

**ECONOMIC RESEARCH REPORTS**

HETEROGENEOUS INFORMATION AND THE  
REAL BUSINESS CYCLE:  
A THEORETICAL AND EMPIRICAL ANALYSIS

by

Boyan Jovanovic

and

Robert A. Shakotko

R.R. #83-17

August 1983

**C. V. STARR CENTER  
FOR APPLIED ECONOMICS**



**NEW YORK UNIVERSITY  
FACULTY OF ARTS AND SCIENCE  
DEPARTMENT OF ECONOMICS  
WASHINGTON SQUARE  
NEW YORK, N.Y. 10003**

HETEROGENEOUS INFORMATION AND THE  
REAL BUSINESS CYCLE:  
A THEORETICAL AND EMPIRICAL ANALYSIS\*

Boyan Jovanovic  
New York University

and

Robert A. Shakotko  
Columbia University

August 1983

\* We thank Jim Albrecht, Clive Bull, Roger Gordon and Jim Heckman for helpful comments. They are not responsible for the views expressed here.



## ABSTRACT

This paper formulates a model of the aggregate labor market, and tests some of its implications using quarterly post-war data. The paper does not aim to explain the autoregression properties of the business cycle. Rather, it takes a step toward a systematic study of agents' information structures as they relate to the determination of aggregates, by analyzing an example in which agents' information structures differ. This is important because different information structures hold different implications for the nature of optimal government intervention.



## 1. Introduction

This paper formulates a model of the aggregate labor market, and tests some of its implications using quarterly post-war data. The paper does not aim to explain the autoregressive properties of the business cycle. Rather, it analyzes a model in which agents' information sets differ, and finds it to be consistent with certain properties of the aggregate time series that are hard to explain with a common information model.

Models in which agents have common information have greatly increased our understanding of business cycles. Recent examples of such models are Kydland and Prescott (1982) and Long and Plosser (1983). In these models, fluctuations around trend in aggregate variables represent agents' optimal responses to aggregate shocks that follow well-specified stochastic laws of evolution. When externalities are absent, equilibria in such models correspond to optimal solutions to the problem of maximizing the welfare of the representative individual. For a general treatment, see Prescott and Mehra (1980).

The power of these models stems to a large measure from their common information assumption: Models with common information about aggregate shocks are generally more manageable than models in which agents have heterogeneous information sets about such shocks. The success of common information models in predicting important features of cycles, such as the co-movement of aggregates and their relative amplitudes, represents a powerful argument against interventionist policy designed to smooth out cycles, because in these models, cycles are Pareto-optimal.

This optimality hinges, however, not only on the assumed absence of externalities, but also on the assumption of common information. Laffont (1983) shows that in a class of heterogeneous information models, the competitive outcome can usually be Pareto-dominated by a mechanism which is incentive-compatible given the information that agents have. The empirical validity of the common information assumption is therefore of great relevance for policy formulation.

Now consider the following simple example involving two different information structures. Let  $y = (y_1, y_2)'$  be two endogenous variables driven by just one exogenous shock  $x$ . Suppose a common information model (in which agents observe  $x$  perfectly) yields the decision rules  $y = a + bx$ . (Here  $y_1$  and  $y_2$  may denote Robinson Crusoe's investment and labor supply, and  $x$  may be the amount of rainfall.) Letting starred values denote deviations from expected values (as of the end of the previous period) yields  $y^* = bx^*$ . Suppose that we determined that  $E y_1^* y_2^* = 0$ . Since

$$E(y^* y^{*'}) = \sigma_x^2 \begin{bmatrix} b_1^2 & b_1 b_2 \\ b_1 b_2 & b_2^2 \end{bmatrix},$$

this restriction implies that either  $b_1 = 0$  or  $b_2 = 0$ , so that at most one of the elements of the variance-covariance matrix of the endogenous variables can be non-zero. Now consider the same setup, but with agents who determine  $y_1$  being different from the agents who determine  $y_2$ . If the agents who determine  $y_2$  must "commit" their decision before they can observe  $x$  (or an imperfect signal of  $x$ ) then the same restrictions on  $E[y^* y^{*'}]$  follow.

The point of this example is that a zero restriction on an off-diagonal entry in the matrix  $E y^* y^{*'}$  can be explained in two different ways. In a common information model, it represents a nonlinear restriction on the parameter vector, while in a heterogeneous information model, it can be generated by a particular assumption about the nature of the informational asymmetry. The same distinction applies to models in which both  $y$  and  $x$  are of higher dimension. With this in mind, consider the following matrix of correlations for the innovations in five variables given in Table 1. We are unable to conceive of a manageable common information model that can account for the pattern of correlations shown in this table. Why, for example, should the unemployment surprise be so strongly correlated with the productivity and investment surprises, and yet be quite unrelated to the labor force participation surprise? Or, why should the participation and investment surprises be unrelated?<sup>1</sup> We found that we could not resort to measurement error as an explanation for the absence of a higher number of significant correlations because each surprise is strongly correlated with at least one other surprise; one would expect measurement error to eliminate all correlations.

Our explanation of this correlation matrix assumes a two-factor model (i.e., two aggregate shocks, so that  $x$  is two dimensional), and two types of participants, workers and employers. The employers know the aggregate demand shock, while workers know the aggregate supply shock. Given their private information, employers commit on investment while workers commit on participation. After this commitment has taken place, wages and employment are determined. In other words, the model relies on a



	Productivity	Investment	Labor-Force Participation	Real Wages	Unemployment
Productivity	1.0				
Investment	.38** (.0001)	1.0			
Labor-Force Participation	-.14 (.107)	-.04 (.67)	1.0		
Real Wages	.04 (.65)	.06 (.52)	-.24* (.004)	1.0	
Unemployment	-.62** (.0001)	-.48** (.0001)	.08 (.38)	-.11 (.19)	1.0

Table 1 : Correlation Coefficients of the Innovations

---

Notes: Residuals generated from vector autoregression with four lags, and with 136 quarterly observations.

Autoregressions include linear and quadratic time trends, and quarterly dummies.

In brackets is the probability of a type two error on the zero correlation hypothesis.

\* Significant at the one percent level.

\*\* Significant at the one-tenth of one percent level.

particular information structure, and a particular sequencing of decisions. The model correctly predicts each of the four starred correlations, and is consistent with all but one of the others.<sup>2</sup> We can not reject the hypothesis that the information contained in the demand-shock surprise is fully private to the employers, and that the labor-supply surprise is fully private to the workers (Table 5).

Our welfare results (contained in the final section of the paper) are tentative, although they show the need for further study of information structures. First, all equilibria in the model are shown to be suboptimal relative to the first-best (full-information) outcome. This, of course, implies nothing for interventionist policy. To make some progress on the policy issue, we simplify the market setup to a two-player game which, we believe, retains the model's essential ingredients. We then show that for certain configurations of the parameters, intervention can make everyone better off. The optimal interventionist policy is not contingent on the shocks, and so imposes no information requirements on the authorities: the example we consider is an unemployment insurance benefit, which improves welfare by increasing labor force participation to its first-best level. We also show that with a common (fully informed) information structure, policy has no role because equilibrium coincides with the first-best outcome. The point of the example is not that equilibria with more information are superior to equilibria with less information (this happens to be true in our example but is not true generally -- see Hirshleifer (1971), for example), but rather that intervention can improve welfare under a particular information structure, a structure that is consistent with the evidence presented in Table 1.

## 2. The Model

This section describes the model by taking up, in turn, the labor-force participation decision, the investment decision, and the wage-setting decision. The section ends with a definition of equilibrium and the proof of its existence.

The participation decision: The worker's value of leisure is  $u + \varepsilon$ . Its first component,  $u$ , is common to all workers, while the second,  $\varepsilon$ , is specific to the worker and is assumed to be independent over workers. In other words,  $u$  is the population average value of leisure, while  $\varepsilon$  is a worker's deviation from that average.

At the beginning of the period, the worker decides either to participate in the labor market or to stay out of it. Having made this choice, he is committed to it for the rest of the period. We shall assume, however, that in making the choice the worker knows  $u$ , but not  $\varepsilon$ . For example,  $\varepsilon$  could reflect the non-pecuniary characteristics of the job that the worker gets if he decides to participate, or it could be the result of other influences on the value of leisure that reveal themselves only after he has committed himself. Since  $\varepsilon$  has zero mean, the expected value of not participating is just  $u$ , and it is equal for all workers. Other than  $u$ , no information is of any use in assessing the value of not participating.

The value of participating, however, will depend on information about the state of the demand for labor, information on the availability of jobs, and information on expected wages. Let  $\theta$  stand for all the relevant information (other than  $u$ ) that the worker has (a precise definition of  $\theta$  will be given shortly), and let  $n$  denote the fraction of workers that participate in

the labor market. The value of participation will depend not only on  $u$  and  $\theta$ , but also on  $n$ ; we shall denote this value by  $v(u, \theta, n)$ .

All workers are identical when they make the participation decision. Equilibrium requires that if  $n \in (0, 1)$ ,

$$u = v(u, \theta, n), \tag{1}$$

so that if some participate while others do not, all are indifferent between the two options. Note that equation (1) is the outcome of a comparison of one-period returns only; if he participates, the worker must leave the labor market at the end of the period; the cost of participating must be incurred again should he decide to participate again the following period.

Equation (1) can be viewed as a condition of an asymmetric Nash equilibrium in which identical workers take different actions. Alternatively, one can view it as a condition of a symmetric mixed-strategy equilibrium in which each worker enters the labor market with probability  $n$ ; if each worker is of measure zero, this results in exactly a fraction  $n$  entering,<sup>3</sup>

Embedded in  $v$  is an assumed fixed cost,  $c$ , of participating in the labor market. This cost is incurred in each period that the worker decides to participate. Cogan (1981) estimates that the costs of labor force participation are important, at least for married women.<sup>4</sup> The mathematical representation of  $v$  will be given in equation (10).

The investment decision: Investment decisions are made by employers who know the state of labor productivity,  $x$ , but who do not know  $u$ . The amount of physical capital jointly created at the beginning of the period by all the employees is denoted by  $k$ . The cost, in terms of the consumption good, of

creating an additional machine is assumed to depend on the total number created and is denoted by  $q(k)$ , a strictly increasing function of  $k$ , with  $q(0) = 0$  and  $q(k) \rightarrow \infty$  as  $k \rightarrow \hat{k} < \infty$ .

While employers do not know  $u$ , they do know  $\theta$ . Let  $\pi(x, \theta, k)$  be the expected value of providing another machine, based on what the employer knows. Equilibrium requires that investment take place up to the point at which

$$\pi(x, \theta, k) = q(k), \quad (2)$$

Each employer can supply at most one unit of capital; being of measure zero, he can have no influence on the cost  $q(k)$ .<sup>5</sup> As we shall show below,  $\pi$  is decreasing in  $k$ : Capital's expected marginal earnings decrease at the aggregate level. The increase in the marginal cost of capital ( $q'(k) > 0$ ) is caused by diminishing returns in the production of capital goods. Alternatively, we could think of  $q(k)$  as the investment cost of the marginal employer, with the best employer always entering first.

As with equation (1), the future does not matter. Capital depreciates entirely by the beginning of the following period. Investment and capital are the same thing in our model. Since capital is durable, this assumption is obviously unrealistic. It would, of course, lead to large bias if our aim was to explain the auto-covariance properties of the business cycle (i.e. the coefficients from the vector autoregression in Table (A.9)). Our aim, however, is to interpret qualitatively the contemporaneous correlation of surprises, and here the investment series should do about the same as the capital series since capital is typically constructed as a weighted average of lagged investments, so that a capital surprise is equal to the

investment surprise.

The marginal cost of capital,  $q(k)$ , is not a market price. It is a physical cost, incurred by the employer directly. No market for savings exists in this model, as each person immediately consumes his earnings. Our theoretical explanation for the regularities described in the introductory section relies on asymmetric information between workers and investors about the aggregate state  $(x,u)$ . It is possible that the asymmetry could disappear if, at the start of the period, both sides could observe the rate of interest; this rate would depend on both  $x$  and  $u$  and would, given monotonicity, reveal  $x$  and  $u$  to anyone who knew only one of the variables.<sup>6</sup> For each side of the market, there is only one aggregate source of uncertainty -- each has a perfect signal on the other source. In the nonmonetary version of their model which has only one aggregate (productivity) shock, Grossman and Weiss (1982) find that the real rate reveals the aggregate shock perfectly. The same is true when an aggregate asset market is added to other simple aggregate shock models such as Lucas' (1975) model [see Karni (1980)]. One can limit the amount of information conveyed by the aggregate asset market by adding more aggregate disturbances (see Barro (1980), and section 3 of Grossman and Weiss (1982)); clearly the larger the importance of "other" shocks, the noisier will be the signal conveyed by interest rates. Although we do not explicitly model the savings market and the processing of noisy signals, an erratic monetary policy may severely limit the value of the nominal rate as a predictor of the real rate (for reasons spelled out by Grossman and Weiss, 1982). Moreover, our empirical results suggest that the information-pooling role of prices is slight -- we can not reject the hypothesis that none of it takes place -- and that it can safely be ignored when analyzing quarterly data for the aggregate variables that we consider (see Tables 5, 6 and 7). Finally, note that from the point of view of revelation, the real wage is not useful -- the wage is determined only after commitment has taken place.

Wage and employment determination: The output technology is one of fixed proportions. We shall choose capital units in such a way that one unit of capital must combine with one worker to produce  $x$  units of output. Assume also that the total number (i.e. measure) of workers is unity, so that  $n$  is both the participation rate and number participating. Then  $k$  is both the number of machines and the number of machines per worker (but not per participant),

During the period, each employer can make contact with at most one worker, and each worker can make contact with at most one employer. When contact is made, the employer makes a wage offer to the worker. If the worker accepts the offer, the match lasts until the end of the period, at which time it is dissolved. If the worker rejects the offer, he is unemployed for the remainder of the period.<sup>7</sup>

Let  $n$  and  $k$  be given. With this search technology,

$\min(1, k/n)$  = probability that a given worker will contact some employer,

and

$\min(1, n/k)$  = probability that a given employer will contact some worker.

So, if  $k < n$ , some workers get no wage offers and will be unemployed as a result. If  $k > n$ , all workers get wage offers, while some employers end up with unfilled vacancies. We should emphasize that our results do not depend on the particular assumptions about the search process. One could, for instance, allow more than one "interview" per period without changing anything of substance.

An incentive exists for long-term commitments to develop; workers and employers would wish to be insured against the danger of being committed to the market, but being left without a partner. For instance, one can envision long-term contracts between the employer and the worker that offer each the right of first refusal to the other's services. Although one can conceive of an equilibrium in which a fraction of the labor market is organized in this way, the key point is that without the possibility of pre-

commitment communication between employers and workers, any equilibrium will imply the same qualitative features for the aggregate investment and participation series as the ones that we study. It is well known, of course, that the degree of long-term commitment to the labor market differs across different members of the labor force, but it is not the objective of this paper to explain such differences.

Before looking at wage determination, let us review the informational assumptions. When making the participation decision, the worker knows  $u$ , but not  $\varepsilon$ . By the time he gets a wage offer, however, he has learned what his  $\varepsilon$  is, and he accepts a wage offer of  $w$  if

$$w > u + \varepsilon. \quad (3)$$

The employer, on the other hand, does not know  $u$  when deciding whether to invest. But by the time he contacts a worker and makes a wage offer, he has learned what  $u$  is, and his wage offer will reflect this. For instance, the employer could learn about  $u$  by observing the number participating, which is a monotone function of  $u$  (see Theorem 2).<sup>8</sup> The employer does not, however, know the value of  $\varepsilon$ .

Let  $F(\varepsilon)$  be the distribution of  $\varepsilon$ . Equation (3) then implies that the worker will accept the wage offer with probability  $F(w - u)$ . If the worker accepts, the employer's profit is  $x - w$ . If he rejects, the profit is zero. Thus the employer will choose the wage so as to maximize expected profits:

$$\max (x - w)F(w - u) \equiv \hat{\pi}(x, u). \quad (4)$$

Note that  $\hat{\pi}(x, u)$  is the expected profit conditional on the employer's having contacted a worker.

The necessary conditions for a maximum are

$$0 = -F(w - u) + (x - w)f(w - u), \quad (5)$$

and



$$0 > -2f(w - u) + (x - w)f'(w - u). \quad (6)$$

In general, these two conditions may hold at several local maxima, and for some distributions  $F(\varepsilon)$ , they may not have a solution. A sufficient condition for a unique solution to exist is that  $F(\varepsilon)$  be weakly log-concave in  $\varepsilon$ .<sup>9</sup> This is not a strong requirement; the normal, the logistic, the  $t$  distribution, and many other distributions are in this class (see Pratt (1981)).

Differentiating in eq. (5), one obtains

$$\frac{\partial w}{\partial x} = \frac{f}{2f - (x - w)f'} > 0 \quad \text{and} \quad \frac{\partial w}{\partial u} = \frac{f - (x - w)f'}{2f - (x - w)f'} > 0 \quad (7)$$

from which it follows that  $\partial w/\partial x > 0$ , and  $\partial w/\partial u < 1$ . Thus, a unit increase in labor productivity brings about an increase in wages, but by less than one unit. Eq. (7) also implies that

$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial u} = 1. \quad (8)$$

Note that the optimal wage, which we shall write as  $w(x, u)$ , does not depend on  $\theta$ ; it depends, rather, only on the current demand and supply shocks.<sup>10</sup> Once the current shocks are controlled for, past shocks (or proxies for past shocks, such as lagged employment) are predicted to have no effect on the current wage. In this sense, our specification of the wage is consistent with the results of Neftci (1978) and of Sargent (1978), who find no evidence that the prediction of current wages can be improved

by using lagged values of aggregate employment (in addition to lagged values of the wage itself). On the other hand, eq. (5) implies that the wage depends on current demand and supply shocks, which means that the wage should be related to current values of variables such as investment and labor force participation.

Our assumptions about the contracting process yield a unique division of the rents associated with the worker-employer contact.<sup>11</sup> The contracting process is a special case of a bidding game (see, e.g., Wilson (1977)) in which the employer has only one bid, and wins a prize of  $x - w$  if his offer exceeds  $u + \varepsilon$ ; the worker's share of the rent is  $w - u - \varepsilon$ . The precise way in which the rent is split will depend on the form of  $F(\cdot)$ : a rearrangement of eq. (5) yields the following expression for the wage:

$$w = x - F(w - u)/f(w - u) ,$$

which implies that  $w < x$ , so that the employer's share of the rent is always positive. Given a particular law of motion for  $x$  and  $u$ , different  $F$ 's will imply different cyclical properties for the wage.<sup>12</sup>

Note that the average ex-post rent to a contact is  $x - u$ . Our major results would remain unaffected if, instead of the rule implied by eq. (4), we had assumed some other, possibly arbitrary rule governing the division of this average rent. Our empirical results do make use of the restriction in eq. (8), but this restriction holds for any rent-sharing rule that specifies that the wage should equal  $u + g(x - u)$ , where  $g$  is some rent-division function. Ideally, of course, the wage would depend on  $\varepsilon$  as well

(see the related discussion in section 4), but  $\varepsilon$  is private information to the worker, and there seems to be no incentive-compatible mechanism that would induce him to reveal it truthfully.

Having defined both  $x$  and  $u$ , we may now define the stochastic process that they follow. This process is assumed to be  $n^{\text{th}}$  order Markov, with cumulative distribution function  $H(x,u;\theta)$ , where  $\theta \in \Theta$  is a  $(2 \times n)$ -dimensional vector of lagged  $x$ 's and  $u$ 's. Note that  $\theta$  represents information that is common to both workers and employers at the start of the period. But workers do not know the current  $x$  and employers do not know the current  $u$  at the start of the period, so that one must define the following two conditional distributions:

$$H^x(x;u,\theta) \equiv \frac{H(x,u;\theta)}{\int H(dx,u;\theta)} ,$$

and

$$H^u(u;x,\theta) \equiv \frac{H(x,u;\theta)}{\int H(x,du;\theta)} .$$

The first conditional distribution,  $H^x$ , represents workers' beliefs about  $x$ , given knowledge of  $u$  and  $\theta$ . The second distribution,  $H^u$ , represents employers' beliefs about  $u$ , given knowledge of  $x$  and  $\theta$ .<sup>13</sup> We emphasize that our analysis remains valid if either  $x$  or  $u$  (or both) are degenerate random variables (i.e., constants). Indeed, the hypothesis that one of the factors is degenerate is tested below, and it is rejected.

Even though the model is static, one still gets persistence in the effects of aggregate shocks. The source of the persistence is the agents' incomplete knowledge of the current values of  $x$  and  $u$ , so that their past values (of which the relevant ones are in  $\theta$ ) matter. Nevertheless, the dynamics of the model are generated exogenously in the sense that if  $x$  and  $u$  were serially independent random variables, the endogenous variables would also be serially independent.

While autocorrelation of exogenous shocks is certainly not the only source of persistence, it is likely that such shocks are strongly autocorrelated. The price of imported oil, for instance, exhibits strong serial correlation and it is one of the determinants of labor productivity (and possibly of the value of leisure as well). The state of the technology, which also influences labor productivity, is also a serially correlated variable. The value of leisure is similarly driven by forces which themselves are autocorrelated: the age structure of the population, the nature of the government's labor-market policies, the propensity of women to participate in the labor market, and so forth. Because such correlation in the levels of exogenous shocks is likely to be important, we allow for it by letting  $H(\cdot)$  depend on  $\theta$ . On the other hand, our explanation for the observed pattern of contemporaneous correlations of the innovations does not require that these shocks be autocorrelated; all one needs is that agents be asymmetrically informed about the current values of  $x$  and  $u$ .

Equilibrium: Employers make investment decisions before they can observe how many workers participate. They do know, however, that the participation

decision is governed by eq. (1). Let  $n(u, \theta)$  be the solution for  $n$  to eq. (1). (If, for some  $(u, \theta)$ ,  $u < v(u, \theta, n)$  for all  $n \in [0, 1]$ , set  $n(u, \theta) = 1$ . Similarly, if for some  $(u, \theta)$ ,  $u > v(u, \theta, n)$  for all  $n \in [0, 1]$ , set  $n(u, \theta) = 0$ .) The expected return to investment, based on what employers know at the beginning of the period is

$$\pi(x, u, k) = \int \hat{\pi}(x, u) \min[1, n(u, \theta)/k] H^u(du; x, \theta) . \quad (9)$$

Similarly, let  $k(x, \theta)$  solve eq. (2) for  $k$ . (Since  $q(0) = 0$ , and since  $\pi$  is bounded, a unique solution for  $k$  exists.) Although workers do not know how much will be invested, they do know the decision rule  $k(x, \theta)$  used by employers. Therefore,

$$\begin{aligned} v(u, \theta, n) = & -c + \int \{ \int \max[w(x, u), u + \varepsilon] F(d\varepsilon) \min[1, k(x, \theta)/n] \\ & + u \{1 - \min[1, k(x, \theta)/n]\} \} H^x(dx; u, \theta) . \end{aligned} \quad (10)$$

Recall that  $\min(1, k/n)$  is the probability that the worker will find a vacancy. If he does not find one, he can expect to end up with utility  $u$ , because  $\varepsilon$  has zero mean. If he does find a vacancy, he will get either the wage,  $w$ , or his value of leisure,  $u + \varepsilon$ , whichever is larger. A useful simplification of eq. (10) is the following:

$$\begin{aligned} v(u, \theta, n) = & -c + u + \int \int_{-\infty}^{w(x, u) - u} [w(x, u) - u - \varepsilon] F(d\varepsilon) \\ & \cdot \min[1, k(x, \theta)/n] H^x(dx; u, \theta) \end{aligned} \quad (11)$$

On the right-hand side of this expression,  $u$  is what the worker can guarantee himself, in expected value, if he participates. The double integral is the expected gain from participating; the probability that the gain takes place is the probability that the worker contacts an employer, times the probability that the wage offer is acceptable. Combining condition (11) with eq. (1) gives us the condition which must be satisfied for the worker to be indifferent between participating and staying out of the labor force,

$$c = \int \int_{-\infty}^{w(x,u)-u} [w(x,u) - u - \epsilon] F(d\epsilon) \min[1, k(x, \theta)/n] H^x(dx; u, \theta) , \quad (12)$$

which states that the cost must equal the expected gain. A similar expression is obtained for the employer when one combines eqs. (2) and (9):

$$q(k) = \int \hat{\pi}(x, u) \min[1, n(u, \theta)/k] H^u(du; x, \theta) \quad (13)$$

Equations (10) and (11) make it clear that labor force participation depends not only on the wage (relative to the value of leisure), but also on the prospects of job availability. Workers and employers must make a commitment to the market based on what they know; the nature of this commitment is described by  $n(u, \theta)$  and  $k(x, \theta)$ .

Before defining equilibrium, we shall rearrange eqs. (12) and (13). Rearrangement of eq. (12) yields

$$n = \frac{1}{c} \int \int_{-\infty}^{w(x,u)-u} [w(x,u) - u - \epsilon] F(d\epsilon) \min[n, k(x, \theta)] H^x(dx; u, \theta) \quad (14)$$

The right-hand side of eq. (14) cannot be negative, but it may exceed unity and thereby violate the constraint that  $n \in [0, 1]$ . We therefore define

$$T(n; k) \equiv \min \left[ 1, \frac{1}{c} \int \int_{-\infty}^{w(x, u) - u} [w(x, u) - u - \epsilon] F(d\epsilon) \right. \\ \left. \cdot \min [n, k(x, \theta)] H^X(dx; u, \theta) \right], \quad (15)$$

which maps functions  $k(\cdot)$  and scalars  $n$  into  $[0, 1]$ . Rearrangement of eq. (13) yields

$$kq(k) = \int \hat{\pi}(x, u) \min [k, n(u, \theta)] H^u(du; x, \theta) . \quad (16)$$

Now let  $\zeta(k) \equiv kq(k)$ . Since  $q(0) = 0$ ,  $\zeta(0) = \zeta'(0)$ . Let  $\zeta^{-1}(\cdot)$  be the inverse function. Then  $\zeta^{-1}(0) = 0$  and  $\lim_{y \rightarrow 0} \partial \zeta^{-1}(y) / \partial y = +\infty$ . Eq. (16) implies that

$$k = \zeta^{-1} \left[ \int \hat{\pi}(x, u) \min [k, n(u, \theta)] H^u(du; x, \theta) \right] \equiv M(k; n) . \quad (17)$$

Since  $q(k)$  is unbounded for  $k \geq \hat{k}$ ,  $\zeta^{-1}(y)$  is bounded by  $\hat{k}$ . Therefore, the operator  $M(k, n)$  maps functions  $n(\cdot)$  and scalars  $k$  into  $[0, \hat{k}]$ .

The proof of existence relies on bounds on wages and profits that are given by the following lemma:

Lemma 1: For all  $x$  and  $u$ ,

$$x - \frac{F(x - u)}{f(x - u)} \leq w(x, u) \leq x , \quad (18)$$

and

$$\frac{x - u}{2} \leq \hat{\pi}(x, u) . \quad (19)$$

Proof: If  $w > x$ , expected profits  $\hat{\pi}(x, u)$  would be negative. Since  $w = x - F(w - u)/f(w - u)$ , and since log-concavity of  $F$  implies that  $F/f$  is increasing, one has that  $F(w - u)/f(w - u) \leq F(x - u)/f(x - u)$ , from which the lower inequality in eq. (18) follows. Turning to  $\hat{\pi}$ , note that it is feasible to set  $w(w, u) = u$ , and that at this wage,  $F(w - u) = F(0) = \frac{1}{2}$ . The inequality in eq. (19) then follows from the definition of  $\hat{\pi}$  in eq. (4). This completes the proof of the lemma.

These results are useful in deriving conditions under which equilibrium investment and labor-force participation are both strictly positive in every state of the world  $(x, u, \theta)$ . Since integration by parts yields<sup>14</sup>

$$\int_{-\infty}^{w(x, u) - u} (w(x, u) - u - \varepsilon) F(d\varepsilon) = \int_0^{\infty} F[w(x, u) - u - s] ds ,$$

eq. (14) implies the following lower bound for the expected gains from participating in the labor market conditional on  $x$  and  $u$ ;

$$\int_{-\infty}^{w(x, u) - u} [w(x, u) - n - \varepsilon] F(d\varepsilon) \geq \int_0^{\infty} F\left[x - \frac{F(x - u)}{f(x - u)} - u - s\right] ds. \quad (20)$$

The right-hand side of eq. (20) is defined entirely in terms of exogenous data, and it is strictly positive for each  $x$  and  $u$ .



Definition of equilibrium: Equilibrium consists of three functions:

$w: \mathbb{R}_+^1 \times \mathbb{R}^1 \rightarrow \mathbb{R}^1$ ,  $k: \mathbb{R}^1 \times \Theta \rightarrow [0, \hat{k}]$ , and  $n: \mathbb{R}^1 \times \Theta \rightarrow [0, 1]$ , satisfying eqs. (4), (15), and (17).

This definition requires that the wage-setting, investment, and participation decision rules, all be consistent with each other.

Theorem: Existence of a non-degenerate equilibrium: For each  $\theta \in \Theta$ , if

- (i) There is a constant  $\delta(\theta) > 0$  such that for each  $x$ ,  $x - E(u|x, \theta) \geq \delta(\theta)$ ,
- (ii) For each  $u$ ,  $\int_0^\infty F[x-u-s-F(x-u)/f(x-u)] ds H^x(dx; u, \theta) \geq c$ ,

then an equilibrium exists in which  $w$ ,  $n$  and  $k$  are continuous in  $x$  and  $u$ , and in which there exists a constant  $\rho(\theta) > 0$  such that  $k(x, \theta) \geq \rho(\theta)$ , and  $n(u, \theta) \geq \rho(\theta)$  for all  $x$  and  $u$ .

Proof: If  $\lim_{\varepsilon \rightarrow \infty} \varepsilon F(-\varepsilon) = 0$ , the maximum to eq. (4) exists for  $w \in (-\infty, x]$ .

The range of  $\varepsilon$  is infinite and the assumption of log-concavity of  $F$  implies that a unique maximum exists and that it satisfies eqs. (5) and (6). If  $f$  is continuous, so is  $w$ . Since  $w(\cdot)$  depends neither on  $u(\cdot)$  nor on  $k(\cdot)$ , the proof of existence has been reduced to finding a solution for  $n(\cdot)$  and  $k(\cdot)$  to the pair of equations  $n = T(n; k)$  and  $k = M(k; n)$ .

Since  $T(0; 0) = M(0; 0) = 0$ , a degenerate equilibrium in which  $n \equiv k \equiv 0$  always exists.

To prove the existence of a non-degenerate equilibrium, we use the results of Lemma 1. Eq. (19) implies that  $\int \hat{\pi}(x, u) H^u(du; x, \theta) \geq \frac{1}{2} [x - E(u|x, \theta)] \geq \delta/2$  from assumption (i) of this theorem. Therefore, for each  $x$ ,

$\zeta^{-1}[\int \hat{\pi} \min(k, n) H^u(du; \dots)] \geq \zeta^{-1}[\min(k, n) \delta/2]$ . On the other hand,

$$\int_{-\infty}^{w(x, u) - u} [w(x, u) - u - \epsilon] F(d\epsilon) H^x(dx; u, \theta) \geq \left[ \int_0^{\infty} F[x - u - s - F(x-u)/f(x-u)] ds \right] \cdot H^x(dx; u, \theta) \geq c$$

(where the last inequality follows from assumption (ii) of the theorem). But this implies that for any  $\rho > 0$ , and for any  $k(\cdot)$  such that  $\inf_x k \geq \rho$ ,  $T(n; k) \geq \rho$ .

Thus for a fixed  $k(\cdot)$  such that  $\inf_x k \geq \rho$ ,  $T(n; k)$  is a mapping from  $[\rho, 1]$

into itself. Now choose  $\rho$  sufficiently small such that  $\zeta^{-1}(\rho \delta/2) \geq \rho$ .

This can be done because  $\lim_{y \rightarrow \infty} \partial \zeta^{-1}(y) / \partial y = +\infty$ . Then for any  $n(\cdot) \geq \rho$

(all  $u$ ),  $M(k; n)$  is a map from  $[\rho, \hat{k}]$  into itself.

Let  $B$  be the set of pairs of functions  $n: R^1 X_0 \rightarrow [\rho, 1]$  and  $k: R^1_+ X_0 \rightarrow [\rho, \hat{k}]$ , where  $n$  and  $k$  are continuous in  $u$  and  $x$  respectively. Then define the mapping  $J(n, k) \equiv \{T(n; k), M(k; n)\}: B \rightarrow B$ . Let  $\|n, k\| \equiv \sup_u |n(u, \theta)| + \sup_x |k(x, \theta)|$ .

With this metric, the normed space  $B$  is compact. But  $J$  is a continuous mapping. Any continuous mapping from a compact space into itself has at least one fixed point. This completes the proof of existence of a non-degenerate equilibrium.

### 3. Observables and the Contemporaneous Correlation of Innovations.

Our discussion of equilibrium concentrated on three observables: the real wage,  $w(x,u)$ , real investment per capita,  $k(x,\theta)$ , and the labor force participation rate,  $n(u,\theta)$ . The rate of unemployment,  $U$ , is equal to the number of rejected wage offers, plus the number of workers who do not find a vacancy, divided by  $n$ :

$$U(x,u,\theta) = \min[1, k(x,\theta)/n(u,\theta)] \{1 - F[w(x,u) - n]\} \\ + \max[0, 1 - k(x,\theta)/n(u,\theta)].$$

Labor productivity (i.e., output per worker) is equal to  $x$  in our model, and it too is observable.

To determine how the observables depend on  $x$  and  $u$ , we shall use the following result:

Theorem 2: If  $x$  and  $u$  are independent conditional on  $\theta$ , so that  $H(x,u;\theta)$  is the product of two distributional functions  $H_1(x;\theta)H_2(u;\theta)$ , then in a non-degenerate equilibrium  $[k(\cdot), n(\cdot) > 0]$ ,  $k$  is strictly increasing in  $x$  everywhere. Moreover,  $n$  decreases with  $u$ , and this decrease is strict at each  $n$  for which  $\inf_x k(x,\theta) \leq n < 1$ .

Before proving the theorem, let us remark that a priori, no restriction can be placed on the covariance of  $x$  and  $u$ . In our empirical work, we can not reject the hypothesis of conditional independence of  $x$  and

$u$ , even though the standard error of the estimate of the conditional covariance of  $x$  and  $u$  is quite small (see Table 5).

Proof: Eq. (4) implies that  $\partial \hat{\pi} / \partial x = F(w - u)$ , by the envelope theorem. By the conditional independence of  $x$  and  $u$ ,  $H^u(u; x, \theta) = H_2(u; \theta)$ . Eq. (9) then implies that  $\partial \pi / \partial x > 0$  everywhere. This equation also implies that  $\pi$  is decreasing in  $k$ . Since  $q(k)$  is strictly increasing, eq. (2) implies that  $k$  is strictly increasing in  $x$ . Eq. (11) implies that  $\partial v / \partial u = 1 - \int (\partial x / \partial u - 1) F(w - u) \min(1, k/n) H$ . But from eqs. (7) and (8),  $\partial w / \partial u < 0$ , so that  $\partial v / \partial u < 1$  everywhere. From theorem 1, if conditions (i) and (ii) hold, no corner solution with  $n = 0$  to eq. (2) exists. For  $n \in (0, 1)$ ,  $\partial v / \partial n < 0$  if  $\inf_x k(x, \theta) < n$ , and the result then follows from eq. (1).

We should remark that conditional independence of  $x$  and  $u$  is sufficient, but not necessary for the stated monotonicity properties of  $k$  and  $n$ . One could perhaps establish a similar result when  $x$  and  $u$  are conditionally negatively correlated. If they are positively correlated, however, it is possible that the monotonicity properties are reversed. A high  $u$ , for instance, discourages the worker from participating because it raises the value of home time by more than it raises the expected wage. But if a high  $u$  signals a high  $x$ , this would raise the expected wage even more, and  $n$  could increase.

Let  $z \equiv [x, k, n, w, U]$  be the vector of observables. We shall now derive the predicted signs of the variance covariance matrix of  $z$  conditional on  $\theta$  alone:  $E\{[z - E(z|\theta)][z - E(z|\theta)]' | \theta\} \equiv \Sigma(\theta)$ . In doing so we maintain the assumption that  $x$  and  $u$  are conditionally independent. The sign

Table 2

Sign Restrictions on the Elements of  $\Sigma(\theta)$ 

	$x$	$k(x,\theta)$	$n(u,\theta)$	$w(x,u)$	$U(x,u,\theta)$
$x$	+				
$k(x,\theta)$	+	+			
$n(u,\theta)$	0	0	+		
$w(x,u)$	+	+	-	+	
$U(x,u,\theta)$	-	-	?	?	+

restrictions and zero restrictions on the covariance are given in the matrix (Table 2).

Since  $u$  and  $x$  are conditionally independent, neither  $x$  nor  $k(x, \theta)$  is correlated with  $n(u, \theta)$ , hence the two zero restrictions in the matrix. All the other correlations except the ones involving  $U$  follow directly from eq. (7) and Theorem 2.

The model can now be tested informally by comparing Tables 1 and 2. The model contains predictions for eight of the ten off-diagonal correlations reported in Table 1. The model correctly predicts the sign of each of the four starred entries in Table 1. The zero restrictions on the correlation of labor-force participation on the one hand with productivity and investment on the other is certainly consistent with the results reported in Table 1. However, the positive predicted correlation between the wages on the one hand, and investment and productivity on the other, is not present in the data: although the estimated correlation coefficients are of the right sign, they are both quite insignificant. The wage thus appears to respond to the aggregate supply shock, but not to the aggregate demand shock. Further discussion of the results is deferred until the next section.

In the correlations involving  $U$ , note that

$$\frac{\partial U}{\partial x} = \begin{cases} -f \frac{\partial w}{\partial x} < 0 & \text{if } n < k \\ -\frac{1}{n} \left[ F \frac{\partial k}{\partial x} + kf \frac{\partial w}{\partial x} \right] < 0 & \text{if } n \geq k \end{cases}$$

so that this derivative is unambiguously negative. On the other hand,

$$\frac{\partial U}{\partial u} = \begin{cases} -f \left( \frac{\partial w}{\partial u} - 1 \right) > 0 & \text{if } n < k \\ -\frac{k}{n} f \left( \frac{\partial w}{\partial u} - 1 \right) + \frac{k}{n^2} F \frac{\partial n}{\partial u} & \text{if } n \geq k, \end{cases}$$

and, since  $\partial w/\partial u < 1$ , and  $\partial n/\partial u < 0$ , the sign of  $\partial U/\partial u$  is indeterminate. If  $u$  increases, participation declines, and on that score, fewer people stand a chance of becoming unemployed. On the other hand, of those who do enter, fewer will accept the wage offers they receive, which works to increase the rate of unemployment. Because of this, correlations between  $n$  and  $U$ , and between  $w$  and  $U$ , are both of indeterminate sign.

An implication that will be used in the empirical work is (using eq. (8)),

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial u} = 0 \quad \text{when } n < k ,$$

and

$$\frac{\partial U}{\partial x} < 0 \quad \text{when } n \geq k . \quad (21)$$

Note that except for the zero-measure event  $n = k$ , either jobs or workers must be rationed. This is a consequence of the imperfect coordination between capital and labor, and of the fixed proportions technology. When  $n < k$ , the only unemployment is "mismatch" unemployment -- unemployment resulting from failed interviews. But when  $n > k$ , there is also a genuine "glut" of workers, a glut which, given the fixed number of jobs in the short run, can not be eliminated. The mismatch unemployment is the result of "micro" uncertainty (i.e., the  $\varepsilon$  which may reflect matching considerations). The glut unemployment is the result of surprises in aggregate shocks.

Turning next to output and employment, note that the level of employment,  $\psi(x, u, \theta)$  is equal to the number of contacts that result in the wage offer being accepted:

$$\psi(x, u, \theta) = \min[k(x, \theta), n(u, \theta)] F[w(x, u) - u].$$

In view of Theorem 2, conditional independence of  $x$  and  $u$  implies that  $\psi$  is increasing in  $x$ . Moreover, eq. (8) implies that  $\partial w / \partial u < 1$ . Therefore  $\psi$  is decreasing in  $u$ . Therefore, employment increases with  $x$  and decreases with  $u$ . The same is true of aggregate output, which is equal to  $x\psi(x, u)$ .



### Empirical Specification

The main implication of the foregoing model is that the contemporaneous covariance structure of a typically observed vector of aggregate labor market variables can be explained by a two-dimensional factor structure, where the latent factors correspond essentially to labor supply and labor demand shocks. Moreover, the model implies certain restrictions on this factor structure: (1) participation rates are independent of the demand factor; (2) demand indicators, in particular the value of output per unit of labor and investment decisions, are independent of the supply factor; (3) the sum of the supply and demand effects on wage rates is unity; (4) the sum of the supply and demand effects on unemployment rates is zero, if capital does not constrain employment; and (5) the supply and demand factors are conditionally independent. We test this model using quarterly time series data for the U.S. for the years 1948 to 1982.

Let  $z(t)$  denote the column vector of variables in period  $t$  which includes the female labor force participation rate (FPR), female unemployment rate (FUR), male participation rate (MPR), male unemployment rate (MUR), real investment in plant equipment (INV), real compensation per man-hour in manufacturing (WAGE), and real value of output per man-hour in manufacturing (OUT). For purposes of this analysis, it is important to have a well-defined output indicator whose measurement is independent of payments to factors, especially labor. This is most easily accomplished in cases where output is physical, and for this reason we choose to examine output and compensation in the manufacturing sector alone. A more detailed description of the data and sources can be found in the Appendix.

The theoretical model discussed above is cast in the context of a static economy recreated each period, with the assumption that choices made each period are conditioned by some information set  $\theta(t)$  available to all agents. Assuming that  $\theta(t)$  spans the same space as  $\{z(t-1), \dots, z(t-n)\}$ , we suppose that the systematic part of  $z(t)$  can be represented by

$$z(t) = a_0 + a_1 t + a_2 t^2 + \tilde{z}(t) \quad (22)$$

$$[I-A(L)]\tilde{z}(t) = \Lambda f(t) + e(t) \quad (23)$$

where  $t$  is a time index and  $L$  denotes a lag operator of length  $n$ , and where  $f(t) = [x(t), u(t)]'$  and  $e(t)$  is a white noise error vector whose elements are mutually independent.

We furthermore suppose that the dynamics of  $f(t)$  are such that agents are able to form ex ante expectations of  $f(t)$ , conditional on  $\theta(t)$ , before committing any capital or labor, and that this expectation can be written

$$E[f(t) | \theta(t)] = B(L)\tilde{z}(t) . \quad (24)$$

In principle, the lag structure on the right hand side of (24) can be infinite, if, for example,  $f(t)$  has an autoregressive component, but for empirical purposes we assume a finite lag of at most length  $n$ . Define

$$f^*(t) = f(t) - E[f(t) | \theta(t)] \quad ; \quad (25)$$

then,  $f^*(t)$  represents the unanticipated innovations in  $x(t)$  and  $u(t)$ ,

denoted  $x^*(t)$  and  $u^*(t)$  henceforth. Assume these innovations have zero expectation and are serially uncorrelated. Then substituting (24) and (25) into (23), we have

$$[I - A(L) - \Lambda B(L)] \tilde{z}(t) = z^*(t) \quad (26)$$

where

$$z^*(t) = \Lambda f^*(t) + e(t) \quad (27)$$

is serially uncorrelated with zero expectation. In this formulation, the sequence  $\{z^*(t), t=1, 2, \dots, T\}$  can be viewed as a set of independent realizations corresponding essentially to a sample of single-period economies.

Letting  $\Sigma_z$ ,  $\Sigma_f$ , and  $\Sigma_e$  denote the covariance matrices of  $z^*(t)$ ,  $f^*(t)$ , and  $e(t)$  respectively, we have from (27)

$$\Sigma_z = \Lambda \Sigma_f \Lambda' + \Sigma_e, \quad (28)$$

and under the assumption of normality, the log likelihood function for  $\{z^*(t), t = 1, 2, \dots, T\}$  is

$$\log L\{\Lambda, \Sigma_f, \Sigma_e / z^*(t), t = 1, 2, \dots, T\} = -\frac{T}{2} (\log |\Sigma_z| + \text{tr } \Sigma_z^{-1} S) \quad (29)$$

where  $S$  is the empirical covariance matrix of  $z^*(\cdot)$ . It will be recognized that (27), (28) and (29) comprise a typical factor analytic model.

It is useful at this stage to consider the restrictions on (28) implied by the model. The matrix of factor loadings  $\Lambda$  transforms unanticipated shocks in  $x(t)$  and  $u(t)$  into shocks in the vector  $z^*(t)$  (or equivalently  $z(t)$ ). Given the linearity of (27), the coefficients in  $\Lambda$  then are partial derivatives of each of the elements of  $z(t)$  with respect to  $x(t)$  and  $u(t)$ . Since the implicit metric for both  $x(t)$  and  $u(t)$  is real output units, we can normalize

$$\frac{\partial \text{OUT}}{\partial x} = 1 ,$$

and since labor is presumed homogenous, so that labor supply does not affect output per unit of labor,

$$\frac{\partial \text{OUT}}{\partial u} = 0 .$$

We furthermore have

$$\frac{\partial \text{WAGE}}{\partial x} + \frac{\partial \text{WAGE}}{\partial u} = 1 \quad (\text{from eq. (8)},$$

and the zero restrictions

$$\frac{\partial \text{INV}}{\partial u} = \frac{\partial \text{FPR}}{\partial x} = \frac{\partial \text{MPR}}{\partial x} = 0 ,$$

and if capital is not a constraining factor,

$$\frac{\partial \text{FUR}}{\partial x} + \frac{\partial \text{FUR}}{\partial u} = \frac{\partial \text{MUR}}{\partial x} + \frac{\partial \text{MUR}}{\partial u} = 0 , \quad (\text{from eq. (21)},$$

where the argument  $t$  has been dropped for notational convenience.

### Estimates

Since the sequence of vectors  $\{z^*(t)\}$  is not directly observed, we adopt a two-stage approach to estimating the covariance structure (4), first estimating (22) and (26) to obtain  $\hat{z}^*(.)$ , and consequently  $\hat{S}$ , and then replacing  $S$  with  $\hat{S}$  in (29). Under the maintained hypothesis that  $z^*(t)$  is serially uncorrelated, least squares is consistent, and the least squares moment estimator of  $\Sigma_z$ ,  $\hat{S}$ , converges in distribution to  $S$ . The second stage entails maximizing (29), which is now a conditional log likelihood function when  $\hat{S}$  replaces  $S$ , appropriately adjusted for the loss in degrees of freedom incurred at the least squares stage.<sup>15</sup> Table 3 shows the correlation matrix of the estimated residuals of  $z(t)$  after detrending (i.e., from (22)), and Table 4 presents the correlation and covariance matrices of  $\hat{z}^*(t)$  from the least squares estimates of (26), estimated from a four-quarter autoregressive lag. The coefficient estimates are shown in the Appendix. It is readily verified that the theoretical qualitative predictions summarized in Table 2 are generally confirmed in Table 4.

The conditional maximum likelihood estimates of the parameters of (29) are reported in Table 5. The estimated  $\chi^2$  statistic for the model ( $-2\log L_{\max}$ ) indicates a strong confirmation of the model.<sup>16</sup> Moreover, we note that the estimated covariance between the supply and demand factors is not statistically different from zero, which supports the hypothesis that  $x(t)$  and  $u(t)$  are distributed independently. Table 6 shows the estimates of factor loadings and variances of the elements of  $e(t)$  when this covariance is constrained to equal zero.

TABLE 3  
CORRELATIONS OF DETRENDED LABOR MARKET VARIABLES

	FPR	FUR	MPR	MUR	INV	WAGE	OUT
FPR	1.000						
FUR	-0.056	1.000					
MPR	0.679	-0.075	1.000				
MUR	0.022	0.927	0.101	1.000			
INV	-0.116	-0.749	-0.079	-0.808	1.000		
WAGE	-0.080	0.199	-0.251	0.036	-0.035	1.000	
OUT	-0.422	-0.264	-0.483	-0.470	0.406	0.343	1.000

TABLE 4  
COVARIANCE AND CORRELATION MATRICES OF  $\hat{z}^*(t)$

## (a) COVARIANCE MATRIX

	FPR	FUR	MPR	MUR	INV	WAGE	OUT
FPR	4.753						
FUR	0.136	7.791					
MPR	1.151	-0.540	3.383				
MUR	-0.323	5.349	0.243	9.216			
INV	-0.015	-1.626	-0.069	-2.393	2.276		
WAGE	-0.428	-0.247	-0.742	-0.433	0.171	2.525	
OUT	0.000	-4.271	-0.344	-6.958	2.263	0.151	13.799

## (b) CORRELATION MATRIX

	FPR	FUR	MPR	MUR	INV	WAGE	OUT
FPR	1.000						
FUR	0.022	1.000					
MPR	0.287	-0.105	1.000				
MUR	-0.049	0.631	0.044	1.000			
INV	-0.005	-0.386	-0.025	-0.523	1.000		
WAGE	-0.124	-0.056	-0.254	-0.090	0.072	1.000	
OUT	0.000	-0.412	-0.050	-0.617	0.404	0.026	1.000

TABLE 5  
TWO-FACTOR MODEL WITH CORRELATED DEMAND AND SUPPLY SHOCKS<sup>a</sup>

Dependent Variable	Factor Loadings		Equation Variance
	$x^*(t)$	$u^*(t)$	
FPR	0	-1.498 (2.00)	4.36 (7.28)
FUR	-0.775 (6.15)	0.994 (1.34)	4.14 (6.13)
MPR	0	-4.406 (2.63)	0 <sup>b</sup>
MUR	-1.165 (6.73)	0.120 (0.18)	1.13 (1.44)
INV	0.349 (5.23)	0	1.55 (6.77)
WAGE	0.055 (0.85)	0.945 (14.6)	2.34 (1.83)
OUT	1.000	0	7.83 (6.31)

$$\hat{\sigma}_x^2 = 5.967 (3.56)$$

$$\hat{\sigma}_u^2 = 0.174 (1.29)$$

$$\hat{\sigma}_{xu} = -0.065 (0.49)$$

$$-2 \log L_{\max} = 3.79 \text{ (d.f. = 11)}$$

<sup>a</sup> Asymptotic  $z$  statistics are reported in parentheses.

<sup>b</sup> Variance constrained to zero because of boundary solution.

TABLE 6

TWO-FACTOR MODEL WITH INDEPENDENT DEMAND AND SUPPLY SHOCKS<sup>a</sup>

<u>Dependent Variable</u>	<u>Factor Loadings</u>		<u>Equation Variance</u>
	<u>x*(t)</u>	<u>u*(t)</u>	
FPR	0	-1.484 (2.00)	4.36 (7.28)
FUR	-0.774 (6.15)	0.855 (1.32)	4.14 (6.13)
MPR	0	-4.364 (2.63)	0 <sup>b</sup>
MUR	-1.164 (6.73)	-0.076 (0.15)	1.13 (1.44)
INV	0.349 (5.23)	0	1.55 (6.77)
WAGE	0.055 (0.85)	0.945 (14.7)	2.34 (1.82)
OUT	1.000	0	7.84 (6.31)

$$\hat{\sigma}_x^2 = 5.958 \quad (3.55)$$

$$\hat{\sigma}_u^2 = 0.178 \quad (1.30)$$

$$-2 \log L_{\max} = 4.03 \quad (\text{d.f.} = 12)$$

<sup>a</sup> Asymptotic  $z$  statistics are reported in parentheses.

<sup>b</sup> Variance constrained to zero because of boundary solution.



Overall, the estimates indicate a dominance of the demand shock in explaining the contemporaneous covariance of labor market variables, by virtue of the substantially larger estimated demand shock variance. A demand shock of one positive standard deviation increases output per man-hour by 2.4 real 1977 cents, increases investment by 0.85 billion 1977 dollars (at an annual rate), and reduces unemployment by approximately 3/10 of a percentage point for males and 2/10 of a percentage point for females. Wage rates, however, respond only slightly, increasing only by 0.13 cents. The indication is that wages are not very sensitive to demand side shocks, while employment is. This pattern of coefficients suggests two possibilities: either committed labor (i.e., labor force participants) is very sensitive to the wage rate in accepting or rejecting wage offers -- the estimates imply a labor supply elasticity of approximately 8 -- and vacancies are in excess supply ( $k > n$ ), or vacancies are a constraining factor and the relatively high unemployment coefficients reflect increased probabilities of getting an interview with a favorable demand shock, by virtue of more vacancies being created with  $k$  being increased. The second alternative seems more plausible, especially in view of the comparative statics result that when  $k$  does constrain employment, the derivative of the unemployment rate with respect to  $x(t)$  tends to be larger in absolute value. We examine this question in greater detail below.

The estimated variance of the supply shock is substantially less than that of the demand shock: comparing estimated standard deviations, we find  $\sigma_u = 0.42$  and  $\sigma_x = 2.44$ . The scale of the supply shock is identified by imposing the restriction that  $\frac{\partial WAGE}{\partial x} + \frac{\partial WAGE}{\partial u} = 1$ , thereby rendering this

restriction untestable (at least without imposing other identifying restrictions). As predicted in the theoretical model, a positive supply shock (measured in terms of a higher value of leisure in real output units) decreases participation rates and increases the wage rate, with the effects on unemployment being ambiguous. We note, however, that a supply shock affects wage rates to a significantly greater extent than does a demand shock: a positive one standard deviation supply shock increases wages by 40 cents, despite the standard deviation being relatively small. The same supply shock decreases male participation rates by 18/100 and female participation rates by 6/100 percentage points.

We might note that Mincer (1966) found that the greater the average participation rate of a group, the lower is the cyclical amplitude of that group's unemployment rate. Comparing the male and female unemployment-rate factor loadings in Table 6, we confirm this finding for that component of the cycle induced by changes in  $u$ , but not for that induced by changes in  $x$ . Recall that in section 3 we had found that aggregate output was increasing in  $x$  and decreasing in  $u$ , so that both unemployment rates (driven primarily by changes in  $x$ ) are countercyclical. We confirm Mincer's finding that female participation rates are procyclical, but to our surprise, we also find that male participation rates are just as procyclical as female rates.

The results also shed some light on the debate over the cyclical content of real wages and participation rates in past empirical work (see Neftçi (1978) and the references therein). Here we find robust support for the notion that real wages have only a weak pro-cyclical response when demand indicators are used to measure cycles, but instead that wages are

substantially supply-determined and vary systematically with participation rates. The failure of  $x$  to significantly affect the wage in Tables 5 and 6 contradicts the implication of the model (see section 3 and Table 2). The factor loadings in the wage equation are essentially estimates of the derivatives in eq. (7). These derivatives depend upon  $F$  (the distribution of  $\varepsilon$ ), and one has some latitude in perhaps choosing  $f$  so that  $\partial w/\partial x$  is small. Nevertheless, the model implies a significant positive effect of  $x$  on the wage, and the significance is a puzzle to us. On the other hand, the predicted positive effect of  $u$  on the wage (see eq. (7)) is quite strongly confirmed in both Tables 5 and 6.

Returning to the question of vacancy-constrained employment, one test of this hypothesis is a test on the sum of the factor coefficients in the unemployment rate equations. The theoretical model implies that if vacancies are not a constraint (i.e.,  $k > n$ ), then the coefficients will sum to zero; otherwise, their sum is less than zero. From the unconstrained estimates reported in Table 6, the conclusions appear to be ambiguous. We note a large disparity in the male unemployment rate coefficients, and a negative sum, but relatively equal and offsetting coefficients in the female unemployment equation, suggesting perhaps that vacancies constrain male employment but not female. Maximizing (5) with the additional restriction that the unemployment equation coefficients sum to zero across factors yields a model  $\chi^2$  of 9.75 with 14 degrees of freedom. This is statistically different from the unconstrained model (Table 6) at the 10 percent level of significance, but not the 5 percent level. Moreover, it is arguable that this test is somewhat misleading, and perhaps biased in favor of the

constrained model, on the grounds that males, who constitute the bulk of the labor force, are weighted equally with females in the model, whereas it is their coefficients which are more disparate.

The two-factor model presented in Table 6 was tested against three alternative specifications of the factor structure. Table 7 summarizes these results; in each case, only the matrix of estimated factor loadings and model Chi-squared statistic is shown. The first alternative, denoted by (a) in Table 7, relaxes the zero restrictions imposed on the demand factor by allowing the participation rate equations to have a demand shock component. For both males and females, the point estimates are close to zero (-0.049 and 0.024 respectively) and not statistically significant. Testing this alternative against the constrained model of Table 6 gives a log likelihood ratio statistic of 0.455, which is not significantly different from zero with two degrees of freedom. Similarly, the second alternative, denoted by (b), relaxes the zero restrictions imposed on the supply factor in the output and investment equations. In this case, the log likelihood ratio is 0.274, again not significant with two degrees of freedom. In all cases, therefore, the zero restrictions are confirmed.

This confirmation of the zero restrictions supports our model of heterogeneous information. Note that our argument is not affected by the existence of long lags between the initiation of capital expenditures and their completion. What matters is whether an increase in investment amounts to an immediate increase in the demand for labor. Survey data indicates that for some forty percent of investment projects, construction begins (and thus new jobs are created) within three months of when the plans are drawn up (Meyer (1960), p.130). This supports our view that current

TABLE 7  
FACTOR LOADINGS IN ALTERNATIVE SPECIFICATIONS<sup>a</sup>

<u>Dependent Variable</u>	(a) Unconstrained Demand Factor		(b) Unconstrained Supply Factor		(c) Single Factor Model
	$x^*(t)$	$u^*(t)$	$x^*(t)$	$u^*(t)$	
FPR	0.024 (0.26)	-1.497 (1.99)	0	-1.467 (2.03)	0.032 (0.35)
FUR	-0.763 (6.02)	0.984 (1.35)	-0.775 (6.15)	0.689 (0.99)	-0.766 (6.06)
MPR	-0.049 (0.49)	-4.357 (2.60)	0	4.314 (2.70)	-0.025 (0.33)
MUR	-1.170 (6.74)	0.121 (0.19)	-1.166 (6.72)	-0.310 (0.45)	-1.175 (6.68)
INV	0.348 (5.22)	0	0.349 (5.22)	0.087 (0.25)	0.349 (5.21)
WAGE	0.065 (0.96)	0.935 (13.76)	0.054 (0.85)	0.946 (14.6)	0.060 (0.90)
OUT	1.000	0	1.000	0.439 (0.52)	1.000
$-2 \log L_{\max}$ :	3.57 (d.f.= 10)		3.75 (d.f.= 10)		23.16 (d.f.= 14)

<sup>a</sup> Asymptotic  $z$  statistics are reported in parentheses.

investment and current labor supply are complementary. Moreover, if there was a significant delay between the timing of investment expenditures and the creation of new jobs, the participation rate would be expected to depend significantly on lagged ratios of investment; Table (A.1) shows that this is not so.

The third alternative is a single factor model of labor market outcomes; the estimated factor loadings are denoted by (c) in Table 7. Not surprisingly, the estimates correspond closely to the demand factor loadings in Table 6, due, of course, to the predominance of demand shocks. While the single factor model can be accepted as a sufficient structural explanation of the covariance structure at conventional levels of significance ( $\chi^2(14) = 23.16$ ; Prob = .058), the two-factor structure is statistically superior on the basis of a likelihood ratio test ( $\chi^2(4) = 23.16 - 4.02 = 19.14$ ).

#### 4. Welfare

Conditional on  $k$  and  $n$ , the wage-setting mechanism described in eq. (4) results in too little employment from the social point of view. Once capital and labor are committed to the market, and once the investment and participation costs have been sunk, the rent of a contract is  $x - u - \epsilon$ . The fraction of contacts which result in positive rents is  $F(x - u)$ , whereas the actual fraction of successful contacts is  $F[w(x, u) - n]$ . The former exceeds the latter because  $w(x, u) < x$ , so that some positive rents are not exploited.

The expected rents from a random contact are  $\int_{-\infty}^{x-u} (x-u-\epsilon)F(d\epsilon)$ . Conditional on  $x$  and  $u$ , and on a commitment of  $n$  workers and  $k$  employers, total surplus is

$$\min(n, k) \int_{-\infty}^{x-u} (x-u-\epsilon)F(d\epsilon) - cn - \int_0^k q(s)ds .$$

This expression can clearly not be at a maximum unless  $n = k$ . If a scheme could be devised such that all positive rents were realized ex post, the first best socially optimal investment and participation satisfies

$$\int_{-\infty}^{x-u} (x-u-\epsilon)F(d\epsilon) = q(z) + c , \quad (30)$$

where  $z$  is the first-best level of investment and of labor force participation.

The first-best rule given in eq. (30) requires that information on  $x$  and  $z$  be pooled. If the planner knew  $x$  and  $u$ , he could allocate an equal number,  $z$ , of men and machines so as to equate the marginal benefit and the

marginal cost of committing capital and labor to the market. The solution for  $z$  depends only on the difference  $x - u$  :  $z = q^{-1} \left[ \int_0^{x-u} (x-u-\varepsilon) F(d\varepsilon) - c \right]$ . Condition (ii) of Theorem 1 ensures that the argument in  $q^{-1}(\cdot)$  is positive for all  $(x,u)$ . Since  $q(\cdot)$  increases monotonically from zero and becomes unbounded at  $\hat{k}$ ,  $q^{-1}(\cdot)$  also increases from zero, and is bounded by  $\hat{k}$ . There is no reason for the planner to allocate more capital than labor to the market, so that the binding constraint on  $z$  may be unity rather than  $\hat{k}$ , as illustrated in Figure 1.

To compare this first-best solution with the uncoordinated competitive outcome, suppose initially that both  $x$  and  $u$  are known to workers and employers alike. Eq. (12) then reads

$$\min(1, k/n) \int_0^{w(x,u)-u} [w(x,u) - u - \varepsilon] F(d\varepsilon) , \quad (12)'$$

while eq. (13) becomes

$$\min(1, n/k) [x - w(x,u)] F[w(x,u) - u] = q(k) . \quad (13)'$$

Adding (12)' and (13)' together, one obtains

$$\int_0^{w(x,u)-u} [x - u - \varepsilon] F(d\varepsilon) \geq q(k) + c . \quad (31)$$

Since  $w(x,u) < x$ , the left-hand side of eq. (30) exceeds the left-hand side of eq. (31), so that investment falls short of its first-best level.

We cannot prove a similar result for labor force participation. Nevertheless,



since  $k < z$ , the number of contacts in the market outcome falls short of the first-best number of contacts,  $z$ . Moreover, the fraction of contacts that are successful is smaller in the market case. On both counts, therefore, employment and output are smaller in the market outcome than in the first-best outcome.

All this says nothing, of course, about the policy implications under different information structures. At this level of generality, not much can be said about whether the government could, without knowledge of  $x$  or  $u$ , set corrective taxes or subsidies so as to improve welfare. In the context of a simpler example, however, we shall now show that a lump-sum financed unemployment-insurance benefit can improve welfare when the information structure is asymmetric, but not when employers and workers know both  $x$  and  $u$ . Although the example is rather specific, it does show that welfare implications hinge on the information structure and that models which allow for heterogeneous information sets should be further explored.

The welfare considerations that arise in the model are studied here in a simpler setting -- that of just one worker and one employer. It costs the worker  $c$  to participate and it costs the employer  $q$  to enter. If they both enter, and if the rents are non-negative, they split the rents equally. Given this fixed rule, we can dispense with  $\epsilon$  altogether -- its function in the earlier sections was to prevent employers from driving the wage down to the (common) value of leisure,  $u$ . Without the presence of  $\epsilon$ , the ex post rent is just  $x - u$ . The payoff matrix for this game, conditional on  $x$  and  $u$ , is:

		<u>Employer</u>	
		Out	In
<u>Worker</u>	Out	0  u	-q  u
	In	0  u - c	$\frac{1}{2}(x+u) - q$  $\frac{1}{2}(x+u) - c$

Although one half of the ex post rent is  $\frac{1}{2}(x-u)$ , the worker's payoff in the lower right-hand corner is  $u + \frac{1}{2}(x-u) = \frac{1}{2}(x+u)$ . The employer, on the other hand, gets  $x - \frac{1}{2}(x-u) = \frac{1}{2}(x+u)$ . If they both enter and the rent is negative ( $x < u$ ), the match is not formed, and their payoffs are  $-c$  and  $-q$ .

Before analyzing this game further, we should point out that regardless of the information structure, the pair {out, out} is always an equilibrium in this game, and as we shall see, it is usually a Pareto-inferior outcome. This is a common problem with Nash equilibrium, but Pareto-inferior equilibria are especially problematic in games where coordination is important in one way or another -- as is the case here.

Suppose now that both  $u$  and  $x$  are serially and mutually independent random variables, and that

$$u = \begin{cases} 0 & \text{with prob. } 1 - p_u \\ 1 & \text{with prob. } p_u \end{cases} ,$$

and

$$x = \begin{cases} 0 & \text{with prob. } 1 - p_x \\ 2 & \text{with prob. } p_x \end{cases} .$$

Let us begin by assuming that both players know  $p_x$  and  $p_u$ , but that  $u$  is private information to the worker, and  $x$  is private information to the employer. Suppose that the parameters obey the following restrictions:

$$c < p_x < 2c \tag{32}$$

and

$$0 < p_u < 1 - q . \tag{33}$$

(Since the probabilities are between zero and one, these conditions imply that  $c < 1$  and  $q < 1$ .) Apart from the {out, out} equilibrium, the only other equilibrium for this game is for the worker to participate only if  $u = 1$ , and for the employer to enter only if  $x = 2$ . Consider the expected payoffs. Let  $v(u)$  be the value of participating in state  $u$  (which also is a sufficient statistic for the worker's information because he does not know  $x$ ), and let  $\pi(x)$  be the employer's value of entry. Given that one's opponent is using the strategy described above, the payoffs to entry are:

$$v(0) = p_x - c ,$$

$$v(1) = 1 - c + p_x/2 ,$$

$$\pi(0) = -q ,$$

and

$$\pi(2) = 1 - q - p_u .$$

Equilibrium requires that  $v(0) > 0$ ,  $v(1) < 1$ ,  $\pi(0) < 0$ , and  $\pi(2) > 0$ , and conditions (32) and (33) ensure this.

Now consider welfare. The worker's steady-state welfare is  $(p_x - c)(1 - p_u) + p_u$ , while the employer's steady-state welfare is  $(1 - q - p_u)p_x$ . It follows, then, from conditions (32) and (33) that this equilibrium is Pareto superior to the equilibrium in which both players always stay out.

How does this compare with average welfare in the full-information case? This is when both players always know both  $x$  and  $u$ . For each pair of realizations  $(x,u)$ , there is now a separate equilibrium. Four possible cases arise:

Case 1:  $x = 0$ ,  $u = 0$ . The unique equilibrium is for both players to stay out. Each player gets zero. The probability of this event is  $(1 - p_x)(1 - p_u)$ .

Case 2:  $x = 0$ ,  $u = 1$ . Again, the unique equilibrium is for both players to stay out. The employer gets zero, the worker gets 1. The probability of this event is  $(1 - p_x)p_u$ .

Case 3:  $x = 2$ ,  $u = 0$ . Apart from the customary {out, out} equilibrium, a Pareto superior equilibrium is for both to enter, in which case the employer gets  $1 - q$ , while the worker gets  $1 - c$ . The

probability of this event is  $p_x(1 - p_u)$ .

Case 4:  $x = 2, u = 1$ . Apart from the {out, out} equilibrium, if  $c < \frac{1}{2}$ , there is a Pareto superior {in, in} equilibrium. (Even if  $\frac{1}{2} < c < 1$ , this is a Pareto superior outcome -- the sum of the payoffs is larger.) The worker gets  $-c + 3/2$ , the employer gets  $-q + 3/2$ . The probability of this event is  $p_x p_u$ .

Comparing the full information case to the private information case, we see that they have the same outcome in all cases except case 4, in which, in the private information regime, the worker stays out and the employer enters; the worker gets 1, while the employer gets  $-q$ . Joint steady-state welfare therefore differs by  $p_x p_u (3/2 + 3/2 - c - q) - p_x p_u (1 - q) = p_x p_u (2 - c)$ .

We shall now show that if lump-sum taxation is feasible, and if  $c < \frac{1}{2}$ , the authorities could make everyone better off by subsidizing labor-force participation. One such subsidy is the unemployment insurance benefit. Financed in a lump-sum fashion, the benefit could be set at a level that is sufficiently high so as to make the worker choose to always participate. Employers would still enter only if  $x = 2$ . Let  $b$  be the insurance benefit. For the worker to choose participation even when  $u = 1$ , one must have

$$(1 - p_x)(1 - c + b) + p_x(3/2 - c) > 1,$$

which implies that

$$b > \frac{c - \frac{1}{2}p_x}{1 - p_x} \quad (34)$$

On the other hand,  $b$  must not be so large as to make the worker prefer

unemployment even when he can get work. This implies that

$$1 - c + b < 3/2 - c ,$$

(Note that if he prefers work to unemployment when  $u = 1$ , he will certainly prefer work to unemployment when  $u = 0$ .) This implies that

$$b < \frac{1}{2} \tag{35}$$

A necessary and sufficient condition for there to exist an unemployment insurance benefit such that both (34) and (35) hold, one must have  $c < \frac{1}{2}$ .

When the unemployment insurance benefit is implemented, steady-state welfare is  $-c + p_u + 2p_x - q$ . Steady-state welfare in the unsubsidized asymmetric information case is  $p_u - (1 - p_u)c + 2p_x(1 - p_u - q/2)$ . Thus welfare in the subsidized case is seen to exceed total welfare in the unsubsidized case by  $p_u(2p_x - c)$ , which, from condition (32), is at least as great as  $p_u c$ . Thus the unemployment insurance benefit improves welfare in the asymmetric information case. It can not, however, improve welfare in the symmetric information case, at least not if one takes Pareto-superior equilibria only, because they coincide with first-best optima.

Could unemployment insurance be privately provided in this setup, thereby eliminating the need (in the asymmetric information case) for government intervention? The risk of insuring against the outcome of  $x$  is an aggregate risk, against which private insurance could not insure.

Although we are assuming that the government knows neither  $x$  nor  $u$ , so that it can not predict in advance the size of the unemployment insurance payment, it could collect the necessary taxes, say in the next period, and break even.

What this example shows is that the information structure among the agents in the economy makes a good deal of difference to the policy implications of a model. If the private information regime is true (and our empirical work indicates that this is indeed so), then the optimal government policy is to subsidize labor-force entry.

References

- Ashenfelter, Orley and Card, David. "Time Series Representations of Economic Variables and Alternative Models of the Labor Market." Rev. Econ. Stud. 69 (1982): 761-82.
- Barro, R.J. "A Capital Market in an Equilibrium Model of the Business Cycle." Econometrica 48, no. 6 (September 1980): 1393-1417.
- Cogan, John F. "Fixed Costs and Labor Supply." Econometrica 49, no. 4 (July 1981): 945-64.
- Grossman, S.J. and Weiss, L. "Heterogeneous Information and the Theory of the Business Cycle." J.P.E. 90, no. 4 (July 1981): 945-64.
- Hirschleifer, Jack. "The Private and Social Value of Information and the Reward to Incentive Activity." A.E.R. 61 (September 1971): 561-74.
- Jovanovic, Boyan. "Entry with Private Information." Bell Journal of Economics, 12, no. 2 (Autumn 1981): 649-60.
- Karni, E. "A Note on Lucas' Equilibrium Model of the Business Cycle." J.P.E. 88, no. 6 (December 1980): 1231-36.
- Kydland, Finn E. and Prescott, Edward C. "Time to Build and Aggregate Fluctuations." Econometrica 50, no. 6 (November 1982): 1345-70.
- Laffont, J. "On the Welfare Analysis of Price-Revealing Equilibria." Toulouse University, June 1983. Presented at the 1983 summer meetings of the Econometric Society.
- Lillien, David M. "The Cyclical Pattern of Temporary Layoffs in U.S. Manufacturing." Review of Econ. and Stat. (1979): 24-31.
- Long, John B., and Plosser, Charles I. "Real Business Cycles." J.P.E. 91, no. 1 (February 1983): 39-69.
- Mayer, Thomas. "Plant and Equipment Lead Times." J. Business 33, no. 2 (April 1960): 127-32.
- Mincer, Jacob. "Labor Force Participation and Unemployment: A Review of Recent Evidence." in R.A. Gordon and M.S. Gordon (eds.), Prosperity and Unemployment (New York, Wiley, 1966).
- Neftçi, Salih N. "A Time Series Analysis of the Real Wages-Employment Relationship." J.P.E. 86, no. 2 (Pt. 1) (April 1978): 281-92.
- Pratt, J. "Concavity of the Log Likelihood." J. American Stat. Assoc. 76 (1981): 103-6.



- Prescott, Edward C., and Mehra, Raj. "Recursive Competitive Equilibrium: The Case of Homogeneous Households." Econometrica 48 (1980): 1365-79.
- Rubinstein, Ariel. "Perfect Equilibrium in a Bargaining Model." Econometrica 50, no. 1 (January 1982): 97-109.
- Sargent, Thomas J. "Estimation of Dynamic Labor Demand Schedules Under Rational Expectations." J.P.E. 86, no. 6 (December 1978): 1009-44.
- Wilson, Robert. "A Bidding Model of Perfect Competition." Rev. Econ. Stud. (1977): 511-18.

Footnotes

1. Ashenfelter and Card (1982, p.765) also express surprise at the lack of contemporaneous correlations of surprises of four different aggregate variables.

2. Existing common information models can not explain the bulk of the entries in Table 1. The Long and Plosser model implies zeros for all correlations involving unemployment and participation (labor supply in their model is constant). Models in which leisure is intertemporally substitutable, such as the Kydland and Prescott (1982) model, imply elastic response of labor supply to (partially temporary) labor-demand shocks. We fail to find any relation between participation and demand-shocks (see the discussion of Tables 5 and 6).

3. In what follows, it makes no difference which type of equilibrium one assumes. For an application of the symmetric mixed strategy equilibrium, see Jovanovic (1981).

4. For the average woman, Cogan estimates that the fixed costs of entry into the labor market amount to 28 percent of annual earnings. Our interpretation of  $c$  differs from Cogan's, however; we think of  $c$  as including only the cost of looking for work; we would exclude from it such costs as those of travelling to and from work.

5. We assume that the required net rate of return on investments is identically zero. The analysis would remain unchanged if a normal rate of return is added on to the right-hand side of eq. (2), so long as this rate does not depend on  $u$ .

6. If the rate of interest was  $q(x,u)$ , and monotone in both  $x$  and  $y$ , then if  $u = u_0$ , the workers could invert  $q(x,u_0)$  and infer the value of

x. The employers could similarly discover  $u$ , and the informational asymmetry would disappear.

7. Given our assumptions, once the worker has committed himself to the labor market, he has no incentive to leave the labor force in the same period. The incentive would be smaller still, if an unemployment insurance benefit were introduced into the analysis. The assumption that only one employer per period can be contacted does not seem restrictive when one considers the relative immobility of unemployed workers on temporary layoff -- only a negligible fraction find alternative employment (Lillien 1980).

8. This is not an unreasonable learning mechanism because the participation decision precedes the wage-offer decision. It cannot, however, be the employers' only source of information on  $u$ , because equilibrium may be such that for a subset of the  $u$ 's,  $n(u, \theta)$  is not strictly monotone (see Theorem 2). Most of the predictions of our model (and certainly those summarized in Table 2) would remain unchanged if, on a subset of the  $u$ 's, employers had only partial knowledge of  $u$ .

9. Log-concavity implies that  $d(\ln F)/d\varepsilon = f(\varepsilon)/F(\varepsilon)$  is monotonically decreasing. This implies that  $Ff' - f^2 \leq 0$ . Substituting from eq. (5) into eq. (6) for  $u - w$  requires that  $-2f + f'F/f$  is negative. But the latter expression is negative by log-concavity. This establishes the existence of a unique maximum on the interval  $(-\infty, x]$ .

10. Although  $x$  is a shock to labor productivity, it ends up being a shock to the demand for labor. In this sense, it is a demand shock.

11. Rubinstein (1981) has recently developed a theory of rent-division

based on costly bargaining that allows for the possibility of sequential counteroffers.

12. We assume for simplicity that  $f(s) > 0$  on the entire real line, so that corner solutions (i.e., maxima that do not satisfy eq. (5)) cannot arise.

13. We shall retain  $\theta$  in our notation throughout, so as to distinguish those observables which depend on  $\theta$  from those that do not; the distinction is important in our empirical work.

14. For any constant  $a$ ,  $\int_{-\infty}^a (a - \epsilon)f(\epsilon)d\epsilon = -\int_{-\infty}^0 zf(a - z)dz$  (after a change of variable  $z \equiv a - \epsilon$ )  $= \int_0^{\infty} zf(a - z)dz = \int_0^{\infty} F(a - z)dz$ , after integrating by parts. The last integral exists, because the mean of  $\epsilon$  exists.

15. A fully efficient procedure would be to jointly estimate (1), (2), and (3), noting that the MA representation of  $z(t)$  can be written as a function of the  $f(t-i)$  and  $e(t-i)$ . The computational effort in this joint methodology makes it impractical for this model.

16. In the unconstrained model, the estimated variance of the MPR equation crossed the zero boundary, a not uncommon problem in factor analytic models. The global maximum of the likelihood function whose model Chi-squared statistic was 3.58 with 10 degrees of freedom, is not significantly different from the model which constrains the variance to lie on the boundary.

AppendixA. Data and Sources

<u>Variable</u>	<u>Description and Source</u>
FPR	: Civilian Female labor force participation rate, age 20+; Bureau of Labor Statistics. Measured in tenths of percentage points.
FUR	: Civilian female unemployment rate, age 20+; Bureau of Labor Statistics. Measured in tenths of percentage points.
MPR	: Civilian male labor force participation rate, age 20+; Bureau of Labor Statistics. Measured in tenths of percentage points.
MUR	: Civilian male unemployment rate, age 20+; Bureau of Labor Statistics. Measured in tenths of percentage points.
INV	: Business capital expenditure for new plant and equipment, at an annual rate; Bureau of Economic Analysis. Deflated by Producer Price Index for capital equipment, 1967 = 100; Bureau of Labor Statistics. Measured in billions of 1967 dollars.
WAGE	: Real compensation per man-hour in manufacturing; BLS index (1977 = 100) multiplied by (Total compensation, mfg, 1977 / Total man-hours, mfg, 1977), from <u>Survey of Current Business</u> , Dept. of Commerce. Measured in 1977 cents per hour.
OUT	: Real output per man-hour in manufacturing; BLS index (1977 = 100) multiplied by (GDP, mfg, 1977 / Total man-hours, mfg, 1977), from <u>Survey of Current Business</u> , Dept. of Commerce. Measured in 1977 cents per hour.

All variables are seasonally adjusted, and reported quarterly for 1948 to 1982.

Table A.1 : Reduced Form Autoregressions

Dep. Variable :	FPR	FUR	MPR	MUR
INTERCEPT	0.099 (0.46)	0.092 (0.33)	0.122 (0.67)	0.219 (0.73)
FPR(-1)	0.822 (8.49)	-0.153 (1.23)	0.196 (2.39)	-0.249 (1.85)
FPR(-2)	0.093 (0.76)	0.169 (1.08)	0.018 (0.17)	0.242 (1.42)
FPR(-3)	-0.077 (0.65)	-0.198 (1.29)	0.047 (0.46)	-0.138 (0.83)
FPR(-4)	-0.009 (0.09)	0.015 (0.12)	-0.081 (0.99)	-0.088 (0.64)
FUR(-1)	0.042 (0.43)	0.721 (5.82)	0.096 (1.17)	0.229 (1.70)
FUR(-2)	-0.252 (2.07)	-0.182 (1.16)	-0.050 (0.48)	-0.340 (2.00)
FUR(-3)	0.123 (0.99)	-0.088 (0.55)	-0.232 (2.22)	-0.204 (1.18)
FUR(-4)	-0.026 (0.24)	-0.131 (0.93)	0.064 (0.70)	0.069 (0.45)
MPR(-1)	-0.264 (2.17)	-0.333 (2.14)	0.442 (4.32)	-0.152 (0.90)
MPR(-2)	0.242 (1.79)	0.303 (1.75)	-0.005 (0.04)	0.185 (0.98)
MPR(-3)	0.063 (0.45)	-0.049 (0.27)	0.109 (0.93)	-0.053 (0.27)
MPR(-4)	-0.006 (0.05)	0.035 (0.24)	0.216 (2.26)	0.255 (1.61)
MUR(-1)	-0.006 (0.05)	0.583 (4.25)	-0.071 (0.79)	1.285 (8.61)
MUR(-2)	0.021 (0.13)	-0.467 (2.30)	0.213 (1.59)	-0.424 (1.91)
MUR(-3)	0.035 (0.22)	-0.038 (0.18)	-0.137 (1.03)	-0.016 (0.07)
MUR(-4)	0.031 (0.29)	0.335 (2.49)	0.061 (0.69)	0.149 (1.02)
INV(-1)	0.133 (0.81)	-0.443 (2.09)	0.094 (0.57)	-0.475 (2.06)
INV(-2)	-0.369 (1.71)	0.119 (0.43)	-0.136 (0.75)	0.280 (0.93)
INV(-3)	0.174 (0.79)	0.115 (0.41)	0.029 (0.15)	-0.068 (0.22)
INV(-4)	0.030 (0.20)	0.175 (0.93)	-0.055 (0.44)	0.121 (0.59)
WAGE(-1)	0.058 (1.31)	0.012 (0.19)	0.052 (1.20)	0.003 (0.04)
WAGE(-2)	-0.081 (1.09)	0.097 (1.02)	0.011 (0.18)	0.045 (0.43)
WAGE(-3)	0.084 (1.15)	-0.097 (1.03)	-0.075 (1.22)	-0.131 (1.29)
WAGE(-4)	-0.005 (0.09)	0.110 (1.59)	0.010 (0.22)	0.172 (2.28)
OUT(-1)	-0.034 (1.20)	-0.034 (0.95)	-0.005 (0.22)	-0.034 (0.88)
OUT(-2)	0.039 (0.98)	0.081 (1.57)	0.037 (1.09)	0.070 (1.26)
OUT(-3)	-0.050 (1.56)	-0.104 (2.12)	-0.025 (0.77)	-0.119 (2.22)
OUT(-4)	0.057 (2.97)	0.050 (2.08)	0.024 (1.25)	0.061 (1.93)
R <sup>2</sup>	0.826	0.922	0.761	0.947

Table A.1 (continued)

Dep. Variable :	INV	WAGE	OUT
INTERCEPT	-0.124 (0.83)	-0.161 (0.39)	-0.346 (0.39)
FPR(-1)	0.059 (1.04)	-0.040 (0.21)	0.028 (0.07)
FPR(-2)	-0.136 (1.60)	-0.008 (0.03)	-0.738 (1.45)
FPR(-3)	-0.010 (0.12)	0.077 (0.33)	0.169 (0.34)
FPR(-4)	0.082 (1.21)	-0.122 (0.65)	0.574 (1.42)
FUR(-1)	-0.002 (0.03)	0.315 (1.70)	0.515 (1.28)
FUR(-2)	0.069 (0.82)	-0.144 (0.62)	0.093 (0.18)
FUR(-3)	-0.029 (0.34)	0.145 (0.61)	0.532 (1.03)
FUR(-4)	-0.095 (1.26)	0.109 (0.52)	0.075 (0.16)
MPR(-1)	0.052 (0.74)	0.255 (1.10)	-0.180 (0.35)
MPR(-2)	-0.038 (0.40)	-0.199 (0.77)	0.224 (0.40)
MPR(-3)	-0.026 (0.27)	-0.096 (0.36)	-0.264 (0.46)
MPR(-4)	0.022 (0.28)	0.138 (0.63)	-0.520 (1.10)
MUR(-1)	-0.139 (1.88)	0.030 (0.14)	0.232 (0.52)
MUR(-2)	0.034 (0.31)	-0.215 (0.70)	-0.731 (1.11)
MUR(-3)	0.106 (0.97)	0.015 (0.05)	0.388 (0.59)
MUR(-4)	-0.006 (0.08)	-0.077 (0.38)	-0.566 (1.30)
INV(-1)	1.014 (8.87)	0.267 (0.84)	1.750 (2.55)
INV(-2)	-0.006 (0.03)	-0.477 (1.16)	-1.056 (1.18)
INV(-3)	0.005 (0.03)	0.182 (0.43)	-1.283 (1.40)
INV(-4)	-0.141 (1.39)	-0.004 (0.01)	0.960 (1.58)
WAGE(-1)	-0.012 (0.36)	0.934 (9.44)	0.166 (0.77)
WAGE(-2)	0.005 (0.09)	-0.143 (1.01)	0.145 (0.47)
WAGE(-3)	0.058 (1.34)	0.123 (0.88)	-0.020 (0.06)
WAGE(-4)	-0.024 (0.64)	-0.119 (1.15)	-0.398 (1.77)
OUT(-1)	0.032 (1.65)	-0.031 (0.59)	1.138 (9.70)
OUT(-2)	-0.034 (1.24)	0.004 (0.04)	-0.724 (4.35)
OUT(-3)	0.052 (1.96)	0.039 (0.53)	0.634 (3.97)
OUT(-4)	-0.029 (1.89)	-0.019 (0.44)	-0.239 (2.53)
R <sup>2</sup>	0.955	0.815	0.897

Asymptotic t statistics are reported in parentheses.

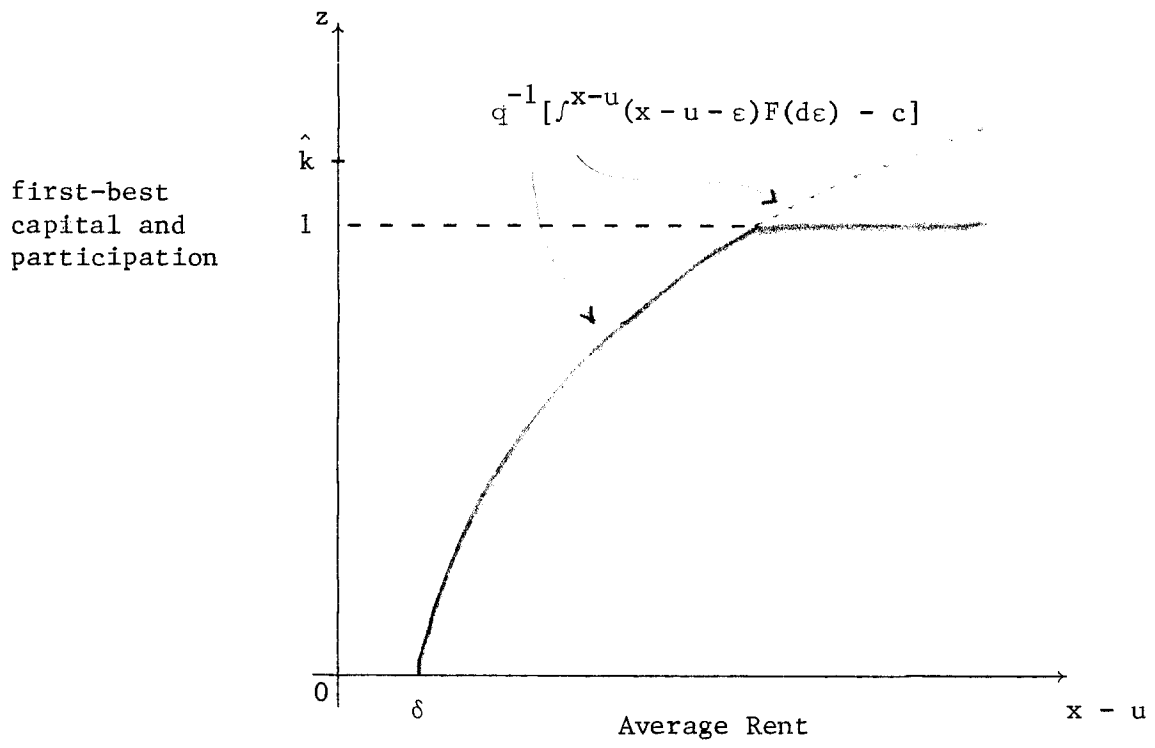


Figure 1. First-Best Capital and Labor



References

- Ashenfelter, Orley and Card, David. "Time Series Representations of Economic Variables and Alternative Models of the Labor Market." Rev. Econ. Stud. 69 (1982): 761-82.
- Barro, R.J. "A Capital Market in an Equilibrium Model of the Business Cycle." Econometrica 48, no. 6 (September 1980): 1393-1417.
- Cogan, John F. "Fixed Costs and Labor Supply." Econometrica 49, no. 4 (July 1981): 945-64.
- Grossman, S.J. and Weiss, L. "Heterogeneous Information and the Theory of the Business Cycle." J.P.E. 90, no. 4 (July 1981): 945-64.
- Hirschleifer, Jack. "The Private and Social Value of Information and the Reward to Incentive Activity." A.E.R. 61 (September 1971): 561-74.
- Jovanovic, Boyan. "Entry with Private Information." Bell Journal of Economics, 12, no. 2 (Autumn 1981): 649-60.
- Karni, E. "A Note on Lucas' Equilibrium Model of the Business Cycle." J.P.E. 88, no. 6 (December 1980): 1231-36.
- Kydland, Finn E. and Prescott, Edward C. "Time to Build and Aggregate Fluctuations." Econometrica 50, no. 6 (November 1982): 1345-70.
- Laffont, J. "On the Welfare Analysis of Price-Revealing Equilibria." Toulouse University, June 1983. Presented at the 1983 summer meetings of the Econometric Society.
- Lillien, David M. "The Cyclical Pattern of Temporary I Manufacturing." Review of Econ. and Stat. (1979):
- Long, John B., and Plosser, Charles I. "Real Business no. 1 (February 1983): 39-69.
- Mayer, Thomas. "Plant and Equipment Lead Times." J. (April 1960): 127-32.
- Mincer, Jacob. "Labor Force Participation and Unemplo Recent Evidence." in R.A. Gordon and M.S. Gordon and Unemployment (New York, Wiley, 1966).
- Neftçi, Salih N. "A Time Series Analysis of the Real Relationship." J.P.E. 86, no. 2 (Pt. 1) (April 1979):
- Pratt, J. "Concavity of the Log Likelihood." J. American (1981): 103-6.

- Prescott, Edward C., and Mehra, Raj. "Recursive Competitive Equilibrium: The Case of Homogeneous Households." Econometrica 48 (1980): 1365-79.
- Rubinstein, Ariel. "Perfect Equilibrium in a Bargaining Model." Econometrica 50, no. 1 (January 1982): 97-109.
- Sargent, Thomas J. "Estimation of Dynamic Labor Demand Schedules Under Rational Expectations." J.P.E. 86, no. 6 (December 1978): 1009-44.
- Wilson, Robert. "A Bidding Model of Perfect Competition." Rev. Econ. Stud. (1977): 511-18.