ON THE NONLINEAR PRICING OF INPUTS

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1. Introduction

There has recently been substantial amount of research devoted to the study of nonlinear pricing in general and two-part tariffs in particular. Leland and Meyer [1976] have demonstrated under fairly general conditions[1] that a two part tariff is welfare superior to a uniform price. Spence [1977] began the study of general nonlinear outlay schedules. Willig [1978] has demonstrated that there exists such a schedule which is Pareto superior to a uniform price not equal to marginal cost. Goldman, Leland, and Sibley [1977] and Roberts [1979] have established the result that a welfare optimal general nonlinear outlay schedule must present the largest buyer with a marginal price equal to marginal cost. This result, which has as its counterpart in the optimal taxation literature the requirement that the marginal tax rate be zero for the highest income bracket (see Cooter [1978], for example), is also a corollary of Willig's result.

However, all of these results have been established using models which posit no economic interactions between purchasers of the nonlinearly priced commodities. In this paper we relax this (implicit) assumption by postulating a model in which the good in question is purchased as an input by firms which operate in a perfectly competitive output market. This generates indirect interactions between purchasers since the marginal price faced by any firm affects the equilibrium output price and hence the input demand of all firms.

In Section 2 we recast the welfare analysis of the simple two-part tariff using the classical model of perfect competition in which all firms are identical and free entry and exit ensures that the equilibrium output price is equal to minimum average cost. In this context we discover that two-part tariffs are not generally desirable from a welfare standpoint, as the Leland and Meyer analysis would suggest.[2] Rather, their desirability depends crucially on the properties of the underlying production technology. We provide readily interpretable sets of necessary and sufficient conditions for two-part tariffs to be welfare superior to a uniform price. Most surprisingly, for the empirically relevant class of production processes in which the purchased input is required in fixed proportion to output, we discover that a two-part tariff is never optimal from either a profit or welfare maximizing standpoint.

In Section 3 we extend our analysis to encompass general nonlinear outlay schedules by introducing firm heterogeneity in the context of a perfectly competitive industry with Ricardian rents. Our principal result here[3] is that the profit or welfare maximizing outlay schedule requires that marginal price be <u>everywhere</u> greater than marginal cost. Thus the efficiency result cited earlier, i.e., marginal price equals marginal cost for the largest user; does not hold in our interactive formulation.

2. Two-Part Tariffs for Inputs: The Case of Identical Firms

The classic case of the perfectly competitive industry characterized by the free entry and exit of identical firms operating at the minimum point of a U-shaped average cost curve provides an ideal starting point for our analysis. Since all firms are identical, the only variables directly affected by pricing policy are the number of firms purchasing the input and the quantity they select. Thus the simple two-part tariff, consisting of an entry fee e and a unit price r, is also the most general nonlinear outlay schedule we need consider.

2.1 The Basic Model

There are many equivalent ways of characterizing downstream industry equilibrium for competitive firms with access to a freely available technology facing a two-part tariff for one of its inputs. For our purposes it proves most convenient to employ the McFadden-Lau profit function $\tilde{\pi}(p,r,w)$, where p is the (endogenously determined) price of the final product and w is the vector (henceforth suppressed) of prices for other factors of production. To ensure against triviality, we assume that the input in question is essential. No positive level of output can be produced without it. Equilibrium in the final product market is then determined by

$$\pi(p,r,e) \equiv \tilde{\pi}(p,r) - e = 0 \tag{2.1}$$

and

$$ny(p,r) - D(p) = 0, \tag{2.2}$$

where n is the equilibrium number of firms in the industry, D(p) is the demand curve for the final product, and y(p,r) is the firm supply function.

At this point it is also worthwhile to recall some of the basic properties of $\tilde{\pi}$ and, hence, π:

$$\frac{\partial \pi}{\partial p} = \frac{\partial \tilde{\pi}}{\partial p} = y(p,r) \tag{2.3}$$

$$\frac{\partial \pi}{\partial r} = \frac{\partial \tilde{\pi}}{\partial r} = -x(p,r) \tag{2.4}$$

$$\frac{\partial \pi}{\partial e} = -1 \tag{2.5}$$

where x is the firm's demand function for the input supplied by the monopolist. It will prove convenient to establish some notation for the structural properties of this characterization of the productive technology in terms of elasticities:

$$\delta \equiv \frac{py_p}{y} \geq 0 \tag{2.6}$$

$$\eta \equiv -\frac{rx_r}{x} \geq 0 \tag{2.7}$$

$$\alpha \equiv \frac{px_p}{x} . \tag{2.8}$$

We shall typically assume sufficient smoothness of the underlying technology so that the inequalities in (2.6) and (2.7) are strict. That is, the elasticity of firm supply δ and the own price elasticity of demand for the input η will, except where noted, be assumed to be strictly positive.

The elasticity of input demand with respect to <u>output</u> price α will play an important role in our analysis. Its qualitative sign is indeterminate a priori, but is readily interpreted in terms of the traditional expansion path of the firm. It can easily be shown that

(2.9) $$\alpha = \eta_y \delta$$

where η_y is the elasticity with respect to output of the Samuelsonian constant-output input demand function $\bar{x}(y,r)$. By standard classification, the input x is said to be <u>inferior</u> if $\eta_y < 0$, <u>normal</u> if $0 \leq \eta_y \leq 1$, and <u>superior</u> if $\eta_y > 1$. Using (2.9), these definitional boundaries are, respectively, given by $\alpha < 0$, $0 \leq \alpha \leq \delta$, and $\alpha > \delta$. Finally, $\varepsilon \equiv -pD'/D > 0$ denotes the elasticity of demand for the <u>final</u> product.

With this notation in hand we are ready to perform some comparative statics analysis in order to describe the response of the equilibrium values of p and n to changes in the parameters r and e. Using (2.1), we have immediately,

(2.10) $$\frac{\partial p}{\partial e} = \frac{1}{y} > 0$$

and

$$\frac{\partial p}{\partial r} = \frac{x}{y} > 0, \tag{2.11}$$

which amount to a slight extension, to the case of a fixed charge, of the usual result (see, e.g. Silberberg [1974]) that the equilibrium output price is an increasing function of the price of an input.

The effects on the number of firms are somewhat more complicated. Totally differentiating (2.2) and substituting (2.10) and (2.11) yields

$$\frac{\partial n}{\partial e} = -\frac{n}{py}(\varepsilon+\delta) < 0 \tag{2.12}$$

and

$$\frac{\partial n}{\partial r} = \frac{nx}{py}(\alpha-\delta-\varepsilon) \tag{2.13}$$

An increase in the fixed charge affects the <u>average</u> but not the <u>marginal</u> cost curve of the firm, and results in increased optimal firm output. But, because $\partial p/\partial e > 0$, the market demand for the final product declines. This smaller demand will be produced by a smaller number of firms each producing a larger output.

An increase in the input price has an ambiguous affect on the equilibrium number of firms because two effects are at work. Total market demand falls, since $\frac{\partial p}{\partial r} > 0$. If the input is inferior or normal, then the equilibrium firm size rises, and both forces, operating in the same direction, ensure that $\frac{\partial n}{\partial r} < 0$. If the input is superior $(\alpha > \delta)$, however, the optimal output per firm shrinks, and the number of firms may increase provided that the final demand is not too elastic. If, in percentage terms, optimal firm size shrinks by more than market demand, the equilibrium number of firms must rise. These are the forces at work in (2.13).

We conclude this discussion of comparative statics results by demonstrating a symmetry property which will be important in subsequent analysis.

Lemma: The change in industry demand, $X \equiv nx$, resulting from an increase in the entry fee is precisely equal to the change in the number of firms due to an increase in the unit price; i.e., $\partial X/\partial e = \partial n/\partial r$.

Proof: $\frac{\partial X}{\partial e} = n\left(x_p \frac{\partial p}{\partial e}\right) + x \frac{\partial n}{\partial e} = nx_p/y + x \frac{\partial n}{\partial e}$

Using (2.12), this becomes

$$\frac{\partial X}{\partial e} = -\frac{nx}{py}(\varepsilon+\delta) + \left(\frac{nx}{py}\right)\frac{pXp}{x} = \frac{nx}{py}(\alpha-\delta-\varepsilon) = \frac{\partial n}{\partial r}. \quad (2.14)$$

Q.E.D.

2.2 Welfare Analysis

We wish to study optimal choices of input two-part tariffs under a variety of objective functions. Therefore, we take as our welfare measure a weighted sum of upstream monopoly profits and the surplus of final consumers:

$$(2.15)\quad W = \gamma \int_{p} D(p')dp' + (1-\gamma)[(r-m)X + en - F]$$

where m and F are, respectively, the (constant) marginal and fixed costs of the input monopolist and $0 \leq \gamma \leq 1/2$. This formulation allows us to encompass the analyses of monopoly profit maximization ($\gamma = 0$), unconstrained surplus maximization ($\gamma = .5$), and Ramsey-type constrained welfare optima ($0 < \gamma < .5$).

Necessary conditions for an optimum are given by

$$(2.16)\quad \frac{\partial W}{\partial e} = -\gamma D(p)\frac{\partial p}{\partial e} + (1-\gamma)\left\{(r-m)\frac{\partial X}{\partial e} + n + e\frac{\partial n}{\partial e}\right\} \leq 0;$$

$$e \geq 0,\ e\frac{\partial W}{\partial e} = 0.$$

$$(2.17)\quad \frac{\partial W}{\partial r} = -\gamma D(p)\frac{\partial p}{\partial r} + (1-\gamma)\left\{(r-m)\frac{\partial X}{\partial r} + X + e\frac{\partial n}{\partial r}\right\} \leq 0;$$

$$r \geq 0,\ r\frac{\partial W}{\partial r} = 0.$$

Using (2.2), (2.10), and (2.11), these immediately simplify to

$$(2.18)\quad \frac{\partial W}{\partial e} = (1-2\gamma)n + (1-\gamma)(r-m)\frac{\partial X}{\partial e} + (1-\gamma)e\frac{\partial n}{\partial e} = 0$$

(2.19) $\frac{\partial W}{\partial r} = (1-2\gamma)X + (1-\gamma)(r-m)\frac{\partial X}{\partial r} + (1-\gamma)e\frac{\partial n}{\partial r} = 0$

The first contrast between two-part tariffs for inputs and those for final products is highlighted by the simple structure of our model. Were the market in question a final product one, the assumptions of identical consumers and essentiality would suffice for the well-known Coasian result in which the unit price is set equal to marginal cost and profits are extracted via the entry fee. In our model, however, the input is essential but firms are not, as revealed by our comparative statics analysis. So that, not surprisingly, this simple result rarely pertains.

Proposition 1: A "perfect" two-part tariff (i.e., $e > 0$, $r = m$) can be optimal only if the purchasing firms are operating in a region where input demand is unresponsive to the level of output and output price; that is, $\alpha = \eta_y = 0$.

Proof: Assume, arguendo, that $e > 0$, $r = m$ are optimal. Then from (2.16)-(2.19), we must have

$$(2.20)\qquad \frac{\partial W}{\partial e} = (1-2\gamma)n + (1-\gamma)e\frac{\partial n}{\partial e} = 0$$

and

$$(2.21)\qquad \frac{\partial W}{\partial r} = (1-2\gamma)nx + (1-\gamma)e\frac{\partial n}{\partial r} = 0.$$

Substituting (2.20) into (2.21) yields the requirement

$$(2.22)\qquad \frac{\partial W}{\partial r} = (1-\gamma)e\left\{\frac{\partial n}{\partial r} - x\,\frac{\partial n}{\partial e}\right\} = 0.$$

Using (2.12) and (2.13), this becomes

$$(2.23)\qquad \frac{\partial W}{\partial r} = (1-\gamma)e\left(\frac{nx}{py}\right)\alpha = 0.$$

Q.E.D.

We turn now to examine whether or not any two-part tariff is preferable to a uniform price. Leland and Meyer [1976] found that, with no income effects, it always paid a final product monopolist to introduce a positive entry fee. Schmalensee [1980] later extended this result to the case of an objective function of the form employed here. However, he also argued that this need not be the case for an input monopolist. We now present a precise, readily interpretable condition for a two-part tariff to dominate a uniform price.

Proposition 2: If the total derivative of input usage with respect to input price is negative ($dx/dr = x_p\, \partial p/\partial r + x_r < 0$), then a uniform price cannot be optimal.

Proof: Suppose $e = 0$, $r > 0$. Then from (2.19), we have

$$(2.24)\qquad \left.\frac{\partial W}{\partial r}\right|_{e=0} = (1-2\gamma)X + (1-\gamma)(r-m)\,\frac{\partial X}{\partial r} = 0$$

Multiplying (2.18) by x yields

$$\left.\frac{\partial W}{\partial e}\right|_{e=0} = (1-2\gamma)X + (1-\gamma)(r-m)x\,\frac{\partial X}{\partial e}\,. \tag{2.25}$$

Substituting from (2.24) yields

$$\begin{aligned}\left.\frac{\partial W}{\partial e}\right|_{e=0} &= (1-\gamma)(r-m)\;x\left[\frac{\partial X}{\partial e} - \frac{\partial X}{\partial r}\right] \\ &= (1-\gamma)(r-m)\;x\left[\frac{\partial X}{\partial e} - x\,\frac{\partial n}{\partial r} - n\,\frac{dx}{dr}\right].\end{aligned}$$

$$\left.\frac{\partial W}{\partial e}\right|_{e=0} = -(1-\gamma)(r-m)n\,\frac{dx}{dr} \tag{2.26}$$

from our Lemma. Thus the necessary condition for an optimum at $e = 0$, $\frac{\partial W}{\partial e} \leq 0$, cannot be satisfied if $dx/dr < 0$.

Q.E.D.

The fact that dx/dr is not always negative and a two-part tariff generally desirable in this model may be surprising at first. After all, don't all input demand functions exhibit negative own-price derivatives? The difference here is that the derivative in question is total rather than partial in that it includes the indirect effect on input demand via the equilibrium output price response to an input price change. As Silberberg [1974] has pointed out, total own-price effects need not be negative when the equalizing role of the output price is recognized. To see this, we note that

(2.27) $$\frac{dx}{dr} = x_p \frac{\partial p}{\partial r} + x_r = \frac{x}{y} x_p + x_r.$$

While $x_r < 0$, x_p may be of either sign. Thus we have immediately that a two part-tariff is required for optimality if the input is inferior. Somewhat surprisingly, this result can be extended.

Proposition 3: A uniform price can be optimal only if the input is strictly normal over the relevant range. That is $0 < \alpha \leq \delta$, or equivalently $0 < \eta_y \leq 1$.

Proof: Inspection of (2.27) establishes the result for the case $\alpha \equiv px_p/x \leq 0$. For superior inputs, we exploit the convexity of $\tilde{\pi}(p,r)$:

(2.28) $$\tilde{\pi}_{rr}\tilde{\pi}_{pp} - \tilde{\pi}^2_{rp} = -x_r y_p - x_p^2 \geq 0,$$

which yields

(2.29) $$x_r \leq -x_p^2/y_p.$$

Upon substituting this into (2.27), we obtain

(2.30) $$\frac{dx}{dr} \leq x_p\left[\frac{x}{y} - \frac{x_p}{y_p}\right] = \frac{xx_p}{py_p}(\delta-\alpha).$$

The r.h.s. of (2.30) is negative only if $\alpha < 0$ or $\alpha > \delta$.

Q.E.D.

We have left the case of homothetic production, $\delta = \alpha$ or $\eta_y = 1$, for special discussion since the nature of the optimal two-part tariff there depends crucially upon the smoothness properties of the underlying technology with respect to x:

Proposition 4: If the underlying production function is strongly quasiconcave and homothetic, there exists a two-part tariff superior to a uniform price.

Proof: Under these hypotheses Silberberg [1974] has shown that $dx/dr < 0$, in which case Proposition 2 applies.

Q.E.D.

For the interesting and empirically relevant case in which the input x (but not all others) is required in fixed proportions for the production of y, we obtain precisely the opposite result!

Proposition 5: Given fixed proportions between x and y; i.e., $C(y,r,w) = rzy + \psi(y,w)$; the optimal entry fee must be zero.

Proof: Under these conditions, $y(p,r,w) = f(p-rz,w)$. Letting f' represent the derivative of f with respect to its first argument, we have

$$\frac{\partial y}{\partial p} = f'; \quad \frac{\partial y}{\partial r} = -zf' = -z\,\frac{\partial y}{\partial p} \qquad (2.31)$$

Substituting (2.31) and the identity zy = x into (2.10) and (2.11) yields

(2.32) $$\frac{\partial p}{\partial r} = z; \; \frac{\partial p}{\partial e} = \frac{1}{y} \,.$$

Totally differentiating (2.2) and solving using (2.31) and (2.32), we obtain

(2.33) $$\frac{\partial n}{\partial r} = \frac{z}{y} D', \; \frac{\partial n}{\partial e} = (D'-nf')/y^2$$

Inserting these results into (2.18) and (2.19), using our Lemma, leaves us with

(2.34) $$\frac{\partial W}{\partial e} = (1-2\gamma)n + (1-\gamma)[(r-m)zD'/y + e(D'-nf')/y^2]$$

and

(2.35) $$\frac{\partial W}{\partial r} = (1-2\gamma)nzy + (1-\gamma)[(r-m)z^2D' + ezD'/y]$$

Substituting $(\partial W/\partial r)/zy = 0$ into (2.34) yields

(2.36) $$\frac{\partial W}{\partial e} = -(1-\gamma)enzf'/y \leq 0$$

Since, with the exception of e, all terms on the r.h.s. of (2.36) are strictly positive, the only way the necessary condition $e(\partial W/\partial e) = 0$ can be satisfied is with e = 0.

Q.E.D.

The intuition behind this result is rather straightforward. It is well-known that under fixed proportions an upstream uniform pricing input monopolist can extract all the profits which an integrated uniform pricing monopolist could reap. Since competition downstream ensures that a uniform price prevails in the final product market, there can be nothing to gain from introducing a two-part tariff; optimal choice of r allows the monopolist to earn the maximum possible under such circumstances. There is something to lose, however, since an entry fee $e > 0$ causes the downstream firms to operate at an inefficiently large scale. Total (upstream plus downstream) costs are not minimized and a portion of this dead-weight burden falls on the monopolist.

The qualitative results thus far are valid for both profit maximizing monopolists and welfare maximizing firms bound by profit requirements. Thus there is a clear implication that pricing rules for profit and (constrained) welfare maximizing monopolists in some sense "look" the same. The policy issue which our analysis has yet to address concerns the desirability, as measured by total surplus, of allowing a uniform pricing profit maximizing monopolist to introduce a two-part tariff. Absent strong regularity assumptions on the underlying structural model it is impossible, in general, to compare welfare levels generated by a profit maximizing monopolist with and without the ability to offer two-part tariffs. (See Leland and Meyer [1976] for some simulation results on this subject.) However, our analysis does allow us to deduce something about the relative marginal social and private incentives to introduce two-part pricing.

Proposition 6: With respect to an initial uniform pricing equilibrium with $r > m$, the private marginal incentive to introduce a two-part tariff always exceeds the social one.

Proof: In our formulation, W equals one half total consumers' plus producer's surplus, $\pi^m + S$ when $\gamma = 1/2$. Therefore, using (2.19) we have

$$\left.\frac{\partial(\pi^m+S)}{\partial e}\right|_{e=0} = 2\left.\frac{\partial W}{\partial e}\right|_{\substack{\gamma=\frac{1}{2}\\ e=0}} = (r-m)\frac{\partial X}{\partial e}\,. \tag{2.37}$$

Whereas

$$\left.\frac{\partial \pi^m}{\partial e}\right|_{e=0} = n + (r-m)\frac{\partial X}{\partial e} > \frac{\partial(\pi^m+S)}{\partial e} \tag{2.38}$$

Q.E.D.

Thus it seems safe to conclude that there may be cases in which it pays a profit-maximizing monopolist to introduce a two-part tariff which lowers total surplus.

3. Optimal Outlay Schedule for an Input: The Case of Heterogeneous Firms

In Section 2 we analyzed the welfare implications of supplanting a uniform price scheme with a two-part tariff (TPT). We noted that when all buyers have identical cost functions, more precisely, identical derived demand functions for the input, a TPT is the only relevant alternative to a uniform price. In this section we relax the homogeneity assumption and postulate instead that firms differ in their cost functions. Once heterogeneity among the input buyers is admitted, the set of possible pricing schemes encompasses not only a uniform price and a TPT but also other more complex arrangements. In fact, a TPT is usually not the best price schedule that can be implemented. Consequently, in this section, we allow the purveyor of the input -- the upstream monopolist -- to choose any pricing schedule, subject, however, to various constraints which we shall spell out below. The resulting price schedule is referred to as the (optimal) outlay schedule for an input and is denoted by $R(x)$. We show that the properties of $R(x)$ differ in important respects from the properties of the optimal outlay schedules for outputs, (see Willig [1978], Roberts [1979]) or, for that matter, from the optimal income tax schedules, (see Mirrlees [1971], Cooter [1978]).

3.1 Preliminaries

Before $R(x)$ can be established, the model of the downstream industry must be cast in a form which will permit us to characterize the diversity in individual firm technologies. We do this by indexing the structural, competitive profit function $\tilde{\pi}$ with the cost reducing parameter $\theta \varepsilon [\underline{\theta},\overline{\theta}]$. Thus $\tilde{\pi} = \tilde{\pi}(p,r,\theta)$, with $\frac{\partial\tilde{\pi}}{\partial\theta} \equiv \tilde{\pi}_\theta > 0$ implying that greater "endowments" of θ make it possible for the firm to achieve higher (maximized) profits, given output and input prices. We also assume that such favored firms are unambiguously "bigger" in the sense that they supply more output and demand more of the monopolist's input. That is; using the derivate properties of $\tilde{\pi}$,

(3.1) $$\frac{\partial^2\tilde{\pi}}{\partial p\partial\theta} \equiv \frac{\partial y(p,r,\theta)}{\partial\theta} \equiv y_\theta > 0 \quad ;$$

$$-\frac{\partial^2\tilde{\pi}}{\partial r\partial\theta} \equiv \frac{\partial x(p,r,\theta)}{\partial\theta} \equiv x_\theta > 0 \quad .$$

We assume that the least efficient firm, $\underline{\theta}$, is viable when the input is priced at marginal cost m; i.e.,

(3.2) $$\tilde{\pi}(p^e,m,\underline{\theta}) \geq 0 \quad ,$$

where p^e is the resulting equilibrium price in the downstream market.

In order to develop a manageable optimal control formulation, let us assume for a moment that the optimal R(x) schedule were known. When confronted with such a schedule a firm of type θ has derived supply and demand functions $\hat{y}(p,\theta)$ and $\hat{x}(p,\theta)$ which result from the solution of the following program:

$$(3.3)\qquad \max_{y,\ell,x}\; py - w\cdot\ell - R(x) \quad \text{s.t.} \quad f(x,\ell,\theta) \geq y$$

where ℓ is the vector of other inputs and $f(\cdot)$ the production function representation of the technology. Let us denote by $r(\theta)$ the slope of the outlay function evaluated at $\hat{x}(p,\theta)$; i.e. $r(\theta) \equiv R'[\hat{x}(p,\theta)]$. (For simplicity, we consider only differentiable outlay schedules.) Thus, $r(\theta)$ is the marginal cost of the input to a firm of type θ.

Observe that if the monopolist could identify a firm's θ it would be able to induce the same supply and demand behavior by the firm. This follows from the fact that, given that the firm is producing, only the marginal properties of the outlay schedule matter. Therefore $\hat{x}(p,\theta) = x(p,r(\theta),\theta) \equiv -\frac{\partial\tilde{\pi}}{\partial r}$ and $y(p,\theta) = y(p,r(\theta),\theta) \equiv \frac{\partial\tilde{\pi}}{\partial p}$. Were firm θ able to purchase all of its x at price $r(\theta)$, its profits would be given by $\tilde{\pi}(p,r(\theta),\theta)$. In general, however, the prices paid for inframarginal units will diverge from $r(\theta)$ and actual profits $\pi^*(\theta)$ will differ from $\tilde{\pi}$; the difference, $\tilde{\pi} - \pi^*$, which accrues to the monopolist can be viewed as a firm-specific entry fee $e(\theta)$. From this perspective, then, the outlay schedule $R(x)$ can be viewed as a set of firm specific two-part tariffs, $(e(\theta),r(\theta))$: personalized entry fees and marginal prices. When a firm is presented with such a tariff, it purchases the same quantity of input, produces the same output, and earns the same profit as it would if it were optimizing against the impersonal outlay schedule $R(x)$.

This observation permits us to convert the problem of choosing the optimal outlay schedule into the problem of choosing the set of optimal firm specific TPT's, (see Roberts [1979], for a similar approach). In fact, in what follows it will prove advantageous to treat $\pi^*(\theta)$ as being subject to choice, i.e., as a state variable, and let $e(\theta)$ be implicitly defined as $\tilde{\pi}(p,r(\theta),\theta) - \pi^*(\theta)$.

However, both formulations obscure the important fact that no firm-specific information is available to the monopolist. When the purveyor of the input does not use any such information regarding buyers, the fully decentralized input allocation process guarantees that

$$\frac{d\pi^*(\theta)}{d\theta} - \frac{\partial\tilde{\pi}}{\partial\theta} \geq 0 \quad . \tag{3.4}$$

This follows from the fact that in a decentralized system a firm of type θ' could behave "as if" it were a firm of type θ, if by so doing it could earn higher profit. Clearly, then we must have

$$\theta = \underset{\theta'}{\text{argmax}}\, [\tilde{\pi}(p,r(\theta),\theta')-e(\theta)-\pi^*(\theta')] \quad . \tag{3.5}$$

Equation (3.4) is necessary for (3.5) to hold and therefore becomes our "law of motion".

3.2 The Optimal Control Formulation

We now have the ingredients needed to characterize the optimal outlay schedule $R(x)$. In the formulation that follows, $\pi^*(\theta)$ is the state variable satisfying the differential equation (3.4). We treat $\{r(\theta)\}$ and p as controls. The market price p is, strictly speaking, not under the direct control of the monopolist, being instead an implicit functional of $\{r(\theta)\}$. However, it is more convenient and instructive to take p as a decision variable and impose as an additional constraint the condition supply equal demand in the downstream market.

Our objective function is a weighted sum of downstream producers' plus consumers' surplus and the monopolists' profits. These latter are given by

$$(3.6) \qquad \pi^m = \int_{\underline{\theta}}^{\overline{\theta}} [\tilde{\pi}(p,r(\theta),\theta) - \pi^*(\theta) + [r(\theta)-m]x(p,r(\theta),\theta)]dH(\theta) - F$$

where $H(\theta)$ is the (strictly increasing) culmulative density over the interval $[\underline{\theta},\overline{\theta}]$. The social welfare function is thus

$$(3.7) \qquad W = \gamma\int_p D(p')dp' + \gamma\int_{\underline{\theta}}^{\overline{\theta}} \pi^*(\theta)dh(\theta) + (1-\gamma)\pi^m$$

The formulation in (3.7) reflects the fact that now, unlike in Section 2, the downstream firms earn producers' surplus and some welfare weight must be attached to it. Since the downstream industry is perfectly competitive, it is natural to value a dollar of consumer's surplus on par with a dollar of (downstream) producer's surplus.

The social welfare function in (3.7) is to be maximized by the appropriate choice of p and $\{r(\theta)\}$. This maximization is subject to the following constraints. First, each downstream firm must earn non-negative profit, that is

(3.8) $$\pi^*(\theta) \geq 0 \quad .$$

Second, in the downstream market supply must equal demand, that is

(3.9) $$D(p) - \int_{\underline{\theta}}^{\overline{\theta}} y(p,r(\theta),\theta)dH(\theta) \geq 0 \quad .$$

Third, the state variable $\pi^*(\theta)$ must satisfy the differential equation in (3.4).

The new maximand, after the constraints are adjoined to the social welfare function (3.7), can be written as

(3.10) $$L = W + \int_{\underline{\theta}}^{\overline{\theta}} \lambda(\theta)\pi^*(\theta)dH(\theta) + \xi[D(p) - \int_{\underline{\theta}}^{\overline{\theta}} y(p,r(\theta),\theta)dH] + \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta) \; [\tilde{\pi}_\theta(p,r(\theta),\theta) - \frac{d\pi^*(\theta)}{d\theta} \;] \; d\theta \quad .$$

At the optimum the following necessary conditions must be satisfied. First, since $\pi^*(\theta)$ is the state variable,

(3.11) $$\frac{\partial L}{\partial \pi^*(\theta)} = -(1 - 2\gamma)h(\theta) + \lambda(\theta)h(\theta) + \frac{\partial \mu(\theta)}{\partial \theta} = 0 \quad .$$

To obtain this condition we first rewrite the last integral in (3.10)

(3.12) $$I = \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta)\tilde{\pi}_\theta d\theta - \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta) \; [\; \frac{d\pi^*(\theta)}{d\theta} \;] \; d\theta$$

and then integrate by parts the second integral in (3.12). The transversality condition $\mu(\overline{\theta}) = 0$ ensures that in the unintegrated part, the term $\mu(\overline{\theta})\pi^*(\overline{\theta})$ vanishes. If every downstream firm is producing in equilibrium then the remaining transversality condition $\mu(\underline{\theta}) = 0$ implies that $\mu(\underline{\theta})\pi^*(\underline{\theta}) = 0$. If some firms do not purchase the input, then for the marginal firm, denoted by $\hat{\theta}$, $\pi^*(\hat{\theta}) = 0$. Hence again $\mu(\hat{\theta})\dot{\pi}^*(\hat{\theta}) = 0$. Putting all those facts together yields the condition (3.11).

It is easy to establish that $\underline{\theta} \leq \hat{\theta} < \overline{\theta}$, that is at the optimum either all firms are producing, or the excluded firms belong to a connected set $[\underline{\theta}, \hat{\theta})$. To prove this assertion we recall that, by assumption, the input is essential and that profits must satisfy the differential equation $d\pi^*/d\theta \geq 0$. Consequently, if a firm of type θ' is using the input and thus earning $\pi^*(\theta') \geq 0$, then all firms with $\theta > \theta'$ must be also using the input. Assume not; then a firm of type $\theta'' > \theta'$ receives $\pi^*(\theta'') = 0$ which given $\tilde{\pi}_\theta > 0$ contradicts equation (3.4). This observation and complementary slackness leads to the conclusion that $\lambda(\theta) = 0$ for $\theta \geq \hat{\theta}$.

Let us return now to the remaining necessary conditions for the optimum. Differentiating (3.10) with respect to $r(\theta)$ and using the fact that $\partial\tilde{\pi}/\partial r = -x(\cdot)$ yields

$$(3.13) \quad \frac{\partial \mathcal{L}}{\partial r(\theta)} = (1-\gamma)(r(\theta)-m)x_r - \xi y_r - \frac{\mu(\theta)}{h(\theta)} x_\theta = 0$$

$$\theta \in [\hat{\theta}, \overline{\theta}] \quad .$$

Differentiating (3.10) with respect to p and using the fact that $\partial\tilde{\pi}/\partial p = y(\cdot)$ yields

$$(3.14)\qquad \frac{\partial \mathcal{L}}{\partial p} = -\gamma D(p) + (1-\gamma)\int_{\underline{\theta}}^{\overline{\theta}} \left[y(\theta) + (r(\theta)-m)\ x_p\right] dH(\theta)$$

$$+ \xi\left[D'(p) - \int_{\underline{\theta}}^{\overline{\theta}} y_p\ dH(\theta)\right] + \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta) y_\theta d\theta = 0\ .$$

We now turn to.the characterization of the optimal outlay schedule $R(x)$.

3.3 Discussion of the First-Order Conditions

Easiest to consider is the case when the social welfare function weighs equally consumer surplus and the monopolist's profits. Not unexpectedly, we have

Proposition 7: If $\gamma = \frac{1}{2}$ then the input price is equal to marginal cost and no firm is excluded from the input market.

Proof: Since $\gamma = \frac{1}{2}$, then from equation (3.16) we note that $\partial\mu(\theta)/\partial\theta = -\lambda(\theta)h(\theta)$. But $\lambda(\theta) = 0$, $\theta \in [\hat{\theta},\overline{\theta}]$. Hence, in view of the transversality condition $\mu(\overline{\theta}) = 0$, it follows that for a producing firm $\mu(\theta) = 0$. When $(\theta) = 0$, equation (3.18) reduces to

$$(3.15)\qquad (1-\gamma)(r(\theta)-m) = \xi\ y_r/x_r$$

Substitution of (3.15) into (3.14) gives

$$(3.16)\qquad \partial\mathcal{L}/\partial p = \xi D'(p) = 0\quad .$$

Since $D'(p) \neq 0$ it follows that $\xi = 0$. This implies, in turn, that $r(\theta)-m = 0$. To see that no firm is excluded, we recall the transversality condition $\mu(\underline{\theta}) = 0$. Since $\mu(\theta) = 0$,

$\theta \in [\hat{\theta},\theta]$ and $\lambda(\theta) \neq 0$, $\theta \in [\underline{\theta},\hat{\theta})$, continuity ensures that $\mu(\theta) = 0$, $\theta \in [\underline{\theta},\hat{\theta})$ as well. Hence all firms must be producing.

Q.E.D.

The intuition behind this proposition is obvious: If, from the welfare standpoint, redistribution of surplus between the monopolist and the downstream consumers, or producers, is treated as a pure transfer there is no equity reason to introduce a distortion into the price schedule. Consequently, the input price must be equal to its marginal cost. A similar conclusion can be reached if, for a particular value of $\gamma < \frac{1}{2}$, in the optimum all firms are purchasing the input.

Proposition 8: If in equilibrium no firm is excluded from the input market, input price is equal to marginal cost. In addition, each firm will be assessed with a lump-sum entry fee.

Proof: The simplest way to proceed is to assume that with each value of γ, $0 \leq \gamma < \frac{1}{2}$, we can associate a unique minimum profit that the monopolist earns at the optimum. We could then rewrite the social welfare function as

$$W' = \int_p D(p')dp' + \int_\theta \pi^*(\theta)dH + \pi^m \quad .$$

This would be maximized subject to constraints (3.4), (3.8), and (3.9) and one additional constraint, viz $\pi^m \geq c$. If we denote by ϕ the Lagrange multiplier associated with this constraint, we can rewrite (3.11) as

$$(3.11') \qquad \frac{\partial \mathcal{L}'}{\partial \pi^*} = \frac{\partial \mu(\theta)}{\partial \theta} - \phi h(\theta) + \lambda(\theta)h(\theta) = 0 \quad .$$

Since, by assumption, $\lambda(\theta) = 0$, it follows that $\phi = 0$. This, in turn, implies that $\xi = 0$ and, consequently, $r(\theta) = r = m$.

Q.E.D.

We can interpret this result in the following way. Given that the input is priced at marginal cost, there is a maximum lump-sum tax that can be imposed on the least inefficient firm without causing it to shut-down: Any entry fee higher than that will cause exit. If the optimal entry fee is below this maximum, its imposition causes no distortions in the downstream industry. Since (productive) efficiency mandates marginal cost-pricing, and since equity goals can be accomplished with a nondistortionary entry fee, marginal cost pricing is the welfare-maximizing solution. Thus we find that, for any γ above some cut-off $\hat{\gamma}$, input should be priced at marginal cost and the entry fee should be used to transfer the requisite net revenue from the downstream firms, and consumers, to the upstream monoplist.

One interesting implication of this discussion is that a profit-maximizing monoplist will always charge some of his customers a price above marginal cost, a point also made by Roberts [1979] in a somewhat different context. This can be seen directly from equation (3.11): If we note that profit-maximization is equivalent to setting $\gamma = 0$ then from (3.11) we deduce that $\mu(\theta)$ cannot be zero for all $\theta > \hat{\theta}$. Consequently, marginal cost pricing cannot be optimal.

In the context of the last two propositions, it is interesting to consider why this implication, $\gamma = \frac{1}{2}$ $(\phi=0) \rightarrow \xi = 0$, holds. Intuitively, the answer is that if the profit constraint upstream is not binding, any demand for the final product can be satisfied. All that is required is a sufficiently low price for the input. In fact, supply (downstream) can always be made to exceed demand at a given market price p. But if the constraint is not binding then the associated Lagrange multiplier, here ξ, must be zero.

Let us now turn to the case where uniform marginal cost pricing is not possible, and as a consequence, some firms are excluded. Note that upon using the first order condition $\partial L/\partial r(\theta) = 0$, the expression for the optimal choice of market price for the final good can be greatly simplified, <u>viz</u>.

$$(3.17) \qquad \frac{\partial L}{\partial p} = (1-2\gamma)D(p) + D'(p) = 0 \quad .$$

Hence, provided that $1-2\gamma$ is not equal to zero,

$$(3.18) \qquad p^* = [\xi/(1-2\gamma)]\varepsilon$$

where ε is the elasticity of demand for the final good. Hence,

<u>Proposition 9</u>: At the optimum the market price is proportional to demand elasticity, and the factor of proportionality is the ratio $\xi/(1-2\gamma)$. Or, setting $1-2\gamma = \phi$, the Lagrange multiplier associated with π^m, the optimal price is proportional to the ratio of Lagrange multipliers, ξ/ϕ.

Turning now to the first-order conditions for the optimal deviation of the marginal input price from marginal cost, we observe

Proposition 10: Let us assume that the input is normal, i.e. $-y_r \equiv x_p > 0$, then the marginal imput price always exceeds the marginal cost, even for the largest buyer.

Proof: For $\theta = \overline{\theta}$, equation (3.13) reduces to

$$(3.19) \qquad r(\overline{\theta})-m = \left[\frac{\phi}{1+\phi}\right]\left[\frac{\xi}{\phi}\right]\frac{y_r}{x_r} > 0 \quad ,$$

where, again, $\phi = 1-2\gamma$. This can be rewritten using (3.18) and (2.9) as

$$(3.20) \qquad \frac{r(\theta)-m}{m} = \left[\frac{\phi}{1+\phi}\right]\left[\frac{py}{mx}\right] \cdot \frac{1}{\varepsilon\eta_y} > 0 \quad .$$

Remark: In an earlier note (see Ordover and Panzar [1980]) we showed that under special cost conditions, a move from the regime of uniform pricing to nonlinear pricing, with quantity discounts to largest buyers, need not be a Pareto improvement. Equation (3.19) further demonstrates that social welfare considerations do not mandate deep discounts to the largest users. This is in contradistinction to well-known studies on optimal pricing wherein it is established that a nonlinear price schedule must approach marginal cost for the largest buyers. The rationale for the difference in results should be apparent from equation (3.13): reduction in the (marginal) input-price causes an increase in output by the firm that benefits from the reduction. This increase, in turn, pushes down the price for the final good. With lower price, the demand for input by the inframarginal firm decreases causing a loss in revenues. Indeed, the term on the right-hand side of equation (3.19) reflects the presence of this (pecuniary) externality. Efficiency alone mandates that $r(\overline{\theta})-m = 0$; however, the spill-over effects that occur when the marginal input price is reduced, forestall efficient pricing of the input even to the largest users.

We must now demonstrate that $r(\theta) > m$ for all values of θ. Examining (3.13), note that the term $\mu(\theta)y_\theta$ is non-positive because $\mu(\theta) \leq 0$ and y_θ is assumed to be positive. Thus, since $\xi y_r/x_r$ is positive, the necessary conditions can be satisfied if and only if $r(\theta) > m$, which establishes the proposition.

One interesting implication of Proposition 10 is that even the marginal firm contributes something to the overhead. We attribute this result to the fact that even a marginal firm buys a strictly positive quantity and produces a finite output. When a marginal firm is permitted to participate in the market, it will exert a downward pressure on the price of the final product. Indubitably, this is beneficial to consumers. However, the monopolist suffers a loss in revenue because other firms reduce their purchases of input. Consequently, the marginal firm must be willing to pay more than its marginal cost for unit of input.

In Proposition 5 we showed that when the production function is of the special one-for-one type, average cost pricing is optimal if all firms are identical. Let us reconsider this result when firms have different costs functions. Now the cost function becomes

$$(3.21) \qquad c(y,r,\theta) = r(\theta)zy + \psi(y,\theta) \quad .$$

We showed in Section 2 that this implies that $x_r/y_r = z$, a constant. Using this to rewrite (3.13) we obtain

$$(3.22) \qquad (1-\gamma)(r(\theta)-m) = \frac{\xi}{z} + \frac{\mu(\theta)}{n(\theta)} \cdot \frac{x_\theta}{x_r}$$

For $\overline{\theta}$ this reduces to

$$(3.23) \qquad r(\theta)-m = \frac{\xi}{z} \; \frac{1}{(1-\gamma)} \quad ,$$

allowing us to state:

<u>Proposition 11</u>: In the case of fixed proportions the mark up over marginal cost paid by the largest user, while strictly positive, is less than that paid by any smaller firm.

<u>Proof</u>: The second term on the right-hand side of equation (3.22) is negative for all $\theta < \overline{\theta}$.

Q.E.D.

We note that this result need not be true in general, because without the fixed proportions relationship the first term on the right-hand side of equation (3.23) is also a function of θ.

4. Concluding Remarks

This paper has demonstrated that there is a fundamental difference in the nonlinear pricing of inputs and final products. However, our analysis has dealt only with industries which are and remain perfectly competitive independent of the form of the price schedule. Future research should address these issues under alternative behavioral assumptions regarding the downstream market structure, and, if possible, examine the impact of the input price schedule on the nature of equilibrium in the downstream industry.

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FOOTNOTES

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1. Their's and all the other results referred to in the text depend on the assumption that buyers cannot resell the commodity, for otherwise implementation of the nonlinear pricing schedule would be frustrated by arbitrage.
2. This phenomenon was recently noted by Schmalensee [1980] in a similar context.
3. In an earlier note we constructed a similar but less general model in which Willig's [1978] Pareto-superiority result does not obtain, see Ordover and Panzar [1980].