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ABSTRACT

This paper analyzes the time consistent and efficient solution for wages and employment in intertemporal insider-outsider models of bargaining with endogenous membership. We show that, in partial equilibrium, the presence of quits causes the only difference in the dynamics of employment between expansion and contraction. If one abstracts from quits, membership dynamics do not generate employment dynamics, and the adjustment to the steady state(s) is immediate. However, when the reservation wage is made endogenous in a general equilibrium context, insider-outsider models can generate a more fundamental asymmetry in the adjustment of employment to the steady state. In an upturn, employment increases gradually toward the new equilibrium, while in a severe downturn, employment first decreases sharply because of layoffs and then declines more gradually due to quits. In a mild recession, the firm and the union rely solely on quits.

INTRODUCTION

Models based on the insider-outsider distinction have made an important contribution to our understanding of why employment may only change slowly, if at all, after productivity or demand shocks. The papers by Blanchard and Summers (1986), Lindbeck and Snower (1984, 1987, 1988a, 1988b), Carruth and Oswald (1987), and Gottfries and Horn (1987) differ considerably in their details, but all rely on the fundamental difference in power and weight given to the interests of insiders and outsiders to explain the persistence of employment levels over the cycle.[1]

The purpose of this paper is to provide additional insights into the nature of the adjustment process in multiperiod insider-outsider models when it is assumed that agents are forward-looking and bargain efficiently about wages and employment after the shock to the firm's profit function has been revealed. In particular, we investigate whether the insider-outsider distinction may give rise to asymmetric responses for employment in expansions and recessions. The issue of which kind of asymmetry can be supported by this type of model is of great relevance, since progress in this area can shed light on the nature of business cycle fluctuations.[2]

We will show that, in partial equilibrium, quits generate the only difference in the adjustment pattern of output between expansions and contractions. More specifically, following a large, permanent, favorable shock, employment jumps to the new steady state. Following a large, unfavorable shock, on the other hand, the firm initially lays off workers and then relies on quits to reach the new steady state. If the negative shock is small, the firm relies solely on quits. If one were to assume that there were no quits, immediate adjustment would characterize both the upturn and the downturn.

Partial equilibrium models suffer from obvious limitations. The main contribution of our paper is to show that, in general equilibrium, insider-outsider models generate a fundamental asymmetry in the way employment adjusts during expansions and recessions. More specifically, when the reservation wage is made endogenous, employment reaches the new equilibrium, in an economic upturn, through a process of gradual adjustment. The downturn continues to be characterized, for large negative shocks, by an initial sharp drop in employment involving layoffs, followed by a more gradual decline caused by attrition.

The sharp initial decline and the gradual upturn are consistent with both Keynes's (1936) and Burns and Mitchell's (1946) view of business cycles. Recently, Neftci (1984), Delong and Summers (1986), and Sichel (1987) have provided empirical evidence in favor of employment and the unemployment rate behaving asymmetrically over the business cycle.

The structure of this paper is as follows. In section I, we set up the infinite horizon partial equilibrium model with forward-looking agents that bargain after the shock to the firm's profit function is known. We also discuss the nature of the contracts that result under these circumstances. In section II, we make the reservation wage endogenous in a general equilibrium framework and show how this generates a fundamental asymmetry in the adjustment of employment to the steady state. We also compare our results with the employment adjustment patterns implied by Kidd and Oswald's (1987) and Lockwood and Manning's (1987) intertemporal models, which allow for membership dynamics, and with the one-period models by Carruth and Oswald (1987) and Lindbeck and Snower (1988a, 1988b). Section III concludes the paper.

Note that these conclusions (and the ones to follow) do not change if we assume that the union maximizes the expected utility of its members. This is because, in the expected utility formulation, we simply divide the Nash maximand in (3) by total membership, $(1-q)\ell_{t-1}$, leaving the solution to the maximization problem unaltered.

What is the relationship between L^N and L^R? We can easily answer this question by comparing the first order conditions for the normal and the recession contracts. When employment exceeds membership ($\ell_t > (1-q)\ell_{t-1}$), the first order conditions for the wage and employment can be written respectively as:

$$u'(w_t)\left[\pi(w_t,\ell_t,s)+\delta\Pi(\ell_t,s)\right] + \left(u(w_t)+\beta\left[(1-q)V(\ell_t,s)+qU^*\right] - U^*\right)\pi_w(w_t,\ell_t,s) = 0 \quad (4)$$

$$\beta(1-q)V_\ell(\ell_t,s)\left[\pi(w_t,\ell_t,s)+\delta\Pi(\ell_t,s)\right] +$$
$$\left(u(w_t)+\beta\left[(1-q)V(\ell_t,s)+qU^*\right] - U^*\right) \left[\pi_\ell(w_t,\ell_t,s)+\delta\Pi_\ell(\ell_t,s)\right] = 0 \quad (5)$$

If employment falls short of the level of membership ($\ell_t < (1-q)\ell_{t-1}$), the first order condition for the wage, (4), remains unaltered, and the first order condition for employment becomes:

$$\frac{1}{\ell_t}\left(u(w_t)+\beta\left[(1-q)V(\ell_t,s)+qU^*\right] - U^*\right) \left[\pi_\ell(w_t,\ell_t,s)+\delta\Pi_\ell(\ell_t,s)\right] +$$
$$\beta(1-q)V_\ell(\ell_t,s)\left[\pi(w_t,\ell_t,s)+\delta\Pi(\ell_t,s)\right] +$$
$$\left(u(w_t)+\beta\left[(1-q)V(\ell_t,s)+qU^*\right] - U^*\right) \left[\pi_\ell(w_t,\ell_t,s)+\delta\Pi_\ell(\ell_t,s)\right] = 0 \quad (5')$$

We will assume throughout the paper that the first and second order conditions for the normal and recession contract yield unique solutions for L^N and L^R. Comparing the first order condition for employment in the recession and normal contracts, one sees that equation (5′) equals (5) with the addition of a (positive) additional term (see the first line of (5′)). As we prove in more detail in the Appendix (see Result 1), this implies that $L^R > L^N$, for all s. The intuitive explanation for this result is that, if a given value of the productivity shock leads to an expansion of employment beyond membership, we must have started with relatively few insiders who have no fear of losing their jobs and push for a high wage, leading to low employment. If, on the contrary, the same value of s causes a downturn, the fear of job loss among the (too many) insiders leads them to demand relatively low wages in order to preserve their jobs, and hence employment is higher.

The fact that L^N is less than L^R implies that there is also a third type of contract. When $L^N < (1-q)\ell_{t-1} < L^R$ neither a level of employment that exceeds membership nor one that falls short of it is a solution to (3). Instead, we get a corner solution to the bargaining problem in which employment is set equal to membership ($\ell_t = (1-q)\ell_{t-1}$). The wage is in this case still determined by (4). We will call this solution the hysteresis solution. Note that under the maintained assumption that $q > 0$, the normal contract is the only steady state in the model. In this steady state, new hirings just replace quits.

Finally, the solution for the borderline cases in which membership equals either L^N or L^R can be seen both as normal and recession contracts respectively, in which no hiring or layoffs occur, or as hysteresis contracts.

How does employment adjust to productivity shocks? In the Appendix (Result 2) we show that the sign of the derivative of L^N with respect to s depends on the specification of the revenue function. The effect would be negative if the effects of s on total revenue far outweighed its effect on marginal revenue ($r_s >> r_{1s}$). Then, in the normal contract, the wage would rise so much that employment would fall. However, we show that for the case where s affects the revenue function multiplicatively and the utility function is iso-elastic with a nonnegative coefficient of risk aversion, L^N increases with s. As for L^R, it is easy to show that it increases unambiguously with s if q = 0. If q > 0, the analysis becomes intractable (because L^R is then no longer a steady state), but we may assume that, for small q, the result still holds (and q is small in practice). In the remainder of the paper we assume that the problem is specified such that both L^N and L^R increase in s.

Assume that there is a permanent, unanticipated, favorable change in productivity such that $(1-q)\ell_{t-1} < L^N$. Then employment immediately jumps to its new steady state value L^N. After that, hiring will simply replace quits. Assume that there is a small negative shock such that $L^N \leq (1-q)\ell_{t-1} \leq L^R$. Then the firm and the union rely on quits to further reduce employment to L^N. Assume finally that there is a large negative shock such that $(1-q)\ell_{t-1} > L^R$. In this case there will be immediate lay-offs to reach a level of employment equal to L^R, and then quits will be used to reduce the number of workers to L^N. The model with an exogenously given reservation utility, therefore, generates a different pattern of adjustment to the steady state in booms and recessions. The source of the difference, however, is essentially the existence of nonzero quits. If we take the limiting (and unrealistic) case of no quits, both L^N and L^R are steady states of the model. This implies that the adjustment to the steady state(s), following large shocks, is immediate both in upturns and in downturns. Moreover, any value of employment that lies between L^N and L^R is also a steady state equilibrium of the model. Another way to look at it is that, starting from any given level of membership and employment, small positive or negative changes in s will leave employment at the initial level. That a range of equilibrium levels of employment exists is a major implication of the one-period models by Carruth and Oswald (1987) and Lindbeck and Snower (1988a, b)[7]. However, as we discussed above, the introduction of quits reduces the steady state employment level to a single value denoted by L^N.[8]

II. ENDOGENOUS RESERVATION LEVEL OF UTILITY AND DYNAMICS

The intertemporal insider-outsider model developed in the previous section is a useful benchmark. However, it suffers from the obvious limitations of being a partial equilibrium model. In this section we extend it to a general equilibrium setting by making the reservation level of utility endogenous. We are particularly interested in what implications the endogenous reservation utility has for the dynamics of employment. Our central objective is to show that it generates a richer dynamic and a fundamental asymmetry between upturns and downturns.

We assume that the economy consists of n+1 identical firm-union pairs, with n large, which all experience the same unexpected permanent productivity shock s. The value of s now affects the macro wage and employment outcomes, and hence, it may affects U^*, the fall-back position of a union member.

We will show that this is the fundamental reason why the adjustment of employment is different during expansions than it is during contractions. We are fully aware that the specification of the fallback position/threat point in bargaining models is open to debate. For instance, it has been suggested that it may be appropriate to use the level of utility during a strike as the threat point.[9] However, our fundamental point remains valid, provided that union utility depends, inter alia, upon the probability of getting a job outside the firm. This would, for example, be the case if we assume directly that union utility equals the rents of its employed members, that is, the difference between their utility under the contract and the utility they would have enjoyed had they started the period in a state of unemployment. With these caveats in mind, we will write the equation for U* as:

$$U^*(\ell_{t-1},s) = p_t[u(w_t) + \beta V(\ell_t,s)] + (1-p_t)[u_h + \beta U^*(\ell_t,s)] \tag{6}$$

where p_t is the probability of finding another job. This equation states that, after a breakdown of the negotiations at a particular firm in period t, its workers have a probability p_t of finding another job. If they succeed, they get the contract wage for the current period plus the expected present discounted utility associated with starting out as a union member (although in another firm) in the next period[10]. Otherwise, they have to resort to household production for the current period, which is assumed to provide a constant utility level u_h per period. Their expected discounted utility, therefore, equals u_h for the current period plus the expected present discounted utility associated with starting out as an outsider in the next period. Since all firms are identical, wage rates are equal across firms in equilibrium.

In a recession, there are no job openings and $p_t = 0$. From (6), workers' fall back position equals $u_h + \beta U^*(\ell_t,s)$, which is independent of past employment levels. Therefore, the first order conditions in a general equilibrium downturn do not depend upon past employment, but only on s. In particular, for a given s, all recession contracts still specify the same level of employment (which we again denote by L^R). This means that, if the change in s is such that the economy experiences a downturn, the results from the partial equilibrium section carry over to the general equilibrium case: the downturn may involve a sudden downward jump in employment achieved through layoffs. However, if layoffs occur, they occur only once and immediately. The firm and union then rely on quits to obtain further reductions in employment.

If employment expands, there are extra new job openings. In each firm there are $\ell_t - m_t$ job openings, and since there are n other firms, there are $n(\ell_t - m_t)$ job openings in the economy. Let f be the total labor force. Then, if any one firm does not reach a contract agreement with its workers, there will be $f-nm_t$ unemployed workers, and the probability of finding a job equals[11]:

$$p_t = \frac{n(\ell_t-m_t)}{f-nm_t} \tag{7}$$

From equation (6) and (7), it follows that U^*_t is now a function of the entire vector of present and future wages and of current membership, m_t. Of course, the fallback utility is still regarded as exogenous by the individual union-firm pair. Equations (6) and (7) are general equilibrium conditions. It is still true, moreover, that ultimately the solution to the infinite horizon problem is a function of $m_t = (1-q)\ell_{t-1}$ only (which we assume to be the same for all firms). However, it is not true that all normal contracts specify the same level of employment and wages given s. Since U^*_t is a function of ℓ_{t-1}, ℓ_{t-1} does not drop out of the first order conditions. Therefore, in an economic upturn, employment will now depend the past level of employment.[12]

This suggests that the adjustment to the steady state is going to be gradual. Indeed we can prove that this is the case under two assumptions. The first one is that, for all $q > 0$, there is a unique steady state value of employment, namely L^N. The second is that the solution for employment in a normal contract is a continuous function of the exogenous quit rate q. The proof consists of three parts. We first prove that employment in an upturn cannot jump to it steady state value L^N. We then show that the adjustment to the steady state in an upturn is gradual if $q = 0$. Finally, we show that employment adjustment in an upturn is gradual for any $q > 0$. Since the proof is rather long, it is confined to the Appendix (see Result 3). However, the intuitive explanation for this result is quite clear. When employment expands, the value of fallback utility exceeds its steady state value, basically because there are a lot of job openings. In the steady state there are only a few job openings; these are created when workers quit. We expect that a higher level of U^* leads to higher wages and lower employment. This is because a higher threat point for the union leads to a stronger union bargaining position and thus to a higher level of utility. Since in an upturn union utility only depends upon the wage, the wage is likely to rise. In an upturn the firm is the dominant agent in employment determination, and we expect, therefore, that a higher wage leads to lower employment, compared to the case where the reservation level of utility has reached its steady state value. If we repeat this argument period after period, it will follow that the complete solution in an upturn consists of a gradual adjustment to the steady state.

How do our results concerning the dynamics of employment relate to the conclusions reached by other authors? The two papers most closely related to ours are by Kidd and Oswald (1987) and Lockwood and Manning (1987).[13] Both describe partial equilibrium models, in which the reservation level of utility is exogenous. Kidd and Oswald use an intertemporal monopoly union model, abstract from the existence of quits, and analyze only the employment response to a negative shock.[14] They conclude that the adjustment to the steady state is immediate. The no-dynamics result follows if the union utility function is additively separable in employment and membership, because then membership does not enter the first order condition for employment. This separability holds if union utility has a utilitarian formulation and equals the sum of the utility of its members, but it does not hold if it equals the expected utility per member. So, in the former case (which they analyze) there is no dynamic adjustment, while in the latter case there is. Moreover, the two formulations lead to different steady states. In our model, the maximand is the product of two gain functions, and, under the special assumption of zero quits, we also find that the adjustment to the steady state in a downturn is immediate. The no-dynamics result depends, however, on the multiplicative separability of employment and membership in the union gain function (so that membership drops out of the first order conditions, and the link between current and past employment, which leads to the dynamics, is broken). As shown above, this separability holds in both the utilitarian and the expected gain formulations, and, in fact, it does not make any difference which formulation is chosen. More generally, if we allow for quits, the downward adjustment to the steady state is not immediate. The initial layoffs are followed by a further reduction in employment generated by quits. This is true both in our general equilibrium and partial equilibrium models.

It is also interesting to compare our model that of Lockwood and Manning (1987), who (numerically) provide a time-consistent solution to a right-to-manage model in which both firm and union are forward-looking.[15] In their paper, employment does not jump immediately to the steady state in an upturn, even if reservation utility is exogenous. There are two reasons for this. First, in order to be able to solve the model, they maximize the sum of the firm and union gain functions instead of their product. As a result, membership no longer enters the maximand multiplicatively, and it no longer drops out of the first order conditions. Moreover, using the sum of the gain functions as a maximand has the unsatisfactory consequence that the type of adjustment (gradual versus oscillatory) depends critically upon whether the utilitarian or the expected union gain function is adopted. The second reason for the dynamic adjustment is that Lockwood and Manning assume that membership depends, at least partly, upon the level of present employment. As a result, membership at the time of contract negotiations is no longer predetermined. Membership (and with it, past employment) would, therefore, enter the first order conditions even if the usual procedure of maximizing the product of the two gain functions had been followed. In our model the fundamental reason for the gradual adjustment in the upturn is, instead, the endogenous nature of the fall back position.

A final difference between these two papers and ours is their assumption that the firm sets employment; we assume that the firm and the union reach an efficient wage and employment bargain. However, if we had used a right-to-manage model as well, our results concerning the dynamics of adjustment would not have changed, since the basic structure of the problem remains unaltered. The intuitive explanation is that membership would still enter the Nash maximand multiplicatively (or not at all) and it would be predetermined during the negotiation of the contract. It follows that membership does not appear in the first order conditions, unless it is a determinant of the reservation level of utility in general equilibrium.[16]

III. CONCLUSIONS

In this paper we have shown that insider-outsider models can generate in general equilibrium a very rich pattern of dynamic employment adjustment during the business cycle. For large shocks, moreover, they give rise to a fundamental asymmetry between expansions and contractions. While upturns are gradual, downturns are characterized by a sharp drop in employment, followed by a further reduction due to quits. In mild recessions, the reduction in employment is achieved only through quits. The basic reason for the difference between expansions and recessions is that the reservation level of utility in an upturn depends upon the nonzero probability of finding another job. This probability is a function of the number of insiders, which, in turn, depends on the level of past employment. If this link between current and past levels of employment is broken by the assumption that the reservation utility is constant, then the convergence to the steady state in an upturn is immediate. Since in a downturn there is no probability of finding a job, the endogenization of reservation utility does not affect the nature of the adjustment, and the only dynamics are generated by quits.

The adjustment pattern implied by our general equilibrium model is consistent with the time series evidence that employment and unemployment changes are more persistent in expansions than in recessions. Certainly, other factors we have abstracted from in our analysis may account for the different nature of upturns and downturns (unequal hiring and firing costs, etc.). However, we have shown in this paper that insider-outsider models can contribute to explaining asymmetries in business cycles.

FOOTNOTES

[1]The asymmetric treatment of different groups of workers is an appealing idea already discussed in Nickell and Andrews (1983) and in McDonald and Solow (1984), and central to papers by Lindbeck and Snower (1984) and Solow (1985).

[2]See Begg et al. (1988) for an initial overview of the issues in this area.

[3]For more complex models of union memberhip, see Grossman (1983) and Booth (1984).

[4]If the union bargaining power is justified on the basis of replacement costs (as in Lindbeck and Snower 1988a, b), we are implicitly assuming that those costs are infinite: the firm simply cannot replace its existing workforce. If we assume less than infinite replacement costs, the alternative for the firm would be to replace all the insiders with outsiders generating a profit of, say, Π^*. If we assume that Π^* is constant, nothing fundamental changes in the conclusions derived from our model, except that an upper limit for the wage is provided.

[5]We assume here that all workers are paid the same wage and that they do not receive private unemployment insurance in order to maintain consistency with most economic models of trade union behavior and because two-tier wage systems and private unemployment insurance are not very frequent in practice. However, we are conscious that the literature has not provided a fully satisfactory explanation of these phenomena.

[6]The crucial element that characterizes the partial equilibrium model in this section is that U^* is constant through time. This would be the case if we assume that the rest of the economy is in a steady state. Alternatively, we could assume that workers who leave the firm for any reason (layoff, quits or contract breakdown) can never find another job again and have to resort to household production, which provides constant utility u_h per period. Then U^* equals $u_h/(1-\beta)$ and is constant as well.

[7]Lindbeck and Snower also allow explicitly for labor turnover costs.

[8]Outside the context of insider-outsider models, McDonal (1987) has also suggested the existence of a range of equilibrium levels of employment in a model that combines trade union wage setting and lump-sum costs in changing prices. Unionized labor markets and oligopolistic price determination also give rise to a continuum of equilibria in Dixon's (1988) model, although in this case the multiplicity is generated by different levels and/or sectoral compositions of government spending.

[9]See, for instance, Binmore et al. (1986) and Sutton (1986). Binmore et al. suggest that the utility under a strike is the apprpriate choice for the threat point if the Nash solution is seen as the limiting solution of a model with impatient agents, when the length of the single bargaining period approaches zero. However, when the model is based on the exogenous risk of a terminal breakdown in negotiations, the use of a weighted average of the utility when employed in another job and when unemployed may be the appropriate threat point for the worker.

[10]Strictly speaking, the variables in equation (6) are the general equilibrium outcomes if a breakdown in the negotiations in one firm had

occurred. We assume that such a breakdown would have a negligible impact on the overall general equilibrium, because the number of firms is large.

[11]Note that, in equilibrium, it must be the case that $p < 1$, since otherwise the utility of a union member and of an unemployed worker would be the same, so that union gain would be zero, and the product of the union and firm gain functions would clearly not have been maximized. This implies that $(n+1)\ell < f$, that is, there must be unemployment in equilibrium, and also $\delta p/\delta m < 0$, that is, a reduction of the number of insiders raises, ceteris paribus, the probability of finding a job.

[12]A different scenario, as suggested by a referee, would be to think of the economy as consisting of a unionized primary sector and a perfectly competitive secondary sector. If labor demand in the secondary sector is downward sloping, the model becomes more complex, because the wage and hence u^*_t and U^*_t will in general equilibrium depend on primary sector employment through its effect on the labor supply in the secondary sector. It remains true, however, that U^*_t depends on membership and hence on past employment because of its effect on the probability of finding a job in the primary sector.

[13]The papers by Blanchard and Summers (1986) and Gottfries and Horn (1987) differ fundamentally from ours in that the wage is fixed before a stochastic shock to the revenue function is known.

[14]Note that, whereas the union in their model is forward-looking, the firm is myopic and hires along its static labor demand function. Moreover, their model simplifies union behavior by assuming that the union can precommit itself to future wage and employment outcomes.

[15]In another paper, Lockwood and Manning (1989) analyze the dynamics of employment generated not by membership considerations but by the presence of convex adjustment costs in changing the workforce.

[16]Note that Π is a function of w_t, ℓ_t and s. Differentiate Π with respect to ℓ_t to obtain the dynamic labor demand function in the right-to-manage model. Solving for the wage, we can write w_t as a function of ℓ_t and s. Using this to substitute out w_t in the Nash maximand shows that this maximand can be written as a function only of ℓ_t and s in the downturn. m_t enters multiplicatively in the upturn. For these reasons our basic conclusions concerning the nature of the adjustment to the steady state still hold.

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APPENDIX

RESULT 1: $L^R > L^N$.

Proof.
The infinite horizon problem can be written as:

$$\max_{w_t,\ell_t} \min(\ell_t,m_t)P(w_t,\ell_t;s) \tag{A1}$$

where P is the product of the gain functions for the individual worker and the firm (see equation (2) in the main text). Dropping the subscript t for notational simplicity, we can write the first order conditions for the normal contract as:

$$P_w = 0 \quad \text{and} \quad P_\ell = 0 \tag{A2}$$

and for the recession contract as:

$$P_w = 0 \quad \text{and} \quad \ell P_\ell + P = 0 \tag{A3}$$

Unless $P = 0$, the two sets of first order conditions will give rise to two different solutions. To show that employment is lower in the normal contract, consider the system:

$$P_w = 0 \tag{A4}$$

$$P_\ell + C = 0 \tag{A5}$$

for any constant C. For $C = 0$ the solution to this system is the normal contract, yielding $\ell = L^N$, while for $C = P/\ell$ evaluated at $\ell = L^R$ we get the recession contract, yielding $\ell = L^R$. We show that $L^R > L^N$ by showing that, for any $C > 0$, (A4) and (A5) imply $\ell > L^N$.

Differentiating the system at N with respect to w, ℓ and C, and using the second order conditions, we find $\partial\ell/\partial C > 0$. Moreover, for any $C \neq 0$, (A4) and (A5) imply that $\ell \neq L^N$. Suppose, for instance, that for some $C = C' \neq 0$, (W',L^N) solves (A4) and (A5). Then (A5) implies that $W' \neq W^N$ because P_ℓ evaluated at (W^N,L^N) equals zero. So, (W',L^N) and (W^N,L^N) are two different solutions to (A4). But this cannot be, since using equation (4) in the text and substituting $\pi_w = -\ell$ twice we can write:

$$P_{ww} = u''\Pi - 2u'\ell$$

and this expression is negative for all (w,ℓ). This means that, for any given level of employment, there cannot be two values of the wage that satisfy $P_w = 0$. Hence, $\ell \neq L^N$ for $C \neq 0$. Concluding, since at L^N $\Delta\ell/\Delta C$ is positive for small ΔC and nonzero for any ΔC, it must be, assuming continuity of the solution of (A4) and (A5) in C, positive for all C. QED.

RESULT 2: $L^N(s)$ increases with s if s enters the revenue function multiplicatively and if the utility function is iso-elastic with a non-negative coefficient of risk aversion.

Proof.
We start with a lemma:

Lemma: The first and second left hand derivatives of $V(\ell_t,s)$ and $\Pi(\ell_t,s)$ with respect to ℓ_t are zero when evaluated at $\ell = L^N$.
Proof: Suppose $\ell_t \le L^N$ in period t. Then, in period t+1, agents will choose the normal contract according to equations (4) and (5) in the main text (with t replaced by t+1). Since ℓ_t drops out of these equations, all $\ell_t \le L^N$ will yield the same solution for the wage and employment in period t+1 (and hence also in all future periods). Thus $V(\ell_t,s) = V(L^N,s)$ and $\Pi(\ell_t,s) = \Pi(L^N,s)$ for all $\ell_t \le L^N$. Hence the left hand derivatives of $V(\ell_t,s)$ and $\Pi(\ell_t,s)$ with respect to ℓ_t are zero at $\ell_t = L^N$.

Using this lemma, the first order conditions for the normal contract, equations (4) and (5) in the main text, reduce to (omitting the arguments of the functions and the time subscripts for simplicity of notation):

$$u'[\pi + \delta\Pi] + \{u + \beta[(1-q)V+qU^*] - U^*\}\pi_w = 0 \tag{A7}$$

$$\pi_\ell = 0 \tag{A8}$$

Since we are in a steady state, $\Pi = \pi/(1-\beta)$. Define $\bar{u}^* = (1-\beta)U^*$. $\bar{u}^*$ can be thought of as the average expected utility over time of an unemployed worker. Since U^* is a constant, so is $\bar{u}^*$. Since in the steady state union members will not be laid off, $V = u + \beta[(1-q)V+qU^*]$. Solving for V and substituting the result into (A7), we can rewrite (A7) as:

$$u'\pi - \alpha(u - \bar{u}^*)\ell = 0 \tag{A7'}$$

where $\alpha = (1-\delta)/(1-\beta+\beta q)$. Differentiating (A7') and (A8) fully with respect to w, ℓ and s and using Cramer's rule we find:

$$\frac{d\ell}{ds} = -\frac{[u''\pi - (1+\alpha)u'\ell]r_{\ell s} + u'r_s}{\Delta} \tag{A9}$$

where Δ is the determinant of the Hessian of (A7') and (A8). Using the above lemma, it is straigtforward to show that Δ is greater than the determinant of the Hessian of the second order conditions for the normal contract. Since the latter is positive, Δ must be positive as well.

Since we assumed that both total and marginal revenue increase with s ($r_s > 0$ and $r_{\ell s} > 0$), the sign of $d\ell/ds$ is ambiguous. The sign of $d\ell/ds$ is negative if the increase in s mostly increases inframarginal revenue ($r_s \gg r_{\ell s}$). Then the increase in profits would through the bargaining process result in a larger increase in the wage than in marginal revenue, and hence steady state employment would fall.

However, $d\ell/ds$ is unambiguously positive if we specify the utility function as $u = (w^{1-\eta})/(1-\eta)$, with $\eta \ge 0$ (non-negative coefficient of relative risk aversion) and the revenue function as $r = sf(\ell)$. Then, using (A9), the condition for $d\ell/ds$ being positive reduces to:

$$\alpha w \ell > (1-\eta)\pi \qquad \text{(A10)}$$

while the first order condition for employment, equation (A7′), can be rewritten as: $\alpha w \ell = (1-\eta)\pi + \alpha \bar{u}^* \ell / u$. Thus, condition (A10) is indeed met, and $dL^N/ds > 0$ for this class of utility and revenue functions. QED.

RESULT 3: If past employment is less than its steady state value L^N, employment will gradually converge to L^N.

Proof.
The proof is by contradiction and consists of three parts. First we prove that employment cannot immediately jump to its steady state value L^N. We then prove that employment adjustment is gradual if $q = 0$. Finally we show that employment adjustment is gradual for any $q \geq 0$. Throughout the proof we assume that the steady state is unique and that the solution for the normal contract is a continuous function of q.

Part I: proof that if employment was less than L^N in the previous period, employment cannot be equal to L^N in the current period.

Suppose that employment was below L^N in the previous period and equal to L^N in the current period. The first order conditions are again represented by (A7) and (A8). They yield a level of employment equal to ℓ (the subscript t is again omitted). (A8) implies that if $\ell = L^N$, $w = W^N$. Morover, $\ell = L^N$ implies that $\ell = L^N$ and $w = W^N$ for all future periods as well. Then, from (A7) it follows that $U^* = U^{*N}$, where U^{*N} is the steady state value of U^*. Equation (6) in the main text now implies that p, the probability of finding a job this period, is at its steady state value as well. However, since employment last period was assumed to be below its steady state value, membership this period is below its steady state value as well, and from equation (7) in the text we see that the probability of finding a job this period must be higher than in the steady state (see also footnote 12). We have, therefore, a contradiction and hence the initial assumption cannot hold, that is, from a previous level of employment lower than L^N, ℓ cannot jump to L^N. QED. Note that this proof holds for all q.

Part II: proof that for the case of $q = 0$, $\ell_{-1} < L^N$ implies $\ell_{-1} < \ell < L^N$, and hence the adjustment to the steady state is gradual.
This part of the proof consists of two sections. In IIa we show that $\ell_{-1} < L^N$ implies $\ell_{-1} < \ell$. In IIb we show that $\ell < L^N$.

IIa: We first prove that $\ell_{-1} < L^N$ implies that $\ell > \ell_{-1}$. We start by remarking that, if $q = 0$, L^N and L^R are steady states to the model. In a steady state with $q = 0$, employment is constant and there are no layoffs, so there are no job openings either. Thus, $U^* = u_h / (1-\beta)$ forever. In both contracts, the solution obeys the relevant first order conditions in the partial equilibrium section of the paper, with $U^* = u_h / (1-\beta)$. Therefore, Result 1 in this Appendix applies, and it must be the case that, in general equilibrium with $q = 0$, L^R exceeds L^N. It follows that there cannot be a recession contract with a level of employment less than L^N, since, as argued in the text, all recession contracts yield the same solution. Therefore, if $\ell_{-1} < L^N$, it cannot be the case that $\ell < \ell_{-1}$, since such a contract would be a recession contract with a level of employment less than L^N. Moreover, ℓ cannot equal ℓ_{-1}, because in that case ℓ_{-1} would be a steady state, and thus U* would equal $u_h/(1-\beta)$. However, given a value of $U^* = u_h/(1-\beta)$, individual firm union pairs set employment equal to L^N. So, a steady state with employment less than L^N is inconsistent with individual firm-union optimization.

IIb: In order to prove that $\ell_{-1} < L^N$ implies that $\ell < L^N$, we first prove that the left hand derivative $d\ell/d\ell_{-1}$ is positive at L^N, that is, if ℓ_{-1} is infinitesimally less than L^N, so is ℓ. We already know from part I that the left hand derivative $d\ell/d\ell_{-1}$ is not zero at L^N. So, we only have to show that the left hand derivative $d\ell/d\ell_{-1}$ cannot be negative at L^N. The argument is again by contradiction.

Suppose the left hand derivative $d\ell/d\ell_{-1}$ is less than zero at L^N. Then, if ℓ_{-1} is marginally below L^N, ℓ is a little above L^N, causing an economy wide upturn in the current period. However, assuming continuity of ℓ with respect to ℓ_{-1}, ℓ is marginally above L^N, and employment has fallen into the hysteresis range, and all future contracts will be hysteresis contracts, in which wage and employment levels remain constant.

The way an economy-wide upturn affects individual firms is through the time path of U*. However, in this case, U* will equal U^{*N} as of next period. So, u*, the expected utility of being unemployed this period, completely summarizes the effect of the upturn on the individual firm. Note that if $q = 0$, the steady state value of u* equals u_h, the per period utility derived from household production. Therefore, it must be that individual firm union pairs, who in the previous period all had a level of employment equal to $\ell_{-1} < L^N$, react to a level of $u^* > u^*_h$, by raising employment above L^N. We know that if $u^* = u^*_h$, then individual firms will set ℓ equal to L^N (since then $U^* = U^{*N} = u_h/(1-\beta)$). It must, therefore, be the case that $d\ell/d\ell_{-1} < 0$ at L^N implies that the right hand derivative $d\ell/du^*$ is positive at L^N in a normal contract. We now show that this cannot be the case. In order to do this, we first need a lemma. To condense the notation we define P again as the product of the gain functions of the individual worker and the firm.

Lemma: As in Result 1, define P as the product of the gain functions of the individual worker and the firm. Then, if a normal contract involves a jump into the hysteresis range, $P_{w\ell} < 0$ and $\pi_\ell + \delta\Pi_\ell > 0$.
proof: by contradiction.

Suppose a normal contract involves a jump into the hysteresis range. Then, as of next period, the same hysteresis contract will result forever, with constant wage and employment levels. Thus a union member's utility as of next period, $V(\ell,s)$, simply equals the discounted utility of receiving that hysteresis contract's wage forever. That wage is determined by the equations $P_w = 0$ (where $P_{ww} < 0$ to satisfy the second order conditions) and by the fact that employment remains equal to current employment ℓ. Since the current contract is a normal contract, current employment is determined by $P_w = 0$ and $P_\ell = 0$.

Suppose that $P_{w\ell} \geq 0$. Then the effect of current employment on future wages is given by $dw/d\ell = -P_{w\ell}/P_{ww} \geq 0$. Thus, $V_\ell(\ell,s) \geq 0$ as well. Since $P_\ell = \beta V_\ell[\pi+\delta\Pi] + \{u+\beta[(1-q)V+qU^*] - U^*\}[\pi_\ell+\delta\Pi_\ell] = 0$, it follows that

$\pi_\ell+\delta\Pi_\ell \leq 0$. Since $P_{w\ell} = \beta V_\ell \pi_w + u'[\pi_\ell+\delta\Pi_\ell] + (u+\beta[(1-q)V+qU^*] - U^*)\pi_{w\ell}$, it follows after substituting $\pi_w = -\ell$ and $\pi_{w\ell} = -1$ that $P_{w\ell} < 0$, which is contrary to our assumption. Therefore, if a normal contract involves a jump into the hysteresis range, then $P_{w\ell} < 0$, which implies $V_\ell(\ell,s) < 0$ and $\pi_\ell+\delta\Pi_\ell > 0$. This proves the lemma.

We now differentiate the system of first order conditions for the normal contract totally with respect to w, ℓ and u*. We find:

$$\begin{bmatrix} F_{ww} & F_{w\ell} \\ F_{w\ell} & F_{\ell\ell} \end{bmatrix} \begin{bmatrix} dw \\ d\ell \end{bmatrix} = \begin{bmatrix} -\ell \\ \pi_\ell+\delta\Pi_\ell \end{bmatrix} du^*$$

from which it is easily seen that $d\ell/du^* < 0$. Thus it indeed cannot be the case that the right hand derivative $d\ell/du^*$ is positive at L^N in a normal contract. Hence, the assumption that, in general equilibrium, the left hand derivative $d\ell/d\ell_{-1}$ is negative at L^N is inconsistent with individual firm-union behaviour. Thus it follows that the left hand derivative $d\ell/d\ell_{-1}$ must be non-negative. We already showed in part I that $\ell \neq L^N$ for any $\ell_{-1} < L^N$. Therefore, assuming that ℓ is continuous in ℓ_{-1} in a normal contract, it follows that, for the case where $q = 0$, $\ell < L^N$ for all $\ell_{-1} < L^N$.

Combining the results of sections IIa and IIb, we conclude that, if $q = 0$ and $\ell_{-1} < L^N$, then $\ell_{-1} < \ell < L^N$. QED.

Part III: proof that for the case of $q \geq 0$, $\ell_{-1} < L^N$ implies $\ell_{-1} < \ell < L^N$.

We assume that L^N is the unique steady state and, furthermore, that current employment, ℓ, and steady state employment, L^N, are continuous functions of q. We know that the result $\ell_{-1} < \ell < L^N$ holds in the case of $q = 0$. Now consider a gradual increase in q. Then both ℓ and L^N may change. But it can never be that $\ell = \ell_{-1}$, because then ℓ_{-1} would be a steady state in addition to L^N, contrary to our assumption that the steady state is unique. Also, ℓ cannot be equal to L^N by Part I of Result 3. Since we assume that both ℓ and L^N are continuous functions of q, it must, therefore, remain true that $\ell_{-1} < \ell < L^N$, as long as $\ell_{-1} < L^N$. QED.