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Charles A.M. de Bartolome

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C. V. STARR CENTER FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

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New York University

ABSTRACT

"The Ramsey Equations," which characterize the optimal indirect tax structure, are interpreted. The price derivatives of compensated demand are important because they measure the gain from substituting the collection of tax revenue from one tax instrument to others.

Proposed running title: The Ramsey Equations.

Address for correspondence: Charles A.M. de Bartolome, Department of Economics, New York University, 269 Mercer Street, New York, NY 10003.

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1. INTRODUCTION

"The Ramsey Equations," which specify the optimal indirect tax structure, are a foundation of modern tax theory. These equations are frequently written using changes in compensated demands (e.g. Atkinson and Stiglitz (1980, Equation 12-53)). At the optimum tax structure, intuition suggests that the welfare loss associated with the collection of marginal tax revenue is the same for each tax instrument. It is hard to reconcile this intuition with the equations because, of course, all changes in compensated demand are made with the utility of each household - and therefore welfare - being held constant. This note provides an intuitive explanation for the way in which changes in compensated demand enter the equations.

Because the distortion of a tax is related to the substitution effect, the presence of compensated demands in The Ramsey Equations is always held to be self-evident. For example, Atkinson and Stiglitz (1980, p.373) write: "The importance in this formula of the *compensated* derivatives accords with intuition: the income effect would arise with any form of taxation, and the distortion stems from the substitution effect." This explanation is incomplete, because it does not explain the precise form in which the compensated derivatives enter. It is also puzzling. The distortion is associated with a fall in household utility, whereas a substitution effect is associated with a change in household consumption holding utility constant. In fact, it is the substitution between tax instruments which is relevant. Compensated derivates enter because tax revenue is affected if one tax instrument is substituted by another.

Since Dupuit (1844), it has been known that a tax structure is to be evaluated by the changes it induces in demand. Ramsey's (1927) contribution was to make this operational, by showing that the utility loss of the representative household is minimized when the tax structure causes an equi-proportionate fall in the consumption of all taxed commodities - ignoring income effects and assuming that the tax revenue requirement is small. As intuitively the first-order conditions would be expected to equalize the marginal utility loss associated with the use of each tax instrument, the reader might be excused for thinking that The Ramsey Equations compare the effect of the induced quantity changes on welfare. As indicated earlier, in this context the importance of compensated derivatives is puzzling, as there is no utility loss if all demand changes are along an indifference curve. My explanation makes clear that The Ramsey Equations are associated with the dual problem, of choosing tax rates to maximize tax revenue subject to the achievement of a given welfare level. The quantity changes present in the formulae represent the marginal effect of the tax rate on the tax base, and reflect the traditional concern of the policy-maker that the effect of raising tax rates is partly offset by shrinkage in the tax base.

This paper extends de Bartolome (1992), who interprets The Ramsey Equations if the economy may be described by a representative household. With a representative household, maximizing utility is equivalent to minimizing the excess burden (Dimaond and McFadden (1974) and Auerbach (1985)), and the interpretation stresses excess burden and consumer surplus loss. This paper interprets The Ramsey Equations when households differ. Both analyses stress the fiscal externality associated with indirect taxes. Compensated demands are important because the substitution of the use of one tax instrument by others affects household consumption. The fiscal externality arises because the change in quantities affects the tax revenue of the government.

An intuitive explanation of The Ramsey Equations seems desirable in view of their wide use in the optimal tax literature. The modern formulation of the equations for the representative household were derived by Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971), and Samuelson (1986). The extension to many households was made by Diamond (1975), and Atkinson and Stiglitz (1976). The equations may be used directly to derive the results of Corlett and Hague (1953-54) and Bradford and Rosen (1976). Closely related to "The Ramsey Equations" of the optimal tax literature are the "The Ramsey Pricing Formulae" of public utilities (e.g., Boiteux (1956) and Baumol and Bradford (1970)). In this context, the planner must choose consumer prices to maximize welfare, subject to the public utility recovering fixed costs. The intuition provided in this paper may be used to explain the optimal mark-up over marginal cost in these regulated industries.

2. THE RAMSEY EQUATIONS

The economy is competitive and consists of H households. A unit of labor of a household of unit productivity is chosen as the numeraire. The productivity of household h is w^h: one unit of its labor produces the same output as w^h units of numeraire labor. The household h therefore receives a wage w^h. There are n commodities. Labor is the only input and the production of each commodity shows constant returns to scale: the production of one unit of good i requires p_i units of numeraire labor. The competitive producer price of good i is therefore p_i, independent of the quantity of production. The government has a tax revenue requirement G; by assumption labor is untaxed, and tax revenue must be collected by the indirect taxation of commodities and a uniform lump-sum tax T. If the tax rate on commodity i is t_i, the consumer price of commodity i is $q_i = p_i(1+t_i)$. It is convenient to use vector notation: producer and consumer price vectors are $\mathbf{p} = (p_1, \ldots, p_n)$ and $\mathbf{q} = (q_1, \ldots, q_n)$. Because $\mathbf{p}$ is fixed by the production technology, $\mathbf{q}$ is a representation of the indirect tax structure.

Each household is price-taking and takes the after-tax price $\mathbf{q}$ and the lump-sum tax T as given[1] when choosing its commodity purchases and labor supply. The non-labor income of household h is denoted M^h and is zero, $M^h=0$. The concern is about how tax rates should be set, and not about expenditure; the public project, on which the tax revenue is spent, is therefore ignored. Household h achieves indirect utility $V^h \equiv V^h(\mathbf{q}, w^h; -T)$, The consumption vector of household h is $\mathbf{x}^h \equiv \mathbf{x}^h(\mathbf{q}, w^h; -T) \equiv (x_1^h(\mathbf{q}, w^h; -T), \ldots, x_n^h(\mathbf{q}, w^h; -T))$, and its labor supply is $L^h(\mathbf{q}, w^h; -T)$. Tax revenue is $R(\mathbf{q}, T) \equiv HT + \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h$.

"The Ramsey Problem" is to choose the tax rates $\mathbf{q}$ and the lump-sum tax T to maximize a Bergson-Samuelson social welfare function, $W(V^1(\mathbf{q}, w^1; -T), \ldots, V^H(\mathbf{q}, w^H; -T))$, subject to the revenue constraint. For ease of notation, I simplify to considering a Benthamite social welfare function,[2] $W \equiv \sum_{h=1}^{H} V^h(\mathbf{q}, w^h; -T)$, for which The Ramsey Problem is

$$\max_{\mathbf{q},T} \sum_{h=1}^{H} V^h(\mathbf{q}, w^h; -T) \quad \text{s.t.} \quad G \le HT + \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h(\mathbf{q}, w^h; -T).$$

The Lagrangian[3] is

$$L = \sum_{h=1}^{H} V^h(\mathbf{q}, w^h; -T) + \Lambda \left[HT + \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p})\cdot\mathbf{x}^h(\mathbf{q}, w^h; -T) - G\right].$$

The typical first-order condition for the tax rate on the kth commodity is

$$\frac{\partial L}{\partial q_k} = \sum_{h=1}^{H} \left[\frac{\partial V^h}{\partial q_k} + \Lambda \left(x_k^h + \sum_{i=1}^{n} (q_i - p_i)\frac{\partial x_i^h}{\partial q_k} \right)\right] = 0, \qquad k=(1, \ldots, n).$$

Using Roy's Identity, denoting the marginal utility of income of household h as $\alpha^h \equiv \partial V^h / \partial M^h$, and rearranging,

$$\frac{\sum_{h=1}^{H} \alpha^h x_k^h}{\sum_{h=1}^{H} [x_k^h + \sum_{i=1}^{n} (q_i - p_i) \frac{\partial x_i^h}{\partial q_k}]} = \Lambda, \qquad k=(1, \ldots, n). (1)$$

The left-hand side of Equation (1) is the marginal welfare loss per unit of additional tax revenue collected by adjustment of instrument q_k, $-(\partial W/\partial q_k)/(\partial R/\partial q_k)$. The right-hand side is the least change in the objective as the constraint is tightened, $\Lambda = -\partial W/\partial G$. Equation (1) is interpreted: at the optimal tax structure, the marginal welfare loss per additional unit of tax revenue collected is the same for each tax instrument, and equals the least welfare loss associated with the collection of an additional unit of tax revenue by the adjustment of *all* tax instruments. Equation (1) is therefore the first-order condition, referred to in the Introduction, which compares the effect of the induced quantity changes on welfare.[4]

Writing $h_i^h(\mathbf{q}; V^h)$ as the compensated demand of household h of the ith good, using the Slutsky Equation and rearranging, Equation (1) becomes

$$-\sum_{h=1}^{H} \sum_{i=1}^{n} (q_i - p_i) \frac{\partial h_i^h}{\partial q_k} = \sum_{h=1}^{H} [1 - \sum_{i=1}^{n} (q_i - p_i) \frac{\partial x_i^h}{\partial M^h} - \frac{\alpha^h}{\Lambda}] \cdot x_k^h, \qquad k=(1, \ldots, n) \text{ (2a)}.$$

The equivalent equation for the instrument T is

$$0 = \sum_{h=1}^{H} [1 - \sum_{i=1}^{n} (q_i - p_i) \frac{\partial x_i^h}{\partial M^h} - \frac{\alpha^h}{\Lambda}]. \qquad (2b)$$

Equations (2a) and (2b) are the "The many-person Ramsey tax rule," and are the equations I interpret.[5]

3. INTERPRETATION

The interpretation of Equation (2a) compares tax instruments by the tax revenue generated, if there is a marginal substitution from the use of instrument q_k to the use of other tax instruments.[6] To elucidate the interpretation, I consider a two stage process. In the first stage, the tax rate q_k is lowered from q_k to q_k+dq_k, ($dq_k < 0$). On its own, this would cause the utility of each household to rise; however, the lump-sum tax on each household h is simultaneously increased dT_k^h so that its utility is unchanged. At the first stage, a distorting tax is replaced by a lump-sum tax, and tax revenue rises. In the second stage, the lump-sum tax on each household h is reduced by the same amount dT_k^h, and *all* tax rates are simultaneously adjusted so as to leave welfare unchanged. At the second stage, a lump-sum tax is replaced by distorting taxes and tax revenue falls. The combined effect of the two stages is to lower tax rate q_k and to raise the other tax rates. Although each household is levied an individualistic lump-sum tax at the first stage, it is removed at the second stage. There is no overall change in the lump-sum tax, and the combined change is feasible for a planner able to use only a uniform lump-sum tax. The combined change is favorable if tax revenue rises as a result of the adjustments - the additional tax revenue may be used to lower the lump-sum levy and to raise welfare.

I consider the first stage. The tax rate changes dq_k ($dq_k < 0$) and the lump-sum tax on each household h is increased dT_k^h , so that its utility is unchanged,

$$V^h(q_1, \ldots, q_k, \ldots, q_n, w^h; -T) = V^h(q_1, \ldots, q_k+dq_k, \ldots, q_n, w^h; -T-dT_k^h).$$

Expanding the right-hand side to first order terms gives

$$0 = \frac{\partial V^h}{\partial q_k} dq_k - \frac{\partial V^h}{\partial M^h} dT_k^h ,$$

or, using Roy's Identity,

$$dT_k^h = \frac{\partial V^h/\partial q_k}{\partial V^h/\partial M^h} dq_k = -x_k^h dq_k .$$

The increase in the lump-sum tax levied on household h exactly equals the fall in the expenditure needed to buy its pre-existing consumption bundle.

The net increase in tax revenue at the first stage is the change in indirect tax revenue plus the increase in the lump-sum tax,

$$dR_1 = d[\sum_{h=1}^{H} (\mathbf{q}-\mathbf{p}) \cdot \mathbf{x}^h)] + \sum_{h=1}^{H} dT_k^h.$$

Quantities change because of the lower tax rate and because of the higher lump-sum tax. However, utility is unchanged. Changes in demand are therefore changes in compensated demand, and hence

$$dR_1 = \sum_{h=1}^{H} [\sum_{i=1}^{n} (q_i - p_i) dh_i^h + h_k^h dq_k + dT_k^h].$$

The first term is the change, due to the adjustment of quantities, in tax revenue collected through the pre-existing tax structure; the second term is the lower tax revenue collected on commodity h_k due to the lower tax rate; the third term is the higher lump-sum tax. But $dT_k^h = -x_k^h dq_k = -h_k^h dq_k$, or the last two terms exactly offset. The net revenue increase per unit fall in tax rate q_k is

$$\left.\frac{dR_1}{-dq_k}\right|_{V^1,\ldots,V^H} = - \frac{\sum_{h=1}^{H}\sum_{i=1}^{n}(q_i-p_i)dh_i^h}{dq_k} .$$

The left-hand side of Equation (2a) is therefore the tax revenue gain if tax rate q_k is lowered one unit, and individualistic lump-sum taxes are imposed so that the utility of each household is unchanged. The tax revenue gain is the change in tax revenue collected through the pre-existing tax structure due to the expansion of the tax base.

The increase in tax revenue arises because of the fiscal externality created by the indirect tax structure (de Bartolome (1992)): a change in household consumption affects not only the household but also the government - because tax revenue is affected. The latter effect is external to the household. The simultaneous reduction in the tax rate q_k and increase in the lump-sum levy dT_k affects the mix of consumption. Household utility is unchanged (by assumption) and the tax revenue gain is the external (fiscal) benefit of the induced quantity changes. This benefit accrues to the government.

At the second stage, the lump-sum tax on each household h is reduced by the amount it was increased in the first stage, dT_k^h, and there is a general adjustment in tax rates to leave welfare unchanged. This stage can be thought of as having two separate parts. In the first part, the lump-sum tax is reduced. This causes tax revenue to fall by the fall in the lump-sum levy, less the associated increase in indirect tax revenue due to the increase in demands,

$$\sum_{h=1}^{H}\left[dT_k^h - \sum_{i=1}^{n}(q_i-p_i)\frac{\partial x_i^h}{\partial M^h}dT_k^h\right].$$

The lump-sum tax reduction in the first part causes the utility of household h to rise $\alpha^h dT_k^h$, and welfare to rise $\Delta W = \sum_{h=1}^{H} \alpha^h dT_k^h$. The second part is the adjustment in tax rates, to lower welfare ΔW so that it returns to its pre-existing level. The Lagrangian multiplier measures the change in the objective as the constraint is tightened, $\partial W/\partial G = -\Lambda$. Therefore the most revenue gain in the second part, due to the adjustment of all tax instruments, is $\Delta G = \Delta W/\Lambda$. The least fall in tax revenue at the second stage is the fall of the first part, less the increase of the second part,

$$dR_2 = \sum_{h=1}^{H} [1 - \sum_{i=1}^{n} (q_i - p_i) \frac{\partial x_i^h}{\partial M^h}] \cdot dT_k^h - \frac{1}{\Lambda} \sum_{h=1}^{H} \alpha^h dT_k^h.$$

But $dT_k^h = -x_k^h dq_k$. The lump-sum tax reduction at the second stage is defined by the change in the tax rate at the first stage. The fall in tax revenue at the second stage, measured per unit fall in q_k at the first stage, is

$$\left. \frac{dR_2}{-dq_k} \right|_W = \sum_{h=1}^{H} [1 - \sum_{i=1}^{n} (q_i - p_i) \frac{\partial x_i^h}{\partial M^h} - \frac{\alpha^h}{\Lambda}] \cdot x_k^h .$$

The right-hand side of Equation (2a) is therefore the least loss in tax revenue if the lump-sum tax is reduced, and all tax rates are adjusted to leave welfare unchanged[7] - measured for lump-sum tax changes associated with a unit fall in tax rate q_k at the first stage. The first term is the lower lump-sum tax, the second term is the increase in tax revenue due to the effect of the increase in disposable income on demand, and the third term is the tax revenue gain due to the general adjustment in tax rates.

Summarizing, the left-hand side of The Ramsey Equations is the tax revenue gained if tax instrument q_k is marginally lowered and a lump-sum tax imposed on each household so that its utility is unchanged. The right-hand side is the least tax revenue lost if the lump-sum tax on each household is lowered by the same amount, and all tax rates are adjusted to leave welfare unchanged. The combined change substitutes the collection of tax revenue from instrument q_k to other instruments; it does not change the lump-sum levy and is feasible. Equation (2a) is interpreted: at the optimum, the revenue gained equals the least revenue foregone. It is impossible to increase tax revenue by substituting any tax instrument for others (to leave welfare unchanged).[8]

Equation (2b) is similarly interpreted, with the relevant tax instrument being T. The left-hand side is the increase in tax revenue if instrument T is reduced by one unit and the lump-sum tax on each household is increased to leave utility unchanged: the lump-sum tax increase is (trivially) a unit, so there is no increase in tax revenue at the first stage. The right-hand side is the least fall in tax revenue if the lump-sum tax is reduced by one unit and all tax rates adjusted to leave welfare unchanged. The first term is the loss in tax revenue due to the reduced lump-sum levy, the second term is the increase in tax revenue due to the induced effect on demand, and the third term is the rise in tax revenue due to the general adjustment in tax rates. At the optimum, the revenue gained equals the revenue foregone, and it is impossible to increase tax revenue by substituting the lump-sum tax for other tax instruments (to leave welfare unchanged).

4. CONCLUSION

In the Introduction, I noted that the presence of compensated demands in the modern formulation of The Ramsey Equations is difficult to reconcile with the intuition that, at the optimal tax structure, the use of each tax instrument to collect marginal tax revenue causes the same welfare loss. The modern form of The Ramsey Equations is associated with the dual problem, of maximizing tax revenue subject to the achievement of a reservation level of welfare. At the optimum tax structure, there is no tax revenue advantage by substituting from the use of one tax instrument to the use of all tax instruments, and therefore from one tax instrument to another. The indirect tax structure causes a fiscal externality to be associated with the consumption choices of households. Compensated demands are important because the substitution of one tax instrument by others affects consumption and, because of the externality, tax revenue.

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FOOTNOTES

[1] With many households, the purchases of an individual household has almost no effect on total tax revenue collected. In the limit of infinitely-many households, there is therefore no feedback from the decisions of an individual household to tax rates and the lump-sum levy.

[2] If the general Bergson-Samuelson social welfare function $W(\cdot)$ is used, α^h is pre-multiplied by $\partial W/\partial V^h$ in Equations (1), (2a) and (2b). The intuitive interpretation is unchanged.

[3] Mirrlees (1986) discusses when the Lagrangian technique is inappropriate.

[4] Many authors compute the marginal welfare loss per unit of tax revenue for different tax instruments, e.g. Decoster and Schokkaert (1990).

[5] In a model of a representative household, the superscipt h may be omitted. Noting that $\partial h_i/\partial q_k \equiv \partial h_k/\partial q_i$, Equation (2a) for a representative household is often written as

$$-\frac{1}{x_k}\sum_{i=1}^{n}(q_i-p_i)\frac{\partial h_k}{\partial q_i} = 1-\sum_{i=1}^{n}(q_i-p_i)\frac{\partial x_i}{\partial M}-\frac{\alpha}{\Lambda}, \qquad k=(1,\ \ldots,\ n). \quad (2')$$

The right-hand side is the same for all k. For small changes, the change in the compensated demand of the kth commodity is $\Delta h_k = \sum_{i=1}^{n}(\partial h_k/\partial q_i)\cdot\Delta q_i$. If the tax rate is small, set $\Delta q_i = q_i - p_i$ and Equation (2′) becomes $\Delta h_k/h_k$ = const.. If an optimal tax structure is imposed, and if compensation is paid, there is an equal percentage change in all taxed goods (Samuelson (1986)). With no income effects, $\partial h_k/\partial q_i \equiv \partial x_k/\partial q_i$, $\Delta h_k = \Delta x_k$, and the optimal tax structure imposes an equi-proportionate fall in the demand of each taxed commodity (Ramsey (1927)).

[6] As noted by Harris and Wildasin (1985), Equations (2a) and (2b) are also the first-order condition of the dual problem, to maximize tax revenue subject

to a suitable reservation welfare, viz.

$$\max_{\mathbf{q},T} \; HT + \sum_{h=1}^{H} (\mathbf{q}-\mathbf{p}) \cdot \mathbf{x}^h(\mathbf{q},\; w^h;\; -T) \quad \text{s.t.} \quad \overline{W} \leq \sum_{h=1}^{H} V^h(\mathbf{q},\; w^h;\; -T).$$

The interpretation follows naturally from this observation.

[7] The interpretation for the representative household is similar and is made by Atkinson and Stern (1974).