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A SURPRISE-QUIZ VIEW OF LEARNING IN ECONOMIC EXPERIMENTS

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Abstract

In this paper we use experimental data to investigate what it is that subjects have learned after participating in an economic experiment for a large number of periods. It is our hypothesis that depending on the manner in which subjects are paid, they will attempt to learn different aspects of the experiment they are placed in and perform differently. At the end of our experiments subjects are given another experiment to perform (for "high stakes") which functions like a surprise quiz since they had not been informed about its existence before. The results of this surprise—quiz round, along with the time—series of their responses before the quiz, allows us to judge what they have learned. Our results have potential consequences for experimental methodology since they indicate that the manner in which subjects are paid may have a direct impact on what they choose to learn about. In addition, we hope to present surprise quizzes as a potentially useful technique that can be used in other experiments where the task facing agents is to learn the static equilibrium of the decision situation they are engaged in.

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Section 1: Introduction

Consider the following economic experiment. I subjects indexed i=1,2,...,I are brought into a laboratory and asked to play a game $\Gamma(N,S_j,\Pi_j;\ j\in N)$, where N is the set of players, S_j is the strategy space for player j, and Π_j is player j's payoff function.¹ The game Γ is played T times in succession with payoff π_{it} for subject i in period t=1,2,...T. At the end of the experiment each subject is paid $\Sigma_t\pi_{it}$ and sent home. The question asked by the experimenters is whether the subjects learn to play a particular static equilibrium of the game repeated in the experiment.

In answering this question it is common practice to watch the time series of decisions that subjects make and look for some type of convergence to the predicted equilibrium. If convergence is observed, then the theory is supported, if not, then the theory is one step closer to falsification. But is the convergence of behavior toward equilibrium evidence of learning? If so what is it assumed these subjects have learned?

In this paper, we use experimental data to investigate what exactly it is that subjects have learned after participating in an economic experiment for 75 periods. It is our hypothesis that depending on the manner in which subjects are paid, they will attempt to learn different aspects of the experiment they are placed in and perform differently.

We call this paper a surprise—quiz view of learning since at the end of our experiments subjects are given another experiment to perform which functions like a surprise quiz since they had not been informed about its existence before.² The results of this surprise—quiz round, along with the time—series of their responses before the quiz, allows us to judge what they have learned. To explain, we run a simple 75—round one—

¹If a computer plays the role of N-1 nonstrategic players leaving only one live subject to make decisions, then the game becomes a one-person maximization problem. Even in that case we will continue to call the situation a game.

²One should not consider this surprise equivalent to lying to subjects since we never indicated that the experiment they were performing would be the only task we ask them to do when recruited. We left that possibility ambiguous.

person decision—making experiment using two different payoff structures. In one, which we call the Learn—While—You—Earn environment (LWYE), we pay subjects the way they are typically paid by having them play a game 75 times for small payoffs and then pay them the sum of their earnings over the 75 rounds. When this experiment is over, we then give these subjects a "surprise quiz" by having them play the same game one more time for "big stakes"; in fact, payoffs that are 75 times the one round payoffs in the game they have just played. If subjects have learned the structure of the decision problem they were just engaged in, then we would expect them to make a choice in this extra round which is close to the optimum defined by the problem (i.e. at the top of the payoff function defined by the experiment). In short, this decision should be their best guess as to what optimal behavior is here since it is a period in which a significant amount of money is on the table.

In our other payoff condition, which we call the Learn-Before-You-Earn (LBYE) environment and which we use as our control environment, subjects play the same game and again play it for 75 rounds. However, here they play 74 rounds for free and only in the 75th round do they receive a payoff. The subjects are informed about this at the beginning of the experiment and the payoff they receive in the last round is of the exact size as that of the big-stakes extra round in the Learn-While-Your-Earn experiment. Hence, supposedly subjects have the same incentive to choose correctly in the last round of the Learn-Before-You-Earn experiment as they did in the extra round of the Learn-While-Your-Earn experiment (abstracting from any income effects we consider to be fairly irrelevant).

What we find is that the decisions made in the Learn-Before-Your-Earn environment are significantly better (i.e. closer to the optimal decision defined by the problem) than are the decisions made in the extra round of the Learn-While-You-Earn experiment. More surprisingly, these differences in the last and extra rounds of these experiments can not be attributed to differences in the sampling subjects engaged in before

the surprise quiz or last round was administered. The subjects appeared to explore the domain of the payoff function in identical ways so that they had the same information at their disposal when they were asked to choose for big stakes. As a consequence, we hypothesize that because of the different payoff environments in which these experiments were run the subjects chose to learn about different objects with the subjects in the Learn-Before-You-Earn experiment concentrating on learning the optimal choice for the big-stakes 75th round, which they knew was approaching, and the subjects in the Learn-While-You-Earn experiment concentrating on learning some type of myopic stimulus-response rule not knowing that a big-stakes surprise quiz was coming up. This conjecture is supported by the results of simple nonparametric tests as well as linear regressions run to approximate the subjects' decision rules.

These results have potential consequences for experimental methodology since they indicate that the manner in which subjects are paid may have a direct impact on what they choose to learn about. More critically, they indicate that the theory of learning in markets may need reevaluation since markets place subjects in exactly those payoff environments (LWYE) which are shown here to be the least friendly to learning optimal decisions. Finally, we hope to present surprise quizzes as a potentially useful technique that can be used in other experiments where the task facing agents is to learn the static equilibrium or optimum of the decision situation they are engaged in.

This does not mean, however, that current experimental practices are wrong or have been used incorrectly in the past. For example, take the case of the double-oral-auction experiments which have a long and glorious history. In these experiments convergence and learning are well known stylized facts. Since these are Learn-While-You-Earn environments how can our criticism be relevant? The answer is that these happen to be good environments for Learn-While-You-Earn payoff structures since such environments tend to focus attention of subjects on the feedback of the experiment and in the case of the oral auctions such feedback provides a clear and unambiguous path to the

competitive equilibrium—those buyers not making a transaction must bid higher while those sellers not making a transaction must ask lower prices. Hence, Learn—While—You—Earn payoff structures, since they foster learning about feedback or stimulus—response rules, are expected to exhibit convergence in those experiments where the feedback subjects get leads them efficiently to the equilibrium (see Milgrom and Roberts (1991) for a discussion of how adaptive impulse—response rules can lead to convergence to Nash equilibria). In environments where the feedback is noisy and harder to interpret, as in the experiments performed here and perhaps many others, such a payoff structure would not be optimal.

In this paper we proceed as follows. In Section 2 we explain the experiments performed and the experimental design. In Section 3 we present our results while in Section 4 we offer a possible explanation of the results. Finally, in Section 5 we offer some conclusions.

Section 2: Experimental Setting

2.1: The Games Played

All of the experiments performed to investigate learning were of the tournament variety and similar to those of Bull, Schotter and Weigelt (1987) and Schotter and Weigelt (1992).³ In those experiments, randomly paired subjects must, in each round, each choose a number, e, between 0 and 100 called their decision number. After this number is chosen a random number is independently generated by each subject from a uniform distribution over the interval [-a,+a]. These numbers (each player's decision number and random number) are then added together and a "total number" defined for each player. Payoffs are determined by comparing the total numbers of the subjects in each pair and awarding that subject with the largest total number a "big" payment of M and that subject with the smallest total number a "small" payment of m, M > m. The cost of the decision

³For a description of the theory of tournaments underlying these experiments see, e.g., Lazear and Rosen (1981).

number chosen, given by a convex function $c(e) = e^2/k$, is then subtracted from these fixed payments to determine a subject's final payoff. Hence, in these experiments there is a trade-off in the choice of decision numbers; higher numbers generate a higher probability of winning the big prize but also imply a higher decision cost.⁴ By letting k = 500, k = 40, k = 20, and k = 17.2, the two-person tournament defined has a unique symmetric Nash equilibrium at 37. By replacing one player with a computerized automaton programmed to always choose the Nash equilibrium decision, we transform the problem for the remaining live player into a one-person maximization problem.

We consider this experiment to be a good one for our purposes for at least two reasons. First, although it presents subjects with a complete information maximization problem for which the optimal action could be calculated a priori, such a problem is sufficiently complex so that a deductive solution should be out of the grasp of most experimental subjects. Such complexity forces subjects to learn inductively and it is this process that we are interested in studying. Second, in spite of the complexity of the decision problem it involves, this experiment is simple to describe to subjects and to understand. This feature is appealing since it should reduce the noise in the data.

2.2: Payoff Environments

We distinguish between two payoff environments which we call Learn-While-You-Earn (LWYE) and Learn-Before-You-Earn (LBYE) since they span the spectrum of environments with differing learning costs. A LWYE payoff structure is the typical payoff structure found in laboratory experiments and markets. In this environment time is divided into discrete periods with a known horizon T and in each period subjects or market participants make decisions. These decisions yield them a payoff at the end of the period, and their final payoff from the experiment or market is the sum of their (possibly discounted) period payoffs. The LWYE environment is then one in which payoffs occur

In the instructions, a sample of which is contained in Appendix B, we take great care in not using such value ladened terms as "winning" or "losing".

each period and cumulate throughout the experiment. The cost of learning is the opportunity cost associated with exploring the environment and a trade-off exists between "exploiting" actions already proven to be satisfactory by using them repeatedly and "exploring" to discover new and possibly better actions.

A LBYE environment is a limit case of a payoff environment with low learning costs. It is an artificial environment created by an experimental administrator, although it has some parallels to real world markets. Here time is again divided into T discrete periods but no payoffs are awarded during the first T-1 periods. Rather, subjects make decisions and observe what they would have earned if these were played for real. What does count is their period T decisions and these payoffs are sufficiently large so that their expected payoff from this last round decision is comparable to the expected sum of the payoffs in the LWYE environment. In short, in a LBYE setting there are T-1 practice rounds and one real and lucrative round so that there is no exploration—exploitation trade—off.

While the LBYE environment is used here strictly as an artificial control environment, it could be given a real-world interpretation. For example consider an infinitely lived firm who interacts repeatedly in a market and who has an extremely low (possibly zero) discount rate. In such an environment, the firm might treat any finite number of periods as free-learning periods since, with zero discount rates, any finite period would have only a negligible influence on their infinite horizon payoff. Under these circumstances, our LBYE environment is a reasonable approximation to reality.

2.3: Experimental Design

We performed two different experiments that were conducted at the Experimental Lab of the C.V. Starr Center for Applied Economics at New York University, using 47 undergraduate students recruited from economics classes at N.Y.U. Each experiment lasted approximately 45 minutes with average payoffs of about \$13.00. No subject engaged in more than one experiment and had previous experience with tournament experiments.

In these experiments, subjects played the tournament game described above 75 times consecutively against a computer whose strategy was known to be that of choosing 37 in each period. These experiments then presented our subjects with a one—person decision problem under stochastic uncertainty. The experiments were performed under two payoff regimes. In Experiment 1 subjects did the experiment 74 times without receiving a payoff but were paid for the decisions that they made in round 75 (LBYE environment). With the parameters specified above, the (equilibrium) expected amount for this one period choice was \$15.27. (Actually, payoffs were denominated in a fictitious currency called francs and converted into dollars at the rate of \$0.75 per franc).

In Experiment 2 subjects performed the same experiment 75 times but received a payoff in each of the 75 rounds. Their final payoff was the sum of their 75 round earnings over the course of the experiment (LWYE environment). In order to keep equilibrium payoffs constant across environments, here we converted francs into dollars at the rate of \$0.01 per franc. After the 75 rounds of Experiment 2 were over, subjects were then informed that they would perform the experiment one more time with increased payoffs--actually, with the same payoffs that were used in the LBYE environment. They had not been told about this extra experiment until after they had finished their 75 round experiment. In other words, subjects in Experiment 2 performed the experiment twice. Once for 75 rounds with small payoffs in each round, and once for one extra round (the surprise-quiz round) with a one round payoff equal to the payoff in the last round of the LBYE experiment. This extra-round with increased payoffs faced LWYE subjects with a decision task identical to the one faced by LBYE subjects in round 75 (their only payoff relevant round). As such, their extra-round choices should serve as a sufficient statistic for all that these subjects have learned during the course of the experiment as does the 75th round decision of subjects in LBYE Experiment 1.

⁵For results of the two-person version of these experiments see Merlo and Schotter (1994).

Our experimental design is summarized below:

Experiment	Opponent	Opponent's Strategy	Payoff Environment	Number of Subjects
1	Computer	Fixed and Known to be 37	LBYE	23
2	Computer	Fixed and known to be 37	LWYE	24

Section 3: Results

Our experimental design allows us to compare the learning of subjects in our two payoff environments by comparing their choices in the 75th round of Experiment 1 (the LBYE experiment) and the extra round of Experiment 2 (the LWYE experiment). These two rounds represent situations in which subjects have approximately the same length of experience in the experiment (74 versus 75 rounds) and play an identical game with substantial and identical stakes. The only difference is how they were paid in the rounds preceding these two "test" rounds and whether they knew there would be a test round. What we find is very different behavior in the last (extra) round of these two experiments despite the fact that subjects appear to have employed indistinguishable sampling strategies during the experiments and hence to have the same information at their disposal to guide their final decision.

Tables 3.1 and 3.2 present the last round and extra round choices of subjects in Experiments 1 and 2, respectively while Figure 3.1 presents histograms of the absolute deviations of these choices from the optimal choice of 37.

[Tables 3.1 and 3.2 and Figure 3.1 About Here]

As we can see, subjects in our LBYE environment made choices in their last round which were substantially closer to the optimal choice of 37 for the one-person decision problem they were engaged in than did the subjects in the surprise quiz round of the LWYE

experiment. More precisely, while the mean last period choice for subjects in LBYE Experiment 1 was 42.61 (5.61 units away from the optimal choice of 37), the mean surprise—quiz round choice for subjects in LWYE Experiment 2 was 51.33 (14.33 units away from the optimal choice of 37).⁶ In addition, while 11 subjects in the LWYE experiment made surprise—quiz choices which were 30 units or more away from 37, in the LBYE experiment only two such choices were made. A Kolmogorov—Smirnov test of equality of the distributions of last and extra round choices rejects the null hypothesis at conventional significance levels (P—value 0.025).

In terms of a money metric, the choices of our subjects in the LBYE environment of Experiment 1 led to a mean expected payoff for the experiment of \$14.68 which was \$1.11 higher than the mean expected payoff of subjects in the extra round of LWYE Experiment 2 (\$13.57). In addition, while 13 subjects in LWYE Experiment 2 had payoff losses (measured by the difference in their expected payoffs when choosing the optimal choice of 37 and their actual last round choice) greater than \$1.00, only 2 subjects in LBYE Experiment 1 had such big losses. Conversely, while 12 subjects in LBYE Experiment 1 had losses of less than \$0.10, only 2 subjects had such small losses in the LWYE environment of Experiment 2. The histograms of these losses are presented in Figure 3.2.

[Figure 3.2 About Here]

One possible explanation for these results would be that these two experiments contain very different learning costs. As a result, the sampling strategies of subjects might

⁶A t-test of the hypothesis that the mean of last period choices by subjects in LBYE Experiment 1 is equal to 37 cannot reject the null at conventional significance levels (P-value 0.122), while a t-test of the hypothesis that the mean of extra period choices by subjects in LWYE Experiment 2 is equal to 37 rejects the null at conventional significance levels (P-value 0.015).

Since the actual payoffs in the last (extra) round of the experiment depend on the particular realizations of the additive shocks in that round, our money metric uses the expected payoffs corresponding to the subjects' last (extra) round choices instead.

differ so dramatically (with subjects in the LWYE environment sampling more conservatively than their counterparts in the LBYE environment) that when they make their choices in the last or extra round they do so using very different information sets. Our data reject this interpretation. To illustrate, we present Tables 3.3 and 3.4 which contain descriptive statistics of the sampling distributions of our subjects in Experiments 1 and 2 over the 74 or 75 rounds of their history prior to making their last choice for big stakes. This was the data at their disposal when they made their extra round or last round choices.

[Tables 3.3 and 3.4 About Here]

Kolmogorov-Smirnov tests of equality of the distributions of such descriptive statistics between experiments find no statistically significant differences between the distributions of either the mean, median, standard deviation, or interquantile range of these samples in LBYE Experiment 1 and LWYE Experiment 2 (P-values 0.098, 0.058, 0.363, and 0.197, respectively).

To summarize, from the data presented in Tables 3.1-3.4 and Figures 3.1 and 3.2 it would appear that subjects in the LBYE environment of Experiment 1 were significantly better at the decision task presented to them than were their counterparts in the LWYE environment of Experiment 2 as measured both by the distance of their last (extra) period choices from the optimal choice and by their payoffs. Furthermore, these differences can not be explained by the different sampling strategies of subjects across these experiments since such differences do not appear to exist. The explanation must lie elsewhere and this is what we turn our attention to next.

Section 4: A Possible Explanation of the Results

Our surprise—quiz and big—stakes rounds test one thing and one thing only which is how well the subjects in our experiments learn to locate the maximum of the payoff

function they faced. The fact that subjects in our LBYE environment did better on this task may simply indicate that that payoff environment, with its big payoff round looming on the horizon, focuses attention on this maximum as the correct object of learning. In the LWYE environment, however, where small payoffs come every period, subjects might think it more natural to attempt to learn an impulse—response or adaptive rule and not concentrate on the static payoff function and its maximum (remember, these subjects had no idea that a big payoff experiment was awaiting them in the extra round). In short, what we are saying is that different payoff environments serve as different framing devises which lead subjects to attempt to learn about different aspects of the problem they face. While the LBYE environment focused attention on the static payoff function and its maximum, the LWYE environment focused attention on a more myopic—reactive behavior to the data generated period by period.

To substantiate this conjecture we perform two calculations. To explain our first calculation, note that one way of testing the hypothesis that subjects in the LBYE environment focus their attention on the static optimization problem they face while those in the LWYE environment do not, is to ask whether the subjects in LBYE Experiment 1 use the data they have generated to estimate the quadratic payoff function they face in the experiment and then make a last round choice which is close to the estimated maximum while subjects in LWYE Experiment 2 do not. Hence, we should observe choices in the last (big-stakes) round of the LBYE experiment closer to the estimated payoff maximizing choices than are the surprise-quiz round choices in the LWYE experiment.

Second, we use the data generated by Experiments 1 and 2 to estimate (reduced form) linear approximations of the decision rules used by subjects in these two experiments. In particular, we regress the decision number chosen by a subject in each round (\det_t^i , t = 3,4,...,74 or 75) on his or her lagged decision numbers ($\det_{t-\tau}^i$, $\tau = 1,2,3$), lagged payoffs ($\operatorname{pay}_{t-\tau}^i$), lagged dummy variables ($\operatorname{win}_{t-\tau}^i$) denoting a win (1) or a loss (0) in the tournament, and a local estimate of the sign of the gradient of the payoff function

(slope¹_t).⁸ In order to identify behavioral regularities in the subjects population, we run one such regression for each of the two pooled samples obtained by combining all individual histories in each of the two experiments. To control for individual heterogeneity, we also include among the covariates player—specific dummy variables (α^{i}).

If subjects in the LWYE experiment were merely responding to the data they were generating in a myopic manner and attempting to learn an appropriate impulse—response rule, then we would expect that they would not look more than one period back in formulating their next period choice and would not have a significant slope coefficient since they were not attempting to locate the peak of their payoff function. Subjects in the LBYE experiments should behave differently in that their slope coefficient should be significant and they might be more likely to look further back at the data in formulating their next move.

Looking at our first calculation, we present Tables 4.1 and 4.2 which report the maxima of the payoff functions that could have been estimated by the subjects in the LBYE (Table 4.1) and LWYE (Table 4.2) experiments using the data they generated before the last or extra round, together with the estimates' standard errors, the subjects' final choices, their absolute differences from the estimated payoff maximizing choices, and the subjects' payoff losses computed using their estimated payoff functions. Figure 4.1 presents histograms of the absolute deviations of last round and extra round choices from the estimated optimal choices.

[Tables 4.1 and 4.2 and Figure 4.1 About Here]

 $[\]frac{1}{\text{*slope}_{t}^{i} = \text{sign}([pay_{t-1}^{i} - pay_{t-2}^{i}]/[dec_{t-1}^{i} - dec_{t-2}^{i}]).}$

⁹Standard errors are computed using the Delta Method (see, e.g., Greene (1993)) from the least—squares estimates reported in Tables A.2 and A.3, as illustrated in Table A.4 in Appendix A. Note that five subjects in LWYE Experiment 2 were eliminated from the subject pool because they could not have estimated a concave payoff function.

As we can see, the last round choices of subjects in the LBYE experiment were closer to their estimated payoff maximizing choices than were the choices of subjects in the LWYE experiment. For example, while the mean absolute deviation of last round choices from the estimated peak of the payoff function was 12.2 in the LBYE experiment, it was 19.3 in the LWYE experiment. Also, while 14 out of 19 deviations in the LBYE experiment were less than 10 units, in the LWYE experiment only 7 out of 19 deviations were this small. A Kolmogorov-Smirnov test of equality of the distribution of last round or extra round choices and the distribution of estimated optimal choices cannot reject the null hypothesis at conventional significance levels for the LBYE experiment (P-value 0.164), while it clearly rejects such a hypothesis for the LWYE experiment (P-value 0.015).

In terms of our money metric, in Figure 4.2 we display histograms of the subjects' payoff losses based on their estimated payoff functions. 11

[Figure 4.2 About Here].

Note that as it was true in Figure 3.2, these losses are on average greater in the LWYE experiment than in the LBYE experiment. For example, 8 subjects had estimated losses greater than \$1.0 in the LWYE experiment while only 2 had such large losses in the LBYE experiment. Conversely, only 8/19 (42.1%) of the subjects had losses less than \$0.25 in the LWYE experiment while 13/23 (56.5%) of the subjects in the LBYE experiment had such small losses. In short, subjects in the LBYE experiment acted as if they were making choices in the last round which were close to the peak of their estimated payoff function far more consistently than did subjects in the LWYE experiment.

 $^{^{10}}$ A t-test of equality of the two means rejects the null hypothesis in favor of the alternative hypothesis that the mean deviation in the LBYE experiment is smaller than the one in the LWYE experiment at the 5% level (t = -1.39).

¹¹Such losses are measured by the differences in subjects' expected payoffs when choosing their estimated optimal choices and their actual last round or extra round choices.

The results of our second calculation are summarized by the following regression results: (To economize on space we present our results without reporting the estimated individual intercept terms.)

LBYE Experiment 1: (Number of Observations = 1633)

LWYE Experiment 2: (Number of Observations = 1728)

The numbers in parentheses are heteroskedasticity-consistent standard errors (White (1980)). Estimates in bold typeface indicate that the coefficients are statistically significant at the 10% level, while estimates that are also underlined indicate that the coefficients are statistically significant at the 5% level.

These results clearly show that the decision rules used by subjects in the two experiments appear to be quite different.¹² In particular, note that in regression [2] (which

¹²An F-test of equality of the coefficients in the two regressions (excluding the intercept terms) rejects the null hypothesis at conventional significance levels (P-value 0.000).

refers to LWYE Experiment 2) the only coefficients that are significant at either the 5% or the 10% confidence level are the ones associated with one—period lagged variables. In contrast, in regression [1] (which refers to LBYE Experiment 1) the coefficients associated with the slope variable and the two—period lagged decision number are also significantly different from zero at the 5% level. Such findings are consistent with our conjecture that subjects in the LWYE environment only look one period back to determine their decision in any given period and react in a purely adaptive way to its outcome by increasing their choice if either they lose the tournament or if they win with a relatively low decision number—i.e. they receive a "relatively high" payoff. Subjects in the LBYE control environment instead, appear to use gradient information about the payoff function and a two—period adjustment rule to guide their search for the peak of the payoff function. 13

Section 4: Conclusion

While it is never wise to generalize on the basis of a small number of experimental results, there are a number of lessons that we can learn from our experiments if they hold up to replication elsewhere. To begin, our results have direct bearing on the methodology of experimental economics. This is true because almost all experiments in economics aim to test static theories using a repeated framework. This is typically justified by the claim that doing an experiment once and only once does not allow subjects to learn. Hence, repetition is recommended to foster learning. In most designs, subjects play games repeatedly and earn payoffs each period—they play in a Learn—While—You—Earn environment. In performing statistical tests on the data generated by these experiments, experimentalists typically use observations collected at the end of the experiment since those supposedly distill all the information learned during the course of

¹³In Merlo and Schotter (1994) we analyze individual behavior in one and two-person tournament experiments and attempt to characterize subjects' learning according to specific rules.

the experiment. Our results, however, indicate that it is exactly in these types of Learn—While-You-Earn environments that learning can be problematic. While such environments can foster convergence when the feedback subjects receive unambiguously pushes them in the direction of the equilibrium (as in the double-oral-auction mechanism), when the feedback inherent in the experiment is noisier, experimentalists might think of using a LBYE payoff structure.

Furthermore, our experiments imply a simple way for experimentalists to proceed. If the experiment performed requires subjects to learn the properties of a static equilibrium, then the extent to which this learning has occurred can be tested using a surprise quiz as performed here. In short, if you want to know what people have learned we suggest simply asking them using a surprise quiz. The only other alternative would be to try to infer their learning (or the behavioral rule of thumb they were using in the experiment) using some type of maximum—likelihood technique as developed by El—Gamal and Grether (1994) (see also Cox, Shachat, and Walker (1995) for an application of this technique to a problem in learning). While such a technique is quite elegant and useful, we are suggesting that there might be instances where no inference needs to be made since we can elicit the actual rules people are using through an appropriately defined surprise quiz. If such a quiz can not be formulated, then clearly inference is the only path left.

Our results also add yet another piece of evidence for the growing view that learning is a situation and institution specific phenomenon (see, e.g., Mookerjee and Sopher (1994)). This view has been recently summarized by Milgrom and Roberts (1991) as follows:

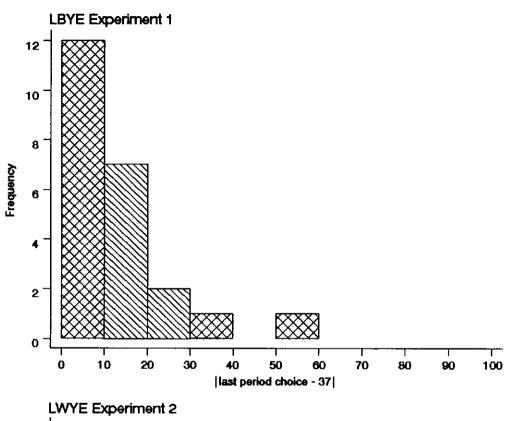
"Taken together, these results [i.e., earlier theoretical results on learning in games, cfr.] raise serious doubts about the validity of Nash equilibrium and its refinements as a general model of the likely outcomes of adaptive learning. More fundamentally, they indicate that the 'rationality' of any particular learning algorithm is situation dependent: An algorithm that performs well in some situations may work poorly in others. Apparently, real biological players tailor rules of thumb to their environment and experience: They learn how to learn. Thus, any single, simple specification of a learning algorithm is unlikely to represent well the behavior that actual players would adopt." (p. 84).

In our view, it is not only that people learn how to learn but they also learn what to learn about and what they learn about is institution specific. If learning is situation or institution dependent, however, it raises the possibility that one would have to construct special learning theories for each and every economic institution—certainly a dismal prospect. This opens the door for experimentalists, however, since if they could classify institutions into equivalence classes across which human learning behavior is similar, then theorists could attempt to characterize these institutions. If successful, a small class of learning theories might be constructed which would explain behavior in a large number of institutions.

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Figure 3.1: Histograms of absolute deviations from 37



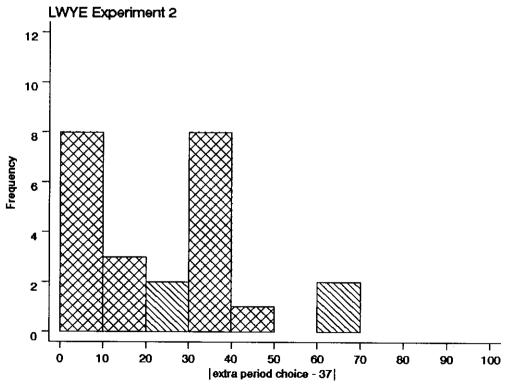


Figure 3.2: Histograms of payoff losses

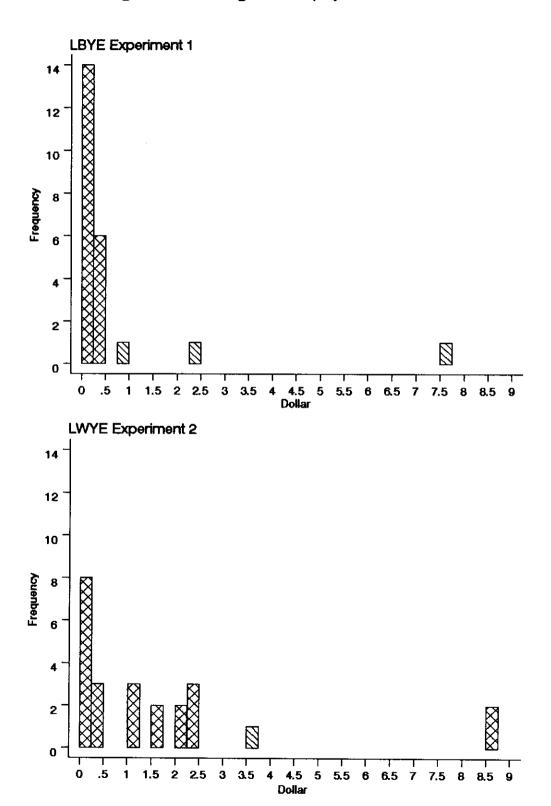
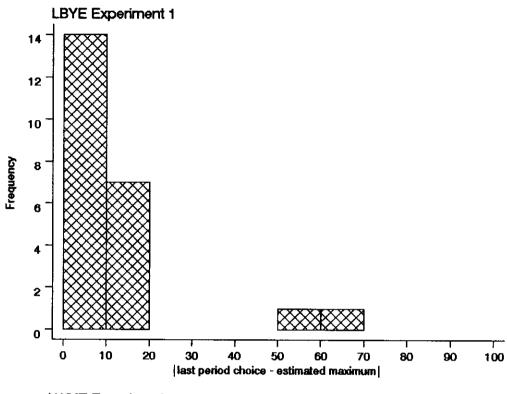


Figure 4.1: Histograms of absolute deviations from estimated maxima



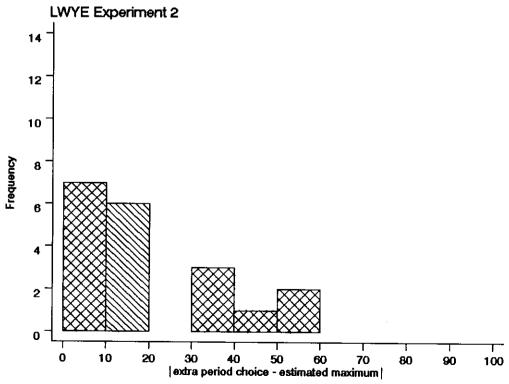


Figure 4.2: Histograms of estimated payoff losses

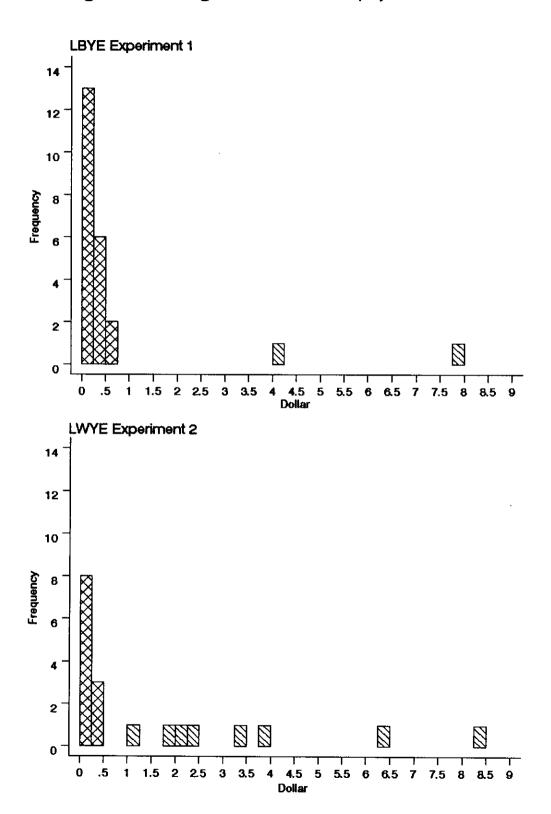


Table 3.1:	: Last Period	d Choices	— LBYE E	Experiment 1
Subject	Last Period	Expected	(2) - 37	Payoff
	Choice	Payoff		1068
1.	40	15.25	က	0.02
2*	38	15.27	1	0.00
က	58	14.30	21	76.0
4	20	14.90	13	0.37
ំ	39	15.26	2	0.01
9	96	7.62	29	7.65
: -	30	15.23	7	0.04
* *	33	15.26	4	0.01
Çî,	23	15.12	14	0.15
10	20	14.90	13	0.37
11.	42	15.21	ro.	90.0
12	15	14.89	22	0.38
13	69	13.02	32	2.25
14*	36	15.27	—	00.0
15*	32	15.25	သ	0.05
16	22	15.10	15	0.17
17*	42	15.21	5	90.0
18	37	15.27	0	00.0
19	20	14.90	13	0.37
20	20	14.90	13	0.37
21*	38	15.27	-	00.0
22*	40	15.25	က	0.03
23	20	14.90	13	0.37
Average	42.61	14.68	11.52	0.59
0				

* indicates that the payoff loss is smaller than \$0.10.

Table 3.	3.2: Extra Period Choices	d Choices	— LWYE E	Experiment 2
Subject	Extra Period	Expected	(2) - 37	Payoff
	Choice	Payoff		Loss
-	65	13.54	28	1.73
2	45	15.13	œ	0.14
က	100	6.55	63	8.72
4	77	11.75	40	3.52
ഹ	0	14.18	37	1.09
9	45	15.13	œ	0.14
7:	41	15.23	4	0.04
∞	89	13.15	31	2.12
6	20	12.87	33	2.40
10	65	13.54	28	1.73
11	45	15.13	∞	0.14
12*	35	15.27	7	00.00
13	0	14.18	37	1.09
14	44	15.16	7	0.11
15	89	13.15	31	2.12
16	0	14.18	37	1.09
17	70	12.87	33	2.40
18	69	13.02	32	2.25
19	20	14.90	13	0.37
20	20	14.90	13	0.37
21	45	15.13	œ	0.14
22	20	14.90	13	0.37
23*	30	15.23	1	0.04
24	100	6.55	63	8.72
Average	51.33	13.57	24.33	1.70

* indicates that the payoff loss is smaller than \$0.10.

Table Subjects'	able 3.3: ects' Cho	Table 3.3: Descriptive Statistics of jects' Choices — LBYE Experime	otive Statist LBYE Exp	atistics of Experiment 1
Subject	Mean	Standard	Median	Interquantile
	Choice	Deviation	Choice	Range
-	45.84	9.93	42	7
2	38.53	23.02	38	29
ಣ	56.52	6.55	57	4
4	45.60	32.92	47	9
ខា	37.25	11.24	39	_
9	52.40	30.22	52	54
7	35.92	13.76	30	10
∞	32.15	17.70	30	17
6	31.40	31.37	29	42
10	54.65	29.25	52	44
11	32.87	18.28	29	24
12	46.44	24.75	40	35
13	64.63	21.00	29	31
14	41.93	28.04	36	38
15	34.51	15.53	32	9
16	28.05	11.52	25	œ
17	42.25	17.48	42	17
18	45.24	16.79	40	14
19	48.93	11.24	20	10
20	48.95	17.89	20	23
21	32.03	15.31	38	œ
22	41.56	20.42	40	10
23	45.99	10.79	47	16
Average	42.77	18.91	41.39	22.35

L		scrij	ve Statist	ics of
Subjects	- 1	Choices — LW	LW YE EXP	Experiment 2
Subject	Mean	Standard	Median	Interquantile
	Choice	Deviation	Choice	Range
_	57.00	11.03	99	15
87	47.73	5.68	20	nọ.
က	73.79	19.37	78	37
.4	61.93	27.47	67	44
2	0.00	0.00	0	0
9	44.11	4.74	45	4
_	9.00	19.47	-	4
∞	64.72	14.38	65	20
g.	48.12	11.33	45	15
10	63.93	6.97	65	S
11	26.99	27.80	6	40
12	53.17	28.22	20	46
13	16.75	34.39	0	0
14	51.86	17.58	45	14
15	59.92	33.05	92	50.5
16	23.47	15.94	35	35
17	61.12	5.32	90	7
18	76.97	17.50	77	22
19	27.49	25.64	35	20
20	39.95	7.57	40	~
21	38.86	11.95	45	2.5
22	50.00	0.00	20	0
23	45.61	21.09	44	21.5
24	69.74	13.05	20	15.5
Average	46.34	15.81	46.33	19.17

_																											
riment 1	Estimated	Pavoff	Loss	0.03	0.02	0.36	0.49	0.28	7.95	0.20	0.20	0.03	0.42	0.64	0.39	4.08	0.01	0.02	0.12	0.03	0.64	0.02	0.32	0.03	0.0	0.16	 0.72
LBYE Experiment	(4) - (2)			က	7	9	17	12	62	7	6	4	16	18	13	51	2	673	&	က	13	S	12	4	-	œ	12.2
	Last Period	Choice		40	38	58	20	39	96	30	33	23	20	42	15	69	36	32	22	42	37	20	20	38	40	20	42.6
Estimated Optimal Choices	Estimate's	Standard	Error	16.9	5.9	4.5	5.0	21.3	5.3	4.9	8.1	11.5	5.4	10.7	8.2	25.1	4.1	7.5	20.0	5.8	4.7	9.2	7.4	9.4	4.2	10.0	9.4
4.1: Estimat	Estimated	Optimal	Choice	43	34	22	33	27	34	37	24	19	34	24	58	18	38	35	30	33	20	45	8 8	34	41	42	34.7
Table 4	Subject			<u>.</u>	5	<u></u>	₹	rc ·	9	<u>.</u>	oc	.	10	11	13	13	14.	15	16	17.	20 }	19*	20	21.	22.	23	 Average

* indicates that the estimated payoff loss is smaller than \$0.10.

Subject	Estimated	Estimate's	Last Period	(4) - (2)	(2) Estimated
	Optimal	Standard	Choice		Payoff
	Choice	Error			Loss
_	46	9.0	65	19	1.12
7	38	20.4	45	~	0.11
es	43	8.6	100	57	8.3
₹*	32	7.8	77	45	3.96
.	43	2.1	45	2	0.10
2	31	80.50	41	10	0.39
œ	*	36.9	89	34	2.22
O	57	1.6	70	13	0.38
10•	63	8.3	65	2	0 0
11	32	5.1	45	13	0.29
12*	34	7.5	35	-	0.0
13	30	5.1	0	30	1.89
14	37	10.7	44	_	0.15
12	32	4.1	89	36	2.48
12	26	3.4	20	14	3.49
6	32	15.7	20	82	0.17
. 02	48	50.7	50	8	0.00
23	33	5.4	30	က	0.01
24	46	10.7	100	54	6.46

(#) subjects 5, 16, 18, 21 and 22 could not have estimated a concave payoff function.
* indicates that the estimated payoff loss is smaller than \$0.10.

APPENDIX A

Table A.1: Coefficients of the Theoretical Payoff Function

payoff function: $\pi = \alpha + \beta e + \gamma e^2$, e: effort level $\frac{\gamma}{-0.0011}$ 0.079 18.94

Table A.2: Least-Squares Estimates of the
Payoff Function — LBYE Experiment 1
(Standard Errors in Parentheses)

	(Standard	Errors in	Parentheses)
Subject	â	β	Ŷ	F-test*
1	12.56	0.371	-0.0043	0.78
	(14.10)	(0.509)	(0.0043)	
2	17.37	0.174	-0.0025	2.06
Ĺ	(1.57)	(0.728)	(0.0007)	
3	-19.57	1.505	-0.0144	2.07
	(23.97)	(0.863)	(0.0077)	
4	18.25	0.161	-0.0024	7.96
<u></u>	(1.20)	(0.058)	(0.0005)	
5	19.40	0.160	-0.0029	1.82
	(5.67)	(0.231)	(0.0021)	
6	17.06	0.193	-0.0028	11.13
	(1.68)	(0.075)	(0.0007)	
7	14.22	0.373	-0.0050	1.20
	(3.41)	(0.173)	(0.0021)	
8	19.78	0.153	-0.0032	1.93
	(1.99)	(0.117)	(0.0016)	
9	20.70	0.067	-0.0018	5.64
	(1.08)	(0.963)	(0.0006)	
10	17.10	0.183	-0.0026	10.35
	(1.63)	(0.067)	(0.0006)	
11	20.24	0.131	-0.0027	2.55
12	(2.23)	(0.116)	(0.0013)	
12	18.50	0.160	-0.0028	6.93
13	(2.19) 21.63	(0.094) 0.076	(0.0009) -0.0021	11.10
13		(0.147)		11.10
14	(4.29) 15.90	0.252	(0.0012)	7.22
1.4	(1.55)	(0.073)	-0.0033 (0.0007)	(.22
15	17.87	0.205	-0.0029	1.11
10	(2.24)	(0.107)	(0.0013)	1.11
16	18.62	0.139	-0.0023	0.54
**	(3.95)	(0.197)	(0.0019)	0.04
17	16.26	0.263	-0.0034	1.48
1 -	(2.54)	(0.110)	(0.0012)	1.70
18	8.42	0.483	-0.0048	2.95
	(4.18)	(0.160)	(0.0014)	2.00
19	13.06	0.338	-0.0037	0.65
1	(7.12)	(0.253)	(0.0022)	3.55
20	16.33	0.220	-0.0029	1.56
	(2.90)	(0.110)	(0.0010)	-
21	17.43	0.157	-0.0023	0.64
	(1.76)	(0.083)	(0.0011)	
22	15.93	0.274	-0.0034	3.38
	(1.61)	(0.067)	(0.0007)	Ì
23	15.99	0.231	-0.0028	0.24
	(5.15)	(0.231)	(0.0027)	

^{*} $H_0: \hat{\alpha} = 18.94, \ \hat{\beta} = 0.079,$ $\hat{\gamma} = -0.0011; \ F_{.95}(3,72) = 2.15.$

Table A.3: Least-Squares Estimates of the Payoff Function — LWYE Experiment 2 (Standard Errors in Parentheses)

	Comment	LAIOIS III	i di Cittleses	<u>, </u>
Subject	â	β	Ŷ	F-test*
1	9.79	0.410	0.0044	2.29
	(10.08)	(0.452)	(0.0045)	
2	15.67	0.244	-0.0032	0.08
	(8.36)	(0.427)	(0.0058)	
3	12.79	0.306	-0.0035	21.74
	(5.24)	(0.159)	(0.0011)	
4	18.29	0.166	-0.0026	12.61
	(2.20)	(0.084)	(0.0007)	
5**	-	-	-	_
6	-33.97	2.579	-0.0301	0.83
	(36.93)	(1.718)	(0.0199)	
7	18.55	0.344	-0.0055	0.84
	(0.78)	(0.247)	(0.0037)	
8	17.32	0.168	-0.0025	5.69
	(11.86)	(0.354)	(0.0025)	
9	11.27	0.304	-0.0027	0.85
	(13.80)	(0.545)	(0.0051)	
10	-12.94	0.987	-0.0078	3.27
	(21.88)	(0.624)	(0.0044)	
11	17.77	0.147	-0.0023	3.38
	(0.87)	(0.057)	(0.0007)	
12	17.98	0.192	-0.0029	8.38
	(2.35)	(0.101)	(0.0009)	
13	18.44	0.165	-0.0027	13.33
	(0.52)	(0.067)	(0.0007)	
14	15.45	0.255	-0.0035	5.33
	(5.01)	(0.175)	(0.0014)	
15	18.51	0.168	-0.0026	15.53
	(1.11)	(0.056)	(0.0006)	
16**				_
17	60.65	2.830	-0.0251	3.42
	(60.70)	(2.010)	(0.0165)	_
18**			<u></u>	
19	19.11	0.061	-0.0009	0.07
	(0.89)	(0.123)	(0.0022)	
20	15.55	0.233	-0.0024	0.53
	(4.93)	(0.333)	(0.0057)	
21**				_
22**		_	-	
23	18.20	0.213	-0.0033	2.20
	(1.92)	(0.090)	(0.0010)	0
~ .	12.88	0.264	-0.0029	6.80
24	12.88			
24	(6.25)	(0.194)	(0.0015)	0.00

^{*} $H_0: \hat{\alpha}=18.94, \, \hat{\beta}=0.079, \ \hat{\gamma}=-0.0011; \, F_{.95}(3,72)=2.15.$ ** Subjects 5, 16, 18, 21, and 22 could not have estimated a concave payoff function.

Table A.4: Delta Method

payoff function: $\pi = \alpha + \beta e + \gamma e^2$, e: effort level true optimal effort level: $e^* = -\beta/2\gamma$ estimated optimal effort level: $\hat{e}^* = -\hat{\beta}/2\hat{\gamma}$

estimated optimal effort level: $\hat{e}* = -\hat{\beta}/2\hat{\gamma}$ estimate's standard error: $\sigma_{\hat{e}*} = \left[(1/4\hat{\gamma}^2)\sigma_{\hat{\beta}}^2 - (\hat{\beta}/2\hat{\gamma}^3)\sigma_{\hat{\beta}\hat{\gamma}} + (\hat{\beta}^2/2\hat{\gamma}^4)\sigma_{\hat{\gamma}}^2 \right]^{1/2}$

APPENDIX B: Instructions for Experiment 1

Introduction

This is an experiment about decision making. The instructions are simple, and if you follow them carefully and make good decisions, you could earn a considerable amount of money, which will be paid to you in cash.

Specific Instructions

As you read these instructions you will be in a room with a number of other subjects. Each subject has been randomly assigned an ID number and a computer terminal. The experiment consists of 75 decision rounds. In each decision round you will be paired with a computerized subject which has been programmed to make the same decision in every round. The computerized subject randomly matched with you will be called your pair member. Your computerized pair member will remain the same throughout the entire experiment.

Experimental Procedure

In the experiment you will perform a simple task. Attached to these instructions is a sheet called your "Decision Cost Table". This sheet shows 101 numbers from 0 to 100 in column A. These are your decision numbers. Associated with each decision number is a decision cost, which is listed in column B. Note that the higher the decision number chosen, the greater is the associated cost. Your computer screen should look as follows as you entered the lab:

PLAYER #__

ROUND DECISION # RANDOM # TOTAL # COST EARNINGS

In each decision round the computer will ask you to choose a decision number. Your computerized pair member will also choose a decision number. Remember that it will always choose the same decision number, which will be 37 in each decision round. You of course, are free to choose any number you wish among those listed in column A of your "Decision Cost Table". Therefore, in each round of the experiment, you and your computerized pair member will each select a decision number separately (and you know that it will always choose 37). Using the number keys, you will enter your selected number and then hit the Return (Enter) key. To verify your selection, the computer will then ask you the following question:

Is ___ your decision number ? [Y/N]

If the number shown is the one you desire, hit the Y key. If not, hit the N key and the computer will ask you to select a number again. After you have selected and verified your number, this number will be recorded on the screen in column 2, and its associated cost will be recorded in column 5. After you have selected your decision number, the computer will ask you to generate a random number. You do this by hitting the space bar (the long key at the bottom of the keyboard). Hitting the bar causes the computer to select one of the 81 numbers that fall between -40 and +40 (including 0). Each of these 81 numbers has an equally likely chance of being chosen when you hit the space bar. Hence, the probability that the computer selects, say, +40 is the same as the probability that it selects -40, 0, -12 or +27. Another random number (again between -40 and +40) will be automatically generated for your computerized pair member as well. The processes that generate your random number and the random number assigned to your computerized pair member are independent - i.e., you should not expect any relationship between the two random numbers generated to exist. After you hit the space bar, the computer will record your random number on the screen in column 3.

Calculation of Payoffs

Although the experiment will have 75 rounds in total, your actual payment will just be determined by your earnings in the last round of the experiment (round 75). In each of the first 74 rounds, however, you will have the chance of observing what your payoff would have been as a consequence of your choice, the choice of your computerized pair member (37), and the two random numbers generated. These hypothetical payments will not be paid, however. Only your period 75 payment will count. Your payment (either real or hypothethical) in each decision round will be computed as follows. After you select a decision number and generate a random number, the computer will add these two numbers and record the sum on the screen in column 4. We will call the number in column 4 your "Total Number". The computer will do the same computation for your computerized pair member as well. The computer will then compare your Total Number to that of your computerized pair member. If your Total Number is greater than your computerized pair member's Total Number, then you will receive the high fixed payment of 29 Fr., in a fictitious currency called Francs. If not, then you will receive the low fixed payment 17.2 Fr. Whether you receive the fixed payment 29 Fr. or the fixed payment 17.2 Fr. only depends on whether your Total Number is greater than your computerized pair member's Total Number. It does not depend on how much bigger it is. The Francs will be converted into dollars at the conversion rate to be stated below. The computer will record (on the screen in column 6) which fixed payment you receive. If you receive the high fixed payment (29 Fr.), then "M" will appear in column 6. If you receive the low fixed payment (17.2 Fr.), "m" will appear. After indicating which fixed payment you receive, the computer will subtract your associated decision cost (column 5) from this fixed payment. This difference represents your (actual or hypothetical) earnings for the round. The amount of your earnings will be recorded on the screen in column 6, right next to the letter ("M" or "m") showing your fixed payment.

Continuing Rounds

After round 1 is over, you will perform the same procedure for round 2, and so on for 75 rounds. In each round you will choose a decision number and generate a random number by pressing the space bar. Your Total Number will be compared to the Total Number of your computerized pair member, and the computer will calculate your earnings for the round. Your final earnings will depend only on your decisions in round 75. When that round is completed, the computer will ask you to press any key on its keyboard. After you do this, the computer will convert your Francs earnings in round 75 in Dollars at the rate of \$.75 per Franc. We will then pay you this amount.

Example of Payoff Calculations

Suppose that the following occurs during one round: pair member A_2 chooses a decision number of 60 and generates a random number of 10, while computerized pair member A_1 selects a decision number of 37 and gets a random number of 5. Pair member A_2 would then receive the high fixed payment of 29 Fr. From this fixed payment, A_2 would subtract 7.2 Fr. (the cost of decision number 60). A_2 's earnings for that round would then be 21.8 Fr. (i.e., 29 Fr. -7.2 Fr.). Note that the decision cost subtracted in column 5 is a function only of your decision number; i.e., your random number does not affect the amount subtracted. Also, note that your earnings depend on the following: the decision number you select (both because it contributes to your Total Number and because it determines the amount - i.e., your Decision Cost - to be subtracted from your fixed payment), your computerized pair member's pre-selected decision number (37), your generated random number, and your computerized pair member's generated random number.

Decision Cost Table

Column A Decision Number	Column Decision (Francs)	B Column A Decision Number	Column B Cost of Decision (Francs)	Column A Decision Number	Column B Cost of Decision (Francs)
0	0.00	36	2.59	72	10.37
1	0.00	37	2.74	73	10.66
$ar{2}$	0.01	38	2.89	74	10.95
3	0.02	39	3.04	75	11.25
4	0.03	40	3.20	76	11.55
5	0.05	41	3.36	77	11.86
6	0.07	42	3.53	78	12.17
7	0.10	43	3.70	79	12.48
8	0.13	44	3.87	80	12.80
9	0.16	45	4.05	81	13.12
10	0.20	46	4.23	82	13.45
11	0.24	47	4.42	83	13.78
12	0.29	48	4.61	84	14.11
13	0.34	49	4.80	85	14.45
14	0.39	50	5.00	86	14.79
15	0.45	51	5.20	87	15.14
16	0.51	52	5.41	88	15.49
17	0.58	53	5.62	89	15.84
18	0.65	54	5.83	90	16.20
19	0.72	55	6.05	91	16.56
20	0.80	56	6.27	92	16.93
21	0.88	57	6.50	93	17.30
22	0.97	58	6.73	94	17.67
23	1.06	59	6.96	95	18.05
24	1.15	60	7.20	96	18.43
25	1.25	61	7.44	97	18.82
26	1.35	62	7.69	98	19.21
27	1.46	63	7.94	99	19.60
28	1.57	64	8.19	100	20.00
29	1.68	65	8.45		
30	1.80	66	8.71		
31	1.92	67	8.98		
32	2.05	68	9.25		
33	2.18	69	9.52		
34	2.31	70	9.80		
35	2.45	71	10.08		