+1 ; \\ \vdots & \\ R(\chi_1((r_m,\ldots,r_{m+\ell})')), & t = m+\ell . \end{cases}$$

Thus in equilibrium each period's rental rate on human capital is an implicit function of itself and the rental rates in ℓ adjacent periods in either direction, or up to the beginning or ending date of the economy if either is within ℓ periods of the reference date.

Definition 2 The operator T(r), where $r \equiv (r_1\ r_2\ \ldots\ r_{m+\ell})'$, is given by

$$[8] \qquad T(r) = \begin{bmatrix} \lambda(1)R(\zeta_1((r_1,\ldots,r_{\ell+1})')) \\ \vdots \\ \lambda(\ell)R(\zeta_\ell((r_1,\ldots,r_{2\ell})')) \\ \lambda(\ell+1)R(\xi((r_1,\ldots,r_{2\ell+1})')) \\ \vdots \\ \lambda(m)R(\xi((r_{m-\ell},\ldots r_{m+\ell})')) \\ \lambda(m+1)R(\chi_\ell((r_{m-\ell+1},\ldots,r_{m+\ell})')) \\ \vdots \\ \lambda(m+\ell)R(\chi_1((r_m,\ldots,r_{m+\ell})')) \end{bmatrix}.$$

Before stating and proving the main analytical result of the paper, we explicitly state the subscripting convention to be used throughout the sequel.

Subscripting Convention: When the functions g or h or their derivatives appear with subscript t, this denotes that the function or derivative is evaluated at the argument approriate for cohort t [*i.e.* the schooling choice made in period t]. For all other functions, a subscript on it or its derivative denotes that it is evaluated at the appropriate argument for period t.

Theorem For every cohort size vector $n \in \mathbb{R}_+^m$ there exists a unique $r^* \in \mathbb{R}_+^{m+\ell}$ for which $r^* = T(r^*)$.

Proof: The proof will employ an induction argument. First consider the determination of an equilibrium first period rental rate r_1 given the rental rate sequence $(r_2,\ldots,r_{m+\ell})' \in \mathbb{R}_+^{m+\ell-1}$. Such an r_1 must satisfy

$$[9] \qquad r_1^* = \lambda(1)R(\zeta_1((r_1^*,r_2,\ldots,r_{\ell+1})')).$$

In what follows, denote $\lambda(t)R(H_t)$ by RHS_t; this quantity corresponds to the t^{th} row of the operator T. We have that:

$$\lim_{x\to 0} RHS_1((x,r_2,\ldots,r_{\ell+1})') = \infty ,$$

$$[10] \qquad \lim_{x\to\infty} RHS_1((x,r_2,\ldots,r_{\ell+1})') = \lambda(1)R(n\ )_1 < \infty.$$

$$\frac{\partial RHS_1}{\partial r_1} = -\lambda(1)R_1'n_1g_1'/\beta Q_2 < 0 ,$$

Since the left-hand side of [9] is strictly increasing on $\mathbb{R}_+$, we have established that there exists a unique first period equilibrium rental rate for every $(r_2,\ldots,r_{\ell+1})' \in \mathbb{R}_+^{\ell}$; we will write this conditional equilibrium value as $r_1^*((r_2,\ldots,r_{\ell+1})')$ [$\in (\lambda(1)R(n_1),\infty)$].

Furthermore, $r_1^*((r_2,\ldots,r_{\ell+1})')$ is a partially differentiable function. RHS_1 can be most succintly written as a function of only r_1 and Q_2. From [9], the explicit form of the partial derivative of $r_1^*(Q_2)$ with respect to the second period rental rate r_2 is

$$[11] \qquad \frac{\partial r_1^*}{\partial r_2} = \frac{\partial r_1^*}{\partial Q_2}\frac{\partial Q_2}{\partial r_2} = \left[\frac{r_1^*}{Q_2}\ \frac{\lambda(1)R_1'n_1g_1'}{\beta Q_2+\lambda(1)R_1'n_1g_1'}\right]\beta > 0 \quad \forall\ (r_2,\ldots,r_{\ell+1})' \in \mathbb{R}_+^{\ell} .$$

Now consider the second period equilibrium condition, conditional on the rental rate sequence $(r_3,\ldots,r_{\ell+2}) \in \mathbb{R}_+^{\ell}$ and on a first period equilibrium. This conditional second period equilibrium is defined by

[12] $$r_2^* = \lambda(2)R(\zeta_2((r_1^*((r_2^*,r_3,\ldots,r_{\ell+1})'),r_2^*,r_3,\ldots,r_{\ell+2})')).$$

By substituting the function $r_1^*(Q_2)$ for the generic value r_1 in $\zeta_2((r_1,\ldots,r_{\ell+2})')$, we ensure that if a unique solution $r_2^*((r_3,\ldots,r_{\ell+2})')$ can be found for [12], a <u>conditional</u> first period equilibrium will exist as well. Partially differentiating RHS_2 with respect to r_2, we have

[13] $$\frac{\partial RHS_2}{\partial r_2} = \lambda(2)R_2'\left[\left[\frac{n_1h_1'g_1'}{Q_2}\right]\left[\frac{\partial r_1^*}{\partial r_2} - \frac{r_1^*}{Q_2}\beta\right] - n_2g_2'/Q_3\right] < 0,$$

since

$$\frac{\partial r_1^*}{\partial r_2} \le \frac{\beta r_1^*}{Q_2}.$$

Now RHS_2 displays the following limiting behavior:

[14] $$\lim_{x\to 0} RHS_2((x,r_3,\ldots,r_{\ell+2})') > 0,$$

so that there exists a unique solution to [12], denoted by $r_2^*((r_3,\ldots,r_{\ell+2})')$. By substitution, there also exists a unique <u>conditional</u> first period equilibrium of the form $r_1^*((r_2^*((r_3,\ldots,r_{\ell+2})'),r_3,\ldots,r_{\ell+1})')$, or simply $r_1^*((r_3,\ldots,r_{\ell+2})')$.

Consider the case of general t when endpoint conditions are not operative, that is when $t \in \{\ell+1,\ldots,m\}$. In this case we write

[15] $$RHS_t = \lambda(t)\{R(1-g(r_t/Q_{t+1}))n_t + h(g(r_{t-1}/Q_t))n_{t-1} + \ldots + h(g(r_{t-\ell}/Q_{t-\ell+1}))n_{t-\ell}\}.$$

Given the <u>conditional</u> equilibrium functions $r_{t-1}^*((r_t,\ldots,r_{t+\ell-1})')$, ..., $r_{t-\ell}^*((r_t,\ldots,r_{t+\ell-1})')$, we have that the partial derivative of RHS_t with respect to r_t is

[16] $$\frac{\partial RHS_t}{\partial r_t} = \lambda(t)R_t'\left[-\frac{g_t'n_t}{Q_{t+1}} + \frac{h_{t-1}'g_{t-1}'n_{t-1}}{Q_t}\left[\frac{\partial r_{t-1}^*}{\partial r_t} - \frac{\beta r_{t-1}^*}{Q_t}\right] + \ldots + \frac{h_{t-\ell}'g_{t-\ell}'n_{t-\ell}}{Q_{t-\ell+1}}\left[\frac{\partial r_{t-\ell}^*}{\partial r_t} - \frac{\beta^\ell r_{t-\ell}^*}{Q_{t-\ell+1}}\right]\right].$$

Now [16] is negative since

$$[17]\qquad \frac{\partial r^*_{t-j}}{\partial r_t} = \frac{\beta^j r^*_{t-j}}{Q_{t-j+1}}\left[\frac{\lambda(t-j)R'_{t-j}g'_{t-j}n_{t-j}}{Q_{t-j+1}+\lambda(t-j)R'_{t-j}g'_{t-j}n_{t-j}}\right]$$

$$\leq \frac{\beta^j r^*_{t-j}}{Q_{t-j+1}}, \qquad \forall\ j,\ j \in \{1,\ldots,\ell\}.$$

Thus since RHS_t is strictly decreasing on $\mathbb{R}_+$ and since

$$\lim_{x\to 0} RHS_t(x, r_{t+1},\ldots,r_{t+\ell}) > 0, \quad \forall \quad (r_{t+1},\ldots,r_{t+\ell})' \in \mathbb{R}_+^{\ell-1}$$

there exists a unique $r^*_t((r_{t+1},\ldots,r_{t+\ell})')$ which solves

$$[18]\qquad r^*_t = \lambda(t)R(\xi((r^*_{t-\ell}((r^*_t,r_{t+1},\ldots,r_{t+\ell-1})'),\ldots,\\ r^*_{t-1}((r^*_t,r_{t+1},\ldots,r_{t+\ell-1})'),r^*_t,r_{t+1},\ldots,r_{t+\ell})')).$$

Now consider the determination of the equilibrium rental rate in period $m+\ell$, defined by

$$[19]\qquad r^*_{m+\ell} = \lambda(m+\ell)R(\chi_1((r^*_m(r_{m+\ell}),r^*_{m+1}(r_{m+\ell}),\ldots,r^*_{m+\ell-1}(r_{m+\ell}))')).$$

The derivative of $RHS_{m+\ell}$ with respect to $r_{m+\ell}$ is

$$[20]\qquad \frac{\partial RHS_{m+\ell}}{\partial r_{m+\ell}} = \frac{\lambda(m+\ell)R'_{m+\ell}h'_m g'_m n_m}{Q_{m+1}}\left[\frac{\partial r^*_m}{\partial r_{m+\ell}} - \frac{r^*_m}{Q_{m+1}}\left(\beta\frac{\partial r^*_{m+1}}{\partial r_{m+\ell}} + \ldots + \beta^\ell\right)\right]$$

$$= \frac{\lambda(m+\ell)R'_{m+\ell}h'_m g'_m n_m}{Q_{m+1}}\left[\frac{\beta^\ell r^*_m}{Q_{m+1}}\left(\frac{\lambda(m)R'_m g'_m n_m}{Q_{m+1}+\lambda(m)R'_m g'_m n_m} - 1\right)\right.$$

$$\left. - \frac{r^*_m}{Q_{m+1}}\left(\beta\frac{\partial r^*_{m+1}}{\partial r_{m+\ell}} + \ldots + \beta^{\ell-1}\frac{\partial r^*_{m+\ell-1}}{\partial r_{m+\ell}}\right)\right] < 0 .$$

Since $\lim_{x\to 0} RHS_{m+\ell}((r^*_m(x),r^*_{m+1}(x),\ldots,r^*_{m+\ell-1}(x),x)') > 0$ and since $RHS_{m+\ell}$ is a strictly decreasing function of $r_{m+\ell}$, there exists a unique solution $r^*_{m+\ell} \in \mathbb{R}_+$ to [19], and hence there exists one and only one fixed point $r^* \in \mathbb{R}_+^{m+\ell}$ of the operator T. □

As mentioned in the Introduction, there exists a sufficient degree of generality in the model to preclude the possibility of performing simple comparative statics exercises. For example, one open empirical question is whether or not increasing the size of a given cohort j will result in an increase or decrease in the schooling attainment of cohort j members. There is not an unambiguous prediction to this question using the framework presented here [which may be a bit surprising given our assumption of perfect substitutability in aggregate production]. However, most of our simulation exercises suggest that the relationship is negative over the "normal" range of structural parameter values. We now turn to the issue of quantifying the effects of a cohort size sequence on intergenerational welfare and schooling attainment comparisons.

3. The Effect of Cohort Size on Schooling

As is obvious from the Theorem, in general the cohort size of each generation will affect the welfare of every other cohort which exists during the life of this economy. Given exogenous schooling decisions, the welfare of a cohort is inversely related to its size and the size of cohorts alive during its life. The direct effects of cohort size, that is those which exist given exogenous schooling decisions, are not the net effects. It is essential to also consider the "indirect" effects of the cohort size sequence on the schooling sequence. The calculations performed below are performed as an exercise in which we determine the magnitude of such indirect effects relative to direct effects for one example economy. The methods utilized can be applied more generally.

We begin by defining the effect of an infinitesimal change in the size of one cohort $j \in \{1,\ldots,m\}$ on the lifetime welfare of cohort t, $t \in \{1,\ldots,m\}$. Without loss of generality, in the sequel we will restrict attention to the case in which all demand shifters are equal to unity [*i.e.* $\lambda(1) = \ldots = \lambda(m+\ell) = 1$]. Then the lifetime welfare of cohort j is given by

[21] $$U_t = R[(1-g_t)n_t + h_{t-1}n_{t-1} + \dots + h_{t-\ell}n_{t-\ell}](1-g_t)$$
$$+ \beta R[(1-g_{t+1})n_{t+1} + h_t n_t + \dots + h_{t-\ell+1}n_{t-\ell+1}]h_t$$
$$+ \dots$$
$$+ \beta^{\ell} R[(1-g_{t+\ell})n_{t+\ell} + h_{t+\ell-1}n_{t+\ell-1} + \dots + h_t n_t]h_t .$$

Making use of the subscripting conventions established in the previous section, we have that the partial derivative of this function with respect to cohort size n_j is

$$\frac{\partial U_t}{\partial n_j} = \{R'_t[-\frac{\partial g_t}{\partial n_j} n_t + h'_{t-1}\frac{\partial g_{t-1}}{\partial n_j} n_{t-1} + \dots + h'_{t-\ell}\frac{\partial g_{t-\ell}}{\partial n_j} n_{t-\ell}](1-g_t)$$
$$+ \beta R'_{t+1}[-\frac{\partial g_{t+1}}{\partial n_j} n_{t+1} + h'_t\frac{\partial g_t}{\partial n_j} n_t + \dots + h'_{t-\ell+1}\frac{\partial g_{t-\ell+1}}{\partial n_j} n_{t-\ell+1}]h_t$$
$$+ \dots$$

[22] $$+ \beta^{\ell} R'_{t+\ell}[-\frac{\partial g_{t+\ell}}{\partial n_j} n_{t+\ell} + h'_{t+\ell-1}\frac{\partial g_{t+\ell-1}}{\partial n_j} n_{t+\ell-1} + \dots + h'_t\frac{\partial g_t}{\partial n_j} n_t]h_t$$
$$+ \frac{\partial g_t}{\partial n_j}[-R_t + h'_t(\beta R_{t+1} + \dots + \beta^{\ell} R_{t+\ell})]\}$$
$$+ \{\mathbb{I}[t-\ell \le j \le t-1][R'_t h_{t-j}(1-g_t) + h_{t-j}h_t(\beta R'_{t+1} + \dots + \beta^{j+\ell-t} R_{j+\ell})]$$
$$+ \mathbb{I}[j = t][R'_t(1-g_t)^2 + h_t^2(\beta R'_{t+1} + \dots + \beta^{\ell} R'_{t+\ell})]$$
$$+ \mathbb{I}[t+1 \le j \le t+\ell][\beta^{j-t} R'_j(1-g_j)h_t + h_j h_t(\beta^{j-t+1} R'_{j+1} + \dots + \beta^{\ell} R'_{t+\ell})]\},$$

where the indicator function $\mathbb{I}[A]$ takes the value 1 if the logical statement A is true and otherwise takes the value 0. The quantities in the first pair of braces ({}) on the RHS of [23] represent the "indirect" effect of the changing cohort size n_j on the welfare of cohort t, and the quantities in the second pair of braces represent the "direct" effect of the cohort size change. As we can readily see, a change in the size of cohort j will only have a direct effect on the welfare of cohort t if $|t-j| \le \ell$. On the other hand, through the adjustment of equilibrium schooling levels, a change in the size of cohort j can, and in general will, have an effect on the welfare of cohort t even when $|t-j| > \ell$.

In order to facilitate comparisons, we report elasticites which are constructed as follows. We symbolically write [23] as

$$[22'] \qquad \frac{\partial U_t}{\partial n_j} = e_{tj} + d_{tj} ,$$

where e_{tj} denotes the indirect, or equilibrium, effect of changing cohort size j on the welfare of cohort t and d_{tj} denotes the corresponding direct effect. The elasticity of cohort t welfare with respect to cohort j size is then

$$[23] \qquad \eta_{tj} = \varepsilon_{tj} + \delta_{tj} \equiv [e_{tj} + d_{tj}] \frac{n_j}{U_t} ,$$

where η_{tj}, ε_{tj}, and δ_{tj} are the total, indirect, and direct elasticities measuring the welfare effects for cohort t of an infinitesimal change in the size of cohort j.

If there are m cohorts, then there will be a total of m^2 elasticites η_{tj} [and also of ε_{tj} and δ_{tj}], so we must find some way to summarize these quantities. By analogy with the computation of parameters characterizing discrete-time stationary stochastic processes[6], we have computed the average elasticity of the welfare of a cohort with respect to an infinitesimal change in the size of a cohort s periods distant from it, where s = -3,...,3, or

$$[24] \qquad \bar{\eta}_s = \sum_{t=\underline{t}}^{\bar{t}} \eta_{t,t-s} / (\bar{t} - \underline{t} + 1),$$

where $\underline{t}$ is the first cohort used in the averaging operator and $\bar{t}$ is the last cohort used in the operator. It is wise to avoid setting $\underline{t}$ = 1 and $\bar{t}$= m for initial and terminal conditions effects; in the exercise below we set $\underline{t}$ = 6 and $\bar{t}$ = 95. The first few cohorts and last few cohorts which exist in the economy in general face very different conditions than do the other "interior" cohorts due to the fact that their members face less competition than do the members of other cohorts. For this reason it makes sense to delete them when constructing a statistic designed to depict average behavior. We can define

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[6]Although the existence of equilibrium does not depend on the stationarity of the cohort size sequence, summarization of cohort size effects on welfare and schooling levels is made substantially easier when this is the case. In the simulation exercise performed below, the cohort size sequence is generated by a stationary first-order autoregressive process.

the statistics $\bar{\varepsilon}_s$ and $\bar{\delta}_s$ in an analagous manner.

We also present the computed values of the elasticity of schooling with respect to cohort size changes; this elasticity is defined by

$$[25] \qquad \bar{\gamma}_s \equiv \sum_{t=\underline{t}}^{\bar{t}} \gamma_{t,t-s} \;/\; (\bar{t} - \underline{t} + 1)$$

where $\gamma_{t,t-s} = \dfrac{\partial g_t}{\partial n_j} \dfrac{n_j}{g_t}$.

In general, no explicit expressions exist for the terms $\varepsilon_{t,t-s}$ [and by extension $\eta_{t,t-s}$] or $\gamma_{t,t-s}$. Therefore, to compute $\bar{\eta}_s$ and $\bar{\gamma}_s$, it is necessary to use numerical methods. For a given cohort size vector $(n_1 \ldots n_m)'$, we first solve for the equilibrium rental rate vector r^* [we discuss how this was done in the next section]. We then successively perturbed each cohort size by the "small" positive amount σ; we denote the cohort size sequence in which the j^{th} cohort size has been increased by σ by $\tilde{n}_j \equiv$ $(n_1,\ldots,n_{j-1},n_j+\sigma,n_{j+1},\ldots,n_m)'$. For each cohort size sequence $\tilde{n}_j$, $j =$ $1,\ldots,m$,
we calculate the equilibrium rental rate vector, which we will denote by $r^*(j)$. By definition,

$$[22''] \qquad \frac{\partial U_t}{\partial n_j} = \lim_{\sigma \to 0_+} \left[\frac{\tilde{U}_t(j) - U_t^*}{\sigma} \right]$$

where $\tilde{U}_t(j) = \tilde{r}_t(j)(1-\tilde{g}_t(j)) + \beta\tilde{r}_{t+1}(j)h(\tilde{g}_t(j)) + \ldots + \beta^{\ell}\tilde{r}_{t+\ell}(j)h(\tilde{g}_t(j))$, $\tilde{g}_t(j)$ denotes the equilibrium schooling choice of cohort t corresponding to the cohort size vector $\tilde{n}_j$, and U_t^* denotes the equilibrium utility level for cohort t given the original cohort size vector n. By choosing a small positive σ we numerically approximate [22] (and of course its alternative representations [22'] and [22"]). These approximations are used in the construction of the average elasticities $\bar{\eta}_s$ and $\bar{\varepsilon}_s$. In the same manner, we use the approximation

$$[26] \qquad \frac{\partial \tilde{g}_t}{\partial n_j} \doteq \left[\frac{\tilde{g}_t(j) - g_t^*}{\sigma} \right]$$

for "small" positive σ to compute the $\bar{\gamma}_s$ (see [25]), where g_t^* denotes the equilibrium schooling choice of the t^{th} cohort given the original cohort size vector n.

4. Simulation Exercise

In calculating cohort size effects for an experimental economy, we require a cohort size vector n, a human capital production function h, and an inverse demand function for human capital R. While any vector containing strictly positive elements can be used for n, the functions h and R must satisfy the restrictions imposed on them in A3 and A5, respectively. We first will discuss the method used to generate the particular cohort size sequence used in this study. We then turn to the method used to construct a function h which satisfies the conditions of A3. Given the specification of the structural parameters, we briefly discuss the method used to find the equilibrium rental rate vector for the economy. Finally, we present the numerical results of the exercise.

In this simulation exercise, we compute cohort size effects on welfare and schooling choice for a cohort size sequence generated by a first-order autoregressive process of the form:

[27] $$n_t = .1 + .9n_{t-1} + .1\omega_t, \quad t = 2,\ldots,100;$$
$$n_1 = 1,$$

where ω_t is an independently and identically distributed standard normal random variable. The theoretical mean of this process is 1 and the standard deviation is .229. In the one sample realization we use to perform all the calculations reported below, the sample mean is .981 and the sample standard deviation is .203. The sample regression function was

[28] $$n_t = .127 + .867n_{t-1},$$

and the estimated conditional variance of n_t was .011 (in comparison with the value .01 for the underlying process).

It is easy to find functional forms for the inverse demand function for human capital which satisfy the requirements of A5. Perhaps the most obvious choice is the one we have selected for use, the inverse of the human capital stock,

[29] $$R(H_t) = H_t^{-1}.$$

The specification of the function h is slightly more involved, though this task is made fairly straightforward when we recognize that the requirements of A3 will be met if we choose for the function g a survivor function for a continuously distributed random variable with support $\mathbb{R}_+$ over which the density is everywhere defined. Distributions commonly used in survival analysis are obvious candidates - in the exercise below we use the survivor function for an exponentially-distributed random variable for g, so that

[30] $$g(x) = \exp(-\alpha x), \ \alpha > 0.$$

To derive the human capital production function which generates g, recognize that

[31] $$s = g(x) = (h')^{-1} = \exp(-\alpha x),$$
$$\Rightarrow h'(s) = -\frac{1}{\alpha}\ell n(s) = x.$$

Then

[32] $$h(s) = -\frac{1}{\alpha}\int_0^s \ell n(y)dy + h(0)$$
$$= -\frac{1}{\alpha}s(\ell n(s)-1), \ s \in [0,1],$$

since $h(0) = 0$. For this choice of g, we see that the upper bound on the amount of human capital possessed by the member of any cohort when they are two periods of age or older is $h(1) = \alpha^{-1}$.

The proof of the Theorem is not constructive in the sense that it does not provide an immediately useful way to find the unique equilibrium rental rate vector corresponding to a given cohort size vector. To compute the equilibrium rental rate vector, we simply used a variant of the method of successive approximation. Though we are not able to establish monotonicity of the operator T, in practice we found that a fixed point could always be readily located using the algorithm

[33] $$\hat{r}_{k+1} = \theta T(\hat{r}_k) + (1-\theta)\hat{r}_k, \ \theta \in (0,1],$$

where $\hat{r}_k$ is the value of the rental rate vector at the k^{th} iteration of the algorithm. A fixed point is located when $d_\infty(\hat{r}_{k+1},\hat{r}_k) \le \upsilon(\theta)$, where $\upsilon(\theta)$ is some arbitrarily small prespecified positive constant [for a fixed value of θ] and $d_\infty(\cdot,\cdot)$ is the sup norm operator; we then call $\hat{r}_{k+1}$ the equilibrium rental rate sequence. The algorithm specified in [33] can be used to locate the fixed point since any fixed point will satisfy $\hat{r}_{k+1} = T(\hat{r}_k) = \hat{r}_k$. The algorithm given in [33] will <u>not</u> converge in general for arbitrary specifications of the weighting parameter θ and starting value $\hat{r}_1$. If we start "far away" (in the sup norm metric) from the fixed point, a "small" value of θ must be chosen for [33] to converge. After some initial experimentation, it was relatively easy to choose values of $\hat{r}_1$ and θ to ensure rapid convergence to the fixed point for the range of structural parameter values and functional forms utilized in this experiment.[7]

In the limited simulation experiment which was conducted, given our functional form assumptions, there are four structural parameters which characterize the economy. First, there is the number of cohorts which exist (m); we set m = 100 and generated one cohort size vector as described above. Second, there is the number of periods of life for each cohort (ℓ+1); we report results for the two cases $\ell = 1$ and $\ell = 2$ in Tables 1 and 2, respectively. Third, there is the discount factor (β); we allow β to assume the two values .5 and .9. Finally, we allow the human capital production function parameter (α) to assume the values .25 and .5.

In Table 1 we present results for the case in which agents live for two periods (ℓ=1). With respect to the effect of cohort size on average schooling elasticities, the effects are in accordance with intuition [which is to say the equilibrium effects do not dominate the more apparent direct effects of cohort size change]. An increase in the size of an immediately prior cohort results in increased schooling, since the amount of human capital supplied to the market in the cohort's initial period of life increases (in equilibrium) for the parameter values examined here, thus lowering the opportunity cost of schooling. Conversely, increases in the size of cohorts immediately following a given cohort result in decreased schooling, since the (equilibrium) return to schooling is diminished. In terms of the size of the cohort effects on schooling, in all cases examined the absolute magnitude of the elasticity of schooling time with respect to previous cohort size was greater than the elasticity associated with own cohort size. The absolute magnitude of the elasticity associated with the size of the next cohort was typically substantially less than either that associated with previous or own cohort size. The elasticities associated with the size of cohorts other than these three were generally negligible, with the possible exception of the cohort born two periods prior.

The welfare elasticities reported in Table 1 are also easy to summarize.

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[7] One has to be careful to modify the convergence criteria utilized to stop the algorithm whenever the weighting parameter θ is changed, otherwise an artificial "fixed point" will be located. The constant ν chosen should be an increasing function of θ.

TABLE 1

Effects of Cohort Size on Schooling Attainment and Welfare
for Two-Period Lived Agents:
Elasticities Evaluated at Equilibrium Values
$\{ n_t = .1 + .9n_{t-1} + .1\omega_t \}$

						s			
α	β	*Elasticity*	*-3*	*-2*	*-1*	*0*	*+1*	*+2*	*+3*
.25	.5	$\bar{\gamma}_s$	.004	.037	.327	-.308	-.057	-.003	-.000
		$\bar{\delta}_s$			-.159	-.755	-.079		
		$\bar{\varepsilon}_s$	.004	-.014	-.032	.132	-.014	.001	.006
		$\bar{\eta}_s$	.004	-.014	-.191	-.623	-.093	.001	.006
.25	.9	$\bar{\gamma}_s$	.000	.010	.220	-.207	-.023	-.001	-.000
		$\bar{\delta}_s$			-.061	-.884	-.054		
		$\bar{\varepsilon}_s$	.009	.005	-.020	.108	-.018	.006	.009
		$\bar{\eta}_s$	.009	.005	-.081	-.776	-.072	.006	.009
.5	.5	$\bar{\gamma}_s$	.019	.092	.407	-.264	-.230	-.025	-.003
		$\bar{\delta}_s$			-.322	-.481	-.161		
		$\bar{\varepsilon}_s$	-.009	-.061	.023	.081	.013	-.012	.002
		$\bar{\eta}_s$	-.009	-.061	-.299	-.400	-.148	-.012	.002
.5	.9	$\bar{\gamma}_s$	.004	.037	.315	-.237	-.107	-.011	-.001
		$\bar{\delta}_s$			-.168	-.682	-.151		
		$\bar{\varepsilon}_s$	.002	-.018	-.021	.129	-.019	-.014	.002
		$\bar{\eta}_s$	.002	-.018	-.189	-.553	-.170	-.014	.002

The direct effects of cohort size changes on welfare ($\bar{\delta}_s$) are usually much greater for own cohort size than for s = ± 1; the only exception occurs when agents are relatively myopic and human capital is more costly to produce (β = .5 and α = .5). The indirect, or equilibrium, elasticities are generally much smaller (in absolute value) than are the elasticites associated with direct effects. For the four sets of structural parameters examined, the elasticities associated with the indirect effects of a change in own cohort size are always positive, and serve to mitigate the direct effect of increases in own cohort size on welfare. However, the indirect effects are only approximately one-sixth or one-seventh the absolute magnitude of the direct effects, so that changes in schooling choices do not completely offset the direct effects of cohort size increases to make cohort size shifts welfare-neutral. Changes in the size of cohort sizes other than own have [proportionately] even smaller effects in terms of the equilibrium elasticites than they do in terms of the direct elasticites. The pattern of signs of these elasticities is a bit difficult to summarize, so given there insignficant magnitudes we will not attempt to do so here.

The simulation results reported in Table 2 for agents who live for three periods provide an interesting contrast to those just described for two-period lived agents [recall that both sets of results were computed using the same cohort size sequence]. Looking first at the average schooling elasticities, we see a similiar sign pattern to that recorded in Table 1; the elasticities associated with previous cohorts are positive and those associated with successive cohorts are negative. However, we now find that the elasticities which are largest in absolute value are those associated with the cohort born two periods prior. The elasticities associated with the previous cohort's size are now less in absolute value than those associated with own cohort size. There is a large negative response to an increase in the size of the next cohort for the four combinations of α and β presented.

The welfare elasticities in Table 2 also reveal an interesting result. The elasticities associated with the direct effects of cohort size increases are all negative as they must be; however, we see that even for the case of a change in own cohort size, the equilibrium effects of schooling choice changes may further decrease the welfare of a cohort instead of mitigating the welfare loss associated with the direct effect. This result is observed for the two cases {α = .25 and β = .5} and {α = .5 and β = .5}, which represent the situation in which agents are relatively myopic. For the most part, the

TABLE 2

Effects of Cohort Size on Schooling Attainment and Welfare for Three-Period Lived Agents: Elasticities Evaluated at Equilibrium Values

$\{ n_t = .1 + .9n_{t-1} + .1\omega_t \}$

α	β	Elasticity	s = -3	-2	-1	0	+1	+2	+3
.25	.5	$\bar{\gamma}_s$	.002	.133	.044	-.126	-.052	-.004	-.000
		$\bar{\delta}_s$		-.043	-.336	-.285	-.156	-.011	
		$\bar{\varepsilon}_s$	-.005	-.004	.046	-.092	.012	.001	.002
		$\bar{\eta}_s$	-.005	-.047	-.290	-.377	-.144	-.010	.002
.25	.9	$\bar{\gamma}_s$	.000	.063	.030	-.061	-.031	-.001	-.000
		$\bar{\delta}_s$		-.010	-.263	-.479	-.236	-.008	
		$\bar{\varepsilon}_s$	.002	.002	.031	.063	.027	.002	.003
		$\bar{\eta}_s$	.002	-.008	-.232	-.426	-.209	-.006	.003
.5	.5	$\bar{\gamma}_s$	.012	.221	.066	-.166	-.124	-.027	-.003
		$\bar{\delta}_s$		-.119	-.332	-.251	-.137	-.030	
		$\bar{\varepsilon}_s$	-.022	-.004	.049	-.040	-.002	.002	.001
		$\bar{\eta}_s$	-.022	-.123	-.283	-.291	-.139	-.028	.001
.5	.9	$\bar{\gamma}_s$	.003	.118	.056	-.103	-.065	-.010	-.001
		$\bar{\delta}_s$		-.033	-.262	-.450	-.235	-.027	
		$\bar{\varepsilon}_s$	-.002	-.004	.025	.060	.022	-.002	-.000
		$\bar{\eta}_s$	-.002	-.037	-.237	-.390	-.213	-.029	-.000

direct elasticities are of an appreciable magnitude only for $s = \{-1,0,1\}$. Note that the magnitude of the elasticities are considerably less than is the case in Table 1. Since each cohort lives for three periods rather than two, the negative effect of an increase in any given cohort size is smoothed and diffused over more periods. For a cohort size sequence which does not display "too much" intertemporal dependence, we would expect that as ℓ increases, cohort size effects at all leads and lags will be go to zero.

5. Conclusion

A number of studies of the cohort size - schooling choice relationship have been conducted in either the context of an economy perturbed from a steady state growth path or in an environment in which the cohort size process in covariance stationary. More often than not, previous researchers have endowed agents with non-rational expectations or sub-optimal decision rules to simplify the analysis. One of the aims of this paper was to show that the problem can be investigated without necessarily resorting to such devices. Within a fairly general setting, we were able to develop a set of restrictions on the information sets of agents, human capital production functions, and human capital demand functions which were sufficient to guarantee the existence of an unique equilibrium mapping from *any* cohort size vector containing positive elements to [simultaneously] a set of human capital rental rates and schooling choice decisions. Virtually all of the assumptions used in this paper have a precedent in earlier studies of the question, although the particular combination used here is unique. From our point of view, the most restrictive assumptions [*i.e.* the most difficult to relax and the most empirically counterfactual] are those of perfect foresight and the perfect substitutability of members of different cohorts in aggregate production. It would be especially be difficult to dispense with the perfect foresight assumption given the level of generality at which the stochastic process generating the cohort size sequence has been specified, although such an extension would probably be required if a model such as the one developed here was to be used in empirical analysis.

In this general setting, we have confronted the issue of summarizing "cohort size effects." Making a distinction between the direct and indirect (equilibrium) effects of cohort size changes, our small simulation study suggested that the direct negative effects of increases in cohort size, which pertain to the situation in which schooling choices are held fixed, would not be entirely offset by equilibrium changes in schooling choices. While in most cases the equilibrium effects mitigated the size of welfare losses caused by increases in (own) cohort size, the absolute magnitudes of these effects were typically much smaller than the corresponding direct effect. In fact, in some cases changes in equilibrium schooling choices worsened the welfare situation of cohorts whose own size was increased. The suggestion that cohort size changes may not substantially affect welfare levels once equilibrium responses are taken into account appears to need further substantiation in light of the limited [but suggestive] results presented here.

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